

# Undermining Scenarios and Determinate Recommendations

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## 1 Introduction

There are certain scenarios which lead to seeming epistemic dilemmas, for example:

### Ring Toss

You are playing ring toss at the carnival and are about to throw your ring at the peg. If your credence that you'll get it on the peg is  $\geq 0.5$ , then you'll be tense and miss. If your credence is not  $\geq 0.5$ , then you'll be relaxed and make it. And you know this about yourself. What is epistemically required of you?

*Closely related to Archer case in Joyce, 2018 or Basketball in Caie, 2013.*

A case like Ring Toss is problematic because every credence undermines its own adoption. Suppose you have adopted credence 0.2. Then you know that you will successfully get the ring on the peg. So perhaps it is appropriate to say that credence 1 is epistemically recommended by your credence of 0.2; the credence of 0.2 undermines its own rational adoption. However, were you to have credence 1, then you'd know that you'll fail. So we might conclude that adopting credence 1 recommends adopting credence 0, also undermining its own adoption too. Similarly, any credence value you might adopt would undermine its rational adoption.

Following this line of thought, it seems, then cases like Ring Toss result in epistemic dilemmas where there are no credences you could adopt which would be rational. This is the position argued for in Konek and Levinstein (2019) and Joyce (2018). There are alternatives offered by Pettigrew (2018), Caie (2013) who say that the way I just spelled out recommendation and underminingness aren't ultimately relevant for rationality judgements. Their position avoids an epistemic dilemma but results in a picture of rationality which is unattractive. For example, it loses intuitive ties between rationality and evidence.<sup>1</sup> I offer a proposal that is similar in spirit and motivation to that of Konek and Levinstein and Joyce.

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<sup>1</sup>This rejects all intuitive rationality constraints outright. (Carr, 2017, Campbell-Moore, 2015).

This recommendation notion will be key to our rationality considerations, so we obtain many desirable features of their account, such as a close tie between rationality and following one's evidence; but we nonetheless avoid the epistemic dilemma.

The key to our proposal is to consider that one's credence may be an indeterminate matter. In our account, it is one's indeterminate state that is judged for rationality. When one's credences are indeterminate, it may be indeterminate whether something is recommended. We propose rationality constraints on one's indeterminate credal state by asking, for example, whether one determinately satisfies determinate recommendations.

In the case of Ring Toss, we consider being indeterminate between the two extremal values of 0 and 1. It is then determinate that either 1 or 0 is recommended, but indeterminate which. This epistemic state thus determinately satisfies any determinate recommendations and will be proposed as rational. The epistemic dilemma has thus been avoided.

If one makes the additional assumption that one's credence is determinately adopted, then our constraints exactly agree with the judgements of Joyce and Konek and Levinstein. However, we drop this requirement that one's epistemic state be a determinate matter and we move the focus from recommendations to determinate recommendations. This allows us to avoid the epistemic dilemmas.

The formal background for this proposal is from relating such cases to the Liar paradox and implementing work accounting for the liar paradox, which is a sentence:

Liar: Liar is not true.

In particular, this account is motivated by a supervaluational variant of the prominent account developed in Kripke (1975), using some of the interpretation from McGee (1989).<sup>2</sup> In this, one considers truth to be vague, or indeterminate. Due to the liar sentence, we cannot have  $\varphi$  is true iff  $\text{Tr}^\Gamma \varphi^\neg$  is true (without giving up classical logic). We therefore also cannot have that it is determinate that  $\varphi$  is true iff  $\text{Tr}^\Gamma \varphi^\neg$  is true. However, we can obtain "narrow-scope" variants such as: if  $\varphi$  is determinately true, then so is  $\text{Tr}^\Gamma \varphi^\neg$ . McGee (1989, 1990) argues that it is such principles which are important for an adequate account of truth.<sup>3</sup>

Similarly here, we cannot maintain that it is determinate that one's credences follow the recommendations. That is the challenge offered by cases such as Ring Toss. But we can obtain closely related constraints such as one's credences determinately follow any determinate recommendations. We propose that these sorts of constraints are the ones that are important for judging one's rationality. In this paper, we will present these "narrow scope" constraints and investigate the resultant picture of rationality. We propose our account to be an attractive general account of rationality in indeterminate settings, and it allows epistemic dilemmas in cases like Ring Toss to be avoided.

<sup>2</sup>Campbell-Moore (forthcoming) can be consulted for further comments on the connections and formal account, though there are a few differences between the setup of that paper and what is proposed here.

<sup>3</sup>McGee in fact talks about *definite* truths.

## The paper is structured as follows:

In section 2 we discuss the recommendation notion applying to credal states, where what is recommended depends on the credal state adopted.

In section 3 we talk about indeterminate credences. We take it that the recommendation notion directly applies to precisifications of one's indeterminate state but that it is one's indeterminate state that is judged for rationality. We propose two rationality constraints on one's indeterminate credal state:

- Recommendation Constraint: Every determinate recommendation should be determinately satisfied.
- Acceptability Constraint: Every determinately satisfied property should be determinately acceptable.

