

Measuring the gravitational constant of Earth, using a micro:bit

In this report we will describe an experiment to measure Earth's gravitational constant, g , using micro:bit, a tiny programmable educational computer.

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1. Gravitation

Gravitation is a physical phenomena when two or more material bodies attract each other. This phenomena can be observed in meteors, comets, planets and even galaxies.

The first who explained the nature of the force by which the bodies attract each other was Isaac Newton in the 17th century.

1.2 Newton law of gravitation

Newton derived the law of gravitational force from empirical observations and inductive reasoning. The law states that "every particle attracts every other particle in the universe with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers" (Newton, 1999).

The above law can be stated as a formula:

$$F = G \frac{m_1 m_2}{r^2}$$

where: F is the gravitational force acting between the masses, m_1 is the first of the first object, m_2 is the second mass of the second object, r is the distance between the centers of the masses, and G is the gravitational constant.

Newton's law has since been superseded by Albert Einstein's theory of general relativity, but it continues to be used as an excellent approximation of the effects of gravity in most applications (ibid).

1.3 Gravitation on Earth

The gravity of Earth, known as g , is the net acceleration that is transmitted to objects as a result of the combined response of gravitation (distribution of mass enclosed by Earth) and the centrifugal force (from the Earth's rotation). In other words, g is the acceleration that the earth conveys to items near or on its surface (Gassim et al., n.d.; Hofmann-Wellenhof, 2006)

From the Newton law of universal gravitation, the force on a body acted upon by Earth's gravity is given by:

$$F = G \frac{m_1 m_2}{r^2} = \left(G \frac{m_1}{r^2} \right) m_2$$

where r is the distance between the centre of the Earth and the center of the body, and here we take m_1 to be the mass of the Earth and m_2 to be the mass of the body.

Newton's second law, $F = ma$, where m is mass and a is acceleration, tells us that:

$$F = m_2 g$$

Comparing the two formulas, it is easy to observe that:

$$g = G \frac{m_1}{r^2}$$

To find the acceleration due to gravity at the sea level, we substitute the values of the gravitational constant, G , the Earth's mass (in kilograms), m_1 , and the Earth's radius (in metres), r , to obtain the value of g :

$$g = G \frac{m_1}{r^2} = 6.67384 \cdot 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2} \frac{5.9722 \cdot 10^{24} \text{ kg}}{(6.371 \cdot 10^6 \text{ m})^2} = 9.8196 \text{ m} \cdot \text{s}^{-2}$$

The value $g = 9.81 \text{ m/s}^2$ is usually taken as good approximation of gravitational constant on the Earth's surface.

The main goal of the experiment we will undertake is to experimentally obtain the value of g .

2. Simple gravity pendulum

A so-called "simple pendulum" is an idealization of a "real pendulum" using the following assumptions:

- * The rod or cord on which the bob swings is massless, inextensible and always remains taut;
- * The bob is a point mass;
- * Motion occurs only in two dimensions, i.e. the bob does not trace an ellipse but an arc.
- * The motion does not lose energy to friction or air resistance.
- * The gravitational field is uniform.
- * The support does not move.

The following picture shows a simple gravity pendulum with its parameters.

