Coherence, Belief Expansion and Bayesian Networks

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Abstract

We construct a probabilistic coherence measure for information sets which determines a partial coherence ordering. This measure is applied in constructing a criterion for expanding our beliefs in the face of new information. A number of idealizations are being made which can be relaxed by an appeal to Bayesian Networks.

Introduction

Suppose that one receives information $\{R_1, \ldots, R_n\}$ from n independent but less than fully reliable sources. Is it rational to believe this information? Following a tradition in epistemology that goes back to John Locke, we let belief correspond to a sufficiently high degree of confidence (Foley 1992, Hawthorne and Bovens 1999). There are three factors that determine this degree of confidence: (i) How surprising is the information? (ii) How reliable are the sources? (iii) How coherent is the information? First, suppose that the sources are halfway reliable and the information is halfway coherent. Then certainly the degree of confidence will be greater when the reported information is less rather than more surprising. Second, suppose that the information is halfway surprising and is halfway coherent. Let *truth-tellers* provide fully reliable information and let *randomizers* flip a coin for each proposition to determine whether they will affirm or deny it. Then certainly the degree of confidence will be greater when the sources are more like truth-tellers than when they are more like randomizers. Third, consider the following story: a scientist runs two independent tests to determine the locus of a genetic disease on the human genome. In the first case, the tests respectively point to two fairly narrow regions that just about overlap in a particular region. In the second case, the tests respectively point to fairly broad regions that have minimal overlap in the very same region. Suppose that the tests are halfway reliable and that this region is a somewhat surprising locus for the disease. Then certainly the degree of confidence that the locus of the disease is in this region is greater in the former case, in which the

information is more coherent, than in the latter case, in which the information is less coherent.

We define measures for each of these determinants of the degree of confidence in a probabilistic framework. The real challenge lies in developing a measure of coherence (cf. Lewis 1946, Bonjour 1985). This measure defines a partial ordering over information sets. Subsequently, we argue that belief expansion is a function of the reliability of the sources and the coherence of the new information with the information that we already believe. We construct an acceptance measure which determines whether newly acquired information can be added to our beliefs under alternative suppositions about the reliability of the sources. Our calculations rest on some results in the theory of Bayesian Networks. Throughout we have made some strong idealizations. We show how these idealizations can be relaxed by directly invoking Bayesian Networks.

The Model

For each proposition R_i (in roman script) in the information set, let us define a propositional variable R_i (in italic script) which can take on two values, viz. R_i and $\overline{R}_i$ (i.e. not-R_i) for $i = 1, \ldots, n$. Let $REPR_i$ be a propositional variable which can take on two values, viz. $REPR_i$, i.e. there is a report from the proper source to the effect that R_i is true, and $\overline{REPR}_i$, i.e. there is a report to the effect that R_i is false. We construct a joint probability distribution P over $R_1, \ldots, R_n, REPR_1, \ldots, REPR_n$ satisfying the constraint that the sources are independent and less than fully reliable.

We model independence by stipulating that P respects the following conditional independences:

$$REPR_i \perp R_j, REPR_j | R_i \text{ for } i \neq j; \ i, j = 1, 2, \ldots, n \tag{1}$$

or, in words, $REPR_i$ is probabilistically independent of $R_j, REPR_j$, given R_i, for $i \neq j$ and $i, j = 1, 2, \ldots, n$. What this means is that the probability that I will receive a report that R_i given that R_i is the case or given that R_i is not the case, is not affected by any additional information about whether R_j is the case or whether there is a report to the effect that R_j is the

case. Each source tunes in on the item of information that it is meant to report on: it may not always provide an accurate report, but its report is not affected by what other sources have to report or by other items of information than the one it reports on (Lewis 1946, Bovens and Olsson 1999).