In section 4 we discuss the consequences of applying these constraints in various cases and we discuss the various features that arise. We are able to obtain “narrow-scope” variants of the various desirable principles. For example, we maintain that if something is determinately part of your evidence, then you should determinately have credence 1 in it.

There are two main remaining challenges when epistemic dilemmas might rearise. Firstly, in section 5.1, I note that to avoid epistemic dilemmas, we might need to move to a slightly more complicated picture where we focus directly on the definite features of one's credence and there may be no precisifications whatsoever. Secondly, in section 5.2, I discuss a revenge challenge: Maybe what happens depends on my *indeterminate* credences in an undermining way. In fact, Recommendation and Acceptability Constraint may still be satisfied in such a case. The case highlights, though, that the move to indeterminacy wasn't the only important move, it was also important that we move from looking at recommendations to determinate recommendations.

## 2 Recommendations

There are various cases closely related to Ring Toss. For example:

### Extremal Bad Navigator

You've come to a crossroads and are wondering whether you need to turn left or right to get to your hotel. You know you're an extremely bad navigator. In particular, if you adopt credence  $\geq 0.5$  that it's to the left, then you know for sure that it's to the right, and if you adopt credence  $< 0.5$  that it's on the left, then you know for sure that it's on the left. (Related to an example in Egan and Elga, 2005)

If you have credence  $\geq 0.5$ , then you know it's to the right, so credence 0 is epistemically recommended. If not, then you know it's to the left and credence 1 is recommended.

What's similar in the cases of Bad Navigator and Ring Toss is this recommendation notion. They have very different phenomena underlying this: in the case of Ring Toss, your credence had causal power, whereas for Bad Navigator, it simply generates additional evidence. Our general account of what is rational depends only on this recommendation notion. It will then apply to Extremal

Bad Navigator exactly as it applies to Ring Toss. There will also be further cases we will mention in sections 4.2 and 4.3 which result in the same notion: a self-referential sentence, **CredLiar**, which is more obviously closely related to the liar paradox, and a case closer to examples considered in Greaves (2013) where adopting a credence generates information about the chances.

We assume that we have a recommendation notion for each case under consideration and will not say much more about it. Formally, this *recommendation notion takes as input a credence function – the one that has been adopted – and returns a collection of the credence functions that are then (epistemically) acceptable*.

When posing our rationality constraints we will focus not directly on which credences are acceptable but instead what properties of one’s credences are recommended or not. (Properties are understood extensionally as characterised by a set of credence functions.)

- A property of one’s credence is *recommended* if every acceptable credence has that property.
- A property of one’s credence is *acceptable* if some credence with that property is acceptable.<sup>4</sup>

These notions are dependent on the adopted credal state. ‘Recommended’ can be understood by analogy to ‘required’, and ‘acceptable’ by analogy to ‘permitted’. The reason we do not simply use the terms ‘required’ and ‘permitted’ is that we will take epistemic requirements to apply to one’s indeterminate credal state, whereas this recommendation notion governs the precisifications of one’s indeterminate credal state. However, in our general account, they will line up when your epistemic state is determinately adopted.

Our general account doesn’t depend on any details of what is acceptable, it simply takes the recommendation notion as an input to the rationality judgements (which will be governed by Recommendation and Acceptability Constraint). However, what the resultant picture of rationality looks like will depend on how one has implemented this recommendation notion. For example which principles one took to underlie the recommendation notion will alter which principles one’s indeterminate credences must satisfy to be rational.

One can encode your favourite principles of rationality as principles regarding what is recommended. We can, for example, encode any principles of the form of “follow your evidence” as principles about recommendation. Alternatively, one might consider the recommendation notion as grounded in other considerations such as maximising the accuracy of one’s credences, where we would say that a particular credence is acceptable iff it is not accuracy-dominated, or maybe iff it maximises estimated accuracy. The implementation should follow Konek and Levinstein (2019), Joyce (2018) rather than the “consequentialism” of Pettigrew (2018), Caie (2013).<sup>5</sup>

The cases of Ring Toss and Extremal Bad Navigator result in the same recommendation notion. With this recommendation notion, no credence is

<sup>4</sup>Note that these are duals: A property is acceptable iff its negation is not recommended.

<sup>5</sup>Though there is nothing stopping one applying our account with the notion of recommendation given by the consequentialist implementation. In that case, what is recommended is not a function of what credences have been adopted and no epistemic dilemma appears on the credence-level. So our account is not required to avoid epistemic dilemmas. I nonetheless would apply our considerations to understand rationality when one’s credences are indeterminate.

compatible with its own recommendations. That is, no credence is acceptable according to itself. So we are in epistemic-dilemma territory. However, we consider moving to a framework where one's credences may be indeterminate.

### 3 Indeterminacy

#### 3.1 Indeterminate credences

We now consider the possibility that it is indeterminate what credence you have. There are various *precisifications*, each of which is a credence function,  $b$ , which assigns numerical degrees of belief between 0 and 1 to each relevant proposition. The recommendation notion that we are given by a scenario, though, still applies to the various credences themselves, i.e., the precisifications. This latter point is an important feature of our account.

