十. GWS 模型

1. 弱力的唯象理论

1) Fermi 作用

Fermi耦合常数 β衰变：$n \to p + e^- + \bar{\nu}_e$ ← 中微子

$$\mathcal{L}_F = -G_F\underbrace{[\bar{p}(x)\gamma^\mu n(x)]}_{J^\mu(x)}\underbrace{[\bar{e}(x)\gamma_\mu \nu_e(x)]}_{j_\mu(x)} + h.c.$$

⟶ 流-流耦合

↓

普适Fermi相互作用：同样适于其它弱力

场的左旋态、

宇称不守恒 → $\bar{\nu}_e$左旋 ⟶ 场的左旋态 $j^\mu = \bar{e}\gamma^\mu(1-\gamma^5)\nu_e + \cdots$

↓

(u, d, c, s 为夸克场量)

2) Cabibbo角

$$J^\mu = \bar{u}\gamma^\mu(1-\gamma^5)d_\theta + \bar{c}\gamma^\mu(1-\gamma^5)s_\theta$$

$$\begin{pmatrix} d_\theta \\ s_\theta \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix}$$

Fermi 相互作用：$\mathcal{L}_F = -\frac{1}{\sqrt{2}}G_F \mathcal{J}_\mu^+ \mathcal{J}^\mu$，

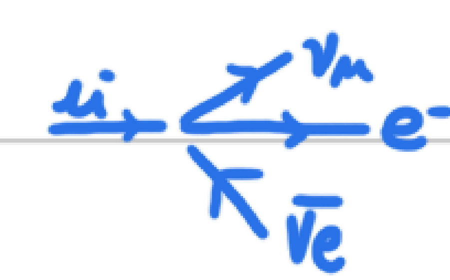
$\mathcal{J}^\mu = J^\mu + j^\mu = \bar{e}\gamma^\mu(1-\gamma^5)$

夸克模型：中子衰变为夸克衰变 $d \to u + e^- + \nu_e$

μ子：$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$

四动量：$s \quad p \quad q \quad k \Rightarrow \vec{s} = \vec{p} + \vec{q} + \vec{k} \quad w_\mu = w_e + q + k$

$$\mathcal{L}_F = -\frac{1}{\sqrt{2}}G_F[\bar{\nu}_\mu\gamma^\lambda(1-\gamma^5)\mu][\bar{e}\gamma_\lambda(1-\gamma^5)\nu_e]$$



平均跃迁振幅：$\overline{P_{fi}} = \overline{\sum} V(2\pi)^4\delta^4(P_f - P_i)|M_{fi}|^2$

⇓

$$\overline{P_{fi}} = \frac{1}{2}\sum_{s}\int \frac{Vd^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\cdot V(2\pi^4)\underbrace{N_{fi}^2}_{\downarrow}|M_{fi}|^2$$

$$N_{fi} = \frac{1}{4\sqrt{w_\mu w_e q k}}$$

$\sum|M_{fi}|^2 \to M_{fi} = \frac{-iG_F}{\sqrt{2}}\left[\bar{u}_{\nu_\mu}(k,\xi_{\nu_\mu})\gamma^\lambda(1-\gamma_5)u_\mu(s,\xi_\mu)\right]\left[\bar{u}_e(p,\xi_e)\gamma_\lambda(1-\gamma^5)v_{\nu_e}(q,\xi_{\nu_e})\right]$ (一阶图为上：)

$$\sum|M_{fi}|^2=\frac{G_F^2}{2}\sum[\bar v_{\nu_e}\gamma_\rho(1-\gamma_5)u_e][\bar u_\mu\gamma^\rho(1-\gamma_5)u_{\nu_\mu}]\cdot[\bar u_{\nu_\mu}\gamma^\lambda(1-\gamma^5)u_\mu][\bar u_e\gamma_\lambda(1-\gamma^5)v_{\nu_e}]$$

$\sum_s u(k,s)\bar u(k,s)=\not k+m$

$\sum v(k,s)\bar v(k,s)=\not k-m$

↓收缩

$$=\frac{G_F^2}{2}\cdot tr[(\not p+m_e)\gamma_\lambda(1-\gamma_5)\not q\gamma_\rho(1-\gamma_5)]\ tr[(\not s+m_\mu)\gamma^\rho(1-\gamma^5)\not k\gamma^\lambda(1-\gamma^5)]$$

$$=128G_F^2(qs)(pk)$$

$$\Gamma_\mu=\bar P_{fi}=4G_F^2\int\frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\frac{S_\sigma P_\rho q^\sigma k^\rho}{w_\mu w_e qk}(2\pi)^4\delta^4(P_f-P_i)$$