We define a less-than-fully-reliable source as a source that is better than a randomizer, but short of being a truth-teller and make the simplifying idealization that the information sources are equally reliable. We specify the following two parameters: $P(\text{REPR}_i|R_i) = p$ and $P(\text{REPR}_i|\overline{R}_i) = q$ for $i = 1, \ldots, n$. If the information sources are truth-tellers, then $p = 1$ and $q = 0$, while if they are randomizers, then $p = q > 0$. We model less-than-full-reliability by imposing the following constraint on P:

$$P(\text{REPR}_i|R_i) = p > q = P(\text{REPR}_i|\overline{R}_i) > 0 \qquad (2)$$

The degree of confidence in the content of the information set is the posterior joint probability after all the reports have come in:

$$P^*(R_1, \ldots, R_n) = P(R_1, \ldots, R_n|\text{REPR}_1, \ldots, \text{REPR}_n) \qquad (3)$$

The motivation for the definition of less-than-full reliability is that we are interested in cases in which incoming information raises our confidence in the content of the information set to different levels. When the sources are randomizers, our confidence will be unaffected (Huemer 1997, Bovens and Olsson 1999), i.e. $P^*(R_1, \ldots, R_n) = P(R_1, \ldots, R_n)$; when they are truth-tellers, our confidence will be raised to certainty, i.e. $P^*(R_1, \ldots, R_n) = 1$; and when they are worse than randomizers, our confidence will drop, i.e. $P^*(R_1, \ldots, R_n) < P(R_1, \ldots, R_n)$.

Expectation, Reliability and Coherence

It can be shown by the probability calculus, that, given the constraints on P in (1) and (2),

$$P^*(R_1, \ldots, R_n) = \frac{a_0}{\sum_{i=0}^{n} a_i x^i}, \qquad (4)$$

in which the *likelihood ratio* $x = q/p$ (note that $0 < x < 1$ for $p > q > 0$) and a_i is the sum of the joint probabilities of all combinations of values of the variables $R_1, \ldots, R_n$ that have i negative values and $n - i$ positive values: e. g. for $n = 3$, $a_2 = P(R_1, \overline{R}_2, \overline{R}_3) + P(\overline{R}_1, R_2, \overline{R}_3) + P(\overline{R}_1, \overline{R}_2, R_3)$. Note that $\sum_{i=0}^{n} a_i = 1$.

We can directly identify the first determinant of the degree of confidence in the information set. Note that $a_0 = P(R_1, \ldots, R_n)$ is the prior joint probability of the propositions in the information set, i.e. the probability before any information was received. This prior probability is lower for more surprising information and higher for less surprising information. Since more surprising information is tantamount to less expected information, let us call this prior probability the *expectation measure*. It is easy to see that $P^*(R_1, \ldots, R_n)$ is

a monotonically increasing function of a_0. We can also directly identify the second determinant, i.e. the reliability of the sources. Note that $P^*(R_1, \ldots, R_n)$ is a monotonically decreasing function of $x = q/p$. Hence, let us call $r := 1 - x$ the *reliability measure*, since $P^*(R_1, \ldots, R_n)$ is a monotonically increasing function of r and this measure ranges from 0 for sources that are randomizers to 1 for sources that are truth-tellers.

It is more difficult to construct a *coherence measure*. Consider the following analogy: to assess the impact of a training program, we consider the rate of the student's actual performance level over the performance level that he would have reached in an ideal training program, all other things equal. Similarly, to assess the impact of coherence, we consider the rate of the present degree of confidence over the degree of confidence that would have been obtained had the information set been maximally coherent, all other things equal. The information set would have been maximally coherent if and only if $R_1, \ldots, R_n$ had all been coextensive. Let P be the actual joint probability distribution. Construct a joint probability distribution P^{max} with the same expectation measure and the same reliability measure as P, but $R_1, \ldots, R_n$ are all coextensive, i.e., on P^{max}, a_0 is the same as on P, but $a_n = 1 - a_0 =: \overline{a}_0$, so that $a_i = 0$, for all $i \neq 0, n$. It follows from (4) that,

$$P^{max*}(R_1, \ldots, R_n) = \frac{a_0}{a_0 + \overline{a}_0 x^n}. \qquad (5)$$

Hence, for $a_0 \neq 0$, the ratio

$$\begin{aligned}
c_x(R_1, \ldots, R_n) &= \frac{P^*(R_1, \ldots, R_n)}{P^{max*}(R_1, \ldots, R_n)} \\
&= \frac{a_0 + \overline{a}_0 x^n}{\sum_{i=0}^{n} a_i x^i} \qquad (6)
\end{aligned}$$

is a measure of the impact of the coherence of the information set on the degree of confidence in the content of the information set. But note that this measure is contingent on the value of the reliability measure: (6) only provides us with a reliability-relative coherence measure. This is unwelcome: there is a pretheoretical notion of the coherence of an information set which has nothing to do with the reliability of the sources that provides us with their content. On the other hand, this pretheoretical notion seems to be an ordinal rather than a cardinal notion. And furthermore, it seems to require a partial rather than a complete ordering over information sets: for certain, though not for all pairs of information sets, we are prepared to pass a judgment that one set in the pair is more or less coherent than the other.

