Undermining Scenarios and Supervaluational Credences

Catrin Campbell-Moore*

November 19, 2019

1 A challenge for rationality

A few scenarios have recently been considered in formal epistemology that cause challenges for various views of rationality, and are closely connected to the liar paradox.

For example, consider the following:

Passport.
If you have credence $\geqslant 1/2$ that you'll remember your passport, then when the time comes you'll end up forgetting it (you'll get on with other things).
And if you has credence $< 1/2$ that you'll remember it then you *will* end up remembering it (you'll spend your time worrying about it).
And you know this about yourself. What should your credence be?
(Similar to basketball case from Caie (2013, footnote 8))

Here's a natural start to reasoning about this sort of case. Consider assigning a credence value, say $1/4$ that you'll forget his passport. Then, you would in fact forget your passport. Moreover, you know this. So credence $1/4$ recommends becoming certain that you'll forget your passport, i.e. adopting credence 1. But, now consider adopting credence 1. If you were to adopt credence 1, then you would know you wouldn't forget his passport, and thus this recommends adopting credence 0. More generally, in this case, every credence value is undermining.

But one cannot have undermining credences and be rational. An epistemic state which undermines its own adoption cannot be relied on and leads to capricious doxastic changes, ruling out its rationality.[1] So, what does rationality

*Thanks to people

[1]This is well accepted (see, e.g., Lewis, 1971; Oddie, 1997; Greaves, 2013; Gibbard, 2008) and has been used to ground epistemic constraints such as probabilism with the so-called accuracy argument (Joyce, 1998; Pettigrew, 2016). What is more controvertial in these cases is that recommendation is spelled out in the kind of way I've spelled it out here rather than in a consequentialist spirit as in Caie (2013); Greaves (2013); Carr (2017); Pettigrew (2018). Joyce (2018) has argued for the notion of recommendation as spelled out here in cases similar

tell us to do in such cases? In this paper we provide an option that is non-undermining by moving to a non-classical setting. In particular, we propose that there are imprecise probabilities which are non-undermining opinion states in such cases.

There are other cases which display the same reasoning pattern, but for different reasons:

> Bad Navigator.
> You've come to a crossroads and are wondering whether you need to turn left or right to get to your hotel. You know you're a really bad navigator. In particular, you believe that if you have credence $\geqslant 1/2$ that left is the way to your hotel, then it's actually right; and if you have credence $< 1/2$ that it's left, then it's actually to the left. What should your credence be that it's actually right?
> (Related to the example from Egan and Elga, 2005)

The reasoning pattern from considering any particular credence you might adopt is the same as in the passport case. However the reason that another credence is recommended is not because the credence you adopt changes what will happen, here it simply gives you evidence about what the world is like.

There is one further case which displays the same reasoning pattern, were we simply consider a sentence:

π: Your credence in π is not $\geqslant 1/2$.

What credence you consider adopting changes its truth value and thus the recommended credence in the same way as in Passport and BadNavigator.

π is very closely related to the liar paradox, which is given by a sentence:

λ: λ is not true.

Is λ true or false? Suppose it were true. Then since it says "λ is not true", and λ is true, we can conclude it is not true. Similarly, suppose it were not true. Then we can conclude it is true. We might describe this as: any truth value we assign to it undermines itself. This underminingness in classical truth values is what we will take to parallel the underminingness in credence functions exhibited in Passport or BadNavigator.

There are a number of options for how to account for the liar paradox. One proposal is a Kripkean account based on a supervaluational evaluation scheme. In this paper I take inspiration from this account for the liar paradox, and apply it to credence. This gives us a proposal for what the rational response to these cases would be. I'll suggest that in such undermining cases, one should adopt so-called imprecise credences, which model our opinions by a set of precise credence functions. These are independently interesting models of belief, and are appropriate for a supervaluational picture of credence. These, I claim, allow for non-undermining opinion states in cases like Passport, BadNavigator and π.

to these, and that this is the appropriate target for the notion of recommendation grounding the non-undermining credences constraint in such cases. See also Konek and Levinstein (2017) for a similar but slightly different analysis in the precise setting.

In particular, the imprecise credal state of $\{0, 1\}$ is, I claim, non-undermining in these cases.

I will also show how this account relates to the standard supervaluational Kripkean account for the liar paradox.

1.1 Summary of the paper

In Section 2 we present the idea of supervaluationism and how it can apply to credences. In particular, we argue (Section 2.1) that we should model credences in the supervaluational setting as sets of precise credence function, and note that this is a well-studied model of belief under the term 'imprecise probability'. This differs from the picture of supervaluational truth where we use an extension and anti-extension.

In Section 3 we discuss how to revise the supervaluational notion of credence: what does reflection on the supervaluational credence adopted recommend adopting? A natural suggestion is provided: what a collection of precise objects recommend is simply the collection of what each of the precise objects recommend (formalised with $\mathcal{R}$). What individual precise credences recommend is given to us as part of the modelling of the cases. This allows for the supervaluational credence of $\{0, 1\}$ to be non-undermining in the cases of Passport, BadNavigator and π. We also discuss how this can be thought of as arising from the supervaluational Kripke construction for truth. However, this natural $\mathcal{R}$ does not allow us to always avoid underminingness (Section 3.4). We propose a more developed solution which takes the closure of $\mathcal{R}$ to be recommended (Section 3.5). This guarantees a non-trivial fixed point. So, with the notion of when a supervaluational credence is undermining as spelled out in Section 3.7, we can always find some supervaluational credence that is non-undermining, thus allowing for candidate rational responses to such challenge cases.

In Section 4 we note that actually the supervaluational Kripke jump for truth can be seen to be doing the work similar to taking closures, and this allows it to avoid triviality. This is done by its choice of representation of supervaluational truth as a partial interpretation giving an extension and anti-extension corresponding to the super-true and super-false sentences rather than directly treating it as a set of precisifications.

In Section 5 we conclude, highlighting what our account says about the epistemic cases we've seen, and noting some general features that this analysis has highlighted about the supervaluational Kripke theory of truth that will help the theory to apply to a range of notions. In particular, we see that the tool we need to apply such a theory to other notions is information on how to revise the precise interpretations, but also that to avoid triviality, closures need to be taken. This might be done for free if one chooses a less-expressive representation more like the extension/anti-extension representation of truth rather than the set of precisifications representation. However, even in that case, if we adopt an admissibility condition that is not closed, such as being ω-consistent, the triviality is no longer avoided.

We start with some technical preliminaries.

1.2 Technical preliminaries

1.2.1 Credences

We start with a collection of sentences, $\mathcal{O}$, which we call our *opinion set*.[2] This could be all sentences of a given language, but it can also be more restrictive, for example we might consider cases where we are only looking at your credence in a single proposition, so where $\mathcal{O}$ is a singleton. For example we might just be interested in the credence that you'll forget your passport, so $\mathcal{O}$ might just contain the single sentence saying that you'll forget your passport.

