

Modal Formalization of Brand Value

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1 Main Definitions

Definition 1 ($\text{Brand}(x)$). *Brand(x) is interpreted as: "x is an entity identified as a brand, with a distinguishable name or identity in the market that carries values, positioning, and image for a target audience."*

Definition 2 ($\text{Consumer}(y)$). *Consumer(y) is interpreted as: "y is an agent (person or group) who acquires or influences the acquisition of goods or services."*

Definition 3 ($\text{Product}(x)$). *Product(x) means: "x is a good or service offered in the market, tangible or intangible, that satisfies needs or desires."*

Definition 4 ($\text{Company}(x)$). *Company(x) is understood as: "x is a business entity or organization dedicated to producing, distributing, or marketing products and/or services."*

Definition 5 ($\text{StrongEmotionalConnection}(x)$). *StrongEmotionalConnection(x) indicates: "the brand x awakens in its audience an intense emotional bond that goes beyond functional utility."*

Definition 6 ($\text{AuraOfBrand}(x)$). *AuraOfBrand(x) refers to: "the immaterial and symbolic quality that surrounds brand x, perceived as carrying an exceptional value that transcends the merely functional. This 'halo' manifests in the admiration, prestige, or fascination that the public projects onto the brand."*

Definition 7 ($\text{CommodityFetish}(x)$). *CommodityFetish(x) indicates: "the brand x acquires an AuraOfBrand(x), that is, a social or symbolic value that transcends its utilitarian function (in line with the 'commodity fetishism' described by Marx)."*

Definition 8 ($\text{ValueBeyondUse}(x)$). $\text{ValueBeyondUse}(x)$ means: "x offers a symbolic or emotional value beyond the practical utility of its product or service."

Definition 9 ($\text{MarketDifferentiation}(x)$). $\text{MarketDifferentiation}(x)$ indicates: "the brand x possesses unique attributes or perceptions that distinguish it from its direct competitors."

Definition 10 ($\text{WillingToPay}(y, x, \text{Higher})$). $\text{WillingToPay}(y, x, \text{Higher})$ means: "the consumer y is willing to pay a higher price for brand x compared to equivalent alternatives, due to the perceived value of the brand."

Definition 11 ($\text{StrongBrand}(x)$). $\text{StrongBrand}(x)$ means: "the brand x exhibits great strength, evidenced by high metrics of recognition, positive associations, loyalty, and 'brand equity'."

Definition 12 ($\text{HasNarrativeEcosystem}(x)$). $\text{HasNarrativeEcosystem}(x)$ indicates: "the brand x develops a cohesive system of stories and symbols (a 'narrative universe') that unfolds across multiple channels."

Definition 13 ($\text{ExpandedOfferings}(x)$). $\text{ExpandedOfferings}(x)$ denotes: "the brand x has extended or plans to extend its portfolio of products and/or services beyond its initial offering."

Definition 14 ($\text{Trusts}(y, x)$). $\text{Trusts}(y, x)$ is interpreted as: "the consumer y trusts brand x and perceives it as credible, ethical, or consistent."

Definition 15 ($\text{LoyalTo}(y, x)$). $\text{LoyalTo}(y, x)$ indicates: "the consumer y maintains sustained loyalty to brand x, resisting offers from competitors."

Definition 16 ($\text{HighEngagement}(y, x)$). $\text{HighEngagement}(y, x)$ is read as: "y shows a high and active commitment to brand x, with frequent and enthusiastic interactions."

Definition 17 ($\neg \text{HasBrandStrategy}(x)$). $\neg \text{HasBrandStrategy}(x)$ describes: "the product x lacks a formal brand strategy, without managed positioning, identity, or differentiation."

Definition 18 ($\text{CompetitiveMarket}(x)$). $\text{CompetitiveMarket}(x)$ indicates: "x is located in an environment with high rivalry and multiple competitors or equivalent alternatives."

Definition 19 ($\text{Obsolete}(x)$). *Obsolete(x) means: "x has become outdated or irrelevant in the market, unable to compete or satisfy current needs."*

Definition 20 ($\text{WantsToTranscendSingleProduct}(x)$). *WantsToTranscendSingleProduct(x) indicates: "the company x intends to stop depending on a single product and diversify."*

Definition 21 ($\text{InvestInBrand}(x)$). *InvestInBrand(x) means: "the company x allocates concrete resources to the development and positioning of its brand."*

Definition 22 ($\text{SuccessfulExpansion}(x)$). *SuccessfulExpansion(x) is interpreted as: "the company x achieves positive results and grows by diversifying its offering, exceeding its market objectives."*

Definition 23 ($\text{BrandResilience}(x)$). *BrandResilience(x) means: "the brand x maintains its relevance and adaptability over time, avoiding obsolescence."*

Definition 24 ($\text{Success}(x)$). *Success(x) refers to: "x has achieved or exceeded the proposed goals and objectives (economic, market, image, etc.), showing a high degree of accomplishment or achievement."*

2 Modal Propositions

Proposition 1 (Emotional connection and commodity fetishism).

$$\forall x \left(\text{Brand}(x) \wedge \text{StrongEmotionalConnection}(x) \rightarrow \Diamond \text{CommodityFetish}(x) \right).$$

Interpretation: *If a brand generates an intense emotional bond, it is possible that it embodies the phenomenon of "commodity fetishism" and acquires a symbolic social aura.*

Proposition 2 (Fetishism and value beyond use).

$$\forall x \left(\text{Brand}(x) \wedge \text{CommodityFetish}(x) \rightarrow \Diamond \text{ValueBeyondUse}(x) \right).$$

