Accuracy and Immodesty for the Imprecise Probabilist

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WARNING: It's in the middle of a rewrite, so it's a bit weird at the moment. Sorry! Updates should be coming soon, but I got distracted by other things right now, so it might take me longer than I hoped. Contact me if you want to talk through anything!!!

Abstract

In the imprecise probabilities framework we represent one's belief state as a set of precise probabilities. This paper considers the following rather natural principle to tell us about what imprecise options an imprecise probability recommends:

IMPRECISE RECOMMENDATION.
What a set recommends is the set of what the individuals recommend.

We will show that this principle has a number of desirable consequences for the imprecise probabilist once we tie it with accuracy-theoretic considerations. It allows for a picture of accuracy for the imprecise where any imprecise probability recommends itself, and we can obtain arguments for various principles of rationality such as Imprecise Probabilism and Imprecise Conditionalization. It can also allow for some rationally permissible options in certain scenarios which have formed a challenge case for accuracy-theory, or more generally for rationality considerations. In such scenarios, which bear a close relationship to the liar paradox, any precise probability undermines itself, but by using Imprecise Recommendation, we see that imprecise probabilities can be self-recommending, and thus candidates for rationally permissible options. We also give some kind of justification to Imprecise Recommendation by providing a story thinking about the imprecise credence as a group, or credal-committee, where each committee member (precise probability) has opinions about what is better or worse, and they come together to make a decision on which groups are better and worse.

*Thanks to lots of people!

1 Accuracy and rationality for the imprecise

In the imprecise probabilities framework we represent one's belief state as a *set* of precise probabilities.[1] This framework has proved very powerful extension of the Bayesian framework, but there are various key principles which are used by the imprecise probabilist which could do with more support. For example:

Imprecise-Probabilism.
One's credal set, $\mathbb{B}$, should only contains probability functions.

I.e. every member of my credal committee should satisfy the axioms of probability. And:

Imprecise-Conditionalization.
After learning E, one's updated belief state should be the set of conditionalized precise probabilities:

$$\mathbb{B}_E = \{b(-|E) \mid b \in \mathbb{B}\}.$$

These principles are often assumed by the imprecise probability theorist.

An influential method of justifying rationality constraints is the accuracy framework.[2] This justifies various rationality principles by their success at pursuing matters of epistemic value, which, specifically, is taken as their success at giving credences which are "close to the truth". This framework has been very powerful in the precise setting, for example there are dominance arguments for an agent's (precise) degrees of beliefs satisfying the axioms of probability: non-probability functions are irrational because there will be some (probabilistic) opinion which is more epistemically valuable, i.e. closer to the truth, whatever the truth turns out to be; similarly there are various arguments surrounding conditionalization as the epistemically optimal update procedure.

Can the imprecise probabilist reap the fruits of this accuracy project? I suggest yes. I try to argue that the imprecise probabilist can directly help herself to the results from the precise framework and get (pretty-much immediate) arguments for her principles such as Imprecise-Probabilism and Imprecise-Conditionalization.

The suggestion I make is that we should consider the opinions of the imprecise probabilist by looking at the opinions of the individual members of the credal-committee. We measure accuracy simply of the precise opinions, then an imprecise probability has opinions about what does best accuracy-wise by supervening on the opinions of the precise members of her credal committee. In particular, the following principle allows us to directly obtain arguments for these normative requirements such as Imprecise-Probabilism and Imprecise-Conditionalization.

Imprecise Recommendation.
What a set recommends is the set of what the individuals recommend.

[1] See, e.g. Mahtani (2019) for an introduction to imprecise probabilities.
[2] Joyce (1998, 2009); Pettigrew (2016).

For our accuracy considerations we are taking 'recommend' understood as doing best at the pursuit of accuracy. This allows accuracy considerations to apply to the imprecise, and allows us to obtain desirable consequences.

This goes against some recent papers providing various impossibility results regarding the possibility of applying accuracy considerations for the imprecise probabilist.[3] Such papers have centred around the difficulty of applying accuracy considerations for the imprecise probabilist in a way that obtains immodesty for the imprecise:

Imprecise-Immodesty.
Every set of probability functions $\mathbb{B}$ should take itself to be doing best accuracy-wise.

These papers have provided impossibility results for providing a way of measuring accuracy that obtain Imprecise-Immodesty. Immodesty has been an immensely important component in the accuracy literature. Formally, it is spelled out as saying that our way of measuring accuracy should be strictly proper. The accuracy literature motivates this as a requirement for an appropriate account of accuracy, and it is technically important for their results. However, the impossibility results assume substantial matters about how accuracy considerations should be developed in the imprecise setting. The way I propose to apply accuracy considerations goes against the settings of the impossibility results primarily by not providing a specific value (precise or imprecise) that measures the accuracy of an imprecise probability at a world. Instead, we have precise accuracy values for the precise members of one's credal committee, and we use Imprecise Recommendation to obtain rationality constraints for the imprecise.

1.1 The contents of the paper to come

Section 3 spells out Imprecise Recommendation in a bit more detail then shows that when we link it with accuracy-theoretic considerations, we can obtain Imprecise-Immodesty (Section 3.1), Imprecise-Probabilism (Section 3.4) and Imprecise-Conditionalization (Section 3.2). In Section 3.3 we show that Imprecise Recommendation allows for some rationally permissible options in certain scenarios which have formed a challenge case for accuracy-theory, or more generally for rationality considerations. In such scenarios, which bear a close relationship to the liar paradox, any precise probability undermines itself, but by using Imprecise Recommendation, we see that imprecise probabilities can be self-recommending, and thus candidates for rationally permissible options.

In Section 4 we try to give more detail for why the idea in Imprecise Recommendation is plausible. We provide a story thinking about the imprecise credence as a group, or credal-committee, where each committee member has a precise degree of belief and opinions about what is better or worse, and they come together to make a decision on which groups are better and worse. This

[3]Seidenfeld et al. (2012); Mayo-Wilson and Wheeler (2016); Schoenfield (2017); Chambers (2008). See also Konek (2019).

section needs more work. Firstly, there's work to be done on the formal side to define how the individual's ordering of credence function is jacked up to the committee's ordering of sets in a way that is satisfactory and does the work we need it to do (Section 4.2). Secondly, more needs to be done to think about the interpretation of the imprecise probabilities that is being presupposed and which can ground the kind of proposal that is being made here (Section 4.3). But hopefully it'll give you a sense for how this might go.

In **??** we finish by tying some loose ends. In **??** we talk specifically about how this avoids the impossibility results in the literature. In **??** we present some considerations regarding obtaining a dominance result for Imprecise Probabilism.

