六.散射振幅与费曼图

1. 相互作用标量场

相互作用 $\mathcal{L}=\mathcal{L}_0+\mathcal{L}_1 \Rightarrow \mathcal{H}=\mathcal{H}_0+\underline{\mathcal{H}_I}$ 相互作用项 $\mathcal{H}_I=-\mathcal{L}_I$

建立模型：$\mathcal{L}=-\frac{\lambda}{4!}\phi^4$ —
1) 相对论定域要求 ⟶ 不存在 $\phi(x)\phi(y)$
2) 能量正定 ⟶ ϕ^n, n为偶（最低为 ϕ^4）
3) 可重正化 ⟶ n>4不可：λ量纲为 $[L]^{n-4}$，故结果含 λk_c^{n-4}，当n=4为不发散。

可重正化：式中不含积分截断 k_c

?门待学重正化

$$\pi=\frac{\partial\mathcal{L}}{\partial\dot\phi}=\frac{\partial\mathcal{L}_0}{\partial\dot\phi}+\frac{\partial\mathcal{L}_1}{\partial\dot\phi}$$ 引入相互作用导致正则量子化可能变化（当 $\mathcal{L}_1$ 不含 $\partial_\mu\phi$ 时无影响）

$$Z[J]=\int D\phi\, e^{i\int dt\,\mathcal{L}_0+\mathcal{L}_1+J\phi+i\epsilon\phi^2} \xrightarrow{\text{中心恒等式}} Z[J]=Ne^{i\mathcal{L}_1(\delta/i\delta J)}Z_0[J],\quad Z_0[J]=e^{-\frac{i}{2}J\Delta_F J}$$

归一化：$Z[0]=1$

ϕ^4模型

微扰论展开：$\mathcal{L}_1(\delta/i\delta J)=-\frac{\lambda}{4!}\left(\frac{\delta}{i\delta J}\right)^4 \xrightarrow{\lambda\text{小量}} e^{i\mathcal{L}_1(\delta/i\delta J)}=1-\frac{i\lambda}{4!}\int dz\left[\frac{\delta}{i\delta J(z)}\right]^4+o(\lambda)$

$$Z[J]=N\left[1-\frac{i\lambda}{4!}\int dz\left(\frac{\delta}{i\delta J(z)}\right)^4+o(\lambda)\right]Z_0[J]$$

奇 图规则：加一条 x— ，同时原图减一个x

$$\left(\frac{\delta}{i\delta J}\right)^4 Z_0[J]=\left(\frac{\delta}{i\delta J_z}\right)^4 e^{-\frac{i}{2}J_x\Delta_F^{xy}J_y}=\left(\frac{\delta}{i\delta J_z}\right)^3\cdot(-\Delta_F^{zy}J_y)e^{-\frac{i}{2}J_x\Delta_F^{xy}J_y}=\left(\frac{\delta}{i\delta J}\right)^2\left((\Delta_F^{zy}J_y)^2+i\Delta_F^{zz}\right)Z_0[J]$$

$$=\frac{\delta}{i\delta J_z}\left(i(\Delta_F^{zy}J_y)^3+-i\Delta_F^{zy}J_y\Delta_F^{zz}+-2i\Delta_F^{zz}\Delta_F^{zy}J_y\right)$$

$$=\left[-3(\Delta_F^{zz})^2+6i\Delta_F^{zz}(\Delta_F^{zy}J_y)^2+(\Delta_F^{zy}J_y)^4\right]Z_0[J]$$

$$Z[J]=N\left[1-\frac{i\lambda}{4!}\int dz\left[-3(\Delta_F^{zz})^2+6i\Delta_F^{zz}(\Delta_F^{zy}J_y)^2+(\Delta_F^{zy}J_y)^4+\cdots\right]Z_0[J]\right.$$ ⟵ $\Delta_F^{zz}=\Delta_F(z-z)=\Delta_F(0)$，$\Delta_F^{zy}J_y=\int dy\,\Delta_F(x-y)J(y)$

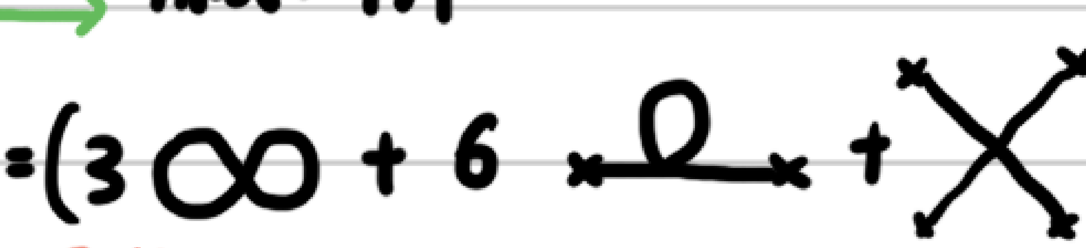
Feynman图表示：传播线：$i\Delta_F(x-y)=$ x——y　2点x−y

圈：$i\Delta_F(z-z)=i\Delta_F(0)=$ ○z　一条传播线两端重合到z

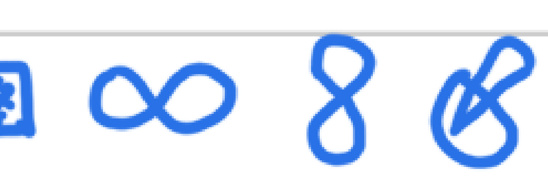
外源：$iJ_x=$ ×x　附在传播线的端点上

顶点：✕　Vertex，也叫顶角

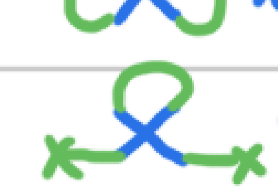
原式表示为：

$\left(\frac{\delta}{i\delta J}\right)^4 Z_0[J]=$  $Z_0[J]$

都仅有1个顶点 ✕

 无外线→真空图 

$C_3^1=3$

 带外线的圈图→蝌蚪图：

$C_4^2=6$

归一化：$N = 1 + \frac{i\lambda}{4!}\int d^4z\, 3\infty + O(\lambda^2) \rightarrow Z[J] = \left[1 - \frac{i\lambda}{4!}\int d^4z\left(6\,\Omega + \times\right)\right] Z_0[J]$

