三.矢量场

1、Maxwell场

Abel规范场：

$\phi' = e^{i\gamma(x)}\phi$　相位变换 → $U(1)$

$A'_\mu(x) = A_\mu(x) + \partial_\mu \chi(x)$ ，　$\chi(x) \propto \gamma(x)$ → $A_\mu(x)$　$U(1)$规范场

A'不改变$\mathcal{L}$

$\mathcal{L}$中含场的部分必有 $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ ，　$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ ← 纯场无粒子

使E正定

习惯约定

δA^μ ：　$\delta\mathcal{L} = (-\frac{1}{2}F_{\mu\nu}\,\delta F^{\mu\nu}) = -\frac{1}{2}F_{\mu\nu}\partial^\mu \delta A^\nu + \frac{1}{2}F_{\nu\mu}\partial^\nu \delta A^\nu = -F_{\mu\nu}\partial^\mu \delta A^\nu = \partial^\mu F_{\mu\nu}\cdot \delta A^\nu - \partial^\mu \underbrace{(F_{\mu\nu}\delta A^\nu)}_{0}$

A^ν与$\partial_\mu A^\nu$为2个独立变量 ↳　$\frac{\partial\mathcal{L}}{\partial A_\nu} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial \partial_\mu A_\nu}\right) = 0 + \partial_\mu\partial^\mu A^\nu - \partial_\mu \partial^\nu A^\mu = \partial_\mu F^{\mu\nu} = 0$

Jacobi恒等式 $\partial_{[\lambda}F_{\mu\nu]} = 0$

规范条件：

Lorenz　$\partial_\mu A^\mu = 0$

Coulomb　$\vec{\nabla}\cdot\vec{A} = 0$

辐射　$A^0 = 0$; $\vec{\nabla}\cdot\vec{A} = 0$

2.场的角动量

空间转动：$\Omega^i{}_j$ ，$x'^i = \Omega^i{}_j x^j$ ，　$A'^i = \Omega^i{}_j A^j$

└ $\Omega^i{}_j = e^{-iJ_k\cdot\theta^k}$ → $J_1 = \begin{bmatrix} 0 & & \\ & 0 & -i \\ & i & 0 \end{bmatrix}$. $J_2 = \begin{bmatrix} & & i \\ & 0 & \\ -i & & \end{bmatrix}$. $J_3 = \begin{bmatrix} 0 & -i & \\ i & 0 & \\ & & 0 \end{bmatrix}$ → $[J_i, J_j] = -i\varepsilon^k{}_{ij}J_k$

$J^2 = J_1^2 + J_2^2 + J_3^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

自旋本征态：$\vec{e}^{\pm} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}(\vec{e}^1 \pm i\vec{e}^2)$ ，$\vec{e}^0 = \vec{e}^3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ ，本征值 $s = \pm 1, 0$.

解形式：$\vec{k} // \vec{e}^3$的波　$\vec{A}(x,t) = (a_1\vec{e}^1 + a_2\vec{e}^2 + a_3\vec{e}^3)\,e^{i(\omega t - \vec{k}\cdot\vec{x})} = (a_+ e^+ + a_- e^- + a_0 e^0)\,e^{-i(\omega t - \vec{k}\cdot\vec{x})}$　$a_+ = \frac{1}{\sqrt{2}}(a_1 - ia_2)$, $a_- = \frac{1}{\sqrt{2}}(a_1 + ia_2)$, $a_0 = a_3$

矢量场自旋为1?

a^-左旋波，a^+右旋波，a^0纵波

转动守恒： $A'^i(x) = a^i{}_j A^j(x) \to \delta A^i = \varepsilon^i{}_{jk} A^j \theta^k$； $\delta x^i = \varepsilon^i{}_{jk} x^j \theta^k$． $\theta^k \ll 1$

$\mathcal{L}$在空间转动不变： $\delta x^0 = \delta A^0 = 0$， $\delta\mathcal{L} = 0$

$$\delta\mathcal{L} = \partial_\mu \mathcal{L}(x)\,\delta x^\mu + \left(\frac{\partial\mathcal{L}}{\partial A^\mu}\bar\delta A^\mu + \frac{\partial\mathcal{L}}{\partial\partial_\mu A^\nu}\bar\delta\,\partial_\mu A^\nu\right)$$

$$\delta S = \int d^4x \left(\frac{\partial\mathcal{L}}{\partial A^\nu} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu A^\nu}\right)\bar\delta A^\mu + \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu A^\nu}\bar\delta A^\nu + \mathcal{L}\,\delta x^\mu\right)$$

