note_quicklens

ketchup

May 23, 2019

Contents

1 quicklens

- 'quicklens/examples/$\text{plot}_{\text{lensreconstructionnoiselevels}}$.py'
- $\text{calc}_{\text{nlqq}}$(qest_{TT}, cltt, cltt, cltt, flt, flt)
- $\text{nlqq}_{\text{fullsky}}$
- $\text{clqq}_{\text{fullsky}}$, $\text{resp}_{\text{fullsky}}$
- qest.$\text{fill}_{\text{clqq}}$, qest.$\text{fill}_{\text{resp}}$
- 'quicklens/qest/qest.py'
- $\text{fill}_{\text{clqq}}$
- $\text{fill}_{\text{clqqfullsky}}$
- $\text{qe}_{\text{covfillhelperfullsky}}$
- def $\text{qe}_{\text{covfillhelperfullsky}}$
- glq = math.wignerd.$\text{gauss}_{\text{legendrequadrature}}$, gp1 =glq.$\text{cf}_{\text{fromcl}}$, gp2 = glq.$\text{cf}_{\text{fromcl}}$

- 'quicklens/math/wignerd.py'
- class $\text{gauss}_{\text{legendrequadrature}}$
- $\text{cf}_{\text{fromcl}}$, $\text{cl}_{\text{fromcf}}$
- $\text{cwignerd.wignerd}_{\text{cffromcl}}$
- 'wignerd.pyf' python mode cwignerd #pyf is a fortran

2 quicklens/qest/qest.py

- class qest(object), full-sky, W^{XY}
- def $\text{eval}_{\text{fulls}}$

3 formula

in 'quicklens' flat sky:

$$\begin{aligned} W^{XY} = & \sum_{i=0}^{N} \int d^2 z (e^{+i*2\pi*s^{i,X}+i*(l_X.z)} W^{i,X}(l_X)) \\ & (e^{+i*2\pi*s^{i,Y}+i*(l_Y.z)} W^{i,Y}(l_Y))(e^{-i*2\pi*s^{i,L}+i*(-L.z)} W^{i,L}(L)) \end{aligned} \tag{1}$$

curved sky:

$$W^{XY} = \sum_{i=0}^{N_i} \int d^2 n_s^{i,X} Y_{l_X m_X}(n) W^{i,X}(l_X)_s^{i,Y} Y_{l_Y m_Y}(n) W^{i,Y}(l_Y)_s^{i,L} Y_{LM}(n) W^{i,L}(L) \tag{2}$$

$$q^{XY}(L) = 1/2 \sum_{l_X} \sum_{l_Y} W^{XY}(l_X, l_Y, L) \bar{X}(l_X) \bar{Y}(l_Y) \tag{3}$$

in 'CMB Lensing Reconstruction on the Full SKy' the general weighted sum of multipole pairs as

$$d_L^{\alpha M} = \frac{A_L^\alpha}{\sqrt{L(L+1)}} \sum_{l_1 m_1} \sum_{l_2 m_2} (-1)^M \begin{pmatrix} l_1 & l_2 & L \\ m_1 & m_2 & -M \end{pmatrix} g_{l_1 l_2}^\alpha(L) a_{l_1}^{m_1} b_{l_2}^{m_2} \tag{4}$$

so the corresponding weighted function is

$$W_{l_1 l_2 L}^{\alpha m_1 m_2 M} = \frac{A_L^\alpha}{\sqrt{L(L+1)}} \sum_{m_1} \sum_{m_2} (-1)^M \begin{pmatrix} l_1 & l_2 & L \\ m_1 & m_2 & -M \end{pmatrix} g_{l_1 l_2}^\alpha(L) \tag{5}$$

$$d_L^{\alpha M} = 1/2 \sum_{l_X} \sum_{l_Y} [\frac{A_L^\alpha}{\sqrt{L(L+1)}} \sum_{m_1} \sum_{m_2} (-1)^M \begin{pmatrix} l_1 & l_2 & L \\ m_1 & m_2 & -M \end{pmatrix} g_{l_1 l_2}^\alpha(L)] \bar{X}(l_X) \bar{Y}(l_Y) \tag{6}$$

