"Lagrange"

(*) Mecánica:

Energias:

- Cinetica (m, J) $\rightarrow$ $\underset{\text{Cine.}}{T} = \frac{1}{2} m V^2 = \frac{1}{2} J \omega^2$
- Potencial $\rightarrow$ mgh, $\underset{K}{\text{resorte}}$ $\rightarrow$ $\underset{\text{Pot.}}{U}$ $\rightarrow$ $U_g = mgh$, $U_e = \frac{1}{2} K X^2 = \frac{1}{2} K \theta^2$

$\dot{x} = \frac{dx}{dt}$

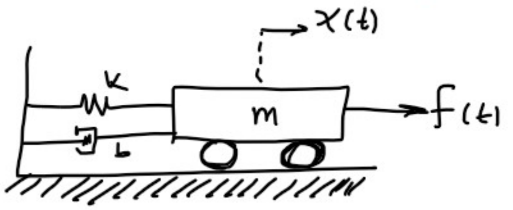
$b \rightarrow \underset{\text{Perdidas}}{R} = \frac{1}{2} b V^2 = \frac{1}{2} b \omega^2$

$\rightarrow$ Grado de libertad (q_i) $\quad i = 1, 2, 3, \ldots \infty$

$$\boxed{\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} + \frac{\partial U}{\partial q_i} = Q_i}$$

$Q_i \equiv$ Señales de entrada al sistema

Ej:



$\longleftrightarrow \underline{\underline{x}}$ $\quad\rightarrow$ Hacer por el método de Lagrange

$T = \frac{1}{2} m V^2 = \frac{1}{2} m \dot{x}^2$

$U = \frac{1}{2} K x^2$

$R = \frac{1}{2} b V^2 = \frac{1}{2} b \dot{x}^2$

$\boxed{q_1 = x}$

EC. Lagrange $\longrightarrow$ $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} + \frac{\partial U}{\partial q_i} = Q_i$

$\underline{q_1 = x}$:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x} + \frac{\partial R}{\partial \dot{x}} + \frac{\partial U}{\partial x} = Q_i$$

$\frac{\partial T}{\partial \dot{x}} = \frac{\cancel{2}}{\cancel{2}} m \dot{x}^1 \cdot (1) = m\dot{x} \longrightarrow \frac{d}{dt}(m\dot{x}) = m \frac{d\dot{x}}{dt} = \boxed{m\ddot{x}}$ ✓ $\quad \frac{d^2x}{dt^2} = \ddot{x}$

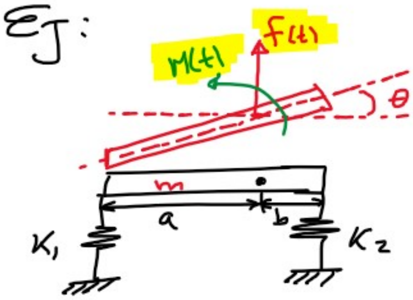
$\frac{\partial T}{\partial x} = \boxed{0}$ $\quad \frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left[\frac{1}{2} m \dot{a}^2\right] = \frac{\partial}{\partial x}\left(\cancel{\frac{1}{2} m a^2}\right) = 0$ KTE

$\frac{\partial R}{\partial \dot{x}} = \boxed{b\dot{x}}$

$$m\ddot{x} - 0 + b\dot{x} + Kx = f(t)$$

$\frac{\partial U}{\partial x} = \boxed{Kx}$ $\boxed{m\ddot{x} + b\dot{x} + kx = f(t)}$

Ej:



→ Hacer lo por Lagrange

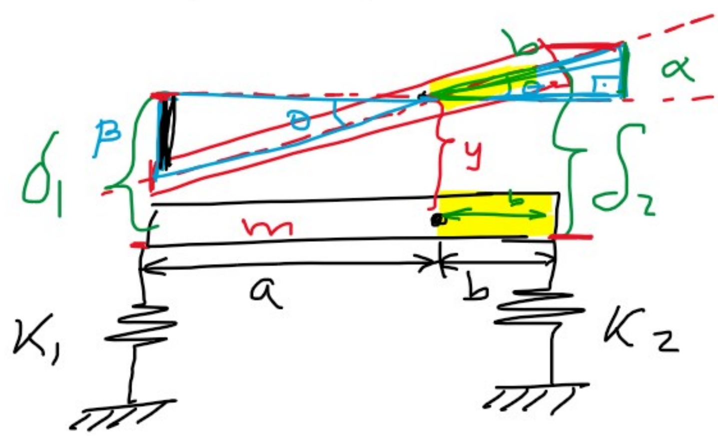
$$T = \frac{1}{2} m \dot{y}^2 + \frac{1}{2} J \dot{\theta}^2$$

$$U = \frac{1}{2} K_1 \delta_1^2 + \frac{1}{2} K_2 \delta_2^2$$

$f_K = kx$

$$R = 0$$

$$q_i = \{ y, \theta \}$$



$\operatorname{Sen}\theta = \frac{\alpha}{b}$ $\boxed{\alpha = b \operatorname{sen}\theta}$