顶点规则：对于顶点，乘以 $-\frac{i\lambda}{4!}$ ⟶ 将此项含入式中

2 相互作用 Green 函数

带 Feynmann 图计算：$Z_0[J] = e^{-\frac{i}{2}J\Delta_F J} = e^{\frac{1}{2}\times\!-\!\times} \longrightarrow Z[J] = A \times B = \left(1 + 6\,\Omega + \times + O(\lambda^2)\right) \times e^{\frac{1}{2}\times\!-\!\times}$

$\downarrow$

Green 函数 $G(x_1 \cdots x_n) = \left.\frac{\delta^n Z[J]}{i^n \delta J_{x_1} \cdots}\right|_{J=0}$

2点 Green 函数：$G(x,y) = x\!-\!\bullet\!-\!y = \left[\left(1 + 6\,\Omega + \times + \cdots\right)\left(e^{\frac{1}{2}\times-\times}\right)\right]''_{xy}\Big|_{J=0}$

$x\!-\!\bullet\!-\!y \rightarrow x\!-\!y + C_2^1 A_4^3\; x\,\Omega\, y$

$= \left(e^{\frac{1}{2}\times-\times}\right)'' \cdot 1\Big|_{J=0} + \left(1 + 6\,\Omega + \times + \cdots\right)'' \cdot (e^0)\Big|_{J=0}$

$= \left(\times\!-\!x \cdot e^{\frac{1}{2}\times-\times}\right)' + \left(12\,x\,\Omega + 4\times + \cdots\right)' = \underbrace{x\!-\!y}_{0阶,自由传播} + \underbrace{12\,x\,\Omega\,y}_{A_4^2 \;\times \rightarrow x\Omega y} + \cdots$ (两外端点 x,y)

4点 Green 函数：$G(x_1,x_2,x_3,x_4) = \times = \left(AB'''' + 6A''B'' + A''''B\right)\Big|_{J=0}$

$= 1 \cdot 3 = + 6 \cdot (2 \cdot 6\,\Omega \cdot -) + 4 \cdot (3!) \times$

$\times = \frac{C_3^1}{=} + C_4^2 \cdot A_4^3\; \underline{\underline{\Omega}} + C_4^2 \cdot A_4^1\; \times$

$= \underbrace{3 =}_{Z_0[J]自由项} + 72\; \underline{\underline{\Omega}} + 24 \times + \cdots$

eg：4点 Green 的 1阶：$= \times = \Rightarrow \times$ ($m=1, n=4$) $A_4^4 = 24$

$\underline{\underline{\Omega}}$ $C_4^2 \cdot A_4^2 \cdot 1 = 72$

$\underline{\underline{\infty}}$ $\frac{C_4^2}{2} \cdot \frac{C_4^2}{2} = 9$ — ∞ — X 解？— 书中为 36 ?

真空图不在归一化泛函中,不作考虑。

一般规则：n点 Green 函数有 n 条自由外线 外 ⟶ 内(接顶角)

m阶图有 m 个顶点 (0阶图无顶点)

高阶图 m阶项 → 有因子 $\frac{1}{m!}$，m个点对换有 m!，相消

质量修正：

对 $G(x,y)$，一阶修正 $G(x,y) = x\!-\!y + \underline{12\,x\,\Omega\,y} + \cdots = i\Delta_F(x-y) + 12 \cdot \frac{\lambda}{4!}\int d^4z\; i\Delta_F(x-z)\, i\Delta_F(0)\, i\Delta_F(z-y) + \cdots$

$$= \int \frac{d^4p}{(2\pi)^4} \frac{ie^{-ip(x-y)}}{p^2 + m^2 + i\varepsilon}\left[1 + \frac{i\lambda\Delta_F(0)}{2} \cdot \int \frac{d^4z\, d^4k}{(2\pi)^4} \frac{e^{-i(k-p)(z-y)}}{k^2 - m^2 + i\varepsilon}\right] + \cdots$$

$$= \int \frac{d^4p}{(2\pi)^4} \cdot \frac{ie^{-ip(x-y)}}{p^2 - m^2 + i\varepsilon}\left[1 + \frac{i\lambda\Delta_F(0)}{2} \cdot \frac{1}{p^2 - m^2 + i\varepsilon}\right] + \cdots$$

$$= \int \frac{d^4p}{(2\pi)^4} \frac{ie^{-ip(x-y)}}{p^2 - m^2 - \frac{1}{2}\lambda\Delta_F(0) + i\varepsilon}$$

m阶λ小量是否可代入？

$\downarrow$

$m_c^2 = m^2 + \frac{i}{2}\lambda\Delta_F(0)$ → 需重正化

连通函数的作用？]

为何取W就可去掉非连通项？]

连通Green函数

连通图：不可分为独立两部分

$$Z[J] = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int dx_1 \cdots dx_n\, G(x_1 \cdots x_n) J_{x_1} \cdots J_{x_n}$$

↓

定义 $W[J]$，$Z[J] = e^{iW[J]}$，$W[J] = -i \ln Z[J]$

↓

$$W[J] = \sum_{n=0}^{\infty} \frac{i^{n-1}}{n!} \int dx_1 \cdots dx_n\, G_c(x_1 \cdots x_n) J_{x_1} \cdots J_{x_n}$$