中微子部分：$I^{\sigma\rho}=\int\frac{d^3\vec q}{(2\pi)^3}\frac{d^3\vec k}{(2\pi)^3}\frac{q^\sigma k^\rho}{qk}(2\pi)^4\delta(Q-q-k)$ → 协变,取质心系，$\vec q+\vec k=0\Rightarrow \begin{matrix}q^i=-k^i\\ q^0=k^0=k\end{matrix}$

$$=\int\frac{d^3\vec k}{(2\pi)^3}\frac{-k^\sigma k^\rho+2kkg^{\sigma0}g^{\rho0}}{kk}2\pi\delta(Q_0-2k)$$

kk为ν_μ能量平方

$$=\frac{1}{24\pi}Q_0^2(g^{\sigma\rho}+2g^{\sigma0}g^{\rho0})=\frac{1}{24\pi}(Q^2g^{\sigma\rho}+2Q^\sigma Q^\rho)$$

$$=4G_F^2\int\frac{d^3p}{(2\pi)^3}\frac{S_\sigma P_\rho I^{\sigma\rho}}{w_\mu w_e}=\frac{G_F^2}{6\pi}\int\frac{d^3p}{(2\pi)^3}\frac{Q^2(SP)+2(SQ)(PQ)}{w_\mu w_e}$$

↓μ静止系，$PQ\approx PS=m_\mu w_e$（略电子质量）

$$=\frac{G_F^2}{12\pi^3}\int_0^{\frac{m_\mu}{2}}w_e^2dw_e(3m_\mu^2-4m_\mu w_e)=\frac{G_F^2m_\mu^5}{192\pi^3}$$

衰变宽度 $\frac{1}{\tau_\mu}=\Gamma_\mu$ ：单位时间衰变率（与宽度成正比）

中间Bose子模型 → $\mathcal L_W=-gJ^\mu W_\mu+h.c.$ ，W_μ为重矢量场

↓

传播子 $iD_{\mu\nu}(k)=i\frac{-1}{k^2-m_W^2}(g_{\mu\nu}-\frac{k_\mu k_\nu}{m_W})$

↓

$$\frac{g^2}{m_W^2}=\frac{G_F}{\sqrt2}\Rightarrow m_W\approx100\text{GeV}$$

2.自发对称性破缺

对称性破缺：复标量场ϕ^4：$\mathcal L=\partial_\mu\phi^\dagger\partial^\mu\phi-V(\phi,\phi^\dagger)=\partial_\mu\phi^\dagger\partial^\mu\phi-m^2\phi^\dagger\phi-\lambda(\phi^\dagger\phi)^2$

规范变换：$\phi'=e^{i\theta}\phi$ 不变

基态→极小→ $\frac{\partial V}{\partial\phi^\dagger}=m^2\phi+2\lambda\phi(\phi^\dagger\phi)=0\Rightarrow\begin{matrix}m^2>0\to|\phi|^2=0\\ m^2<0\to|\phi|^2=-\frac{m^2}{2\lambda}=a^2\end{matrix}$ $(\lambda>0)$ ⟶ $|\langle0|\phi|0\rangle|^2=a^2$，真空不为0

Goldstone 粒子：$\phi(x)=a+\frac{1}{\sqrt2}(h(x)+i\rho(x))$ → $\langle0|h|0\rangle=\langle0|\rho|0\rangle=0$

$$\mathcal L=\frac12(\partial_\mu h)^2+\frac12(\partial_\mu\rho)^2-\underbrace{\lambda v^2h^2}_{h\text{获得质量}\sqrt{2\lambda}v}-\lambda vh(h^2+\rho^2)-\frac{\lambda}{4}(h^2+\rho^2)^2\qquad(v=\sqrt2a)$$

h 质量 $2\lambda a$，与 $|\langle k|\phi|0\rangle| = a$ 正比 $\Rightarrow$ Higgs场的 Higgs 粒子

ρ 无质量： Goldstone 场

Goldstone定理：

$\mathcal{L} = \partial_\mu\phi^+\partial^\mu\phi - V(\phi,\phi^+)$，$\phi$为N维内部空间矢，$\phi=\phi_0$使$V(\phi)$极小 $\left.\frac{\partial V}{\partial\phi_a}\right|_{\phi=\phi_0} = 0$

$V(\phi) = V_0 + \frac{1}{2}m_{ab}\chi^a\chi^b + O(\chi^3)$

$(\chi = \phi(x) - \phi_0)$，$m_{ab}$ 非负定 $\left.\frac{\partial^2 V}{\partial\phi_a\partial\phi_b}\right|_{\phi_0} \geq 0$

转动 $\phi' = e^{i\theta_a T^a}\phi \Rightarrow \delta V|_{\phi=\phi_0} = \frac{1}{2}m_{ab}\delta\phi^a\delta\phi^b = 0$ $(\delta V=0)$

$\delta\phi^a \neq 0 \Rightarrow m_{aa}=0$，无质量

$\delta\phi^a = 0 \Rightarrow m_{aa}$可不为0，对称性简并，可有质量

(n个$\delta\phi^a$为0，即n简并，有n个有质量粒子)

QM证法： Noether定理：对称性 $\rightleftarrows$ 守恒量

H不是哈密顿量 H Q

$[H,Q]=0$

真空：$H|0\rangle = 0$ 对称性：$e^{i\theta Q}|0\rangle = |0\rangle$，

$Q|0\rangle = 0$

对称性破缺：$Q|0\rangle \neq 0 \rightarrow H Q|0\rangle = (HQ - QH)|0\rangle = [H,Q]|0\rangle = 0$