Interpretation: *The brand that manifests this "fetishistic" quality can translate it into a symbolic value that exceeds the merely utilitarian.*

Proposition 3 (Aura and willingness to pay more).

$$\forall x \forall y \left(Brand(x) \wedge StrongEmotionalConnection(x) \wedge Consumer(y) \rightarrow \Diamond WillingToPay(y, x, Higher) \right)$$

Interpretation: *When there is a strong emotional bond, it is possible that the consumer is willing to pay a higher price for the brand.*

Proposition 4 (Symbolic value and differentiation).

$$\forall x \left(Brand(x) \wedge ValueBeyondUse(x) \rightarrow \Diamond MarketDifferentiation(x) \right).$$

Interpretation: *Symbolic value beyond function can drive brand differentiation in the market.*

Proposition 5 (Differentiation and brand strength).

$$\forall x \left(Brand(x) \wedge MarketDifferentiation(x) \rightarrow \Diamond StrongBrand(x) \right).$$

Interpretation: *A brand that is clearly distinguished can become a strong brand.*

Proposition 6 (Strong brand and future expansion).

$$\forall x \left(Brand(x) \wedge StrongBrand(x) \rightarrow \Diamond F(ExpandedOfferings(x)) \right).$$

Interpretation: *If the brand is strong, there is a possibility that, in the future, it will expand its portfolio.*

Proposition 7 (Narrative and future expansion).

$$\forall x \left(Brand(x) \wedge HasNarrativeEcosystem(x) \rightarrow \Diamond F(ExpandedOfferings(x)) \right).$$

Interpretation: *A narrative ecosystem can enable expansion to new products or services in the future.*

Proposition 8 (Narrative and high engagement).

$$\forall x \forall y \left(Brand(x) \wedge TargetConsumer(y, x) \wedge HasNarrativeEcosystem(x) \rightarrow \Diamond HighEngagement(y, x) \right)$$

Interpretation: *For the target consumer, a brand with a narrative universe can generate a high level of engagement.*

Proposition 9 (Engagement and trust).

$$\forall x \forall y \left(\text{HighEngagement}(y, x) \rightarrow \Diamond \text{Trusts}(y, x) \right).$$

Interpretation: *If the consumer is highly involved with the brand, they could come to trust it.*

Proposition 10 (Trust and future loyalty).

$$\forall x \forall y \left(\text{Trusts}(y, x) \rightarrow \Diamond G(\text{LoyalTo}(y, x)) \right).$$

Interpretation: *A brand that is trusted can, over time, retain consumer loyalty.*

Proposition 11 (Investment in brand and resilience).

$$\forall x \left(\text{Brand}(x) \wedge \text{InvestInBrand}(x) \rightarrow \Diamond \text{BrandResilience}(x) \right).$$

Interpretation: *By allocating resources to branding, it is possible that the brand becomes resilient and survives changes in the environment.*

Proposition 12 (Absence of brand strategy and obsolescence).

$$\forall x \left(\text{Product}(x) \wedge \neg \text{HasBrandStrategy}(x) \wedge \text{CompetitiveMarket}(x) \rightarrow \Diamond F(\text{Obsolete}(x)) \right).$$

Interpretation: *Without a brand strategy in a competitive environment, there is a possibility that the product will become obsolete.*

Proposition 13 (Transcending a single product through the brand).

$$\forall x \left(\text{Company}(x) \wedge \text{WantsToTranscendSingleProduct}(x) \rightarrow \Diamond (\text{InvestInBrand}(x) \rightarrow \text{SuccessfulExp}) \right).$$

Interpretation: *The company that wants to move from a single product to a portfolio can, if it invests in its brand, achieve successful expansion.*

Proposition 14 (Relationship between resilience and success).

$$\forall x \left(\text{Brand}(x) \wedge \text{BrandResilience}(x) \rightarrow \Diamond \text{Success}(x) \right).$$

Interpretation: *A brand that becomes resilient can achieve or exceed the goals set, that is, achieve success (whether in terms of market share, profitability, image, etc.).*

3 Summary

1. **(Prop. 1) Emotional connection \rightarrow Fetishism:** if the brand produces an intense emotional bond, it can acquire a symbolic value (CommodityFetish).
2. **(Prop. 2) Fetishism \rightarrow Value beyond use:** the symbolic aura translates into ValueBeyondUse.
3. **(Prop. 3) Emotional bond \rightarrow Higher payment:** the consumer might pay more.
4. **(Prop. 4) Symbolic value \rightarrow Differentiation:** the brand distinguishes itself.
5. **(Prop. 5) Differentiation \rightarrow Strong brand:** over time, the brand strengthens.
6. **(Prop. 6) Strong brand \rightarrow Expansion:** the possibility of Expanded-Offerings emerges.
7. **(Prop. 7) Narrative \rightarrow Expansion:** a narrative universe also drives diversification.
8. **(Prop. 8) Narrative \rightarrow High engagement:** target consumers might become more involved.
9. **(Prop. 9) Engagement \rightarrow Trust:** engagement can lead to trust.
10. **(Prop. 10) Trust \rightarrow Loyalty:** over time, loyalty is consolidated.
11. **(Prop. 11) Investment in brand \rightarrow Resilience:** with continuous branding, relevance is maintained.
12. **(Prop. 12) Without strategy \rightarrow Obsolescence:** in competitive markets, the lack of a brand strategy facilitates decline.
13. **(Prop. 13) Transcending a product \rightarrow Successful expansion:** the company that invests in its brand can successfully diversify.
14. **(Prop. 14) Resilience \rightarrow Success:** a resilient brand *can* achieve the proposed goals (Success).