MTE 204 – Numerical Methods – S16

Project Ib (due Friday, June 10th @ 11:59 pm; 7.5% of total course mark)

Introduction

Project I was designed to help students develop their mathematical modelling and algorithm design skills. In the previous section, students developed finite element models of simple static structures to solve for static equilibrium. The purpose of Project Ib is to build upon the student's current knowledge to be able to solve structural dynamic systems. As part of this project, you will be required to mesh, build, assemble and numerically solve the global stiffness, mass and damping matrices.

Each group will write a code, each of which should be generically capable of solving the types of problems shown below (i.e., no hard-coding). In other words, you should write a code that is capable of solving up to 2-degrees of freedom and apply enough boundary conditions to solve a 1-degree of freedom problem. For this project, you are permitted to use built-in libraries. However, for Question #3, you are not allowed to use matrix solvers to perform inversion. You must implement your own method for this question. All other questions, you may use the matrix solver functions.

Recommended Procedure:

In Project 1a, your groups created a basic structure to solving a static finite element problem. In this project, you can extend your static code to introduce dynamics. The following is strongly recommended:

1) For boundary conditions (either fixed or prescribed forces/displacements), it is recommended that you create a file in the format of a "load curve". A loadcurve specifies the displacements/forces vs time in the form of:

Time	Displacement/Force
0.0000	1.0000
0.0001	1.0000
…	…
1.000	1.000

2) Modify the initialization matrices to include damping and mass matrices
3) Increase the "`process_material_props(PROPS)`" to determine damping and density values.
4) Create functions to build global mass and damping matrices.
5) Create a function to initialize the displacements for all time steps.
6) Modify "`solveKU(KGLOBAL,FGLOBAL,UGLOBAL,FIXED,FREE)`" to be solving dynamics in the form of "`solveMCKU(KGLOBAL, MGLOBAL, CGLOBAL,FGLOBAL,UGLOBAL,FIXED,FREE,OPTION)`" In commercial finite element codes, it is standard to specify an "option" for the type of dynamic solvers to use (i.e.: implicit dynamics vs explicit dynamics, damping, no damping, etc.). You should be able to specify the option that corresponds to the type of dynamic solver to use.

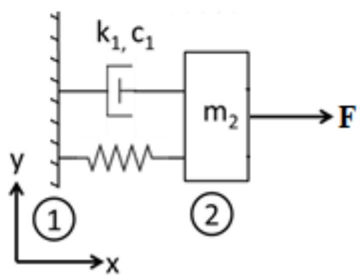
Marking Criteria:

The following marking metrics will be weighted as follows for Project Ib:

- Implementation of Mathematical Model into Code/Software (25%)
- Presentation of Results (25%)
- Discussion of Solution (50%)

Question 1: Modeling a Single Degree of Freedom Mass-Spring-Damper System

The following problem is a single mass-spring-damper in one-dimension. A force, F, is applied to the structure at node 2. Between the mass, is a single spring, k_1, and a damper c_1. Assume that gravity does not act on the structure. The location of Node 1 and Node 2 are (0,0) and (1,0) respectively in meters.



From ordinary differential equations, the closed form analytical solution for the motion of node 2 is:

$$u_2(t) = \frac{F}{k} - \frac{F}{k\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\cos(\omega_d t - \Theta) + e^{-\xi\omega_n t}\left[\frac{\dot{u}_2(0) + \xi\omega_n u_2(0)}{\omega_d}\sin\omega_d t + u_2(0)\cos\omega_d t\right]$$

$$\Theta = \tan^{-1}\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\xi = \frac{c}{2m\omega_n}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

Let $E = 10Pa, A = 1m^2$, $\upsilon = 0, k_1 = \frac{AE}{L} = \frac{10N}{m}, c_1 = 1\frac{N}{s}, m_2 = \rho AL = 10kg, F = 10N$

a) [6 marks] Using a time step of $\Delta t = \{0.01, 0.1, 1.0\}\ seconds$, determine the displacement response of node 2 using the explicit dynamics (central differences theorem). Present a plot that overlays all three results. Solve for 5 seconds assuming zero initial conditions.