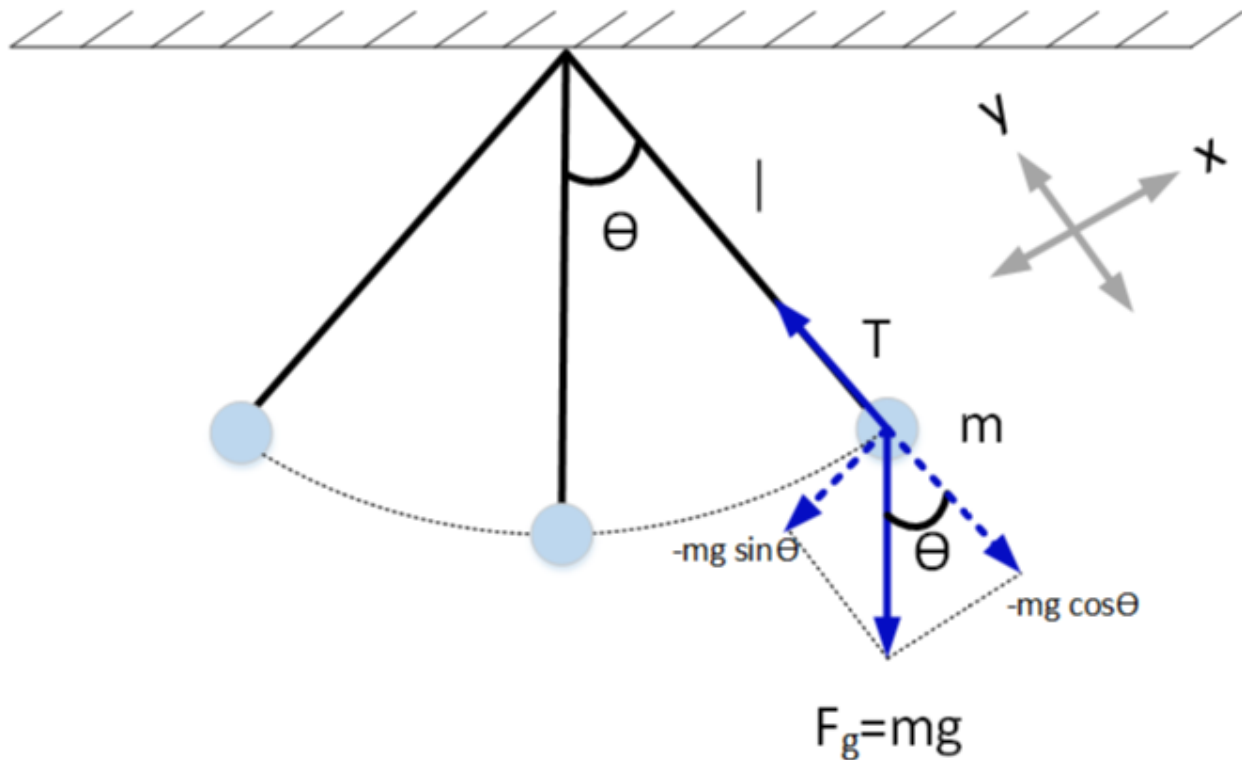


Fig. 1: Simple gravity pendulum

2.1 Pendulum equation

The differential equation describing the motion of a simple pendulum is:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

where g is acceleration due to gravity, l is the length of the pendulum, and θ is the angular displacement (Belénde et al, 2007).

For small $\theta \ll 1$, we can approximate $\sin\theta$ by θ , so the equation becomes

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

Given the initial conditions $\theta(0) = 0$, and $\frac{d\theta(0)}{dt} = 0$, the solution becomes:

$$\theta(t) = \theta(0) \sin\left(\sqrt{\frac{l}{g}}t\right)$$

2.2 Pendulum period of oscillation (T)

This solution $\theta(t) = \theta(0) \sin\left(\sqrt{\frac{l}{g}}t\right)$ is simple harmonic motion where $\theta(0)$ is the amplitude of the oscillation.

The period of the oscillation, the time for a complete oscillation (outward and return), is:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

2.3 Idea to measure g by measuring T

We can rewrite

$$T = 2\pi\sqrt{\frac{l}{g}}$$

as:

$$g = l\left(\frac{2\pi}{T}\right)^2$$

Thus, we can calculate g using the above formula by knowing the length of pendulum, l , and measuring the time T (for the period of oscillation).