2 Accuracy, Imprecision and Immodesty

In order to obtain its results, accuracy-theorists need to say more about how to measure accuracy. One of the most common measures of accuracy for precise credences is the Brier score, which gives us:[4]

$$\mathsf{Acc}(c, w) = 1 - \sum_{A \in \mathcal{A}} (c(A) - w(A))^2;$$

Consider a rational agent who has degree of belief 0.5 in A. We can ask how accurate she thinks it would be to adopt various degrees of belief in a proposition A. A credence of .8 in A would be more accurate than .5 if A is true, but less accurate if A is false:

x	$\mathsf{Acc}(x, \mathsf{true})$	$\mathsf{Acc}(x, \mathsf{false})$
.5	.75	.75
.8	.96	.36

But .5 thinks that it is overall doing best at the pursuit of accuracy: the *expected accuracy* of .5 is higher than .8, as evaluated according to a probability function assigning .5 to A. This is defined as:

$$\mathrm{Exp}_b\mathsf{Acc}(c) = \sum_w b(w) \times \mathsf{Acc}(c, w).$$

Then:[5]

$$\mathrm{Exp}_{.5}\mathsf{Acc}(.5) = .5 \times .75 + .5 \times .75 = .75$$
$$\mathrm{Exp}_{.5}\mathsf{Acc}(.8) = .5 \times .96 + .5 \times .36 = .66$$

[4] *In*accuracy is typically considered, measuring distance from truth. We stick to accuracy to have that optimal relates to highest value.

[5] We are abusing notation since .5 is itself not a probability function. This is simply defined as $\mathrm{Exp}_x\mathsf{Acc}(y) = x \times \mathsf{Acc}(y, \mathsf{true}) + (1 - x) \times \mathsf{Acc}(y, \mathsf{false})$, and is what would result by calculating the expectation from the perspective of any probability function which assigns x to A.

If the expected accuracy of .8 were higher than .5, this would have undermined the rationality of .5: our agent who adopts .5 would think that it would be better to change her degree of belief in A even without additional evidence, and her degree of belief .5 in A could not be rationally relied on.[6]

One can show that when using the Brier score to measure accuracy, all probability functions are immodest – they expect themselves to be doing best at the pursuit of accuracy. This property of the Brier score is called *strict propriety*:

Strict Propriety.
For b probabilistic, if $c \neq b$, $\mathrm{Exp}_b\mathsf{Acc}(c) < \mathrm{Exp}_b\mathsf{Acc}(b)$.

This property is of fundamental importance for the accuracy project. The majority of the accuracy-theoretic arguments, such as the arguments for conditionalization, rely on the property of strictly properiety. In fact they typically work for any accuracy measure which is strictly proper.[7]

Strict propriety is often justified by rational immodesty: rational agents should evaluate themselves to be doing best accuracy-wise.

Rational-Immodesty.
Every rational epistemic state takes itself to be doing best accuracy-wise.

And if we suppose that any probability function could be rational given the right circumstances; and evaluation is to be done by consideration of expected accuracy in those circumstances; then a measure of accuracy for the precise that obtains immodesty has to be strictly proper.[8]

Now, if we allow imprecise credences to be epistemic options, we would like to consider how good these imprecise credences are accuracy-wise. We know that .5 thinks it is better than any other *precise* credence, but perhaps it thinks some imprecise credence would be better, undermining its rationality. Also if any imprecise credences are to be rationally permissible, they should think that they are better than any other precise or imprecise credence.

There are a few recent papers which provide challenges for apply accuracy to the imprecise that can allow for immodesty. (Seidenfeld et al., 2012; Mayo-Wilson and Wheeler, 2016; Schoenfield, 2017; Chambers, 2008). These papers provide impossibility results for ways of measuring accuracy that might allow imprecise probabilities to be immodest. These results typically show that for any way of measure of accuracy (with a precise numerical score as the accuracy score for the imprecise credences) there will be two credal states which have the same accuracy-profile, i.e. the same accuracy score in each world. So the one

[6]Whilst this argument only motivates *weak* propriety, strict propriety is a consequence of weak propriety and strict truth-directedness (and additivity) (**?**).

[7]Satisfying some basic additional principles, in particular, continuity.

[8]In the presence of extensionality, we can formulate this as: for every probability space $\langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$, there is an isomorphic space $\langle \Omega', \mathcal{F}', \mathrm{Pr}' \rangle$ such that Pr' is rationally permissible (in the right circumstances). This is then compatible with further restrictions to probabilism. See Joyce (2009, Footnote 17). This is also then compatible with not using expectations to do evaluations in certain cases, such as those considered in Section 3.3 where some deference considerations are also used.

cannot think it is better than the other. Thus, one has to either reject that to be rational one should take oneself to be doing best at the pursuit of accuracy, or one has to reject imprecise probabilities as epistemic options.

There are various ways to respond to such arguments.[9] The way I will apply accuracy considerations to the imprecise avoids such impossibility results. In fact we will be able to obtain:

Imprecise-Immodesty.
Every set of probability functions $\mathbb{B}$ takes itself to be doing best accuracy-wise.

The main reason I avoid the impossibility results is that I do not directly measure the accuracy of an imprecise credence at a world. Instead accuracy measures only directly apply to the precise credences.

Consider an agent who has precise credence .5. How should .5 evaluate how well $\{.5\}$ or $\{.3, .8\}$ are doing at the pursuit of accuracy? It is only directly the precise credences, .5, .3 or .8 that get accuracy values. And taking expectations should also be done primarily just to the precise probabilities.

x	$\mathsf{Acc}(x, \mathsf{true})$	$\mathsf{Acc}(x, \mathsf{false})$	$\mathrm{Exp}_{.5}\mathsf{Acc}(.5)$
.5	.75	.75	.75
.8	.96	.36	.66
.3	.51	.91	.71

So when evaluating $\{.3, .8\}$, our agent will assign a set of epistemic values, $\{.66, .71\}$. We will not reduce this to a single numerical accuracy value. So our agent will consider $\{\mathrm{Exp}_{.5}\mathsf{Acc}(.3), \mathrm{Exp}_{.5}\mathsf{Acc}(.8)\} = \{.66, .71\}$. And she will compare this to $\mathrm{Exp}_{.5}\mathsf{Acc}(.5) = .75$, and will be able to conclude that .5 is preferable to $\{.2, .8\}$.