It turns out that the reliability-relative coherence measure indeed induces a partial ordering over informations sets which is not contingent on the reliability of the sources. Consider two information sets of size n. These sets can be represented by the marginal probability distributions P and P' over $R_1, \ldots, R_n$. It can be shown that for some P with $\langle a_0, \ldots, a_n \rangle$ and

P' with $\langle a'_0, \ldots, a'_n \rangle$, the difference $c_x(\mathrm{R}_1, \ldots, \mathrm{R}_n) - c'_x(\mathrm{R}_1, \ldots, \mathrm{R}_n)$ has the same sign for any value of x ranging from 0 to 1. Hence, the reliability-relative coherence measure $c_x(\mathrm{R}_1, \ldots, \mathrm{R}_n)$ induces a partial coherence ordering over information sets that is not contingent on the reliability of the sources. For information *pairs*, i.e. for information sets containing exactly two propositions, it can be shown that the following is a necessary and sufficient condition for inclusion in the partial coherence ordering: P and P' are such that (i) $a_0/a'_0 \leq a_1/a'_1$ and $a_1 \geq a'_1$, or, (ii) $a_0/a'_0 \geq a_1/a'_1$ and $a_1 \leq a'_1$. For information sets in general, it can be shown that the following is a sufficient condition for inclusion in the partial coherence ordering: P and P' are such that (i) $a_i/a'_i < a_0/a'_0 < 1$, or, (ii) $a_i/a'_i > a_0/a'_0 > 1$, for $i = 1, \ldots, n-1$.

We provide an example of this condition for information pairs. Suppose that we are trying to locate a corpse of a murder somewhere in Tokyo. We draw a grid of 100 squares over the map of the city so that it is equally probable that the murder occurred in each grid. We interview two independent less-than-fully-reliable sources. Source 1 reports that the corpse is somewhere in squares 41 to 60 and source 2 reports that the corpse is somewhere in squares 51 to 70. In this case, $a_0 = .10$ and $a_1 = .20$. This our base case. Now consider alternate case A in which source 1 reports squares 50 to 60 and source 2 reports squares 51 to 61. In this case, $a'_0 = .10$ and $a'_1 = .02$. The information set in alternate case A is clearly more coherent than in the base case. Notice that the condition for a partial ordering is indeed satisfied. But now consider alternate case B: source 1 reports squares 26 to 60 and source 2 reports squares 41 to 75. In this case $a''_0 = .20$ and $a''_1 = .30$. Is the information set in alternate case B more coherent than in the base case? The proportion of the reported squares that overlap in each report is greater in the alternate case, which suggests that there is more coherence. But on the other hand, the price of getting more proportional overlap is that the overlapping area is less precise and that both sources make a much broader sweep over the map, suggesting less coherence. Indeed, in this case, we cannot pass judgment whether the information set in alternate case B is more coherent than in the base case. Notice that the condition for a partial ordering is indeed not justified.

Belief Expansion

Suppose that we acquire various items of background information from various sources and that our degree of confidence in the content of the information set is sufficiently high to believe the information. Now a new item of information is being presented. Are we justified to add this new item of information to what we already believe? The answer to this question has something to do (i) with the reliability of the information source as well as (ii) with the plausibility of the new information, given what we already believe, or in other words,

with how well the new information coheres with the background information. The more reliable the source is, the less plausible the new information needs to be, given what we already believe, to be justified to add the new information. The more plausible the new information is, given what we already believe, the less reliable the source needs to be, to be justified to add the new information. The challenge is: can a precise account of this relationship be provided?