$\bar\delta A^\nu = \delta A^\nu - \partial_\nu A^\mu \delta x^\nu$

$$j^\mu = \frac{\partial\mathcal{L}}{\partial\partial_\mu A^\nu}\bar\delta A^\nu + \mathcal{L}\,\delta x^\mu \longrightarrow j^0 = g^0{}_\mu j^\mu = \frac{\partial\mathcal{L}}{\partial\partial_0 A^i}\delta A^i - \left(\frac{\partial\mathcal{L}}{\partial\partial_0 A^\mu}\partial_i A^\mu - \mathcal{L} g^0{}_i\right)\delta x^i = \left[-\underbrace{\left(\frac{\partial\mathcal{L}}{\partial\partial_0 A^\mu}\partial^i A^\mu - \mathcal{L} g^0{}_i\right)}_{P^i}\varepsilon_{ijk}x^j + \frac{\partial\mathcal{L}}{\partial\partial_0 A^i}\varepsilon^i{}_{jk}A^j\right]\theta^k$$

$\delta x^0 = \delta A^0 = 0$

$\downarrow$

$$j^0 = (M_k + S_k)\theta^k, \quad M_k = \varepsilon_{ijk} x^i P^j, \quad S_k = \frac{\partial\mathcal{L}}{\partial\partial_0 A^i}\varepsilon^i{}_{jk}A^j$$

轨道角动量 $\vec{x}\times\vec{p}$　　内禀角动量

ε_{ijk}中ij反对称

自旋角动量： 对 $\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$． $S_k = (\partial_i A^0 - \partial^0 A_i)A_j\varepsilon^{ij}{}_k = -(\partial^0 A^i)A^j\varepsilon_{ijk} = \frac{1}{2}(A^i\partial^0 A^j - \partial^0 A^i A^j)\cdot\varepsilon_{ijk}$

$= -\frac{i}{2} A^i\, i\overleftrightarrow{\partial_0} A^j\,\varepsilon_{ijk}$

A展开： $\vec{A}(x,t) = \int\frac{d^3k}{\sqrt{(2\pi)^3 2\omega}}\cdot\vec{e}^s_k\underbrace{(a_{ks}e^{-ik_\mu x^\mu} + a^\dagger_{ks}e^{ik_\mu x^\mu})}_{实化}$ ， $\vec{e}^s_k\cdot\vec{e}^{s'}_k = \delta_{ss'}$ → 规正基

☆李结构？$\boxed{}$

$[\vec{A},\vec{B}] = \vec{C}$

$\langle A_i, B_j\rangle\varepsilon^{ij}{}_k = C_k$

$\downarrow$

约定： $\vec{e}^1_{-k} = -\vec{e}^1_k$． $\vec{e}^2_{-k} = -\vec{e}^2_k$　($\vec{k}/\!/\vec{e}_3$，$\nabla\cdot\vec{A} = 0$ → 纵项为0)

$$\underbrace{S_k}_{总自旋角动量} = \int d^3x\cdot S_k = \int d^3x\cdot -\frac{i}{2}A^i\partial_0 A^j\varepsilon_{ijk} = -\frac{i}{2}\int\frac{d^3k}{(2\pi)^3 2\omega}e^s_{k,i}e^{s'}_{k,j}\int d^3x\,(a_{ks}e^{-ik_\mu x^\mu} + a^\dagger_{ks}e^{ik_\mu x^\mu})\,i\overleftrightarrow{\partial_0}\,(a_{ks'}e^{-ik_\mu x^\mu} + a^\dagger_{ks'}e^{ik_\mu x^\mu})\,\varepsilon^{ij}{}_k$$

$$= \frac{i}{2}\int d^3k\cdot e^s_{k,i}e^{s'}_{k,j}\varepsilon^{ij}{}_k(a_{ks}a^\dagger_{ks'} - a^\dagger_{ks}a_{ks'})$$

$$S_3 = \frac{i}{2}\int d^3k\left[(a^\dagger_{k1}a_{k2} + a^\dagger_{k2}a_{k1}) - (a_{k2}a^\dagger_{k1} + a^\dagger_{k1}a_{k2})\right.$$

$$= \frac{1}{2}\int d^3k\left[\underbrace{(a_{k+}a^\dagger_{k+} + a^\dagger_{k+}a_{k+})}_{+1自旋} - \underbrace{(a_{k-}a^\dagger_{k-} + a^\dagger_{k-}a_{k-})}_{-1自旋}\right.$$