We are not assigning an expected epistemic value to $\{.3, .8\}$. Perhaps you respond: yes we are, it just happens to be a set-valued expected accuracy value. Whilst I would perhaps accept taking $\{.66, .71\}$ to be the expected accuracy value of $\{.3, .8\}$ according to .5, I note that it is not found by taking an expectation over the accuracy value of $\{.3, .8\}$ at the various worlds. We would usually define the expected accuracy of $\mathbb{C}$ as:

$$.5 \times \mathsf{Acc}(\mathbb{C}, \mathsf{true}) + .5 \times \mathsf{Acc}(\mathbb{C}, \mathsf{false}) \tag{1}$$

But this is not what we are doing here, $\mathsf{Acc}(\mathbb{C}, \mathsf{true})$ never appears in the calculations. Our picture of evaluation of goodness is redeveloped. Accuracy scores, and expectations, directly apply to the precise objects. Only at the end do we take sets. This is important, because there is coordination information as important.

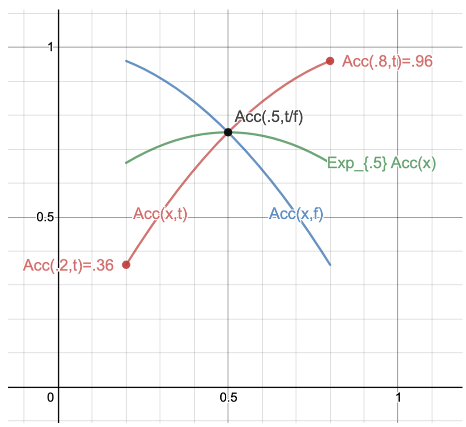
[9] A prominent alternative response is by **?**, who argues that we shouldn't be looking for a single measure of accuracy, but that different rational agents (with more or less precise credences) will use different accuracy measures, as one's accuracy measure encodes one's attitudes.

To push this further, you might continue to try to argue that what we are doing is taking expected value in the sense of Eq. (1), but where $\mathsf{Acc}(\{.2,.8\},\mathsf{true}) = \{.96,.51\}$ and $\mathsf{Acc}(\{.3,.8\},\mathsf{false}) = \{.36,.91\}$, so we should consider

$$.5 \times \{.96,.51\} + .5 \times \{.36,.91\} \tag{2}$$

Now, what is the result of this equation? What we want it to be is $\{.66,.71\}$, but to get this it is important that the .96 is linked with .36 (coming from .8) and .51 is associated with .91 (coming from .2). This information isn't encoded directly in $\mathsf{Acc}(\{.2,.8\},w)$ values. Indeed, if we associate .96 with .91 we would get something better than .75 which arises from credence .5.

This is closely related to Schoenfield (2017) who considers assigning set-valued accuracy values at worlds. She assumes a principle: if $\mathbb{C}$ has a guaranteed set-valued accuracy value, and so does $\mathbb{C}'$, then the comparison of $\mathbb{C}$ and $\mathbb{C}'$ is given just by comparing these guaranteed set-valued accuracy values, and this will hold whatever perspective one has, i.e. whether $\mathbb{C}$ or $\mathbb{C}'$ is doing the expectation. Whilst this of course sounds plausible, it is something I will reject. She compares [.2, .8] to .5.



Now, $\mathsf{Acc}([.2,.8],t) = [.36,.96]$ and $\mathsf{Acc}([.2,.8],f) = [.36,.96]$, so it seems that there is a guaranteed accuracy outcome of [.36, .96], so to evaluate how [.2, .8] compares to .5, which has an accuracy value of .75 we need to compare [.36, .96] to .75. Note that there is no 'perspective' here. What we want to say is that from .5's perspective, .5 looks best, and from [.2, .8]'s perspective [.2, .8] looks best. But since there are guaranteed accuracy-theoretic outcomes there is no room to say this. The way we avoid this conclusion is by dropping the first principle that says that if there is a guaranteed set-valued accuracy value that's what the expected value is. The reason is that we have to associate members of the set of accuracy values when true with members of the set of accuracy values when false. If we look at $\{\mathrm{Exp}_{.5}\mathsf{Acc}(x) \mid x \in [.2,.8]\} = [.66,.75]$ and compare that to just .75, we might have some reason to say that .75 is better, and thus .5 recommends itself over [.2, .8], whereas [.2, .8] will look at different expected values and compare those.

This is a general feature of imprecise probabilities: one should be wary about taking sets too early in a process. There are various arguments that we shouldn't consider an imprecise probability as a function from propositions to sets, but instead as a set of functions. Just looking at the imprecise probabilities for each proposition, $B(A) = \{b(A) \mid b \in \mathbb{B}\}$, does not encode all the information available in $\mathbb{B}$, for example it cannot encode comparative confidences (**?**, Example 2). This is also a lesson that a number of authors have taken dilation to show us (see Joyce, 2010). Dilation is not a failure of reflection when we consider the epistemic state not as a set-valued function but instead as a set of functions.[10]

Once we take into account this cross worldly structure we can claim that .5 thinks that $\{.5\}$ is doing better than $\{.2, .8\}$ accuracy-wise.

We would also then like to allow imprecise probabilities to be rationally permissible, and think that they are doing best accuracy-wise. What about an agent who adopts $\{.2, .8\}$? We should consider:

$$\{\mathrm{Exp}_b \mathsf{Acc}(\mathbb{C}) \mid b \in \mathbb{B}\} = \{\{\mathrm{Exp}_b \mathsf{Acc}(c) \mid c \in \mathbb{C}\} \mid b \in \mathbb{B}\}.$$

So in our case, we are comparing:

$$\begin{aligned}
\mathrm{Exp}_{\{.2,.8\}} \mathsf{Acc}\{.2, .8\} &= \{\mathrm{Exp}_{.2} \mathsf{Acc}\{.2, .8\}, \mathrm{Exp}_{.8} \mathsf{Acc}\{.2, .8\}\} \\
&= \left\{ \begin{array}{l} \{\mathrm{Exp}_{.2} \mathsf{Acc}(.2), \mathrm{Exp}_{.2} \mathsf{Acc}(.8)\}, \\ \{\mathrm{Exp}_{.8} \mathsf{Acc}(.2), \mathrm{Exp}_{.8} \mathsf{Acc}(.8)\} \end{array} \right\} \\
&= \{\{.84, .48\}, \{.48, .84\}\} = \{\{.48, .84\}\} \\
\mathrm{Exp}_{\{.2,.8\}} \mathsf{Acc}\{.5\} &= \{\mathrm{Exp}_{.2} \mathsf{Acc}\{.5\}, \mathrm{Exp}_{.8} \mathsf{Acc}\{.5\}\} \\
&= \{\{\mathrm{Exp}_{.2} \mathsf{Acc}(.5)\}, \{\mathrm{Exp}_{.8} \mathsf{Acc}(.5)\}\} \\
&= \{\{.75\}, \{.75\}\} = \{\{.75\}\}
\end{aligned}$$

Now we have to compare sets of sets. Which is better? I will argue that we should have $\{\{.48, .84\}\} \succ \{\{.75\}\}$, thus can say that $\{.2, .8\}$ thinks it is better than $\{.5\}$, indeed more generally, I argue that $\{.2, .8\}$ thinks it's better than any other set.

