Honours Analysis Notes

Anthony Catterwell

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1 The Real Number System

1.1 Introduction

1.2 Ordered Field Axioms

- **Postulate 1** *Field Axioms*
  There are functions $+$ and $\cdot$ defined on $\mathbb{R}^2 := \mathbb{R} \times \mathbb{R}$, which satisfy the following properties $\forall a, b, c \in \mathbb{R}$
  - *Closure Properties*: $a + b,\ a \cdot b \in \mathbb{R}$
  - *Associative Properties*: $a + (b + c) = (a + b) + c$ and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
  - *Commutative Properties*: $a + b = b + a$ and $a \cdot b = b \cdot a$
  - *Distributive Law*: $a \cdot (b + c) = a \cdot b + a \cdot c$
  - *Existence of Additive Identity*: There is a unique element $0 \in \mathbb{R}$ such that $0 + a = a$ for all $a \in \mathbb{R}$
  - *Existence of Multiplicative Identity*: There is a unique element $1 \in \mathbb{R}$ such that $1 \neq 0$ and $1 \cdot a = a$ for all $a \in \mathbb{R}$
  - *Existence of Additive Inverses*: For every $x \in \mathbb{R}$ there is a unique element $-x \in \mathbb{R}$ such that
  $$x + (-x) = 0$$
  - *Existence of Multiplicative Inverses*: For every $x \in \mathbb{R}\backslash\{0\}$ there is a unique element $x^{-1} \in \mathbb{R}$ such that
  $$x \cdot (x^{-1}) = 1$$
- **Postulate 2** *Order Axioms*
  There is a relation $<$ on $\mathbb{R} \times \mathbb{R}$ that has the following properties:
  - *Trichotomy Property* Given $a, b \in \mathbb{R}$, one and only one of the following statements hold:
  $$a < b,\ b < a,\ \text{or } a = b$$
  - *Transitive property* For $a, b, c, \in \mathbb{R}$
  $$a < b \text{ and¡}c \implies a < c$$

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