A *credence function* is a function, c, from $\mathcal{O}$ to $[0,1]$; i.e. it associates with each sentence in $\mathcal{O}$ a degree of belief, which is a real number between 0 and 1 (including the endpoints).[3] $\mathsf{Creds}_{\mathcal{O}}$ is the space of all credence functions, i.e. all functions from $\mathcal{O}$ to $[0,1]$. If $\mathcal{O}$ just contains a single sentence, a credence function can be thought of simply as a value in $[0,1]$ and we will call this a *credence value.*

We will not assume that these credence functions are probabilistic. One could change all our definitions to fix attention to finitely additive probabilistic credences without affecting our results and discussion.[4]

1.2.2 Truth

In the case of truth, we use a standard setup. Let $\mathcal{L}$ be a base language in which we have the ability to code sentences, for ease we take this to be the language of Peano Arithmetic. Let $\mathsf{Sent}_{\mathsf{T}}$ denote the sentences of this language extended by the addition of a unary predicate T. We will assume we have a fixed model of our base language, which we assume is the standard model of arithmetic, denoted $\mathbb{N}$.

Our classical, precise, interpretations of truth are given by a set of sentences, $Q \subseteq \mathsf{Sent}_{\mathsf{T}}$. ($\wp(\mathsf{Sent}_{\mathsf{T}})$ gives the collection of all such precise interpretations.)

We then talk about $(\mathbb{N}, Q) \models \varphi$ which should be understood as: by expanding our base model $\mathbb{N}$ with Q providing the sentences whose codes are in the extension of the truth predicate, we have a resultant classical model where φ is true. So $(\mathbb{N}, Q) \models \mathsf{T}\ulcorner\varphi\urcorner$ iff $\varphi \in Q$.

2 Supervaluationism

Kripke (1975) provides a prominent account of the liar paradox following the underlying idea that what we should do is adopt some non-classical notion of

[2]This terminology is following Pettigrew (2016). It would not affect our account if we took them to be propositions understood in a different way, e.g. they could even be a set of possible worlds.

[3]We restrict to $[0,1]$ rather than $\mathbb{R}$ for technical ease (so our space is compact), but one might also think, that the choice of values 0 and 1 as the bounds is arbitrary (Pettigrew, 2016, ch.6).

[4]We could not restrict attention to *countably*-additive probabilities as that generates a non-compact space as limits of countably additive probabilities might be merely finitely additive. We say c is probabilistic if it is extendable to a probability function on all sentences of the language.

truth, and allow λ to be gappy: neither true nor false. We have to describe how to model this non-classical setting and a notion of a revision of a non-classical interpretation, a so-called 'jump'. Then we can hopefully show that the non-classical setting allows for fixed points.

In our epistemic setting, then, we can try to apply the same tools. We will move to some non-classical setting and describe how to reason about and revise these non-classical opinion states, which, hopefully, will allow for non-underminingness.

To do this, we first need to think about what the non-classical setting is. In the case of truth, we take the non-classical setting still to be given by an account of the truth status of each sentence: '$0 = 0$' is true; λ is gappy, etc. This is usually formalised by associating with our truth predicate two sets of sentences: S^+ which gives the *extension* of the truth predicate (the sentences that are true), and S^- which gives its *anti-extension* (sentences that are false). Sentences that are neither in the extension or anti-extension are gappy, such as λ.[5]

We then need to determine a 'jump': under consideration of (S^+, S^-), if that provides the interpretation of the truth predicate, what is satisfied and not? This will then determine the revised extension/anti-extension. There are various evaluation schemes which can be used to characterise different jumps. The evaluation scheme which provides the basis for the work in this paper is the supervaluational scheme. In this scheme, our partial interpretation of the truth predicate gives rise to various precisifications of it, Q.

Definition 2.1. $Q \subseteq \mathsf{Sent}_\mathsf{T}$ is a *precisification* of $S = (S^+, S^-)$, written $Q \in \mathsf{Precs}(S)$, iff

- If $\varphi \in S^+$ then $\varphi \in Q$.
- If $\varphi \in S^-$ then $\varphi \notin Q$.

Additional restrictions on Q might be imposed for it to be an *admissible precisification*, for example we might add the conditions:

- Q is consistent.
- Q is consistent and closed under logical deduction.
- Q is maximally consistent (i.e. consistent and complete).

The notion of supervaluational satisfaction, and thus the supervaluational jump is then defined by considering what is classically satisfied by each precisification of S. Before presenting that more carefully, we discuss modelling our supervaluational notion of credence.

2.1 Modelling supervaluational credence

In the case of truth we model our supervaluational notion of truth with a partial interpretation, given by an extension and anti-extension. What is the

[5]We might also think of allowing sentences to be both in the extension and anti-extension, but in the supervaluational setting this results in triviality as there are no precisifications.

appropriate supervaluational picture of credences? I suggest we should think of a supervaluational credence as a *set of (precise) credence functions*, $\mathbb{C} \subseteq \mathsf{Creds}_{\mathcal{O}}$ ($\mathbb{C} \neq \varnothing$). This model of belief is one that is familiar in formal epistemology under the term 'imprecise probabilities'. It has been proposed for a range of reasons, including being able to represent incomparability of opinion, distinguishing between lack of evidence and symmetric evidence, allowing for suspension of judgement, and rationalising intuitively rational responses to the Ellsberg decision problem (Joyce, 2010; Bradley, 2015; Levi, 1978; Jeffrey, 1983).

There are some other options that we could have chosen to use, which would be more analogous to what is done in the case of partial truth. For partial truth, we essentially assigned each sentence a truth value, true, false or gappy. We might try to match this in the case of credence by assigning non-classical credal values to each sentence. We could, for example, (1) assign to each sentence two credal values: a lower credence value, $\underline{c}(\varphi)$, and an upper credence value $\overline{c}(\varphi)$, e.g. perhaps my credal state that it'll rain tomorrow is given by the two values $\underline{c}(\varphi) = 0.6, \overline{c}(\varphi) = 0.9$. Or (2) we could associate with each sentence a set of real numbers, e.g., my degree of the belief that a ball drawn from an urn containing 10 balls is red is $\{0, 1/10, \ldots, 9/10, 1\}$.[6] A further alternative would be (3) to associate with various predicates $\mathsf{P}_{\geqslant r}$ an extension and anti-extension.[7]

The former two are models of belief that have been considered in the imprecise probability literature; the third lies between them.[8]Such models will not allow us to encode relationships between sentences; for example both φ and ψ might get $[0.3, 0.4]$ (or have lower credence 0.3 and upper credence 0.4), but the agent might also might think that φ is at least as likely as ψ; or might think that φ provides no evidence for ψ (i.e. that they're probabilistically independent). These sorts of relationships aren't encoded if we simply focus on attitudes towards individual propositions separately from the whole. And, if one thinks these are the sorts of opinions one can have (e.g. Walley, 2000; Joyce, 2010) we should model our opinions in a way that accounts for them.[9]

So the more general sets-of-credence-functions framework has been proposed as an appropriate model of belief within formal epistemology under the term 'imprecise probabilities'. Thus, by using the model of set of precise credence functions as our supervaluational credences, we are making use of an independently motivated and well-liked model of belief. We stick with the term 'supervaluational credences' rather than 'imprecise probabilities' partly because we don't only restrict to functions satisfying the probability axioms.[10]

[6]This follows the presentation of the notion of supervaluational truth where we assign to each sentence a set of classical truth values, where $\{\mathsf{true}, \mathsf{not\text{-}true}\}$ would be interpreted as neither.