To develop this ordering, we go back to what it is representing and use an idea which is encoded in Imprecise Recommendation.

3 Imprecise Recommendation and its Accuracy-Theoretic Consequences

Imprecise Recommendation is the idea that what a set thinks is best is the set of what the individuals think are best. It can be spelled out a bit more precisely:

IMPRECISE RECOMMENDATION.
If each precise b thinks $\mathsf{rec}(b)$ to be uniquely best, then the imprecise object that $\mathbb{B}$ thinks is best is $\{\mathsf{rec}(b) \mid b \in \mathbb{B}\}$.

[10] Bradley (2015): "one lesson we should learn from dilation is that imprecise belief is represented by sets of functions rather than by a set-valued function"

This principle allows us to move from a notion of individual-recommendation to a notion of set-recommendation.

We try to motivate this principle by using the picture of a credal committee. Each committee member has an opinion about which other committee members are better or worse, and the committee comes together to decide which other committee the vote in.

Consider the following analogy (thanks to Julia Staffel!). Suppose a family is decorating their Christmas tree. Here is their policy for what the best Christmas tree decoration is, given the box of decorations they have. The best collection on the tree is just the collection of all the family's favourite decorations. If some member of the family thinks the angel is best, the angel will go on the tree, even if someone else doesn't like it. If a decoration is noone's favourite then it stays in the box.

An imprecise probabilities' opinion about which imprecise option is best is like this family's decoration policy.

When applying accuracy considerations in general we want to know not only which imprecise option is best, but more generally whether one options is better than another. This is analogous to our family ranking whether one choice of decorations is better than another choice.

If Katie thinks the angel is best, and Joe thinks the star is best, then the tree that just has the star will be incomparable to a tree that just has the angel. But

The family might also have more general opinions about when one decorated-tree is better than another. Sometimes trees are incomparable. A tree which just has one person's favourite will typically be incomparable to a tree just with someone else's. Each individual thinks that the best tree is the tree that just has her favourite decoration on it. But their opinions are brought together in a more subtle way, which

Stuff added here

To determine its consequences, then, we need to provide it with a notion of individual-recommendation. The principle of Imprecise Recommendation is quite general, but we'll specifically be applying it in the case where recommendation on the precise side is given by accuracy-theoretic considerations. Each of our different applications of the accuracy argument for the imprecise will involve a slightly different consideration here, so we move directly to considering the applications.

3.1 Imprecise Immodesty

Our accuracy measure being strictly proper gets us that each probability function thinks that it is doing best, accuracy-wise. At least when they 'think that' in accordance with expected utility theory, which we assume for now. We thus say $\mathsf{rec}(b) = b$ and can thus use Imprecise Recommendation to tell us that what the imprecise credence, $\mathbb{B}$, thinks is best is $\{\mathsf{rec}(b) \mid b \in \mathbb{B}\}$. But this is just $\mathbb{B}$. This thus allows us to immediately obtain Imprecise-Immodesty:

Imprecise-Immodesty.

Every set of probability functions $\mathbb{B}$ takes itself to be doing best accuracy-wise.

3.2 Imprecise Conditionalization

We now consider conditionalization.

There are different ways of giving accuracy arguments for this. Are we judging what credal state is best now that I know E but I haven't yet updated my credences, or what update strategy is best? Either way, the idea is: conditionalization is recommended by each probabilistic b; so an imprecise $\mathbb{B}$ will recommend conditionalizing each of its members. And this is basically what Imprecise-Conditionalization says.

3.2.1 Restricting Possibilities

The move to the imprecise setting is most natural using the approach of Leitgeb and Pettigrew (2010) for the precise notion of recommendation. A way to view their proposal maintains that the immediate, arational effect of learning E is that all $\neg E$ possibilities drop out from the agent's credal state. It is only once this has happened that the rational aspect of belief-revision can take place. The agent is left with her previous credences in the various possibilities compatible with E, which (remaining – as yet – unmodified) fail to add up to 1 and hence fail to constitute a probability distribution. She then uses these (non-probabilistic) credences to decide what credences to adopt, by maximizing expected accuracy. This means that she will adopt the credences that maximize expected accuracy, as calculated using her prior credences over the worlds consistent with E.

$$\mathrm{Exp}_b \mathsf{Acc}(c)\restriction_E = \sum_{w \in E} b(w) \times \mathsf{Acc}(c, w)$$

And, they show that what maximises this is the conditionalized credence.

So, each b recommends $b(\cdot \mid E)$. So, Imprecise Recommendation tells us that $\mathbb{B}$ will recommend $\{b(\cdot \mid E) \mid b \in \mathbb{B}\}$. This was exactly our desired Imprecise-Conditionalisation principle.

3.2.2 Update Strategies

This argument has been criticised for the picture of updating that it encodes and how these arational credences are used to evaluate options. An alternative is from Greaves and Wallace (2006) where our prior credences b evaluate various update strategies for what credence to adopt after learning one proposition from a partition. And they show that the strategy of updating by conditionalizing on the learned member of the partition is the optimal update strategy.

To apply this to the imprecise we have to think about what the right notion of an update strategy is in the imprecise setting. Imprecise-Recommendation can directly apply to tell us that an imprecise probability $\mathbb{B}$ recommends the set of precise update strategies: $\{\langle b(\cdot \mid E), b(\cdot \mid \neg E)\rangle \mid b \in \mathbb{B}\}$. But, it is not clear how to interpret this and whether that tells us anything about how an imprecise probability $\mathbb{B}$ should be updated. Instead, then we want strategies that say: if you learn E adopt thus-and-so imprecise probability, and if you learn $\neg E$ adopt thus-and-so imprecise probability. If we turn the above set into such a strategy by just taking its projections, we will obtain: $\langle \{b(\cdot \mid E) \mid b \in \mathbb{B}\}, \{b(\cdot \mid \neg E) \mid b \in \mathbb{B}\}\rangle$. There's certainly more to be said here. But I am just going to move on now.

Before moving to Imprecise Probabilism, which is a little more subtle, I want to mention one other, non-standard, application, which shows something new arising from Imprecise Recommendation.

