一. 引言

1. 变分 $x^\mu \to x^\mu + \delta x^\mu$, $d^4x \to d^4x\, J$, $J = \frac{\partial(x^{\mu\prime})}{\partial(x^\nu)} = \det\left(\frac{\partial x^{\mu\prime}}{\partial x^\nu}\right) = \det(g^\mu{}_\nu + \partial_\nu \delta x^\mu) \approx 1 + \partial_\mu \delta x^\mu$

同时：$\phi(x) \to \phi(x) + \delta\phi(x)$ $\longrightarrow$ $\delta\mathcal{L} = \mathcal{L}'(x') - \mathcal{L}(x) \approx \partial_\mu \mathcal{L}(x)\delta x^\mu + \bar{\delta}\mathcal{L}(x')$

坐标不变的变分

$\mathcal{L}'(x') - \mathcal{L}(x') = \frac{\partial \mathcal{L}}{\partial \phi}\bar{\delta}\phi + \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi}\bar{\delta}\partial_\mu\phi$

$\downarrow$

$$\delta S = \frac{1}{c}\int d^4x\, \delta\mathcal{L} = \frac{1}{c}\int d^4x \left(\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi}\right)\bar{\delta}\phi + \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi}\bar{\delta}\phi + \mathcal{L}\delta x^\mu\right)$$

$\downarrow$ 坐标不变：$\delta x^\mu = 0$，边界 $\bar{\delta}\phi = 0$

Lagrange 方程：$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} = 0$

$\downarrow$ 当 $\phi(x)$ 为其解时看后半部分：

守恒流 $\delta S = \frac{1}{c}\int d^4x\, \partial_\mu j^\mu$. $j^\mu = \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi}\bar{\delta}\phi + \mathcal{L}\delta x^\mu$ 对某一变换 $\phi(x) \to \phi'(x')$，对应的 j^μ 有：$\partial_\mu j^\mu = 0$ 守恒流

书上误

守恒量 空间积分：$\int d^3x \frac{1}{c}\frac{\partial}{\partial t} j^0 - \int d^3x\, \vec{\nabla}\cdot\vec{j} = 0 \longrightarrow \frac{dQ}{dt} = qc\int_\infty d\vec{s}\cdot\vec{j} = 0$, $Q = q\int d^3x\, j^0$

二.标量场

1. 场建模：

场量$\phi(x)$ → 标量场的特征，标量协变

↓ 适应经典拉氏量：$\mathcal{L}=\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+\frac{m^2}{2}\phi^2$ （=次型使力学伴性）

↓ 拉方程：$\partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}+\frac{\partial\mathcal{L}}{\partial\phi}=0 \rightarrow (\partial^2+m^2)\phi=0$ Klein-Gordon方程

↓ 解的形式：$\phi_k=\frac{1}{\sqrt{(2\pi)^3 2w}}e^{-i(wt-\vec{k}\cdot\vec{x})}$，$w=\sqrt{k^2+m^2}$ → $\partial^2\phi=(-w^2+k^2)\phi=m^2\phi$，$\phi$归一化得系数

↓ 正交归一关系：$(\varphi_{k'},\varphi_k)=\int d^3x\,\varphi_{k'}^*(x)\,i\overleftrightarrow{\partial_0}\varphi_k(x)$，$A\overleftrightarrow{\partial_\mu}B=A\frac{\partial B}{\partial x^\mu}-\frac{\partial A}{\partial x^\mu}B$

1) $k\neq k'$：$(\varphi_k,\varphi_{k'})=\int d^3x\,i\left(\varphi_k(x)\frac{\partial\varphi_{k'}}{\partial t}-\varphi_{k'}(x)\frac{\partial\varphi_k}{\partial t}\right)=i\int d^3x\,(iw'-iw)\varphi_{k'}\varphi_k=(w-w')\int d^3x\,AA'\,e^{-i(w+w')t-i(\vec{k}+\vec{k'})\cdot\vec{x}}=0$

