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An Introduction to Formal Logic

Contents

Part I

Arguments

Chapter 1

Arguments

1.1 Introduction

Much of philosophical practice is about argument and analysis. Arguing in support of or against some position, or understanding someone else's argument. Logic is the study of the practice of argument and analysis, abstracted from the specific details of a particular case.

In everyday language, we sometimes use the word 'argument' to talk about belligerent shouting matches. If you and a friend have an argument in this sense, things are not going well between the two of you. Logic is not concerned with such teeth-gnashing and hair-pulling. They are not arguments, in our sense; they are disagreements.

An argument, as we will understand it, is something more like this:

- If I acted of my own free will, then I could have acted otherwise.
- I could not have acted otherwise.
- Therefore: I did not act of my own free will.

add a reference to SEP compatibilism?

We here have a series of sentences which may either be true or false. The final sentence, "I did not act of my own free will." expresses the **conclusion** of the argument. The two sentences before that are the **premises** of the argument. In a good argument, the conclusion follows from the premises. If you believe the premises then the argument should lead you to believing the conclusion.

Logic provides the ideal model of good argument: rational argument without rhetoric. The logical study of an argument can show whether it supports its conclusion or is flawed. Logic focuses only on the statements presented and the relationships between them. Extraneous factors are set aside: unspoken assumptions, additional connotations of words, appeal to emotions.

Logical thinking can help us to work out the intended interpretation of a text, and to find alternative unintended interpretations. This can be helpful when reading someone else's writing, and essential when we are trying to write unambiguously. Logical analysis can help us to find ambiguity and alternative interpretations, and to write in a precise and unambiguous way that can only be interpreted as we intend. These are vital skills used in all philosophy as well as in life more generally.

This Part discusses arguments in natural languages like English. Throughout this textbook we will also consider arguments in formal languages and say what it is for those to be valid or invalid. We want formal validity, as defined in the formal language, to have at least some of the important features of natural-language validity.

1.2 Finding the components of an argument

Arguments consist of a list of **premises** along with a **conclusion**. In a good argument, the conclusion will follow from the premises.

Often arguments are presented simply in a paragraph of text, or in a speech or article, and we first have to work out what the premises and conclusions are. Sometimes it's easy, for example:

> If I acted of my own free will, then I could have acted otherwise. And I could not have acted otherwise. So, I did not act of my own free will.

But often it is a significant piece of work to work out the premises and conclusion of an argument.

Many arguments start with premises, and end with a conclusion, but not all of them. It might start with the conclusion:

> <u>We should not have a second Brexit referendum</u>.
> A second Brexit referendum would erode the very basis of democracy by suggesting that rule by the majority is an insufficient condition for democratic legitimacy.

Or it might have been presented with the conclusion in the middle:

> Since the first Brexit referendum was made under false pretences,
> <u>the voters deserve a further say on any final deal ag</u>
> After all, decisions as big as this need to have the public support, which has to come from a referendum.

Sometimes premises or the conclusion may be clauses in a sentence. A complete argument may even be contained in a single sentence:

> The butler has an alibi; so they cannot have done it.

This argument has one premise, followed immediately by its conclusion.

One particular kind of sentence can be confusing. Consider:

- If the murder weapon was a gun, then Prof. Plum did it.

These conditional, or “if–then”, statements might look like it expresses the argument, but in itself it does not. It’s just stating a fact, albeit a conditional fact. It might also be used in an argument, even as the conclusion of the argument:

- If I have free will, then there is some event that I could have caused to go differently.

- If determinism is true, then there is no event that I could have caused to go differently.
- Therefore: If determinism is true, I do not have free will.

As a guideline, there are some words you can look for which are often used to indicate whether something is a premise or conclusion:

Words often used to indicate an argument's conclusion:

so, therefore, hence, thus, accordingly, consequently

Words often used to indicate a premise:

since, because, given that

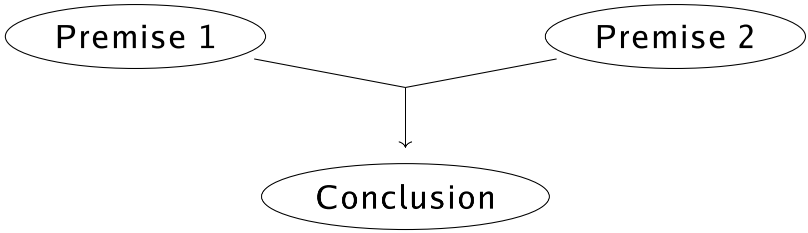
In analysing an argument, there is no substitute for a good nose. Whenever you come across an argument in a piece of philosophy you read, be it lecture notes, primary text, or secondary text, or in a newspaper article or on the internet, practice identifying the premises and conclusion.

Sometimes, though, people aren't giving arguments but are simply presenting facts or stating their opinion. For example, the following do not contain arguments, they're not trying to convince us of anything.

- I don't like cats. I think they're evil.
- Hundreds of vulnerable children as young as 10, who have spent most of their lives in the UK, are having their applications for British citizenship denied for failing to pass the government's controversial 'good character' test.