[7]This can distinguish between closed and open intervals, but it can only represent intervals rather than non-convex sets such as $\{0, 1/10, \ldots, 9/10, 1\}$.

[8]At least insofar as they determine sets of precise probabilities.

[9]There have been arguments in the "IP community" that sets-of-probabilities is not general enough, although their models of belief as sets of desirable gambles are not ones we want to use (Walley, 2000). However, Campbell-Moore and Konek (2019) develops a model which might lead to natural extensions of the work in this paper and might bear a close tie to a partial notion of truth regarding sentences talking about probability.

[10]Also the term 'imprecise probability' is used differently by different authors, some of whom

The model of supervaluational credence as a set of precise credences is the most general model of credence that is appropriate for supervaluationism. Since our supervaluational model of credence needs to determine a collection of precisifications, the most general account is to take this collection of precisifications as the model itself. And that is what we propose doing.

2.2 Alternative model of supervaluational truth

So we are going to use a representation for supervaluational probabilities that is not directly analogous to that used for truth. We note that one might consider also using a representation of supervaluational truth by a set-of-precise-interpretations model. The model we used for supervaluational truth was to give an extension and anti-extension, which associate with each sentence a truth value – true, false or gappy. These determine a collection of precisifications, each of which is a classical interpretation of truth. We could instead model the supervaluational notion of truth as a set of precisifications: $\mathbb{Q}$.

We can move between these two models by using Precs which gives us the set of precisifications corresponding to a partial interpretation; and Det which lists the determinate truths/falsities agreed on by all $Q \in \mathbb{Q}$ and thus gives a partial interpretation arising from a set of precise interpretations:

Definition 2.2. For a set of precise interpretations of truth $\mathbb{Q}$, $\mathsf{Det}(\mathbb{Q})$ is a partial interpretation, $(\mathsf{Det}(\mathbb{Q})^+, \mathsf{Det}(\mathbb{Q})^-)$ given by

- $\varphi \in \mathsf{Det}(\mathbb{Q})^+$ iff $\varphi \in Q$ for all $Q \in \mathbb{Q}$
- $\varphi \in \mathsf{Det}(\mathbb{Q})^-$ iff $\varphi \notin Q$ for all $Q \in \mathbb{Q}$

As in the case of probability, the set of precisifications model has additional representational power; for example they can encode relationships between sentences that are not encoded in the partial interpretation.

Consider some sentences τ_1, τ_2, and

$$\mathbb{Q} = \{Q \mid \tau_1 \in Q \text{ or } \tau_2 \in Q\}$$

and Q^* which alters some $Q \in \mathbb{Q}$ by making $\tau_1 \notin Q^*$ and $\tau_2 \notin Q^*$.

There is a relationship between τ_1 and τ_2 which every $Q \in \mathbb{Q}$ agrees on, but which is violated by Q^*. However, adding Q^* to the set does not alter any determinate verdicts on truth values: τ_1 and τ_2 are still both gappy. That is, $\mathsf{Det}(\mathbb{Q}) = \mathsf{Det}(\mathbb{Q} \cup \{Q^*\})$. Q^* is a precisification of the partial interpretation corresponding to $\mathbb{Q}$, i.e. $Q^* \in \mathsf{Precs}(\mathsf{Det}(\mathbb{Q}))$.

So the representation of supervaluational truth using a partial interpretation giving definite verdicts on various sentences can lose information encoded in the set of precise interpretations framework.

If we add admissibility conditions we might have to be a bit more careful constructing an example. For example, Q^* as above is not maximally consistent, so this example wouldn't work if we adopt the maximal consistency admissibility

reject to thinking about them in supervaluational terms (Bradley, 2015, §1.2).

condition. However, a slightly more complicated example will work. Consider some sentence μ and truth iterations of it: $\mathsf{T}\ulcorner\mu\urcorner$, $\mathsf{T}\ulcorner\mathsf{T}\ulcorner\mu\urcorner\urcorner$, $\ldots$. Consider the collection of precise interpretations

$$\mathbb{Q} = \{Q \in \mathsf{MaxCons} \mid \text{for some } n \geqslant 0,\ \mathsf{T}^n\ulcorner\mu\urcorner \notin Q\}.$$

Each $\mathsf{T}^n\ulcorner\mu\urcorner$ is indeterminate. We can find some $Q_{\mathsf{all\text{-}true}}$ containing each $\mathsf{T}^n\ulcorner\mu\urcorner$ which is maximally consistent and which doesn't disagree with any determinate truth values resulting from $\mathbb{Q}$. Thus, $\mathsf{Det}(\mathbb{Q}) = \mathsf{Det}(\mathbb{Q} \cup \{Q_{\mathsf{all\text{-}true}}\})$. And $Q_{\mathsf{all\text{-}true}} \in \mathsf{Precs}(\mathsf{Det}(\mathbb{Q}))$. The partial interpretation framework cannot ensure that $Q_{\mathsf{all\text{-}true}}$ is ruled out. The relationship between the sentences that every $Q \in \mathbb{Q}$ agrees on — that at least one of the $\mathsf{T}^n\ulcorner\mu\urcorner$ is not true — is not representable in the partial interpretation framework.

In the case of our epistemic states, we think that such additional information that goes beyond opinions on individual sentences are legitimately part of one's opinion state. In the case of truth, these additional relationships are treated as spurious by representing it as a partial interpretation. It is worth investigating the consequences of adopting alternative models, the most general of which is the sets-of-precise-interpretations framework. Such investigation allows us to parallel the supervaluational Kripke theory as it applies to truth with a theory applicable to the credence case where we want to use the sets-of-precisifications framework.It also allows us to highlight features that allow the construction to avoid triviality, which are typically hidden by the choice of representation as partial truth and thus get a sense for how to avoid triviality in other applications of the supervaluational Kripke theory.

3 Supervaluational jump

The next task is to determine what the 'jump' of this supervaluational notion of credence is. We first consider the usual account for truth.

3.1 Supervaluational jump for truth

The supervaluational jump of a partial notion of truth is given as follows. We start with a partial interpretation S and define $\mathcal{J}(S) = (\mathcal{J}(S)^+, \mathcal{J}(S)^-)$ where

$$\begin{aligned} \varphi \in \mathcal{J}(S)^+ &\text{ :iff } (\mathbb{N}, Q) \models \varphi \text{ for all } Q \in \mathsf{Precs}(S) \\ \varphi \in \mathcal{J}(S)^- &\text{ :iff } (\mathbb{N}, Q) \not\models \varphi \text{ for all } Q \in \mathsf{Precs}(S) \end{aligned} \tag{1}$$

We can think of the '$(\mathbb{N}, Q) \models \varphi$' component of the definition as providing a way to revise a precise interpretation to find a new precise interpretation. This is the notion that is used as the revision step in the revision theory of truth:

$$\varphi \in R_T(Q) \text{ :iff } (\mathbb{N}, Q) \models \varphi.$$

If $\lambda \in Q$, then $\lambda \notin R(Q)$ and visa-versa. This is a way to formalise our informal talk in Section 1: for the liar sentence, true recommends not-true, and not-true recommends true; the two classical truth values undermine themselves.

