Fast Wavelet-Based Visual Classification

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Abstract

We investigate a biologically motivated approach to fast visual classification, directly inspired by the recent work [20]. Specifically, trading-off biological accuracy for computational efficiency, we explore using wavelet and grouplet-like transforms to parallel the tuning of visual cortex V1 and V2 cells, alternated with max operations to achieve scale and translation invariance. A feature selection procedure is applied during learning to accelerate recognition. We introduce a simple attention-like feedback mechanism, significantly improving recognition and robustness in multiple-object scenes. In experiments, the proposed algorithm achieves or exceeds state-of-the-art success rate on object recognition, texture and satellite image classification, language identification and sound classification.

1 Introduction

Automatic object recognition and image classification are important and challenging tasks. This paper is inspired by the remarkable recent work of Poggio, Serre, and their colleagues [20], on rapid object categorization using a feedforward architecture closely modeled on the human visual system. The main directions it departs from that work are twofold. First, trading-off biological accuracy for computational efficiency, our results exploit more engineeringmotivated mathematical tools such as wavelet and grouplet transforms [13, 12], allowing faster computation and limiting ad-hoc parameters. Second, the approach is generalized by adding a degree of *feedback* (another known component of human perception), yielding significant performance and robustness improvement in multiple-object scenes. In experiments, the resulting scale- and translation-invariant algorithm achieves or exceeds state-of-the-art performance in object recognition, but also in texture and satellite image classification, and in language identification.

2 Algorithm description

2.1 Feature computation and classification

As in [20], the algorithm is hierarchical. In addition, motivated in part by the relative uniformity of cortical anatomy [14, 21], the two layers of the hierarchy are made to be computationally similar, as shown in Fig. 1. Layer one performs a wavelet transform [13] in the $\mathbf{S_1}$ unit followed by a local maximum operation in the $\mathbf{C_1}$ unit. The transform in the $\mathbf{S_2}$ unit in layer two is similar to the grouplet transform [12], and is followed by a global maximum operation in the $\mathbf{C_2}$ unit.

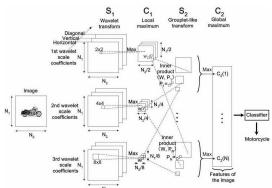


Figure 1. Algorithm overview.

 $\mathbf{S_1}$: Wavelet transform. The frequency and orientation tuning of cells in visual cortex V1 can be interpreted as performing a wavelet transform of the retinal image [13]. Let us denote f(x,y) a gray-level image of size $N_1 \times N_2$. A translation-invariant wavelet transform is performed on the image:

$$Wf(u, v, j, k) = \sum_{x=1}^{N_1} \sum_{y=1}^{N_2} f(x, y) \frac{1}{2^j} \psi^k \left(\frac{x - u, y - v}{2^j} \right),$$

where k=1,2,3 denotes the orientation (horizontal, vertical, diagonal), $\psi^k(x,y)$ is a wavelet function and Wf are the wavelet coefficients. Scale invariance is achieved by a

normalization

where
$$||f||_{\sup(\psi_j^k)}^2$$
 is the image energy within the support

of the wavelet $\psi^k\left(\frac{x-u,y-v}{2^j}\right)$. One can verify that

$$S_1(u, v, j, k) \sim S_1'(2^{\beta}u, 2^{\beta}v, \beta j, k)$$
 (3)

where S_1 and S'_1 are the coefficients of f(x,y) and of its 2^{β} -time zoomed version $f(x/2^{\beta}, y/2^{\beta})$. The normalization also makes the recognition invariant to global linear illumination change.

 C_1 : Local maximum Limited translation invariance is achieved at this stage by keeping the local maximum of S_1 coefficients in a subsampling procedure:

$$C_1(u, v, j, k) = \max_{u' \in [2^j(u-1)+1, 2^j u), v' \in [2^j(v-1)+1, 2^j v)} S_1(u', v', j, k),$$

the maximum being taken at each scale j and orientation kwithin a spatial neighborhood of size proportional to 2^{j} × 2^{j} . The resulting C_1 map at scale j and orientation k is thus of size $N_1/2^j \times N_2/2^j$.

 S_2 : Grouplet-like transform. Cells in visual cortex V2 and V4 have larger receptive fields comparing to those in V1 and are tuned to geometrically more complex stimuli such as contours and corners [19]. The geometrical grouplets recently proposed by Mallat [12] imitate this mechanism by grouping and re-transforming the wavelet coeffi-

The procedure in S_2 is similar to the grouplet transform. Instead of grouping the wavelet coefficients with a multiscale geometrically adaptive association field and then retransforming them with Haar-like functions as in [12], responses of S_2 are obtained via inner products between C_1 coefficients and sliding patch functions of different sizes:

$$S_2(u, v, j, i) = \sum_{u'=1}^{N_1/2^j} \sum_{v'=1}^{N_2/2^j} \sum_{k=1}^3 C_1(u', v', j, k) P_i(u'-u, v'-v, k),$$

where P_i of support size $M_i \times M_i \times 3$ are patch functions that group the 3 wavelet orientations in a square of size $M_i \times M_i$.

While the grouplet functions are adaptively chosen to fit the geometry in the image [12], the patch functions P_i , $i = 1, \dots, N$ are learned with a simple random sampling as in [20]: each patch is extracted at a random scale and a random position from the C_1 coefficients of a randomly selected training image, the rationale being that patterns that appear with high probability are likely to be learned.

