On the semantics of merging

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Abstract

Intelligent agents are often faced with the problem of trying to merge possibly conflicting pieces of information obtained from different sources into a consistent view of the world. We propose a framework for the modelling of such merging operations with roots in the work of Spohn (Spohn 1988; Spohn 1991). Unlike most approaches we focus on the merging of epistemic states, not knowledge bases. We construct a number of plausible merging operations and measure them against various properties that merging operations ought to satisfy. Finally, we discuss the connection between merging and the use of infobases (Meyer 1999; Meyer, Labuschagne, & Heidema 2000).

Introduction

To be able to operate in its environment it is necessary for an intelligent agent to have a consistent view of the world. This demand is often complicated by the fact that such agents receive conflicting pieces of information from different sources. The process of combining possibly inconsistent pieces of information, known as *merging*, has many applications and has started to receive more attention recently (Borgida & Imielinski 1984; Lin 1996; Baral, Kraus, & Minker 1991; Baral *et al.* 1992; Konieczny & Pino-Pérez 1998; Liberatore & Schaerf 1998; Revesz 1993; Revesz 1987; Subrahmanian 1994). In this paper we propose a framework for the modelling of merging operations. The proposal has its roots in the work of Spohn (Spohn 1988; Spohn 1991). Unlike most approaches we adopt a description of merging on the level of *epistemic states* instead of *knowledge bases*.

First we give a brief introduction to the merging of knowledge bases, focussing on the work of Konieczny and Pino-Pérez (Konieczny & Pino-Pérez 1998). This is followed by a description of our framework for the merging of epistemic states. Then we construct a number of merging operations and show how they measure up to proposed properties of merging. Finally, we discuss links between merging and the *infobases* of Meyer (Meyer 1999).

We assume a finitely generated propositional language L closed under the usual propositional connectives, and with a classical model-theoretic semantics. U is the set of interpretations of L and $M(\alpha)$ is the set of models of $\alpha \in L$. Classical entailment is denoted by $\vDash$. We use $\sqcup$ to denote the concatenation of lists. We let x^n denote the list consisting of n versions of x. The length of a list l is denoted by $|l|$.

Merging knowledge bases

In the spirit of the work of Katsuno and Mendelzon (Katsuno & Mendelzon 1991), approaches to the merging of knowledge bases usually represent the beliefs of an agent as a single wff ϕ of L, known as a *knowledge base*, where ϕ represents the set of all wffs entailed by ϕ. The goal is to construct, from a finite list of such knowledge bases, an appropriate consistent knowledge base in some rational fashion. Konieczny and Pino-Pérez (Konieczny & Pino-Pérez 1998) have proposed a general framework for the merging of knowledge bases. A *knowledge list* e is a finite list of consistent knowledge bases $[\phi_1, \ldots, \phi_{|e|}]$. Two knowledge lists e_1 and e_2 are *element-equivalent*, written as $e_1 \approx e_2$, iff for every element ϕ_1 of e_1 there is a unique element ϕ_2 (position-wise) of e_2 such that $\phi_1 \equiv \phi_2$ and for every element ϕ_2 of e_2 there is a unique element ϕ_1 (position-wise) of e_1 such that $\phi_2 \equiv \phi_1$. A *KP-merging operation* δ is a function from the set of all knowledge lists to the set of all knowledge bases satisfying the following postulates (the KP-postulates):

(KP1) $\delta(e) \nvDash \bot$

(KP2) If $\bigwedge_{i=1}^{|e|} \phi_i \nvDash \bot$ then $\delta(e) = \bigwedge_{i=1}^{|e|} \phi_i$

(KP3) If $e_1 \approx e_2$ then $\delta(e_1) \equiv \delta(e_2)$

(KP4) If $\phi_1 \wedge \phi_2 \vDash \bot$ then $\delta([\phi_1] \sqcup [\phi_2]) \nvDash \phi_1$