故 $Q|0\rangle$有：$H(Q|0\rangle)=0$，$Q|0\rangle$为真空态，

真空发生简并

对量子场，$Q = \int d^3x\, j^0(x,t)$，j守恒流，Q守恒$\rightarrow$与t无关

$|k\rangle = \int d^3x\, e^{i\vec{k}\cdot\vec{x}} j^0(x,t)|0\rangle$

$\lim_{k\to 0}|k\rangle = Q|0\rangle$

真空态，$H|k=0\rangle = 0$

无质量

3. Higgs 机制

1. Abel场：荷为g：$D_\mu = (\partial_\mu - igA_\mu) \Rightarrow \mathcal{L} = D_\mu\phi^+ D^\mu\phi - m^2\phi^+\phi - \lambda(\phi^+\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

$= (\partial_\mu\phi^+\partial^\mu\phi - m^2\phi^+\phi - \lambda(\phi^+\phi)^2) + (-ig\phi^+\overleftrightarrow{\partial_\mu}\phi A^\mu + g^2A_\mu A^\mu\phi^+\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu})$

$(v=\sqrt{2}a)$ 取$\phi = a + \frac{1}{\sqrt{2}}(h(x) + i\rho(x)) \Rightarrow \mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2(h^2+\rho^2)A_\mu A^\mu + g^2vhA_\mu A^\mu + \frac{1}{2}g^2v^2A_\mu A^\mu + gv\partial_\mu\rho A^\mu + gh\overleftrightarrow{\partial_\mu}\rho A^\mu$

Goldstone与A^μ可互换

定域规范：$\phi' = e^{i\gamma}\phi$，$A_\mu' = A_\mu - \frac{1}{g}\partial_\mu\gamma$

$\gamma(x)\to 0$，$\delta\phi = i\gamma\phi \rightarrow \delta h = -\gamma\rho$，$\delta\rho = \gamma h + \gamma v \rightarrow (h,\rho)$变换为平移+转动

取$\gamma(x)$使$\rho(x)=0 \Rightarrow$ U规范

$\mathcal{L}_A = -\frac{1}{4}F^2 + \frac{1}{2}g^2v^2A_\mu A^\mu$ ←ρ耦合项消去，获得质量 g^2v^2，与h耦合

↑ Higgs 机制

$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2h^2 - \lambda vh^3 - \frac{1}{4}\lambda h^4$

$\mathcal{L}_I = (g^2vh + \frac{g^2}{2}h^2)A_\mu A^\mu$

R_ξ规范：取$\Omega[A_\mu] = \partial_\mu A^\mu - \xi gv\rho$ 't Hooft规范/可重正化规范

Faddev-Popov 规范固定项：$-(\partial_\mu A^\mu - \xi gv\rho)^2/2\xi$ $\rightarrow$ $gv\partial_\mu\rho\cdot A^\mu - \frac{1}{2\xi}(\partial_\mu A^\mu - \xi gv\rho)^2 = -\frac{1}{2\xi}(\partial_\mu A^\mu)^2 - \frac{1}{2}\xi g^2v^2\rho^2 + gv\partial_\mu(\rho A^\mu)$

↓

$$\mathcal{L}=\underbrace{\tfrac{1}{2}(\partial_\mu h)^2-\lambda v^2h^2}_{\substack{H子\\ \mu=\sqrt{2\lambda}v}}+\underbrace{\tfrac{1}{2}(\partial_\mu\rho)^2-\tfrac{1}{2}\xi g^2v^2\rho^2}_{\substack{Gs子\\ m=\sqrt{\xi}gv}}-\underbrace{\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}g^2v^2A_\mu A^\mu-\tfrac{1}{2\xi}(\partial_\mu A^\mu)^2}_{规范子\to 质量\ M=gv}$$

$m=\sqrt{\xi}gv$，与规范选择有关
$\downarrow$
含 ξ 项可消去

U规范传播子：$\dfrac{-i}{k^2-\mu^2}$ $\qquad\qquad$ $\dfrac{-i}{k^2-M^2}\left(g_{\mu\nu}-\dfrac{k_\mu k_\nu}{M^2}\right)$

R_ξ规范传播子：$\dfrac{-i}{k^2-\mu^2}$ $\qquad$ $\dfrac{-i}{k^2-\xi M^2}$ $\qquad$ $\dfrac{-i}{k^2-M^2}\left(g_{\mu\nu}-(1-\xi)\dfrac{k_\mu k_\nu}{k^2-\xi M^2}\right)$

2. 非Abel场

eg：SU(2)场 $\phi=\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$，$U=e^{i\theta_k T^k}=e^{i\theta_k\frac{\tau^k}{2}}$，$\tau$为Pauli矩阵，$[T_i,T_j]=i\epsilon_{ijk}T^k$

$\downarrow$

$\mathcal{L}=(D_\mu\phi)^\dagger D^\mu\phi-m^2\phi^\dagger\phi-\lambda(\phi^\dagger\phi)^2-\frac{1}{4}F^i_{\mu\nu}F_i^{\mu\nu}$ $\quad$，$D_\mu=\partial_\mu+igA^i_\mu T^i$；$F^i_{\mu\nu}=\partial_\mu A^i_\nu-\partial_\nu A^i_\mu-g\epsilon^{ijk}A^j_\mu A^k_\nu$