2) $k=k'$：$(\varphi_k^*,\varphi_{k'})=\int d^3x\,i\left(\varphi_k^*\frac{\partial\varphi_{k'}}{\partial t}-\varphi_k\frac{\partial\varphi_k^*}{\partial t}\right)=(w'+w)\int e^{i(w'-w)t}\cdot e^{i(\vec{k}-\vec{k'})\cdot\vec{x}}\frac{1}{2\pi^3}\frac{1}{2\sqrt{ww'}}d^3x=\frac{w+w'}{2\sqrt{ww'}}\cdot e^{i(w'-w)t}\delta(k-k')$

↓

内积：$\langle\varphi_{k'},\varphi_k\rangle=(\varphi_{k'},\varphi_k)$，$\langle\varphi_k,\varphi_k\rangle=1$，else $\langle\varphi_k,\varphi_{k'}\rangle=0$ → $\int d^3k\langle\varphi_{k_0},\varphi_k\rangle=\frac{w_0+w_0}{2\sqrt{w_0w_0}}e^{i(w_0-w_0)}=1$

故$\langle\varphi_k,\varphi_{k'}\rangle=\delta(k-k')$　$\langle\varphi_k^*,\varphi_{k'}^*\rangle=-\delta(k-k')$

正则量子化：$\pi=\frac{\partial\mathcal{L}}{\partial\dot\phi}=\dot\phi$，$\dot\phi=\partial_t\phi$

因此会随观者变

↓

哈氏方程：$\mathcal{H}=\pi\dot\phi-\mathcal{L}=\frac{1}{2}[\pi^2+(\nabla\phi)^2+m^2\phi^2]$ → 正定：$\mathcal{H}\geq0$

量子化 ⟶ 等时对易关系 $[\phi(x,t),\phi(x',t)]=0$，$[\pi(x,t),\pi(x',t)]=0$，$[\phi(x,t),\pi(x',t)]=i\delta(x-x')$

因此这里不是"好"的对易关系

这里t相等，故不协变

动量空间

平面波展开：$\phi(x,t)=\int d^3k\cdot\pi_k(k,t)\cdot\phi_k(x,t)=\int\frac{d^3k}{\sqrt{(2\pi)^3 2w}}\left[a_k e^{-i(wt-\vec{k}\cdot\vec{x})}+a_k^\dagger e^{i(wt-\vec{k}\cdot\vec{x})}\right]=\int d^3k\left[a_k\varphi_k(x)+a_k^\dagger\varphi_k^*(x)\right]$

构造的一个使$\phi(x)$厄米的形式

claim：$\int\frac{d^3k}{2w}=\int d^4k\,\delta(k^2-m^2)\theta(k^0)$，$\theta(\xi)=\begin{cases}0 & \xi<0\\ 1 & \xi>0\end{cases}$，阶越函数

pf：$\int d^4k\,\delta(k^2-m^2)\theta(k^0)f(k)=\int d^3k\int dk_0\,\delta(m^2+k_s^2-k_0^2)\theta(k_0)=\int\frac{d^3k}{2w}$

$\int d^4k\,\delta(k^2-m^2)\theta(k_0)$ 协变（有效域 $k_0^2=m^2+k_3^2$，$k_0>0=+\omega$ → $ch^2k_0^2+sh^2k_i^2+2chsh\,k_0k_i=m^2+ch^2k_i^2+sh^2k_0^2+2chsh\,k_0k_i$（同上式的有效域））

故有：正交底表示：$\phi(x,t)=\sum_k\langle\varphi_k,\phi\rangle\cdot\varphi_k+\langle\phi,\varphi_k\rangle\varphi_k^*$

$ch\,k_0+sh\,k_i>0$，$ch\,k_0+sh\,k_i=\sqrt{m^2+(ch\,k_i+sh\,k_0)^2}$

对应 $k_0=+\omega$，（有效域也相同）

这里 φ_k 与 φ_k^* 一并作正交完备基底

a_k 为中算符分量 → $\langle\varphi_k,\phi\rangle=a_k$ $\langle\phi,\varphi_k\rangle=a_k^\dagger$ $\langle\phi,\varphi_k\rangle\varphi_k^*=\frac{\langle\varphi_k^*,\phi\rangle}{\langle\varphi_k^*,\varphi_k^*\rangle}\varphi_k^*$