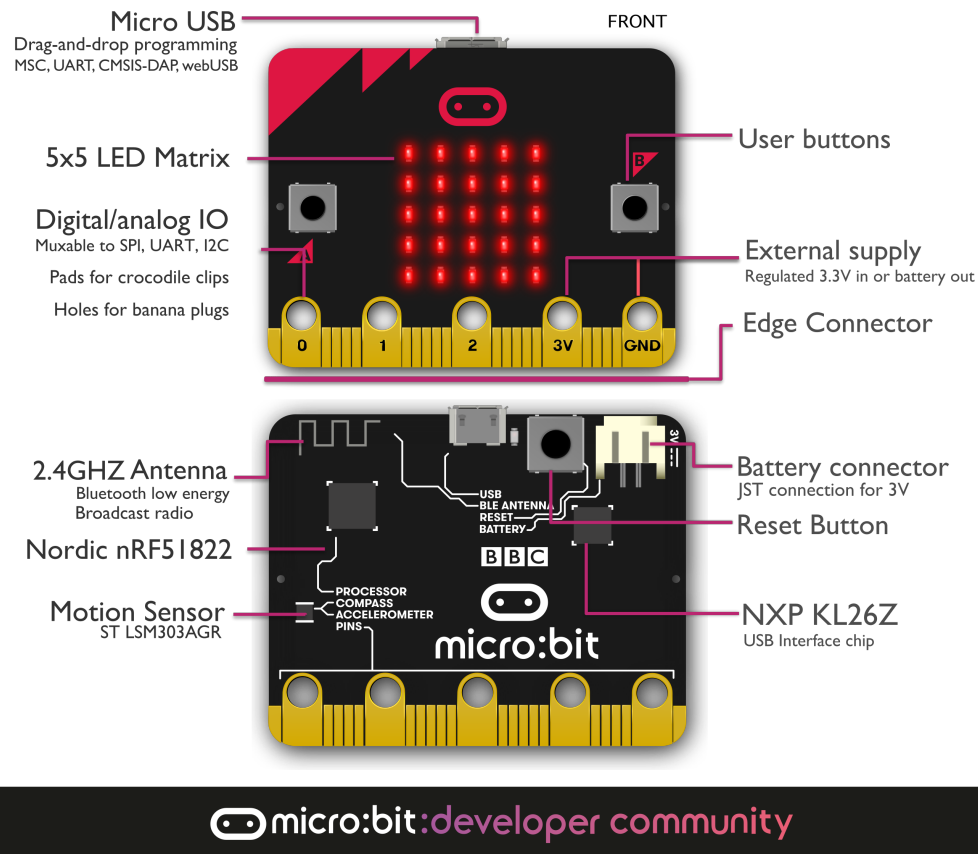
This is the basic idea for our experiment explained in details later.

3. Tiny computer micro:bit

The BBC micro:bit is widely used in schools around the world. It is designed to be held in the hand. It is a micro-computer that can be programmed and used for numerous applications just as robots musical instruments, etc (Micro:bit Educational Foundation, n.d.).

3.1 Hardware

The following picture gives an overall view of micro:bit components. We will shortly describe all of them.



instead of

Fig. 2: Micro:bit hardware. (Picture from <https://tech.microbit.org/hardware/#hardware-description> (<https://tech.microbit.org/hardware/#hardware-description>))

Processor:

Nordic nRF51 application processor, Flash ROM 256 KB, RAM 16 KB, speed 16 MHz.

Bluetooth Wireless Communication

2.4GHz ISM (Industrial, Scientific and Medical) 2.4GHz. 2.41GHz.

Low Level Radio Communications,

Freq band 2.4GHz, Protocol Nordic Gazell, Channels 101.

Buttons

Two programable buttons (A and B) on the front of the micro:bit, and the one button on the back (reset button).

Display

The display is a 5x5 array of LEDs. This LED matrix may also be used to sense ambient light.

Motion sensor

Combined accelerometer and magnetometer chip that provides 3 – axis sensing and magnetic field strength sensing.

Temperature Sensing

Sensing range -25°C . 75°C , Resolution 0.25°C steps, Accuracy $\pm 4^{\circ}\text{C}$ (uncalibrated).