3.3 Undermining Probabilities

There are some particular cases in the literature that form a challenge for rationality, but where moving to imprecise probabilities and using Imprecise Recommendation results in a solution that is intuitive. This is interesting because it shows something new that Imprecise Recommendation can bring to the table, in addition to its allowing for justification of otherwise accepted principles of rationality.

Typically, accuracy considerations are applied in cases where the opinions that one adopts in a proposition has no direct impact on the world and doesn't itself create evidence. But once that assumption is dropped some challenges arise.

Consider *Passport*:

If I have degree of belief $\geqslant 1/2$ that I'll forget my passport then I won't.
If I have degree of belief $< 1/2$ that I'll forget it, then I will.

Suppose our agent knows this about herself. If she starts off with degree of belief $1/4$ that she'll forget it, then by reflecting on her opinion, she knows that she'll actually forget it, so should adopt degree of belief 1. But adopting 1 then recommends adopting 0. In this case every credal state is undermining in this way: recommending some other credal state.

However, we can find an imprecise credal state that is self-recommending: $\{0, 1\}$. 0 recommends 1; 1 recommends 0. So, using our idea encoded in Imprecise Recommendation, $\{0, 1\}$ recommends $\{\mathsf{rec}(0), \mathsf{rec}(1)\} = \{1, 0\} = \{0, 1\}$. So $\{0, 1\}$ is self-recommending.

This provides an interesting account of such cases.[11] For further discussion of the application of Imprecise Recommendation to such cases, see Campbell-Moore (ms), we now return to the main theme, how Imprecise Recommendation can be used to justify usual epistemic requirements for the imprecise.

3.4 Imprecise Probabilism

A very desirable principle is:

[11] One can also formally connect such scenarios to the liar paradox and such an account can be seen as an analogue of a supervaluational logic based Kripkean account for truth (?).

Imprecise-Probabilism.
One should have a credal set $\mathbb{B}$ that only contains probability functions.

The argument for probabilism on the precise side is a dominance argument: If b is non-probabilistic, there is some probabilistic c that is more accurate whatever the world is like.

This, then, doesn't directly talk about what these (possibly non-probabilistic) credences think are best. So it's not immediately obvious how we use it with Imprecise Recommendation which tells us what the imprecise credences think are best once we plug in a notion of what the precise credences think are best.

We can, however,

We could, of course, encode such recommendation talk in. We could say:

- If b is non-probabilistic and c accuracy-dominates b, then b recommends (possibly non-uniquely) c.

Which would be added to the usual recommendation for the precise probabilities

- If b is probabilistic and then it recommends the c that maximises $\mathrm{Exp}_b\mathsf{Acc}(c)$ (namely, itself).

Imprecise Recommendation tells us that $\mathbb{B}$ recommends $\{\mathsf{rec}(b) \mid b \in \mathbb{B}\}$. For each probabilistic b, it will recommend itself; but non-probabilistic b's will recommend some alternative credence c. So if $\mathbb{B}$ contains some non-probabilistic b; then $\mathbb{B}$ will not recommend itself, but will recommend some alternative set where b is replaced with c.

There's a slight subtlety that we're passing over: the non-probabilistic members needn't uniquely recommend someone, but Imprecise Recommendation only applied when precise credences had unique recommendations. But that was just to keep it simple. We will extend Imprecise Recommendation to:

Imprecise Recommendation (Extended).
(Suppose each $\succ_b$ is chain-complete.[12]) If

- Every $b \in \mathbb{B}$ has some $c \in \mathbb{C}$ which is amongst its recommendations.
- Every $c \in \mathbb{C}$ is a recommendation of some $b \in \mathbb{B}$

Then $\mathbb{C}$ is amongst $\mathbb{B}$'s recommendations.

Using Imprecise Recommendation (Extended), our argument here will still work: the non-probabilistic $b \in \mathbb{B}$ will not be contained in any set that's recommended by $\mathbb{B}$ because it's not recommended by anyone.

We might, however, hope to get a *dominance* argument for Imprecise-Probabilism where we don't have to talk about what the non-probabilistic b recommend. How might a dominance argument go?

[12] That is, every c is either 'amongst b's recommendations' or can be improved on by some such c', that is, a c' which cannot itself be improved on. Or, equivalently, that there is no continual sequence of improvements with no maximal member.

Well, we consider the accuracy value of $\mathbb{C}$ at w as:

$$\mathsf{Acc}(\mathbb{C}, w) = \{\mathsf{Acc}(c, w) \mid c \in \mathbb{C}\}.$$

We know from the usual precise result that: If $\mathbb{B}$ contains some non-probabilistic b, then there is some dominator, $\mathsf{dom}(b)$ which is more accurate in each world. If b is probabilistic, so has no such dominator, then we let $\mathsf{dom}(b) = b$.

Consider $\{\mathsf{dom}(b) \mid b \in \mathbb{B}\}$. Compare:

$$\begin{array}{l} \{\quad \mathsf{Acc}(b, w) \quad \mid b \in \mathbb{B}\} \\ \{\mathsf{Acc}(\mathsf{dom}(b), w) \mid b \in \mathbb{B}\} \end{array}$$

We know $\mathsf{Acc}(b, w) \leqslant \mathsf{Acc}(\mathsf{dom}(b), w)$; and sometimes $<$. But the challenge is that these are supposed to be unstructured sets of numerical values. The explanation for why one is better than the other involves this pairing: knowing that $\mathsf{dom}(b)$ should be paired with b. But these should be thought of simply as sets and this fact of how they are to be paired up is not one that we can use.

4 Extending and justifying Imprecise Recommendation

4.1 The idea

Imprecise Recommendation focused on the question of which set is evaluated as *most* accurate. But in general, accuracy considerations allow us to see which belief-objects are better or worse than which others. We want to develop an account which provides us with an ordering of sets, which will be grounded in the individual's orderings over individuals.

To give this story, we will use the credal-committee analogy. If you have an imprecise credence we can think of you as a group of credal-committee members. Each comittee member has an opinion about which object is best, or, more generally, a ranking of certain objects; typically these objects will be other precise credences. The committee comes together and discusses whether they think that this set is better than this other set, based on the opinions of the committee members.

We will focus on the question of how the imprecise probability ranks other imprecise probabilities, i.e. whether $\mathbb{C}' \succ_{\mathbb{B}} \mathbb{C}$. To extend this to ranking of other imprecise credal objects is straightforward. So the individuals start with orderings over the precise objects: $c' \succ_b c$. This might come from expected accuracy considerations, $c' \succ_b c$ iff $\mathrm{Exp}_b\mathsf{Acc}(c') > \mathrm{Exp}_b\mathsf{Acc}(c)$, or if b is non-probabilistic, at least her ranking will follow dominance considerations, if c' dominates c then $c' \succ_b c$. The group of $b \in \mathbb{B}$ then come together and discuss whether $\mathbb{C}' \succ_{\mathbb{B}} \mathbb{C}$ based on their rankings of the individuals in $\mathbb{C}'$ and $\mathbb{C}$.

