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Taller 1: Análisis en el dominio del tiempo.

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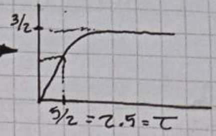
Sistemas de control 1.

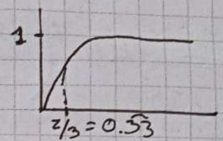
Taller 1 ⇒ Análisis en el dominio del tiempo.

1)

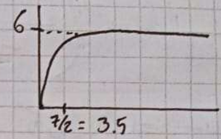
a) $\frac{C_s}{R_s} = \frac{3}{5s+2} \rightarrow \frac{3/2}{5/2 s+1} \rightarrow$



b) $\frac{C_s}{R_s} = \frac{2/3}{s+2/3} \Rightarrow \frac{1}{2/3 s+1} \rightarrow$



c) $\frac{C_s}{R_s} = \frac{6}{7/2 s+1} \longrightarrow$



El sistema "b" es el que se estabiliza más rápido. Según la ecuación, $\tau = 2/3$; esto indica que el sistema llegaría al 63% de la respuesta en 0.33s. Si a los sistemas se le aplican un escalón unitario, la relación ganancia y Tao aplicaron una respuesta en el sistema "b" más rápida.

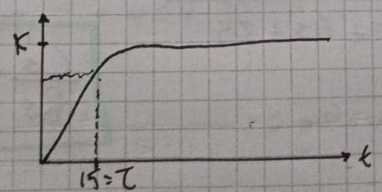
2)
- Termómetro = 4τ = 98% = 60s
- sistema de primer orden.

$$G(s) = \frac{K}{\tau s+1}$$

$$4\tau = 60s$$

$$\tau = \frac{60s}{4} = 15s$$

$$G_s = \frac{K}{15s+1} =$$



3) a) $\frac{C_s}{R_s} = \frac{720}{10s^2 + 504s + 360} = \frac{72}{s^2 + 50.4s + 36}$ $\frac{Y_s}{R_s} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$\omega_n^2 = 36 \rightarrow \omega_n = 6 \;||\; \zeta = \frac{50.4s}{2*6} = 4.2 \;||\; K = \frac{72}{36} = 2$ / sobreamort.

b) $\frac{C_s}{R_s} = \frac{40}{5s^2 + 160s + 80} = \frac{8}{s^2 + 32s + 16}$

$\omega_n^2 = 16 \rightarrow \omega_n = 4 \;||\; \zeta = \frac{32}{2*4} = 4 \;||\; K = \frac{8}{36} = 0.22$ / Sobreamorti.

c) $\frac{C_s}{R_s} = \frac{18}{2s^2 + 72s + 18} = \frac{9}{s^2 + 36s + 9} \rightarrow \omega_n^2 = 9 \rightarrow \omega_n = 3$

$\zeta = \frac{36}{2*3} = 6 \;||\; K = \frac{9}{9} = 1$ / Sobreamortiguado.

d) $\frac{C_s}{R_s} = \frac{441}{3s^2 + 29.4s + 147} = \frac{147}{s^2 + 9.8s + 49} \rightarrow \omega_n^2 = 49 \rightarrow \omega_n = 7$

$\zeta = \frac{9.8}{2.7} = 0.7 \;||\; K = \frac{147}{49} = 3$ / Subamortiguado.

e) $\frac{C_s}{R_s} = \frac{252}{7s^2 + 42s + 63} = \frac{36}{s^2 + 6s + 9} \rightarrow \omega_n^2 = 9 \rightarrow \omega_n = 3.$

$\zeta = \frac{6}{2*3} = 1 \;||\; K = \frac{36}{9} = 4$ / Criticamente amortiguado.

f) $\frac{C_s}{R_s} = \frac{48}{4s^2 + 6.4s + 16} = \frac{12}{s^2 + 1,6s + 4} \rightarrow \omega_n^2 = 4 \rightarrow \omega_n = 2.$