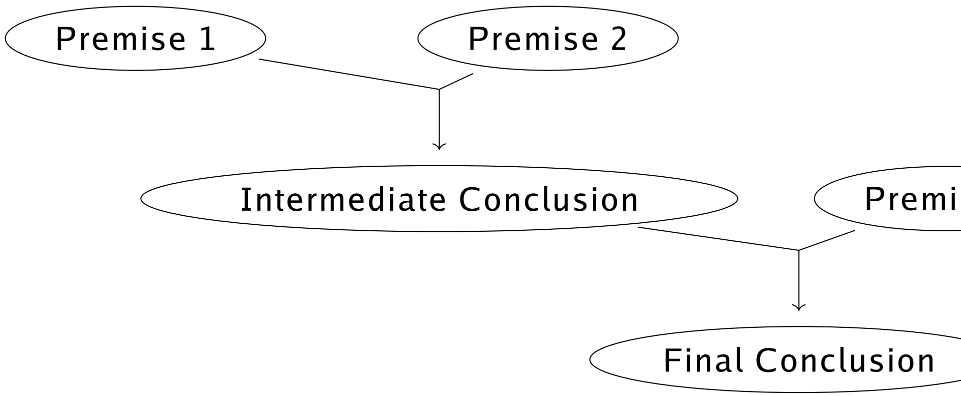
1.3 Intermediate Conclusions

We said an argument is given by a collection of **premises** along with a single **conclusion**. We might represent this as something like:



The premises are working together to lead to the conclusion.

But sometimes in the process of someone making an argument someone will make use of **intermediate conclusions.** Such arguments might have a structure more like:



However, we say that an argument is only something of the first kind. So what do we say about the second kind of thing? We can consider it two ways. We might could consider it as an argument from premise 1, 2 and 3 to the conclusion. Or alternatively we can think of it as two arguments of the first kind chained together, one from premise 1 and 2 to the intermediate conclusion, and the second from the intermediate conclusion and premise 3 to the final conclusion.

1.4 Sentences

What kinds of things are the premises and conclusions of arguments? They are sentences which can either be true or false. Such sentences are called **declarative sentences**.

is anyone ever confused about this anyway?? Also Richard thinks that it's not

There are many other kinds of sentences, for example:

Questions 'Are you sleepy yet?' is an interrogative sentence. Although you might be sleepy or you might be alert, the question itself is neither true nor false. For this reason, questions will not count as declarative sentences. Suppose you answer the question: ‘I am not sleepy.’ This is either true or false, and so it is a declarative sentences. Generally, **questions** will not count as declarative sentences, but **answers** will.

‘What is this course about?’ is not a declarative sentence (in our sense). ‘No one knows what this course is about’ is a declarative sentence.

Imperatives Commands are often phrased as imperatives like ‘Wake up!’, ‘Sit up straight’, and so on. These are imperative sentences. Although it might be good for you to sit up straight or it might not, the command is neither true nor false and it is thus not a declarative sentence. Note, however, that commands are not always phrased as imperatives. ‘You will respect my authority’ **is** either true or false— either you will or you will not— and so it counts as a declarative sentences.

Exclamations ‘Ouch!’ is sometimes called an exclamatory sentence, but it is not the sort of thing which is true or false. ‘That hurt!’, however, is a declarative sentence.

Our focus is only on **declarative sentences** — those sentences which can be true or false — for example ‘spiders have eight legs’. We typically drop the term ‘declarative’ and simply call them sentences, but bear in mind that it is only these sorts of sentences that are relevant in this textbook.

You should not confuse the idea of a sentence that can be true or false with the difference between fact and opinion. Often, sentences in logic will express things that would count as facts— such as ‘spiders have eight legs’ or ‘Kierkegaard liked almonds.’ They can also express things that you might think of as matters of opinion—such as, ‘Almonds are tasty.’ In other words, a sentence is not disqualified from being part of an argument because we don’t know if it is true or false, or because its truth or falsity is a matter of opinion. All that matters is whether it is the sort of thing that could be true or false. If it is, it can play the role of premise or conclusion.

Should I talk about ambiguity at all?

Indexica and context dependence?

Practice exercises

At the end of some chapters, there are exercises that review and explore the material covered in the chapter. The problem sheet you need to complete is constructed from these exercises, but the book offers some additional practice if you want more. There is no substitute for actually working through some problems. This course isn't about memorizing facts but about developing a way of thinking.

So here's the first exercise.

A. Highlight the phrase which expresses the conclusion of each of these arguments:

1. It is sunny. So I should take my sunglasses.
2. It must have been sunny. I did wear my sunglasses, after all.

3. No one but you has had their hands in the cookie-jar. And the scene of the crime is littered with cookie-crumbs. You're the culprit!
4. Miss Scarlett and Professor Plum were in the study at the time of the murder. Reverend Green had the candlestick in the ballroom, and we know that there is no blood on his hands. Hence Colonel Mustard did it in the kitchen with the lead-piping. Recall, after all, that the gun had not been fired.
5. Since I do not know that I am not under the spell of a malicious demon, I do not know that this table exists. After all, if I know that this table exists, then I know that I am not under the spell of a malicious demon.
6. Cutting the interest rate will have no effect on the stock market this time round as people have been expecting a rate cut all along. This factor has already been reflected in the market.
7. Virgin would then dominate the rail system. Is that something the government should

worry about? Not necessarily. The industry is regulated, and one powerful company might at least offer a more coherent schedule of services than the present arrangement has produced. The reason the industry was broken up into more than 100 companies at privatisation was not operational, but political: the Conservative government thought it would thus be harder to renationalise. **The Economist 16.12.2000; used on critical thinking web**

8. The idea that being vegetarian is better for the environment has, over the last forty years, become a piece of conventional wisdom. But it is simply wrong. A paper from Carnegie Mellon University researchers published this week finds that the diets recommended by the Dietary Guidelines for Americans, which include more fruits and vegetables and less meat, exacts a greater environmental toll than the typical American diet. Shifting to the diets recommended by Dietary Guidelines for American would increase energy use by

38 percent, water use by ten percent and greenhouse gas emissions by six percent, according to the paper.