2点连通Green函数：$G_c(x,y) = \frac{i\delta^2 W[J]}{i^n\, \delta J_1 \delta J_2}\Big|_{J=0} = \frac{\delta}{i\delta J_x} \frac{1}{Z[J]} \frac{\delta Z[J]}{i\delta J(y)}\Big|_{J=0} = \frac{-1}{Z[J]^2}\left(\frac{\delta Z[J]}{\delta J}\right)^2 + \frac{1}{Z[J]} \frac{\delta^2 Z[J]}{i^2 \delta J^2}$

$= -\underbrace{G(x)}_{0}\underbrace{G(y)}_{0} + G(x,y) = G(x,y)$

↳ 减去非连通项

4点连通 Green函数：$iW[J] = \ln Z[J] = (Z-1) - \frac{1}{2}(Z-1)^2 + \frac{1}{3}(Z-1)^3 + \cdots$

$$G_c(x_1, x_2, x_3, x_4) = Z_{1234}'''' - Z_{12}'' Z_{34}'' - Z_{13}'' Z_{24}'' - Z_{14}'' Z_{23}''$$

与拓扑学的关系？] 欧拉定理

$= G(1,2,3,4) - G(12)G(34) - G(13)G(24) - G(14)G(23)$

$= (3 = + 72\, \underline{\Omega} + 24X) - 3(- + 12\,\underline{\Omega})^2 + \cdots$

Gc中仅包含核=1 的图

$= 24X + \cdots$

3. S矩阵与散射

1) S矩阵

对入态 $|\alpha, in\rangle$ 与出态 $|\beta, out\rangle$，$\{|\alpha, in\rangle\}$ 与 $\{|\beta, out\rangle\}$ 均构成完备基 → 可相互表示

散射振幅 $\langle\beta, out|\alpha, in\rangle$ 可写作 $S_{\beta\alpha} = \langle\beta, in|S^{-1}|\alpha, in\rangle = \langle\beta, out|S|\alpha, out\rangle = \langle\beta, out|\alpha, in\rangle$

由此定义 S矩阵：$S|\alpha, out\rangle = |\alpha, in\rangle$；$S^{\dagger}|\beta, in\rangle = |\beta, out\rangle$

↳ 出入射变换算子

性质：1) $S_{00} = 1$ 真空具有不变性（$|0\rangle$态唯一）设 $S|0, out\rangle = e^{i\varphi}|0, in\rangle$ → $S_{00} = \underbrace{\langle 0, out|0, in\rangle}_{为同一态} = 1 = e^{i\varphi}$

2) 单粒子态具有不变性 对单子态→ $S_{pp'} = \langle p, out|p', in\rangle = e^{i\varphi}\delta(p-p') = \delta(p-p')$ （$S_{pp'} = \delta(p-p')$）

3) S幺正，$SS^{\dagger} = S^{\dagger}S = 1$ $\langle\beta, in|S^{\dagger}S|\alpha, in\rangle = \langle\beta, out|\alpha, out\rangle = \delta_{\beta\alpha}$，易证

出入射场变换

$t \to -\infty$	$\|t\| < \infty$	$t \to +\infty$
$\phi_{in}(x)$	$\phi(x)$	$\phi_{out}(x)$

，满足K-G方程 $(\partial^2 + m^2)\phi_{in}(x) = (\partial^2 + m^2)\phi_{out}(x) = 0$

对出入场展开：$\phi_{in}=\int d^3k\,[a_{k,in}\varphi_k(x)+a^+_{k,in}\varphi^*_k(x)]$　　$(\partial^2+m^2)\phi(x)=\frac{\partial\mathcal{L}_I(\phi)}{\partial\phi}$

$\phi_{out}=\int d^3k\,[a_{k,out}\varphi_k(x)+a^+_{k,out}\varphi^*_k(x)]$　　场源，用Δ_R与Δ_A可得ϕ

$\phi(x)=\phi_{in}(x)-\int dy\,\Delta_R(x-y)K(y)\phi(y)$

$=\phi_{out}(x)-\int dy\,\Delta_A(x-y)K(y)\phi(y)$ ， $K(x)=\partial_x^2+m^2$ (K-G算子)

强渐近条件：$\phi(x)\xrightarrow{t\to-\infty}\phi_{in}(x)$ ， $\phi(x)\xrightarrow{t\to+\infty}\phi_{out}(x)$ 不可使用：

$[\phi(x),\phi(y)]\xrightarrow{t\to-\infty}[\phi_{in}(x),\phi_{in}(y)]=i\Delta(x-y)$　　$i\Delta(x-y)\xrightarrow{t\to+\infty}i\Delta(x-y)$

$[\phi(x),\phi(y)]\xrightarrow{t\to+\infty}[\phi_{out}(x),\phi_{out}(y)]=i\Delta(x-y)$

两粒子未相互作用，不能用

矢量场中也用到此类近似，有何共同点？

化作振幅？

弱渐近条件：$\langle\beta|\phi(x)|\alpha\rangle\xrightarrow{t\to-\infty}\langle\beta|\phi_{in}(x)|\alpha\rangle$　将场算子渐近化为期望渐近

$\langle\beta|\phi(x)|\alpha\rangle\xrightarrow{t\to+\infty}\langle\beta|\phi_{out}(x)|\alpha\rangle$

$\langle 0|\phi(x)|P\rangle\xrightarrow{|t|\to\infty}\begin{matrix}\langle 0|\phi_{in}(x)|P\rangle\\ \langle 0|\phi_{out}(x)|P\rangle\end{matrix}=Ce^{-ipx}$　归一化系数不含时

梳理 $Z[J]$ 的生成链 $(G, G_c, S\cdots)$ ？

S矩阵变换：$S\phi_{out}|\alpha,out\rangle=\phi_{in}|\alpha,in\rangle=\phi_{in}S|\alpha,out\rangle$