Our approach is markedly different from AGM belief revision. In the AGM approach, the question is not *whether* to accept new information or not, but rather, once we have made the decision to accept the new information, *how* we should revise our beliefs in the face of inconsistency (Makinson 1997, Olsson 1997). Our approach shares a common motivation with the program of non-prioritized belief revision. According to Hansson (1997), we may not be willing to accept the new information because "it may be less reliable (...) than conflicting old information." Makinson (1997) writes that "we may not want to give top priority to new information (...) we may wish to weigh it against old material, and if it is really just too far-fetched or incredible, we may not wish to accept it." However, whereas the program of non-prioritized belief revision operates within a logicist framework, we construct a probabilistic model. The cost of this approach is that it is informationally more demanding. The benefit is that it is empirically more adequate, because it is sensitive to degrees of reliability and coherence and to their interplay in belief acceptance. In non-prioritized belief revision, the reliability of the sources does not enter into the model itself and the lack of coherence of an information set is understood in terms of logical inconsistency, which is only a limiting case in our model. To introduce the approach, we address the question of belief expansion. We believe that our model also carries a promise to handle belief revision in general, but this project is beyond the scope of this paper.

We need to make some simplifying assumptions about the origin of the background information and the new information: (a) the propositions in the background information are provided by independent sources, which are (b) less than fully reliable, (c) equally reliable as the new source, and (d) independent of the new source.

Our background information is contained in $\{\mathrm{R}_1, \ldots, \mathrm{R}_n\}$. Now suppose that we have a certain threshold level for belief and that the degree of confidence for the background information after having received a report to this effect from independent less than fully reliable sources is right at this level. (This stipulation is not required if we model actual cases by means of Bayesian Networks.) Now we are handed a new item of information R_{n+1} by an independent less than fully reliable source. Then we will expand our belief set from $\{\mathrm{R}_1, \ldots, \mathrm{R}_n\}$ to $\{\mathrm{R}_1, \ldots, \mathrm{R}_{n+1}\}$ if and

only if

$$P(\text{R}_1, \ldots, \text{R}_{n+1} | \text{REPR}_1, \ldots, \text{REPR}_{n+1}) \geq$$
$$P(\text{R}_1, \ldots, \text{R}_n | \text{REPR}_1, \ldots, \text{REPR}_n). \qquad (7)$$

Our sources are independent:

$$REPR_i \perp R_j, REPR_j | R_i \text{ for } i \neq j; \ i, j = 1, \ldots, n+1 \qquad (8)$$

(4) defines an *acceptance measure* for an information set:

$$e_x(\text{R}_1, \ldots, \text{R}_m) = P^*(\text{R}_1, \ldots, \text{R}_m) = \frac{a_0}{\sum_{i=0}^m a_i x^i} \qquad (9)$$

Considering (6) and (9), we can define this acceptance measure in terms of the reliability-relative coherence measure c_x, provided that $a_0 \neq 0$:

$$e_x(\text{R}_1, \ldots, \text{R}_m) = \frac{a_0}{a_0 + \overline{a}_0 x^m} c_x(\text{R}_1, \ldots, \text{R}_m) \qquad (10)$$

¿From (7) and (9), it follows that we can expand our belief set with a new item of information if and only if

$$e_x(\text{R}_1, \ldots, \text{R}_{n+1}) \geq e_x(\text{R}_1, \ldots, \text{R}_n). \qquad (11)$$

We can make the following two observations:

(i) From (9) and (11), it is clear that whether we can expand our beliefs or not, is a complex function of the reliability of the sources and the dependence of new on earlier information as expressed in the probability distribution over the variables $R_1, \ldots, R_{n+1}$. The reliability of the sources is reflected in the likelihood ratio x and the dependence of new on earlier information is reflected in the series $\langle a_0, \ldots, a_n \rangle$ for $e_x(\text{R}_1, \ldots, \text{R}_n)$ and in the series $\langle a'_0, \ldots, a'_{n+1} \rangle$ for $e_x(\text{R}_1, \ldots, \text{R}_{n+1})$.

(ii) From (10), it is clear that the acceptance measure is a weighted reliability-relative coherence measure. The weight tends to 1 for smaller values of x, i.e. for more reliable sources, and for greater values of n, i.e. for larger information sets, so that the acceptance measure will coincide with c_x. We have shown that this measure lets us construct a coherence ordering over a pair of information n-tuples, if certain conditions are met. We conjecture that such an ordering can also be constructed over pairs containing an information n-tuple and an expansion of this n-tuple, i.e. over pairs of the form $\{\{\text{R}_1, \ldots, \text{R}_n\}, \{\text{R}_1, \ldots, \text{R}_{n+1}\}\}$, if certain conditions are met. Contingent on this conjecture, we can make a substantial point: if there exists a determinate answer to the relative coherence of the old and the new information sets, then the more reliable the sources are and the larger the information set is, the more the question of belief expansion is determined by whether the new information set is or is not more coherent than the old information set, and not by the reliability of the sources.