9. There are no hard numbers, but the evidence from Asia's expatriate community is unequivocal. Three years after its handover from Britain to China, Hong Kong is unlearning English. The city's gweilos (Cantonese for "ghost men") must go to ever greater lengths to catch the oldest taxi driver available to maximize their chances of comprehension. Hotel managers are complaining that they can no longer find enough English- speakers to act as receptionists. Departing tourists, polled at the airport, voice growing frustration at not being understood.
   The Economist 20.1.2001, used in Critical Thinking Web

B. Now see if you can also identify the premises of the arguments in the previous part. For example:

1. - It is sunny.
   - Therefore: I should take my sunglasses.

Chapter 2

The scope of logic

2.1 Consequence and validity

In §1, we talked about arguments, i.e., a collection of sentences (the premises), followed by a single sentence (the conclusion). We said that some words, such as “therefore,” indicate which sentence in is supposed to be the conclusion. “Therefore,” of course, suggests that there is a connection between the premises and the conclusion, namely that the conclusion **follows from**, or **is a consequence of**, the premises.

This notion of consequence is one of the primary things logic is concerned with. One might even say that logic is the science of what follows from what. Logic develops theories and tools that tell us when a sentence follows from some others.

What about the following argument:

- Either the butler or the gardener did it.
- The butler didn't do it.
- Therefore: The gardener did it.

We don't have any context for what the sentences in this argument refer to. Perhaps you suspect that "did it" here means "was the perpetrator" of some unspecified crime. You might imagine that the argument occurs in a mystery novel or TV show, perhaps spoken by a detective working through the evidence. But even without having any of this information, you probably agree that the argument is a good one in the sense that whatever the premises refer to, if they are both true, the conclusion cannot but be true as well. If the first premise is true, i.e., it's true that "the butler did it or the gardener did it," then at least one of them "did it," whatever that means. And if the second premise is true, then the butler did not "do it." That leaves only one option: "the gardener did it" must be true. Here, the conclusion follows from the premises. We call arguments that have this property **valid**.

By way of contrast, consider the following argument:

- If the driver did it, the maid didn't do it.
- The maid didn't do it.
- Therefore: The driver did it.

We still have no idea what is being talked about here. But, again, you probably agree that this argument is different from the previous one in an important respect. If the premises are true, it is not guaranteed that the conclusion is also true. The premises of this argument do not rule out, by themselves, that someone other than the maid or the driver "did it." So there is a case where both premises are true, and yet the driver didn't do it, i.e., the conclusion is not true. In this second argument, the conclusion does not follow from the premises. If, like in this argument, the conclusion does not follow from the premises, we say it is **invalid**.

2.2 Validity

How did we determine that the second argument is invalid? We pointed to a case in which the premises are true and in which the conclusion is not. This was the scenario where neither the driver nor the maid did it, but some third person did. Let's call such a case a **counterexample** to the argument. If there is a counterexample to an argument, the conclusion cannot be a consequence of the premises. For the conclusion to be a consequence of the premises, the truth of the premises must guarantee the truth of the conclusion. If there is a counterexample, the truth of the premises does not guarantee the truth of the conclusion.

As logicians, we want to be able to determine when the conclusion of an argument follows from the premises. And the conclusion is a consequence of the premises if there is no counterexample—no case where the premises are all true but the conclusion is not. This motivates a definition:

> A sentence Y is a **consequence** of sentences $X_1, \ldots, X_n$ if and only if there is no case where $X_1, \ldots, X_n$ are all true and Y is not true. (We then also say that Y **follows from** $X_1, \ldots, X_n$ or that $X_1, \ldots, X_n$ **entail** Y.)

We said that arguments where the conclusion is a consequence of the premises are called valid, and those where the conclusion isn't a consequence of the premises are invalid.

> An argument is **valid** if and only if the conclusion is a consequence of the premises. That is, there is no case where all the premises are true but the conclusion not true.

An argument is **invalid** if and only if it is not valid. That is, there is some case where all the premises are true and the conclusion false.

2.3 Cases and types of validity

The “definitions” from the previous section are incomplete: it does not tell us what a “case” is or what it means to be “true in a case.” So far we’ve only seen an example: a hypothetical scenario involving three people. Of the three people in the scenario—a driver, a maid, and some third person—the driver and maid didn’t do it, but the third person did. In this scenario, as described, the driver didn’t do it, and so it is a case in which the sentence “the driver did it” is not true. The premises of our second argument are true, but the conclusion is not true: the scenario is a counterexample.

Logicians are in the business of making the notion of “case” more precise, and investigating which arguments are valid when “case” is made precise in one way or another. If we take “case” to mean “hypothetical scenario” like the counterexample to the second argument, it’s clear that the first argument counts as valid. If we imagine a scenario in which either the butler or the gardener did it, and also the butler didn’t do it, we are automatically imagining a scenario in which the gardener did it. So any hypothetical scenario in which the premises of our first argument are true automatically makes the conclusion of our first argument true. This makes the first argument valid.