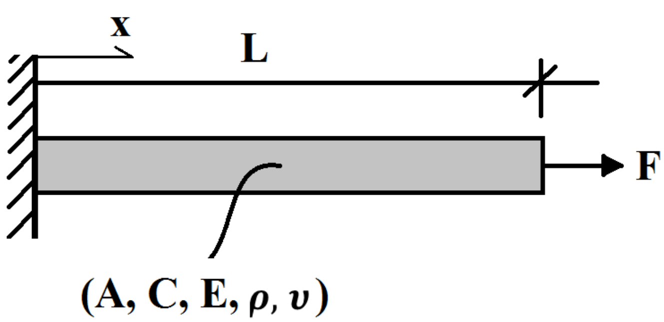
b) [6marks] Using a time step of $\Delta t = \{0.01, 0.1, 1.0\}\ seconds$, determine the displacement response of node 2 using the implicit dynamics (Newmark method, $\gamma = \frac{3}{2}, \beta = \frac{8}{5}$) formulation. Present a plot that overlays all three results. Solve for 5 seconds assuming zero initial conditions.

c) [6marks] Compare the results with the analytical predictions. Comment on the results. If you were to select a time step, which one would you chose and why? Justify your answer.

d) [5 marks] Let us say that we want to load the structure from rest with a 10N load. This load will remain constant and never change or be removed. If the yield stress of the spring is $\sigma_y = 12.5Pa$, is it safe to use this spring for this eventual static load?

Question 2: Modeling the Dynamic Loading of an 1D Elastic Bar as a Multiple Degree of Freedom Mass-Spring-Damper System

Below is a schematic of a steel bar that is rigidly clamped and pulled in tension. The mechanical properties of aluminum are $E = 70GPa$, $\nu = 0.30$, and $\rho = 2700kg/m^3$. The rod also has some internal damping, $C = 0.001N/s$. The rod end is fixed on the left end. A load that is a function of time, $F(t) = tN$, is applied on the right hand in the positive x-direction for 5 seconds. The rod has an initial length, L, and cross sectional area equal to $1m$ and $1\times10^{-6}m^2$ respectively.



We wish to perform a finite element analysis to determine the dynamic response of the structure using 10 equal bar (truss) elements for these 5 seconds, assuming zero initial conditions.

a) [5 marks] Write out the equations of motion for this model in the form:

$$[M][\ddot{U}] + [C][\dot{U}] + [K][U] = [F]$$

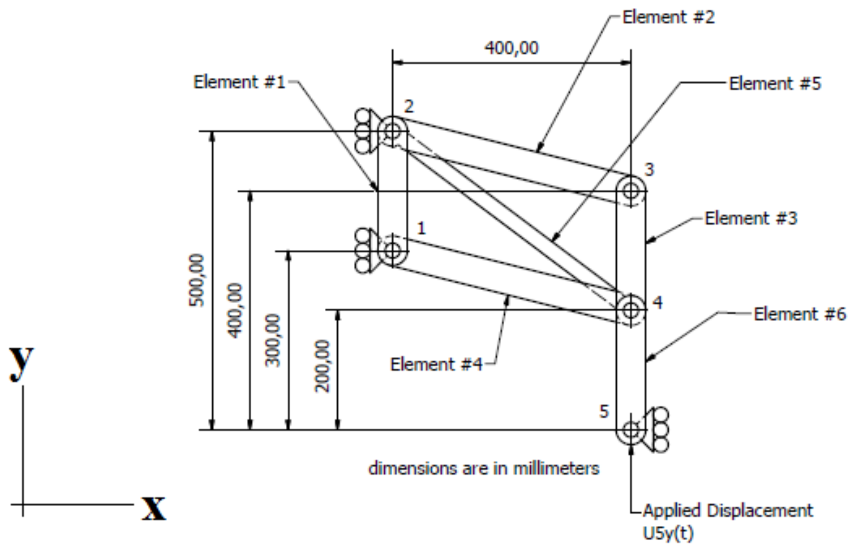
b) [6 marks] Using the explicit dynamics (central differences theorem) formulation, plot the nodal displacement response of the end node where the load is applied. Use the following time steps: $\Delta t = \{1\times10^{-1}s,\ 1\times10^{-3}s, 1\times10^{-5}s\}$. Present each plot individually. Determine how much computational time is required to generate the result once your code begins performing the explicit dynamics solving.