这里选 $\varphi_k=i\varphi_k^*$，$\langle\varphi_k,\varphi_k\rangle=0$

$\langle\varphi_k,\varphi_k\rangle=1$

$[a_k,a_k^\dagger]=i\delta(k-k')$

a_k 为场算符，有：$[a_k,a_{k'}]=0$，$[a_k^\dagger,a_{k'}^\dagger]=0$，$[a_k,a_{k'}^\dagger]=\delta(k-k')$

协变对易关系：（与等时区分）

$[\phi(x),\phi(x')]=i\Delta(x-x')$，$\Delta(x-x')=-i\int d^3k[\varphi_k(x)\varphi_k^*(x')-\varphi_k^*(x)\varphi_k(x)]=-i\int d^4k\,\theta(k^0)\delta(k^2-m^2)\cdot 2m[\quad]$

$$=-i\int\frac{d^4k}{(2\pi)^3}\underbrace{(\theta(k_0)-\theta(-k_0))}_{\text{sgn}(k_0)}e^{-ik_\mu(x^\mu-x'^\mu)}$$

1) 有协变性 2) 有 $e^{-ik_\mu(x^\mu-x'^\mu)}$，为 K-G 方程解 $(\partial_\mu\partial^\mu+m^2)\Delta(x-x')=0$，

3) $\Delta(x-x')=-\Delta(x'-x)$，$t=t'$ 时 $\Delta(\vec r,0)=0$（由解的形式得）

4) $\frac{\partial}{\partial t}\Delta(x-x')|_{t=t'}=-\delta(x-x')$

↓

前对易关系 3) 解决 1,2，4) 解决 3

2. 实标量场粒子性

？] 有限⇒无限可否用结构

箱归一化：$\int d^3k\leftrightarrow\sum_k\frac{(2\pi)^3}{V}$，$k\leftrightarrow\frac{2\pi}{L}(m,n,l)$

（这里忽略时间）

↓ $\delta(k-k')\leftrightarrow\frac{V}{(2\pi)^3}\delta_{kk'}$，$\phi(x)=\sum_k\frac{1}{\sqrt{2\omega V}}(a_ke^{-ikx}+a_k^\dagger e^{ikx})$

↓ $a_k\leftrightarrow\sqrt{\frac{V}{(2\pi)^3}}a_k$ $a_k^\dagger\leftrightarrow\sqrt{\frac{V}{(2\pi)^3}}a_k^\dagger$，$[a_k,a_{k'}^\dagger]=\delta_{kk'}$

↓

粒子数表象 $N_k=a_k^\dagger a_k$，$N_k|n_k\rangle=n_k|n_k\rangle$，$[N_k,a_k^\dagger]=a_k^\dagger$，$[N_k,a_k]=-a_k$

故：$N_ka_k^\dagger|n_k\rangle=(n_k+1)\underbrace{a_k^\dagger|n_k\rangle}_{|n_k+1\rangle}$ → $a_k^\dagger$ 作用：产生湮灭算符 $a_k^\dagger|n_k\rangle=\sqrt{n_k+1}|n_k+1\rangle$，$a_k|n_k\rangle=\sqrt{n_k}|n_k-1\rangle$

$n_k=\langle n_k|N_k|n_k\rangle=\langle n_k|a_k^\dagger a_k|n_k\rangle=\|a_k|n_k\rangle\|^2\geq0$

$a_k|0\rangle=0$ → $|n_k\rangle=\frac{(a_k^\dagger)^{n_k}}{\sqrt{n_k!}}|0\rangle$

能动张量：　变分：$\phi(x) \to \phi'(x') = \phi(x)$，时空平移不变性 $\to \delta\phi = 0 \to \bar{\delta}\phi + \partial_\mu\phi \cdot \delta x^\mu = 0$

守恒流：$j^\mu = -\left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \cdot \partial_\nu\phi - \mathcal{L} g^\mu{}_\nu\right) e^\nu$　　$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \partial^\nu \phi - \mathcal{L} g^{\mu\nu}$　能动张量