General Purpose Input/Output Pins

19 assignable GPIO pins, ADC resolution 10 bit (0..1023).

Power Supply

Operating range $1.8V \dots 3.6V$, USB current $120mA$ max.

Interface

The interface chip handles the USB connection, and is used for flashing new code to the micro:bit, sending and receiving serial data back and forth to your main computer. Flash ROM 256 KB , RAM 16 KB , speed 16 MHz .

USB Communications

USB micro, Speed $12Mbit/sec$.

Mechanical

Dimensions $5cm(w)$ $4cm(h)$, weight $5g$.

3.2 Software

Software for the micro:bit consists of two main groups:

Software that runs on your main computer (the 'host' computer, left hand side in the diagram)

Software that runs on the micro:bit (the 'target' computer, right hand side in the diagram)

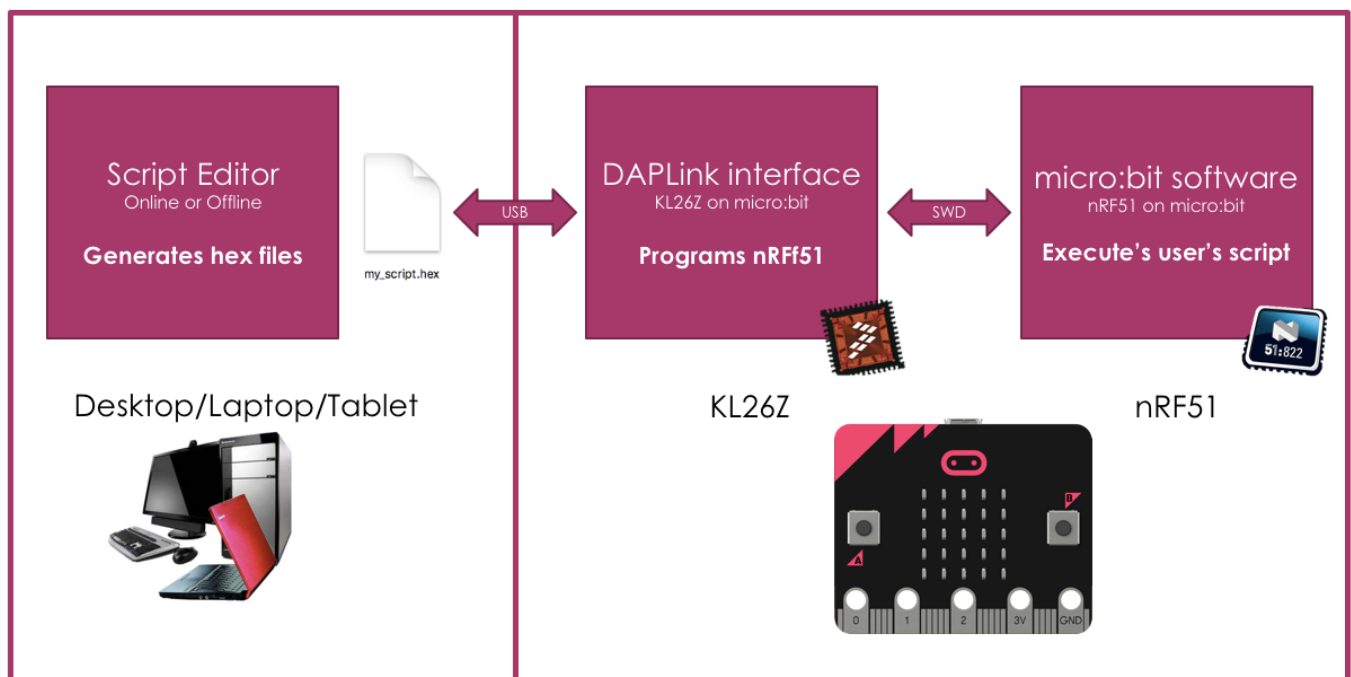


Fig. 3: Micro:bit software (picture from <https://tech.microbit.org/software/> (<https://tech.microbit.org/software/>))

Typically, a micro:bit program is written on the host computer and then transferred to the micro:bit over USB. There are actually two chips on the micro:bit, one that is running software entirely to facilitate the flashing (the KL26Z) and one that actually runs the user's code (the nRF51822).

