Undermining Scenarios and Supervaluational Credences

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WARNING: Updates coming in the next week or two!!

1 Introduction

Consider the following unfortunate scenario:

> PASSPORT.
> If you have credence $\geqslant 0.5$ that you'll remember your passport, then when the time comes you'll end up forgetting it (you'll get on with other things).
> And if you have credence < 0.5 that you'll remember it then you *will* end up remembering it (you'll spend your time worrying about it).
> And you know this about yourself. What should your credence be?
> (Similar to basketball case from Caie (2013, footnote 8))

Suppose you adopt credence 0.2 that you'll remember your passport. Then you'll get on with your other work and ultimately will forget your passport. And you know this. So you should be certain that you'll forget your passport, i.e. adopt credence 1. But were you to adopt credence 1 that you'll forget your passport, you would spend all your time worrying about it, and thus will remember it. And since you know this about yourself, this would recommend adopting credence value 0. More generally, any credence value you could assign to you remembering your passport will undermine itself.

Such scenarios have recently been discussed as a challenge for rationality. A rational opinion state should not undermine its own adoption. If it does, it cannot be stably relied on for decision making and evaluation. We suggest that moving to a supervaluational model of credences allows for non-undermining credal states. This account can be seen as very close to the supervaluational Kripkean account of truth (Kripke, 1975).

We parallel the kind of underminingness found in PASSPORT to that found in the liar paradox. Consider the liar paradox:[1]

*Thanks to people

[1]Note I have chosen to use the weak-liar.

`Liar`: `Liar` is false.

While staying in the classical setting, we might reason about the liar sentence as follows: Is it true or false? Suppose it were false. Then since it says "`Liar` is false", and it is false, we can conclude it is true. Suppose it were true. Then since it says "`Liar` is false", and `Liar` is true, we can conclude that it is false. Whatever (classical) truth value one applies to it, by reflecting on the truth value it has, we obtain a contradictory truth value.

Kripke (1975) provided a very influential account of truth where one moves to working with a partial notion of truth. Having described how a stage of reflection should apply to such a partial interpretation—a so-called Kripkean jump—one then shows that fixed points can be found, where a stage of reflection does not affect the assigned truth value. We want to similarly move to a more general notion of credence and obtain 'fixed points', where a stage of reflecting on one's opinions does not undermine the attitude adopted.

In particular, we suggest that the *supervaluational* Kripkean construction can be very fruitfully applied to the case of credences. Modelling one's opinions as a set of credence functions is a popular account of belief, under the term 'imprecise probabilities'. We suggest that these models of belief can be seen as supervaluational accounts of credence. And a reflection on these credences can be provided by taking inspiration from the supervaluational Kripkean jump for truth. This then allows for fixed points, which are then candidates for being rational opinions to adopt in such cases. There are two kinds of supervaluational jumps one could apply to this setting, both of which can be seen as being inspired by the usual supervaluational Kripkean jump for truth. The most natural and obvious supervaluational jump is to revise a set of precisifications simply by revising each of the individual precisifications. But this can sometimes result in triviality due to cases which bear some similarity to the McGee phenomenon for truth. We also propose an alternative jump by also allowing for the addition of some 'limits' of the resultant precise interpretations, more carefully: we take the topological closure of the resultant set of revised interpretations. This jump avoids triviality, but at the cost of some intuitiveness. However, we show that this alternative jump is much closer to what is usually done in the supervaluational Kripke account of truth, viewed through the lenses of sets-of-precisifications, due to its choice of focusing on partial interpretations.

In the process of developing this account, we will obtain a clearer picture of the supervaluational Kripkean account of truth viewed through the lenses of the resultant sets-of-precisifications. We obtain a very general account that could apply to a wide range of target notions. All that needs to be specified is a way of revising the classical interpretations, which act as the precisifications of the supervaluational interpretations. A supervaluational interpretation can simply be taken to be given by a set of precise interpretations, and revision of the supervaluational interpretations can be specified by considering how each of the precisifications are revised. This will often result in non-trivial fixed points. More precisely: if one adds topological closures when applying the supervaluational jump, and the underlying space of precise interpretations is

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topologically compact, then there will be non-trivial fixed points.

2 Kripkean supervaluational account of truth

We first present the usual supervaluational Kripkean account. We assume this is relatively familiar to the reader, so don't add much further comment. Although, we highlight that it can be seen as related to a revision step of classical truth, as familiar from the revision theory of truth. This observation will be important for determining the revision of supervaluational credences.

2.1 Classical truth

The following setup is relatively standard:

Setup 2.1. Let $\mathcal{L}$ be a base language in which we have the ability to code sentences, for ease we take this to be the language of Peano Arithmetic. Let $\mathcal{L}_\mathsf{T}$ extend this with the addition of a unary predicate, T. Sent_T denote the sentences of this language. We will assume we have a fixed model of our base language, which we assume is the standard model of arithmetic, denoted $\mathbb{N}$.

Definition 2.2. A *classical interpretation of truth*, Q, is given by a collection of sentences, i.e. $Q \subseteq \mathsf{Sent}_\mathsf{T}$. $(\mathbb{N}, Q)$ refers to the classical model of $\mathcal{L}_\mathsf{T}$ resulting from expanding the base model $\mathbb{N}$ with Q providing the sentences whose codes are in the extension of the truth predicate. So we have $(\mathbb{N}, Q) \models \mathsf{T}\ulcorner\varphi\urcorner$ iff $\varphi \in Q$.

We then define a way to 'revise' a classical interpretation, Q, to a new classical interpretation, $\tau(Q)$.

Definition 2.3. $\tau(Q) \subseteq \mathsf{Sent}_\mathsf{T}$ is defined by: $\varphi \in \tau(Q)$:iff $(\mathbb{N}, Q) \models \varphi$.

So, for example, if $\varphi \in Q$ then $\mathsf{T}\ulcorner\varphi\urcorner \in \tau(Q)$, and if $\varphi \notin Q$ then $\mathsf{T}\ulcorner\varphi\urcorner \notin \tau(Q)$. This is the revision step that's often used in the revision theory of truth. It can be understood as a stage of reflecting on the supposed truth values to see if any alterations are required.To link with our discussion of credences, we will say that Q is undermining if $\tau(Q) \neq Q$. A interpretation of truth which is undermining is plausibly not an interpretation of truth for the very language in which it is contained.

Ideally, one would find a classical interpretation Q, which is non-undermining, i.e. a fixed point of τ. But, the liar paradox shows us that this is not possible because for the liar sentence, $\texttt{Liar}$, with $\mathsf{PA} \vdash \texttt{Liar} \leftrightarrow \neg\mathsf{T}\ulcorner\texttt{Liar}\urcorner$, we have that $\texttt{Liar} \in Q$ iff $\texttt{Liar} \notin \tau(Q)$.

2.2 Partial truth

Definition 2.4. A *partial interpretation* of truth, S, is given by *two* sets of sentences, the extension, S^+, and antiextension S^-.

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We could equivalently think of this as providing an account of the truth status of each sentence: '$0 = 0$' is true; the liar sentence is gappy; '$0 = 1$' is false; etc.

Since we will be working with the supervaluational evaluation scheme, it is important to associate a partial interpretation with a set of precisifications. We do this as follows:

Definition 2.5. $Q \subseteq \mathsf{Sent}_\mathsf{T}$ is a *precisification* of $S = (S^+, S^-)$:iff

- If $\varphi \in S^+$ then $\varphi \in Q$.
- If $\varphi \in S^-$ then $\varphi \notin Q$.

The collection of precisifications of S is $\mathsf{Precs}(S)$.

One might also consider adding 'admissibility conditions' which restrict the precisifications to, e.g., those that are consistent. We will return to that later.

We will also make use of a converse of Precs:

Definition 2.6. Given a set of precisifications, $\mathbb{Q} \subseteq \wp(\mathsf{Sent}_\mathsf{T})$, $\mathsf{Partial}(\mathbb{Q})$ is a partial interpretation, $(\mathsf{Partial}(\mathbb{Q})^+, \mathsf{Partial}(\mathbb{Q})^-)$, given by

- $\varphi \in \mathsf{Partial}(\mathbb{Q})^+$:iff $\varphi \in Q$ for all $Q \in \mathbb{Q}$
- $\varphi \in \mathsf{Partial}(\mathbb{Q})^-$:iff $\varphi \notin Q$ for all $Q \in \mathbb{Q}$

This puts in the extension any sentence which is true in all precisifications, i.e. which is determinately true, and in the antiextension any sentence which is determinately not true.

Now we need to define the supervaluational Kripke jump. This is defined by:

Definition 2.7. $\mathcal{J}(S) := (\mathcal{J}(S)^+, \mathcal{J}(S)^-)$ where

$$\begin{aligned} \varphi \in \mathcal{J}(S)^+ &:\text{iff } (\mathbb{N}, Q) \models \varphi \text{ for all } Q \in \mathsf{Precs}(S) \\ \varphi \in \mathcal{J}(S)^- &:\text{iff } (\mathbb{N}, Q) \not\models \varphi \text{ for all } Q \in \mathsf{Precs}(S) \end{aligned} \tag{1}$$

Given our definitions of τ and $\mathsf{Partial}$, we can immediately see this is equivalent to:

Proposition 2.8.

$$\mathcal{J}(S) = \mathsf{Partial}(\{\tau(Q) \mid Q \in \mathsf{Precs}(S)\})$$

We can further simplify this by making another definition:

Definition 2.9. For $\mathbb{Q} \subseteq \wp(\mathsf{Sent}_\mathsf{T})$,

$$\Theta_\tau(\mathbb{Q}) := \{\tau(Q) \mid Q \in \mathbb{Q}\}$$

We then immediately have:

Proposition 2.10.

$$\mathcal{J}(S) = \mathsf{Partial}(\Theta_\tau(\mathsf{Precs}(S)))$$

We can describe this jump as follows: (i) use Precs to move from a partial interpretation of truth to the corresponding set of precisifications, (ii) use τ to revise each of these, and (iii) use $\mathsf{Partial}$ to move from the resultant collection of revised classical interpretations to a partial interpretation.

One then shows that $\mathcal{J}$ has fixed points, and these are then proposed as candidates for being legitimate interpretations of the truth predicate.

3 Classical credences

We first start just by specifying the notion of credence function that we're working with in the classical setting.

Setup (Credence functions)**.** We start with a non-empty collection of sentences, $\mathcal{A}$, which we call our *agenda*.[2] This could be all sentences of a given language, but it can also be more restrictive, for example we might consider cases where we are only looking at your credence in a single sentence, so where $\mathcal{A}$ is a singleton. For example we might just be interested in the credence that you'll forget your passport, so $\mathcal{A}$ might just contain the single sentence saying that you'll forget your passport.

A *credence function* is a function, c, from $\mathcal{A}$ to $[0, 1]$; i.e. it associates with each sentence in $\mathcal{A}$ a degree of belief, which is a real number between 0 and 1 (inclusive). $\mathsf{Creds}_{\mathcal{A}}$ is the space of all credence functions, i.e. all functions from $\mathcal{A}$ to $[0, 1]$. If $\mathcal{A}$ just contains a single sentence, a credence function can be thought of simply as a value in $[0, 1]$ and we will call this a *credence value.*

We will not assume that these credence functions are probabilistic. One could change all our definitions to fix attention to finitely additive probabilistic credences without affecting our results and discussion.[3]

In the case of truth, we presented a 'revision function', τ. For credences, we will assume that we have a revision function given.[4] However, we will not say more about what this revision function will be. Our supervaluational account will be a general one that can apply to any given revision function.

Formally:

Definition 3.1. A *revision function* is a function, ρ, from $\mathsf{Creds}_{\mathcal{A}}$ to $\mathsf{Creds}_{\mathcal{A}}$.

Passport is a story which directly describes to us how one should revise one's credence in the target-proposition—that you'll remember your passport. If

[2]It would not affect our account if we took them to be propositions understood in a different way, e.g. they could be a set of possible worlds.

[3]We could not restrict attention to *countably*-additive probabilities as that generates a non-compact space, as limits of countably additive probabilities might be merely finitely additive.

[4] In fact, many situations will not give rise to a recommendation *function.* Instead, maybe there are ties. Our whole account can be expanded to deal with ties, we would say: $\mathbb{C}$ recommends $\mathbb{B}$ iff each $c \in \mathbb{C}$ has (at least) one of their maximally recommended credences in $\mathbb{B}$, and nothing else is in $\mathbb{B}$. However, this would complicate the presentation and the parallel to the truth case, so I assume that we always have this revision function.

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we let $\mathcal{A}$ simply contain this one proposition, then credence functions are values $x \in [0, 1]$, and the revision function that PASSPORT gives rise to is:

$$\rho_{\text{PASSPORT}}(x) = \begin{cases} 1 & x < 1/2 \\ 0 & x \geqslant 1/2 \end{cases}$$

And, one can see that there are no fixed points of ρ_{PASSPORT}. That is, every classical credence function undermines its own adoption. In this sense PASSPORT can be related to the liar sentence: in both cases, revision of precise values are undermining.

There are other cases that also give rise to ρ_{PASSPORT}. For example:

BAD NAVIGATOR.
You've come to a crossroads and are wondering whether you need to turn left or right to get to your hotel. You know you're a really bad navigator. In particular, you believe that if you have credence $\geqslant 1/2$ that left is the way to your hotel, then it's actually right; and if you have credence $< 1/2$ that it's left, then it's actually to the left.What should your credence be that it's actually right? (Related to the example from Egan and Elga, 2005)

In this case, the change in one's credence isn't due to the causal structure, but instead simply that the credence that one adopts affects the evidence that one has about the situation. But the same revision function describes this case. The same revision function would also arise when considering a sentence

`PrLiar`: Your credence in `PrLiar` is not $\geqslant 1/2$.

In this case, the revision is due to semantic features of the sentence.

We do not assume any further modelling of this revision funtion, we simply assume that any scenario given to us will give rise to such a revision funtion. The revision funtion

Simply encoding the scenarios using a revision function is useful for being able to model the wide range of cases that have been discussed in the literature. Consider, for example, Greaves (2013) who describes the following scenario:

PROMOTION.
"Alice is up for promotion. Her boss, however, is a deeply insecure type: he is more likely to promote Alice if she comes across as lacking in confidence. Furthermore, Alice is useless at play-acting, so she will come across that way iff she really does have a low degree of belief that shes going to get the promotion. Specifically, the chance of her getting the promotion will be $1 - x$, where x is whatever degree of belief she chooses to have in the proposition P that she will be promoted. What credence in P is it epistemically rational for Alice to have?" (Greaves, 2013, pp.1–2)

(Moreover Greaves assumes "that the agent is aware of the specification of [...] her case".) If Alice considers adopting credence 0.2 in P; then the chance of P would be 0.8, and she knows that, so that would recommend adopting credence

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0.8. More generally, the description of this case directly provides us with the revision funtion

$$\rho_{\text{Promotion}}(x) = 1 - x.$$

This function does have a fixed point: 0.5. So this scenario doesn't guarantee undermining credences in the way that Passport and BadNavigator do.

This scenario describes how Alice's credences affect the chances, rather than directly affecting the truth values (or providing evidence about the truth value) as in the cases of Passport, BadNavigator and `PrLiar`.

Consider also:

Basketball.
You think there's 1% chance that whether you'll be able to shoot this free-throw is dependent on the credence you adopt in it in a Passport style way, i.e., where if you have credence $\geqslant 0.5$ then you'll fail, and if you have credence < 0.5 then you'll succeed. But you're 99% sure that it's just a normal case where you have a 50% chance of success.

This leads to the revision function

$$\rho_{\text{Basketball}}(x) = \begin{cases} 0.01 \times 1 + 0.99 \times 0.5 = 0.505 & x < 0.5 \\ 0.01 \times 0 + 0.99 \times 0.5 = 0.495 & x \geqslant 0.5 \end{cases}$$

If we want to try to find a unified modelling that can account for the range of cases mentioned so far, it will have to be very complicated. It has to allow for logical, causal and evidential impact (as in `PrLiar`, Passport (and also Promotion) and BadNavigator respectively), and this might go via chance (like Promotion) or directly about the proposition (as in the other cases), or be associated with further uncertainty. But each of them naturally gives rise to a revision function. And that is enough information for us to already start reasoning about what various supervaluational credences recommend.

For the case where one if focusing on a language which can talk about probabilities, we can do a little more in the modelling. In that case we can consider a possible world structure, with various worlds and probabilistic accessibility relations between them. We should then move to working with not credence functions but a credence function at each world — a credal-evaluation-function. We then can define the revision of a credal-evaluation-function by taking the weighted proportion of the accessible worlds where the sentence is evaluated as true with the initial credal-evaluation-function providing the interpretation of the credence function symbol at the various worlds.

But this kind of analysis cannot obviously deal with Promotion. And insofar as it deals with Passport or BadNavigator it just treats them like `PrLiar`, for example, we wouldn't represent 'the hotel is to the left' as an atomic sentence, as would be more natural, but instead as a sentence that refers to itself. Whilst this leads to the right revision function, it does not seem to be the right analysis of the sentence itself. We thus find it valuable to not encode further modelling such as this, but to simply provide the supervaluational Kripkean analysis for any given revision function.

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It is valuable to make one final comment about the notion of a revision function. When we spell out an 'undermining credence function' as one which is not a fixed point of the given revision function, it should be that undermining credence functions are not appropriate candidates for rational opinions. Joyce (2018) spells out $\rho(b)$ as the credence function which maximises expected accuracy, or closeness to truth, relative to the function b updated with the information that b are your credences. After adopting a credence function b, if one is given a further choice about what function to use to guide one's actions, or evaluate estimates of truth values, it is $\rho(b)$ that you will choose. Joyce then argues that credence functions which are undermining (he calls this 'non-ratifiable') are "sham credences" as they do not perform the constitutive roles of one's credences.

4 Supervaluational Credences

In the case of truth we model our supervaluational notion of truth with a partial interpretation, given by an extension and anti-extension. What is the appropriate analogy for credences?

Kripke's picture underlying a move to the partial setting was that some sentences, such as the liar sentence, do not "express a proposition". Following that picture, perhaps one should consider credence functions which can be gappy, and some sentences—those that don't express propositions—don't get credence values at all. However, we think this isn't the right way to go. Firstly, in a case like BadNavigator, the sentence 'the hotel is to the left' seems to express a normal proposition. What's weird in this case is not the proposition that the sentence expresses, but instead, it's what rationality says about such a case that's weird. Secondly, the account will be then be very weak. For example, a case like Basketball would not get assigned a credence value at all. But there are some facts about its probability which are determinately true, for example, that the probability is $\geqslant 0.1$. And that can be used in decision making.

I alternatively propose that we should model our supervaluational credences simply as sets of precise credence functions, that is, it is given by some $\mathbb{C} \subseteq \mathsf{Creds}_{\mathcal{A}}$. So, for example, if our agenda contains a single proposition, a classical probability function is some real number between 0 and 1, whereas a supervaluational probability is a set of numbers, e.g., $\{0.2, 0.3\}$, or $[0.2, .0.3]$. This model can also encode relationships between sentences, for example, it can encode the fact that I think that φ is more likely than ψ by having $\mathbb{C} \subseteq \{c \mid c(\varphi) > c(\psi)\}$, and it can encode that I take φ to count as evidence for ψ by $\mathbb{C} \subseteq \{c \mid c(\psi \mid \varphi) > c(\varphi)\}$, and it can do both of these without precision on φ or ψ individually.

The precise credence functions in the set are called the 'precisifications' of the supervaluational credence.

This model of belief is one that is familiar in formal epistemology under the term 'imprecise probabilities'.[5]It has been proposed for a range of reasons,

[5]This can refer to a few different models of belief, but within philosophy, the most prominently considered is sets of probabilities. There have been some arguments that this model is

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including being able to represent incomparability of opinion, distinguishing between lack of evidence and symmetric evidence, allowing for suspension of judgement, and rationalising intuitively rational responses to certain decision problems (Joyce, 2010; Bradley, 2015; Levi, 1978; Jeffrey, 1983).

5 Revision of supervaluational credences

Recall our presentation of the supervaluational Kripkean jump for truth in Proposition 2.10:

$$\mathcal{J}(S) = \mathsf{Partial}(\Theta_\tau(\mathsf{Precs}(S)))$$

We described that as following the following procedure: (i) use Precs to move from a partial interpretation of truth to the corresponding set of precisifications, (ii) use τ to revise each of these, and (iii) use Partial to move from the resultant collection of revised classical interpretations to a partial interpretation.

So it seems that what motivates this definition of $\mathcal{J}$ is just Θ, but since $\mathcal{J}$ is to be defined on partial interpretations rather than sets of precisifications directly, we consider how Θ acts on partial interpretations. So we have to also introduce stages (i) and (iii).

Now, in the case of credences, we are not working with a partial interpretation, but are instead working directly with the set of precisifications and want to know how to revise that. So stages (i) and (iii) aren't needed. Perhaps we should just revise our set of precisifications by revising each of the precisifications, i.e. by following Θ. We defined Θ_τ for the case of truth as $\Theta_\tau(\mathbb{Q}) := \{\tau(Q) \mid Q \in \mathbb{Q}\}$. We now simply present this as a more general definition that can apply to any revision function:

Definition 5.1. For a given (fixed) revision function ρ,

$$\Theta_\rho(\mathbb{C}) = \{\rho(c) \mid c \in \mathbb{C}\}.$$

We will generally drop the subscript as it's typically clear which ρ is used.

This simply takes the collection of revised individuals. It is a very intuitive notion of revision applied to a supervaluational credence and seems to follow naturally from the idea of supervaluationism that what happens on the supervaluational-side supervenes on what happens on the precise side.

Consider Passport, BadNavigator or `PrLiar`. We simply focus on an algebra consisting of the single sentence at stake in each of these sentences, so credence functions are given by real numbers between 0 and 1. Supervaluational credences are given by sets of precise credences, so this will be a set of real numbers between 0 and 1. Consider adopting $\{0, 1\}$. Revision of the precise credences is spelled out by ρ_{Passport}. In particular, credence 0 recommends adopting credence 1; and 1 recommends 0. Now, $\Theta(\{0, 1\}) = \{\rho(0), \rho(1)\} = \{1, 0\} = \{0, 1\}$. I.e. the supervaluational credence $\{0, 1\}$ is a fixed point of Θ. Whilst each precisification

not general enough, see Walley (2000).

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is undermining, the set, as a whole, is a non-undermining opinion to adopt in these cases. At least if one takes revision to follow Θ.

Some reasons that undermining credences were bad are that one's credences couldn't be used as evaluations of truth values or to guide one's actions. Is that problematic feature of undermining precise credences avoided when we find a supervaluational credence which is non-undermining in the sense given by Θ? Suppose one adopts supervaluational credence $\mathbb{C}$, and then gets a further choice: what supervaluational credence function ($\mathbb{C}'$) would you like to use for the purposes of evaluating estimated truth-values. It's not so clear how to answer this question. One suggestion is to ask each $c \in \mathbb{C}$ what credence function they think would be best, and take the set of these. How about choosing which credence function $\mathbb{C}'$ one would like to use for the purposes of guiding one's action? Any

However, whilst plausible, Θ does not guarantee a fixed point. Thus, if our notion of recommendation is spelled out with Θ we still might end up with a situation where every credal state, precise or imprecise, recommends another, that is, it is undermining, and thus not a candidate for the rational response to the situation.

5.1 Θ doesn't always have a fixed point — Spring

Consider the following kind of scenario:

> SPRING.
> You know that you're always overconfident in this type of situation. Except you also know that a credence value of 0 would be wrong.

What revision function does this lead to? It will have that $\rho(0) > 0$, and for all $x > 0$, $\rho(x) < x$. We need further details about this case: 'how overconfident?' 'how wrong?' in order to determine a revision function.[6] In fact any revision function with these properties leads to a Θ which has no fixed points. To work with a concrete example, we suppose that additional details are added to the case so that we obtain the following revision function:[7]

$$\rho_{\textsc{Spring}}(x) = \begin{cases} 1 & x = 0 \\ \frac{x}{2} & 0 < x \leqslant 1 \end{cases}$$

See Fig. 1 for an illustration.

As in the cases like PASSPORT, every credence value is undermining. But, unlike in PASSPORT, there is also no set of precise credence values which is a fixed point of Θ.

Proposition 5.2. *There is no (non-empty) fixed point of* $\Theta_{\textsc{Spring}}$

[6] In fact, I think natural ways of adding to this story will not guarantee a notion of a particular credence value being recommended, instead it might allow for ties. See Footnote 4.

[7] Our choice of $\rho(0) = 1$ is an extreme way to spell out the details of the story: you assign credence 0 in such cases only when it's definitely true. We have made this choice to parallel the McGee sentence which we will discuss in Section 6.1.

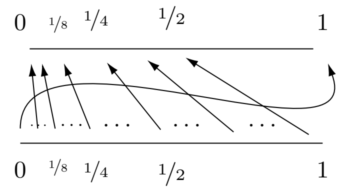

Figure 1: Illustration of ρ_{Spring}.

Proof. We first observe that for any $\mathbb{C}$ and $n \geqslant 1$, any $x \in \Theta^n(\mathbb{C})$ has $0 < x \leqslant {}^{1}\!/_{2^{n-1}}$.

Base case: For any $0 < x \leqslant 1$, $\rho(x) = {}^{x}\!/_{2}$; so $0 < \rho(x) \leqslant \rho(1) = {}^{1}\!/_{2}$. Also $\rho(0) = {}^{1}\!/_{2}$, so also $0 < \rho(0) \leqslant {}^{1}\!/_{2}$. Thus, any $x \in \mathbb{C} \subseteq [0, 1]$ has $0 < \rho(x) \leqslant 1$.

Inductive step: for any $0 < x \leqslant {}^{1}\!/_{2^{n-1}}$,

$$0 < \rho(x) = \frac{x}{2} \leqslant \frac{{}^{1}\!/_{2^{n-1}}}{2} = {}^{1}\!/_{2^{n}}.$$

so for any $x \in \Theta^n(\mathbb{C})$, $0 < \rho(x) \leqslant {}^{1}\!/_{2^{n}}$; and thus for any $x \in \Theta^{n+1}(\mathbb{C}) = \Theta(\Theta^n(\mathbb{C}))$, $0 < x \leqslant {}^{1}\!/_{2^{n}}$

Now, suppose $\mathbb{C} = \Theta(\mathbb{C})$. Then $\mathbb{C} = \Theta^n(\mathbb{C})$ for all n. So any $x \in \mathbb{C}$ has $x \leqslant {}^{1}\!/_{2^{n-1}}$ for all n. But the only such x is 0, and we also require that $x > 0$. So $\mathbb{C} = \varnothing$. □

So, this supervaluational jump does not guarantee that undermining credal states can be avoided.

We will propose an alternative supervaluational jump which will allow for fixed points. In order to do this, we will consider the supervaluational Kripkean jump for truth, viewed through the lenses of sets-of-precisifications.

6 Supervaluational Kripkean jump for truth as it applies to sets-of-precisifications

Recall the supervaluational Kripkean jump for truth was given by:

$$\mathcal{J}(S) = \mathsf{Partial}(\Theta_\tau(\mathsf{Precs}(S)))$$

In fact this jump does not lead to triviality. Whereas in the case of credences, Θ does. There are two places we might look to see the explanation for this difference.

Firstly, some revision functions, such as ρ_{Passport}, did lead to fixed points of Θ, whereas ρ_{Spring} rules them out. Perhaps τ is like ρ_{Passport} rather than ρ_{Spring} and does allow for fixed points of Θ?

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Secondly, perhaps $\mathcal{J}$ does not in fact match Θ, that is, insofar as it acts on sets of precisifications, $\mathcal{J}$ leads to a jump that differs from Θ?

The 'secondly' explanation is the right one. In fact, Θ_T does not have any fixed points either. We can identify a case that leads to Spring-style phenomenon by using the McGee sentence. Instead, how it treats sets of precisifications differs from Θ. We will use this insight to define an alternative jump on sets of precisifications which we can apply to credences in cases like Spring in order to obtain fixed points.

We now discuss these two points in more detail:

6.1 There are no non-trivial fixed points of Θ_T

Truth has its own case which leads to Spring-like phenomena: this can be seen by considering the McGee sentence, $\texttt{McGee}$, where

$$\mathsf{PA} \vdash \texttt{McGee} \leftrightarrow \neg\forall n{>}0\, \mathsf{T}^n\ulcorner\texttt{McGee}\urcorner.$$

Informally, we can describe $\texttt{McGee}$ as a sentence saying

$\texttt{McGee}$: Some truth iteration of $\texttt{McGee}$ is not true.

We can use this to show:

Proposition 6.1 (Halbach, 2014, Thereom 14.11)**.** *There is no (non-empty) fixed point of* Θ_T

Proof. We first observe that for any $\mathbb{Q}$ and $n \geqslant 1$, any $Q \in \Theta^n(\mathbb{Q})$ has $\texttt{McGee}, \mathsf{T}\ulcorner\texttt{McGee}\urcorner, \ldots, \mathsf{T}^{n-1}\ulcorner\texttt{McGee}\urcorner \in Q$, but also has some $\mathsf{T}^k\ulcorner\texttt{McGee}\urcorner \notin Q$. (See Fig. 2.)

	Q in $\Theta(\mathbb{Q})$	Q in $\Theta^2(\mathbb{Q})$	Q in $\Theta^3(\mathbb{Q})$	Q in $\Theta^4(\mathbb{Q})$	$\cdots$
$\texttt{McGee}$	some not-true	true	true	true	
$\mathsf{T}\ulcorner\texttt{McGee}\urcorner$		some not-true	true	true	
$\mathsf{T}^2\ulcorner\texttt{McGee}\urcorner$			some not-true	true	
$\mathsf{T}^3\ulcorner\texttt{McGee}\urcorner$				some nt	
$\vdots$					

Figure 2: Illustration of Θ with the McGee sentence

Base case: We need to show that any $Q \in \Theta(\mathbb{Q})$, some $\mathsf{T}^k\ulcorner\texttt{McGee}\urcorner \notin Q$. Since we always have $\varphi \in Q$ iff $\mathsf{T}\ulcorner\varphi\urcorner \in \tau(Q))$, if $\mathsf{T}^k\ulcorner\texttt{McGee}\urcorner \notin Q$, then $\mathsf{T}^{k+1}\ulcorner\texttt{McGee}\urcorner \notin \tau(Q)$. So we just need to consider Q where there is no such k. For such Q, $(\mathbb{N}, Q) \not\models \texttt{McGee}$, so $\texttt{McGee} \notin \tau(Q)$.

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Inductive step: by our inductive hypothesis we have that for any $Q \in \Theta^n(\mathbb{Q})$ has $\texttt{McGee} \in Q$, $\mathsf{T}\ulcorner\texttt{McGee}\urcorner \in Q$, ..., $\mathsf{T}^{n-1}\ulcorner\texttt{McGee}\urcorner \in Q$, but also has some $\mathsf{T}^k\ulcorner\texttt{McGee}\urcorner \notin Q$. So, since $\mathsf{T}\ulcorner\varphi\urcorner \in \tau(Q)$ iff $\varphi \in Q$, we have $\mathsf{T}\ulcorner\texttt{McGee}\urcorner \in Q$, $\mathsf{T}^2\ulcorner\texttt{McGee}\urcorner \in Q, \ldots, \mathsf{T}^n\ulcorner\texttt{McGee}\urcorner \in Q$, and $\mathsf{T}^{k+1}\ulcorner\texttt{McGee}\urcorner \notin Q$. Also, since some $\mathsf{T}^k\ulcorner\texttt{McGee}\urcorner \notin Q$, $(\mathbb{N}, Q) \models \texttt{McGee}$, so $\texttt{McGee} \in \tau(Q)$.

Now, suppose $\mathbb{Q} = \Theta(\mathbb{Q})$. Then also $\mathbb{Q} = \Theta^n(\mathbb{Q})$ for all n. So any $Q \in \mathbb{Q}$ has $\mathsf{T}^n\ulcorner\texttt{McGee}\urcorner \in Q$ for all n. But also $\mathsf{T}^k\ulcorner\texttt{McGee}\urcorner \notin Q$ for some k. So $\mathbb{Q} = \varnothing$. □

6.2 How $\mathcal{J}$ acts on sets of precisifications

So the explanation for why $\mathcal{J}$ does not result in triviality is not because the revision stage for truth allows for fixed points of Θ. It has to instead be that $\mathcal{J}$ does not lead to revision of sets of precisifications in accordance with Θ.

Instead, it leads to Δ:

Definition 6.2. For a set of precisifications, $\mathbb{Q}$,

$$\Delta(\mathbb{Q}) := \mathsf{Precs}(\mathsf{Partial}(\Theta(\mathbb{Q})))$$

Δ is how $\mathcal{J}$ acts on sets-of-precisifications, in the following sense:

Proposition 6.3. *If* $\mathsf{Precs}(S) = \mathbb{Q}$*, then* $\mathsf{Precs}(\mathcal{J}(S)) = \Delta(\mathbb{Q})$*.*

Proof. Suppose $\mathsf{Precs}(S) = \mathbb{Q}$. Then

$$\begin{aligned}
\mathsf{Precs}(\mathcal{J}(S)) &= \mathsf{Precs}(\mathsf{Partial}(\Theta(\mathsf{Precs}(S)))) && \text{Proposition 2.10} \\
&= \mathsf{Precs}(\mathsf{Partial}(\Theta(\mathbb{Q}))) && \text{as } \mathsf{Precs}(S) = \mathbb{Q} \\
&= \Delta(\mathbb{Q}) && \text{definition of } \Delta
\end{aligned}$$
□

Δ first applies Θ, and then translates the resultant set of precisifications into a partial model, then recovers the set of precisifications corresponding to that partial model. One might have expected that this just recovers $\Theta(\mathbb{Q})$, but that is not the case. It also adds some additional precise interpretations — those that agree with any definite opinions of $\Theta(\mathbb{Q})$. See Fig. 3 for illustration.

Proposition 6.4. $Q' \in \mathsf{Precs}(\mathsf{Partial}(\mathbb{Q}))$ *iff for any* φ*,*

- *if* $\varphi \in Q$ *for all* $Q \in \mathbb{Q}$ *then* $\varphi \in Q'$*, and*
- *if* $\varphi \notin Q$ *for all* $Q \in \mathbb{Q}$ *then* $\varphi \notin Q'$*.*

Proof. Using the definition of Precs, $Q' \in \mathsf{Precs}(\mathsf{Partial}(\mathbb{Q}))$ iff for any φ,

- if $\varphi \in \mathsf{Partial}(\mathbb{Q})^+$ then $\varphi \in Q'$, and
- if $\varphi \in \mathsf{Partial}(\mathbb{Q})^-$ then $\varphi \notin Q'$.

which, using the definition of $\mathsf{Partial}$ is equivalent to our criterion. □

Corollary 6.5. $\mathsf{Precs}(\mathsf{Partial}(\mathbb{Q})) \supseteq \mathbb{Q}$ *for any* $\mathbb{Q}$*. Thus, also,* $\Delta(\mathbb{Q}) \supseteq \Theta(\mathbb{Q})$

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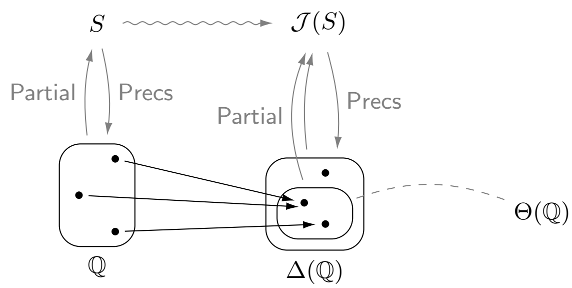

Figure 3: Revising partial interpretation and revising the set of precisifications

Proof. Follows immediately from Proposition 6.4. □

This addition of additional precise interpretations is what allows $\mathcal{J}$, and thus Δ, to have fixed points, where Θ has none.

To see this, consider how Δ treats the McGee sentence, which led to triviality of Θ.

Proposition 6.6. *There is some $Q \in \Delta(\wp(\mathsf{Sent}_\mathsf{T}))$ with all $\mathsf{T}^n\ulcorner\mathtt{McGee}\urcorner \in Q$. There is no such Q in $\Theta(\wp(\mathsf{Sent}_\mathsf{T}))$.*

Proof. □

6.3 How to account for Spring— add extra credences

We propose reconsidering the way of revising our supervaluational credences, replacing Θ with a jump that always allows for fixed points.[8]

Consider the claim that $[0,1]$ recommends $\{\rho_{\text{Spring}}(c) \mid c \in [0,1]\} = (0, 1/2]$. $0 \notin \Theta([0,1])$; it is not recommended by any individual. But, 0 is very close to being recommended, that is, it is the limit of a sequence of recommended credence values, or, equivalently, it is in the closure of $\Theta([0,1])$, understood in a topological sense. We might say that 0 is nearly recommended by $[0,1]$. So the proposal is that we liberalise the notion of recommendation to consider the idea that $[0,1]$ weakly-recommends $[0, 1/2]$, i.e. including 0 as well as all individually recommended credence values.

Liberalising the notion of recommendation to include additional credence values, like 0, who are not individually recommended, but are limits of those who are recommended will allow us to find a fixed point in the case of spring. In particular $\{0, 1/2, 1/4, 1/8, \ldots\}$ will be a fixed point of this notion (see Fig. 4)

To present this idea more carefully, we use the idea of taking the *closure* of the set of recommendations, in a topological sense, and define

$$\Gamma(\mathbb{C}) := \mathsf{closure}(\Theta(\mathbb{C})).$$

[8] Rivello (2018) proposes altering the limit stage, although to do that more information is required: we don't collapse to a set at each stage, but look at sequences of how the revision process takes us through the sets.

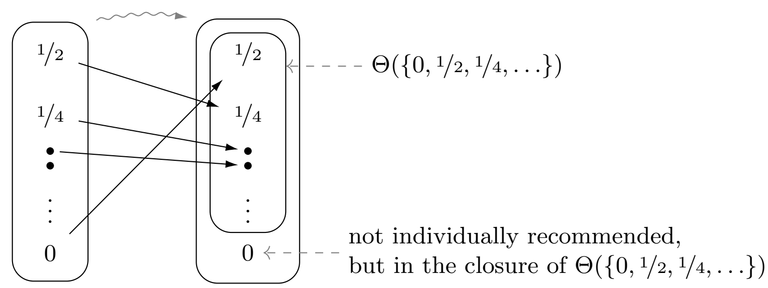

Figure 4: By including the additional credence, 0, we find a fixed point in the case of Spring.

Again this is relative to a ρ, which we suppress. One way of spelling out what it is to take a closure is to add any credences which are limits of some sequence from within the recommendations. For example 0 is the limit of the sequence $^1/_2, ^1/_4, ^1/_8, \ldots$, each of which is a member of $\Theta(\{0, ^1/_2, ^1/_4, ^1/_8, \ldots\})$; and thus $0 \in \mathsf{closure}(\Theta(\{0, ^1/_2, ^1/_4, ^1/_8, \ldots\})) = \Gamma(\{0, ^1/_2, ^1/_4, ^1/_8, \ldots\})$. We will be more precise in Definition 6.7

6.4 There is always a fixed point of Γ

Γ provides us with a jump where there will always be a fixed point, whatever ρ we start with. To prove for this claim we need to say a bit more about how to apply Γ in general. That is, we should say how to take a closure. In the case of spring we were focusing on a single proposition, and our credences are then just real numbers. So the notion of limits, closed sets and taking a closure is relatively familiar. For example an interval that includes its endpoints, say $[0.2, 0.6]$, is closed, whereas $(0.2, 0.6)$ is not as it doesn't include the endpoints 0.2 or 0.6. But in general we might have an opinion set $\mathcal{A}$ containing more than one proposition. So our credence functions are functions from $\mathcal{A}$ to $\mathbb{R}$. We have to describe what it is to take a closure of a set of such credence functions.

To do this we will first describe the notion of being a limit of a sequence of credence functions in this more general setting.

Definition 6.7. $x^* \in \mathbb{R}$ is the *limit* of a sequence $\langle x_\alpha \rangle$, if for any $\epsilon > 0$ there is β such that for all $\alpha > \beta$, $|x^* - x_\alpha| < \epsilon$. That is, the sequence gets arbitrarily close to x^*.

A credence function $c^* : \mathcal{A} \to \mathbb{R}$ is the *limit* of a sequence of credence functions $\langle c_\alpha \rangle$ if for each $\varphi \in \mathcal{A}$, $c^*(\varphi)$ is the limit of the sequence of $\langle c_\alpha(\varphi) \rangle$.

c^* is in the *closure* of $\mathbb{C}$ iff c^* is the limit of some sequence $\langle c_\alpha \rangle$ with each $c_\alpha \in \mathbb{C}$.

$\mathbb{C}$ is *closed* if it contains any limits of sequences in $\mathbb{C}$.[9]

[9]This is equivalent to the usual definition of a closed set in the product topology (Willard, 1970, Theorems 11.9)

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This gives us the so-called topology of pointwise convergence, which is also called the product topology.[10] In general, we might want to allow for non-countable opinion sets, so we extend the notion to be limits of longer sequences.

Theorem 6.8. *For any function ρ, Γ has a non-empty fixed-point.*

Proof. We first show:

Sublemma 6.8.1. *Γ is monotone, that is $\mathbb{C} \subseteq \mathbb{C}' \implies \Gamma(\mathbb{C}) \subseteq \Gamma(\mathbb{C}')$.*

Proof.

$$\begin{aligned}
\mathbb{C} \subseteq \mathbb{C}' &\implies \{\rho(c) \mid c \in \mathbb{C}\} \subseteq \{\rho(c) \mid c \in \mathbb{C}'\} \\
&\implies \Theta(\mathbb{C}) \subseteq \Theta(\mathbb{C}') \\
&\implies \mathsf{closure}(\Theta(\mathbb{C})) \subseteq \mathsf{closure}(\Theta(\mathbb{C}')) \\
&\implies \Gamma(\mathbb{C}) \subseteq \Gamma(\mathbb{C}') \qquad \square
\end{aligned}$$

Take a sequence: $\mathbb{C}_0 = \mathsf{Creds}_{\mathcal{A}}$, $\mathbb{C}_{\alpha+1} = \Gamma(\mathbb{C}_\alpha)$, $\mathbb{C}_\mu = \bigcap_{\alpha<\mu} \mathbb{C}_\alpha$ for μ a limit ordinal. Since Γ is monotone, the sequence of $\mathbb{C}_\alpha$ is decreasing, $\alpha < \beta \implies \mathbb{C}_\beta \subseteq \mathbb{C}_\alpha$. So this process will reach a fixed point, a set $\mathbb{C}$ where $\Gamma(\mathbb{C}) = \mathbb{C}$. What went wrong in the case of Θ is that the fixed point that is reached might be $\varnothing$, which was not a candidate for a legitimate opinion state. We need to ensure that in the case of Γ, it does not result in $\varnothing$.

We can never result in $\varnothing$ at a successor stage: if $\mathbb{C} \neq \varnothing$, then $\Gamma(\mathbb{C}) \supseteq \Theta(\mathbb{C}) \neq \varnothing$.

What went wrong with Θ is that at the limit stage we resulted in $\varnothing$. We have to ensure this will not happen with Γ.[11]

We say a family $\mathcal{E}$, of subsets of $\mathsf{Creds}_{\mathcal{A}}$ is *finitely consistent* iff any finite subcollection of $\mathcal{E}$ has non-empty intersection. In a compact space, any family of closed sets which are finitely consistent is consistent, that is, it has non-empty intersection (Willard, 1970, Theorem 17.4).[12] $\mathsf{Creds}_{\mathcal{A}}$, with the topology of pointwise convergence, is compact by Tychonoff's theorem (recalling that we defined this as functions from $\mathcal{A}$ to the compact set $[0, 1]$).[13]

Now each $\{\mathbb{C}_\alpha \mid \alpha < \mu\}$ is a family of closed sets which is finitely consistent (the intersection of any finite collection is just the final $\mathbb{C}_\alpha$ which is in the collection because the sets are decreasing). And thus, compactness gives us that the intersection of them all, that is, $\mathbb{C}_\mu$, will also be non-empty, as required. $\square$

[10] This is the same as I used in Campbell-Moore (2019).

[11] We can give an alternative argument by using the fact that in a compact space every sequence has some subsequence which has a limit. (see Willard, 1970, Theorem 17.4).

[12] We usually talk about 'having the finite intersection property', which I have called 'finite consistent' to make the parallel with **?**.

[13] See Willard (1970, pp. 120, 278). This means that the non-empty closed subsets of $\mathsf{Creds}_{\mathcal{A}}$ form a ccpo, in the sense of **?**.

6.5 Obtaining non-undermining credal states

Θ wasn't able to always provide fixed points. Whereas Γ is. However, I don't propose Γ is the right characterisation of recommendation.

In everyday cases of belief we have $\rho(c) = c$, at least for probabilistic c. For example, suppose someone is adopting a degree of belief in whether it's going to rain tomorrow. The opinion that she adopts typically provides no additional information about whether it will rain or not and $\rho(c) = c$. In that case, also every supervaluational credence (containing only probability functions) has $\Theta(\mathbb{C}) = \mathbb{C}$. So a supervaluational credence such as $(0, 1)$ is plausibly self-recommending. However, if we apply Γ to $(0, 1)$ we additionally add the extra limit points, $\Gamma((0, 1)) = [0, 1]$. So, if recommendation was spelled out with Γ, we would say that $(0, 1)$ is undermining. Since in this case we do not need to add the extras, I would want to say that $(0, 1)$ is not-undermining.

Instead, we should use both Θ and Γ to characterise when a supervaluational credence is undermining.

Definition 6.9. $\mathbb{C}$ is *non-undermining* if

$$\Theta(\mathbb{C}) \subseteq \mathbb{C} \subseteq \Gamma(\mathbb{C}).$$

Otherwise, it is undermining.

If $\Theta(\mathbb{C}) \nsubseteq \mathbb{C}$ it recommends including any $\rho(c)$ that are left out. If $\mathbb{C} \nsubseteq \Gamma(\mathbb{C})$, it recommends removing the additional members who are not even nearly supported by a member of $\mathbb{C}$. Those are undermining credal states to adopt. Fixed points of Θ or Γ, then, are non-undermining.

One might want to liberalise Γ further. Any credences that are in the closure were said to be nearly recommended, and were included. We might want to liberalise this notion further to, for example, the convex closure: credence functions that are in the closed convex hull of $\Theta(\mathbb{C})$ are also optional extras that can be added without being undermining. Liberalising Γ this way would allow that $[0, 1]$ is a non-undermining response to PASSPORT.

We have now got an account that allows us to always find some non-undermining opinion state for any situation, as spelled out by giving a revision funtion on precise credences. Since opinion states cannot be undermining and rational, this means that we now have an option for what to do in challenging cases such as PASSPORT and BADNAVIGATOR and SPRING. In PASSPORT and BADNAVIGATOR only $\{0, 1\}$ is non-undermining. In SPRING, only $\{0, 1/2, 1/4, \ldots\}$ is non-undermining. In a case like PROMOTION, any set which is symmetric around 0.5 is non-undermining, for example $\{0.5\}$, $\{0.2, 0.8\}$, or $[0.2, 0.8]$. In usual cases, where $R(c) = c$ for probabilistic c, this does nothing new, every set of probabilistic credence functions is non-undermining.[14]

This account treats such cases seriously and provides candidates for the rational response in the form of supervaluational opinion states, which are modelled as sets of precise credence functions, so-called imprecise probabilities.

[14]If each non-probability function recommends a probability function, as we might want since non-probability functions are accuracy-dominated, it will tell us that sets containing non-probability functions are undermining.

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7 Conclusion

7.1 Avoiding undermining epistemic states

Various scenarios have been posed as a challenge for what it takes to be rational. In these scenarios, we want to see which epistemic states are rationally permissible. A criterion for an epistemic state to be rationally permissible is that it doesn't undermine its own adoption as such credences can't be relied on.

We have given a proposal for the epistemic state to adopt in such cases which is not undermining when we move to the supervaluational setting. We proposed to think about supervaluational credences as a *set of* precise credence functions. This is an account of belief which has independent support in epistemology, under the term 'imprecise probabilities'.

To spell this out we then thought about when a supervaluational credence is undermining. At a first pass, we reason about such a supervaluational credence by seeing what each individual credence recommends. We see that in cases like Passport, $\{0, 1\}$ is non-undermining as 0 recommends 1 and 1 recommends 0. However, in a case like Spring, this way of reasoning did not avoid undermining opinions. So we proposed that if everything in the set is in the closure of the set of recommended precise credences, then the set is also not undermining. That allowed for non-undermining opinion states in every scenario, where the scenarios are spelled out with a revision funtion R.

One important question left open by this analysis is how to think about revenge challenges. Can there be a scenario which says: if you adopt $\{0, 1\}$, then it's true. The solution of adopting sets of precise credences to allow non-undermining opinions then does not work. That only works if recommendation is appropriately spelled out on the precise site. We leave investigation of how to think about this to future work.

7.2 What we've learned about the supervaluational Kripke construction

We have thought about how to apply the supervaluational Kripke construction in the case of credences. This development has shown us quite general features of the supervaluational Kripkean account which will be beneficial in applying in a range of cases beyond truth. In particular:

We can show that the supervaluational account really just needs to be told how to revise each of the precise objects, any further formulation of how that revision happens is not important for the final account. This helps us understand the supervaluational Kripkean account in a way that will allow it to apply to more notions of interest.

We also showed that the choice of modelling truth as a partial interpretation is of technical importance to avoid triviality. If we model our supervaluational notion in the most general way possible, namely, simply giving the set of precisifications themselves, then the application of the supervaluational Kripke account leads to contractions. We discussed what one should do in such cases

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where one is interested in other models – we permitted (though did not require) one to take a closure of the set of precisifications. This guarantees fixed points when the underlying space is compact. (This is why ω-consistency cannot be used: that does not give a compact space.)

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