守恒量：$P^\nu = \int d^3x P^0 = \int d^3x T^{0\nu}$　　(守恒量正比于 j^0)　　连续性：$\partial_\mu T^{\mu\nu} = 0$

$$P^\nu = T^{0\nu} = \frac{\partial \mathcal{L}}{\partial \partial_0 \phi} \partial^\nu \phi - \mathcal{L} g^{0\nu} \to P^0 = \pi\dot{\phi} - \mathcal{L}$$

Klein-Golden方程 $(\partial^2 + m^2)\phi = 0$

场的能动算符　$H = \int d^3x \mathcal{H} = \int d^3x \left(\frac{1}{2}\pi^2 + (\nabla\phi)^2 + m^2\phi^2\right) = \int d^3x \frac{1}{2}\left[(\partial_0\phi)^2 + \nabla\cdot(\phi\nabla\phi) - \phi\nabla^2\phi + m^2\phi^2\right] = \int d^3x \cdot \frac{1}{2}\left[(\partial_0\phi)^2 - \phi\partial_0^2\phi\right]$

无穷远为0

即：$H = \frac{i}{2}\langle \phi^*, \pi\rangle = \frac{i}{2}\langle \phi, \pi\rangle$　　$= \frac{1}{2}\int d^3x\, i\phi\, i\overleftrightarrow{\partial_0}(\partial_0\phi)$

$P^\mu = \frac{i}{2}\langle \phi, \partial^\mu \phi\rangle$

$\partial^\mu\phi = \partial^\mu \int d^3k (a_k\phi_k + a_k^+\phi_k^*) = -i\int d^3k\, k^\mu [a_k\phi_k - a_k^+\phi_k^*]$

这里 $\eta = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$

动量空间：$P^\mu = \frac{-i}{2}\int d^3x \int d^3k (a_k\varphi_k + a_k^+\varphi_k^*) i\overleftrightarrow{\partial_0} i k^\mu (a_k\varphi_k - a_k^+\varphi_k^*)$

$= \frac{-i}{2}\int d^3k \cdot ik^\mu\left[(\langle a_k\varphi_k, a_k\varphi_k\rangle - \langle a_k^+\varphi_k^*, a_k^+\varphi_k^*\rangle) + (\langle a_k^+\varphi_k^*, a_k\varphi_k\rangle - \langle a_k\varphi_k, a_k^+\varphi_k^*\rangle)\right]$

$= \int d^3k \left(\frac{k^\mu}{2}\right)\left(a_k^+ a_k \langle\varphi_k, \varphi_k\rangle - a_k a_k^+ \langle\varphi_k^*, \varphi_k^*\rangle + 0\right)$

$= \int d^3k \frac{k^\mu}{2}(a_k a_k^+ + a_k^+ a_k)$

用到内积规则：$\langle a_k\varphi_k, \varphi_k'\rangle = \int d^3x\, a_k^+\left(\varphi_k^* i\frac{\partial\varphi_k'}{\partial t} - \varphi_k' i\frac{\partial\varphi_k^*}{\partial t}\right) - i\varphi_k'\varphi_k^*\frac{\partial a_k^+}{\partial t}$

$= a_k^+\langle\varphi_k, \varphi_k'\rangle \underbrace{- i\frac{\partial a_k^+}{\partial t}\delta(k-k')}_{0}$

$\langle a_k\varphi_k, a_k'\varphi_k'\rangle = \int d^3x\, a_k^+ a_{k'}\left(\varphi_k^* i\frac{\partial\varphi_k'}{\partial t} - \varphi_k' i\frac{\partial\varphi_k^*}{\partial t}\right) - i a_{k'}\varphi_k'\varphi_k^*\frac{\partial a_k^+}{\partial t} + i a_k^+\varphi_k\varphi_k'\frac{\partial a_{k'}}{\partial t}$

$= a_k^+ a_{k'}\langle\varphi_k, \varphi_k'\rangle + \underbrace{i\left(-a_{k'}\frac{\partial a_k^+}{\partial t} + a_k^+\frac{\partial a_{k'}}{\partial t}\right)\delta(k-k')}_{0}$

