Assignment_2

June 3, 2019

1 Assignment 2

Realfagslektormaster, programing course
Author: Paul Knutson

1.1 1a

$$P(x) = x^3 - 2x^2 + x - 2$$

The user can change the x (input value). In this case, $P(3) = 10$.

```
In [14]: def P(x):
             return x**3 - 2*x**2 + x - 2


         print(P(3))
```

```
10
```

1.2 1b

Here, $P(p, x)$ is generic polynomial function, described by the coefficient list p, giving

$$P(p, x) = p_0 x^0 + p_1 x^1 + p_2 x^2 + ... + p_n x^n$$

where n is the size of the coefficient list. In the code, p is reversed (i.e. the index of p is actually going from n to 0). The example function is the same as described in 1 a.

```
In [15]: # Vector describing/defining the polynomial function.
         p = [-2, 1, -2, 1]


         # Function of x (scalar) and p (vector)
         def P(p, x):
             y = 0
             for i in range(len(p)):
                 y += p[i] * x**i
             return y
```

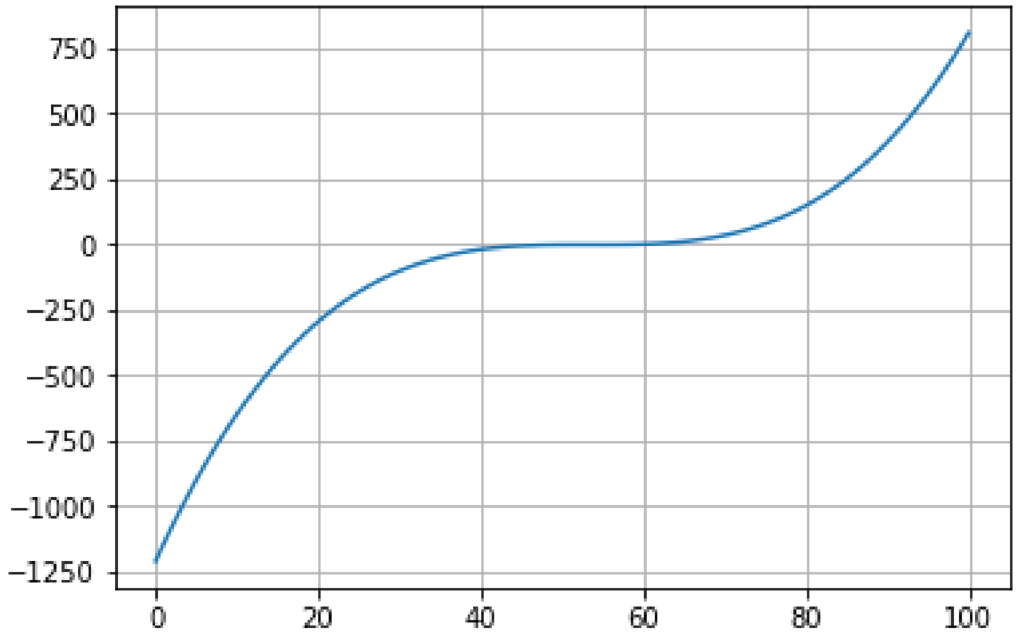
```
print(P(p, 3))
```

```
10
```

1.3 1c

Here, u is a list of x-values as used in the previous task. The results from $P(x)$, or in this case, $P(u_i)$ is stored and plotted.

```
In [16]: # For maths and plotting
         import matplotlib.pyplot as plt
         import numpy as np

         # Vector describing/defining the polynomial function.
         p = [-2, 1, -2, 1]


         # Function of x (scalar) and p (vector)
         def P(p, x):
             y = 0
             for i in range(len(p)):
                 y += p[i] * x**i
             return y


         def P_vect(u):
             v = [0]*len(u)
             for i in range(len(u)):
                 v[i] = P(p, u[i])
             return v


         u = np.linspace(-10, 10, 101)
         y_values = P_vect(u)
         x_values = np.linspace(0, len(y_values) -1, len(y_values))

         # Plotting
         #fig = plt.figure()
         plt.grid(True)
         plt.plot(x_values, y_values)
         plt.show()
```



1.4 1d

This one is similar to c, but where P is used recursive instead of using a list u. As can be seen below, the values become huge extremely quickly.

```
In [17]: # Vector describing/defining the polynomial function.
         p = [-2, 1, -2, 1]


         # Function of x (scalar) and p (vector)
         def P(x, n):
             y = 0
             for i in range(len(p)):
                 y += p[i] * x**i
             if n == 0:
                 return x
             elif n == 1:
                 return y
             else:
                 return P(y, n-1)


         def P_rec(x, n):
             result = [0]*n
             for i in range(n):
                 result[i] = P(x, i)
```

```
    return result


a = 3    # Input number
n = 5    # Vector size/factorial "loops".
print(P_rec(a, n))
```

```
[3, 10, 808, 526209190, 145705278066681752856855988]
```

1.5 1e

This is the same as 1d, but with an input vector of size n, similar to c. This return an $n \times n$ matrix of values.