But we need to be careful about how to do this: Consider b_1 and b_2, and suppose they think themselves are optimal. b_1 will evaluate b_1 to be better than

b_2 ($b_1 \succ_{b_1} b_2$), and she will thus have the opinion that $\{b_1\} \succ_{b_1} \{b_1, b_2\}$. But b_2 disagrees; she thinks that $\{b_1\} \prec_{b_2} \{b_1, b_2\}$, after all, $b_2 \succ_{b_2} b_1$. So in what sense is $\{b_1, b_2\}$ the right compromise for the group rather than $\{b_1\}$? In applying accuracy for the imprecise, Levinstein (2019) allows that either $\{b_1\}$ or $\{b_1, b_2\}$ is recommended by the set, thus allowing that imprecise probabilities think that they are among the optimal. I'm aiming for the stronger conclusion that $\{b_1, b_2\}$ uniquely evaluates itself to be optimal. This can get us stronger conclusions such as that updating by conditionalization is rationally required rather than merely permitted.

Consider again:

$$\{b_1\} \succ_{b_1} \{b_1, b_2\}$$
$$\{b_1\} \prec_{b_2} \{b_1, b_2\}$$

But we want to overall say that $\{b_1, b_2\}$ is preferred by the group, $\{b_1, b_2\}$, to $\{b_1\}$? We think that there is a different strength of opinion encoded in the two preferences above. b_2's opinion is a strong, unmovable demand, whereas b_1's opinion is weaker. Our committee member's first priority is to get their favourite person in the group. The set $\{b_1\}$ fails this from b_2's perspective. There is no way he can get on board with supporting $\{b_1\}$. Whereas $\{b_1, b_2\}$ at least has b_1's favourite member. Sure she thinks that $\{b_1\}$ is better, but she's happy to compromise on this in order to come to an agreement with her committee members.

We will encode these different strengths of opinions: $\gg$ is a strong preference that the individuals are unwilling to compromise on. These are unmovable demands of our members. $\rhd$, on the other hand, is the member's preferences that they are open to compromising on. We then have the preferences:

$$\{b_1\} \rhd_{b_1} \{b_1, b_2\}$$
$$\{b_1\} \ll_{b_2} \{b_1, b_2\}$$

Since the preference $\gg$ takes priority, we thus get that the committee's joint preference has $\{b_1, b_2\} \succ_{\{b_1, b_2\}} \{b_1\}$.

We need to describe how to construct such $\gg_{\mathbb{B}}$ and $\rhd_{\mathbb{B}}$. Then the overall $\succ_{\mathbb{B}}$ first checks for any $\gg_{\mathbb{B}}$ preferences, then breaks any ties using $\rhd_{\mathbb{B}}$.

We want to do all this in a way that obtains Imprecise Recommendation as a consequence. We will also want to obtain:

Imprecise Recommendation (Extended).
If

- Every $b \in \mathbb{B}$ has some $c \in \mathbb{C}$ which is amongst its recommendations.
- Every $c \in \mathbb{C}$ is a recommendation of some $b \in \mathbb{B}$

Then $\mathbb{C}$ is amongst $\mathbb{B}$'s recommendations.[13]

[13] It's still not clear to me whether we want this to hold in general or only when $\prec_b$ is chain complete, i.e. that there's no continual sequence of improvements with no maximal member.

which is important to derive Imprecise Probabilism, when not all b have unique recommendations as the non-probabilistic b only have ordering-opinions in accordance with dominance.

We want to argue for Imprecise Recommendation (Extended) based on the story of the group coming together and voting. The argument will follow the following lines:

We start by considering all $b \in \mathbb{B}$'s unmovable demands, which will be encoded by $\gg_b$. If b has none of her recommended members in the $\mathbb{C}$, then she is going to refuse to accept $\mathbb{C}$; she will demand that one of her recommended members be included in $\mathbb{C}$. The other b' think that this will make the set worse, however, it will not destroy their primary goal: to get their recommended members in the set. So they will accept b's demand to include one of her recommended members.

Now, the sets that are acceptable according to all $b \in \mathbb{B}$'s unmovable demands will be those that include some recommended member for each $b \in \mathbb{B}$. But this can include some additional members who are not recommended by anyone. Everyone will agree that the set can be made better by removing this c who is not recommended by anyone. So the sets that are in the end recommended are those that only include c who are recommended by some b.

We need to develop a nice definition of the orderings that allow us to make this argument.

4.2 Defining the ordering

Here's a new new idea:
Suppose we're in the case where each b in fact has utility values of the various c ($U_b(c) := \mathrm{Exp}_b \mathsf{Acc}(c)$).
We compare $\mathbb{C}$ to $\mathbb{C}'$ by thinking: she'll get one of the members of the set, but doesn't know which. And how to decide under uncertainty?

- use maximax — that gives us $\gg_b$:
  - The best-case outcome from $\mathbb{C}'$ is higher than that of $\mathbb{C}$.
- maybe something with expected utility will gives us $\rhd_b$. However, what distribution over the cs?
  - If for every distribution over the probabilities,

$$\mathrm{Exp}_{p(\cdot|c\in\mathbb{C}')}U(c) > \mathrm{Exp}_{p(\cdot|c\in\mathbb{C})}U(c)$$

- This will possibly ultimately get us that a set recommends its convex closure...

Of course these need to be worked out with details,
and it's only going to work in special cases, but...

So, how to define these orderings in a way that allows us to obtain Imprecise Recommendation (Extended)? We should also be able to ensure that these are strict partial orders (i.e. irreflexive and transitive), and a desirable property is that by taking $\succ_b$ equivalences, we shouldn't break any $\gg_b$ or $\rhd_b$ orderings.

The following definition will allow us to obtain Imprecise Recommendation, but not Imprecise Recommendation (Extended): [NB: this whole thing is still sketchy. I've got various attempts at trying to get Imprecise Recommendation

(Extended), something should work, but this was a quick fall-back, that I think (/hope) does work at least for Imprecise Recommendation.]

Definition 4.1.

- If

  $$\exists c'_* \in \mathbb{C}' \text{ such that } c'_* \succ_b c \text{ for all } c \in \mathbb{C}$$

  then $\mathbb{C}' \gg_b \mathbb{C}$. (And if $\succeq$ then $\ggeq$.)