粒子性：$P^\mu = \sum_k \frac{k^\mu}{2}(a_k a_k^+ + a_k^+ a_k) = \sum_k k^\mu\left(N_k + \frac{1}{2}\right)$

1) 能量 $H = \sum_k \omega\left(N_k + \frac{1}{2}\right) \to \omega$ 为能量子　$\omega > 0$

2) 由于同一态 n_k 无限制 $\to$ Bose子　　$a_{k_1}^+ a_{k_2}^+|0\rangle = a_{k_2}^+ a_{k_1}^+|0\rangle$

单粒子波函数　$\langle 0|\phi(x,t) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega}} e^{-i(\omega t - \vec{k}\cdot\vec{x})}\langle k|$，　$\langle k| = \langle 0|a_k$

$\langle 0|\phi(x,t)|\psi\rangle = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega}} e^{-i(\omega t - \vec{k}\cdot\vec{x})}\psi_k$，　$\psi_k = \langle k|\psi\rangle$

此处略去归一化

$\langle 0|\phi(x, t=0) = \langle x|$，　$\langle x|k\rangle = \int \frac{d^3k'}{\sqrt{(2\pi)^3 2\omega}} e^{i\vec{k}'\cdot\vec{x}}\langle k'|k\rangle \propto \frac{e^{i\vec{k}\cdot\vec{x}}}{\sqrt{(2\pi)^3}}$　(薛定谔不确定关系)

相对论下Bose子满足K-G方程，有 ∂_t，→ 必须用场的量子理论

零点能：$P^0_0 = E_0 = \int \frac{V d^3k}{(2\pi)^3} \frac{1}{2}\omega = \frac{V}{16\pi^3} \lim_{k_c\to\infty} \int_0^{k_c} 4\pi k^2 dk \sqrt{k^2+m^2}$

$k_c\to\infty$时发散，故截断

$H \to H - E_0 = \sum_k \frac{\omega}{2} N_k = \mathcal{N}(H)$ ← 称为"取正规乘积"

即：H的$a_k^\dagger$总在a_k左，可使$E_0=0$

3.复标量场

无$\frac{1}{2}$：$\phi^\dagger$与ϕ是2个量

拉氏量 $\mathcal{L} = \partial_\mu\phi^\dagger\partial^\mu\phi - m^2\phi^\dagger\phi = \dot\phi^\dagger\dot\phi - (\nabla\phi^\dagger)\cdot(\nabla\phi) - m^2\phi^\dagger\phi$ 这里ϕ与$\phi^\dagger$独立

$\partial_\mu \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^\dagger} - \frac{\partial\mathcal{L}}{\partial\phi^\dagger} = 0 \to (\partial^2+m^2)\phi = 0$

$\partial_\mu \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi} - \frac{\partial\mathcal{L}}{\partial\phi} = 0 \to (\partial^2+m^2)\phi^\dagger = 0$

$\to \pi = \dot\phi^\dagger,\quad \pi^\dagger = \dot\phi$

$\mathcal{H} = \pi\dot\phi + \pi^\dagger\dot\phi^\dagger - \mathcal{L} = \pi^\dagger\pi + (\nabla\phi^\dagger)(\nabla\phi) + m^2\phi^\dagger\phi$，正定

正则量子化：(等时对易关系) $[\phi(x,t), \pi(x',t)] = i\delta(x-x')$ $[\phi^\dagger(x,t), \pi^\dagger(x',t)] = i\delta(x-x')$

↓ 协变对易关系 $[\phi(x), \phi^\dagger(x')] = i\Delta(x-x')$，else 0

取 $\phi = \frac{1}{\sqrt2}(\phi_1 + i\phi_2)$，$\phi^\dagger = \frac{1}{\sqrt2}(\phi_1 - i\phi_2)$

$[\phi_i(x,t), \phi_j(x',t)] = i\delta_{ij}\Delta(x-x')$

$\int d^3x\, \varphi_k^* i\overleftrightarrow{\partial_0}\phi = a_k\langle\varphi_k,\varphi_k\rangle$