In [18]:
```
# Vector describing/defining the polynomial function.
p = [-2, 1, -2, 1]


# Function of x (scalar) and p (vector)
def P(x, n):
    y = 0
    for i in range(len(p)):
        y += p[i] * x**i
    if n == 0:
        return x
    elif n == 1:
        return y
    else:
        return P(y, n-1)


def P_rec(x, n):
    result = [0]*n
    for i in range(n):
        result[i] = P(x, i)
    return result


def P_rec_vect(u, n):
    v = [0]*len(u)
    for i in range(len(u)):
        v[i] = P_rec(u[i], n)
    return v


u = [0, 1, 2, 3, 4]
n = 5    # Vector size/factorial "loops".
print(P_rec_vect(u, n))
```

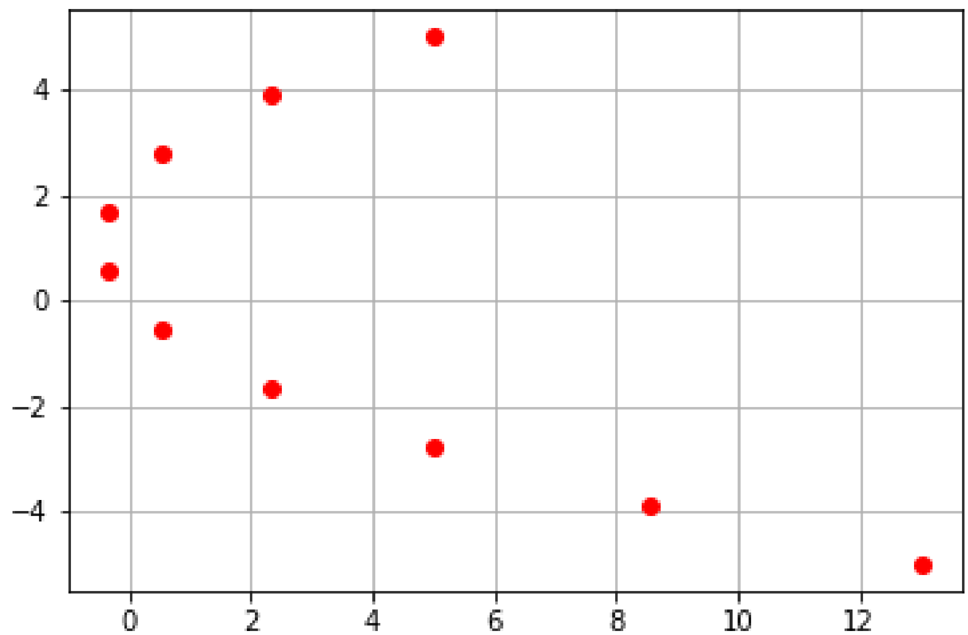
```
[[0, -2, -20, -8822, -686751492440], [1, -2, -20, -8822, -686751492440], [2, 0, -2, -20, -8822],
```

1.6 2a

This plots a single point. The values can be changed to get different points.
$u_0 = -u_1 \implies v = [0,0]^T$

```
In [19]: import matplotlib.pyplot as plt
         import numpy as np

         # Input matrix and vector
         #A = [[1, 0.5], [1, 0.5]]
         #u = [5, 6]
         A = [[1, 2], [3, 4]]
         u = [2, 1]


         # Hard coded dot product (and transpose) function for (2x2) x (2x1) = (2x1).
         def dot_product(A, u):
             v = [0]*len(u)

             v[0] = A[0][0] * u[0] + A[0][1] * u[1]
             v[1] = A[1][0] * u[0] + A[1][1] * u[1]

             return v


         v_lst = []
         def add(A, u):
             v_lst.append(dot_product(A, u))


         def plot(v_lst):
             fig = plt.figure()
             ax = fig.add_subplot(111)
             ax.grid(True)

             for i in range(len(v_lst)):
                 plt.plot(v_lst[i][0], v_lst[i][1], 'ro')

             fig.show()


         #v_lst = []
         #v_lst.append(plot_point([[1, 2], [3, 4]],       [2, 1]))
         #v_lst.append(plot_point([[1, 0.5], [1, 0.5]],   [5, 6]))
         #v_lst.append(plot_point())
```

```
x = np.linspace(-1, 1, 10)
for i in range(len(x)):
    add([[x[i], x[i]], [-x[i], x[i]]], [i, i - len(x)/2])

#add([[1, 2], [3, 4]],        [2, 1])
#add([[1, 0.5], [1, 0.5]],    [5, 6])
#add([[-1, 1], [-1, 1]],      [-3, -3])

plot(v_lst)
```