The acceptance measure depends, at least to some extent, on the value of the likelihood ratio x. But what, one might ask, should we do when we have no clue whatsoever about the reliability of the sources, except that they are better than mere randomizers and yet less than fully reliable? Let us model our limited knowledge as a uniform distribution over the values p and q under the constraint that $p > q$. Then we can construct the following *averaged acceptance measure*:

$$\begin{aligned} E(\text{R}_1, \ldots, \text{R}_m) &= \int_0^1 \int_0^p e_{q/p}(\text{R}_1, \ldots, \text{R}_m) \, dq dp \\ &= \int_0^1 e_x(\text{R}_1, \ldots, \text{R}_m) \, dx \qquad (12) \end{aligned}$$

We can formulate a general criterion for belief acceptance: when we have limited knowledge about the reliability of our information sources, we can expand our belief set from $\{\text{R}_1, \ldots, \text{R}_n\}$ to $\{\text{R}_1, \ldots, \text{R}_{n+1}\}$ if and only if

$$E(\text{R}_1, \ldots, \text{R}_{n+1}) \geq E(\text{R}_1, \ldots, \text{R}_n). \qquad (13)$$

Bayesian Networks

Bayesian Networks represent (conditional) independences between variables and when implemented on a computer they perform complex probabilistic calculations at the touch of a keystroke. We are assuming here that the reader has some familiarity with Bayesian Networks (Cowell et. al. 1999, Jensen 1996, Neapolitan 1990, Pearl 1988).

We construct a Bayesian Network that permits us to read off the reliability-relative coherence measure of an information set $\{\text{R}_1, \ldots, \text{R}_n\}$ in Figure 1. First, we construct a Bayesian Network with nodes for the variables $R_1, \ldots, R_n$ which represents the marginal probability distribution over these variables. Then we add nodes for the variables $REPR_1, \ldots, REPR_n$ and draw in an arrow from each node for the variable R_i to the node for the variable $REPR_i$ and specify the conditional probabilities in (2) for each arrow. By the standard criterion of d-separation, we can now read off the conditional independences in (1) from the network. Subsequently, we construct a node for the variable $R_1 \& \ldots \& R_n$: we draw in the arrows and specify conditional probabilities such that $\text{R}_1 \& \ldots \& \text{R}_n$ holds if and only if $\text{R}_1, \ldots,$ and R_n hold. We can now read off $P^*(\text{R}_1, \ldots, \text{R}_n)$: it is the probability of $\text{R}_1 \& \ldots \& \text{R}_n$ after instantiating $\text{REPR}_1, \ldots, \text{REPR}_n$. To read off $P^{max*}(\text{R}_1, \ldots, \text{R}_n)$, more construction is needed. Notice that $P^{max}(\text{R}_i) = P^{max}(\text{R}_1 \& \ldots \& \text{R}_n)$ for $i = 1, \ldots, n$ in the counterfactual case of maximal coherence, is equal to $P(\text{R}_1 \& \ldots \& \text{R}_n)$ in the actual case where the information set may not be maximally coherent. Hence $P^{max}(\text{R}_1, \ldots, \text{R}_n)$ is the posterior joint probability of $\text{R}_1, \ldots, \text{R}_n$, had we been informed in the actual case by n less than fully reliable independent sources that $\text{R}_1 \& \ldots \& \text{R}_n$. So we add nodes for the variables $REP_i \& R$ (whose positive values states that
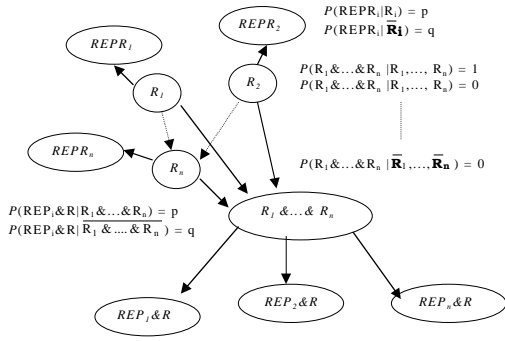
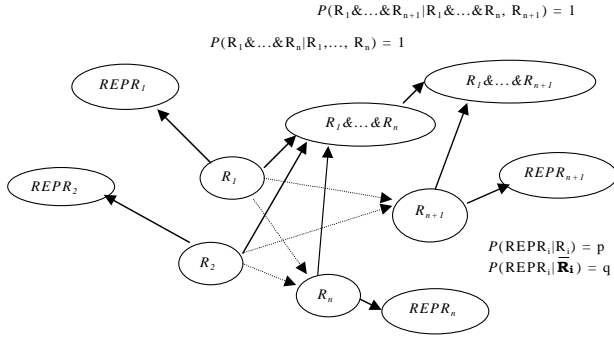Figure 1: Bayesian Network for coherence measure