$\hookrightarrow$ $S\phi_{out}=\phi_{in}S\to\phi_{in}=S\phi_{out}S^{-1}$

Lorentz变换：$SU\phi_{out}U^{-1}=U\phi_{in}U^{-1}S\to\phi_{in}=\underbrace{U^{-1}SU}_{B}\phi_{out}U^{-1}S^{-1}U=B\phi_{out}B^{-1}$

$B=S\to U^{-1}SU=S=USU^{-1}$ → Lorentz不变性

2) 计算S矩阵

$SU[J]$的方程：

对$Z[J]=\langle 0|U[J]|0\rangle$ ， $U[J]=\mathcal{T}e^{i\int dx\,J(x)\phi(x)}$

泛函微分后 $\frac{\delta U[J]}{i\delta J(x)}=\mathcal{T}[\phi(x)U[J]]$ —— $\begin{cases}=U[J]\phi_{in}(x)-\int dy\,\Delta_R(x-y)K(y)\frac{\delta U[J]}{i\delta J(y)}\\ =\phi_{out}U[J]-\int dy\,\Delta_A(x-y)K(y)\frac{\delta U[J]}{i\delta J(y)}\end{cases}$

↓　$\phi_{in}=S\phi_{out}S^{-1}$　　S可直入表达为$SU[J]$

$\phi_{out}U[J]-U[J]\phi_{in}=i\int dy\,\Delta(x-y)K(y)\frac{\delta U[J]}{\delta J(y)}$ → $[\phi_{in}(x),SU[J]]=i\int dy\,\Delta(x-y)K(y)\frac{\delta}{\delta J(y)}SU[J]$

$U[0]=1$ ，先解 $SU[J]$：　　$[\phi_{in}(x),SU[J]]=i\int dy\,\Delta(x-y)\underbrace{K(y)\frac{\delta}{\delta J(y)}}_{f(y)}SU[J]$

猜解：$[\phi_{in}(x), e^{\int dy\,\phi^{(-)}_{in}(y)f(y)} e^{\int dy\,\phi^{(+)}_{in}(y)f(y)}]$ $= [A, B+C]e^B e^C$

$= \int dy [\phi_{in}(x), \phi_{in}(y)] f(y)\, e^{\int dz\,\phi^{(-)}_{in}(z)f(z)} e^{\int dz\,\phi^{(+)}_{in}(z)f(z)}$

$= \int dy\, \Delta(x-y) f(y) \cdot e^{\int dz\,\phi^{(-)}_{in}(z)f(z)} e^{\int dz\,\phi^{(+)}_{in}(z)f(z)}$ $\Rightarrow SU[J] = e^{\int dy\,\phi^{(-)}_{in}(y)f(y)} e^{\int dy\,\phi^{(+)}_{in}(y)f(y)} F[J]$

$= \mathcal{N}\{e^{\int dy\,\phi_{in}(y)f(y)}\} F[J]$

Baker-Hausdorff公式：$e^B A e^{-B} = A + [B, A] + \frac{1}{2!}[B, [B, A]] + \cdots$

$[A, B]$为c数时 $e^B A e^{-B} = [A, B] e^B$

$[A, B], [A, C]$为c数时：$[A, e^B e^C] = [A, B+C] e^B e^C$

确定$F[J]$：

$\langle 0|\mathcal{N}[e^A]|0\rangle = 1$　　$S|0\rangle = |0\rangle$

$\langle 0|SU[J]|0\rangle = F[J] = \langle 0|U[J]|0\rangle = Z[J]$

$\mathcal{N}\{A\} = :A:$　A的正规乘积（产生算符在湮灭之右）

$\forall A\ \langle 0|\mathcal{N}[e^A]|0\rangle = 0$. Pf: $\mathcal{N}[e^A] = 1 + \sum_{k} \omega\, a^\dagger a + \cdots$, $\langle 0|a_k^\dagger a_k|0\rangle = 0$

故原式 $= \langle 0|0\rangle = 1$

$SU[J] = \mathcal{N}[e^{\int dx\,\phi_{in}(x)K(x)\frac{\delta}{\delta J(x)}}] Z[J]$，　$S = SU[J]|_{J=0}$

含Green函数

展开：$S = \sum_n S(x_1, x_2 \cdots x_n)$，　$\underline{S(x_1, \cdots x_n)} = \frac{1}{n!}\mathcal{N}\{(\phi_{in} iK \frac{\delta}{\delta J})^n\} Z[J]|_{J=0}$

n粒子S矩阵元 $= \mathcal{N} \prod_{m=1}^{n} \phi_{in}(x_m) iK(x_m) \cdot G(x_1, \cdots, x_n)$

$\frac{1}{n!}$将 x_m 排序的 $n!$ 消去

传播子 x—●—y $= i\Delta_F(x-y)$ → $iK(x)\, i\Delta_F(x, y) = \delta(x-y)$ → $iK(x)$消去一条线

$\phi_{in} iK(x)$：从ϕ_{in}引出一条线

Feynmann图外线规则：有一个自由端点的外线，标自由场ϕ_{in}

微扰展开：$Z[J] = N e^{i\mathcal{L}_1(\delta/i\delta J)} e^{-\frac{i}{2}J\Delta_F J}$

$S = \mathcal{N}\{e^{\phi_{in}K\frac{\delta}{\delta J}}\} N e^{i\mathcal{L}_1(\delta/i\delta J)} e^{-\frac{i}{2}J\Delta_F J}|_{J=0} = \mathcal{N}\{N e^{i\mathcal{L}_1(\delta/i\delta J)} e^{\phi_{in}K\frac{\delta}{\delta J}} e^{-\frac{i}{2}J\Delta_F J}\}_{J=0}$

