(4)

Given error function: $\frac{1}{2}\sum_i^N \|y-\hat{y}\|^2$
We have our $g(x) = \frac{1}{1+e^{-x}}$, and $g'(x) = g(x)(1-g(x))$
Let $z_k = b_k + \sum_h g(z_h)W_{hk}$,
where $z_h = b_h + \sum_i x_i W_{dh}$
Find $\frac{\partial E}{\partial W_{hk}}$:

$$\begin{aligned}
\frac{\partial E}{\partial W_{hk}} &= \frac{1}{2}\sum_k \|y-\hat{y}\|^2 \\
&= \sum_k -(y-g(z_k))\frac{\partial g(z_k)}{\partial z_k}\frac{\partial z_k}{\partial w_{hk}} \\
&= \sum_k (g(z_k)-y)g'(z_k)\frac{\partial z_k}{\partial w_{hk}}
\end{aligned}$$

We have $z_k = b_k + \sum_h g(z_h)W_{hk}$, so that $\frac{\partial z_k}{\partial w_{hk}} = \sum_h g(z_h)$:

$$\begin{aligned}
\frac{\partial E}{\partial W_{hk}} = \sum_k (\frac{1}{1+e^{z_k}} - y)\frac{e^{z_k}}{1+e} \cdot \sum_h \frac{1}{1+e^{z_h}} \\
\text{where, } z_h = b_h + \sum_i x_i W_{dh} \\
z_k = b_k + \sum_h g(z_h)W_{hk}
\end{aligned}$$

Find $\frac{\partial E}{\partial b_k}$:

$$\begin{aligned}
\frac{\partial E}{\partial b_k} &= \sum_k -(y-g(z_k))\frac{\partial g(z_k)}{\partial z_k}\frac{\partial z_k}{\partial b_k} \\
&= \sum_k (g(z_k)-y)g'(z_k)\frac{\partial z_k}{\partial b_k}
\end{aligned}$$

We have $z_k = b_k + \sum_h g(z_h)W_{hk}$, so that $\frac{\partial z_k}{\partial b_k} = 1$:

$$\begin{aligned}
\frac{\partial E}{\partial b_k} &= \sum_k (g(z_k)-y)g'(z_k) \\
&= \sum_k (\frac{1}{1+e^{z_k}} - y)\frac{e^{z_k}}{1+e^{z_k}}
\end{aligned}$$

(5)

As same as part 4, we have
Given error function: $\frac{1}{2}\sum_i^N \|y-\hat{y}\|^2$
We have our $g(x) = \frac{1}{1+e^{-x}}$, and $g'(x) = g(x)(1-g(x))$
Let $z_k = b_k + \sum_h g(z_h)W_{hk}$,
where $z_h = b_h + \sum_i x_i W_{dh}$
Find $\frac{\partial E}{\partial W_{dh}}$:

$$\begin{aligned}\frac{\partial E}{\partial W_{dh}} &= \frac{1}{2}\sum_k \|y-\hat{y}\|^2 \\ &= \sum_k -(y-g(z_k))\frac{\partial g(z_k)}{\partial z_k}\frac{\partial z_k}{\partial w_{dh}} \\ &= \sum_k (g(z_k)-y)g'(z_k)\frac{\partial z_k}{\partial w_{dh}}\end{aligned}$$

We have $z_k = b_k + \sum_h g(z_h)W_{hk}$, and we have that $z_h = b_h + \sum_i x_i W_{dh}$,
So we have $\frac{\partial z_h}{\partial w_{dh}} = \sum_i x_i$
We also have $\frac{\partial z_k}{\partial w_{dh}} = \frac{\partial z_k}{\partial a_h}\frac{\partial a_h}{\partial w_{dh}}$, where $a_h = \sum_h g(b_h + \sum_i \vec{x_i} W_{dh})$
Now we have $\frac{\partial z_k}{\partial w_{dh}} = w_{dh}\frac{\partial a_h}{\partial w_{dh}} = w_{hk} g'(z_h) \cdot \sum_i x_i$
Then we have:

$$\begin{aligned}\frac{\partial E}{\partial W_{dh}} &= g'(z_h)x_i \sum_k (g(z_k)-y)g'(z_k)w_{hk} \\ &= \frac{e^{z_h}}{1+e^{z_h}} x_i \sum_k (\frac{1}{1+e^{-z_k}} - y)\frac{e^{z_k}}{1+e^{z_k}} w_{hk}\end{aligned}$$

Find $\frac{\partial E}{\partial b_h}$:

$$\begin{aligned}\frac{\partial E}{\partial b_h} &= \frac{1}{2}\sum_k \|y-\hat{y}\|^2 \\ &= \sum_k -(y-g(z_k))\frac{\partial g(z_k)}{\partial z_k}\frac{\partial z_k}{\partial b_h} \\ &= \sum_k (g(z_k)-y)g'(z_k)\frac{\partial z_k}{\partial b_h}\end{aligned}$$

Since we have: $\frac{\partial g(z_k)}{\partial z_k}\frac{\partial z_k}{\partial b_h} = w_{hk} g'(z_h)$:

$$\begin{aligned}\frac{\partial E}{\partial b_h} &= \sum_k (g(z_k)-y)g'(z_k)w_{hk}g'(z_h) \\ &= \frac{e^{z_h}}{1+e^{z_h}} \sum_k (\frac{1}{1+e^{-z_k}} - y)\frac{e^{z_k}}{1+e^{z_k}} w_{hk}\end{aligned}$$