- If $\mathbb{C}' \gg_b \mathbb{C}$ for some $b \in \mathbb{B}$ and $\mathbb{C}' \ggeq_b \mathbb{C}$ for all $b \in \mathbb{B}$, then $\mathbb{C}' \gg_{\mathbb{B}} \mathbb{C}$.

- If

  $\mathbb{C}' \subset \mathbb{C}$ such that all $c \in \mathbb{C} \setminus \mathbb{C}'$ have some $c' \in \mathbb{C}'$ with $c' \succ_b c$.

  this needs improving to get $\mathbb{C}' \gg_b \mathbb{C} \implies \mathbb{C} \rhd_b \mathbb{C}$ and to be more natural.

  Or, by closing this under taking equivalents (which is thus equivalent to the previous when $c \sim_b c' \implies c = c'$):

  * every $c' \in \mathbb{C}'$ has some $c \in \mathbb{C}$ with $c \sim_b c'$
  * there is some $c \in \mathbb{C}$ with no $c' \in \mathbb{C}'$ with $c \sim_b c'$
  * every $c \in \mathbb{C}$ has some $c' \in \mathbb{C}'$ with $c' \succeq_b c$

  then $\mathbb{C}' \rhd_b \mathbb{C}$.

- If $\mathbb{C}' \rhd_b \mathbb{C}$ for all $b \in \mathbb{B}$ then $\mathbb{C}' \rhd_{\mathbb{B}} \mathbb{C}$.

- If $\mathbb{C}' \gg_{\mathbb{B}} \mathbb{C}$, or $\mathbb{C}' \approx_{\mathbb{B}} \mathbb{C}$ but $\mathbb{C}' \rhd_{\mathbb{B}} \mathbb{C}$, then $\mathbb{C}' \succ_{\mathbb{B}} \mathbb{C}$.

Proposition 4.2. *$\gg_b$ and $\rhd_b$ are irreflexive and transitive. So are $\gg_{\mathbb{B}}$ and $\rhd_{\mathbb{B}}$. I don't think $\succ_{\mathbb{B}}$ is necessarily transitive (which should of course be fixed)*

Remark. $\gg_b$: The restricted choice of quantifier order $\exists c' \forall c$ in $\gg_b$ allows it to be irreflexive. $\forall c \exists c'$ would not be irreflexive. For example $\{c_1, c_2, \ldots\}$.

Also note that this definition of $\gg_b$ is much too strong for what we'll want in the end, there's no hope to use it to get Imprecise Recommendation (Extended).[14]

$\gg_{\mathbb{B}}$: The choice of $\mathbb{C}' \ggeq_b \mathbb{C}$ rather than $\mathbb{C}' \not\ll_b \mathbb{C}$ allows $\gg_{\mathbb{B}}$ to be transitive. The difference this makes: improvement in one component and incomparability in another isn't enough for an improvement. We can see this in the notion of weak dominance. Suppose G and H are incomparable. Consider:

	A	B	C
w_1	G	G-\$1	H
w_2	G	H+\$1	H

[14] What we should try to use:

$$\forall c \in \mathbb{C}\ \exists c' \in \mathbb{C}'\ c' \succeq_b c \text{ and } \forall c' \in \mathbb{C}'\ \exists c \in \mathbb{C}\ c' \succeq_b c$$
$$\text{and } \exists c'_* \in \mathbb{C}'\ \exists c_* \in \mathbb{C} \begin{pmatrix} & c'_* \succ_b c_* \\ \wedge & \neg \exists c \in \mathbb{C}\ c \succeq_b c'_* \end{pmatrix}$$

Suppose we defined weak dominance with $\not<$, then we would have: A weakly dominates B (it's better in w_1 and incomparable in w_2), and B weakly dominates C, but A is simply incomparable with C.

$\succ_{\mathbb{B}}$: To ensure that $\succ_{\mathbb{B}}$ is transitive we should extend $\rhd_b$ to a transitive relation such that $\mathbb{C}' \gg_b \mathbb{C} \implies \mathbb{C}' \rhd_b \mathbb{C}$, which is something we'd independently want anyway then we could drop the "$\rhd$" and just call this "$\succ$", with $\gg$ simply being the strong preferences.

$\rhd_b$: Some desiderata/notes for ideas to extend it to get $\gg \implies \rhd$ (think of the numbers as utility values):

- Removing not-top improves: $\{3, 2, 1\} \prec \{3, 1\}$.
- Improving the top improves ($\gg$): $\{3, 1\} \succ \{2, 1\}$. Also $\{\text{Integers}\} \cup \{\infty\} > \{\text{Integers}\}$.
- A definition like $\exists c'_*$ which lies $\geqslant$ everything, and $>$ something would allow that $\{2, 1\} \not\succ \{2, 1\}$: 2 is better than 1.
- Simply requiring that what it improves on is not in $\mathbb{C}$ won't work:

$$\{\text{Even integers}\} \cup \{\infty\} \not\succ \{\text{Odd integers}\} \cup \{\infty\}$$

Proposition 4.3. *Suppose each b has a unique* $\mathsf{rec}(b)$ *($\succ_b$ is complete). Then* $\{\mathsf{rec}(b) \mid b \in \mathbb{B}\}$ *is uniquely $\succ_{\mathbb{B}}$ maximal.*

Proof. One can check:

1. If $\mathsf{rec}(b) \notin \mathbb{C}$ and $\mathsf{rec}(b) \in \mathbb{C}'$ then $\mathbb{C}' \gg_b \mathbb{C}$.
   If $\mathsf{rec}(b) \in \mathbb{C}$ and $\mathsf{rec}(b) \in \mathbb{C}'$ then $\mathbb{C}' \approx_b \mathbb{C}$.
2. So, if $\mathbb{C} \not\supseteq \{\mathsf{rec}(b) \mid b \in \mathbb{B}\}$, then $\{\mathsf{rec}(b) \mid b \in \mathbb{B}\} \gg_{\mathbb{B}} \mathbb{C}$.
   and if $\mathbb{C} \supseteq \{\mathsf{rec}(b) \mid b \in \mathbb{B}\}$, then $\{\mathsf{rec}(b) \mid b \in \mathbb{B}\} \approx_{\mathbb{B}} \mathbb{C}$.
3. If $\mathbb{C} \supset \{\mathsf{rec}(b) \mid b \in \mathbb{B}\}$ then $\{\mathsf{rec}(b) \mid b \in \mathbb{B}\} \rhd_b \mathbb{C}$ for all b,
   so $\{\mathsf{rec}(b) \mid b \in \mathbb{B}\} \rhd_b \mathbb{C}$.
4. Thus, if $\mathbb{C} \neq \{\mathsf{rec}(b) \mid b \in \mathbb{B}\}$ then $\{\mathsf{rec}(b) \mid b \in \mathbb{B}\} \succ_{\mathbb{B}} \mathbb{C}$. □

This all becomes a little more fiddly and delicate to obtain Imprecise Recommendation (Extended). This will be done asap.