(KP5) $\delta(e_1) \wedge \delta(e_2) \vDash \delta(e_1 \sqcup e_2)$

(KP6) If $\delta(e_1) \wedge \delta(e_2) \nvDash \bot$ then $\delta(e_1 \sqcup e_2) \vDash \delta(e_1) \wedge \delta(e_2)$

Konieczny and Pino-Pérez also distinguish between two subclasses of merging operations. An *arbitration* operation tries to take as many differing opinions as possible into account, while the intuition associated with *majority* operations is that the opinion of the majority should

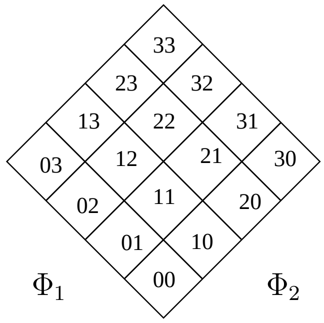

Figure 1: A pictorial representation of an epistemic list containing two epistemic states Φ_1 and Φ_2. The sequence of two digits in each cell above indicates the natural numbers associated with interpretations by the two epistemic states. A cell containing the sequence ij indicates the placement of those interpretations assigned the value i by Φ_1 and assigned the value j by Φ_2.

prevail. They initially propose the following postulates for arbitration and majority operations.

(arb) $\forall n\ \delta(e \sqcup \phi^n) = \delta(e \sqcup [\phi])$

(maj) $\exists n\ \delta(e \sqcup \phi^n) \vDash \phi$

It turns out that there is no KP-merging operation satisfying (arb). Unlike Konieczny and Pino-Pérez we are of the opinion that it is not (arb) that is at fault, but some of the KP-postulates. Below we argue against the inclusion of (KP4) and (KP6) as postulates that need to be satisfied by all merging operations.

Merging epistemic states

In this section we discuss merging on the level of epistemic states. We see an epistemic state as providing a plausibility ranking of the interpretations of L; the lower the number assigned to an interpretation, the more plausible it is deemed to be.

Definition 0.1 An epistemic state Φ is a function from U to the set of natural numbers. Given an epistemic state Φ, the knowledge base associated with Φ, denoted by ϕ_Φ, is some $\alpha \in L$ such that $M(\alpha) = \{u \mid \Phi(u) = 0\}$. □

This representation of an epistemic state and its associated knowledge base can be traced back to the work of Spohn (Spohn 1988; Spohn 1991). It should be clear that an epistemic state with an inconsistent associated knowledge base still contains useful information.

An *epistemic list* $E = [\Phi^E_1, \ldots, \Phi^E_{|E|}]$ is a finite list of epistemic states. It is instructive to view an epistemic list pictorially as in figure 1. While such a pictorial view is only useful in representing epistemic lists containing two elements, it serves as a good foundation for understanding the principles underlying the merging of epistemic states in general.