真空选择：$\phi=\begin{pmatrix}\chi_1+i\chi_2\\ \chi_3+i\chi_4\end{pmatrix}\Rightarrow\phi^\dagger\phi=\chi_1^2+\chi_2^2+\chi_3^2+\chi_4^2=a^2\Rightarrow$ 圆上简并

U规范：取真空 $\phi(x)=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h(x)\end{pmatrix}$

$$V=m^2\phi\phi^\dagger+\lambda(\phi^\dagger\phi)^2=\lambda(\phi^\dagger\phi)(\phi^\dagger\phi-v^2)=\frac{\lambda}{4}\left[(h^2+2vh)^2-v^4\right]$$

$$D_\mu\phi^\dagger D^\mu\phi=\partial_\mu\phi^\dagger\partial^\mu\phi+ig\partial_\mu\phi^\dagger A^\mu\phi-ig\phi^\dagger A_\mu\partial^\mu\phi+g^2\phi^\dagger A_\mu A^\mu\phi$$
$$=\frac{1}{2}(\partial_\mu h)^2+\frac{1}{2}g^2(v+h)^2A_\mu A^\mu$$

故 $\mathcal{L}=T-V=\underbrace{-\frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu}+\frac{1}{2}g^2v^2A_\mu A^\mu}_{规范粒子}+\underbrace{\frac{1}{2}(\partial_\mu h)^2-\lambda v^2h^2}_{Higgs子}-\underbrace{\lambda vh^3-\frac{\lambda}{4}h^4}_{Higgs子自耦合}+\underbrace{g^2vhA_\mu A^\mu+\frac{1}{2}g^2h^2A_\mu A^\mu}_{Higgs子与规范子耦合}+\underbrace{\frac{1}{4}\lambda v^4}_{可略}$

粒子谱：SU(2) → 4种 Higgs子 + 3种规范子 × 2自由度 = 10种

规范后：3个 Goldston Bose子被吃 ⇒ 1 + (3种规范子 × 3自由度) = 10种

4. 弱同位旋空间

手征态：$\psi_L=P_L\psi=\frac{1}{2}(1-\gamma^5)\psi$，$\psi_R=P_R\psi=\frac{1}{2}(1+\gamma^5)\psi$，$\bar{\psi}_L=(P_L\psi)^\dagger\gamma^0=\psi^\dagger P_L\gamma^0$

↓

对于弱流，$j^\mu=\bar{\psi}\gamma^\mu(1-\gamma^5)\psi=2\bar{\psi}\gamma^\mu P_L\psi=2\psi^\dagger\gamma^0\gamma^\mu P_L^2\psi=2\psi_L^\dagger\gamma^0\gamma^\mu\psi_L=2\bar{\psi}_L\gamma^\mu\psi_L$ ⇒仅与左旋量有关

$j^\mu=\sum_{l=e,\mu,\tau}\bar{l}_L\gamma^\mu\nu_{lL}$ ⇒定义矢量 $L_e=\begin{pmatrix}\nu_e\\e\end{pmatrix}_L$，取左旋分量

↳ 弱同位旋空间

取 $j^{\mu i}=\sum\bar{L}_L\gamma^\mu\tau^i L_L$，$\tau$为Pauli矩阵

↳ $j^{\mu1}=\sum(\bar{e}_L\gamma^\mu\nu_{eL}+\bar{\nu}_{eL}\gamma^\mu e_L)$

$j^{\mu2}=i\sum(\bar{e}_L\gamma^\mu\nu_{eL}-\bar{\nu}_{eL}\gamma^\mu e_L)$] 电荷变

$j^{\mu3}=\sum(\bar{\nu}_{eL}\gamma^\mu\nu_{eL}-\bar{e}_L\gamma^\mu e_L)$ → 中性流 （1973年发现）

$\tau^-=\frac{1}{2}(\tau^1-i\tau^2)=\begin{pmatrix}0&0\\1&0\end{pmatrix}$，$\tau^+=\frac{1}{2}(\tau^1+i\tau^2)=\begin{pmatrix}0&1\\0&0\end{pmatrix}$

$j^{\mu+}=\sum\bar{l}_L\gamma^\mu\nu_{lL}$，$j^{\mu-}=\sum\bar{\nu}_{lL}\gamma^\mu l_L$

规范对称性：

拉氏密度 $\mathcal{L}_1=\sum_{l=e,\mu,\tau}i\bar{l}\gamma^\mu\partial_\mu l+\sum_{e,\mu,\tau}i\bar{L}_L\gamma^\mu\partial_\mu L_L$?如何来（保证手征对称）

$=\sum i(\bar{L}_L\gamma^\mu\partial_\mu L_L+\bar{R}_L\gamma^\mu\partial_\mu R_L)$，R为单态，L为2重态。$(R_e=e_R)$

U(1) 相位变换：$l'=e^{i\alpha\Upsilon}l$，Υ为变换算符：$\Upsilon R=\frac{1}{2}Y_R R$，$\Upsilon L=\frac{1}{2}Y_L L$