Using this R_T we can rewrite Eq. (1) as:[11]

$$\varphi \in \mathcal{J}(S)^+ \text{ iff } \varphi \in R_T(Q) \text{ for all } Q \in \mathsf{Precs}(S)$$
$$\varphi \in \mathcal{J}(S)^- \text{ iff } \varphi \notin R_T(Q) \text{ for all } Q \in \mathsf{Precs}(S)$$

Which, recalling the definition of Det, gives us:

$$\mathcal{J}(S) = \mathsf{Det}(\{R_T(Q) \mid Q \in \mathsf{Precs}(S)\})$$

To spell out the Kripkean supervaluational jump in the case of truth our main tools were: (i) using Precs to move from a partial interpretation of truth to a collection of precise interpretations, (ii) using R_T to revise each of these, then (iii) using Det to move from the revised collection of precise interpretations back to a partial interpretation by looking at what is determinately true/false. In the case of credences, we have proposed simply modelling our supervaluational notion as a collection of precise credence functions, so stages (i) and (iii) are not needed, we just need information on how to revise the precise credence functions.

3.2 Modelling our cases

To describe our supervaluational jump for credences we just need to provide a way to revise precise credences to obtain new (recommended) precise credences. We do this by providing a *recommendation function*, R.

Our cases of Passport and BadNavigator were similar to π in that they all led to the same way to reason about precise credence functions, or, more carefully, they each provide us with the same recommendation function. We introduced that informally in the introduction as "the natural way to reason about these scenarios" when the options for one's opinions are precise credence functions.

More carefully, a *recommendation function* is a function, R, from $\mathsf{Creds}_{\mathcal{O}}$ to $\mathsf{Creds}_{\mathcal{O}}$. For the cases of Passport, BadNavigator and π we can focus on $\mathcal{O}$ just containing a single sentence, so think of our credence functions just with a real number $x \in [0, 1]$. These three cases all give rise to the same recommendation function:

$$R_1(x) = \begin{cases} 1 & x < 1/2 \\ 0 & x \geqslant 1/2 \end{cases}$$

We do not assume any further modelling of this recommendation function, we simply assume that any scenario given to us will give rise to such a recommendation function.

Simply encoding the scenarios using a recommendation function is useful for being able to model the wide range of cases that have been discussed in the literature. Consider, for example, Greaves (2013) who describes the following scenario:

[11] See also Burgess (1986, p.666).

Promotion.
"Alice is up for promotion. Her boss, however, is a deeply insecure type: he is more likely to promote Alice if she comes across as lacking in confidence. Furthermore, Alice is useless at play-acting, so she will come across that way iff she really does have a low degree of belief that shes going to get the promotion. Specifically, the chance of her getting the promotion will be $1 - x$, where x is whatever degree of belief she chooses to have in the proposition P that she will be promoted. What credence in P is it epistemically rational for Alice to have?" (Greaves, 2013, pp.1-2)

(Moreover Greaves assumes "that the agent is aware of the specification of [...] her case".) If Alice considers adopting credence 0.2 in P; then the chance of P would be 0.8, and she knows that, so that would recommend adopting credence 0.8. More generally, the description of this case directly provides us with the recommendation function

$$R_2(x) = 1 - x.$$

This function does have a fixed point: 0.5. So this scenario doesn't guarantee undermining credences in the way that Passport and BadNavigator do.

This scenario describes how Alice's credences affect the chances, rather than directly affecting the truth values (or providing evidence about the truth value) as in the cases of Passport, BadNavigator and π. If we want to try to find a unified modelling that can account for the range of cases mentioned so far, it will have to be very complicated. It has to allow for logical, causal and evidential impact (as in π, Passport (and also Promotion) and BadNavigator respectively), and this might go via chance (like Promotion) or directly about the proposition (as in the other cases). But each of them naturally gives rise to a recommendation function. And that is enough information for us to already start reasoning about what various supervaluational credences recommend.

We can later add further information to our modelling. We might try to explain what gives rise to this recommendation function by providing a possible worlds model (more like Campbell-Moore (2015)). Or we might think that what's encoded is a way that an individual credence evaluates the 'accuracy', or *closeness to truth*, of other credence functions; and that this gives rise to the recommendation function. However, since we are just interested in determining what the supervaluational credences recommend, and this will supervene on what the individual precise credences recommend, this further modelling is not required for presentation of our supervaluational account.

There is one extension which I think will be important, but we will not pursue for simplicity: sometimes our credences may not identify a uniquely recommended alternative credence, maybe there are competitors (or perhaps an infinitely descending chain). Thus, I do think we will have to extend our account to deal with these cases, but the ideas of how that will go are simpler in the function case, so I focus my attention on that case.

So, we start off with some such recommendation function on precise credence functions and will show how to construct a supervaluational account that allows for non-undermining supervaluational credences. This allows us to provide some candidates for rational responses in the cases like Passport and BadNavigator.

3.3 A natural notion of recommendation for supervaluational credences

Consider Passport, BadNavigator or π, encoded as R_1. Supervaluational credences are given by sets of precise credences. Consider adopting $\{0, 1\}$. Going with the idea of supervaluationism, the way to consider the effect of adopting this supervaluational credence is to just evaluate the effect of each and then summarise. The precise credence value 0 recommends 1; and 1 recommends 0. We might propose that the supervaluational credence $\{0, 1\}$ then recommends $\{R_1(0), R_1(1)\} = \{1, 0\} = \{0, 1\}$. So the opinion state of $\{0, 1\}$ is self-recommending. Whilst each precisification is undermining, the set, as a whole, has a self-recommending nature to it. $\{0, 1\}$ is a non-undermining opinion to adopt in these cases.

This reasoning gives us a general jump to apply to any $\mathbb{C} \subseteq \mathsf{Creds}_{\mathcal{O}}$, given a fixed recommendation function R:

$$\mathcal{R}_R(\mathbb{C}) = \{R(c) \mid c \in \mathbb{C}\}.$$

which simply takes the collection of $R(c)$ for each $c \in \mathbb{C}$. (We will generally drop the subscript as it's typically clear which R is used.) This is plausible and intuitive as a notion of recommendation for supervaluational credences. It naturally comes out of the idea of supervaluationism where what happens on the supervaluational side supervenes on the what happens on the precise side. It is also a natural modification of the usual supervaluational Kripke jump to apply when our representation is as sets of precise interpretations rather than a partial notion of truth so the translations to and from the partial setting are not required. In the usual Kripkean supervaluational jump we determined a collection of precisifications, revised each of them, then summarised the result. For supervaluational credences we are directly given the set of precisifications in our model and no further summary is required, all that is needed to be done is to revise the individual precisifications.

Whilst plausible, $\mathcal{R}$ does not guarantee a fixed point. Thus, if our notion of recommendation is spelled out with $\mathcal{R}$ we still might end up with a situation where every credal state, precise or imprecise, recommends another, that is, it is undermining, and thus not a candidate for the rational response to the situation.

3.4 $\mathcal{R}$ doesn't always have a fixed point — Spring

Consider the following kind of scenario:

Spring.
You know that you're always overconfident in this type of situation. Except you also know that a credence value of 0 would be wrong.