For any epistemic state Φ, let

$$\min(\Phi) = \min\{\Phi(u) \mid u \in U\}$$

let

$$\max(\Phi) = \max\{\Phi(u) \mid u \in U\}$$

and for an epistemic list E, let

$$\max(E) = \max\{\max(\Phi^E_i) \mid 1 \le i \le |E|\}.$$

For an epistemic list E and $u \in U$ we let $\min^E(u)$ be equal to

$$\min\{\Phi^E_i(u) \mid 1 \le i \le |E|\}$$

and we let $\max^E(u)$ be equal to

$$\max\{\Phi^E_i(u) \mid 1 \le i \le |E|\}.$$

We denote by $seq(E)$ the set of all sequences of length $|E|$ of natural numbers, ranging from 0 to $\max(E)$. We denote by $seq_{\le}(E)$ the subset of $seq(E)$ of all sequences that are ordered non-decreasingly, and by $seq_{\ge}(E)$ the subset of $seq(E)$ of all sequences that are ordered non-increasingly. For $u \in U$, we let $s^E(u)$ be the sequence containing the natural numbers $\Phi^E_1(u), \ldots, \Phi^E_{|E|}(u)$ in that order, we let $s^E_{\le}(u)$ be the sequence $s^E(u)$ ordered non-decreasingly, and we let $s^E_{\ge}(u)$ be the sequence $s^E(u)$ ordered non-increasingly. Clearly $s^E(u) \in seq(E)$, $s^E_{\le}(u) \in seq_{\le}(E)$ and $s^E_{\ge}(u) \in seq_{\ge}(E)$. Given any set seq of finite sequences of natural numbers and a total preorder $\sqsubseteq$ on seq, we define the function $\Omega^{seq}_{\sqsubseteq} : seq \to \{0, \ldots, |seq| - 1\}$ by assigning natural numbers to the elements of seq in the order imposed by $\sqsubseteq$, starting by assigning 0 to the elements lowest down in $\sqsubseteq$. We denote the *lexicographic* ordering on seq by $\sqsubseteq_{lex}$.

A *merging operation on epistemic states* Δ is a function from the set of all non-empty epistemic lists to the set of all epistemic states. We propose the following basic properties for the merging of epistemic states:

(E1) $\exists u$ s.t. $\Delta(E)(u) = 0$

(E2) If $\Phi^E_i(u) = \Phi^E_j(u)\ \forall i, j$ such that $1 \le i, j \le |E|$ and $s^E_{\le}(u) \sqsubset_{lex} s^E_{\le}(v)$ then $\Delta(E)(u) < \Delta(E)(v)$

(E3) If $\Phi^E_i(u) \le \Phi^E_i(v)\ \forall i$ such that $1 \le i \le |E|$ then $\Delta(E)(u) \le \Delta(E)(v)$

(E4) If $\Delta(E)(u) \le \Delta(E)(v)$ then $\Phi^E_i(u) \le \Phi^E_i(v)$ for some i such that $1 \le i \le |E|$

(E1) is a restatement of (KP1) and (E2) generalises (KP2). (E3) states that if all epistemic states in E agree that u is at least as plausible as v, then so should the resulting epistemic state. (E4) expects justification for regarding an interpretation u as at least as plausible as v: there has to be at least one epistemic state in E which regards u as at least as plausible as v. The following fundamental principle for the merging of epistemic states follows easily from (E3):

(Unit) If $\Phi^E_i(u) = \Phi^E_i(v)\ \forall i$ such that $1 \le i \le |E|$ then $\Delta(E)(u) = \Delta(E)(v)$

(Unit) requires interpretations that are treated identically by all epistemic states in an epistemic list to be treated identically in the epistemic state resulting from a merging operation.

Two epistemic lists E_1 and E_2 are *element-equivalent*, written as $E_1 \approx E_2$, iff for every element Φ_1 of E_1 there is a unique element Φ_2 (position-wise) of E_2 such that $\Phi_1 = \Phi_2$ and for every element Φ_2 of E_2 there is a unique element Φ_1 (position-wise) of E_1 such that $\Phi_2 = \Phi_1$. The following property is a generalisation of (KP3). It requires merging to be commutative.

(Comm) $E_1 \approx E_2$ implies $\Delta(E_1) = \Delta(E_2)$

We do not think that (Comm) should hold for all merging operations. Instead, (Comm) should be seen as a postulate picking out an interesting subclass of merging operations.

For a finite list of epistemic lists $\mathcal{E} = [E_1, \ldots, E_{|\mathcal{E}|}]$, let $\Delta(\mathcal{E})$ denote the epistemic list $[\Delta(E_1), \ldots, \Delta(E_{|\mathcal{E}|})]$. We consider the following properties:

(E5) If $\Delta(E_i)(u) \leq \Delta(E_i)(v)$ $\forall i$ such that $1 \leq i \leq |\mathcal{E}|$ then $\Delta(\bigsqcup_{i=1}^{|\mathcal{E}|} E_i)(u) \leq \Delta(\bigsqcup_{i=1}^{|\mathcal{E}|} E_i)(v)$

(E6) If $\Delta(\bigsqcup_{i=1}^{|\mathcal{E}|} E_i)(u) \leq \Delta(\bigsqcup_{i=1}^{|\mathcal{E}|} E_i)(v)$ then for some i such that $1 \leq i \leq |\mathcal{E}|$, $\Delta(E_i)(u) \leq \Delta(E_i)(v)$ for some $1 \leq i \leq |\mathcal{E}|$

(E5) generalises (E3) and (E6) generalises (E4). In fact, (E5) also implies (KP5).

The arbitration postulate (arb) and the majority postulate (maj) can be generalised as follows:

(Arb) $\forall n\ \Delta(E \sqcup [\Phi])(u) = \Delta(E \sqcup \Phi^n)(u)$

(Maj) $\exists n$ s.t. $\forall u, v \in U$, $\Phi(u) \leq \Phi(v)$ if $\Delta(E \sqcup \Phi^n)(u) \leq \Delta(E \sqcup \Phi^n)(v)$

We have not provided a generalised version of (KP4). The reason is that we do not regard it as a suitable postulate for merging. Our basic argument is that the models of a knowledge base associated with an epistemic state Φ_1 may sometimes be given such an implausible ranking by an epistemic state Φ_2 that it would seem reasonable to exclude all these models from the models of $\phi_{\Delta([\Phi_1] \sqcup [\Phi_2])}$. It is worthwhile noting that none of the merging operations we consider below satisfies (KP4). Similarly, we have not provided a generalised version of (KP6) since we regard it as too strong a condition to impose on all merging operations.[1] Below we shall encounter a number of reasonable merging operations which do not satisy (KP6).

Constructing merging operations

Konieczny and Pino-Pérez (Konieczny & Pino-Pérez 1998) discuss several merging operations on knowledge bases using Dalal's measure of distance between interpretations (Dalal 1988). For any two interpretations u and v, let $dist(u, v)$ denote the number of propositional atoms on which u and v differ. The distance $Dist(\phi, u)$ between a knowledge base ϕ and an interpretation u is defined as follows: $Dist(\phi, u) = \min\{dist(u, v) \mid v \in M(\phi)\}$. It is clear that this distance measure can be used to define an epistemic state Φ as follows:

$$\forall u \in U,\ \Phi(u) = Dist(\phi, u).$$

It is easily seen that $\Phi(u) = 0$ iff $u \in M(\phi)$ and therefore $\phi_\Phi \equiv \phi$. Many of the merging operations on epistemic states that we propose below are appropriate generalisations of these merging operations on knowledge bases.

When reading through the remainder of this section, the reader should observe that the construction of every merging operation consists of two steps. In the first step natural numbers are assigned to interpretations. After the completion of this step it will often be the case that *none* of the interpretations have been assigned the value 0. To ensure compliance with (E1) the second step performs an appropriate uniform subtraction of values which we shall refer to as *normalisation*.

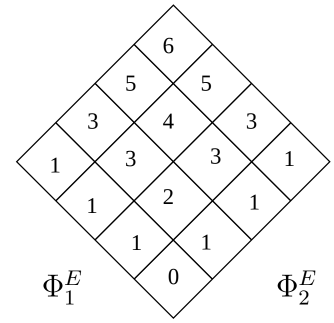

Figure 2: A representation of the merging operation Δ_{ls}. The number in a cell represents the numbers that the appropriate merging operation assigns to the interpretations contained in that cell before normalisation.

Arbitration

Inspired by an arbitration operation proposed by Liberatore and Schaerf (Liberatore & Schaerf 1998) we propose the following two merging operations on epistemic states.

Definition 0.2 1. Let $\Phi_{ls}^E(u) = 2\min^E(u)$ if $\Phi_i^E(u) = \Phi_j^E(u)$ for $1 \leq i, j \leq |E|$, and $\Phi_{ls}^E(u) = 2\min^E(u) + 1$ otherwise. Then $\Delta_{ls}(E)(u) = \Phi_{ls}^E(u) - \min(\Phi_{ls}^E)$.

2. Let $\Phi_{Rls}^E(u) = \Omega_{\sqsubseteq_{lex}}^{seq_{\leq}(E)}(s_{\leq}^E(u))$. Then $\Delta_{Rls}(E)(u) = \Phi_{Rls}^E(u) - \min(\Phi_{Rls}^E)$.

$\Box$

Figure 2 contains a pictorial representation of Δ_{ls} and figure 3 a pictorial representation of Δ_{Rls}. It can easily be shown that Δ_{Rls} is a refined version of Δ_{ls}. Both satisfy (E1)-(E6) and (Comm), neither satisfies (Maj), and only Δ_{Rls} satisfies (KP6). Moreover, Δ_{ls} satisfies

[1](E6) can be regarded as a generalised version of a weaker form of (KP6), but (KP6) does not follow from (E6).

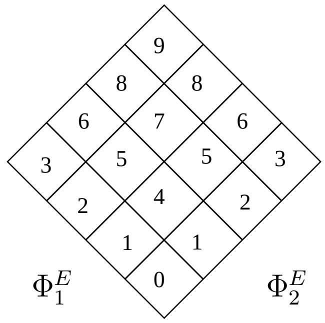

Figure 3: A representation of the merging operation Δ_{Rls} The number in a cell represents the numbers that the appropriate merging operation assigns to the interpretations contained in that cell before normalisation.

(Arb) but Δ_{Rls} does not. So, while both are valid merging operations, Δ_{Rls} should not be seen as an arbitation operation.

Next we consider two merging operations that are generalisations of the $\delta_{\max}$ and δ_{Gmax} operations of Konieczny and Pino-Pérez. The former was inspired by an example of Revesz's model-fitting operations (Revesz 1987).

Definition 0.3 1. Let $\Phi^E_{\max}(u) = \max^E(u)$. Then $\Delta_{\max}(E)(u) = \Phi^E_{\max}(u) - \min(\Phi^E_{\max})$.

2. Let $\Phi^E_{Gmax}(u) = \Omega^{seq_{\geq}(E)}_{\sqsubseteq_{lex}}(s^E_{\geq}(u))$. Then $\Delta_{Gmax}(E)(u) = \Phi^E_{Gmax}(u) - \min(\Phi^E_{Gmax})$.

□

Figure 4 contains a pictorial representation of $\Delta_{\max}$ and figure 5 a pictorial representation of Δ_{Gmax}. Both satisfy (E1)-(E6), neither satisfies (Maj), and only Δ_{Gmax} satisfies (KP6). Moreover, $\Delta_{\max}$ satisfies (Arb), but Δ_{Gmax} does not. So, analogous to the case above, both are valid merging operations but Δ_{Gmax} should not be seen as an arbitation operation. The fact that

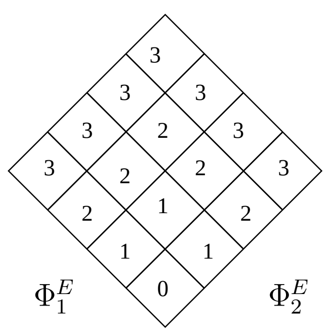

Figure 4: A representation of the merging operation $\Delta_{\max}$. The number in a cell represents the numbers that the appropriate merging operation assigns to the interpretations contained in that cell before normalisation.

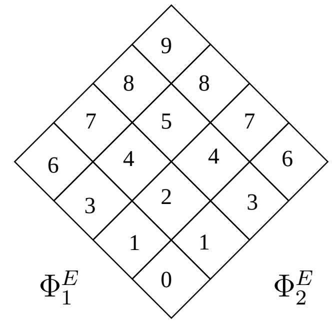

Figure 5: A representation of the merging operation Δ_{Gmax}. The number in a cell represents the numbers that the appropriate merging operation assigns to the interpretations contained in that cell before normalisation.

we do not regard Δ_{Gmax} as an arbitration operation is in conflict with the view of Konieczny and Pino-Pérez who regard δ_{Gmax} as an arbitration operation on knowledge bases even though it does not satisfy (arb). Conversely, Konieczny and Pino-Pérez do not regard $\delta_{\max}$ as a merging operation on knowledge bases since it fails to satisfy (KP6). But we regard it as a valid arbitration operation since it satisfies the postulates (E1)-(E6), (Comm) and (Arb).

Consensus

In this section we consider the idea of a *consensus* operation, where agreement on the ranking of interpretations, instead of the ranking itself, is of overriding importance.

Definition 0.4 For $s \in seq(E)$, let

$$d^E(s) = \sum_{i=1}^{|E|} \sum_{j=i+1}^{|E|} |s_i - s_j|$$

where s_i denotes the ith element of s.

1. Define the total preorder $\sqsubseteq$ on $seq(E)$ as follows: $s \sqsubseteq t$ iff $d^E(s) \leq d^E(t)$. Let $\Phi^E_{cons}(u) = \Omega^{seq(E)}_{\sqsubseteq}(s^E(u))$. Then $\Delta_{cons}(E)(u) = \Phi^E_{cons}(u) - \min(\Phi^E_{cons})$.
2. Define the total preorder $\sqsubseteq$ on $seq_{\leq}(E)$ as follows: $s \sqsubseteq t$ iff $d^E(s) < d^E(t)$ or ($d^E(s) = d^E(t)$ and $s \sqsubseteq_{lex} t$). Now, let $\Phi^E_{Rcons}(u) = \Omega^{seq_{\leq}(E)}_{\sqsubseteq}(s^E_{\leq}(u))$. Then $\Delta_{Rcons}(E)(u) = \Phi^E_{Rcons}(u) - \min(\Phi^E_{Rcons})$.

□

Figure 6 contains a pictorial representation of Δ_{cons} and figure 7 a pictorial representation of Δ_{Rcons}. We do not regard these two operations as suitable candidates for merging, primarily because both fail to satisfy (E3) and (E4). Both satisfy (Unit), though. The problem with these consensus operations seems to be that they place too strong an emphasis on agreement and do not take the ranking of interpretations seriously enough.

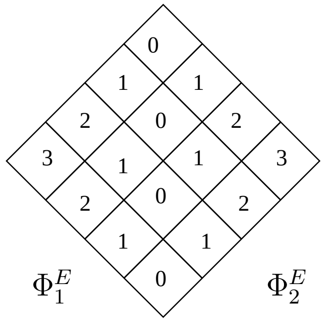

Figure 6: A representation of the merging operation Δ_{cons}. As usual, the number in a cell represents the numbers that the appropriate merging operation assigns to the interpretations contained in that cell before normalisation.

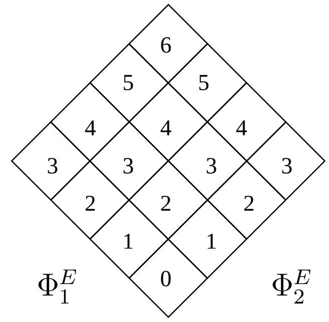

Figure 8: A representation of the merging operation Δ_{Σ}. As usual, the number in a cell represents the numbers that the appropriate merging operation assigns to the interpretations contained in that cell before normalisation.

Majority

We consider the following two majority operations.

Definition 0.5 For $s \in seq(E)$, let

$$sum^E(s) = \sum_{i=1}^{|E|} s_i$$

where s_i is the ith element of s.

1. Let $\Phi^E_{\Sigma}(u) = sum^E(s^E(u))$. Then $\Delta_{\Sigma}(E)(u) = \Phi^E_{\Sigma}(u) - \min(\Phi^E_{\Sigma})$.
2. Define the total preorder $\sqsubseteq$ on $seq(E)$ as follows: $s \sqsubseteq t$ iff $sum^E(s) < sum^E(t)$ or $(sum^E(s) = sum^E(t)$ and $d^E(s) \leq d^E(t))$. Now, let $\Phi^E_{R\Sigma}(u) = \Omega^{seq(E)}_{\sqsubseteq}(s^E(u))$. Then $\Delta_{R\Sigma}(E)(u) = \Phi^E_{R\Sigma}(u) - \min(\Phi^E_{R\Sigma})$.

□

Figure 8 contains a pictorial representation of Δ_{Σ} and figure 9 a pictorial representation of $\Delta_{R\Sigma}$. Δ_{Σ} is an appropriate generalisation of an example by Lin and Mendelzon (Lin & Mendelzon). It was independently proposed by Revesz (Revesz 1993) as an example of weighted model fitting. The idea is simply to obtain the new plausibility ranking of an interpretation by summing the plausibility rankings given by the different epistemic states. $\Delta_{R\Sigma}$ is Δ_{Σ} refined by using consensus. Both Δ_{Σ} and $\Delta_{R\Sigma}$ satisfy (E1)-(E4), (Comm) and (Maj), and neither satisfies (Arb). But while Δ_{Σ} satisfies (E5)-(E6) and (KP5)-(KP6) as well, $\Delta_{R\Sigma}$ does not.

Non-commutative merging

Thus far we have restricted ourselves to the construction of *commutative* merging operations – i.e., satisfying (Comm) – but a complete description of merging ought to take into account constructions such as that of Nayak (Nayak 1994), in which the merging of two epistemic states is obtained by a lexicographic refinement of one by the other. We present here a generalised version of Nayak's proposal. For this case the epistemic

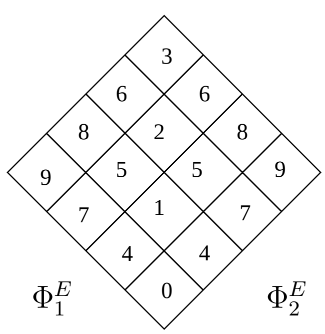

Figure 7: A representation of the merging operation Δ_{Rcons}. As usual, the number in a cell represents the numbers that the appropriate merging operation assigns to the interpretations contained in that cell before normalisation.

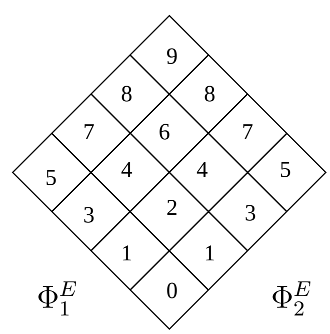

Figure 9: A representation of the merging operation $\Delta_{R\Sigma}$. As usual, the number in a cell represents the numbers that the appropriate merging operation assigns to the interpretations contained in that cell before normalisation.

states in an epistemic list are assumed to be ranked according to reliability. That is, given an epistemic list $E = [\Phi^E_1, \ldots, \Phi^E_{|E|}]$, Φ^E_i is at least as reliable as Φ^E_j iff $i \leq j$.

Definition 0.6 Let $\Phi^E_{lex}(u) = \Omega^{seq(E)}_{\sqsubseteq_{lex}}(s^E(u))$. Then $\Delta_{lex}(E)(u) = \Phi^E_{lex}(u) - \min(\Phi^E_{lex})$. □

Δ_{lex} does not satisfy (Comm), but it satisfies (E1)-(E6), as well as (KP5)-(KP6). By exploiting the non-commutativity of Δ_{lex}, both (Arb) and (Maj) can be phrased in a way to ensure that Δ_{lex} fails to satisfy them.

Merging and infobases

Our description of merging uses a representation of epistemic states as functions assigning a plausibility ranking to the interpretations of L, but where do these plausibility rankings come from? One way in which to generate them is by using the *infobases* of Meyer (Meyer 1999). An infobase is a finite list of wffs. Intuitively it is a structured representation of the beliefs of an agent with a foundational flavour. It is assumed that every wff in an infobase is obtained independently. Meyer uses an infobase to define a total preorder on U, which is then used to perform belief change. However, we can also use an infobase to define an epistemic state. The idea is to consider the number of times that an interpretation occurs as a model of one of the wffs in an infobase: the more it occurs, the higher its plausibility ranking.

Definition 0.7 For $u \in U$, define the IB-number u_{IB} of u as the number of elements α in an infobase IB such that $\nvDash \alpha$ and $u \in M(\alpha)$, and let

$$\max(IB) = \max\{u_{IB} \mid u \in U\}.$$

Now we define the epistemic state associated with IB as follows: for $u \in U, \Phi^{IB}(u) = \max(IB) - u_{IB}$. □

Observe that the knowledge base associated with an epistemic state Φ^{IB} is always consistent, regardless of whether the wffs in IB are jointly consistent. We show that infobases seem to provide a natural setting in which to apply merging.

Firstly, define an *infobase list* $EB = [IB_1, \ldots, IB_{|EB|}]$ as a finite non-empty list of infobases and let E^{EB} denote the epistemic list $[\Phi^{IB_1}, \ldots, \Phi^{IB_{|E|}}]$ of epistemic states associated with the infobases occurring in EB. Then it can be verified that $\Delta_\Sigma(E^{EB}) = \Phi^{IB}$ where $IB = \bigsqcup_{i=1}^{|EB|} IB_i$.

Secondly, Konieczny and Pino-Pérez (Konieczny & Pino-Pérez 1998) give a convincing example to show that we may sometimes want to include, as models of $\delta(e)$, interpretations other than the models of the knowledge bases in e. Below is a scaled down version of their example.

Example 0.8 We want to speculate on the stock exchange and we ask two equally reliable financial experts about two shares. Let the atom p denote the fact that share 1 will rise and q the fact that share 2 will rise. The first expert says that both shares will rise: $\phi_1 = p \wedge q$, while the second one believes that both shares will fall: $\phi_2 = \neg p \wedge \neg q$. Intuitively it seems reasonable to conclude that both experts are right (and wrong) about exactly one share, although we don't know which share in either case. That is, we require the result of the merging of these two knowledge bases to be such that $M(\delta([\phi_1] \sqcup [\phi_2])) = \{10, 01\}$.[2] Observe that $M(\delta([\phi_1] \sqcup [\phi_2])) \nsubseteq M(\phi_1) \cup M(\phi_2)$. □

An analysis of this example shows that both experts are assumed to make an implicit assumption of independence of the performance of the shares. Thus the beliefs of the first expert is best expressed as the infobase $IB_1 = [p, q]$ and the beliefs of the second expert as the infobase $IB_2 = [\neg p, \neg q]$. The epistemic states obtained from these two infobases are: $\Phi^{IB_1}(11) = 0, \Phi^{IB_1}(10) = \Phi^{IB_1}(01) = 1, \Phi^{IB_1}(00) = 2$, and $\Phi^{IB_2}(00) = 0, \Phi^{IB_2}(10) = \Phi^{IB_2}(01) = 1, \Phi^{IB_2}(11) = 2$. It can be verified that $\Delta_{\max}(E^{EB}) = \Delta_{Gmax}(E^{EB}) = \Delta_{R\Sigma}(E^{EB}) = \Phi$, where $EB = [IB_1, IB_2]$, $\Phi(10) = \Phi(01) = 0$ and $\Phi(11) = \Phi(00) = 1$. So $\Delta_{R\Sigma}$, $\Delta_{\max}$ and Δ_{Gmax} yield the results corresponding to our intuition for this example.

Conclusion

The merging operations we have constructed provide evidence that (E1)-(E4) may be regarded as basic postulates for merging operations on epistemic states. Furthermore, we regard (Arb) as an appropriate postulate for the subclass of arbitration operations, (Maj) for the subclass of majority operations, and (Comm) for the subclass of commutative merging operations. The status of (E5) and (E6) is less clear. While all but one of the valid merging operations we have considered satisfy both, the fact that $\Delta_{R\Sigma}$ does not, suggests that they are not as universally applicable as (E1)-(E4). Perhaps they should be seen as picking out particular subclasses of merging operations in the way that (Arb), (Maj) and (Comm) do.

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[2] We represent interpretations as sequences consisting of 0s (representing falsity) and 1s (representing truth), where the first digit in a sequence represents the truth value of p and the second one the truth value of q.

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