$a_{k\pm} = \frac{1}{\sqrt{2}}(a_{k1} \mp i a_{k2})$ ， $a^\dagger_{k\pm} = \frac{1}{\sqrt{2}}(a^\dagger_{k1} \pm i a^\dagger_{k2})$

3. Maxwell 场正则量子化

1. 辐射规范

正则动量：$\pi^\mu = \frac{\partial \mathcal{L}}{\partial \dot{A}_\mu} = F^{\mu 0} = \partial^\mu A^0 - \partial^0 A^\mu$

辐射规范：$A^0 = 0 \to \pi^0 = 0,\ \pi^i = -\dot{A}^i$ ；$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}[(\dot{A})^2 - (\nabla\times A)^2]$

$\mathcal{H} = \pi_\mu \dot{A}^\mu - \mathcal{L} = \frac{1}{2}[(\dot{A})^2 + (\nabla\times A)^2]$

物理想象：量子化产生？$\boxed{}$
的描述，何空间，何作用？
算符为何

横场正则量子化：$A^0=0,\ \pi^0=0 \to$ 不对易

$(\pi_i = -\pi^i)$ 等时对易：$\underline{[A_i(x,t),\ \pi^j(x',t)] = i g^j{}_i\,\delta(x-x')}$ ；$[A_i(x,t), A_j(x',t)] = 0$，$[\pi^i(x,t), \pi^j(x',t)] = 0$

$$\frac{\partial}{\partial x^i}[A^i, \pi_j] = [\nabla\cdot A, \pi_j] = 0$$

$$= \frac{\partial}{\partial x^i}\, i g^j{}_i\,\delta(x-x') = i\int\frac{d^3k}{(2\pi)^3} k_j\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \neq 0 \to \text{使用无散度}\ \bar{\delta}(x,x')$$

$$\bar{\delta}^j{}_i(x,x') \equiv \int\frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}\left(g^j{}_i - \frac{k_i k^j}{k_l k^l}\right) = \left(g^j{}_i + \frac{\partial_i\partial^j}{\nabla^2}\right)\delta(x-x')$$

$$[A_i(x,t),\ \pi^j(x',t)] = i\bar{\delta}^j{}_i(x-x')$$

$\langle \varphi_k(x), [\vec{e}^s_k\cdot\vec{A}(x)]\rangle = \int d^3x\, e^s_k e^s_k \langle\varphi_k(x), a_{ks}\varphi_k(x) + a^\dagger_{ks}\varphi^*_k(x)\rangle$
$= a_{ks}$

动量空间：满足 d'Alembert 方程：$\vec{A}(x) = \int d^3k\, \vec{e}^s_k[a_{ks}\varphi_k(x) + a^\dagger_{ks}\varphi^*_k(x)]$

└ $a_{ks} = \langle\varphi_k,\ \vec{e}^s_k\cdot\vec{A}(x)\rangle$，$a^\dagger_{ks} = \frac{\langle\varphi^*_k,\ \vec{e}^s_k\cdot\vec{A}(x)\rangle}{\langle\varphi^*_k,\ \varphi^*_k\rangle} = -\langle\varphi^*_k, (\vec{e}^s_k\cdot\vec{A}(x))\rangle$

$i\bar{\delta}^j{}_i(x-x') = [A_i, \pi^j] = \int d^3k\, e^s_i e^{s'}_j\left(\frac{1}{i} a_k a^\dagger_k\langle\varphi^*, \varphi^*\rangle + a_k a_k\langle\varphi^*, \varphi\rangle + a^\dagger a^\dagger\langle\varphi, \varphi^*\rangle + a^\dagger a\langle\varphi, \varphi\rangle\right)$

可倒推？]

$= \frac{1}{i}\int d^3k\, e^s_i e^{s'}_j\left([a^\dagger_k, a_k]\,\delta(x-x')\right) \Longrightarrow [a^\dagger_k, a_{k'}] = -\delta(k,k')\cdot\delta_{ss'}$

$[A_i, A_j] = 0,\ [\pi^i, \pi^j] = 0 \Longrightarrow [a_k, a_{k'}] = 0,\ [a^\dagger_k, a^\dagger_{k'}] = 0$

$$H = \int d^3x\,\mathcal{H} = \int d^3x\,\frac{1}{2}[(\dot{A})^2 + (\nabla\times A)^2] = \int d^3x\,\frac{1}{2}[(\dot{A})^2 - \vec{A}\cdot\nabla^2\vec{A}] = \frac{1}{2}\int d^3x\,(\dot{A})^2 - \vec{A}\cdot\ddot{\vec{A}} = \frac{1}{2}\int d^3x\, A_i\overleftrightarrow{\partial_0}\vec{A} = \frac{1}{2}\langle\vec{A}, \vec{A}\rangle$$

$(\nabla\times A)^2 + \vec{A}\nabla^2\vec{A} = \nabla\cdot\underbrace{\left(\nabla\left(\frac{A^2}{2}\right) - 2(A\cdot\nabla)\vec{A}\right)}_{\int\text{下为}0} + 2(\vec{A}\cdot\nabla)(\nabla\cdot A)$

$\partial_\mu\partial^\mu A_\nu = 0$

$$= \frac{1}{2}\int d^3x\int d^3k\, \vec{e}^s_k\cdot\vec{e}^{s'}_k(a_k\varphi_k + a^\dagger_k\varphi^*_k)\, i\overleftrightarrow{\partial_0}\,(i\omega(a^\dagger_k\varphi^*_k - a_k\varphi_k))$$

$$= \int d^3k \sum_{s=1}^{3}\frac{\omega}{2}(a^\dagger_k a_k + a_k a^\dagger_k)$$ → 自旋为1，质量为0 Bose子 → 光子

↳ $s\neq 3$

一般对易：$[A_i(x), A_j(x')] = \int d^3k\sum_{s\neq 3} e^s_{ki} e^s_{kj}[\varphi_k(x)\varphi^*_k(x') - \varphi^*_k(x)\varphi_k(x')] = -\int\frac{d^3k}{(2\pi)^3 2\omega}\left(g_{ij} + \frac{k_i k_j}{k^2}\right)[e^{-ik(x-x')} - e^{ik(x-x')}]$

$\sum_s e^s_{ki} e^s_{kj} = -g_{ij}$ $\quad = -i\left(g_{ij} + \frac{\partial_i \partial_j}{\nabla^2}\right) D(x-x')$

$D(x-x') = -i\int \frac{d^3k}{(2\pi)^3 2\omega}\left[e^{-ik(x-x')} - e^{ik(x-x')}\right] = \Delta(x-x')|_{m=0}$

2. Lorentz规范

使$\pi^0 \neq 0$，保持协变性 → $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}(\partial_\mu A^\mu)^2$ ← 规范固定项

$\frac{\partial \mathcal{L}}{\partial \partial_\mu A_\nu} = -\partial^\mu A^\nu + \partial^\nu A^\mu - \lambda g^{\mu\nu}\partial_\sigma A^\sigma$; $\frac{\partial \mathcal{L}}{\partial A_\mu} = 0$

E-L方程：$\partial_\mu \partial^\mu A^\nu + (1-\lambda)\partial^\nu(\partial_\sigma A^\sigma) = 0$ $\xrightarrow{\lambda=1}$ Feynman 规范 （并非规范） → 使E-L方程为d'Alembert方程

正则量子化： $\lambda=1$时，$\mathcal{L} = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu$

$\mathcal{L} = -\frac{1}{4}(2\partial_\mu A_\nu \partial^\mu A^\nu - 2\partial_\mu A_\nu \partial^\nu A^\mu) - \frac{1}{2}(\partial_\mu A^\mu)^2 = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu - \frac{1}{2}\left(\partial_\mu(A^\mu \partial_\nu A^\nu) - A^\mu \partial_\mu \partial_\nu A^\nu - \partial_\mu(A_\nu \partial^\nu A^\mu) + A_\nu \partial^\nu \partial_\mu A^\mu\right)$

$= -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu - \underbrace{\frac{1}{2}\partial_\mu(A^\mu \partial_\nu A^\nu - A_\nu \partial^\nu A^\mu)}_{\text{积分为}0}$

$\pi^\mu = \frac{\partial \mathcal{L}}{\partial \partial_0 A_\mu} = -\dot{A}^\mu$ ， 等时对易：$[A_\mu(x,t), \pi^\nu(x',t)] = i g_\mu^{\ \nu}\delta(x-x')$

$[A_\mu(x,t), A_\nu(x',t)] = 0$ ，$[\pi^\mu(x,t), \pi^\nu(x',t)] = 0$

与Lorenze规范：$\partial_\mu A^\mu = 0 \to \dot{A}^0 = \nabla \cdot \vec{A}$

与对易式冲突 → 不是算符的约束

平均值约束：$\langle \psi | \partial_\mu A^\mu | \psi \rangle = 0$ → 弱Lorenze规范条件

极化矢量 $e^\sigma_{k\mu}$ ，$e^0_{k\mu}$类时，$e^i_{k\mu}$类空 ，$e^\sigma_k \cdot e^{\sigma'}_k = g^{\mu\nu} e^\sigma_{k\mu} e^{\sigma'}_{k\nu} = g^{\sigma\sigma'}$ 基底变换 $\mu,\nu \to \sigma,\sigma'$

取$k // e^3_k$，有：$k = (\omega, 0, 0, k)$，e^1与e^2为横向极化矢量 （e是完备的）$\sigma,\sigma' \to \mu\nu$也可

$A_\mu(x) = \int d^3k \sum_{\sigma=0}^{3} e^\sigma_{k\mu}\left[a_{k\sigma}\varphi_k(x) + a^\dagger_{k\sigma}\varphi^*_k(x)\right]$ $\longrightarrow$ $a_{k\sigma} = \langle \varphi_k, g_{\sigma\sigma'} e^{\sigma'}_{k\mu} A^\mu(x)\rangle = \langle \varphi_k, e_{k\sigma\mu} A^\mu \rangle$

$a^\dagger_{k\sigma} = -\langle \varphi^*_k, g_{\sigma\sigma'} e^{\sigma'}_{k\mu} A^\mu(x)\rangle = -\langle \varphi^*_k, e_{k\sigma\mu} A^\mu \rangle$

对易关系

$[a_{k\sigma}, a^\dagger_{k'\sigma'}] = -g_{\sigma\sigma'}\delta(k-k')$ else $=0$ → $\sigma=0$为标量光子，1,2为横光子，3为纵光子

协变对易关系：$[A_\mu(x), A_\nu(x')] = -i g_{\mu\nu} D(x-x')$ ← 方法同上

动量空间 Lorenze 规范条件

$A_\mu(x) = A^{(+)}_\mu(x) + A^{(-)}_\mu(x)$ ，$A^{(+)}_\mu(x) = \int d^3k \sum_{\sigma=0}^{3} e^\sigma_{k\mu} a_{k\sigma}\varphi_k(x)$ ，$A^{(-)}_\mu(x) = [A^{(+)}_\mu(x)]^\dagger$

个人理解：可以认为 $\partial_\mu A^\mu=0$，但为保证 $\partial_0 A_0$ 同为算符，故用态矢等式保持地位

$$\langle\psi|\partial_\mu A^\mu(x)|\psi\rangle=\langle\psi|\partial_\mu A^{(+)\mu}(x)|\psi\rangle+h.c.$$

$$\partial^\mu A^{(+)}_\mu(x)|\psi\rangle=0 \rightarrow e^{\lambda}_{k\mu}k^\mu a_{k\lambda}|\psi\rangle=0 \rightarrow (a_{k0}-a_{k3})|\psi\rangle=0$$

纵光子与标量光子同时存在，物理态平均为0 → 测量贡献相反

$$\langle\psi|a^\dagger_{k0}a_{k0}-a^\dagger_{k3}a_{k3}|\psi\rangle=0 \rightarrow N_{k0}=-a^\dagger_{k0}a_{k0},\ N_{k3}=a^\dagger_{k3}a_{k3}$$

$-g_{\mu\nu}$

能量动量：$P^\mu=\frac{\partial\mathcal{L}}{\partial\partial_0A^\nu}\cdot\partial^\mu A^\nu-\mathcal{L}g^{0\mu}=-\frac{i}{2}A_\nu i\overleftrightarrow{\partial_0}\partial^\mu A^\nu \rightarrow P^\mu=\frac{i}{2}\langle A_\nu,\partial^\mu A^\nu\rangle$

$$P^\mu=\int d^3k\cdot\left(-\frac{1}{2}\right)k^\mu g^{\lambda\lambda'}(a_{k\lambda}a^\dagger_{k\lambda'}+a^\dagger_{k\lambda}a_{k\lambda'})$$

不定度规：

$\langle n_{k0}|n_{k0}\rangle=(-1)^{n_{k0}}\langle0|0\rangle$

$$\langle n_{k0}|a^\dagger_{k0}a_{k0}|n_{k0}\rangle=-\langle n_{k0}|N_{k0}|n_{k0}\rangle=-n_{k0}\langle n|n\rangle$$

$$\langle n_{k0}-1|\sqrt{n_{k0}}\sqrt{n_{k0}}|n_{k0}-1\rangle=n_{k0}\langle n_{k0}-1|n_{k0}-1\rangle$$

$\vdots$

不定度规的물理论？ 1 Hilbert空间有不定度规

(宏观上 $g_{\mu\nu}A^\mu A^\nu$ 同为不定度规

定义态矢量模方：$\langle\psi|\eta|\psi\rangle$，$\eta$ 为度规算符 $\eta^\dagger=\eta$

原：自厄：$F=F^\dagger$ $(\eta=1)$

解决了时空不定度规带入态矢的问题

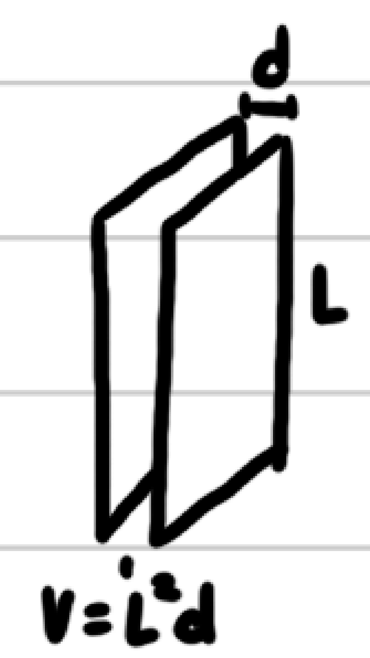
$$\langle F\rangle=\frac{\langle\psi|\eta F|\psi\rangle}{\langle\psi|\eta|\psi\rangle}\in\mathbb{R} \rightarrow F=\underbrace{\eta^{-1}F^\dagger\eta}_{伴算符}$$ → 自伴条件

3. 几个问题：

无 m，$w=|k|$

零点能：$E=\sum_k\sum_{s=1}^{2}\frac{w}{2}(a_{ks}a^\dagger_{ks}+a^\dagger_{ks}a_{ks})=\sum_{k}\sum_{s}w\left(N_{ks}+\frac{1}{2}\right)\rightarrow E_0=\sum_{k,s}\frac{w}{2}=\sum_k w$

积分：$E_0=\frac{V}{(2\pi)^3}\int_0^\infty 4\pi k^2dk\cdot k=\lim_{k_c\to\infty}\frac{Vk_c^4}{8\pi^2}$，$k_c$ 为截断波上限（4次方发散，紫外）



Casimir力：

$d\ll L$，故 k_z 取离散值：$k_z=n\frac{\pi}{d}$

$$E_0(d)=\frac{L^2d}{8\pi^3}\sum_{n=-\infty}^{\infty}\frac{\pi}{d}\int_{-\infty}^{\infty}dk_xdk_y\,k_d=\sum_{n=0}^{\infty}\theta_n\frac{L^2}{\pi^2}\int_0^\infty dk_xdk_y\,k_d$$

$k_d=|k|=\sqrt{k_x^2+k_y^2+|\frac{n\pi}{d}|^2}$，$\theta_n=\begin{cases}1 & n>0\\ \frac{1}{2} & n=0\end{cases}$

光滑截断函数

与自由空间之差：$\delta E_0=\frac{L^2}{\pi^2}\left(\sum_{n=0}^{\infty}\theta_n g\left(\frac{\pi n}{d}\right)-\frac{d}{\pi}\int_0^\infty dk_z\,g(k_z)\right)$

$k=\sqrt{k_x^2+k_y^2+k_z^2}$，$g(k_z)=\int_0^\infty dk_xdk_y\,k\cdot f\left(\frac{k}{k_c}\right)$，$f(x)=\begin{cases}1 & x\ll1\\ 0 & x\gg1\end{cases}$

令 $k=\sqrt{k_x^2+k_y^2}=\frac{\pi\alpha}{d}$，$k_z=\frac{\pi\nu}{d}$，$\xi=\alpha^2+\nu^2$

$$g(k_z)=\frac{1}{4\pi}\int_0^\infty 2\pi x\,dx\sqrt{k^2+\left(\frac{\pi\nu}{d}\right)^2}\,f\left(\sqrt{k^2+\left(\frac{\pi\nu}{d}\right)^2}/k_c\right)=\frac{\pi^4}{4d^3}\underbrace{\int_{\nu^2}^\infty d\xi\sqrt{\xi}\,f\left(\frac{\pi\sqrt{\xi}}{k_c d}\right)}_{F(\nu)}$$

特殊函数概论?

$$\delta E_0=\frac{\pi^2L^2}{4d^3}\left(\sum_{n=0}^\infty\theta_n F(n)-\int_0^\infty d\nu F(n)\right)$$

Eular-Maclaurin公式 $\longrightarrow$ $=\frac{\pi^2L^2}{4d^3}\left(-\frac{1}{6\times 2!}F'(0)+\frac{1}{30\times 4!}\underbrace{F'''(0)}_{-4f(0)=-4}-\cdots\right)\approx\frac{\pi^2L^2}{-720d^3}$

$$F_z=-\frac{1}{L^2}\frac{\partial\delta E_0}{\partial d}\approx\frac{-\pi^2}{240d^4}$$，吸引

表明零点能可观测

单光子无质量 → 必然相对论 → 同2.2结论：无坐标表象波函数

↓

无单光子的空间概率分布 → 可用光子数本征态组 $\{|n_{k\sigma}\rangle\}$

?证实后改黑色

量子光学用 $a_{k s}$ 的本征态（相干态），$a=e^{i\phi}N^{\frac{1}{2}}$，$a^\dagger=N^{\frac{1}{2}}e^{-i\phi}$，$[a,a^\dagger]=[N,i\phi]=1$

粒子数与相位不能同时测准

4. 重矢量场

质量项：$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m^2A_\mu A^\mu$

↓

$\partial_\mu\partial_\nu F^{\mu\nu}=\partial_\mu\partial_\nu F^{\nu\mu}=0$

Proca方程：$\partial_\mu F^{\mu\nu}+m^2A^\nu=0$ $\xrightarrow{\text{取散度}}$ $m^2\partial_\nu A^\nu=0$ （内含Lorentz规范，故无法规范变换）

$\partial^\nu\underbrace{\partial^\mu A_\mu}_{0}+\partial_\mu\partial^\mu A^\nu+m^2A^\nu=0\to\begin{cases}(\partial_\mu\partial^\mu+m^2)A^\nu=0 & \text{Klein-Gordon方程}\\ \partial_\mu A^\mu=0\end{cases}$

↓

正则量子化：动量 $\pi^\mu=\frac{\partial\mathcal{L}}{\partial\partial_0 A_\mu}=F^{\mu 0}=\partial^\mu A^0-\partial^0A^\mu$　$\pi^0=0$

$$\begin{cases}[A_i(x,t),\pi^j(x',t)]=i g^j_i\,\delta(x-x')\\ [A_i(x,t),A_j(x',t)]=0,\ [\pi^i(x,t),\pi^j(x',t)]=0\end{cases}$$

$\pi^0=0$，$A^0(x)=-\frac{1}{m}\nabla\cdot\vec{\pi}$，$\dot{A}^0=-\vec{\nabla}\cdot\vec{A}$

重矢量场仅有3个独立变量 A^i

解Proca方程($\nu=0$)：$\begin{cases}A^0=-\frac{1}{m^2}\partial_\mu F^{\mu 0}=-\frac{1}{m^2}\partial_i\pi^i=-\frac{1}{m^2}\nabla\cdot\pi & \to A^0\text{为}\pi\text{函数}\\ \text{同时：}\dot{A}^0=-\partial_iA^i=-\vec{\nabla}\cdot\vec{A}\end{cases}$

何意义?コ

纤维丛,启动!

?コ 嵌入子流形?

$$[A^0(x,t), A^i(x',t)] = \frac{i}{m^2}\partial^i\delta(x-x')$$

$$[\dot{A}^0(x,t), A^0(x',t)] = -\frac{i}{m^2}\partial_i\partial^i\delta(x-x')$$

$$\to \pi^i = \partial^i A^0 - \dot{A}^i = -\frac{1}{m^2}\partial^i\partial_j\pi^j - \dot{A}^i, \quad \dot{A}^i = -\pi_i - \frac{1}{m^2}\partial_i\partial^j\pi_j$$

类诱导度规!

$$[\dot{A}^i(x,t), A^j(x',t)] = i\left(g_{ij} + \frac{\partial_i\partial_j}{m^2}\right)\delta(x-x')$$

$\to -\dot{A}$ 与 π 的区别: $-\dot{A}_i = \left(g^j{}_i + \frac{1}{m^2}\partial_i\partial^j\right)\pi_j$?コ