$\int d^3x\, \varphi_k i\overleftrightarrow{\partial_0}\phi^\dagger = b_k\langle\varphi_k^*,\varphi_k^*\rangle \to b_k^\dagger = \langle\varphi_k^*,\phi\rangle$

?コ

动量空间

$\phi(x,t) = \int d^3k\, \underbrace{\langle\varphi_k,\phi\rangle}_{a_k}\varphi_k + \underbrace{\langle\varphi_k^*,\phi\rangle}_{b_k^\dagger}\varphi_k^* = \int d^3k[a_k\varphi_k(x) + b_k^\dagger\varphi_k^*(x)]$

↓ $\pi = \partial_0\phi^\dagger = \int d^3k\, i\omega(a_k^\dagger\varphi_k^*(x) - b_k\varphi_k(x)) \to$

$[\phi,\pi] = \iint d^6k\, i\omega([a_k, a_{k'}^\dagger]\cdot\varphi_k\varphi_{k'}^* - [b_k^\dagger, b_{k'}]\varphi_k^*\varphi_{k'})$

$[\phi^\dagger,\pi^\dagger] = \iint d^6k - i\omega([a_k^\dagger, a_{k'}]\cdot\varphi_k\varphi_{k'}^* - [b_k, b_{k'}^\dagger]\varphi_k\varphi_{k'}^*)$

$)) \; \underset{x=x'\text{时为}\frac{i}{(2\pi)^3}\int d^3k}{\underline{i\delta(x-x')}}$

$[\phi,\phi^\dagger] = \iint d^6k\cdot([a_k, a_{k'}^\dagger])\varphi_k\varphi_{k'}^* + [b_k, b_{k'}^\dagger]\varphi_k\varphi_{k'}^*) = 0$

$[a_k, a_{k'}^\dagger] = \delta(k-k')$，$[b_k, b_{k'}^\dagger] = \delta(k-k')$

场的能量动量算符：$P^\nu = \frac{\partial\mathcal{L}}{\partial\partial_0\phi}\partial^\nu\phi + \partial^\nu\phi^\dagger\frac{\partial\mathcal{L}}{\partial\partial_0\phi^\dagger} - \mathcal{L}g^{0\nu} = \partial^0\phi^\dagger\partial^\nu\phi + \partial^\nu\phi^\dagger\partial^0\phi - \mathcal{L}g^{0\nu}$

$\downarrow P^0 = \pi^\dagger\pi + (\nabla\phi^\dagger)\cdot(\nabla\phi) + m^2\phi^\dagger\phi$, $\mathcal{H} = P_0 = P^0$ [1,-1,-1,-1]

$$H = \int d^3x\, \mathcal{H} = \int d^3x\,(\pi\pi^\dagger + (\nabla^2\phi\cdot\phi^\dagger - \partial_0^2\phi\phi^\dagger) + (\nabla\phi^\dagger)\cdot(\nabla\phi))$$

$$= \int d^3x\,(\partial_0\phi\partial_0\phi^\dagger - \phi^\dagger\partial_0^2\phi) + \underbrace{\nabla(\nabla\phi\cdot\phi^\dagger)}_{\text{全空间为0}} = \int d^3x\, i\phi^\dagger i\overleftrightarrow{\partial_0}(\partial_0\phi) = \frac{i}{2}(\langle\phi,\pi^\dagger\rangle - \langle\phi^\dagger,\pi\rangle)$$

$\downarrow P^\mu = i\langle\phi, \partial^\mu\phi\rangle = \frac{i}{2}(\langle\phi,\partial^\mu\phi\rangle - \langle\phi^\dagger,\partial^\mu\phi^\dagger\rangle)$