One final note: we might have hoped for our orderings is that by improving all members, the overall set cannot be made worse. But this is incompatible with our criterion of $\rhd_b$: Consider $\{c_0, c_1, \ldots\}$ with $c_0 \succ_b c_1 \succ_b c_2 \ldots$. Then using our definition of $\rhd_b$, $\{c_0, c_2, c_3, \ldots\} \rhd_b \{c_0, c_1, c_2, c_3, \ldots\}$. But by mapping $c_0 \mapsto c_0$ and $c_n \mapsto c_{n-1}$ for $n \geqslant 2$, which is a (weak-)improvement to each member, we get from $\{c_0, c_2, c_3, \ldots\}$ to $\{c_0, c_1, c_2, c_3, \ldots\}$. So it shouldn't be that this latter set is worse than the former.

4.3 Interpreting our Imprecise Probabilities

So, we have given some definitions which try to spell out a picture on which Imprecise Recommendation is plausible. We used an analogy with a group making a decision. It is still unclear whether the use of Imprecise Recommendation assumes that we have a particular interpretation of the imprecise probabilities. A simply supervaluational picture, there's nothing more than what the individuals agree on, doesn't have the complexity to obtain Imprecise Recommendation, it merely obtains a weak form of it as is used in Levinstein (2019). A picture where they're simply a different kind of opinion that should be treated on a parallel with precise probabilities wouldn't motivate the idea that ultimately accuracy theoretic considerations only directly apply to the precise, with the imprecise probabilities' opinion supervening on these. We have used a kind of mixed picture of what imprecise probabilities are to justify this. Thus, we are perhaps assuming a slightly complicated picture of what imprecise probabilities are in order to justify it. It is still open what exactly this interpretation is, and how important it is to our adoption of Imprecise Recommendation.

5 Conclusions and Further Thoughts

There are still gaps and further work to develop and justify the proposal. The imprecise ranking we've given is still quite sparse and it should be filled out with more conditions. And at the end of Section 4 we mentioned that we are perhaps assuming a slightly complicated picture of what imprecise probabilities are in order to justify it. But the proposal is more powerful than one might have thought, and seems intuitively quite plausible, at least in its Imprecise Recommendation form. We thus think it is interesting and worth further investigation.

Of course, there are objections one might make to Imprecise Recommendation. One such objection would argue that there will be certain desirable/undesirable features of imprecise probabilities that aren't reducible to features of the precise ones (Moss). That seems right. So then what is recommended similarly might not be reducible to the precise recommendations. I'm not quite sure what to say about this. There are certain global features that could be added to Imprecise Recommendation: we might allow a set to recommend the *closure* of what the individuals recommend (see **??**). Perhaps there is some systematic split where some global constraints are OK, so long as they supervene on the set of individual recommendations. But we can't have anything-goes, because that would directly result in revenge problems in the undermining scenarios applications. Will that capture the examples that Moss is thinking about?

There is also more to do on investigating the consequences and applications of Imprecise Recommendation. For example, in aggregating imprecise probabilities. Konek (2019) has recently proposed using accuracy in the imprecise setting for justifying aggregation methods. This would be similarly motivated but a very different kind of instantiation of Konek's ideas. What would it result in? On the precise side we get accuracy arguments for linear pools. On the imprecise side, then, perhaps we get an argument that it should be contained in the convex hull of the union of the $\mathbb{B}_i$s. Can we get more than that?

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A Aggregation

A la Jason, aggregate with accuracy.

Suppose you have $\mathbb{B}_i$.

Usual result, if c is not a linear pool of b_i, then there is some c' which is a linear pool which each b_i expect to be accuracy improvement.

Now, each $\mathbb{B}_i$ has its own orderings of the IPs $\mathbb{C} \succ_{\mathbb{B}_i} \mathbb{C}'$ according to expected accuracy considerations as above. What can we say about which $\mathbb{C}$ are non-dominated from a pooling perspective, i.e. which $\mathbb{C}$ are such that there is no $\mathbb{C}'$ where $\mathbb{C} \prec_{\mathbb{B}_i} \mathbb{C}'$ for all i?

Conjecture A.1. *If $\mathbb{C}$ is a subset of the convex hull of the union of the $\mathbb{B}_i$s, then it is dominated.*

Proof Attempt. Suppose $c \in \mathbb{C}$ is not in the convex hull of the union of the $\mathbb{B}_i$s.

Then it is not in the convex hull of any $\langle b_1, \ldots, b_n \rangle \in \mathbb{B}_1 \times \ldots \times \mathbb{B}_n$. Thus for each $\langle b_1, \ldots, b_n \rangle \in \mathbb{B}_1 \times \ldots \times \mathbb{B}_n$ there is some $c_{\langle b_1,\ldots,b_n \rangle}$ such that $\mathrm{Exp}_{b_i}\mathsf{Acc}(c_{\langle b_1,\ldots,b_n \rangle}) > \mathrm{Exp}_{b_i}\mathsf{Acc}(c)$.

Consider $\mathbb{C}'$ which replaces c by the collection of $c_{\langle b_1,\ldots,b_n \rangle}$. Noone thinks that c is better than all $c_{\langle b_1,\ldots,b_n \rangle}$, after all, b_i thinks that $c_{\langle b_1,\ldots,b_n \rangle}$ is better.

So $\mathbb{C} \not\prec^*_{\mathbb{B}_i} \mathbb{C}'$.

Now consider their weak preferences:

Consider $\mathbb{C} \cup \{c_{\langle b_1,\ldots,b_n \rangle} \mid b_i \in \mathbb{B}_i\} \cup \{c^*\}$ and $\mathbb{C} \cup \{c_{\langle b_1,\ldots,b_n \rangle} \mid b_i \in \mathbb{B}_i\}$. There is no $\succ^*_{\mathbb{B}}$ preferences between these two sets. But: each b_i^* has a weak preference for $\mathbb{C} \cup \{c_{\langle b_1,\ldots,b_n \rangle} \mid b_i \in \mathbb{B}_i\}$, after all, there's something there, namely any $c_{b_1,\ldots,b_i^*,\ldots b_n}$.

BLAH