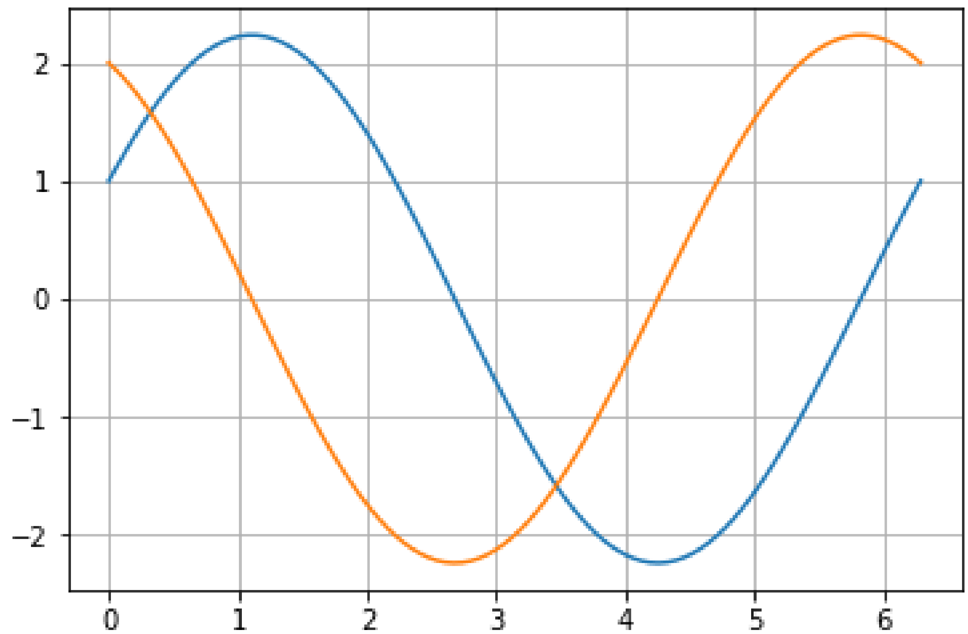


1.7 2b

This plots the values of the trigonometric matrix function T, with the variable input θ, $[0, 2\pi]$.

```
In [20]: import matplotlib.pyplot as plt
         import numpy as np
         import math

         u = [1,2]


         # Hard coded dot product (and transpose) function for (2x2) x (2x1) = (2x1).
         def dot_product(A, u):
             v = [0]*len(u)
```

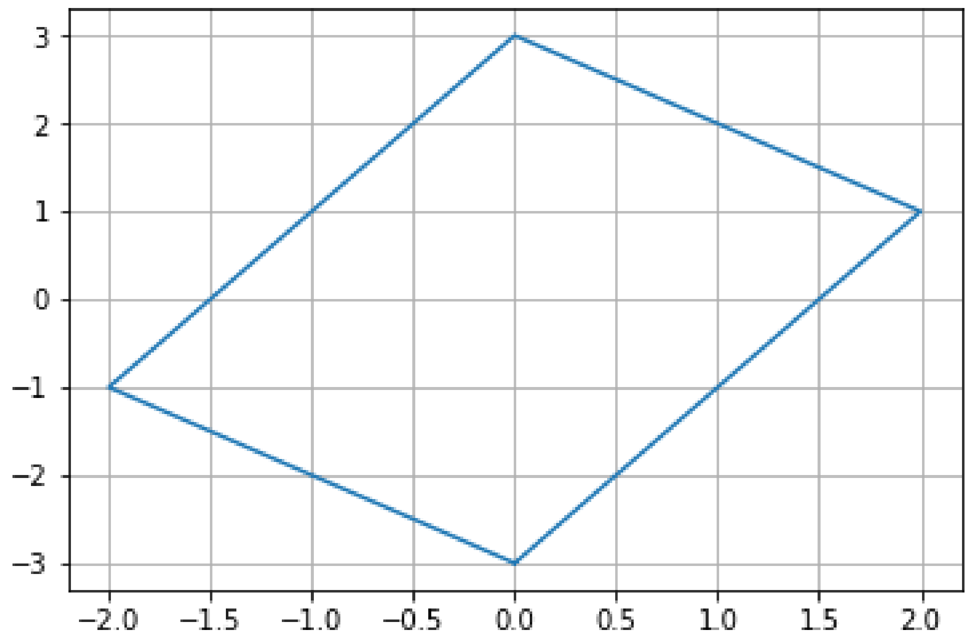
```
    v[0] = A[0][0] * u[0] + A[0][1] * u[1]
    v[1] = A[1][0] * u[0] + A[1][1] * u[1]

    return v


# Returns a matrix based on the given trigonometric variables.
def matrix(theta):
    T = [[math.cos(theta),   math.sin(theta)],
            [-math.sin(theta),  math.cos(theta)]]
    return T


# Set steps, make x (linspace) and empty y-list
steps = 100
x = np.linspace(0, 2*math.pi, steps)
y = [None]*steps


# Loop the function through all theta, dot_product(matrix(theta), u).
for idx in range(steps):
    theta = x[idx]
    T = matrix(theta)
    y[idx] = dot_product(T, u)




# Plotting
fig = plt.figure()
ax = fig.add_subplot(111)
ax.grid(True)
plt.plot(x, y)
fig.show()
```