$\delta_2 = y + \alpha = \boxed{y + b \operatorname{Sen}\theta}$

$d_2 = \boxed{y + b\theta}$

$\boxed{\beta = a \operatorname{Sen}\theta}$

$\delta_1 = y - \beta = \boxed{y - a \operatorname{sen}\theta}$

$\boxed{d_1 = y - a\theta}$

$\theta <<<$ $\operatorname{Sen}\theta \simeq \theta$

$$q_i = \{ y, \theta \}$$

a) y: $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right) - \frac{\partial T}{\partial y} + \frac{\partial R}{\partial \dot{y}} + \frac{\partial U}{\partial y} = Q_i$

$$\begin{cases} T = \frac{1}{2} m \dot{y}^2 + \frac{1}{2} J \dot{\theta}^2 \\ U = \frac{1}{2} K_1 (y - a\theta)^2 + \frac{1}{2} K_2 (y + b\theta)^2 \\ R = 0 \end{cases}$$

$\frac{\partial T}{\partial \dot{y}} = m\dot{y} \longrightarrow \frac{d}{dt}(m\dot{y}) = \boxed{m\ddot{y}}$

$(a \cdot b)' = a'b + ab'$

$\frac{d}{dt}(m\dot{y}) = \cancel{\frac{dm}{dt}}\dot{y} + m\frac{d\dot{y}}{dt} = m\frac{d\dot{y}}{dt} = m\frac{d}{dt}\left(\frac{dy}{dt}\right)$

$\frac{d^2y}{dt^2} = \ddot{y}$

$\frac{\partial T}{\partial y} = \boxed{0}$

$\rightarrow \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} + \frac{\partial U}{\partial q_i} = Q_i$

$\frac{\partial R}{\partial \dot{y}} = \boxed{0}$

$\frac{\partial U}{\partial y} = K_1(y - a\theta)(1) + K_2(y + b\theta)(1) = \boxed{K_1(y - a\theta) + K_2(y + b\theta)}$

(1) $m\ddot{y} + K_1(y - a\theta) + K_2(y + b\theta) = f(t)$

b) $\underline{\theta}$: $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta}\right) - \frac{\partial T}{\partial \theta} + \frac{\partial R}{\partial \dot\theta} + \frac{\partial U}{\partial \theta} = Q_i$

$$\begin{cases} T = \frac{1}{2} m \dot{y}^2 + \frac{1}{2} J \dot\theta^2 \\ U = \frac{1}{2} K_1 (y-a\theta)^2 + \frac{1}{2} K_2 (y+b\theta)^2 \\ R = 0 \end{cases}$$

$\frac{\partial T}{\partial \dot\theta} = J\dot\theta \longrightarrow \frac{d}{dt}(J\dot\theta) = \boxed{J\ddot\theta}$

$\frac{\partial T}{\partial \theta} = \boxed{0}$

$\frac{\partial R}{\partial \dot\theta} = \boxed{0}$

$\frac{\partial U}{\partial \theta} = K_1(y-a\theta)(-a) + K_2(y+b\theta)(b)$

$= \boxed{-aK_1(y-a\theta) + bK_2(y+b\theta)}$

$$(2) \quad \boxed{J\ddot\theta - aK_1(y-a\theta) + bK_2(y+b\theta) = M(t)}$$

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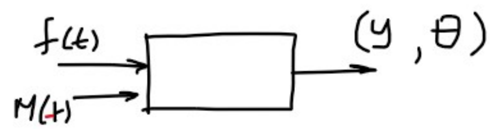
$$\begin{cases} m\ddot{y} + K_1(y-a\theta) + K_2(y+b\theta) = f(t) \\ J\ddot\theta - aK_1(y-a\theta) + bK_2(y+b\theta) = M(t) \end{cases}$$



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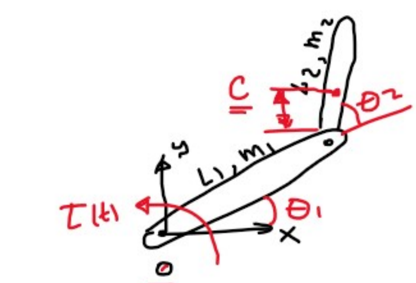


→ Ec. dinámica del sistema en el punto C

Viendo el movimiento en el punto O

(Lagrange)

→ (Variables generalizadas)

Lagrange → "Eléctrico"