箱归一化 $P^\mu = \sum_k k^\mu(a_k a_k^\dagger + b_k^\dagger b_k)$

4. 规范不变性与粒子荷

规范变换 $\phi' = \phi\cdot e^{i\gamma}$ $\longrightarrow$ 取 $\delta\phi = i\gamma\phi$, $\delta\phi^\dagger = i\gamma\phi^\dagger$ $\rightarrow$ $\delta\mathcal{L} = i\gamma\Big(\underbrace{\frac{\partial\mathcal{L}}{\partial\phi}\phi - \phi^\dagger\frac{\partial\mathcal{L}}{\partial\phi^\dagger}}_{0} + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\partial_\mu\phi - \partial_\mu\phi^\dagger\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^\dagger}\Big)$

$$= i\gamma\Big(\partial_\mu\Big(\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\phi - \phi^\dagger\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^\dagger}\Big) - \underbrace{\phi\partial_\mu\Big(\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\Big) + \phi^\dagger\partial_\mu\Big(\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^\dagger}\Big)}_{0}\Big)$$

$$= i\gamma\partial_\mu\Big(\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\phi - \phi^\dagger\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^\dagger}\Big) \qquad (\partial_\mu\partial^\mu\phi^\dagger = m^2\phi^\dagger)$$

$$j^\mu = \frac{q}{i}\Big(\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\phi - \phi^\dagger\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^\dagger}\Big), \quad \partial_\mu j^\mu = 0$$

$\downarrow$

$\partial_\mu j^\mu = \frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0$ $\qquad j^\mu = iq\,[\phi^\dagger\partial^\mu\phi - (\partial^\mu\phi^\dagger)\phi]$

$$\llcorner\, Q = \int d^3x\,\rho = \int d^3x\, iq[\phi^\dagger\partial^0\phi - (\partial^0\phi^\dagger)\phi] = \int d^3x\, q\cdot\phi^\dagger i\overleftrightarrow{\partial_0}\phi = q\langle\phi,\phi\rangle$$

$$[Q,\phi] = \int d^3x\, iq[(\underbrace{\phi\pi^\dagger}_{0} - \underbrace{\pi\phi)}_{[\phi,\pi]\phi},\phi] = \int d^3x\cdot iq\cdot i\delta(x-x')\phi = -q\phi$$

$[Q,\phi^\dagger] = q\phi^\dagger$

作用于态矢: $Q|Q'\rangle = Q'|Q'\rangle$, $\quad Q\phi|Q'\rangle = (\phi Q - q\phi)|Q'\rangle = (Q'-q)\phi|Q'\rangle$

$\qquad Q\phi^\dagger|Q'\rangle = (\phi^\dagger Q + q\phi^\dagger)|Q'\rangle = (Q'+q)\phi^\dagger|Q'\rangle$

$\downarrow$ $\phi^\dagger$ 为Q的产生算符

动量空间: $Q = \int d^3k\cdot q(a_k a_k^\dagger - b_k^\dagger b_k)$

定域规范不变性: $\gamma \rightarrow \gamma(x)$ $\Rightarrow$ $\partial_\mu(e^{i\gamma}\phi) = e^{i\gamma}(\partial_\mu + i\partial_\mu\gamma)\phi$

$\downarrow$ 适应的偏导

协变导数 $D_\mu = \partial_\mu + iqA_\mu(x)$ $\quad \begin{cases} D'_\mu = \partial_\mu + iqA'_\mu \\ A'_\mu = A_\mu - \frac{1}{q}\partial_\mu\gamma \end{cases}$ $\rightarrow$ $D'_\mu(\phi') = D'_\mu(e^{i\gamma}\phi) = e^{i\gamma}(\partial_\mu + i\partial_\mu\gamma)\phi + iq(A_\mu - \frac{1}{q}\partial_\mu\gamma)$

$$= e^{i\gamma}(D_\mu\phi) = (D_\mu\phi)'$$

$\downarrow$

$\mathcal{L} = (D_\mu\phi)^\dagger D^\mu\phi - m^2\phi^\dagger\phi$ ，可使 ϕ 形式不变

规范场：

$\mathcal{L} = (D_\mu\phi)^\dagger D^\mu\phi - m^2\phi^\dagger\phi$ ，场变：δA_μ ，$\delta\mathcal{L} = \delta\Big(\overbrace{(\partial_\mu\phi + iqA_\mu\phi)^\dagger}^{(\partial_\mu\phi^\dagger - iqA_\mu\phi^\dagger)}(\partial^\mu\phi + iqA^\mu\phi)\Big) + 0 = -iq\,\delta A_\mu\phi^\dagger\partial^\mu\phi + iq\,\partial_\mu\phi^\dagger\delta A^\mu\phi$