α为常时，$\mathcal{L}'$不变。

SU(2)弱同位旋空间变换：$L'=e^{i\beta^i T^i}L=e^{\frac{i\beta^i\tau^i}{2}}T$；$R'=R$ $(T^iL=\frac{1}{2}\tau^iL,\ T^iR=0)$

两种变换对易：$[T^i,\Upsilon]=0$ ⇒ $U=e^{i\alpha\Upsilon+i\beta^iT^i}$

定域：$D_\mu=\partial_\mu+ig_1\Upsilon B_\mu+ig_2T^iW^i_\mu$，$g_1,g_2$为耦合常数

$D_\mu R_e=(\partial_\mu+\frac{ig_1}{2}Y_RB_\mu)R_e$

$D_\mu L_e=(\partial_\mu+\frac{ig_1}{2}Y_LB_\mu+\frac{ig_2}{2}\tau^iW^i_\mu)L_e=\partial_\mu L_e+\frac{i}{2}\begin{pmatrix}g_1Y_LB_\mu+g_2W^3_\mu & g_2(W^1_\mu-iW^2_\mu)\\ g_2(W^1_\mu+iW^2_\mu) & g_1Y_LB_\mu-g_2W^3_\mu\end{pmatrix}\begin{pmatrix}\nu_e\\e\end{pmatrix}_L$

↳ 有意义项为：$W^\pm_\mu=\frac{1}{\sqrt{2}}(W^1_\mu\mp iW^2_\mu)$，$W^0_\mu=W^3_\mu$

Weinberg转动：

B_μ与轻子, ν耦合：不为电磁场
W_μ^0与轻子, ν耦合

$\Rightarrow$ 共作用： $A_\mu = \cos\theta_w B_\mu + \sin\theta_w W_\mu^0$
$Z_\mu = -\sin\theta_w B_\mu + \cos\theta_w W_\mu^0$

$$g_1 Y_L B_\mu + g_2 W_\mu^0 = \underbrace{(g_1 Y_L \cos\theta_w + g_2 \sin\theta_w)}_{}A_\mu + (-g_1 Y_L \sin\theta_w + g_2 \cos\theta_w) Z_w$$

当A_μ可代表电磁场，A_μ与ν无作用： 为0 $\Rightarrow$ $\theta_w = \arcsin\left(\frac{-g_1 Y_L}{\sqrt{g_1^2 Y_L^2 + g_2^2}}\right)$ (实验测得 $\sin^2\theta_w = 0.23$)

此时协变微商： $D_\mu = \partial_\mu + (i g_1 Y \cos\theta_w + i g_2 T^3 \sin\theta_w) A_\mu$
$+ (-i g_1 Y \sin\theta_w + i g_2 T^3 \cos\theta_w) Z_\mu$
$+ i g_2 (T^2 W_\mu^2 + T^1 W_\mu^1)$

$= \partial_\mu + i g_1 \cos\theta_w (Y - Y_L T^3) A_\mu$
$+ \cdots$

对比有：当A_μ代表电磁场，取 $g_1 \cos\theta_w = e$，
$Q = -Y_L T^3 + Y$

联立 Gell-Mann-Nishijima公式： $Q R_e = -R_e$

$Q L_e = Q\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L = \begin{pmatrix}0\times\nu_e \\ -1\times e\end{pmatrix}_L$

$Y_L = -1,\ Y_R = -2,\ Q = T^3 + Y$

$\sin\theta_w = \frac{g_1}{\sqrt{g_1^2+g_2^2}},\ e = \frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}}$

$$D_\mu L_e = \partial_\mu L_e + \frac{i}{2}\begin{pmatrix} \sqrt{g_1^2+g_2^2} Z_\mu & \sqrt{2} g_2 W^+ \\ \sqrt{2} g_2 W_\mu^- & -2eA_\mu + \frac{g_1^2 - g_2^2}{\sqrt{g_1^2+g_2^2}} Z_\mu \end{pmatrix} L_e$$

轻子拉氏密度 $\mathcal{L}_{LG} = \sum(i\bar{L}_e \gamma^\mu D_\mu L_e + \bar{R}_e \gamma^\mu D_\mu R_e) = \mathcal{L}_l + \mathcal{L}_{lI}$

$\mathcal{L}_l = \sum(i\bar{L}_e \not\partial L_e + \bar{R}_e \not\partial R_e)$.

$$\mathcal{L}_{lI} = -\frac{1}{2}\sum \bar{l}_L \begin{pmatrix} \sqrt{g_1^2+g_2^2} \not Z & \sqrt{2} g_2 \not W^+ \\ \sqrt{2} g_2 \not W^- & -2e\not A + \frac{g_1^2-g_2^2}{\sqrt{g_1^2+g_2^2}} \not Z \end{pmatrix} l_L + \sum \bar{l}_R \frac{g_1 g_2 \not A + g_1^2 \not Z}{\sqrt{g_1^2+g_2^2}} l_R$$

$$= \sum e[\bar{l}_L \not A l_L + \bar{l}_R \not A l_R] - \frac{1}{2}\sum\left(\bar{l}_L \begin{pmatrix} \sqrt{g_1^2+g_2^2}\not Z & 0 \\ 0 & \frac{g_1^2-g_2^2}{\sqrt{g_1^2+g_2^2}}\not Z \end{pmatrix} l_L + 2\bar{l}_R \frac{g_1^2 \not Z}{\sqrt{g_1^2+g_2^2}} l_R\right) + \sum -\frac{1}{2}\bar{l}_L \begin{pmatrix} & \sqrt{2} g_2 \not W^+ \\ \sqrt{2} g_2 \not W^- & \end{pmatrix} l_L$$