L: $T = \frac{1}{2} L \dot{q}^2$

C: $U = \frac{1}{2}\frac{1}{C} q^2$

R: $R = R\dot{q}\,\delta\dot{q}$

$\frac{dq}{dt} = \dot{q}_{(t)} = i_{(t)}$

$V = Ri$

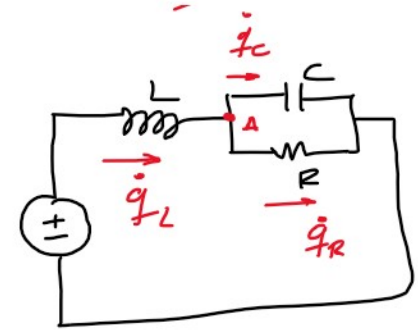
$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} + \frac{\partial U}{\partial q_i} = Q_i \rightarrow 0$

$\dot{q}_c$

Ej: L C

→ Resolver por Lagrange

Eg:



$\longrightarrow$ Resolver por Lagrange

$T = \frac{1}{2} L \dot{q}_L^2$ ✓

$U = \frac{1}{2}\frac{1}{C} q_C^2 = \frac{1}{2}\frac{1}{C}(q_L - q_R)^2$ ✓

$R = \underline{R \dot{q}_R}\, \delta \dot{q}_R - \underline{V_1}\, \delta \dot{q}_L$ ✓

LCK(A)

$\dot{q}_L = \dot{q}_C + \dot{q}_R$

$\int$ $\boxed{\dot{q}_C = \dot{q}_L - \dot{q}_R}$ ✓

$\boxed{q_C = \underline{q_L} - \underline{q_R}}$ ✓

$q_i = \{q_L, q_R\}$

* q_L:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} + \frac{\partial U}{\partial q_i} = \cancelto{0}{Q_i}$$

$$\frac{\partial T}{\partial \dot{q}_L} = L\dot{q}_L \rightarrow \frac{d}{dt}(L\dot{q}_L) = \boxed{L\ddot{q}_L}$$

$T = \frac{1}{2} L \dot{q}_L^2$

$$\frac{\partial T}{\partial q_L} = \boxed{0}$$

$U = \frac{1}{2}\frac{1}{C}(q_L - q_R)^2$

$R = \underline{R \dot{q}_R}\, \delta \dot{q}_R - \underline{V_1}\, \delta \dot{q}_L$

$$\frac{\partial R}{\partial \dot{q}_L} = \frac{\partial}{\partial \dot{q}_L}\left(R\dot{q}_R\, \delta \dot{q}_R - V_1 \delta \dot{q}_L\right)$$

$$= \cancelto{0}{\frac{\partial}{\partial \dot{q}_L}\left(R\dot{q}_R\, \delta\dot{q}_R\right)} - \frac{\partial}{\partial \dot{q}_L}\left(V_1 \delta \dot{q}_L\right)$$

$$= -V_1 \cancelto{1}{\frac{\partial(\dot{q}_L)}{\partial \dot{q}_L}} = \boxed{-V_1}$$ ✓

$L\ddot{q} - V_1 + \frac{1}{C}(q_L - q_R) = 0$

$$\boxed{L\ddot{q} + \frac{1}{C}(q_L - q_R) = V_1}$$

$$\frac{\partial U}{\partial q_L} = \frac{1}{C}(q_L - q_R)(1) = \boxed{\frac{1}{C}(q_L - q_R)}$$

* q_R:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} + \frac{\partial U}{\partial q_i} = \cancelto{0}{Q_i}$$

$T = \frac{1}{2} L \dot{q}_L^2$

$$\frac{\partial T}{\partial \dot{q}_R} = \boxed{0} \longrightarrow \frac{d}{dt}(0) = \boxed{0}$$ ✓

$$T = \frac{1}{2} L \dot{q}_L^2$$

$$U = \frac{1}{2}\frac{1}{C}(q_L - q_R)^2$$

$$\mathcal{R} = \underline{R\,\dot{q}_R}\,\delta\dot{q}_R - \underline{V_1}\,\delta\dot{q}_L$$

$$\frac{\partial T}{\partial \dot{q}_R} = \boxed{0} \longrightarrow \frac{d}{dt}(0) = \boxed{0} \checkmark$$

$$\frac{\partial T}{\partial q_R} = \boxed{0} \checkmark$$

$$\frac{\partial \mathcal{R}}{\partial \dot{q}_R} = \boxed{R\,\dot{q}_R}$$

$$\frac{\partial U}{\partial q_R} = \frac{1}{C}(q_L - q_R)(-1) = \boxed{-\frac{1}{C}(q_L - q_R)}$$

$$\boxed{R\,\dot{q}_R - \frac{1}{C}(q_L - q_R) = 0}$$

S/.

$$\begin{cases} L\ddot{q} + \frac{1}{C}(q_L - q_R) = V_1 \\ R\,\dot{q}_R - \frac{1}{C}(q_L - q_R) = 0 \end{cases}$$

$$\xrightarrow{V_1} \boxed{\quad\quad} \longrightarrow (q_L, q_R)$$