the normalization is

$$A_L^\alpha = L(L+1)(2L+1) \left\{ \sum_{l_1 l_2} g_{l_1 l_2}^\alpha(L) f_{l_1 L l_2}^\alpha \right\}^{-1} \tag{7}$$

$$g_{l_1 l_2}^\alpha(L) = \frac{C_{l_2}^{aa} C_{l_1}^{bb} f_{l_1 L l_2}^{\alpha *} - (-1)^{L+l_1+l_2} C_{l_1}^{ab} C_{l_2}^{ab} f_{l_2 L l_1}^{\alpha *}}{C_{l_1}^{aa} C_{l_2}^{aa} C_{l_1}^{bb} C_{l_2}^{bb} - \left(C_{l_1}^{ab} C_{l_2}^{ab} \right)^2} \tag{8}$$

forA_L^α, I define the summation in it as

$$B_L^\alpha = \sum_{l_1 l_2} g_{l_1 l_2}^\alpha(L) f_{l_1 L l_2}^\alpha \tag{9}$$

so

$$B_L^\alpha = g_{l_1 l_2}^\alpha(L) = \frac{C_{l_2}^{aa} C_{l_1}^{bb} f_{l_1 L l_2}^{\alpha *} f_{l_1 L l_2}^\alpha - (-1)^{L+l_1+l_2} C_{l_1}^{ab} C_{l_2}^{ab} f_{l_2 L l_1}^{\alpha *} f_{l_1 L l_2}^\alpha}{C_{l_1}^{aa} C_{l_2}^{aa} C_{l_1}^{bb} C_{l_2}^{bb} - \left(C_{l_1}^{ab} C_{l_2}^{ab} \right)^2} \tag{10}$$

$$f_{l_1 L l_2}^\alpha =_{s_a} F_{l_1 L l_2} \left[\epsilon_{l_1 l_2 L} \tilde{C}_{l_2}^{ab} + \beta_{l_1 l_2 L} \tilde{C}_{l_2}^{b\bar{a}} \right] +_{s_b} F_{l_2 L l_1} \left[\epsilon_{l_1 l_2 L} \tilde{C}_{l_1}^{ab} - \beta_{l_1 l_2 L} \tilde{C}_{l_1}^{a\bar{b}} \right] \tag{11}$$

$$\epsilon_{l l' L} = \frac{1 + (-1)^{L+l+l'}}{2} \tag{12}$$

$$\beta_{l l' l L} = \frac{1 - (-1)^{L+l+l'}}{2i} \tag{13}$$

$$\pm s F_{l L l'} = \left[L(L+1) + l' \left(l'+1 \right) - l(l+1) \right] \sqrt{\frac{(2L+1)(2l+1)\left(2l'+1\right)}{16\pi}} \begin{pmatrix} l & L & l' \\ \pm s & 0 & \mp s \end{pmatrix} \tag{14}$$

in the B_L^α, we need to calculate $f_{l_1 L l_2}^{\alpha *} f_{l_1 L l_2}^\alpha$ and $f_{l_2 L l_1}^{\alpha *} f_{l_1 L l_2}^\alpha$. in these two terms above, there are terms of products of two wigner D-matrices. Each this sort of term can be converted into an integral, in which the integrand ois multiplication of three wigner d-functions. In my coming code, the ff terms shown will be given a function $\mathrm{f_f}$(a,b,l1,l2,l3).

$$f^{\alpha *}_{l_1 L l_2} f^{\alpha}_{l_1 L l_2} = \left\{ {}_{s_a}F_{l_1 L l_2} \left[\epsilon_{l_1 l_2 L} \tilde{C}^{ab}_{l_2} + \beta^*_{l_1 l_2 L} \tilde{C}^{b\bar{a}}_{l_2} \right] + s_b F_{l_2 L l_1} \left[\epsilon_{l_1 l_2 L} \tilde{C}^{ab}_{l_1} - \beta^*_{l_1 l_2 L} \tilde{C}^{a\bar{b}}_{l_1} \right] \right\}$$
$$\left\{ {}_{s_a}F_{l_1 L l_2} \left[\epsilon_{l_1 l_2 L} \tilde{C}^{ab}_{l_2} + \beta_{l_1 l_2 L} \tilde{C}^{b\bar{a}}_{l_2} \right] + s_b F_{l_2 L l_1} \left[\epsilon_{l_1 l_2 L} \tilde{C}^{ab}_{l_1} - \beta_{l_1 l_2 L} \tilde{C}^{a\bar{b}}_{l_1} \right] \right\} \tag{15}$$