Making “case” more specific by interpreting it as “hypothetical scenario” is an advance. But it is not the end of the story. The first problem is that we don’t know what to count as a hypothetical scenario. Are they limited by the laws of physics? By what is conceivable, in a very general sense? What answers we give to these questions determine which arguments we count as valid.

Suppose the answer to the first question is “yes.” Consider the following argument:

- The spaceship **Rocinante** took six hours to reach Jupiter from Tycho space station.
- Therefore: The distance between Tycho space station and Jupiter is less than 14 billion kilometers.

A counterexample to this argument would be a scenario in which the **Rocinante** makes a trip of over 14 billion kilometers in 6 hours, exceeding the speed of light. Since such a scenario is incompatible with the laws of physics, there is no such scenario if hypothetical scenarios have to conform to the laws of physics. If hypothetical scenarios are not limited by the laws of physics, however, there is a counterexample: a scenario where the **Rocinante** travels faster than the speed of light.

Suppose the answer to the second question is “yes,” and consider another argument:

- Priya is an ophthalmologist.
- Therefore: Priya is an eye doctor.

If we’re allowing only conceivable scenarios, this is also a valid argument. If you imagine Priya being an ophthalmologist, you thereby imagine Priya being an eye doctor. That's just what "ophthalmologist" and "eye doctor" mean. A scenario where Priya is an ophthalmologist but not an eye doctor is ruled out by the conceptual connection between these words.

When we consider cases of various kinds in order to evaluate the validity of an argument, we will make a few assumptions. The first assumption is that every case makes every sentence true or false—at least, every sentence in the argument under consideration. So imagined scenarios have to specify all relevant facts. Any imagined scenario which leaves it undetermined if a sentence in our argument is true will not be considered as a potential counterexample.

Depending on what kinds of cases we consider as potential counterexamples, then, we arrive at different notions of consequence and validity. We might call an argument **nomologically valid** if there are no counterexamples that don't violate the laws of nature, and an argument **conceptually valid** if there are no counterexamples that don't violate conceptual connections between words. For both of these notions of validity, aspects of the world (e.g., what the laws of nature are) and aspects of the meaning of the sentences in the argument (e.g., that "ophthalmologist" just means a kind of eye doctor) figure into whether an argument is valid.

One distinguishing feature of **logical** consequence, however, is that it should not depend on the content of the premises and conclusion, but only on their logical form. In other words, as logicians we want to develop a theory that can make finer-grained distinctions still. For instance, both

- Either Priya is an ophthalmologist or a dentist.
- Priya isn't a dentist.
- Therefore: Priya is an eye doctor.

and

- Either Priya is an ophthalmologist or a dentist.
- Priya isn't a dentist.
- Therefore: Priya is an ophthalmologist.

are valid arguments. But while the validity of the first depends on the content (i.e., the meaning of "ophthalmologist" and "eye doctor"), the second does not. The second argument is **formally valid**. We can describe the "form" of this argument as a pattern, something like this:

- Either a is an F or a G.
- a isn't an F.
- Therefore: a is a G.

Here, a, F, and G are placeholders for appropriate expressions that, when substituted for a, F, and G, turn the pattern into an argument consisting of sentences. For instance,

- Either Mei is a mathematician or a

botanist.
- Mei isn't a botanist.
- Therefore: Mei is a mathematician.

is an argument of the same form, but the first argument above is not: we would have to replace F by different expressions (once by "ophthalmologist" and once by "eye doctor") to obtain it from the pattern.

Moreover, the first argument is not formally valid. **Its** form is this:

- Either a is an F or a G.
- a isn't an F.
- Therefore: a is a H.

In this pattern we can replace F by "ophthalmologist" and H by "eye doctor" to obtain the original argument. But here is another argument of the same form:

- Either Mei is a mathematician or a botanist.

- Mei isn't a botanist.
- Therefore: Mei is an acrobat.

This argument is clearly not valid, since we can imagine a mathematician named Mei who is not an acrobat.

Our strategy as logicians will be to come up with a notion of "case" on which an argument turns out to be valid if it is formally valid. Clearly such a notion of "case" will have to violate not just some laws of nature but some laws of English. Since the first argument is invalid in this sense, we must allow as counterexample a case where Priya is an ophthalmologist but not an eye doctor. That case is not a conceivable situation: it is ruled out by the meanings of "ophthalmologist" and "eye doctor."

2.4 Sound arguments

Before we go on and execute this strategy, a few clarifications. Arguments in our sense, as conclusions which (supposedly) follow from premises, are of course used all the time in everyday, philosophical and scientific discourse. When they are, arguments are given to support or even prove their conclusions. Now, if an argument is valid, it will support its conclusion, but **only if** its premises are all true. Validity rules out the possibility that the premises are true and the conclusion is not true at the same time. It does not, by itself, rule out the possibility that the conclusion is not true, period. In other words, it is perfectly possibly for a valid argument to have a conclusion that isn't true!

Consider this example:

- Oranges are either fruit or musical instruments.
- Oranges are not fruit.
- Therefore: Oranges are musical instruments.

The conclusion of this argument is ridiculous. Nevertheless, it follows from the premises. **If** both premises are true, **then** the conclusion just has to be true. So the argument is valid.

Conversely, having true premises and a true conclusion is not enough to make an argument valid. Consider this example:

- London is in England.
- Beijing is in China.
- Therefore: Paris is in France.