c) [6 marks] Using the implicit dynamics (Newmark's Method) formulation, plot the nodal displacement response of the end node where the load is applied. Use Galerkin's parameters for the time integration of $\gamma = \frac{3}{2}, \beta = \frac{8}{5}$. Use the following time steps: $\Delta t = \{1\times10^{-1}s,$ $1\times10^{-3}s, 1\times10^{-5}s\}$. Present each plot individually. Determine how much computational time is required to generate the result once your code begins performing the implicit dynamics solving.

d) [6 marks] Comment on your results above. What do you notice about the results? For this type of problem, which numerical time integration would you recommend? Justify your answer based on your results.

Question 3: Modeling the Dynamic Loading of a 2D Structure

Below is a schematic of a mechanical structure.



In this structure, Nodes 1, 2, and 5 are fixed in the x-direction.

The elements have the following properties:

Element ID	Elastic Modulus, E, [MPa]	Area, A, $[mm^2]$	Density $[g/mm^3]$	Damping [N/msec] (10^{-3})
1	100	25	0.100	1.00
2	100	25	0.001	1.00
3	100	25	0.001	1.00
4	100	25	0.001	1.00
5	1	25	0.001	100
6	100	25	0.001	1.00

To maintain a consistent set of units, our unit of measurements are [MPa],[mm],[ms] and [N].

We wish to analyze the shock loading of the structure. The shock is applied to structure through a prescribed displacement of Node 5 in the y-direction as a function of time, such that;

$$V_5 = 50mm\,(\sin \omega t)$$

where ω is an input frequency in rad/ms and t is time in milliseconds. We are interested in exploring the motion of Node 2 in the y-direction for various frequencies in increments of 0.1:

$$\omega = \{0.1: 0.1: 10\}\,rad/msec$$

Three finite element mesh files are provided for this structure:

1) Nodes.txt : Contains the x,y coordinates of each node in the specified order (ie: 1st row is Node#1)
2) Sctr.txt : Contains the first and second global node number in the specified order (ie: 1st row is Element #1)
3) Props.txt : Contains the elemental information in the specified order. The format of the text file is [ELASTIC MODULUS], [AREA],[DENSITY],[DAMPING]

To help visualize the structure movement, a post-processing script is also provided:

```
postprocesser(x,y,sctr,d)
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The function takes the initial nodal locations in x and y, the element connectivity matrix, and the global displacement vector and will display the deformed structure.

Assume a small displacements and small rotation formulation. Analyze the structure and answer the following questions:

a) [4 marks] Which time integration method do you think is most appropriate for this application? Explain.

b) [6 marks] Which method for performing matrix inversion (or matrix solving) is most appropriate for this problem: Gauss-Elimination, LU Decomposition, Gauss-Jacobi, or Gauss-Seidel? Justify and explain your answer.

c) [10 marks] Using your answers from above, write a code to solve for the structural response using your recommended time integration scheme. Select appropriate time step and integration parameters. Simulate the deformation of the structure from rest for 500 milliseconds. Plot the nodal displacement, velocity and acceleration of Node 2 for the input frequencies of $\omega = \{0.1, 1.0, 10\}\ rad/msec$. Determine the peak acceleration for of Node 2 and plot the results on a semilogx (x-axis is input frequency of Node 5, y-axis peak acceleration). Comment on this result. Do these results make physical sense or is a result of numerical error?

d) [9 marks] Is our assumption of small displacements and small rotations valid? If it is not completely valid, what regime of input frequencies is it valid for? Explain. Use the results from your post processing to justify your answer. (**Hint: It is much easier to show through the use of the post processor script provided in the package.)**

Submission:

Prepare a document that presents your results for each question. State the question you are being asked and present your answer for each part in an organized manner. Answer each discussion question using complete sentences. Use your results to justify your answer. Do not use vague wording to justify your answer. Your report should be approximately 10 pages. You will be required to submit a hardcopy of the solutions for each problem as determined by your code. In addition to your solutions, you will be required to submit your code on LEARN. Please reference the course outline for submission details.