There are many code editors that can be used to program micro:bit:

Microsoft MakeCode
 MicroPython.
 codethemicrobit.com, a Blocks/JavaScript editor
 Mu, a Python editor
 Espruino, a JavaScript interpreter
 EduBlocks, a block editor for MicroPython

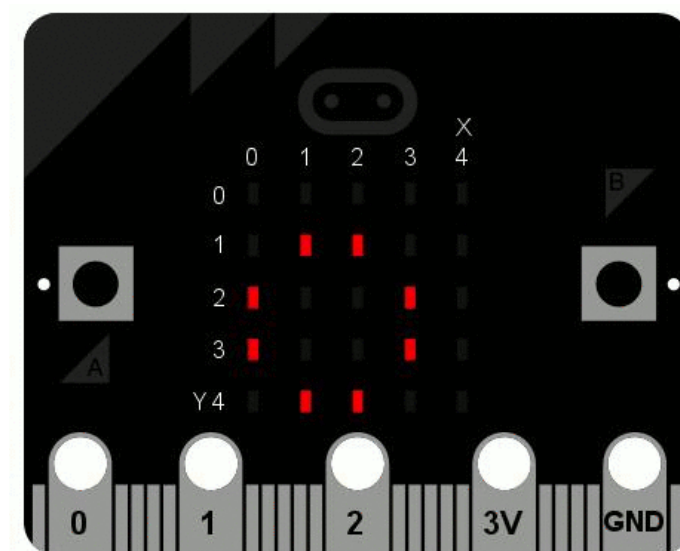
It is, also, possible to program micro:bit in other programming languages:

Free Pascal
 Simulink
 C++
 Forth
 Lisp
 Ada

In our experiment we will use Mu Python editor. As the name suggests, we are going to use Python as programming language.

3.3 Micro:bit light sensors

Microbit has the 5×5 LED display on the front of the board. It can be used to display images, animations and even text. As well as, it can sense the level of light coming to its leds. This feature will be used in our experiments for measuring period of pendulum oscillation, and thus measuring gravitation g , using the formula in (2.3).



Functions of display

Microbit Python library has many methods for displaying texts and images on Microbit 5x5 display.

In this context, we are interested only in microbit method that provides the light measurement:

display.read_light_level()

It uses the display's LEDs in reverse mode to sense the amount of light falling on the display. The method returns an integer between 0 and 255 representing the light level, with larger mean more light.

Using this method we can measure ambient (background) light in the room. Following is the Python script that measure the light level (during the entire experiment).

In []:

```
from microbit import * # import microbit library
while True: # Measure forever
    print(display.read_light_level()) # print light level on REPL
    sleep(200) # wait 200 msec
```