The premises and conclusion of this argument are, as a matter of fact, all true, but the argument is invalid. If Paris were to declare independence from the rest of France, then the conclusion would no longer be true, even though both of the premises would remain true. Thus, there is a case where the premises of this argument are true without the conclusion being true. So the argument is invalid.

The important thing to remember is that validity is not about the actual truth or falsity of the sentences in the argument. It is about whether it is **possible** for all the premises to be true and the conclusion to be not true at the same time (in some hypothetical case). What is in fact the case has no special role to play; and what the facts are does not determine whether an argument is valid or not.[1] Nothing about the way things are can by itself determine if an argument is valid. It is often said that logic doesn't care about feelings. Actually, it doesn't care about facts, either.

When we use an argument to prove that its conclusion **is true**, then, we need two things. First, we need the argument to be valid, i.e., we need the conclusion to follow from the premises. But we also need the premises to be true. We will say that an argument is **sound** if and only if it is both valid and all of its premises are true.

The flip side of this is that when you want to rebut an argument, you have two options: you can show that (one or more of) the premises are not true, or you can show that the argument is

[1]Well, there is one case where it does: if the premises are in fact true and the conclusion is in fact not true, then we live in a counterexample; so the argument is invalid.

not valid. Logic, however, will only help you with the latter!

2.5 Missing premises

If someone you disagree with makes an invalid argument, you might be tempted just to dismiss it as obviously incorrect. But it's more useful (and more charitable) to consider whether there are missing premises that could be filled in that make the argument better. Perhaps the author was assuming that this premise was so obvious that it didn't need to be stated.

For example an author might make the following argument:

- I could not have acted otherwise.
- Therefore: I did not act of my own free will.

This argument is invalid. But, it can be made valid by addition of the premise:

- If I could not have acted otherwise, I did not act of my own free will.

But be careful when you're filling in 'missing' premises. The aim is to help improve the argument, to make it more convincing, so you can assess it fairly. Only add extra premises that seem reasonable, or that you think the original author would agree with. There's no point in adding absurd or unreasonable premises, or premises that the author wouldn't endorse. Then you just create a **strawman** argument – a caricature of the original argument.

> "Just how charitable are you supposed to be when criticizing the views of an opponent? If there are obvious contradictions in the opponent's case, then of course you should point them out, forcefully. If there are somewhat hidden contradictions, you should carefully expose them to view—and then dump on them. But the search for hidden contradictions often crosses the line into nitpicking, sea-lawyering,

> and—as we have seen—outright parody. The thrill of the chase and the conviction that your opponent has to be harboring a confusion somewhere encourages uncharitable interpretation, which gives you an easy target to attack. But such easy targets are typically irrelevant to the real issues at stake and simply waste everybody's time and patience, even if they give amusement to your supporters. "
>
> **Daniel C. Dennett (2013). "Intuition Pumps And Other Tools for Thinking".**

Dennett formulates the following four rules (named after Anatol Rapoport) for "how to compose a successful critical commentary":

1. You should attempt to re-express your target's position so clearly, vividly, and fairly that your target says, "Thanks, I wish I'd thought of putting it that way."

2. You should list any points of agreement (especially if they are not matters of general or widespread agreement).

3. You should mention anything you have learned from your target.

4. Only then are you permitted to say so much as a word of rebuttal or criticism

2.6 Ampliative Arguments

Sometimes there are no plausible missing premises you could add to someone's argument to make it valid. However, this doesn't necessarily mean that the author was wrong or mistaken. Deductively valid arguments with plausible premises are good arguments, but they aren't the only good arguments there are. This is just as well, since many arguments we give in our everyday lives are not deductively valid, even after filling in plausible missing premises. Here's an example:

- In January 1997, it rained in London.

- In January 1998, it rained in London.
- In January 1999, it rained in London.
- In January 2000, it rained in London.
- Therefore: It rains every January in London.

This argument generalises from observations about several cases to a conclusion about all cases—in each year listed, it rained in January, so it does in every year. Such arguments are called **inductive** arguments. The argument could be made stronger by adding additional premises before drawing the conclusion: In January 2001, it rained in London; In January 2002.... But, however many premises of this form we add, the argument will remain invalid. Even if it has rained in London in every January thus far, it remains possible that London will stay dry next January. The point of all this is that inductive arguments—even good inductive arguments—are not (deductively) valid. They are not watertight. The premises might make the conclusion very likely, but they don't absolutely guarantee its truth. Unlikely though it might be, it is possible for their conclusion to be false, even when all of their premises are true.

Inductive arguments of the sort just given belong to a species of argument called **ampliative arguments**. This means that the conclusion goes beyond what you find in the premises. That is, the premises don't guarantee, or entail, the conclusion. They do, however, provide some support for it. These arguments are deductively invalid. They may be good and useful, however it is important to know the difference.

In this book, we will set aside the question of what makes for a good ampliative argument and focus instead on sorting the deductively valid arguments from the deductively invalid ones. But we pause here to mention some further forms of ampliative argument.