$\zeta = \frac{1.6}{2*2} = 0.4 \;||\; K = \frac{12}{4} = 3$ / Sub amortiguado.

g) $\frac{C_s}{R_s} = \frac{150}{3s^2 + 75} = \frac{50}{s^2 + 25.} \rightarrow s^2 + 5^2 = (s+5)^2 = (s+5)(s+5)$

$s^2 + 2 \times 5 \times s + 25$

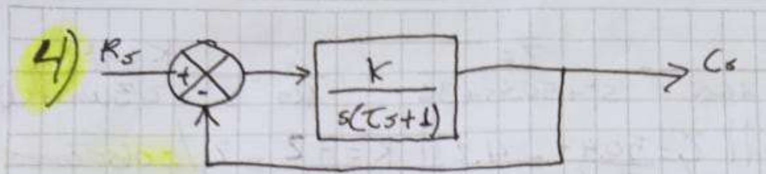
$s^2 + 10s + 25$

- $\frac{50}{s^2 + 10s + 25} \rightarrow \omega_n^2 = 25 \rightarrow \omega_n = 5 \;||\; \zeta = \frac{10}{2*5} = 1$

$K = \frac{50}{25} = 2.$ / Criticamente amortiguado.

Design

4)



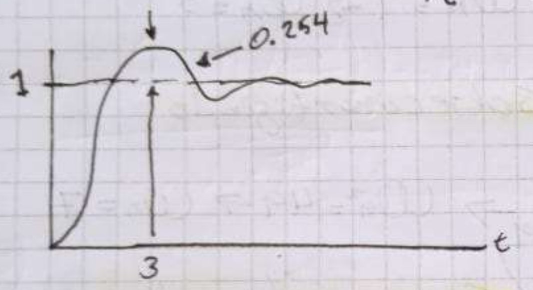
$$\frac{Y_s}{R_s} = \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\frac{\frac{K}{s(\tau s+1)}}{1+\frac{K}{s(\tau s+1)}} = \frac{K/\cancel{s(\tau s+1)}}{\frac{s(\tau s+1)+K}{\cancel{s(\tau s+1)}}} = \frac{K}{s(\tau s+1)+K} = \frac{K}{\tau s^2+s+K}$$

Forma Canonica $= \frac{K/\tau}{s^2+\frac{1}{\tau}s+K/\tau}$

$\frac{1}{\tau}s = 0.9s \rightarrow \tau = \frac{1}{0.9}$

$\tau = 1.11$



$$M_p = \frac{C(\infty)-C(t_p)}{C(\infty)} = \frac{1.254-1}{1} = 25.4\%$$

$$\zeta = \frac{-\ln(M_p)}{\sqrt{\pi^2+\ln^2(M_p)}}$$

$$\zeta = \frac{\ln(0.254)}{\sqrt{\pi^2+\ln^2(0.254)}} = 0.399.$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \omega_n = \frac{\pi}{t_p\sqrt{1-\zeta^2}}$$

$\omega_n = 1.142$.

$\omega_n^2 = 1.30.$

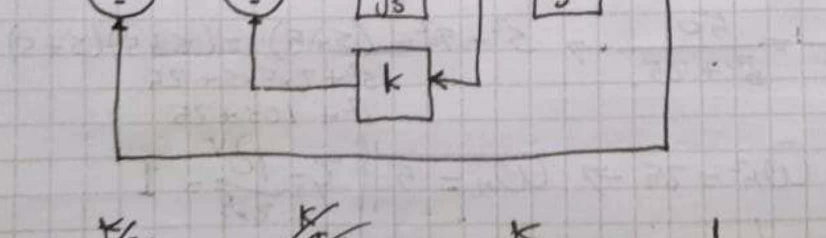
$\cancel{\tau = \frac{1}{0.9}}$

$K = AK \rightarrow \frac{1}{1} = K \Rightarrow 1$

$$\frac{1 \cdot 1.30}{s^2+2\cdot 0.399\cdot 1.142\cdot s+1.30} = \frac{1.30}{s^2+0.9s+1.30}$$