$\mathcal{N}$仅作用于ϕ_{in}：　$K\phi_{in} = 0$.　$K\Delta_F = -1$

$\phi_{in}K\frac{\delta}{\delta J}[(i\phi_{in}J)^n e^{-\frac{i}{2}J\Delta_F J}] = [n(i\phi_{in}J)^{n-1} i\phi_{in}K\phi_{in} + (i\phi_{in}J)^n \phi_{in}K(-i\Delta_F J)] e^{-\frac{i}{2}J\Delta_F J}$

$= (i\phi_{in}J)^{n+1} e^{-\frac{i}{2}J\Delta_F J}$

$e^{\phi_{in}K\frac{\delta}{\delta J}} e^{-\frac{i}{2}J\Delta_F J} = \sum_n \frac{(i\phi_{in}J)^n}{n!} e^{-\frac{i}{2}J\Delta_F J} = e^{i\phi_{in}J - \frac{i}{2}J\Delta_F J}$

故 $S = \mathcal{N}\{e^{i\mathcal{L}_1(\delta/i\delta J)} Z[\phi_{in}, J]\}_{J=0}$　（归一化常数已略）

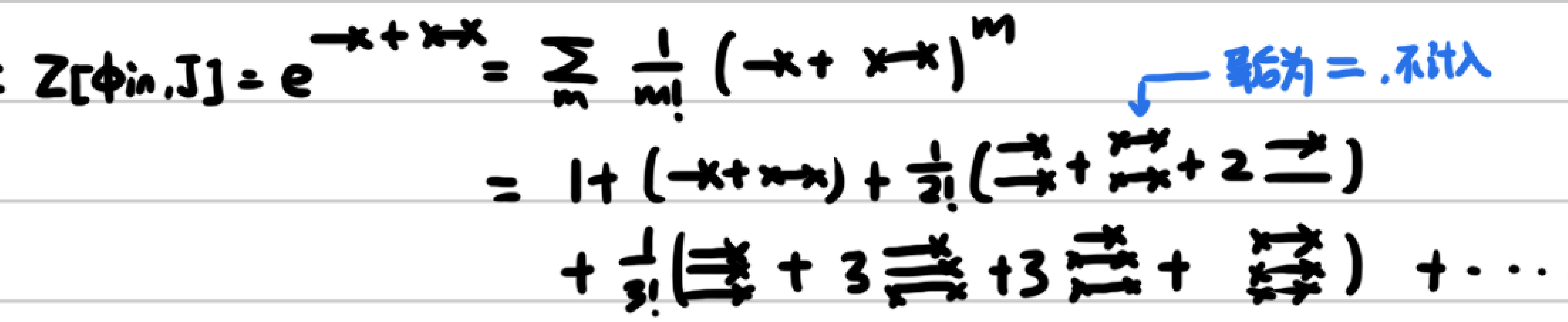
$Z[\phi_{in}, J] = e^{i\phi_{in}J - \frac{i}{2}J\Delta_F J}$　　体现了Feynmann图去除真空图

微扰展开 $S=\sum_n S_n$. $S_n = \mathcal{N}\left[\frac{1}{n!}\left[i\mathcal{L}_I\left(\frac{\delta}{i\delta J}\right)\right]^n Z[\phi_{in}, J]\right]_{J=0}$

$S_0 = Z[\phi_{in}, J]_{J=0} = 1$ 　$\mathcal{L}_I$ 0阶无依赖

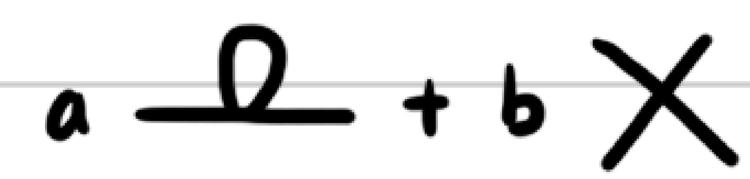
对 ϕ^4 模型：$i\mathcal{L}_I\left(\frac{\delta}{i\delta J}\right) = -i\frac{\lambda}{4!}\left(\frac{\delta}{i\delta J}\right)^4$

$Z[\phi_{in}, J]$ Feynman图：



[illegible]为二，不计入

拓扑等价图的等价方式 ?]

$S_1 =$  （无∞）

5. π-N散射

唯象Yukawa理论

核子间相互作用：核力

低能 — 不考虑核内部结构

核子 → Fermi子，有2个态 $\psi = \begin{pmatrix}\psi_p \\ \psi_n\end{pmatrix}$ → 质子湮灭/反质子产生 → 中子湮灭/反中子产生

π介子 → 0自旋Bose子，属于K-G场 → π^+, π^0, π^- 3种电荷 → $\phi = \begin{pmatrix}\phi_1 \\ \phi_2 \\ \phi_3\end{pmatrix}$ 　$\phi^0 = \phi_3$，$\phi^{\pm} = \frac{1}{\sqrt{2}}(\phi_1 \mp i\phi_2)$

$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$. 　$\mathcal{L}_0 = \mathcal{L}_N + \mathcal{L}_\pi$ 　$\mathcal{L}_N = \bar{\psi}(i\gamma^\mu\partial_\mu - m_N)\psi$ ；$\mathcal{L}_\pi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - m_\pi^2\phi^2$

Yukawa 耦合：$\mathcal{L}_I = \mathcal{L}_{\pi N} = -ig\bar{\psi}\gamma^5\vec{\tau}\psi\cdot\vec{\phi}$ ，τ 为同位旋空间的Pauli矩阵 τ_1, τ_2, τ_3 — 与 ϕ_i 缩并，作用于 ψ

$= -i\sqrt{2}g(\bar{\psi}_p\gamma^5\psi_n\phi^+ + \bar{\psi}_n\gamma^5\psi_p\phi^-) - ig(\bar{\psi}_p\gamma^5\psi_p - \bar{\psi}_n\gamma^5\psi_n)\phi^0$

电荷守恒 ： 中空间转动 ?]