4. Pendulum experiment for measuring g

4.1 Experiment setup

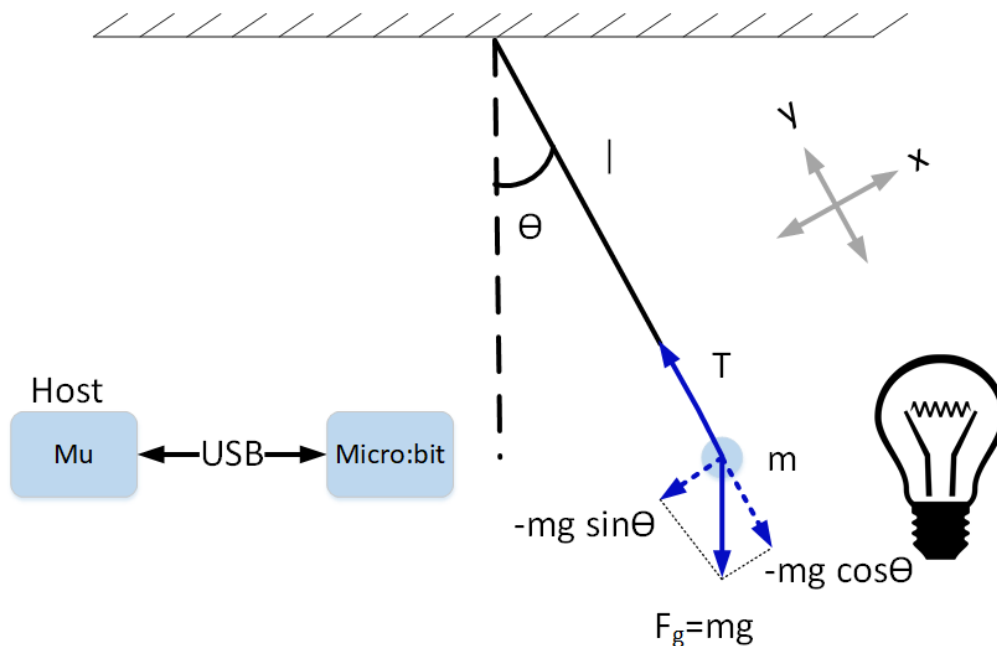
For this experiment we used a micro:bit attached to the pendulum. From 2.3 we know that $g = l \left(\frac{2\pi}{T} \right)^2$.

So if we measure time period T , we can calculate g for a known pendulum length l .

In our experiment which measure T and calculate g we have used the following:

1. 1x BBC Micro:bit device.
2. Light source from mobile phone.
3. Pendulum made of a string of length of 1.7 m .
4. Laptop with Mu Python editor.

The following figure illustrates our experiment.



The micro:bit attached to PC is placed in such a way that the pendulum in middle position is blocking the light from the light source (the bulb on the figure) to fall on microbit display LEDs. In that cases display leds detects only ambient (background) light. When pendulum move from middle point the light from light source is falling to LEDs and the LED light level detected increases. By using using a Python program which measure the light level, given in 3.3, we can detect changes of light level during pendulum movement.

The logic of experiment is the following:

From the above figure we can see that during pendulum movement, micro:bit display can be described within two states:

1. Light from source is falling on 5×5 LEDs (we call it 'light on' state)
2. Light from source is not falling on 5×5 LEDs (we call it 'light off' state)

"Light on" state is the state when the pendulum is away from middle point. "Light off" state when the pendulum is in the middle point (the equilibrium point).

By measuring the light level falling on the 5×5 LEDs we can detect those two states.

To measure the period T of pendulum, we have to measure the start time when display enter 'light on' state for the first time, and end time when display enter 'light on' state for the third time. The difference between end and start time is the period T .

Why third time? Suppose the pendulum is in middle point, display is then in 'light off' state. When pendulum move away, display will enter 'light on' state for the first time. Pendulum will then move to one extreme point, and go back to middle point when display enter 'light off' state. The pendulum then move away from middle point again (toward the other extrem) and display enter 'light on' state (for the second time). After reaching the extreme point, the pendulum goes back to middle point (display enter 'light off' state) and continue to move until display enter 'light on' state (for the third time). But this is the end of full period of pendulum movement (from middle point to one extreme point, back the other extreme point and finally back to middle point).

(What if the pendulum is not initially in the middle point. The first result will be wrong!)

That is exactly what the following program is doing (as it is on). It detects first, second and third change from 'light off' to 'light on' and measures duration from first to third appearance of 'light on' state. The program then calculate g and print the result to REPL (Read, Evaluate, Print and Loop) in Mu Python editor used to run program. Measurement is then repeated.