//   //

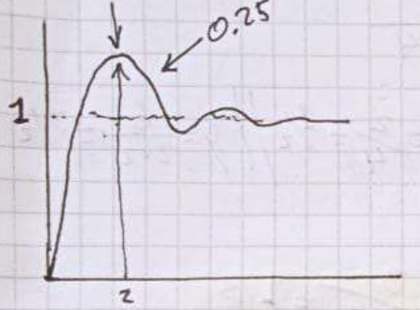
5)



$$\frac{K/Js}{1+\frac{Kk}{Js}} = \frac{\cancel{K/Js}}{\frac{Js+Kk}{\cancel{Js}}} = \frac{K}{Js+Kk} * \frac{1}{s} = \frac{K}{Js^2+Kks}$$

DD MM AA

$$\frac{\frac{K}{Js^2+K_k s}}{1+\frac{K}{Js^2+K_k s}} = \frac{K}{Js^2+K_k s+K} \Rightarrow \frac{K/J}{s^2+\frac{K_k}{J}s+\frac{K}{J}}$$



$t_p = 2$ $\quad \omega_n = \frac{\pi}{2*\sqrt{1-0.4^2}} = 1.71$

$M_p = 0.25$ $\quad \omega_n^2 = 2.93$

$\zeta = 0.40$ $\quad KA = 1 \Rightarrow K = 1$

$$\frac{C_s}{R_s} = \frac{1 * 2.93}{s^2 + 2*0.40*1.71 + 2.93}$$

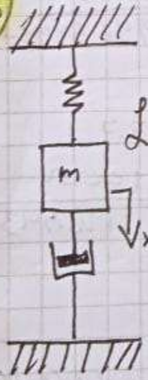
$$\frac{K/J}{s^2+\frac{K_k}{J}s+K/J} = \frac{2.93}{s^2+1.368s+2.93}$$

$\Rightarrow K/J = 2.93 \Rightarrow K = \frac{2.93}{1\ kg/m^2} = 2.93$

$\frac{K_k}{J}\cancel{s} = 1.368\cancel{s} \Rightarrow \frac{1.368 * J}{K} = \frac{1.368\ \cancel{kg/m^2}}{2.93\ \cancel{kg/m^2}} = 0.466$

$k = 0.466$

6)



$f_{(t)} - f_b - f_K = m a_x$

$\mathcal{L}\{\ f(t) = m\ddot{x} + b\dot{x} + Kx$

$f(s) = ms^2X(s) + bsX(s) + KX(s)$

$f(s) = X(s)\left[s^2m + bs + k\right]$

$$\frac{X(s)}{f(s)} = \frac{1}{ms^2+bs+K}$$

$\sum f_x = m a_x$ $\quad \frac{X(s)}{f(s)} = \frac{1/m}{s^2+\frac{b}{m}s+\frac{K}{m}}$

$M_p = \frac{0.1095-0.1}{0.1} = 9.5\%$ $\quad 0.095$

$\zeta = \cancel{1.15}\ 0.59 \quad || \quad \omega_n = 1.94$

$KA = 0.1 \to K = \frac{0.1}{2} \cancel{0.1}$ $\quad 0.05$

$\omega_n^2 = 3.76$

$$\frac{X(s)}{f(s)} = \frac{0.05*3.76}{s^2+2*0.59*1.94+3.76}$$

$$\frac{X(s)}{f(s)} = \frac{0.188}{s^2+2.28s+3.76}$$

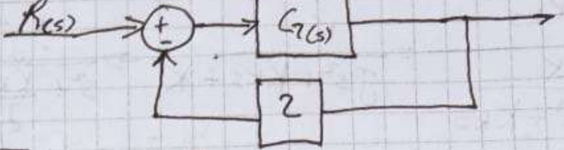
$1/m = 0.188 \Rightarrow m = 1/0.188 = 5.319.16$

$K/m = 3.76 \Rightarrow K = 3.76 * 5.319 = 19.99$

$b/m = 2.28 = b = 2.28 * 5.319 = 12.12$

7)

$G_{(s)} = \frac{2}{s(s+6)} =$



$$\frac{\frac{2}{s(s+6)}}{1+\frac{2\times 2}{s(s+6)}} = \frac{\frac{2}{\cancel{s(s+6)}}}{\frac{s(s+6)+4}{\cancel{s(s+6)}}} = \frac{2}{s^2+6s+4}$$

$\omega_n^2 = 4 \Rightarrow \omega_n = 2 \;//\; K\omega_n^2 = 2 \rightarrow K = \frac{2}{4} = 1/2 \;//\; \zeta = \frac{6}{2\times 2} = 3/2$

$t_d = \frac{1+0.7\zeta}{\omega_n} = \frac{1+(0.7\times 1,5)}{2} = 1,025 s$

$t_r = \frac{\pi - \left(\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)}{\omega_n\sqrt{1-\zeta^2}} = -24.66 \;//\; t_s = \frac{3}{\zeta\omega_n} = 1\, s$

$t_s = \frac{4}{\zeta\omega_n} = 1,14 s$

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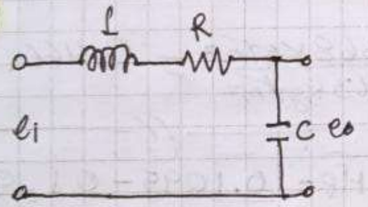
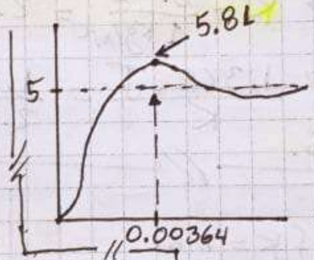
8)





$M_p = \frac{5.81-5}{5} = 16.2\%$

$\zeta = 0.5 \;//\; \omega_n = \frac{\pi}{0.00364\times\sqrt{1-0.5^2}}$

996.593

$KA = 5 \rightarrow K = 1$

$$\frac{1\times 996.593}{s^2 + 2\times 0.5\times 996.593s + 993197.7}$$

$$\frac{993197.7}{s^2 + 996.593s + 993197.7}$$

$R/L = 996.593 \rightarrow L = \frac{100\Omega}{996.593} =$

$L = 0.100 H$

$1/LC = 993197.7$

$C = \frac{1}{0.100\times 993197.7} = 0.0000100 f$

$R = 100\Omega \quad L = 100mH \quad C = 10\mu f$

//

$V_{ei} = V_L + V_R + V_C$

$V_{ei} = L\frac{di}{dt} + R\times i + V_{eo}$

$V_{ei} = L\frac{d}{dt}\left[C\frac{dV_{eo}}{dt}\right] + R\times\left[C\frac{dV_{eo}}{dt}\right] + V_{eo}$

$\mathcal{L}\; V_{ei} = LC\frac{d^2V_{eo}}{dt^2} + RC\frac{dV_{eo}}{dt} + V_{eo}$

$V_{ei(s)} = LCs^2 V_{eo(s)} + RCsV_{eo(s)} + V_{eo(s)}$

$V_{ei(s)} = V_{eo(s)}\left[LCs^2 + RCs + 1\right]$

$$\frac{V_{eo(s)}}{V_{ei(s)}} = \frac{1}{LCs^2 + RCs + 1} \rightarrow \frac{1/LC}{s^2 + \frac{R\cancel{C}}{L\cancel{C}}s + 1/LC} \rightarrow \frac{1/LC}{s^2 + \frac{R}{L}s + 1/LC}$$

Design