对 $\pi^+ p$ 散射，$\mathcal{L}_{\pi^+ p} = -i\sqrt{2}g(\bar{\psi}_p\gamma^5\psi_n\phi^+ + \bar{\psi}_n\gamma^5\psi_p\phi^-)$

同位旋空间

生成泛函 $Z_0[J,\bar\eta,\eta] = Z_\pi[J]Z_N[\bar\eta,\eta] = e^{-J\Delta_F J - i\bar\eta S_F\eta}$

└ 相互作用 $Z[J\bar\eta\eta] = Ne^{g\frac{\delta}{i\delta\eta}\gamma^5\tau\frac{\delta}{i\delta\bar\eta}\cdot\frac{\delta}{i\delta J}} Z_0$

S矩阵 $S = \mathcal{N}\{e^{g\frac{\delta}{i\delta\eta}\gamma^5\tau\frac{\delta}{i\delta\bar\eta}\cdot\frac{\delta}{i\delta J}} \underline{Z[\phi_{in},\psi_{in},\bar\psi_{in};J,\bar\eta,\eta]}\}$。

$= Z_\pi[\phi_{in},J]Z_N[\psi_{in},\bar\psi_{in},\bar\eta,\eta]$

$= e^{i\phi_{in}J - \frac{1}{2}J\Delta_F J}\, e^{i\bar\psi_{in}\eta + i\bar\eta\psi_{in} - i\bar\eta S_F\eta}$

Feynman图： Dirac场规则： $iS_F(x-y) = \langle 0|T\psi(x)\bar\psi(y)|0\rangle =$ y⟶x

$-i\bar\eta S_F(x-y)\eta = \int dxdy\ i\bar\eta(x)iS_F(x-y)i\eta(y) =$ ×⟶×

$i\bar\psi_{in}\eta =$ ×⟶ （$i\bar\eta\psi_{in} =$ ⟶× ）

$i\eta$，$i\bar\eta = \times$

入外线为$\bar\psi_{in}$，出外线为ψ_{in}
入源为$i\eta$，出源为$i\bar\eta$
沿线为粒子，逆线为反粒子

标量场用虚线： $Z[\phi_{in},\psi_{in},\bar\psi_{in};J,\bar\eta,\eta] = e^{\cdots\times + \times\cdots\times}\, e^{\times\to + \to\times + \times\to\times}$

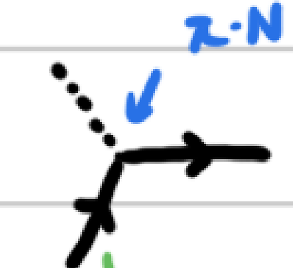
$e^{g\frac{\delta}{i\delta\eta}\gamma^5\tau\frac{\delta}{i\delta\bar\eta}\cdot\frac{\delta}{i\delta J}} = \sum_n \frac{1}{n!}\left(g\frac{-\delta}{i\delta\eta}\gamma^5\tau\frac{\delta}{i\delta\bar\eta}\frac{\delta}{i\delta J}\right)^n$

$g\gamma^5\tau$

↓

S_1 贡献项： ，π[ ]

π-N顶点

$S_1 = \mathcal{N}\{g\frac{-\delta}{i\delta\eta}\gamma^5\tau\frac{\delta}{i\delta\bar\eta}\frac{\delta}{i\delta J}(\ )\}_0 =$  $= g\mathcal{N}\int dx\,\bar\psi_{in}(x)\underline{\gamma^5\tau}\psi_{in}\phi_{in}(x)$

核子-进-出→数守恒
↳ $\frac{\delta}{i\delta\eta}$与$\frac{\delta}{i\delta\bar\eta}$成对存在

跃迁振幅

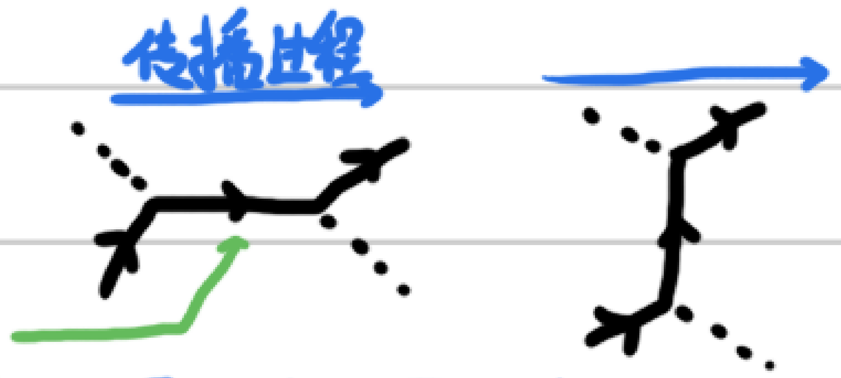
S_2：2个π-N顶点、6个内边→

S_4：4个π-N顶点，12个粘点（×— 去×的单侧点）
2N1次成对出现

散射过程：π-进-出，核-进-出 → 4条外线（2阶图）

高阶微扰：对传修正（见左红字）

传播过程



内线（无法观测，为虚粒子）

（两图表示同过程）

π^+p散射振幅：考虑2阶微扰

散射初态与末态： $|i\rangle = a_k^\dagger c_{ps}^\dagger|0\rangle$

$|f\rangle = a_{k'}^\dagger c_{p's'}^\dagger|0\rangle$

附：Feynman 动量图

$S_{fi}^{(2)} = \langle f|S_2|i\rangle$， $S_2 = (\sqrt{2}g)^2\mathcal{N}\int d^4x d^4y\ \bar\psi_{p'}^{(-)}(x)\gamma^5\phi_{\pi^+}^{(+)}(x)iS_F(x-y)\gamma^5\psi_p^{(+)}(y)\phi_{\pi^+}^{(-)}(p)$