$$= \delta A_\mu\cdot\big(-iq(\phi^\dagger\partial^\mu\phi + \partial^\mu\phi^\dagger\phi)\big)$$

$$\frac{\partial\mathcal{L}}{\partial A_\mu} = -j^\mu$$

实标量场：$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + m^2\phi^2 \rightarrow \frac{1}{2}(D_\mu\phi)(D^\mu\phi) + m^2\phi^2$

$\frac{\partial\mathcal{L}}{\partial A_\mu} = j^\mu = 0$ $\quad \frac{\partial\mathcal{L}}{\partial\phi} = m^2\phi$

故无耦合的场

物变：$\delta\phi, \delta\phi^\dagger$ $\quad \delta\mathcal{L} = (\delta\partial_\mu\phi^\dagger - iqA_\mu\delta\phi^\dagger)(\partial^\mu\phi + iqA^\mu\phi) + (\partial_\mu\phi^\dagger - iqA_\mu\phi^\dagger)(\delta\partial^\mu\phi + iqA^\mu\delta\phi)$

$$= \delta\phi^\dagger\big(-iqA_\mu\partial^\mu\phi + q^2A^\mu A_\mu\phi + \partial_\mu(\partial^\mu\phi + iqA^\mu\phi)\big) + \underbrace{\partial_\mu\big(\delta\phi^\dagger(\partial^\mu\phi + iqA^\mu\phi)\big)}_{0}$$

$$+ \big(iqA^\mu\partial_\mu\phi^\dagger + q^2A^\mu A_\mu\phi^\dagger + \partial_\mu(\partial^\mu\phi^\dagger - iqA^\mu\phi^\dagger)\big)\delta\phi + \cdots$$

$$= \delta\phi^\dagger(q^2A^\mu A_\mu + m^2 + iq\partial_\mu A^\mu)\phi + \phi^\dagger(q^2A^\mu A_\mu + m^2 - iq\partial_\mu A^\mu)\delta\phi$$

$\downarrow$

$$\frac{\partial\mathcal{L}}{\partial\phi^\dagger} - m^2\phi = \phi(q^2A_\mu A^\mu + iq\partial_\mu A^\mu)$$

$$\frac{\partial\mathcal{L}}{\partial\phi} - m^2\phi^\dagger = \phi^\dagger(q^2A_\mu A^\mu - iq\partial_\mu A^\mu)$$

$\mathcal{L}$ 中仅写明粒子场的量，而：

$$\begin{cases} \frac{\partial\mathcal{L}}{\partial A_\mu} = -j^\mu \\ \frac{\partial\mathcal{L}}{\partial\phi^\dagger} - m^2\phi = \phi(q^2A_\mu A^\mu + iq\partial_\mu A^\mu) \\ \frac{\partial\mathcal{L}}{\partial\phi} - m^2\phi^\dagger = \phi^\dagger(q^2A_\mu A^\mu - iq\partial_\mu A^\mu) \end{cases}$$

场 A^μ 与粒子相互作用！：规范场

Weyl 规范场（见 CFT）

实标量场 $\longrightarrow$ 确定一种粒子

复标量场 $\longrightarrow$ 2 种粒子，用守恒荷区分 $\longrightarrow$ 复数 $\dim\mathbb{C} = 2\dim\mathbb{R}$

$\uparrow$

额外维度 $\rightarrow$ 粒子内部空间 $\rightarrow$ 粒子种类对应空间方向

$\searrow$ 相位变换对应复平面转动

$\downarrow$

整体规范不变性对应 转动不变 $\rightarrow$ 方向任意、

$\downarrow$

与守恒荷的定义相对应

定域规范变换

$\downarrow$

同一点的夹角才有意义，即：不同时空点守恒荷不同

规范场 $\rightarrow$ 抵消不同时空点相位差

$\downarrow$

粒子内部空间平衡相位变化的相位场