Figure 2: Bayesian Network for belief expansion

the i-th source informs us that $R_1 \& \ldots \& R_n$), draw in the proper arrows and specify the proper conditional probabilities. We can now read off $P^{max*}(R_1, \ldots, R_n)$: it is the probability of $R_1 \& \ldots \& R_n$ after instantiating $REP_1 \& R, \ldots, REP_n \& R$. The measure $c_x(R_1 \& \ldots \& R_n)$ follows by (6).

We construct a Bayesian Network in Figure 2 to determine whether belief expansion is warranted or not. The construction of the nodes for the variables $R_1, \ldots, R_{n+1}$ and $REPR_1, \ldots, REPR_{n+1}$ should be clear from our construction of the Bayesian Network in Figure 1. This part of the Bayesian Network respects the conditional independences in (8). Now we add a node for the variable $R_1 \& \ldots \& R_n$ and a node for the variable $R_1 \& \ldots \& R_{n+1}$ and specify the conditional probabilities so that $R_1 \& \ldots \& R_n$ holds if and only if $R_1, \ldots,$ and R_n hold and $R_1 \& \ldots \& R_{n+1}$ holds if and only if $R_1, \ldots, R_n$ and R_{n+1} hold. We instantiate $REPR_1, \ldots, REPR_n$ and propagate the evidence throughout the network. We can now read off the acceptance measure $e_x(R_1, \ldots, R_n)$ which is the posterior probability of $R_1 \& \ldots \& R_n$. To raise the question of belief expansion, this value should be greater than or equal to our threshold value for belief. Subsequently, we instantiate $REPR_{n+1}$ and propagate the evidence

throughout the network. We can now read off the acceptance measure $e_x(R_1, \ldots, R_{n+1})$ which is the posterior probability of $R_1 \& \ldots \& R_{n+1}$. Depending on our treshold value for belief, we can determine whether we are justified to expand our beliefs with the proposition R_{n+1}.

It is easy to see how the idealizations can be relaxed in the networks. We can stipulate alternative reliability parameters for the sources. We can let one source report on two items of information. We can add arrows between the $REPR_i$ variables or between some $REPR_i$ and R_j variables (for $i \neq j$) to model certain types of dependence between the sources. It suffices that $P(R_1, \ldots, R_n)$ is equal to or greater than the threshold value for belief. Furthermore, even if $P(R_1, \ldots, R_{n+1})$ is below the threshold value for belief, the model yields a marginal probability distribution over $R_1, \ldots, R_n$. Hence, the general question of belief revision becomes a question of defining a function which maps joint probability distributions over a set of propositional variables into sets of propositions that are values of a subset of these variables and that can reasonably be believed. Defining such a function is beyond the scope of this paper.

Conclusion

(i) We have designed a procedure to determine a partial coherence ordering over a set of information sets of size n. If one information set is more coherent than another on this ordering, then our degree of confidence in the content of the former set will be greater than in the content of the latter set, after having been informed by independent and less than fully reliable sources, ceteris paribus. (ii) We have designed a probabilistic criterion for (non-prioritized) belief expansion, which determines whether it is rational to believe new information, considering how reliable the sources are and how well the new information coheres with the old information. (iii) If either the sources are sufficiently reliable or the information set sufficiently large, then the question of belief expansion is largely determined by whether the expanded information set is more coherent than the original information set (provided that there exists an ordering of this pair of information sets), and only marginally by the reliability of the sources. (iv) We have shown how a coherence ordering over information sets can be constructed by means of Bayesian Networks and how belief expansion can be modeled by means of Bayesian Networks in an empirically adequate manner.

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