Inductive arguments, like the one we saw above, allow one to infer from a series of

At end of 2.1. I said

observed cases to a generalization that covers them: from all observed Fs have been Gs, we infer all Fs are Gs. We use these all the time. Every time I've drunk water from my tap, it's quenched my thirst; therefore, every time I ever drink water from my tap, it will quench my thirst. Every time I've stroked my neighbour's cat, it hasn't bitten me; therefore, every time I ever stroke my neighbour's cat, it won't bite me. And it's a form of arguments much beloved by scientists. Every time we've measured the acceleration of a body falling, it's matched Newton's theory, therefore, all bodies are governed by Newton's theory. The premises of these argument seem to make their conclusions likely without guaranteeing them. The areas of philosophy called inductive logic or confirmation theory try to make precise what that means and why it's true. And of course inductive arguments can go wrong. Before I visited Australia, every swan I'd every seen was white, and so I concluded that all swans were white; but when I visited Australia, I realised my conclusion was wrong, because some swans there are black.

A closely related, but different form of argument, is **statistical**. Here, we start with an observation about the proportion of Fs that are Gs in a sample that we've observed, and we infer that the same proportion of Fs are Gs in general. So, for instance, if I poll 1,000 people in Scotland eligible to vote in a second independence referendum, and 600 say that they'll vote yes, I might infer that 60% of all eligible voters will vote yes. Or if I test 1,000,000 people in England for an active infection, and 20,000 test positive, I might infer that 2% of the whole population has an active infection. How good these argument are depends on a number of things, and these are studied by statisticians. For instance, suppose you picked the 1,000 Scottish voters entirely at random from an anonymised version of the electoral register. But suppose that, when you deanonymised, you learned that, by chance, all of the people you'd picked were over 65, or they all lived on the Isle of Skye. Then you might worry that your sample, though random, was unrepresentative of the population as a whole. This question is a genuine concern for randomised controlled trials in medicine.

Abductive arguments provide an inference from a phenomenon you've observed to the **best explanation** of that phenomenon: from E, and the best explanation of E is H, you might conclude H. Again, this is extremely widespread. A classic sort of example would be the inferences that detectives draw during their investigations. They look at the evidence and the possible explanations of it, and they tend to conclude in favour of the best one. And similarly for doctors looking at a patient's suite of symptoms and trying to discover what ails them. Another important example comes from science. Here is Charles Darwin explaining what convinces him of his theory of natural selection:

> "It can hardly be supposed that a false theory would explain, in so satisfactory a manner as does the theory of natural selection, the

several large classes of facts above specified. It has recently been objected that this is an unsafe method of arguing; but it is a method used in judging of the common events of life, and has often been used by the greatest natural philosophers."

(Charles Darwin, On the origin of species by means of natural selection (6th ed.). London: John Murray)

Practice exercises

A. Which of the following arguments are conceptually valid? Which are conceptually invalid?

- Socrates is a man.
- All men are carrots.
- Therefore: Socrates is a carrot.

- Abe Lincoln was either born in Illinois or he was once president.
- Abe Lincoln was never president.

- Therefore: Abe Lincoln was born in Illinois.

- If I pull the trigger, Abe Lincoln will die.
- I do not pull the trigger.
- Therefore: Abe Lincoln will not die.

- Abe Lincoln was either from France or from Luxemborg.
- Abe Lincoln was not from Luxemborg.
- Therefore: Abe Lincoln was from France.

- If the world were to end today, then I would not need to get up tomorrow morning.
- I will need to get up tomorrow morning.
- Therefore: The world will not end today.

- Joe is now 19 years old.
- Joe is now 87 years old.
- Therefore: Bob is now 20 years old.

B. Could there be:

1. A valid argument that has one false premise and one true premise?
2. A valid argument that has only false premises?
3. A valid argument with only false premises and a false conclusion?
4. An invalid argument that can be made valid by the addition of a new premise?
5. A valid argument that can be made invalid by the addition of a new premise?

In each case: if so, give an example; if not, explain why not.

C. !!!

Chapter 3

Other logical notions

In §2, we introduced the ideas of consequence and of valid argument. This is one of the most important ideas in logic. In this section, we will introduce are some similarly important ideas. They all rely, as did validity, on the idea that sentences are true (or not) in cases. For the rest of this section, we'll take cases in the sense of conceivable scenario, i.e., in the sense in which we used them to define conceptual validity. The points we made about different kinds of validity can be made about our new notions along similar lines: if we use a different idea of what counts as a "case" we will get different notions. And as logicians we will, eventually, consider a more permissive definition of case than we do here.

3.1 Joint possibility

Consider these two sentences:

B1. Jane's only brother is shorter than her.
B2. Jane's only brother is taller than her.

Logic alone cannot tell us which, if either, of these sentences is true. Yet we can say that **if** the first sentence (B1) is true, **then** the second sentence (B2) must be false. Similarly, if B2 is true, then B1 must be false. There is no possible scenario where both sentences are true together. These sentences are incompatible with each other, they cannot all be true at the same time. This motivates the following definition:

> Sentences are **jointly possible** if and only if there is a case where they are all true together.

B1 and B2 are **jointly impossible**, while, say, the following two sentences are jointly possible:

B1. Jane's only brother is shorter than her.
B2. Jane's only brother is younger than her.

We can ask about the joint possibility of any number of sentences. For example, consider the following four sentences:

G1. There are at least four giraffes at the wild animal park.
G2. There are exactly seven gorillas at the wild animal park.
G3. There are not more than two martians at the wild animal park.
G4. Every giraffe at the wild animal park is a martian.