Details of this case would need to be spelled out to provide our recommendation function.[12] One way these details could be spelled out would give the following notion of recommendation:

$$R_3(x) = \begin{cases} x/2 & x > 0 \\ 1/2 & x = 0 \end{cases}$$

As in the cases like PASSPORT, no credence value is self-recommending. But, unlike in PASSPORT, there is also no set of precise credence values which is self-recommending in the sense of $\mathcal{R}$. To see this, consider the following attempt to construct such a self-recommending supervaluational credence: start with the collection of all possible credence values and keep ruling out ones that are not recommended by anything. (See Fig. 1 for an illustration.)

$$\begin{aligned} \mathbb{C}_0 &= \mathsf{Creds}_{\mathcal{O}} \\ \mathbb{C}_{\alpha+1} &= \mathcal{R}(\mathbb{C}_\alpha) = \{R(c) \mid c \in \mathbb{C}\} \\ \mathbb{C}_\mu &= \bigcap_{\alpha<\mu} \mathbb{C}_\alpha \end{aligned}$$

At the limit stage, we take the intersection. Applied to our case we have:

$$\begin{aligned} &\mathbb{C}_0 = [0,1] \\ &\mathbb{C}_1 = \mathcal{R}([0,1]) = \{R_3(c) \mid c \in [0,1]\} = (0, 1/2] \\ &\mathbb{C}_2 = (0, 1/4] \\ &\mathbb{C}_3 = (0, 1/8] \\ &\quad\vdots \\ &\mathbb{C}_\omega = \varnothing \end{aligned}$$

$\varnothing$ does not represent a legitimate opinion state. It results in triviality. So there is no (non-trivial) fixed point of $\mathcal{R}$.

3.5 How to account for Spring— add extra credences

We propose reconsidering the way of revising our supervaluational credences, replacing $\mathcal{R}$ with a jump that always allows for fixed points.[13]

Consider the claim that $[0,1]$ recommends $\{R_3(c) \mid c \in [0,1]\} = (0, 1/2]$. $0 \notin \mathcal{R}([0,1])$; it is not recommended by any individual. But, 0 is very close

[12]In fact, I think natural ways of adding to this story will not guarantee a notion of a particular credence value being recommended, instead it might allow for ties. Our whole account can be expanded to deal with such non-functional notions of revision, we would say: $\mathbb{C}$ recommends $\mathbb{B}$ iff each $c \in \mathbb{C}$ has (at least) one of their maximally recommended credences in $\mathbb{B}$, and nothing else is in $\mathbb{B}$. However, this would take us beyond this paper.

[13]Rivello (2018) proposes altering the limit stage, although to do that more information is required: we don't collapse to a set at each stage, but look at sequences of how the revision process takes us through the sets.

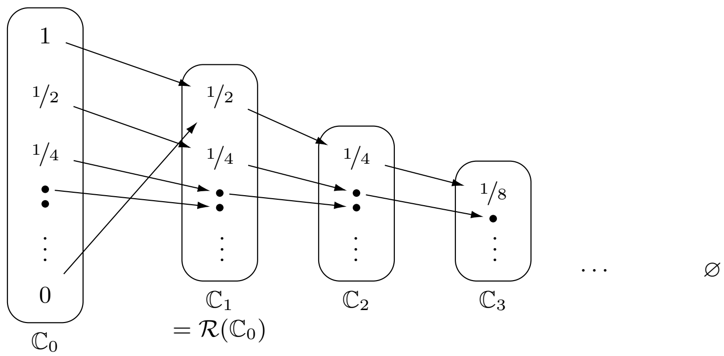

Figure 1: Illustration of $\mathcal{R}$ resulting in triviality for Spring.

to being recommended, that is, it is the limit of a sequence of recommended credence values, or, equivalently, it is in the closure of $\mathcal{R}([0,1])$, understood in a topological sense. We might say that 0 is nearly recommended by $[0,1]$. So the proposal is that we liberalise the notion of recommendation to consider the idea that $[0,1]$ weakly-recommends $[0, 1/2]$, i.e. including 0 as well as all individually recommended credence values.

Liberalising the notion of recommendation to include additional credence values, like 0, who are not individually recommended, but are limits of those who are recommended will allow us to find a fixed point in the case of spring. In particular $\{0, 1/2, 1/4, 1/8, \ldots\}$ will be a fixed point of this notion (see Fig. 2)

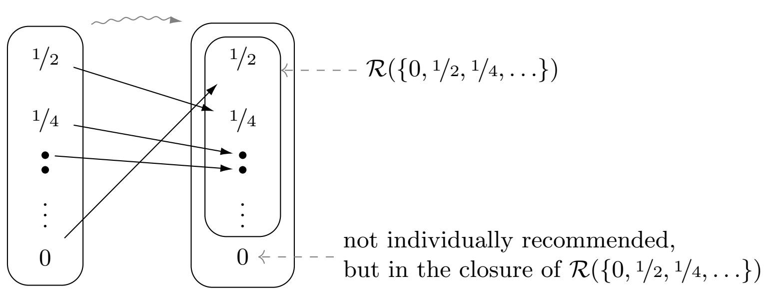

Figure 2: By including the additional credence, 0, we find a fixed point in the case of Spring.

To present this idea more carefully, we use the idea of taking the *closure* of the set of recommendations, in a topological sense, and define

$$\Gamma(\mathbb{C}) := \mathsf{closure}(\mathcal{R}(\mathbb{C})).$$

Again this is relative to a R, which we suppress. One way of spelling out what it is to take a closure is to add any credences which are limits of some sequence from within the recommendations. For example 0 is the limit of the sequence $1/2, 1/4, 1/8, \ldots$, each of which is a member of $\mathcal{R}(\{0, 1/2, 1/4, 1/8, \ldots\})$; and thus $0 \in \mathsf{closure}(\mathcal{R}(\{0, 1/2, 1/4, 1/8, \ldots\})) = \Gamma(\{0, 1/2, 1/4, 1/8, \ldots\})$. We will be more precise in Definition 3.1

3.6 There is always a fixed point of Γ

Γ provides us with a jump where there will always be a fixed point, whatever R we start with. To prove for this claim we need to say a bit more about how to apply Γ in general. That is, we should say how to take a closure. In the case of spring we were focusing on a single proposition, and our credences are then just real numbers. So the notion of limits, closed sets and taking a closure is relatively familiar. For example an interval that includes its endpoints, say $[0.2, 0.6]$, is closed, whereas $(0.2, 0.6)$ is not as it doesn't include the endpoints 0.2 or 0.6. But in general we might have an opinion set $\mathcal{O}$ containing more than one proposition. So our credence functions are functions from $\mathcal{O}$ to $\mathbb{R}$. We have to describe what it is to take a closure of a set of such credence functions.

To do this we will first describe the notion of being a limit of a sequence of credence functions in this more general setting.

Definition 3.1. $x^* \in \mathbb{R}$ is the *limit* of a sequence $\langle x_\alpha \rangle$, if for any $\epsilon > 0$ there is β such that for all $\alpha > \beta$, $|x^* - x_\alpha| < \epsilon$. That is, the sequence gets arbitrarily close to x^*.

A credence function $c^* : \mathcal{O} \to \mathbb{R}$ is the *limit* of a sequence of credence functions $\langle c_\alpha \rangle$ if for each $\varphi \in \mathcal{O}$, $c^*(\varphi)$ is the limit of the sequence of $\langle c_\alpha(\varphi) \rangle$.

c^* is in the *closure* of $\mathbb{C}$ iff c^* is the limit of some sequence $\langle c_\alpha \rangle$ with each $c_\alpha \in \mathbb{C}$.