The follwing link demonstrates how the experiment is conducted: <https://youtu.be/35rvDAKxUdk>
(<https://youtu.be/35rvDAKxUdk>)

4.2 Python program

In []:

```

from microbit import * # import microbit library
import utime # import utime module

L = 1.7 # The length of pendulum (m)
n = 0 # number of entry in 'light on' period

while True: # Measure forever

    # Wait for 'light on' state
    while display.read_light_level() < 50:
        pass

    if n == 0: # if this is the first change to 'light on' state
        time1 = utime.ticks_ms() # record start time
        n += 1 # increase n

    elif n == 2: # if this is the third change to 'light on' state
        time2 = utime.ticks_ms() # record end time
        # Compute the difference between start and end time (in sec)
        T = utime.ticks_diff(time2, time1)/1000
        g = L*(6.28/T)**2 # Calculate g, for given L and T
        print("g = "+str(g)) # print result on REPL
        n = 0 # start new measurement

    else: # if it is the second change to 'light on' state
        n += 1 # increase n

    # Wait for light off
    while display.read_light_level() >= 50:
        pass

```

4.3 Experimental results

Using the above mentioned experimental setup we have made several experiments with various pendulum lengths.

We are listing the results for one experiment that we believe is representative for the other experiments:

L = 1.7 m

Results

Experimental g	values	Experimental g	values
g=	10.2064	g=	10.0179
g=	9.83454	g=	10.1114
g=	10.0179	g=	10.0179
g=	10.0256	g=	10.0256
g=	10.0179	g=	10.0256
g=	10.1114	g=	10.0179
g=	10.0179	g=	10.0256

Experimental g	values	Experimental g	values
g=	10.1114	g=	10.0256
g=	10.0256	g=	9.92556
g=	9.92556	g=	10.0179
g=	10.0179	g=	10.0179
g=	10.0179	g=	10.0256
g=	10.1114	g=	10.0256
g=	10.0256	g=	9.84208
g=	10.1114	g=	10.0256
g=	9.83454	g=	10.1114
g=	10.0179	g=	10.0179
g=	10.0256	g=	10.0179
g=	10.0179	g=	9.93321
g=	10.0179	g=	10.1114
g=	9.92556	g=	10.0179
g=	10.0179	g =	9.92556
g=	10.0256	g =	10.0179
g=	10.0179	g =	10.0256
g=	10.0179	g =	10.1114
g=	10.0179	g =	9.92556
g=	10.0179	g =	10.1114
g=	9.84208	g =	9.92556
g=	10.0179	g =	9.93321
g=	10.0179	g =	10.0179
g=	10.0179	g =	10.0179
g=	10.0179	g =	9.83454
g=	10.1114	g =	10.2064
g=	10.0179	g =	9.84208
g=	9.92556	g =	10.1114
g=	10.1114	g =	10.0179
g=	10.0256	g =	9.92556
g=	10.0179	g =	10.2064
g=	10.1114	g =	9.93321
g=	9.83454	g =	10.0179
g=	10.1114	g =	10.0179
g=	10.0256	g =	10.0179
g=	10.0179	g =	10.0256
g=	10.1114	g =	9.93321
g=	10.0179	g =	10.3026
g=	10.1114	g =	9.92556

Experimental g	values	Experimental g	values
g=	9.93321	g =	10.1114
g=	10.0179	g =	10.0256
g=	10.2064	g =	9.84208

Average value: $g = 10.01851398 \text{ m/s}^2$

Relative error $\% = \frac{|9.81 - 10.018|}{9.81} \cdot 100 = 2.1\%$.

5. Conclusion

The experimental results show that it is possible to accurately measure Earth's gravitation g in a very simple Experimental setup.

Our pendulum was a primitive one, and not fully compliant with requirements given in 2. Despite of that, the relative error of measuring g is very small, not more than *2.1% off*.

It is possible to improve the experiment by using better pendulum (like the one used in old wall watches) and better light source, with with a more focused beam. As well as more accurate measurements for both the rod length and the light level as the micro:bit proved to be fast enough in detecting light level changes.

This experiment, although very simple, involves different kinds of required knowledge from physics and mechanics to electronics and real time programming.

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