1.8 2c

```
In [21]: import matplotlib.pyplot as plt
         import numpy as np


         A = [[1, -1], [2, 1]]


         def dot_product(A, u):
             v = [0]*len(u)

             v[0] = A[0][0] * u[0] + A[0][1] * u[1]
             v[1] = A[1][0] * u[0] + A[1][1] * u[1]

             return v


         x = [-1, 1, 1, -1, -1]
         y = [-1, -1, 1, 1, -1]


         def dot_list():
             for i in range(len(x)):
                 p = dot_product(A, [x[i], y[i]])
```

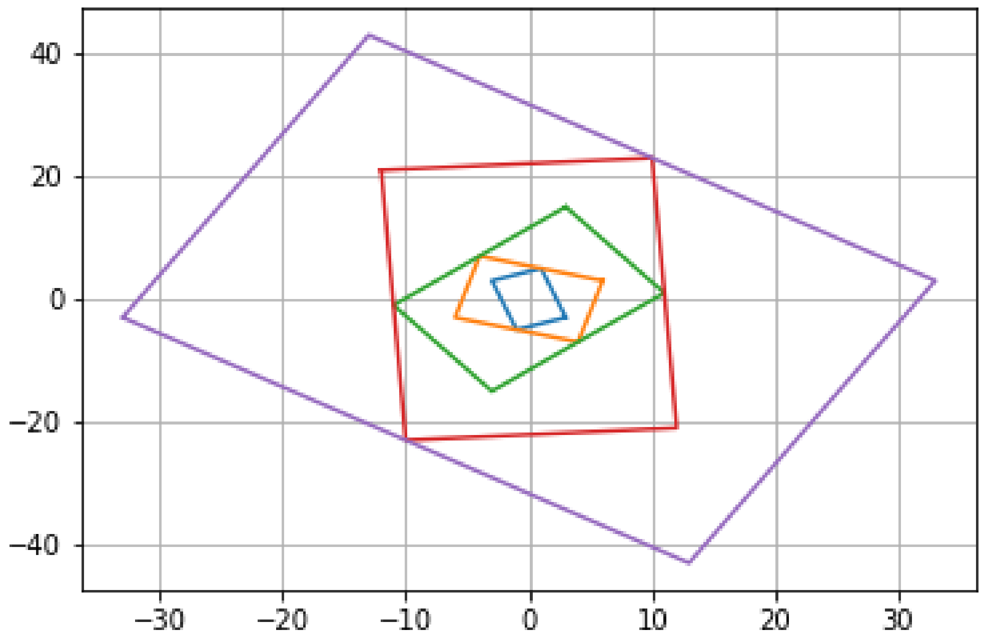
```
        x[i] = p[0]
        y[i] = p[1]


dot_list()

fig = plt.figure()
ax = fig.add_subplot(111)
ax.grid(True)
plt.plot(x, y)
fig.show()
```



1.9 2d

Same as c, but iterate the dot product.

In [22]:
```
fig = plt.figure()
ax = fig.add_subplot(111)
ax.grid(True)

for i in range(5):
    dot_list()
    plt.plot(x, y)

fig.show()
```