与A^μ耦合，
$= -q\sum \bar{l}\gamma^\mu l A_\mu$
QED

与Z^μ耦合
$-\frac{1}{2}\frac{g_2}{\cos\theta_w}\sum(\bar{L}_e \gamma^\mu \tau^3 L_e + 2\sin^2\theta_w \bar{e}\gamma^\mu e) Z_\mu$
预言的中性流

与$W^\pm$耦合
$-\frac{g_2}{2\sqrt{2}}(j^{\mu -} W_\mu^- + j^{\mu +} W_\mu^+) \Rightarrow g_2^2 = \frac{8 G_F m_w^2}{\sqrt{2}}$
弱作用

5. GWS模型

轻子弱同位旋空间 + Higgs机制赋予质量

弱同位旋复标量场中，$V(\phi)=m^2\phi^\dagger\phi+\lambda(\phi^\dagger\phi)^2$

取真空：$\phi(x)=\frac{1}{\sqrt2}\begin{pmatrix}0\\ v+H(x)\end{pmatrix}$，$v^2=-\frac{m^2}{\lambda}$，$V(\phi)=\lambda v^2H^2+\lambda vH^3+\frac{\lambda}{4}H^4\underbrace{-\frac14\lambda v^4}_{忽略}$　$m_H=\sqrt{2\lambda}\,v$

U(1)对称性：$Y\phi=\frac12 Y_H\phi$，由GMN公式：$Q\phi=(T^3+\frac{Y_H}{2})\phi=\frac12\begin{pmatrix}(1+Y_H)\phi^+\\(-1+Y_H)\phi^0\end{pmatrix}$，真空不带电：$\phi^0$为电中性：$Y_H=1$

自由规范场：$B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu=\cos\theta_w A_{\mu\nu}-\sin\theta_w Z^{(a)}_{\mu\nu}$　（角标(a)表示交换性部分）

$W^{0(a)}_{\mu\nu}=\partial_\mu W^0_\nu-\partial_\nu W^0_\mu=-\sin\theta_w A_{\mu\nu}+\cos\theta_w Z^{(a)}_{\mu\nu}$

$\mathcal{L}_B=-\frac14 B_{\mu\nu}B^{\mu\nu}=-\frac{\cos^2\theta_w}{4}A_{\mu\nu}A^{\mu\nu}-\frac{\sin^2\theta_w}{4}Z^{(a)}_{\mu\nu}Z^{(a)\mu\nu}+\frac{\sin2\theta_w}{4}Z^{(a)}_{\mu\nu}A^{\mu\nu}$

$\mathcal{L}_W=-\frac{\sin^2\theta_w}{4}A_{\mu\nu}A^{\mu\nu}-\frac{\cos^2\theta_w}{4}Z^{(a)}_{\mu\nu}Z^{(a)\mu\nu}+\frac{\sin2\theta_w}{4}Z^{(a)}_{\mu\nu}A^{\mu\nu}\underbrace{-\frac14 W^1_{\mu\nu}W^{1\mu\nu}-\frac14 W^2_{\mu\nu}W^{2\mu\nu}}_{-\frac12 W^+_{\mu\nu}W^{-\mu\nu}}$

$\mathcal{L}_G=\mathcal{L}_W+\mathcal{L}_B=-\frac12 W^+_{\mu\nu}W^{-\mu\nu}-\frac14 A^{\mu\nu}A_{\mu\nu}-\frac14 Z^{(a)}_{\mu\nu}Z^{(a)\mu\nu}+\frac{\sin2\theta_w}{2}Z^{(a)}_{\mu\nu}A^{\mu\nu}$

$W^\pm$与Z^0获得质量：$D_\mu=\partial_\mu+\underbrace{\frac{i}{2}(g_1Y_HB_\mu+g_2\tau^iW^i_\mu)}_{\mathfrak{D}_\mu}$，$\mathfrak{D}_\mu=\mathfrak{D}_\mu^\dagger$

$(D_\mu\phi)^\dagger D^\mu\phi=\underbrace{\partial_\mu\phi^\dagger\partial^\mu\phi}_{\frac12(\partial_\mu H)^2}+\underbrace{\mathrm{Im}(\phi^\dagger\mathfrak{D}_\mu\partial^\mu\phi)}_{A_\mu,W_\mu,Z_\mu与H耦合项}+\underbrace{\frac14\phi^\dagger\mathfrak{D}_\mu\mathfrak{D}^\mu\phi}_{无H下的真空}$