G1 and G4 together entail that there are at least four martian giraffes at the park. This conflicts with G3, which implies that there are no more than two martian giraffes there. So the sentences G1–G4 are jointly impossible. They cannot all be true together. (Note that the sentences G1, G3 and G4 are jointly impossible. But if sentences are already jointly impossible, adding an extra sentence to the mix cannot make them jointly possible!)

3.2 Necessary truths, necessary falsehoods, and contingency

In assessing arguments for validity, we care about what would be true **if** the premises were true, but some sentences just **must** be true. Consider these sentences:

1. It is raining.
2. Either it is raining here, or it is not.
3. It is both raining here and not raining here.

In order to know if sentence 1 is true, you would need to look outside or check the weather channel. It might be true; it might be false. A sentence which is capable of being true and capable of being false (in different circumstances, of course) is called **contingent**.

Sentence 2 is different. You do not need to look outside to know that it is true. Regardless of what the weather is like, it is either raining or it is not. That is a **necessary truth**.

Equally, you do not need to check the weather to determine whether or not sentence 3 is true. It must be false, simply as a matter of logic. It might be raining here and not raining across town; it might be raining now but stop raining even as you finish this sentence; but it is impossible for it to be both raining and not raining in the same place and at the same time. So, whatever the world is like, it is not both raining here and not raining here. It is a **necessary falsehood**.

Something might **always** be true and still be contingent. For instance, if there never were a time when the universe contained fewer than seven things, then the sentence 'At least seven things exist' would always be true. Yet the sentence is contingent: the world could have been much, much smaller than it is, and then the sentence would have been false.

3.3 Necessary equivalence

We can also ask about the logical relations **between** two sentences. For example:

> John went to the store after he washed the dishes.
> John washed the dishes before he went to the store.

These two sentences are both contingent, since John might not have gone to the store or washed dishes at all. Yet they must have the same truth-value. If either of the sentences is true, then they both are; if either of the sentences is false, then they both are. When two sentences have the same truth value in every case, we say that they are **necessarily equivalent**.

Summary of logical notions

- An argument is **valid** if there is no case where the premises are all true and the conclusion is not; it is **invalid** otherwise.
- A **necessary truth** is a sentence that is true in every case.
- A **necessary falsehood** is a sentence that is false in every case.
- A **contingent sentence** is a sentence that is neither a necessary truth nor a necessary falsehood; a sentence that is true in some case and false in some other case.
- Two sentences are **necessarily equivalent** if, in every case, they are both true or both false.
- A collection of sentences is **jointly possible** if there is a case where they are all true together; it is **jointly impossible** otherwise.

Practice exercises

A. For each of the following: Is it a necessary truth, a necessary falsehood, or contingent?

1. Caesar crossed the Rubicon.
2. Someone once crossed the Rubicon.
3. No one has ever crossed the Rubicon.
4. If Caesar crossed the Rubicon, then someone has.
5. Even though Caesar crossed the Rubicon, no one has ever crossed the Rubicon.
6. If anyone has ever crossed the Rubicon, it was Caesar.

B. For each of the following: Is it a necessary truth, a necessary falsehood, or contingent?

1. Elephants dissolve in water.
2. Wood is a light, durable substance useful for building things.
3. If wood were a good building material, it would be useful for building things.
4. I live in a three story building that is two stories tall.

5. If gerbils were mammals they would nurse their young.

C. Which of the following pairs of sentences are necessarily equivalent?

1. Elephants dissolve in water.
   If you put an elephant in water, it will disintegrate.
2. All mammals dissolve in water.
   If you put an elephant in water, it will disintegrate.
3. George Bush was the 43rd president.
   Barack Obama is the 44th president.
4. Barack Obama is the 44th president.
   Barack Obama was president immediately after the 43rd president.
5. Elephants dissolve in water.
   All mammals dissolve in water.

D. Which of the following pairs of sentences are necessarily equivalent?

1. Thelonious Monk played piano.

John Coltrane played tenor sax.

2. Thelonious Monk played gigs with John Coltrane.
   John Coltrane played gigs with Thelonious Monk.
3. All professional piano players have big hands.
   Piano player Bud Powell had big hands.
4. Bud Powell suffered from severe mental illness.
   All piano players suffer from severe mental illness.
5. John Coltrane was deeply religious.
   John Coltrane viewed music as an expression of spirituality.

E. Consider the following sentences:

G1 There are at least four giraffes at the wild animal park.

G2 There are exactly seven gorillas at the wild animal park.

G3 There are not more than two Martians at the wild animal park.

G4 Every giraffe at the wild animal park is a Martian.

Now consider each of the following collections of sentences. Which are jointly possible? Which are jointly impossible?

1. Sentences G2, G3, and G4
2. Sentences G1, G3, and G4
3. Sentences G1, G2, and G4
4. Sentences G1, G2, and G3

F. Consider the following sentences.

M1 All people are mortal.

M2 Socrates is a person.

M3 Socrates will never die.

M4 Socrates is mortal.

Which combinations of sentences are jointly possible? Mark each “possible” or “impossible.”

