虽然作者在 gsolve.m 文件中填充A矩阵用了 $N\times P+254+1$ ,但是在原论文中,作者解释了公式 (3),也说明了用 $N\times P+256+2$ 的原因,需要用欧拉公式g''(z)=g(z-1)-2g(z)+g(z+1)进行平滑。论文中的解释如下:

We wish to recover the function g and the irradiances  $E_i$  that best satisfy the set of equations arising from Equation 2 in a least-squared error sense. We note that recovering g only requires recovering the *finite* number of values that g(z) can take since the domain of Z, pixel brightness values, is finite. Letting  $Z_{min}$  and  $Z_{max}$  be the least and greatest pixel values (integers), N be the number of pixel locations and P be the number of photographs, we formulate the problem as one of finding the  $(Z_{max} - Z_{min} + 1)$  values of g(Z) and the N values of  $\ln E_i$  that minimize the following quadratic objective function:

$$\mathcal{O} = \sum_{i=1}^{N} \sum_{j=1}^{P} \left[ g(Z_{ij}) - \ln E_i - \ln \Delta t_j \right]^2 + \lambda \sum_{z=Z_{min}+1}^{Z_{max}-1} g''(z)^2$$
(3)

The first term ensures that the solution satisfies the set of equations arising from Equation 2 in a least squares sense. The second term is a smoothness term on the sum of squared values of the second derivative of g to ensure that the function g is smooth; in this discrete setting we use g''(z) = g(z-1) - 2g(z) + g(z+1). This smoothness term is essential to the formulation in that it provides coupling between the values g(z) in the minimization. The scalar  $\lambda$  weights the smoothness term relative to the data fitting term, and should be chosen appropriately for the amount of noise expected in the  $Z_{ij}$  measurements.

而 gsolve.m 文件中也实现了这个部分,k从1循环到n-2即 $k\in [1,254]$ :

```
%% Include the smoothness equations
for i=1:n-2
    A(k,i)=l*w(i+1); A(k,i+1)=-2*l*w(i+1); A(k,i+2)=l*w(i+1);
    k=k+1;
end
```