Institución Universitaria Antonio José Camacho

SISTEMAS DINÁMICOS

TALLER 4 – VARIABESL DE ESTADO.

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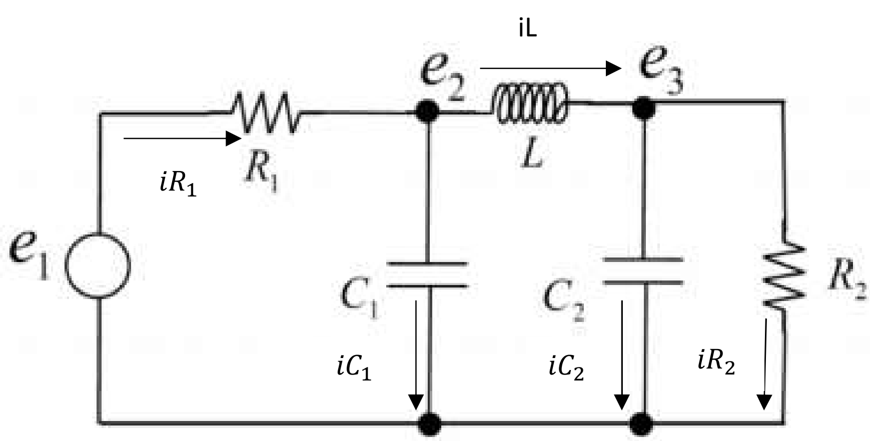
Realice el modelamiento matemático de los sistemas que se presentan a continuación utilizando la técnica de variables de estado.

1. **Variables de interés (IR1, IR2 y VL)**



$\boldsymbol{Variables\ de\ estado}\ \rightarrow f\{VC_1, VC_2, iL\}$

$\boldsymbol{Sea\ e_1 = Vin},\qquad \boldsymbol{e_2 = Vc_1},\qquad \boldsymbol{e_3 = Vc_2}$

$LCK(Vc_1)$	$LCK(Vc_2)$
$iR_1 = iC_1 + iL$ $iC_1 = iR_1 - iL$ $C_1\frac{dVC_1}{dt} = \frac{Vin - VC_1}{R_1} - iL$ $\frac{dVC_1}{dt} = \frac{Vin - VC_1}{C_1R_1} - \frac{1}{C_1}iL$	$iL = iC_2 + iR_2$ $iC_2 = iL - iR_2$ $C_2\frac{dVC_2}{dt} = iL - \frac{VC_2}{R_2}$ $\frac{dVC_2}{dt} = \frac{1}{C_2}iL - \frac{VC_2}{C_2R_2}$
$\boldsymbol{LVK}\ para\ obtner\ la\ función\ de \Rightarrow \boldsymbol{L\frac{di_L}{dt}}$	
$-Vin + VR_1 + VL + VR_2 = 0$	

$$-Vin + i_{R1}R_1 + L\frac{di_L}{dt} + i_{R2}R_2 = 0$$
$$L\frac{di_L}{dt} = Vin - i_{R1}R_1 - i_{R2}R_2$$
$$L\frac{di_L}{dt} = Vin - \left(\frac{Vin - VC_1}{R_1}\right) * R_1 - \left(\frac{VC_2}{R_2}\right) * R_2$$
$$L\frac{di_L}{dt} = Vin - Vin + VC_1 - VC_2$$
$$L\frac{di_L}{dt} = VC_1 - VC_2$$
$$\frac{di_L}{dt} = \frac{1}{L} * VC_1 - \frac{1}{L} * VC_2$$

$\boldsymbol{Ecuaci}$ó$\boldsymbol{n\ de\ entrada}$:

$\dot{X} = Ax + Bu$

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dVC_1}{dt} \\ \frac{dVC_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} & -\frac{1}{L} \\ -\frac{1}{C_1} & -\frac{1}{C_1R_1} & 0 \\ \frac{1}{C_2} & 0 & \frac{1}{C_2R_2} \end{bmatrix} \begin{bmatrix} iL \\ VC_1 \\ VC_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C_1R_1} \\ 0 \end{bmatrix} Vin$$

$$\boldsymbol{Varibles\ de\ interes}\ \{iR_1, iR_2, VL\} \rightarrow f\{VC_1, VC_2, iL\}$$

$$\boldsymbol{Construimos\ la\ salida}\ con\ y = Cx + Du$$

1) LVK $$-Vin + VR_1 + VL + VR_2 = 0$$ $$VL = Vin - \left(\frac{Vin - VC_1}{R_1}\right) * R_1 - \left(\frac{VC_2}{R_2}\right) * R_2$$ $$VL = VC_1 - VC_2$$
2) $$iR_1 = \frac{Vin - VC_1}{R_1}$$
3) $$iR_2 = \frac{VC_2}{R_2}$$

$\boldsymbol{Ecuaci}$ó$\boldsymbol{n\ de\ salida}$:

$y = Cx + Du$

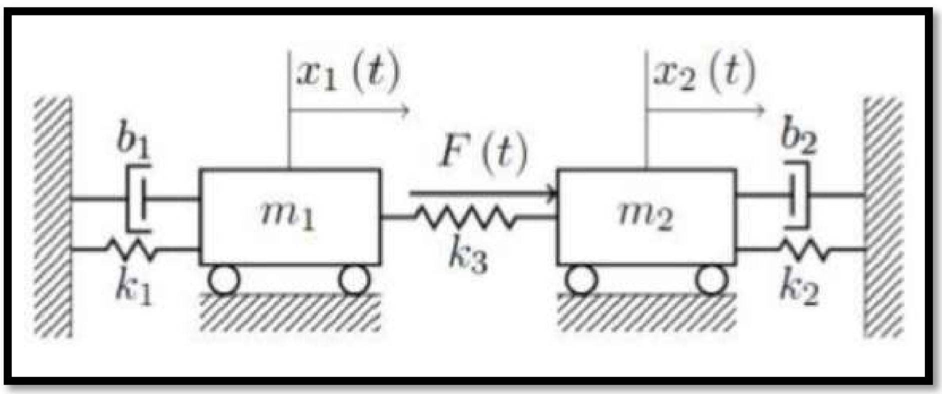
$$\begin{bmatrix} VL \\ iR_1 \\ iR_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 0 & -\frac{1}{R_1} & 0 \\ 0 & 0 & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} iL \\ VC_1 \\ VC_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{R_1} \\ 0 \end{bmatrix} Vin$$

Solución

$$\dot{X} = Ax + Bu \rightarrow \left\{ \begin{bmatrix} \dfrac{di_L}{dt} \\ \dfrac{dVC_1}{dt} \\ \dfrac{dVC_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{1}{L} & -\dfrac{1}{L} \\ -\dfrac{1}{C_1} & -\dfrac{1}{C_1R_1} & 0 \\ \dfrac{1}{C_2} & 0 & \dfrac{1}{C_2R_2} \end{bmatrix} \begin{bmatrix} iL \\ VC_1 \\ VC_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{C_1R_1} \\ 0 \end{bmatrix} Vin \right\}$$

$$y = Cx + Du \rightarrow \left\{ \begin{bmatrix} VL \\ iR_1 \\ iR_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 0 & -\dfrac{1}{R_1} & 0 \\ 0 & 0 & \dfrac{1}{R_2} \end{bmatrix} \begin{bmatrix} iL \\ VC_1 \\ VC_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{R_1} \\ 0 \end{bmatrix} Vin \right\}$$

6. Se presenta un sistema mecánico.



1. $m_1\ddot{x}_1(t) + b_1\dot{x}_1(t) + k_1x_1(t) - k_3[x_1(t) - x_2(t)] = 0$
2. $m_2\ddot{x}_2(t) + b_2\dot{x}_2(t) + k_2x_2(t) + k_3[x_2(t) - x_1(t)] = f(t)$

$$x_1 \begin{cases} z_1 = x_1 \\ z_2 = \dot{z}_1 = \dot{x}_1 \\ \dot{z}_2 = \ddot{x}_1 \end{cases}$$

$$x_2 \begin{cases} z_3 = x_2 \\ z_4 = \dot{z}_3 = \dot{x}_2 \\ \dot{z}_4 = \ddot{x}_2 \end{cases}$$

Ecuación de masa 1.	Ecuación de masa 2.
$m_1\dot{z}_2 + b_1z_2 + k_1z_1 - k_3[z_1 - z_3] = 0$	$m_2\dot{z}_4 + b_2z_4 + k_2z_3 + k_3[z_3 - z_1] = f(t)$
$\dot{z}_2 = -\frac{b_1}{m_1}z_2 - \frac{k_1}{m_1}z_1 + \frac{k_3}{m_1}[z_1 - z_3]$	$\dot{z}_4 = \frac{1}{m_2}f(t) - \frac{b_2}{m_2}z_4 - \frac{k_2}{m_2}z_3 - \frac{k_3}{m_2}[z_3 - z_1]$
$\dot{z}_2 = -\frac{b_1}{m_1}z_2 - \frac{k_1}{m_1}z_1 + \frac{k_3}{m_1}z_1 - \frac{k_3}{m_1}z_3$	$\dot{z}_4 = \frac{1}{m_2}f(t) - \frac{b_2}{m_2}z_4 - \frac{k_2}{m_2}z_3 - \frac{k_3}{m_2}z_3 + \frac{k_3}{m_2}z_1$
Ordenar:	Ordenar:
$\dot{z}_2 = -\frac{b_1}{m_1}z_2 + \frac{(k_3 - k_1)}{m_1}z_1 - \frac{k_3}{m_1}z_3$	$\dot{z}_4 = \frac{1}{m_2}f(t) - \frac{b_2}{m_2}z_4 - \frac{(k_2 + k_3)}{m_2}z_3 + \frac{k_3}{m_2}z_1$
$\dot{z}_1 = z_2$	$\dot{z}_3 = z_4$

Solución:

- $\dot{x} = Ax + Bu$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(k_3 - k_1)}{m_1} & -\frac{b_1}{m_1} & -\frac{k_3}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_3}{m_2} & 0 & -\frac{(k_2 + k_3)}{m_2} & -\frac{b_2}{m_2} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_2} \end{bmatrix} f(t)$$

- $y = Cx + Du$

$$x_1 = z_1$$
$$x_2 = z_3$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} f(t)$$