动量展开：(箱归一化) $\phi_p^{(+)}(x)=\sum_k \frac{1}{\sqrt{2\omega_k V}}a_k e^{-ikx} \rightarrow \phi_x^{(+)}a_k^\dagger|0\rangle=\frac{1}{\sqrt{2\omega_k V}}e^{-ikx}|0\rangle$

$\psi_p^{(+)}(y)=\sum_k \frac{1}{\sqrt{2\omega_p V}}c_{p,s}u(p,s)e^{-ipy} \rightarrow \psi_p^{(+)}(y)c_{ps}^\dagger|0\rangle=\frac{1}{\sqrt{2\omega_p V}}u(p,s)e^{-ipy}|0\rangle$

$\langle 0|c_{p's'}\bar{\psi}_p^{(-)}(x)=\langle 0|\bar{u}(p',s')e^{ip'x}\frac{1}{\sqrt{2\omega_p V}}$

$\downarrow$

$$S_{fi}^{(2)}=(\sqrt{2}g)^2\int\frac{d^4x\,d^4y\,d^4q}{(2\pi)^4}e^{i(p'-k)x}e^{-i(p-k')y}N_{fi}\,\bar{u}(p',s')\gamma^5\underbrace{\frac{ie^{-iq(x-y)}}{\not{q}-m_p+i\varepsilon}}_{iS_F(x-y)}\gamma^5 u(p,s)$$

参附录

$$=\int d^4q(2\pi)^4\delta^4(P_f)\delta(P_i)\,2g^2N_{fi}\,\bar{u}(p',s')\gamma^5\frac{i}{\not{q}-m_p+i\varepsilon}\gamma^5u(p,s)\quad,\quad N_{fi}=\frac{1}{\sqrt{2\omega_p\omega_k\omega_{p'}\omega_{k'}}V^2}$$

$p'-k-q=0,\ p-k'+q=0\Rightarrow p+k=p'+k'$
$P_i=P_f$ (动量守恒)

$$=-i(2\pi)^4\delta(P_f-P_i)\,2g^2N_{fi}\,\bar{u}\not{k}'u\,\frac{1}{2pk'-m_\pi^2}$$?コ

$(\not{q}-m_p+i\varepsilon)(\not{q}+m_p)=q^2-m_p^2+i\varepsilon \Rightarrow$ 原式 $=\bar{u}\gamma^5\frac{\not{q}+m_p}{q^2-m_p^2+i\varepsilon}\gamma^5u$

$q=k'-p$
$\Rightarrow (\not{p}-m_p)u(p,s)=0 \Rightarrow$ 原式 $=\bar{u}\gamma^5\frac{\not{k}'}{q^2-m_p^2+i\varepsilon}\gamma^5u$

$=\bar{u}\not{k}'u\cdot\frac{1}{(k'-p)^2-m_p^2+i\varepsilon}$

$\xRightarrow[p^2-m_p^2=0]{k'^2-m_\pi^2=0} =\bar{u}\not{k}'u\cdot\frac{-1}{2pk'-m_\pi^2}$

动能关系：$p^{0\,2}-\mathbf{p}^2=m^2 \rightarrow$ 质壳

虚粒子不严格满足 $\rightarrow$ 离壳

6. 散射截面

1. 跃迁概率

反应矩阵R　　$S=1+iR$　相互作用时S无影响

$S_{fi}=\langle f|S|i\rangle=\langle f|i\rangle+i\underbrace{\langle f|R|i\rangle}_{R_{fi}}$ ，$R_{fi}=\langle f|R|i\rangle=-(2\pi)^4\delta(P_f-P_i)\mathcal{M}_{fi}$　动量空间F图

不变振幅 $\mathcal{M}_{fi}=N_{fi}M_{fi}$

对 π^+p 散射：$\mathcal{M}_{fi}=2g^2N_{fi}\frac{\bar{u}(p',s)\not{k}'u(p,s)}{2pk'-m_\pi^2}$

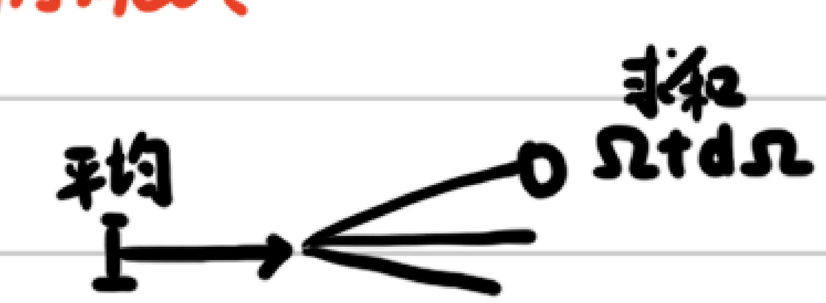
跃迁率：$W_{fi}=|\langle f|S|i\rangle|^2$

$\downarrow$ 对 f, i 不同时，$\langle f|i\rangle=0$，$W_{fi}=|R_{fi}|^2=(2\pi)^8\delta^4(0)\,\delta^4(P_f-P_i)|\mathcal{M}_{fi}|^2$

$\delta^4(0)=\lim_{k\to 0}\int\frac{d^4x}{(2\pi)^4}e^{-ikx}=\lim_{T,V\to\infty\ \text{时空}}\frac{TV}{(2\pi)^4}$

单位时间跃迁率　$P_{fi}=\lim_{T\to\infty}\frac{W_{fi}}{T}$　?コ为何为∞?