$$f^{\alpha *}_{l_2 L l_1} f^{\alpha}_{l_1 L l_2} = \left\{ {}_{s_a}F_{l_2 L l_1} \left[\epsilon_{l_2 l_1 L} \tilde{C}^{ab}_{l_1} + \beta^*_{l_2 l_1 L} \tilde{C}^{b\bar{a}}_{l_1} \right] + s_b F_{l_1 L l_2} \left[\epsilon_{l_2 l_1 L} \tilde{C}^{ab}_{l_2} - \beta^*_{l_2 l_1 L} \tilde{C}^{a\bar{b}}_{l_2} \right] \right\}$$
$$\left\{ {}_{s_a}F_{l_1 L l_2} \left[\epsilon_{l_1 l_2 L} \tilde{C}^{ab}_{l_2} + \beta_{l_1 l_2 L} \tilde{C}^{b\bar{a}}_{l_2} \right] + s_b F_{l_2 L l_1} \left[\epsilon_{l_1 l_2 L} \tilde{C}^{ab}_{l_1} - \beta_{l_1 l_2 L} \tilde{C}^{a\bar{b}}_{l_1} \right] \right\} \tag{16}$$

so it is a story of ${}_{s_a}F_{l_1 l_2 l_3} {}_{s_b}F_{l_1 l_2 l_3}$ and ${}_{s_a}F_{l_1 l_2 l_3} {}_{s_b}F_{l_3 l_2 l_1}$

I need a function which can calculate them.

We have:

$$\int_{-1}^{1} d(\cos\theta) d^{\ell_1}_{s_1 s_1'}(\theta) d^{\ell_2}_{s_2 s_2'}(\theta) d^{\ell_3}_{s_3 s_3'}(\theta) = 2 \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ s_1 & s_2 & s_3 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ s_1' & s_2' & s_3' \end{pmatrix} \tag{17}$$

where $d^{\ell_i}_{s_i s_i'}$ are Wigner d-functions such that $s_1 + s_2 + s_3 = s_1' + s_2' + s_3' = 0$.

Define:

$$H(l_1, l_2, l_3) = [-l_1(l_1+1) + l_2(l_2+1) + l_3(l_3+1)] \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{16\pi}} \tag{18}$$

so

$${}_{s_a}F_{l_1 l_2 l_3} {}_{s_b}F_{l_1 l_2 l_3} = H(l_1, l_2, l_3)^2 \begin{pmatrix} l_1 & l_2 & l_3 \\ s_a & 0 & -s_a \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l_3 \\ s_b & 0 & -s_b \end{pmatrix} \tag{19}$$

we also know:

$$\begin{pmatrix} l_3 & l_2 & l_1 \\ s_a & 0 & -s_a \end{pmatrix} = \begin{pmatrix} l_1 & l_2 & l_3 \\ s_a & 0 & -s_a \end{pmatrix} \tag{20}$$

therefore:

$${}_{s_a}F_{l_1 l_2 l_3} {}_{s_b}F_{l_3 l_2 l_1} = H(l_1, l_2, l_3) H(l_3, l_2, l_1) \begin{pmatrix} l_1 & l_2 & l_3 \\ s_a & 0 & -s_a \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l_3 \\ s_b & 0 & -s_b \end{pmatrix} \tag{21}$$

$$\begin{pmatrix} l_1 & l_2 & l_3 \\ s_a & 0 & -s_a \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l_3 \\ s_b & 0 & -s_b \end{pmatrix} = \frac{1}{2} \int_{-1}^{1} d(\cos\theta) d^{\ell_1}_{s_a s_b}(\theta) d^{\ell_2}_{00}(\theta) d^{\ell_3}_{\mathrm{s}_b s_b o}(\theta) \tag{22}$$

the integral above is what we need to calculate using the Gauss-Legendre quadrature.

$$\begin{aligned} a &= b + c - d \\ &\quad + e - f \\ &= g + h \\ &= i \end{aligned} \tag{23}$$