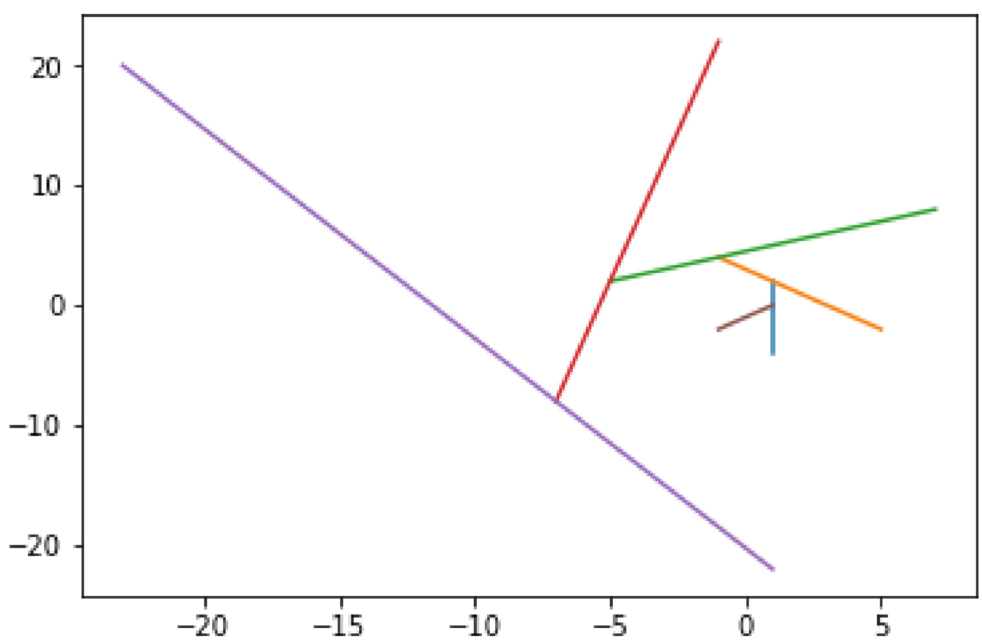
1.10 2e

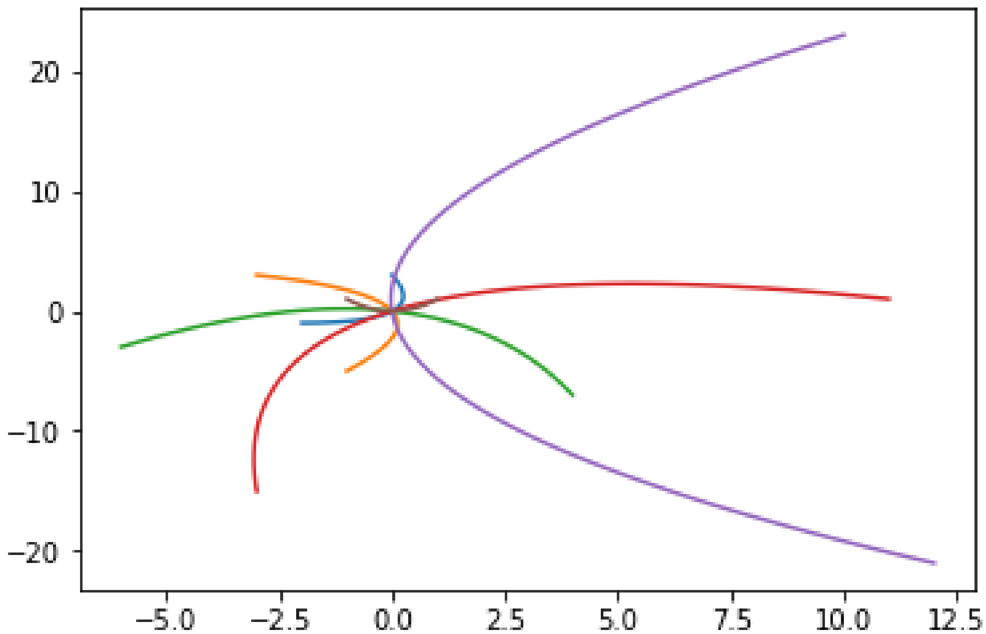
This rotates and misforms the functions oridinal shape. $f(x) = x^3, x \in [-1, 1]$

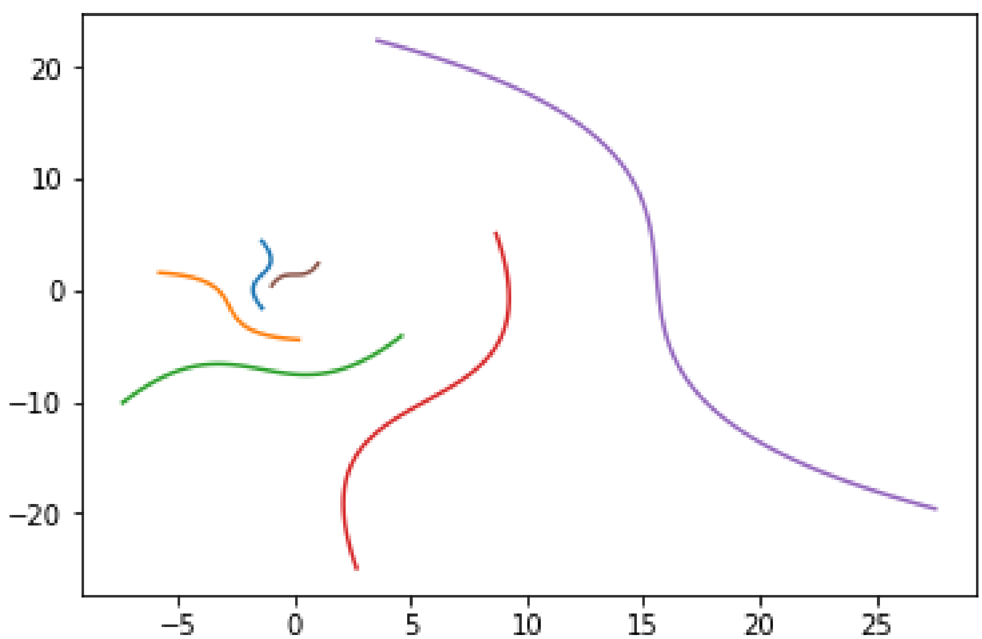
```
In [23]: def f(x):
             return x - 1

         def g(x):
             return x**2

         def h(x):
             return x**3 + np.sqrt(2)

         def k(x):
             return 1/(100*x)

         def r(x):
             return np.sin(x) * np.cos(x)

         def s(x):
             return np.e**x

         def plot(f, n=1):
             X = np.linspace(-1, 1, 100)
             Y = f(X)
             X_orig = X.copy()
```