1. Sentences M1, M2, and M3
2. Sentences M2, M3, and M4
3. Sentences M2 and M3
4. Sentences M1 and M4
5. Sentences M1, M2, M3, and M4

G. Which of the following is possible? If it is possible, give an example. If it is not possible, explain why.

1. A valid argument that has one false premise and one true premise
2. A valid argument that has a false conclusion
3. A valid argument, the conclusion of which is a necessary falsehood
4. An invalid argument, the conclusion of which is a necessary truth
5. A necessary truth that is contingent

6. Two necessarily equivalent sentences, both of which are necessary truths
7. Two necessarily equivalent sentences, one of which is a necessary truth and one of which is contingent
8. Two necessarily equivalent sentences that together are jointly impossible
9. A jointly possible collection of sentences that contains a necessary falsehood
10. A jointly impossible set of sentences that contains a necessary truth

H. Which of the following is possible? If it is possible, give an example. If it is not possible, explain why.

1. A valid argument, whose premises are all necessary truths, and whose conclusion is contingent
2. A valid argument with true premises and a false conclusion
3. A jointly possible collection of sentences that contains two sentences that are not necessarily equivalent

4. A jointly possible collection of sentences, all of which are contingent
5. A false necessary truth
6. A valid argument with false premises
7. A necessarily equivalent pair of sentences that are not jointly possible
8. A necessary truth that is also a necessary falsehood
9. A jointly possible collection of sentences that are all necessary falsehoods

Appendices

Appendix A

Symbolic notation

1.1 Alternative nomenclature

Truth-functional logic. TFL goes by other names. Sometimes it is called **sentential logic**, because it deals fundamentally with sentences. Sometimes it is called **propositional logic**, on the idea that it deals fundamentally with propositions. We have stuck with **truth-functional logic**, to emphasize the fact that it deals only with assignments of truth and falsity to sentences, and that its connectives are all truth-functional.

First-order logic. FOL goes by other names. Sometimes it is called **predicate logic**, because it allows us to apply predicates to objects. Sometimes it is called **quantified logic**, because it makes use of quantifiers.

Formulas. Some texts call formulas **well-formed formulas.** Since 'well-formed formula' is such a long and cumbersome phrase, they then abbreviate this as **wff.** This is both barbarous and unnecessary (such texts do not countenance 'ill-formed formulas'). We have stuck with 'formula'.

In §6, we defined **sentences** of TFL. These are also sometimes called 'formulas' (or 'well-formed formulas') since in TFL, unlike FOL, there is no distinction between a formula and a sentence.

Valuations. Some texts call valuations **truth-assignments**, or **truth-value assignments**.

Expressive adequacy. Some texts describe TFL as **truth-functionally complete**, rather than expressively adequate.

n-place predicates. We have chosen to call predicates 'one-place', 'two-place', 'three-place', etc. Other texts respectively call them 'monadic', 'dyadic', 'triadic', etc. Still other texts call them 'unary', 'binary', 'ternary', etc.

Names. In FOL, we have used 'a', 'b', 'c', for names. Some texts call these 'constants'. Other texts do not mark any difference between names and variables in the syntax. Those texts focus simply on whether the symbol occurs **bound** or **unbound**.

Domains. Some texts describe a domain as a 'domain of discourse', or a 'universe of discourse'.

1.2 Alternative symbols

In the history of formal logic, different symbols have been used at different times and by different authors. Often, authors were forced to use notation that their printers could typeset.

This appendix presents some common symbols, so that you can recognize them if you encounter them in an article or in another book.

Negation. Two commonly used symbols are the **hoe**, '$\neg$', and the **swung dash** or **tilda**, '$\sim$.' In some more advanced formal systems it is necessary to distinguish between two kinds of negation; the distinction is sometimes represented by using both '$\neg$' and '$\sim$'. Older texts sometimes indicate negation by a line over the formula being negated, e.g., $\overline{A \,\&\, B}$. Some texts use '$x \neq y$' to abbreviate '$\neg x = y$'.

Disjunction. The symbol '$\vee$' is typically used to symbolize inclusive disjunction. One etymology is from the Latin word 'vel', meaning 'or'.

Conjunction. Conjunction is often symbolized with the **ampersand**, '&'. The ampersand is a decorative form of the Latin word 'et', which means 'and'. (Its etymology still lingers in certain fonts, particularly in italic fonts; thus an italic ampersand might appear as '*&*'.) This symbol is commonly used in natural English writing (e.g. 'Smith & Sons'), and so even though it is a natural choice, many logicians use a different symbol to avoid confusion between the object and metalanguage: as a symbol in a formal system, the ampersand is not the English word '&'. The most common choice now is '$\wedge$', which is a counterpart to the symbol used for disjunction. Sometimes a single dot, '•', is used. In some older texts, there is no symbol for conjunction at all; 'A and B' is simply written 'AB'.

Material Conditional. There are two common symbols for the material conditional: the **arrow**, '$\rightarrow$', and the **hook**, '$\supset$'.

Material Biconditional. The **double-headed arrow**, '$\leftrightarrow$', is used in systems that use the arrow to represent the material conditional. Systems that use the hook for the conditional typically use the **triple bar**, '$\equiv$', for the biconditional.

Quantifiers. The universal quantifier is typically symbolized as a rotated 'A', and the existential quantifier as a rotated, 'E'. In some texts, there is no separate symbol for the universal quantifier. Instead, the variable is just written in parentheses in front of the formula that it binds. For example, they might write '$(x)Px$' where we would write '$\forall x\, Px$'.

These alternative typographies are summarised below:

negation	$\neg$, $\sim$
conjunction	$\wedge$, &, $\bullet$
disjunction	$\vee$
conditional	$\rightarrow$, $\supset$
biconditional	$\leftrightarrow$, $\equiv$
universal quantifier	$\forall x$, (x)