实验中：初态需平均，末态需求和：



$$\overline{P}_{fi} = \overline{\sum} P_{fi} = \overline{\sum} V(2\pi)^4 \delta^4(P_f - P_i)|M_{fi}|^2$$

$\pi^+ p$ 散射：$\overline{P}_{fi} = \frac{1}{2}\sum_{s,s'}\int \frac{V d^3k'}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3} V(2\pi)^4\delta^4(P_f - P_i)|M_{fi}|^2$

质心系：总 P 为 0 $\rightarrow$ $P_i = 0,\ k = -p \rightarrow \delta^4(P_f - P_i) = \delta(E_f - E_i)\,\delta^3(k' + p')$

$d^3k' = |k'|^2 d|k'|\, d\Omega_{k'} = |k'|\,\omega_\pi' d\omega_\pi' d\Omega_{\pi'}$ $(\omega d\omega = |k| d|k|)$

$$\frac{1}{2}\sum_{s,s'}\int \frac{d^3k' d^3p'}{(2\pi)^6} V^3 (2\pi)^4 \delta^4(P_i - P_f)|M_{fi}|^2 = \frac{V^3}{8\pi^2}\sum_{s,s'}\int d^3p' |k'|\omega_\pi' d\omega_\pi' d\Omega_\pi \left(\delta(\omega_\pi' + \omega_p' - \omega_\pi - \omega_p)\underbrace{\delta^3(k' + p')}_{\delta(p'+k')}|M_{fi}|^2\right.$$

$$= \frac{V^3}{8\pi^2}\sum\int d\omega_\pi d\Omega_\pi\, \omega_\pi' |k'|\, \delta(\omega_\pi' + \omega_p' - \omega_\pi - \omega_p)|M_{fi}|^2$$

能量守恒

$$\left[\begin{aligned} &N_{fi} = \frac{1}{4\omega_\pi \omega_p V^2} \\ &M_{fi} = 2g^2 N_{fi}\frac{\bar{u}(p',s')\not{k}\,u(p,s)}{2pk' - m_\pi^2} \\ &\overline{P}_{fi} = \frac{1}{8\pi^2}\sum_{s,s'}\int d\Omega_k V^3 \omega_\pi |k| |M_{fi}|^2 = \frac{g^2}{32\pi^2}\frac{|k|}{\omega_\pi \omega_p^2 V}\int \frac{d\Omega_{k'}}{2pk' - m_\pi^2}\underbrace{\sum_{s,s'}|\bar{u}(p',s')\not{k}'u(p,s)|^2} \end{aligned}\right.$$

对自旋求和： 对 $\forall$ Dirac 矩阵 A：$(\bar{u}'Au)^\dagger = u^\dagger A^\dagger \gamma^{0\dagger} u' = \bar{u}\underline{A}u'$ $\quad \underline{A} = \gamma^0 A^\dagger \gamma^0$

$$|\bar{u}'\not{k}'u|^2 = \bar{u}\underline{A}u'\bar{u}'Au = \sum_{ijkl}\bar{u}_i \underline{A}_{ij} u_j' \bar{u}_k' A_{kl} u_l = \sum u_j' \bar{u}_k' A_{kl} u_l \bar{u}_i \underline{A}_{ij} = tr(u'\bar{u}'Au\bar{u}\underline{A})$$

$\sum_{s,s'} u(p,s)\bar{u}(p,s) = \not{p} + m_p$

$\gamma^0 \not{k}'^\dagger \gamma^0 = \not{k}'$

$$= tr[(\not{p}' + m_p)\not{k}'(\not{p} + m_p)\not{k}'] = tr(\not{p}'\not{k}'\not{p}\not{k}') + m_p^2 tr(\not{k}'^2)$$

$$= -tr(\not{k}'\not{p}'\not{p}\not{k}') + 2tr(\not{p}'\not{k}')tr(\not{p}\not{k}') + 4m_p^2 k'^2$$

$$= -4p'pk'^2 + 8p'k'pk' + 4m_p k'^2 = 4(2(pk')(p'k') + m_\pi^2[m_p^2 - pp'])$$

$$\overline{P}_{fi} = \frac{g^4}{8\pi^2}\frac{|k|}{\omega_\pi \omega_p^2 V}\int \frac{d\Omega_{k'}}{(2pk' - m_\pi^2)^2}(2pk'p'k' + m_\pi^2[m_p^2 - pp'])$$

散射截面： σ 面积量纲 来流强度 $j = \frac{v}{V}$ 相对速 $v = \frac{|k|}{\omega_\pi} + \frac{|p|}{\omega_p} = \frac{|k|(\omega_p + \omega_\pi)}{\omega_p \omega_\pi}$

$$\sigma = \frac{\overline{P}_{fi}}{j} = \frac{\overline{P}_{fi} V}{v} \rightarrow \frac{d\sigma}{d\Omega} = \frac{g^4}{8\pi^2}\frac{2pk'p'k' + m_\pi^2[m_p^2 - pp']}{\omega_p(\omega_p + \omega_\pi)(2pk' - m_\pi^2)^2}$$

耦合常数计算：低能：$p \ll m \rightarrow \frac{d\sigma}{d\Omega} \approx \frac{g^4}{8\pi^2}\frac{2m_p^2 m_\pi^2 + m_\pi^2[0]}{m_p m_\pi^2(m_p + m_\pi)(2m_p - m_\pi)^2} = \frac{g^2}{4\pi^2}\frac{m_p}{(m_p + m_\pi)(2m_p - m_\pi)^2} \overset{m_\pi \ll m_p}{\approx} \frac{g^2}{16\pi^2 m_p^2}$

$$\sigma = \int d\sigma = 4\pi\frac{g^4}{16\pi^2 m_p^2} = \frac{g^4}{4\pi m_p^2} \xrightarrow{\text{数量级}} \frac{g^2}{4\pi} = 1\sim5 \rightarrow \pi N \text{耦合为强相互作用}$$