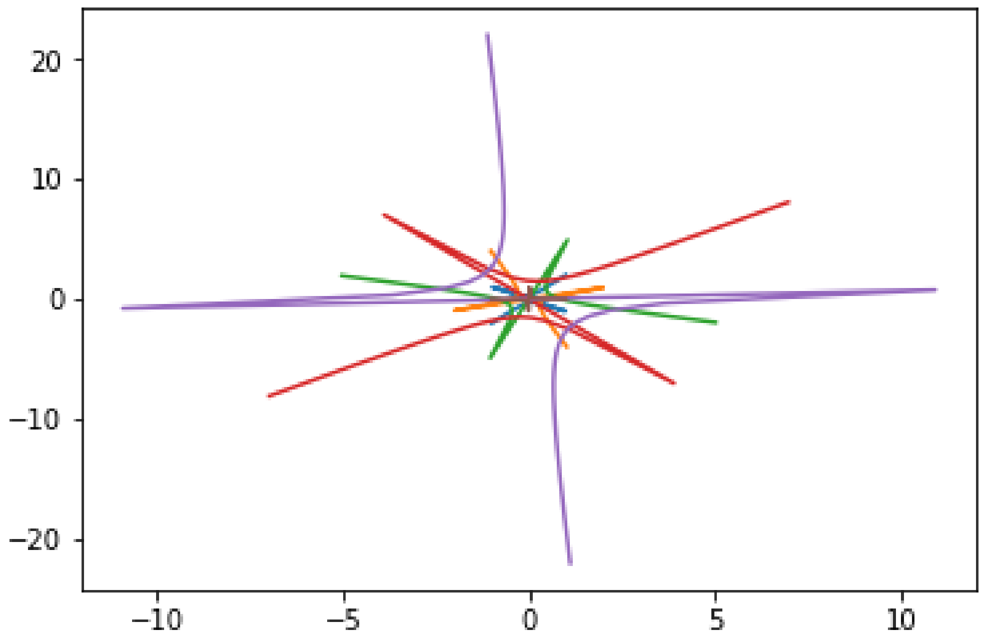
```
    Y_orig = Y.copy()

    fig = plt.figure()


    for j in range(n):
        for i in range(len(X)):
            p = dot_product(A, [X[i], Y[i]])
            X[i] = p[0]
            Y[i] = p[1]
        plt.plot(X, Y)
    plt.plot(X_orig, Y_orig)
    fig.show()
```

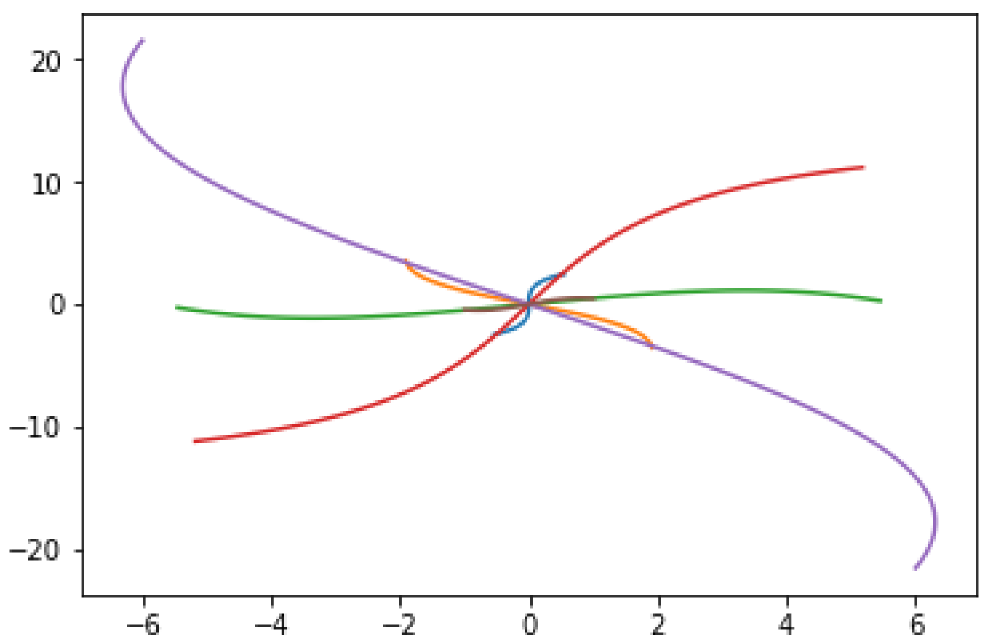
```
plot(f, 5)
plot(g, 5)
plot(h, 5)
plot(k, 5)
plot(r, 5)
plot(s, 5)
```