哈密度: $\pi^i\dot{A}_i - \mathcal{L} = \frac{1}{2}\left[\underbrace{\pi^2 + (\nabla\times A)^2}_{E^2+B^2} + \underbrace{\frac{1}{m^2}(\nabla\cdot\pi)^2 + m^2A^2}_{\frac{\rho^2}{m^2} + m^2A^2}\right]$

动量空间: $A_\mu = \int d^3k \sum_{s=1}^{3} e^s_{k\mu}\left[a_{ks}\varphi_k(x) + a^\dagger_{ks}\varphi^*_k(x)\right]$

$w = \sqrt{k^2+m^2}$, 有: Lorentz条件: $k^\mu e^s_{k\mu} = 0$

取 $k^\mu = (w,0,0,k) \to e^0_k = \begin{pmatrix} w/m \\ 0 \\ 0 \\ k/m \end{pmatrix}, e^1_k = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, e^2_k = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, e^3_k = \begin{pmatrix} k/m \\ 0 \\ 0 \\ w/m \end{pmatrix}$

↓ 完备性: $g^{\mu\nu}e^\sigma_{k\mu}e^\rho_{k\nu} = g^{\sigma\rho}$; $g_{\sigma\rho}\cdot e^\sigma_{k\mu}e^\rho_{k\nu} = g_{\mu\nu}$

$$\pi_i(x,t) = \partial_i A^0(x,t) - \dot{A}_i(x,t) = -i\int d^3k \sum_{s=1}^{3}(k_i e^s_{k0} - k_0 e^s_{ki})\left[a_{ks}\varphi_k(x) - a^\dagger_{ks}\varphi^*_k(x)\right]$$

?コ为何为负?,是否诱导度规?

↓ $a_{ks} = \underbrace{\langle \varphi_k, g_{ss'}e^{s'}_{k\mu}A^\mu\rangle}_{s=1,2,3,\ g_{ss}=-1 \to a_{ks} = -\langle\varphi_k, e^s_{k\mu}A^\mu\rangle}$ 或 $\left(g^{ss'} - \frac{k^sk^{s'}}{m^2}\right)a_{ks'} = \langle\varphi_k, e^s_{ki}A^i\rangle$

$[a_{ks}, a^\dagger_{k's'}] = -g_{ss'}\delta(k-k')$, else$=0$

能量动量密度:

$$P^\mu = \frac{\partial\mathcal{L}}{\partial\partial_0A^\nu}\partial^\mu A^\nu - \mathcal{L}g^{0\mu} = F_{\nu0}\partial^\mu A^\nu - \mathcal{L}g^{0\nu} = \pi^i\partial^\mu A_i - \mathcal{L}g^{0\mu}$$

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu A_\nu)F^{\mu\nu} + \frac{1}{2}A_\nu A^\nu = -\frac{1}{2}\partial_\mu(A_\nu F^{\mu\nu}) + \frac{1}{2}A_\nu(\underbrace{\partial_\mu F^{\mu\nu} + m^2A^\nu}_{\text{Proca方程}}) = -\frac{1}{2}\partial_\mu(A_\nu F^{\mu\nu})$$

负 ?コ

$$\downarrow = -\frac{1}{2}\partial_0(A_\nu F^{0\nu}) - \frac{1}{2}\partial_i(A_\nu F^{i\nu}) = \frac{1}{2}\partial_0(A_i\pi^i) - \frac{1}{2}\partial_i(A_\nu F^{i\nu})$$

$\mathcal{H} = \frac{1}{2}[(\partial_0A_i)\pi^i - A_i\partial_0\pi^i] = \frac{i}{2}A_i\, i\overleftrightarrow{\partial_0}\pi^i$, $H = \frac{i}{2}\langle A_i, \pi^i\rangle = \frac{1}{2}\int d^3k\sum_{s=1}^{3} w(a_{ks}a^\dagger_{ks} + a^\dagger_{ks}a_{ks})$

$P^i = \frac{i}{2}\langle A_j, \partial^i A^j\rangle$; $S_k = -\frac{i}{2}\langle A_i, A_j\rangle\varepsilon^{ij}{}_k$

$\vec{P} = \frac{1}{2}\int d^3k\sum_{s=1}^{3}\vec{k}(a_{ks}a^\dagger_{ks} + a^\dagger_{ks}a_{ks})$ $\quad S_3 = \frac{1}{2}\int d^3k\left((a_{k+}a^\dagger_{k+} + a^\dagger_{k+}a_{k+}) - (a_{k-}a^\dagger_{k-} + a^\dagger_{k-}a_{k-})\right)$