代入$\phi=\frac{1}{\sqrt2}(v+H(x))$：

$\frac14\begin{pmatrix}0\\ \frac{v}{\sqrt2}\end{pmatrix}^\dagger\mathfrak{D}_\mu\mathfrak{D}^\mu\begin{pmatrix}0\\ \frac{v}{\sqrt2}\end{pmatrix}=\frac{\frac{v}{\sqrt2}}{4}\left(\sqrt2 g_2W^-_\mu\quad -\sqrt{g_1^2+g_2^2}Z_\mu\right)\begin{pmatrix}\sqrt2 g_2W^+_\mu\\ -\sqrt{g_1^2+g_2^2}Z_\mu\end{pmatrix}\frac{v}{\sqrt2}=\underbrace{\frac{v^2g_2^2}{4}W^-_\mu W^{+\mu}+\frac{v^2(g_1^2+g_2^2)}{8}Z_\mu Z^\mu}$

获得质量：$m_W=\frac{vg_2}{2}$　$m_Z=\frac12\sqrt{g_1^2+g_2^2}=\frac{m_W}{\cos\theta_w}$

轻子质量：用Yukiwa耦合：$\mathcal{L}_{lH}=-\sum g_e(\bar{L}_e\phi R_e+\bar{R}_e\phi^\dagger L_e)$

$=-\frac{1}{\sqrt2}\sum g_e[\bar{e}_L(v+H)e_R+\bar{e}_R(v+H)e_L]=\sum(m_e\bar{e}e+\frac{m_e}{v}\bar{e}eH)$

$m_e=\frac{g_e}{\sqrt2}v$，为轻子赋予质量

夸克质量：

夸克SU(2)对称性：3代夸克 $\begin{pmatrix}u\\d\end{pmatrix},\begin{pmatrix}c\\s\end{pmatrix},\begin{pmatrix}t\\b\end{pmatrix}$ 六味 → $L_u=\begin{pmatrix}u\\d\end{pmatrix}_L$，$L_c$，$L_t$，R态为6个单态　$\begin{pmatrix}u\to c,t\\ d\to s,b\end{pmatrix}$

上夸克 $q=\frac23$，下夸克 $q=-\frac13$　$Q=T^3+Y\Rightarrow Y_{uL}=\frac13$，$Y_{uR}=\frac43$，$Y_{dR}=\frac{-2}{3}$

弱超荷：$YL_u=\frac{Y_{uL}}{2}L_u=\frac16 L_u$，$YR_u=\frac46 R_u$，$YR_d=-\frac26 R_d$

Yukawa 耦合： 中电荷共轭 $\phi_c = i\tau^2\phi^* = \begin{pmatrix}\phi^{0*}\\ -\phi^-\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}v+H\\ 0\end{pmatrix}$

$\bar{l}_e\phi_c R_e = \bar{\nu}_{eL} e_R \phi^0$ ⇒ 电荷不守恒，不考虑。？ 为何

$$\mathcal{L}_{qH} = -\sum(g_d\bar{L}_u\phi R_d + g_u\bar{L}_u\phi_c R_d) + h.c. = -\sum_{q取6味}\left(m_q\bar{q}q + \frac{m_q}{v}\bar{q}qH\right)$$

$m_q = \sqrt{2}g_q v$

考克与规范粒子耦合：

夸克混合态：3代叠加： $\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}_{L,R} = U_{L,R}\begin{pmatrix}u\\c\\t\end{pmatrix}_{L,R}$ $\begin{pmatrix}d_1\\d_2\\d_3\end{pmatrix}_{L,R} = D_{L,R}\begin{pmatrix}d\\s\\b\end{pmatrix}_{L,R}$

↳ $L_i = \begin{pmatrix}u_i\\d_i\end{pmatrix}_L$, $R_{ui} = u_{iR}$, $R_{di} = d_{iR}$

$$\mathcal{L}_{qG} = \sum_i i(\bar{L}_i\gamma^\mu D_\mu L_i + \bar{R}_{ui}\gamma^\mu D_\mu R_{ui} + \bar{R}_{di}\gamma^\mu D_\mu R_{di}) = \mathcal{L}_{q0} + \mathcal{L}_{qA} + \mathcal{L}_{qZ} + \mathcal{L}_{qW}$$

$\mathcal{L}_{q0} = \sum i(\bar{L}_i\not\partial L_i + \bar{R}_{ui}\not\partial R_{ui} + \bar{R}_{di}\not\partial R_{di})$ ，由于 $D_{L,R}$ 与 $U_{L,R}$ 幺正： $U_R^\dagger U_R = 1$

$= \sum i(\bar{L}_u\not\partial L_u + \bar{R}_u\not\partial R_u + \bar{R}_d\not\partial R_d) = \sum\bar{q}\, i\gamma^\mu\partial_\mu q$ ，加质量： $\mathcal{L}_q = \sum\bar{q}(i\gamma^\mu\partial_\mu - m_q)q$

$$\mathcal{L}_{qI} = -\frac{1}{2}\sum\left(\bar{L}_i\gamma^\mu(g_1 Y_{uL}B_\mu + g_2\tau^j W_\mu^j)L_i + \bar{R}_{ui}\gamma^\mu g_1 Y_{uR}B_\mu R_{ui} + \bar{R}_{di}\gamma^\mu(g_1 Y_{dR}B_\mu)R_{di}\right)$$