$\mathbb{C}$ is *closed* if it contains any limits of sequences in $\mathbb{C}$.[14]

This gives us the so-called topology of pointwise convergence, which is also called the product topology.[15] In general, we might want to allow for non-countable opinion sets, so we extend the notion to be limits of longer sequences.

Theorem 3.2. *For any function R, Γ has a non-empty fixed-point.*

Proof. We first show:

Sublemma 3.2.1. *Γ is monotone, that is $\mathbb{C} \subseteq \mathbb{C}' \implies \Gamma(\mathbb{C}) \subseteq \Gamma(\mathbb{C}')$.*

Proof.

$$\begin{aligned}
\mathbb{C} \subseteq \mathbb{C}' &\implies \{R(c) \mid c \in \mathbb{C}\} \subseteq \{R(c) \mid c \in \mathbb{C}'\} \\
&\implies \mathcal{R}(\mathbb{C}) \subseteq \mathcal{R}(\mathbb{C}') \\
&\implies \mathsf{closure}(\mathcal{R}(\mathbb{C})) \subseteq \mathsf{closure}(\mathcal{R}(\mathbb{C}')) \\
&\implies \Gamma(\mathbb{C}) \subseteq \Gamma(\mathbb{C}')
\end{aligned}$$

□

[14] This is equivalent to the usual definition of a closed set in the product topology (Willard, 1970, Theorems 11.9)

[15] This is the same as I used in Campbell-Moore (2019).

Take a sequence: $\mathbb{C}_0 = \mathsf{Creds}_{\mathcal{O}}$, $\mathbb{C}_{\alpha+1} = \Gamma(\mathbb{C}_\alpha)$, $\mathbb{C}_\mu = \bigcap_{\alpha<\mu} \mathbb{C}_\alpha$ for μ a limit ordinal. Since Γ is monotone, the sequence of $\mathbb{C}_\alpha$ is decreasing, $\alpha < \beta \implies \mathbb{C}_\beta \subseteq \mathbb{C}_\alpha$. So this process will reach a fixed point, a set $\mathbb{C}$ where $\Gamma(\mathbb{C}) = \mathbb{C}$. What went wrong in the case of $\mathcal{R}$ is that the fixed point that is reached might be $\varnothing$, which was not a candidate for a legitimate opinion state. We need to ensure that in the case of Γ, it does not result in $\varnothing$.

We can never result in $\varnothing$ at a successor stage: if $\mathbb{C} \neq \varnothing$, then $\Gamma(\mathbb{C}) \supseteq \mathcal{R}(\mathbb{C}) \neq \varnothing$.

What went wrong with $\mathcal{R}$ is that at the limit stage we resulted in $\varnothing$. We have to ensure this will not happen with Γ.[16]

We say a family $\mathcal{E}$, of subsets of $\mathsf{Creds}_{\mathcal{O}}$ is *finitely consistent* iff any finite subcollection of $\mathcal{E}$ has non-empty intersection. In a compact space, any family of closed sets which are finitely consistent is consistent, that is, it has non-empty intersection (Willard, 1970, Theorem 17.4).[17] $\mathsf{Creds}_{\mathcal{O}}$, with the topology of pointwise convergence, is compact by Tychonoff's theorem (recalling that we defined this as functions from $\mathcal{O}$ to the compact set $[0,1]$).[18]

Now each $\{\mathbb{C}_\alpha \mid \alpha < \mu\}$ is a family of closed sets which is finitely consistent (the intersection of any finite collection is just the final $\mathbb{C}_\alpha$ which is in the collection because the sets are decreasing). And thus, compactness gives us that the intersection of them all, that is, $\mathbb{C}_\mu$, will also be non-empty, as required. □

3.7 Obtaining non-undermining credal states

$\mathcal{R}$ wasn't able to always provide fixed points. Whereas Γ is. However, I don't propose Γ is the right characterisation of recommendation.

In everyday cases of belief we have $R(c) = c$, at least for probabilistic c. For example, suppose someone is adopting a degree of belief in whether it's going to rain tomorrow. The opinion that she adopts typically provides no additional information about whether it will rain or not and $R(c) = c$. In that case, also every supervaluational credence (containing only probability functions) has $\mathcal{R}(\mathbb{C}) = \mathbb{C}$. So a supervaluational credence such as $(0,1)$ is plausibly self-recommending. However, if we apply Γ to $(0,1)$ we additionally add the extra limit points, $\Gamma((0,1)) = [0,1]$. So, if recommendation was spelled out with Γ, we would say that $(0,1)$ is undermining. Since in this case we do not need to add the extras, I would want to say that $(0,1)$ is not-undermining.

Instead, we should use both $\mathcal{R}$ and Γ to characterise when a supervaluational credence is undermining.

Definition 3.3. $\mathbb{C}$ is *non-undermining* if

$$\mathcal{R}(\mathbb{C}) \subseteq \mathbb{C} \subseteq \Gamma(\mathbb{C}).$$

[16] We can give an alternative argument by using the fact that in a compact space every sequence has some subsequence which has a limit. (see Willard, 1970, Theorem 17.4).

[17] We usually talk about 'having the finite intersection property', which I have called 'finite consistent' to make the parallel with Visser (1984).

[18] See Willard (1970, pp. 120, 278). This means that the non-empty closed subsets of $\mathsf{Creds}_{\mathcal{O}}$ form a ccpo, in the sense of Visser (1984).

Otherwise, it is undermining.

If $\mathcal{R}(\mathbb{C}) \nsubseteq \mathbb{C}$ it recommends including any $R(c)$ that are left out. If $\mathbb{C} \nsubseteq \Gamma(\mathbb{C})$, it recommends removing the additional members who are not even nearly supported by a member of $\mathbb{C}$. Those are undermining credal states to adopt. Fixed points of $\mathcal{R}$ or Γ, then, are non-undermining.

One might want to liberalise Γ further. Any credences that are in the closure were said to be nearly recommended, and were included. We might want to liberalise this notion further to, for example, the convex closure: credence functions that are in the closed convex hull of $\mathcal{R}(\mathbb{C})$ are also optional extras that can be added without being undermining. Liberalising Γ this way would allow that $[0, 1]$ is a non-undermining response to PASSPORT.

We have now got an account that allows us to always find some non-undermining opinion state for any situation, as spelled out by giving a recommendation function on precise credences. Since opinion states cannot be undermining and rational, this means that we now have an option for what to do in challenging cases such as PASSPORT and BADNAVIGATOR and SPRING. In PASSPORT and BADNAVIGATOR only $\{0, 1\}$ is non-undermining. In SPRING, only $\{0, 1/2, 1/4, \ldots\}$ is non-undermining. In a case like PROMOTION, any set which is symmetric around 0.5 is non-undermining, for example $\{0.5\}$, $\{0.2, 0.8\}$, or $[0.2, 0.8]$. In usual cases, where $R(c) = c$ for probabilistic c, this does nothing new, every set of probabilistic credence functions is non-undermining.[19]

This account treats such cases seriously and provides candidates for the rational response in the form of supervaluational opinion states, which are modelled as sets of precise credence functions, so-called imprecise probabilities.