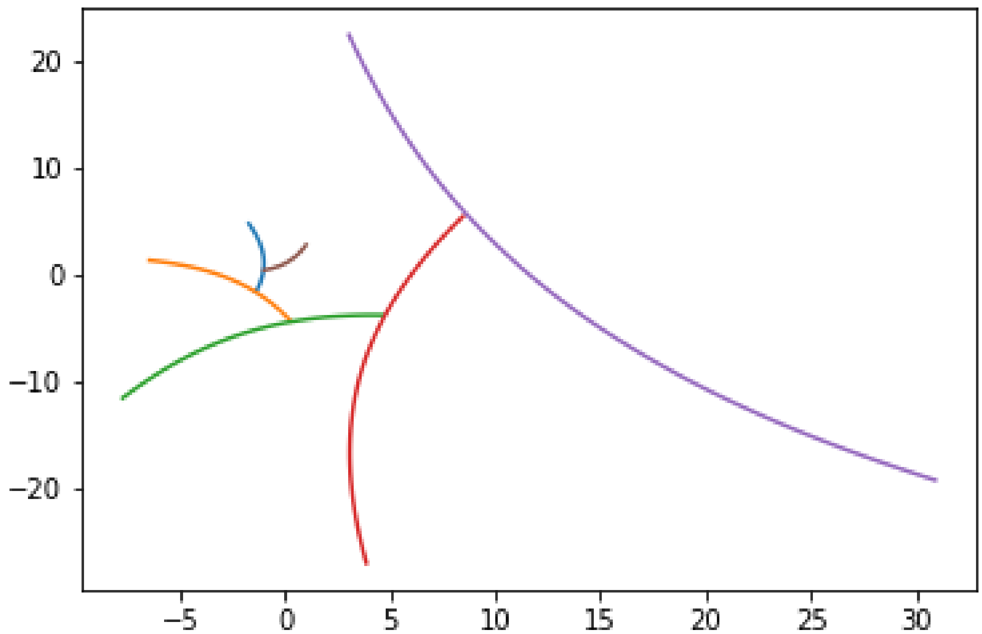



20
10
0
−10
−20
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
12.5


20
10
0
−10
−20
−5
0
5
10
15
20
25


20
10
0
−10
−20
−10
−5
0
5
10


20
10
0
−10
−20
−6
−4
−2
0
2
4
6



1.11 3a

This is a converging function for $1 < \lambda < 2.9$. x is just an initial value and has to be $0 < x < 1$ for the function to work properly.

$L^n(x)$ becomes smaller, as $n \to \infty$ (or 4), when $\lambda < 1.5$.

With $\lambda > 1.5$, L first grows bigger, then smaller, as $n \to \infty$

$$\lim_{n\to\infty} L^n(x) = \infty$$

In [24]:
```
# Initial values
n = 20           # List size
lst = [0]*n       # Make empty list
x = 0.5
lmda = 1.5


# Function L
def L(x, lmda):
    return lmda * x*(1-x)


# Use L "recursively"
def L_rec(x, lmda, rec_level):
    for i in range(rec_level):
        x = L(x, lmda)
    return x
```

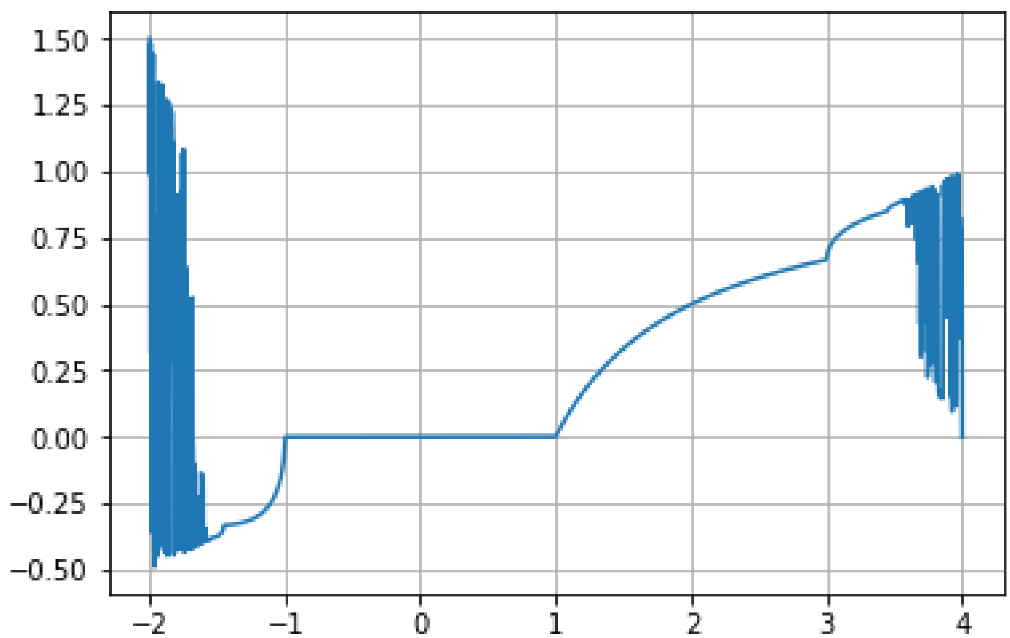
```
        # Make list
        for i in range(n):
            lst[i] = L_rec(x, lmda, i)


        # Print list
        for i in range(n):
            print(lst[i])
```

```
0.5
0.375
0.3515625
0.341949462890625
0.3375300415791571
0.3354052689160944
0.33436286174912516
0.3338465076480908
0.33358952546889614
0.3334613309494993
0.3333973075663317
0.33336531431077876
0.3333493222878816
0.33334132742713746
0.33333733028437706
0.33333533178489183
0.33333433255312184
0.33333383294172997
0.3333335831371573
0.33333345823515165
```

