

$$\epsilon_{l'l'} = \frac{1 + (-1)^{l+l'+L'}}{2} \quad (1)$$

$$\beta_{l'l'} = \frac{1 - (-1)^{l+l'+L'}}{2i} \quad (2)$$

This is the original formula for the lensed field in Hu's paper.

$$\delta X_l^m \approx \sum_{L'M'} \sum_{l'm'} \phi_{L'}^{M'} (-1)^{m'} \begin{pmatrix} l & l' & L' \\ m & -m' & -M' \end{pmatrix} F_{l'l'}^{s_x} \left[\epsilon_{l'l'} X_{l'}^{m'} + \beta_{l'l'} \bar{X}_{l'}^{m'} \right] \quad (3)$$

$$\delta X_l^m \approx \sum_{L'M'} \sum_{l'm'} \phi_{L'}^{M'} (-1)^{m'} \begin{pmatrix} l & l' & L' \\ m & -m' & -M' \end{pmatrix} F_{l'l'}^{s_x} \left[\epsilon_{l'l'} X_{l'}^{m'} + \beta_{l'l'} \bar{X}_{l'}^{m'} \right] \quad (4)$$

$$= \sum_{L'M'} \sum_{l'm'} \phi_{L'}^{-M'} (-1)^{M'} (-1)^{m+m'+M'} \begin{pmatrix} l & l' & L' \\ m & m' & M' \end{pmatrix} F_{l'l'}^{s_x} \left[\epsilon_{l'l'} (-1)^{m'} X_{l'}^{-m'} + \beta_{l'l'} (-1)^{m'} \bar{X}_{l'}^{-m'} \right] \quad (5)$$

We know that

$$(-1)^{M'} \phi_{L'}^{-M'} = \phi_{L'}^{M'*} \quad (6)$$

$$(-1)^{m'} X_{l'}^{-m'} = X_{l'}^{m'*} \quad (7)$$

$$(-1)^{m'} \bar{X}_{l'}^{-m'} = \bar{X}_{l'}^{m'*} \quad (8)$$

Then I'll manipulate the formular to give a form easier to use. ...

(9)

$$B_{\ell_1 m_1}^{\text{len}} = \sum_{\ell'_1 m'_1 \ell' m'} f_{\ell_1 \ell'_1 \ell'}^{EB} \begin{pmatrix} \ell_1 & \ell'_1 & \ell' \\ m_1 & m'_1 & m' \end{pmatrix} \left(E_{\ell'_1 m'_1}^* \phi_{\ell' m'}^* + \right) \quad (10)$$

$$B_{\ell_1 m_1}^{\text{len}} = \sum_{\ell'_1 m'_1 \ell' m'} f_{\ell_1 \ell'_1 \ell'}^{EB} \begin{pmatrix} \ell_1 & \ell'_1 & \ell' \\ m_1 & m'_1 & m' \end{pmatrix} \left(E_{\ell'_1 m'_1}^* \phi_{\ell' m'}^* + \right) \quad (11)$$