We sometimes talk about *your indeterminate credal state*, which is the collection of precisifications,  $\mathbb{B}$ .<sup>6</sup> Indeterminate probabilities is a model of belief that is independently popular, though a number of authors working with indeterminate (or “imprecise”) probability would not accept the heavy reliance on the notion of indeterminacy that this paper takes.

#### 3.2 The rationality constraints on one's indeterminate credences

We take it that judgements of rationality are attached to one's indeterminate credences. But we hold that the notion of recommendation, or acceptability, that we introduced governs the *precisifications* of one's indeterminate epistemic state. If one has an indeterminate credal state it may thus be indeterminate what is recommended. We give the following requirement of rationality:

##### Recommendation Constraint

Every determinate recommendation must be determinately satisfied.

In the case of Ring Toss suppose your credence is indeterminate between 0 and 1. It is then determinately recommended that your credence be either 0 or 1. To follow Recommendation Constraint, it is then required that your credences are determinately either 0 or 1, which in this case they are. A more specific property saying that you have credence 0 is not determinately recommended, after all the precisification where you have credence 0 does not recommend credence 0; it recommends having credence 1. Recommendation Constraint then doesn't require one to determinately have credence 0. Your credences can be indeterminate between 0 and 1 and Recommendation Constraint is satisfied.

Recommendation Constraint is the key constraint which allows us to obtain an attractive picture of rationality for the imprecise. However, I will also want to add a further constraint which offers something of a converse to Recommendation Constraint. It is not plausible to simply require the converse of Recommendation Constraint. If one's credence determinately has some property it needn't be determinately recommended that it has that property. For example, if one

<sup>6</sup>We also talk about you adopting an indeterminate credal state interchangeably with it being indeterminate which credence you have adopted.

credence function,  $b_1$  evaluates both itself and some other credence,  $b_2$ , as acceptable, one may rationally determinately hold  $b_1$  without it being determinately recommended that  $b_1$  is your credence; it is only determinately recommended that your credence is either  $b_1$  or  $b_2$ . But if your credence determinately has some property, we propose that that property should be determinately acceptable.

### Acceptability Constraint

Every determinately satisfied property must be determinately acceptable.

We can also phrase these two constraints by looking at the views of the individual precisifications in what they take to be acceptable:

Every precisification evaluates some precisification as acceptable.  
(Recommendation Constraint).

Every precisification is acceptable according to some precisification.  
(Acceptability Constraint)

Recommendation Constraint means that the indeterminacy shouldn't be wider than is permitted. Acceptability Constraint means that the indeterminacy shouldn't be narrower than is required: no precisification's views can be ignored. These comments are clearer when we think about evaluating indeterminate credal states as acceptable, holding one's adopted state fixed. We turn to this in the next section.

There is also another phrasing of Acceptability Constraint in terms of recommendations:<sup>7</sup>

If some precisification makes some recommendation, then some precisification satisfies it.

I propose Recommendation and Acceptability Constraint to be the key requirements of rationality. In the Ring Toss case, having one's credence indeterminate between 0 and 1 satisfies both these constraints. Indeed, it is the only way to do so.

This doesn't say all there is to be said about rationality because there is a lot being encoded in the "recommendations" notion which governs the precisifications. The suggestion is that most discussions about rationality should be understood instead as discussions about what is recommended (must it follow the evidence, etc). But once that has been settled, it is just Recommendation and Acceptability Constraint which then determine whether one's indeterminate epistemic state is rational or not.

<sup>7</sup>To see this is equivalent, with abbreviations, one might reason:

Acceptability Constraint:	$\text{Det } X$	$\implies$	$\text{Det Acc } X$
iff	$\neg \text{Det Acc } X$	$\implies$	$\neg \text{Det } X$
iff	$\neg \text{Det } \neg \text{Rec } \neg X$	$\implies$	$\neg \text{Det } X$
iff	$\text{SomePrec Rec } \neg X$	$\implies$	$\text{SomePrec } \neg X$

One might use the term "indeterminate" as a shorthand for "some precisification" but this might mislead one to thinking that it excludes being determinate.

### 3.3 Is this enough for rationality?

There is a stronger version of Recommendation Constraint which we will not require for rationality:

It must be determinate that every recommendation is satisfied.

The difference is the scope of ‘determinate’. In our Recommendation Constraint, ‘determinate’ has narrow scope; the determinate recommendations must be determinately satisfied. In this stronger constraint, ‘determinate’ has wide scope, so the precisifications that are doing the recommendations have to be coordinated with the precisifications that are satisfying them.

This strengthening then means that in cases like Ring Toss, it cannot be satisfied. No credences whatsoever satisfy their own recommendations, so it certainly can’t be determinate that one’s credences satisfy their recommendations.

Whilst we propose that the stronger version is not required for rationality, an interlocutor might want to hold on to it. Such an interlocutor would then be committed to epistemic dilemmas in cases like Ring Toss. They might nonetheless be interested in investigating the consequences of the weaker principles of Recommendation and Acceptability Constraint. Whilst this interlocutor would not accept such epistemic states as rational, they have some “thumbs up” which the irrational ones which do not satisfy these principles do not have. This may nonetheless be interesting to our interlocutor who holds on to the wide-scope variants and thus avoids the epistemic dilemma.

For this paper, we take it that the stronger constraints are not requirements of rationality, and that Recommendation and Acceptability Constraint suffice, and thus, that the epistemic dilemmas are avoided.

### 3.4 Evaluation

When thinking about ‘recommendation’ we also considered holding fixed what credences have been adopted and then evaluate what (potentially different) credences are recommended, or acceptable. Similarly here, we can hold fixed the indeterminate credal state adopted and consider what (potentially different) indeterminate states satisfy Recommendation and Acceptability Constraint.

Consider an abstract case: Suppose  $b_1$  evaluates both  $c_1$  and  $c_2$  as acceptable, and  $b_2$  evaluates  $c_3$  as uniquely acceptable. (As drawn in fig. 1)

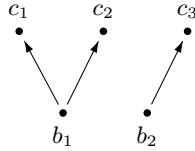


Figure 1: An abstract example of imprecise evaluation. The arrows describe what is evaluated as acceptable according to  $b_1$ ,  $b_2$  and  $b_3$ ; e.g.,  $b_1$  evaluates both  $c_1$  and  $c_2$  as acceptable.

If we hold fixed that one’s indeterminate credal state is given by  $\mathbb{B} = \{b_1, b_2\}$ , the indeterminate credences that satisfy these constraints are  $\{c_1, c_3\}$ ,  $\{c_2, c_3\}$  and

$\{c_1, c_2, c_3\}$ .<sup>8</sup> It is worth talking through this example in a little more depth: For it to be the case that any of  $\mathbb{B}$ 's determinate recommendations are determinately satisfied by  $\mathbb{C}$ , we need that the only precisifications of  $\mathbb{C}$  are  $c_1$ ,  $c_2$  or  $c_3$ . For example, a state which is determinately identical to  $c_1$  satisfies this constraint. However, this would not satisfy Acceptability Constraint. Being identical to  $c_1$  is not determinately acceptable, for example  $b_2$  does not evaluate it as acceptable, but is determinately satisfied by the state that's determinately identical to  $c_1$ . To satisfy Acceptability Constraint,  $c_3$  needs to be a precisification, and so does at least one of  $c_1$  and  $c_2$ . The combination of the two constraints, then, give us  $\{c_1, c_3\}$ ,  $\{c_2, c_3\}$  and  $\{c_1, c_2, c_3\}$  as rationally acceptable.

## 4 Applying this

We now consider what happens when we apply these constraints and considerations in various cases.

### 4.1 Extremal Bad Navigator and Evidence

In the case of Extremal Bad Navigator, the credence you adopt results in additional evidence regarding the location of your hotel, and in fact does so in an extremal and undermining way.

The particular recommendation notion of Extremal Bad Navigator was identical to that of Ring Toss. So we result in the same answer for what is rational: One can satisfy Recommendation and Acceptability Constraint when one's credence is indeterminate between the two extremal values of 0 or 1. Moreover, this is the only way to do so.

It is also valuable to think about how this looks when we focus on desirable epistemic rationality principles. We might have expected that we have the rationality principle:

**Ev1** If  $E$  is part of your evidence, then your credence in  $E$  should be 1. ✗

But what your evidence is can depend on what your adopted credence is. If this happens then it might be that we satisfy hold onto principles like this. This is what happens in the case of Bad Navigator.

However, we will maintain the idea of this principle as a putative epistemic requirement and can impose it as a constraint on what is *recommended*. If  $E$  is part of your evidence, then we can require that it is recommended that you have credence 1 in  $E$  and by imposing Recommendation Constraint we obtain:

**Ev2** If  $E$  is determinately part of your evidence, then you should determinately have credence 1 in  $E$ . ✓

This constraint is satisfiable. I propose it contains the key important content of the original unsatisfiable principle, Ev1.

Extremal Bad Navigator is a modification of a case provided by Egan and Elga (2005) where one's all-things-considered judgements anti-correlate with the truth. Egan and Elga say in such case, one should suspend judgement. They say:

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<sup>8</sup>If we suppose that actually  $c_1 = b_1$  and  $c_2 = b_2$ , then this shows us that Acceptability Constraint does add an additional restriction even for self-acceptance.



“Moral: When one becomes convinced that one’s all-things-considered judgments in a domain are produced by an anti-reliable process, one should suspend judgment in that domain.” Egan and Elga (2005, p83)

There are some relationships between having indeterminate credences and suspending judgement. So whilst our proposal is different to that of Egan and Elga, it does bear some similarity in spirit.

## 4.2 Extremal Promotion and Chance

A range of cases where one’s credences affect the evidence one has were given by Greaves (2013), presented as a challenge for epistemic utility theory. Her examples are formulated involving “chance”. Consider for example a variant on a case she provides:

### Extremal-Promotion

Alice is up for promotion. Her boss, however, is a deeply insecure type: he is more likely to promote Alice if she comes across as lacking in confidence. Furthermore, Alice is useless at play-acting, so she will come across that way iff she really does have a low degree of belief that she’s going to get the promotion. Specifically, if she has credence  $\geq 0.5$ , then the chance she’ll be promoted is 0, and if not, then the chance she’ll be promoted is 1. *This example is considered by Konek and Levinstein (2019).*

No credence you could assign matches the chances. Were you to assign 0, or 0.2, or anything that is not  $\geq 0.5$ , then the chance you’ll get the promotion is 1. So these values don’t match the chances. And similarly if you assign credence 0.8, or 1, or anything that is  $\geq 0.5$  then the chance you’ll get the promotion is 0. So there is no credence value whatsoever which you could adopt and it would match the chances.

However, we can obtain a match between one’s indeterminate credal state and the indeterminate chances. If you adopt the extremal indeterminate credal state with 0 and 1 as precisifications, then it is indeterminate whether the chance is 1 or 0. So your indeterminate state matches the indeterminate chances. The matching is not a precisification-by-precisification match. Each credence is maximally far away from the chances on that precisification, but the indeterminacy in one’s opinions is acceptable according to itself.

What principles do we then obtain regarding deference-to-chance? We can only expect one’s credence to match the chances when you know what those chances are. We may have usually expected:

### Ch1

If you know that the chance is  $x$ , then you have credence  $x$ . ✗

We can encode it as a principle regarding what is recommended and we then obtain:

### Ch2

If you determinately know that the chance is  $x$ , then you determinately have credence  $x$ . ✓

Note that this would equally well apply when we think not about individual propositions and chance values but about the entire chance function. If you determinately know the chance function is some particular function, *ch*, then your credence should determinately be *ch*.

This actually doesn't tell us much about Extremal Promotion. You determinately know what the chance function is, but it's either 0 or 1 depending on which precisification of your credences is under consideration. We can describe a general principle that would also cover this case:

### Ch3

If you determinately know what the chances are, then your indeterminate credences should match the indeterminate chances. ✓

However, it does not suffice to simply determinately know that it has some particular property without determinately know what the chance function is. For example, it is not a putative epistemic constraint that if you know that the chance is either 0 or 1 then you should have credence 0 or 1. A middling response might be perfectly rational. However, we would usually expect this to hold for *convex* properties. For example, that the chance is in an interval  $[a, b]$ . Or that the chance of one proposition is higher than another. That is, one would usually expect:

### Ch4

If  $X$  is a closed, convex property, and you know that the chance function has property  $X$ , then you have credence satisfying  $X$ . ✗

What we obtain instead is:

### Ch5

If  $X$  is a closed, convex property, and you *determinately* know that the chance function has property  $X$ , then you *determinately* have credence satisfying  $X$ . ✓

For example, if you know that the chance of something happening is  $\geq 0.5$ , then you should determinately have credence  $\geq 0.5$ .

## 4.3 Credal Liar

There is a further case that's been mentioned in the literature which leads to the same notion of recommendation as Ring Toss.

Consider the sentence, **CredLiar**:

**CredLiar**: Your credence in **CredLiar** is not  $\geq 0.5$ .

In this case, by adopting a particular credence, you semantically ensure the truth value of **CredLiar**, rather than in the case of Ring Toss where it is a causal process. But the same reasoning exactly follows. Any credence value you could assign would undermine itself. But the credal state which is indeterminate between 0 and 1 satisfies our demands and can be admitted as a rational response.

There is then a nice match between your credence and its truth value. It is indeterminate whether **CredLiar** is true or false. And you have a credal state that is indeterminate between 1 and 0.

Do we obtain this as a general constraint? If you know that some particular proposition is indeterminate, should your credence be indeterminate between 0 and 1? With a very careful formulation, this will come out as a consequence of Recommendation Constraint:

If, in every precisification you know its truth value, and in some precisifications is true and some false, then your credence must be indeterminate between equalling 0 and 1.

This does, though, require the assumption that you know its truth value in each precisification for reasons similar to the care we took in the case of chances.

#### 4.4 Leap

Each of the cases we have considered so far have the same recommendation notion, and they all result in the way to satisfy Recommendation and Acceptability Constraint being to have the extremal indeterminate state of being indeterminate between credence 0 and 1.

We now move to considering cases which have different recommendation notions and will then result in different rational responses.

##### Leap

You are hiking in the Alps and have had the bad luck to work yourself into a position from which the only escape is by a terrible leap. If you are highly confident that you'll make it across, you'll approach your leap with enthusiasm and will sail across successfully. If, on the other hand, you adopt a low degree of belief in your successfully leaping, you'll start trembling and miss your foothold, falling into the abyss. In general, the chance that you'll make it is identical to your degree of belief.

*Greaves (2013)*<sup>9</sup>

In this case, every particular credence you could assign will evaluate just itself as acceptable. If we hold fixed your indeterminate credal state as  $\mathbb{B}$ , to satisfy Recommendation Constraint, any precisification of  $\mathbb{C}$  must also be a precisification of  $\mathbb{B}$ . We can represent our indeterminate credal states as sets of their precisifications, so this tells us that  $\mathbb{C} \subseteq \mathbb{B}$ . To satisfy Acceptability Constraint, any precisification of  $\mathbb{B}$  must also be a precisification of  $\mathbb{C}$ , i.e.,  $\mathbb{C} \supseteq \mathbb{B}$ . The combination of the two principles requires that  $\mathbb{C} = \mathbb{B}$ .

In the leap case, then, any indeterminate credal state,  $\mathbb{B}$ , will satisfy these constraints. For example, determinately adopting any particular credence satisfies them.<sup>10</sup>

#### 4.5 Normal cases

Cases where adopting some credence generates additional information are the unusual cases. Most epistemic scenarios aren't like this but are more like:

<sup>9</sup>James (1897) discusses a case like this.

<sup>10</sup>For the purposes of this paper, we are considering Recommendation and Acceptability Constraint to suffice for rationality. In discussing this case, Joyce (2018) adds further constraint to rule out some such states as irrational. It is not clear exactly how such constraints would apply in our indeterminate setting, but we are open to there being further constraints on rationality so long as they do not reduce the situation to one of an epistemic dilemma.

## Rain

The credence that you adopt that it is going to rain tomorrow provides no additional evidence about the likelihood of rain.

How do our considerations apply to such cases?

There are two ways one might implement the notion of recommendation applying to the precisifications in such a case.

The first of these says in these normal cases, the credence you have adopted doesn't affect what is recommended. It is only in cases where one's adopted credence changes the evidence that the recommendations are altered. So in a normal case like this, a credence of 0.2 is either acceptable or not, and this doesn't depend on what credence has been adopted. Following this implementation, then, an imprecise credal state of  $\mathbb{C}$  satisfies our Recommendation and Acceptability Constraint (according to any  $\mathbb{B}$ ) iff its precisifications are themselves acceptable in this non-relative sense.

The second natural implementation follows the idea that recommendations should follow the epistemic evaluations of the adopted credences. Suppose you've adopted the credence value 0.8 that it will rain tomorrow. What is now recommended of you? A credence of 0.8 evaluates itself as epistemically better than any other particular credence.<sup>11</sup> If recommendations follow such evaluations, then 0.8 is uniquely acceptable according to 0.8. This implementation says that any credence will recommend itself; at least if it is in the range of appropriate credences, such as being probabilistic or appropriately deferring to chances. The implementation will typically match the first implementation by having that the range of credences that were acceptable in the non-relative sense under the first implementation will here be the one's that evaluate themselves as uniquely acceptable; with the others always evaluating themselves as unacceptable. What we then result in is a situation very close to that of Leap. If we hold fixed the adopted state as  $\mathbb{B}$ , and assume that the precisifications of  $\mathbb{B}$  are all appropriate, and thus recommend themselves, it is only  $\mathbb{B}$  itself which satisfies our two constraints: Recommendation Constraint requires that  $\mathbb{C} \subseteq \mathbb{B}$  and Acceptability Constraint requires that  $\mathbb{C} \supseteq \mathbb{B}$ .<sup>12</sup>

So either way one implements this, any  $\mathbb{B}$  can be adopted in accordance with Recommendation and Acceptability Constraint; so long as the individual precisifications of  $\mathbb{B}$  are in a range of appropriate members and thus will evaluate themselves as appropriate. (In the former implementation they will also evaluate others as appropriate, in the latter, they will uniquely evaluate themselves as appropriate.)

## 4.6 Promotion

The original promotion case was offered by Greaves:

### Original Promotion

Alice is up for promotion. Her boss, however, is a deeply insecure type: he

<sup>11</sup>At least if 0.8 is in accordance with any other information one has in the situation. The difference between the normal cases and the other cases we've been considering is that in this case the information is not dependent on the credences that have been adopted.

<sup>12</sup>See also Horowitz (forthcoming) for a related formalism permitting contraction but not expansion. Their formalism can be related to mine by the notion of a guess being licenced as it not being determinately unacceptable.

is more likely to promote Alice if she comes across as lacking in confidence. Furthermore, Alice is useless at play-acting, so she will come across that way iff she really does have a low degree of belief that she's going to get the promotion. Specifically, the chance of her getting the promotion will  $1 - x$ , where  $x$  is whatever degree of belief you choose to have in the proposition that she will be promoted. What credence is it epistemically rational for Alice to have?

*Greaves (2013)*

The recommendation notion this leads to says that if you have adopted credence  $x$ ,  $1 - x$  is uniquely acceptable.

In this case, determinately adopting 0.5 is rational according to our constraints. But there are also indeterminate credal states which could be rationally adopted and satisfy Recommendation and Acceptability Constraint, for example,  $\{0.2, 0.8\}$  or  $[0.2, 0.8]$ . Any set which is symmetric, which contains  $1 - x$  iff it contains  $x$ , is an indeterminate credal state which can be rationally adopted according to Recommendation and Acceptability Constraint.

The key important difference between Extremal Promotion and Original Promotion is that Original Promotion gives us a continuous notion of support. In Original Promotion, the chances continuously depend on one's credences, whereas in Extremal Promotion there is a point of discontinuity at 0.5.

It is a general fact that whenever the chances depend continuously on one's adopted credences, then there will be some credence which would equal the resultant chances were it to be adopted.<sup>13</sup> And thus, there would be some rational determinate credal state. This doesn't mean that one's credences *must* be determinate. When we have considered the indeterminate framework there will also typically be indeterminate credences which satisfy Recommendation and Acceptability Constraint.

However sometimes the chances might depend in a discontinuous way on one's credence as, for example, we considered in Extremal Promotion. Once we allow for such discontinuous acceptability notions, there is no guarantee of credences which evaluate themselves as acceptable. Typically, though, we can find indeterminate epistemic states satisfying Recommendation and Acceptability Constraint, which I propose are then rational responses.

## 4.7 Imps and Bribes

Greaves also provides us with the following case, which offers an *epistemic bribe*:

### Imps

You are in the garden and the sun is shining brightly on you. There are ten impish children in a playhouse. Whether each child comes out to play depends in part on your credence that that it is sunny. In particular the chance that each child comes out to play is  $(1 - \frac{x}{2})$ , where  $x$  is your credence that it is sunny today.

<sup>13</sup>The Intermediate Value Theorem gives us that any continuous function  $f : [0, 1] \rightarrow [0, 1]$  has a fixed point. For single-proposition cases this is enough for our claim. Joyce claims that  $(1 - x)^{1/2}$  has no solutions in  $[0, 1]$ , and thus calls this the "chance paradox". But it has the solution of  $x \approx 0.628$ . More generally, our claim follows from the Brouwer fixed point theorem which generalises this to continuous functions on convex, compact spaces, such as the set of probability functions.

This case offers an epistemic bribe: if you deny the manifest, and adopt credence 0 that it is sunny, you can ensure perfect credences in each proposition regarding the children playing. A consequentialist implementation of epistemic value considerations, such as Caie (2013), Pettigrew (2018), would say that one is required to take such epistemic bribes.<sup>14</sup> However, Joyce, Konek and Levinstein argue that it is never rationally permissible to take such bribes.

Our account also denies taking the epistemic bribe. Whatever credence you have adopted, it is recommended to have credence 1 that it is sunny. After all, that is part of your evidence. And so, to satisfy Recommendation Constraint you have to determinately have credence 1 that it is sunny. You must also, then have credence  $1/2$  that each child will come out to play.

More generally, as we discussed in section 4.1, our account maintains:

If  $E$  is determinately part of your evidence, then it is determinate that one's credence in  $E$  is 1. ✓

So epistemic bribes that ask us to deny determinate evidence must be rejected.

## 4.8 Basketball

Caie considers a case that's closely related to Ring Toss

### Basketball

You are an basketball player in the process of taking your free throw. You will make the shot iff you are less confident that she will make it than that she will miss.

(Caie, 2013).

Here we can't just focus on one's credence that she'll make it, we also have to consider her credence that she'll fail.<sup>15</sup>

Suppose she has adopted some credence  $x$  that she'll make it, and  $y$  that she'll fail, which we can write as  $\langle x, y \rangle$ . If  $x < y$ , then she'll make it, and she knows that, so this would evaluate  $\langle 1, 0 \rangle$  as uniquely acceptable. If  $x \geq y$ , then she'll miss, and she knows that so  $\langle 0, 1 \rangle$  would be uniquely acceptable. This is very similar to the Ring Toss case, and the only indeterminate credences which then satisfy our constraints are being indeterminate between  $\langle 1, 0 \rangle$  and  $\langle 0, 1 \rangle$ .

An alternative implementation of this case is not to set it up with real-valued credences as our precisifications but instead the precisifications are comparative confidence relations. The proposal of Recommendation and Acceptability Constraint would equally well apply to such a setup. Each precisification would be a comparative confidence relation, and we would have that it is indeterminate whether  $\text{Make} \succ \text{Fail}$  or  $\text{Make} \prec \text{Fail}$ .<sup>16</sup>

More generally, we chose to set up our account so that the precisifications are credence functions, assigning real numbers to each proposition. But this was not necessary. Whatever the range of initial opinions we consider, we would then offer a further level of indeterminacy between them.

<sup>14</sup>Though Joyce and Weatherston (2019) argue that in fact the consequentialist shouldn't take the bribe in this particular case because when we consider one's credence in a whole Boolean algebra, taking the bribe reduces one's overall epistemic value.

<sup>15</sup>Unless one encodes by fiat that one's credence in failure is one minus the credence in success. But that seems quite rich.

<sup>16</sup>This should probably be taken as distinct from incomparability. In this case they're determinately comparable, it's just indeterminate which is more likely.

## 5 Outstanding issues

We have proposed Recommendation and Acceptability Constraint as the key constraints on rationality, and investigated their consequences. By having moved from looking at credences which are self-recommending to indeterminate credal states following Recommendation and Acceptability Constraint, we have seen that there are rational epistemic states that could be rationally adopted in these undermining scenarios. However, there are two outstanding issues where epistemic dilemmas may reappear. I will here only be able to briefly discuss them.

### 5.1 Spring

#### Spring

You know that you're always overconfident in this type of situation. Except you also know that a credence value of 0 would be wrong, then you are certain that it will happen.

In particular, if you have credence  $x > 0$ , then it is recommended to have credence  $x/2$ ; and if you have credence 0, then 1 is recommended.

In this case, every credence function is undermining. But unlike in our Ring Toss case, there is also no indeterminate credal state which satisfies Recommendation Constraint, given that it is adopted. To see this, observe the following:

- Whatever has been adopted, credence  $> 0$  and  $\leq 1$  is determinately recommended.
- If one has adopted an epistemic state where determinately one's credence is  $> 0$  and  $\leq 1$ , then credence  $> 0$  and  $\leq 1/2$  is determinately recommended.
- If one has adopted an epistemic state where determinately one's credence is  $> 0$  and  $\leq 1$ , then credence  $> 0$  and  $\leq 1/2$  is determinately recommended.
- If one has adopted an epistemic state where determinately one's credence is  $> 0$  and  $\leq 1$ , then credence  $> 0$  and  $\leq 1/2$  is determinately recommended.
- Etc

So whenever we try to satisfy the determinate recommendations, Recommendation Constraint gives us a further more restrictive determinate recommendation. There is no credence function which satisfies them all.

In Campbell-Moore (forthcoming) I offer a proposal which in the setting of this paper would be the claim that even Recommendation Constraint is too strong and that we should weaken it so that some determinate recommendations needn't be determinately satisfied. In particular, we should weaken Recommendation and Acceptability Constraint to properties of one's credences giving closed sets of credences. This then says that whilst it's determinately recommended to be  $> 0$ , 0 is not ruled out. This weakening allows for  $\{0, 1, 1/2, \dots\}$  to be a rational response to the situation.

There is an alternative approach, which I briefly sketch here. We have been working of a picture of indeterminate credences given by a collection of precisifications. We might instead directly define it via any definite properties.

We could consider some epistemic state where:<sup>17</sup>

- Definitely, one's credence is  $> 0$  and  $\leq 1$ .
- Definitely, one's credence is  $> 0$  and  $\leq 1/2$ .
- Definitely, one's credence is  $> 0$  and  $\leq 1/4$ .
- Definitely, one's credence is  $> 0$  and  $\leq 1/8$ .
- Etc

There is no credence that satisfies all these definite properties, but they are finitely consistent. This picture of belief is interestingly powerful, it has close relationships to some more general “imprecise probability” models studied in the literature.<sup>18</sup>

We characterise when a property is definitely recommended or acceptable by looking at whether the collection of credences which recommend it, or make it acceptable, are definitely held. We then show that any definite recommendation is definitely held, and any definitely held property is definitely acceptable, i.e., the analogues of Recommendation and Acceptability Constraint are satisfied. We propose this as a rational epistemic response in the Spring case.

## 5.2 Archer and Revenge

Joyce (2018) uses a case very similar to Ring Toss. However, in his setup he explicitly says that if you have indeterminate credences, it is true, and you know that. This parallels the definite-liar, which is offered as a revenge challenge for this account of the liar paradox.<sup>19</sup>

### Archer

You are an archer in the process of taking your shot. If your credence that you'll hit the bullseye is determinately  $\geq 0.5$ , then you'll be tense and miss it. And if not, then you'll hit it. And you know this about yourself.

*Joyce, 2018.*

The recommendation notion encoded in this case applies not to one's credences but directly to one's indeterminate state.

The problem here is that if you have the epistemic state indeterminate between credence 1 and 0, as we proposed for Ring Toss, it seems that one should then be certain. So this indeterminate state does not satisfy its own recommendations. However, even if we accept that the recommendation notion appropriate for this case governs one's indeterminate credences, what this shows us is that there is no epistemic state which satisfies its own recommendations.

But in our analysis here we focused not on looking for epistemic states which satisfy their recommendations, but something that follows Recommendation and Acceptability Constraint. Archer doesn't immediately cause a problem for these constraints. The challenge would remain if it was determinate that one has the indeterminate epistemic state  $\{0, 1\}$  as then having determinate credence 1 would be determinately recommended, so this would not be in line

<sup>17</sup>There will be further definite properties obtained by closing these under supersets. More generally, the definite properties should form a ‘filter’.

<sup>18</sup>See Campbell-Moore and Konek (2019), though we present these models using different a different interpretation.

<sup>19</sup>And our response parallels that of, e.g., McGee (1989).



with Recommendation Constraint. But it needn't be determinate that your epistemic state is  $\{0, 1\}$ ; it might be indeterminate whether one's credence is determinate or not.

The resultant account and model of belief needs to be further developed and investigated.<sup>20</sup> But it is at least potentially possible to avoid such criticisms by noting that it not really the move to indeterminate credences that was important but the move from looking at recommendations to determinate recommendations, in the form of Recommendation and Acceptability Constraint.

## 6 Conclusion

We have proposed an account of rationality that applies to one's indeterminate credal state and focuses on following Recommendation and Acceptability Constraint. We have investigated some of the consequences of this account, and have found an attractive picture of rationality which seems to avoid epistemic dilemmas.

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<sup>20</sup>This should be done with an eye to how the definite-liar is treated within the accounts of truth.

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