4 Supervaluational Truth

We have claimed that the account we have given for credences is related to the supervaluational account of the liar. In this section we will relate what we have done to the supervaluational Kripkean account of truth and show that the idea of adding extras as we did in Γ parallels a move that is done implicitly in the case of truth.

4.1 The same challenge as it arises in the case of truth

In the case of truth we might focus on the alternative model of truth as a collection of precise interpretations. We then could consider $\mathcal{R}$, which is the result of revising each precise interpretation in accordance with R_T, the revision step used in the revision theory of truth. Halbach (2014) in fact considered this process as implementation of the revision theory of truth rather than the supervaluational Kripke theory. And he similarly noted that it results in the $\varnothing$.

[19]If each non-probability function recommends a probability function, as we might want since non-probability functions are accuracy-dominated, it will tell us that sets containing non-probability functions are undermining.

	$R(Q)$	$R^2(Q)$	$R^3(Q)$	$R^4(Q)$	$\cdots$
μ		true	true	true	
$\mathsf{T}\ulcorner\mu\urcorner$			true	true	
$\mathsf{T}^2\ulcorner\mu\urcorner$	some nt			true	
$\mathsf{T}^3\ulcorner\mu\urcorner$		some nt	some nt		
$\vdots$				some nt	

Figure 3: Illustration of McGee challenge

This is because truth has its own case which works like Spring in the form of the McGee sentence, μ, where[20]

$$\mathsf{PA} \vdash \mu \leftrightarrow \neg\forall n{>}0\, \mathsf{T}^n\ulcorner\mu\urcorner.$$

This sentence can be used to show that $\mathcal{R}$ has no non-trivial fixed points (This is shown in Halbach (2014, Theorem 14.11) and is already in Gupta and Belnap (1993)).

To show this, we start constructing

$$\begin{aligned}
\mathbb{Q}_0 &= \wp(\mathsf{Sent}_\mathsf{T}) \\
\mathbb{Q}_{\alpha+1} &= \mathcal{R}(\mathbb{Q}_\alpha) = \{R(Q) \mid Q \in \mathbb{Q}\} \\
\mathbb{Q}_\mu &= \bigcap_{\alpha<\mu} \mathbb{Q}_\alpha
\end{aligned}$$

One can show that for every Q, there is some $n \geqslant 0$ such that $\mathsf{T}^n\ulcorner\mu\urcorner \notin R(Q)$. (If $\mathsf{T}^n\ulcorner\mu\urcorner \notin Q$, then $\mathsf{T}^{n+1}\ulcorner\mu\urcorner \notin R(Q)$. If there is no such n, then $(\mathbb{N}, Q) \not\models \mu$ so $\mu \notin R(Q)$.) And thus this holds for all $Q \in \mathbb{Q}_1$. After two iterations of the revision process ($\mathbb{Q}_2$), every interpretation contains μ but there is some n such that it does not contain $\mathsf{T}^n\ulcorner\mu\urcorner$. After three iterations ($\mathbb{Q}_3$), they all contain μ and $\mathsf{T}\ulcorner\mu\urcorner$; after four ($\mathbb{Q}_4$) they all contain $\mu, \mathsf{T}\ulcorner\mu\urcorner$ and $\mathsf{T}\ulcorner\mathsf{T}\ulcorner\mu\urcorner\urcorner$; and so on. More and more iterations of $\mathcal{R}$ guarantee more and more $\mathsf{T}^n\ulcorner\mu\urcorner$ are contained; but they also all have some $\mathsf{T}^n\ulcorner\mu\urcorner$ missing. See Fig. 3 for an illustration.

By ensuring that more and more $\mathsf{T}^n\ulcorner\mu\urcorner$ are contained in successive iterations of $\mathcal{R}$, we might describe this as that it requires our interpretations to get closer and closer to some $Q_{\text{all-}\mathsf{true}}$ which contains all $\mu, \mathsf{T}\ulcorner\mu\urcorner, \mathsf{T}\ulcorner\mathsf{T}\ulcorner\mu\urcorner\urcorner, \ldots$. However, this limit point, $Q_{\text{all-}\mathsf{true}}$ is not in any of these $\mathbb{Q}_n$; it is ruled out at the first stage.

This parallels our Spring-case. 0 was ruled out in the first stage, but successive iterations of $\mathcal{R}$ guaranteed our credal value gets closer and closer to 0.

[20] μ: Some truth iteration of μ is not true.

4.2 McGee avoided by using partial interpretations

In Section 3.3 I described $\mathcal{R}$ as the way to apply the supervaluational Kripke jump when one's representation of the supervaluational notion is as the set of precisifications. But $\mathcal{R}$ leads to triviality, whereas the usual supervaluational Kripke construction for truth does not. How does it avoid such triviality? The key is its choice of representation not as a set of precisifications but as a partial interpretation.

In constructing fixed points for the supervaluational Kripke jump, we start with some S_0, and construct a sequence: $S_{\alpha+1} = \mathcal{J}(S_\alpha) = \mathsf{Det}(\mathsf{Precs}(S_\alpha))$, $S_\mu = \bigcup_{\alpha<\mu} S_\alpha$.[21]

If we are interested in how this treats the sets of precisifications, we can consider the sequence of $\mathbb{Q}_\alpha := \mathsf{Precs}(S_\alpha)$. We can show that $\mathbb{Q}_{\alpha+1} = \mathsf{Precs}(\mathsf{Det}(\mathcal{R}(\mathbb{Q}_\alpha)))$ and $\mathbb{Q}_\mu = \bigcap_{\alpha<\mu} \mathbb{Q}_\alpha$. So we can simply consider this as a construction on the set of precisifications side using:

$$\Delta(\mathbb{Q}) = \mathsf{Precs}(\mathsf{Det}(\mathcal{R}(\mathbb{Q}))).$$

See Fig. 4.

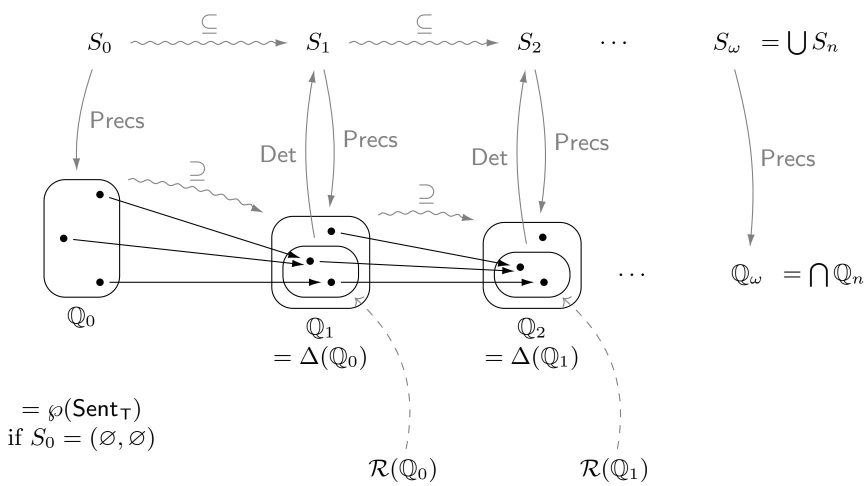

Figure 4: Revising partial interpretation and revising the set of precisifications

We can also consider which Q^* are members of $\Delta(\mathbb{Q})$. Any $Q \in \mathcal{R}(\mathbb{Q})$ is a member of $\Delta(\mathbb{Q})$, but there are some additional interpretations too. $Q^* \in \Delta(\mathbb{Q})$

[21] Since the partial interpretations are pairs, this more carefully means $S^+_\mu = \bigcup_{\alpha<\mu} S^+_\alpha$ and $S^-_\mu = \bigcup_{\alpha<\mu} S^-_\alpha$.

if it agrees with any definite verdicts that the members of $\mathcal{R}(\mathbb{Q})$ all agree on.[22]

So, whilst the supervaluational Kripke construction makes use of $\mathcal{R}$, it can add additional interpretations because of the expressive limitations of the partial interpretation framework. By translating to the partial interpretations framework and back to the set of precise interpretations framework, additional interpretations get added in.