1.12 3b

```
In [25]: import matplotlib.pyplot as plt
         import numpy as np

         # Function L
         def L(x, lmda):
             return lmda * x*(1-x)


         # Use L "recursively"
         def L_rec(x, lmda, rec_level):
             for i in range(rec_level):
                 x = L(x, lmda)
             return x
```

```
depth = 1001
x = 0.5
lambdas = np.linspace(-2, 4, 2001)
Ls = []


for i in range(len(lambdas)):
    Ls.append(L_rec(x, lambdas[i], depth))


fig = plt.figure()
ax = fig.add_subplot(111)
ax.grid(True)

plt.plot(lambdas, Ls)
fig.show()
```



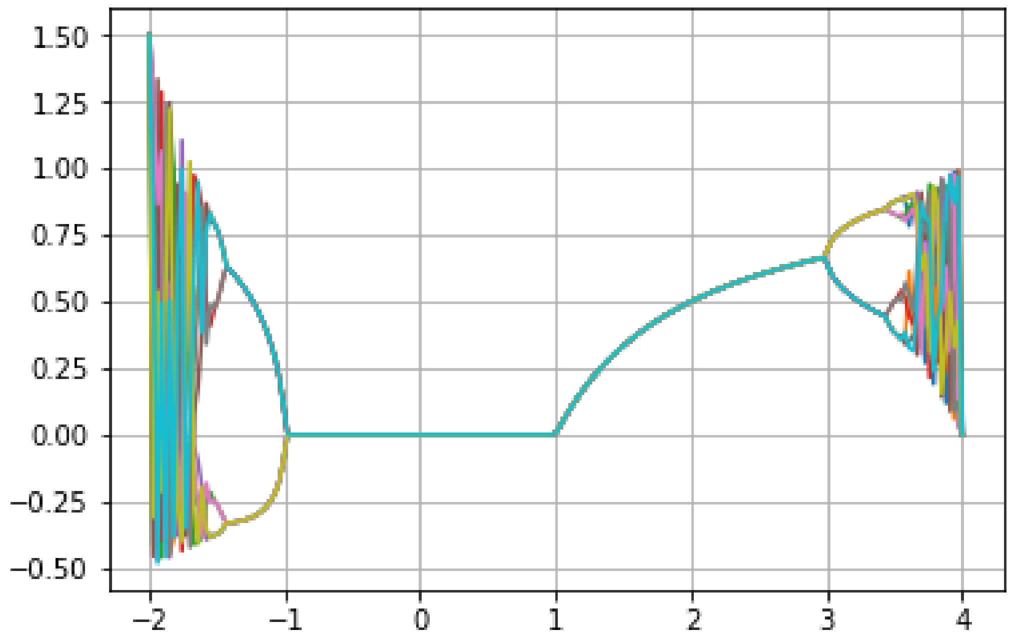
2 3c

The series converges to a predictable, single, non-zero number around $1 < \lambda \lessapprox 2.9$, (or between 2.9 and 3.0).

$-1 \leq \lambda \leq 1$ will cause $L = 0$.

The diagram seems to become a bifurcation diagram. Change the depth by 1, and the lines flip, indicating the lines would split up every time it seems to "jump" a bit. This can indeed also be seen when letting λ become lower than -1, as seen below. $\lambda > 4$ or $\lambda < -2$ will cause double scalar overflow (at least in Jupyter), so keep $-2 \leq \lambda \leq 4$. The plots below show a range of lines, and how they split.

In [26]:
```
import matplotlib.pyplot as plt
import numpy as np

# Function L
def L(x, lmda):
    return lmda * x*(1-x)


# Use L "recursively"
def L_rec(x, lmda, rec_level):
    for i in range(rec_level):
        x = L(x, lmda)
    return x

fig = plt.figure()
lines = 10
for j in range(lines):
    n = 201 + j
    x = 0.5
    lambdas = np.linspace(-2, 4, 201)
    Ls = []


    for i in range(len(lambdas)):
        Ls.append(L_rec(x, lambdas[i], n))



    ax = fig.add_subplot(111)
    ax.grid(True)

    plt.plot(lambdas, Ls)
fig.show()
```



In [ ]: