A note on Darwiche and Pearl

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Abstract

It is shown that Darwiche and Pearl's postulates imply an interesting property, not noticed by the authors.

1 A short remark

In [**?**], Darwiche and Pearl propose postulates for iterated revisions, noted (R*1) to (R*6) and (C1) to (C4). In particular, the postulate (C3) reads:

$$(C3) \quad \text{If } \Psi \circ \alpha \models \mu, \text{ then } (\Psi \circ \mu) \circ \alpha \models \mu.$$

It will be shown that, in the presence (R*1) to (R*6), (C1) and (C3) imply:

$$(**) \quad \text{If } \Psi \circ \alpha \models \mu, \text{ then } (\Psi \circ \mu) \circ \alpha \equiv \Psi \circ \alpha.$$

First, a lemma.

Lemma 1 *Assuming (R*1) to (R*6), if* $\Psi \circ \mu \models \varphi$*, then* $\Psi \circ \mu \equiv \Psi \circ (\mu \wedge \varphi)$*.*

Proof: Since $\Psi \circ \mu \models \varphi$, $\Psi \circ \mu \models (\Psi \circ \mu) \wedge \varphi$. By (R*4), $(\Psi \circ \mu) \wedge \varphi \models \Psi \circ (\mu \wedge \varphi)$. Therefore $\Psi \circ \mu \models \Psi \circ (\mu \wedge \varphi)$.

If $\Psi \circ \mu$ is satisfiable, then, since $\Psi \circ \mu \models \varphi$, $(\Psi \circ \mu) \wedge \varphi$ is satisfiable and, by (R*5), $\Psi \circ (\mu \wedge \varphi) \models (\Psi \circ \mu) \wedge \varphi$ and therefore $\Psi \circ (\mu \wedge \varphi) \models \Psi \circ \mu$.

If $\Psi \circ \mu$ is not satisfiable, then, by (R*3), μ is not satisfiable, and $\mu \wedge \varphi$ is not satisfiable. By (R*1), then, $\Psi \circ (\mu \wedge \varphi) \models \Psi \circ \mu$. ∎

Lemma 2 *Assuming (R*1) to (R*6), (C1) and (C3), if* $\Psi \circ \alpha \models \mu$*, then* $(\Psi \circ \mu) \circ \alpha \equiv \Psi \circ \alpha$*.*

Proof: Suppose $\Psi \circ \alpha \models \mu$. By Lemma 1, $\Psi \circ \alpha \equiv \Psi \circ (\alpha \wedge \mu)$. By (C1), $\Psi \circ (\alpha \wedge \mu) \equiv (\Psi \circ \mu) \circ (\alpha \wedge \mu)$. But, by (C3), $\Psi \circ \mu \circ \alpha \models \mu$ and, by Lemma 1, $\Psi \circ \mu \circ \alpha \equiv \Psi \circ \mu \circ (\alpha \wedge \mu)$.

We conclude that $\Psi \circ \alpha \equiv \Psi \circ \mu \circ \alpha$. ∎