$\mathcal{R}$ results in triviality, but by considering Δ which adds additional interpretations, it can avoid the McGee challenge and triviality as follows: Whilst each $Q \in \mathcal{R}(\mathbb{Q}_0)$ does not contain some $\mathsf{T}^n\ulcorner\mu\urcorner$, an interpretation $Q_{\textsf{all-true}}$ which contains all $\mathsf{T}^n\ulcorner\mu\urcorner$ is a member of $\Delta(\mathbb{Q}_0)$. This is because although each $Q \in \mathcal{R}(\mathbb{Q}_0)$ does not contain some $\mathsf{T}^n\ulcorner\mu\urcorner$, they do not agree on which $\mathsf{T}^n\ulcorner\mu\urcorner$ to leave out. Each $\mathsf{T}^n\ulcorner\mu\urcorner$ is indeterminate, some interpretations in $\mathcal{R}(\mathbb{Q}_0)$ contain it, others do not. So, at least in so far as these $\mathsf{T}^n\ulcorner\mu\urcorner$ are treated, S_1 is exactly like S_0: they are all gappy. So when we return to looking at $\mathsf{Precs}(S_1)$, $Q_{\textsf{all-true}}$ is re-included.

This is similar to our solution to avoiding triviality in the case of Spring. We changed our notion of revising the set of precisifications from $\mathcal{R}$ to Γ which allowed the limit point of 0 to be reincluded. Even though no individual recommended it, it was close enough to the members of $\mathcal{R}$. Similarly, even though no individual interpretation revises to obtain $Q_{\textsf{all-true}}$, it is sufficiently close to the collection of $\mathcal{R}(\mathbb{Q}_0)$ as it doesn't disagree with any of the unanimous verdicts of the members of that set. Any sentences they agree are determinately true or false, are agreed on by $Q_{\textsf{all-true}}$.

The choice of the supervaluational Kripke construction of truth using the partial interpretation thus makes a substantial technical difference: it allows the construction to avoid triviality by adding extra members in the jump than just those in $\mathcal{R}$. This is reminicient of our construction for credences: we moved from $\mathcal{R}$ to Γ which allowed for additional interpretations which are sufficiently similar to those in $\mathcal{R}$. In fact, we can spell out the notions of limit and closure for precise interpretations,[23] and we see that $\Delta(\mathbb{Q}) \supseteq \mathsf{closure}(\mathcal{R}(\mathbb{Q}))$. So our argument that by taking closures, in our definition of Γ, we obtain non-trivial fixed points, applies to the truth setting too (which is compact). This argument can also be applied when we adopt admissibility conditions, so long as the collection of *admissible* precisifications of a partial interpretation is closed and (equivalently) the resultant space of all admissible precise interpretations is compact. This holds for the admissibility conditions of consistency, deductively-closed, and maximal consistency. However, ω-consistency does not lead to a closed set of

[22]

$$\begin{aligned} Q^* \in \Delta(\mathbb{Q}) \iff & \text{If } \varphi \in Q \text{ for all } Q \in \mathcal{R}(\mathbb{Q}), \text{ then } \varphi \in Q^*, \\ & \text{and if } \varphi \notin Q \text{ for all } Q \in \mathcal{R}(\mathbb{Q}), \text{ then } \varphi \notin Q^* \end{aligned}$$

[23] Q^* is a limit of $\langle Q_\alpha \rangle$ if for all φ there is β such that for all $\alpha > \beta$, $Q^\alpha(\varphi) = Q^*(\varphi)$. This is equivalent to the product topology on $\{\mathsf{true}, \mathsf{not\text{-}true}\}^{\mathsf{Sent}_\mathsf{T}}$, with the discrete topology on $\{\mathsf{true}, \mathsf{not\text{-}true}\}$ as described in (Campbell-Moore, 2019). This topology is compact by Tychonoff's theorem.

interpretations, and in fact $Q_{\textsf{all-true}}$ is not admissible. So the move to the partial interpretations does not add enough extras if we are restricted to ω-consistent interpretations, and the construction results in triviality.

5 Conclusion

5.1 Avoiding undermining epistemic states

Various scenarios have been posed as a challenge for what it takes to be rational. In these scenarios, we want to see which epistemic states are rationally permissible. A criterion for an epistemic state to be rationally permissible is that it doesn't undermine its own adoption as such credences can't be relied on.

We have given a proposal for the epistemic state to adopt in such cases which is not undermining when we move to the supervaluational setting. We proposed to think about supervaluational credences as a *set of* precise credence functions. This is an account of belief which has independent support in epistemology, under the term 'imprecise probabilities'.

To spell this out we then thought about when a supervaluational credence is undermining. At a first pass, we reason about such a supervaluational credence by seeing what each individual credence recommends. We see that in cases like Passport, $\{0, 1\}$ is non-undermining as 0 recommends 1 and 1 recommends 0. However, in a case like Spring, this way of reasoning did not avoid undermining opinions. So we proposed that if everything in the set is in the closure of the set of recommended precise credences, then the set is also not undermining. That allowed for non-undermining opinion states in every scenario, where the scenarios are spelled out with a recommendation function R.

One important question left open by this analysis is how to think about revenge challenges. Can there be a scenario which says: if you adopt $\{0, 1\}$, then it's true. The solution of adopting sets of precise credences to allow non-undermining opinions then does not work. That only works if recommendation is appropriately spelled out on the precise site. We leave investigation of how to think about this to future work.

5.2 What we've learned about the supervaluational Kripke construction

We have thought about how to apply the supervaluational Kripke construction in the case of credences. This development has shown us quite general features of the supervaluational Kripkean account which will be beneficial in applying in a range of cases beyond truth. In particular:

We can show that the supervaluational account really just needs to be told how to revise each of the precise objects, any further formulation of how that revision happens is not important for the final account. This helps us understand the supervaluational Kripkean account in a way that will allow it to apply to more notions of interest.

We also showed that the choice of modelling truth as a partial interpretation is of technical importance to avoid triviality. If we model our supervaluational notion in the most general way possible, namely, simply giving the set of precisifications themselves, then the application of the supervaluational Kripke account leads to contractions. We discussed what one should do in such cases where one is interested in other models – we permitted (though did not require) one to take a closure of the set of precisifications. This guarantees fixed points when the underlying space is compact. (This is why ω-consistency cannot be used: that does not give a compact space.)

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