$= \mathcal{L}_{qA} + \mathcal{L}_{qZ} + \mathcal{L}_{qW}$

与A耦合： $\mathcal{L}_{qA} = -\frac{1}{2}\sum\left[\bar{u}_{Li}\gamma^\mu u_{Li}\left(\frac{1}{3}g_1\cos\theta_w + g_2\sin\theta_w\right) + \bar{d}_{Li}\gamma^\mu d_{Li}\left(\frac{1}{3}g_1\cos\theta_w - g_2\sin\theta_w\right) + \left(\frac{4}{3}\bar{u}_{Ri}\gamma^\mu u_{Ri} - \frac{2}{3}\bar{d}_{Ri}\gamma^\mu d_{Ri}\right)g_1\cos\theta_w\right]A_\mu$

$= \sum\frac{2}{3}e\bar{u}\gamma^\mu u A_\mu - \frac{1}{3}e\bar{d}\gamma^\mu d A_\mu$ ⇒ 电磁场作用，无叠加态

与 Z^0 耦合： $\mathcal{L}_{qZ} = -\frac{1}{2}\sum\left[\bar{u}_{Li}\gamma^\mu u_{Li}\left(-\frac{1}{3}g_1\sin\theta_w + g_2\cos\theta_w\right) + \bar{d}_{Li}\gamma^\mu d_{Li}\left(-\frac{1}{3}g_1\sin\theta_w - g_2\cos\theta_w\right) - \left(\frac{4}{3}\bar{u}_{Ri}\gamma^\mu - \frac{2}{3}d_{Ri}\right)g_1\sin\theta_w\right]Z_\mu$

$= -\frac{g_2}{\cos\theta_w}\sum_q\left[\bar{q}_L\gamma^\mu(T_q^3 - Q_q\sin^2\theta_w)q_L - \bar{q}_R\gamma^\mu(Q_q\sin^2\theta_w)q_R\right]Z_\mu$

与 $W^\pm$ 耦合：

$\mathcal{L}_{qW} = -\frac{1}{2}\sum_i\sqrt{2}g_2(\bar{u}_{Li}\gamma^\mu d_{Li}W_\mu^+ + \bar{d}_{Li}\gamma^\mu u_{Li}W^-)$

$= -\frac{g_2}{\sqrt{2}}\sum\bar{u}_L\gamma^\mu V_{ud}d_L W_\mu^+ + \bar{d}_L\gamma^\mu V_{du}^* u_L W^-$.

$\sum_i\bar{u}_{Li}\gamma^\mu d_{Li} = \begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}_L^\dagger\gamma^\mu\begin{pmatrix}d_1\\d_2\\d_3\end{pmatrix}_L = \begin{pmatrix}u\\c\\t\end{pmatrix}^\dagger U_L^\dagger\gamma^0\gamma^\mu D_L\begin{pmatrix}d\\s\\b\end{pmatrix}$ ⇒ $V = U_L^\dagger D_L$

V： Cabibbo-Kobayashi-Maskawa 混合矩阵 CKM

实验数据近似： $V \approx \begin{pmatrix}\cos\theta_c & \sin\theta_c & 0\\ -\sin\theta_c & \cos\theta_c & 0\\ 0 & 0 & 1\end{pmatrix}$ ⇒ Cabibbo角

总结GWS模型： 非第一性原理，为人工假设

$$\mathcal{L}_{GWS} = \underbrace{-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu}}_{\text{规范场}} + \underbrace{\sum_{e\to\mu,\tau} i\left(\bar{L}_e\gamma^\mu D_\mu L_e + \bar{R}_e\gamma^\mu D_\mu R_e\right)}_{\text{轻子}} + \underbrace{\sum_{i=1}^{3} i\left(\bar{L}_i\gamma^\mu D_\mu L_i + \bar{R}_{ui}\gamma^\mu D_\mu R_{ui} + \bar{R}_{di}\gamma^\mu D_\mu R_{di}\right)}_{\text{夸克}}$$

$$+ \underbrace{(D_\mu\phi)^\dagger(D^\mu\phi) - m^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2}_{\text{Higgs子（标量场）}} + \underbrace{\sum_{e\to\mu,\tau} g_1\left(\bar{L}_e\phi R_e + \bar{R}_e\phi L_e\right)}_{\text{轻子-H耦合}} - \underbrace{\sum_{\substack{u\to c,t\\ d\to s,b}}\left(g_d\bar{L}_u\phi R_d + g_u\bar{L}_u\phi_c R_u + h.c.\right)}_{\text{夸克-H耦合}}$$

$$D_\mu = \partial_\mu + ig_1 Y B_\mu + ig_2 T^i W^i_\mu$$

假设：

1. 夸克与轻子无质量，Higgs子有质量
2. 夸克与轻子满足 $SU(2)\times U(1)$ 对称性
3. 夸克弱作用为三代本征值的叠加
4. Higgs场自发对称性破缺 使规范场带质量
5. H与轻，夸 Yukawa耦合，赋予轻，夸质量

含强作用： $D_\mu = \partial_\mu + ig_1 Y B_\mu + ig_2 T^i W^i_\mu + ig_3 T^a A^a_\mu$ ． $U(1)\times SU(2)\times SU(3)$，标准模型