 C_2 : Global maximum. A global maximum operation in space and in scale is applied on S_2 and the resulting C_2 coefficients

$$C_2(i) = \max_{u,v,j} S_2(u,v,j,i)$$
 (6)

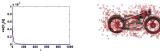
are thus invariant to image translation and scale change.

Classification The classification uses C_2 coefficients as features and thus inherits the translation and scale invariance. While various classifiers such as SVMs can be used, a simple but robust nearest neighbor classifier will be applied in the experiments.

Feature selection

Structures that appear with a high probability are likely to be learned as patch functions through random sampling. However, they are not necessarily salient and neither are the resulting C_2 features. This suggests active selection of the learned patches.¹ For example, Lowe and Mutch have constructed sparse patches by retaining one salient direction at each position [18].

A simple patch selection is proposed here by sorting the variances of the C_2 coefficients of the training images. A small $C_2(i)$ variance implies that the corresponding patch P_i is not salient. Fig. 2-a plots the variance of the C_2 coefficients of the motorcycle and the background images in the Caltech5 database (see Fig. 4), the S_2 patches being learned from the same images. Out of the 1000 patches, 200 salient ones whose resulting C_2 have non-negligible variance are selected. Other patches usually correspond to nonsalient structures such as a common background and are therefore excluded. Fig. 2-b and c show that after patch selection the 200 C_2 coefficients are mainly positioned around the object, as opposed to the 1000 C_2 coefficients spreading over all the image prior to patch selection. The recognition using these salient patches is not only more robust but also 5 times faster.





a Figure 2. a. Variance of the C_2 coefficients before patch selection. b and c. Positions of the C_2 coefficients before and after patch selection (marked by crosses).

2.3 Feedback

Feedback [19, 4, 22] allows tracing back object positions, focusing attention on the objects one by one and thus improving recognition performance in multiple-object

Object positioning For simplicity the feedback procedure is discussed in a two-object scene but can be applied in the case of multiple objects. C_2 coefficients are placed around the two objects after selection, as shown in Fig. 3-a. Using a clustering algorithm such as the K-means algorithm, one

¹Besides improving computational efficiency of the algorithm, such reorganization is inspired both by a similar process thought to occur after immediate learning, notably during sleep, and by the relative uniformity of cortical anatomy [14] which suggests enhancing computational similarity between the two layers.

is able to locate the two objects as illustrated in Fig. 3-b.

Object identification While one could recalculate the features of the attended object cropped out from the whole image, i.e., concentrate all the visual cortex resource on a single object, a faster procedure identifies the attended object, say object A, using directly the lower-dimensional feature vector C_{2A} , composed of the C_2 coefficients corresponding to A already calculated in the feedforward pathway. This can be implemented by reclassifying C_{2A} using subsets of the C_2 coefficients of the training images extracted at the same coordinates of C_{2A} , as shown in Fig. 3-c. Discarding the coordinates that are located on the irrelevant object B in the test image disambiguates the classification and improves the recognition of the object A.

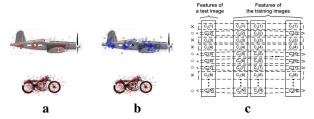


Figure 3. Feedback in a two-object scene. a. Positions of C_2 coefficients are marked by crosses. b. C_2 coefficients are clustered (represented by circles vs crosses). c. Feature coefficients of the *training* images are grouped, the coordinates being in line with the clustering of the coefficients of the *test* image. Rectangles and ellipses represent the two groups.

3 Experiments

All the experiment results were obtained with the same algorithm configuration. Daubechies 7-9 wavelets of 3 scales were used in $\mathbf{S_1}$. In $\mathbf{S_2}$ 1000 patches P_i of 4 different sizes $M \times M \times 3$ with M = 4, 8, 12, 16, 250 for each, were learned from the training images. The classifier was the simple nearest neighbor classification algorithm.

For texture and satellite image classification as well as for language identification, one sample image of size 512×512 was available per image class and was segmented to 16 non-overlapping parts of size 128×128 . Half were used for training and the rest for test.

3.1 Object recognition

For the object recognition experiments we used 4 data sets that are airplanes, motorcycles, cars (rear) and leaves, plus a background class from the Caltech5 database², some sample images being shown in Fig. 4. The images are turned to gray-level and rescaled in preserving the aspect ratio so that the minimum side length is of 140 pixels. A set of 50 positive images and 50 negative images were used for training and another set for test.

Table 1 summarizes the object recognition. The performance measure reported is the ROC accuracy.³ Results obtained with the proposed algorithm are superior to previous approaches [2, 24] and comparable to [20] but at a lower computational cost (in Matlab code about 6 times faster with feature selection). Fig. 5-d shows that the performance is improved when the number of C_2 features increases and is in general stable with 200 features.



Figure 4. Sample images from Caltech5. From left to right: airplanes, motorcycles, cars (rear), leaves and background.

Data sets	Proposed	Serre [20]	Others
Airplanes	96.0	96.7	94.0 [2]
Motorcycles	98.0	98.0	95.0 [2]
Cars (Rear)	96.0	99.8	84.8 [2]
Leaves	92.0	97.0	84.0 [24]

Table 1. Object recognition performance.

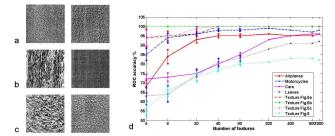


Figure 5. a-c. Pairs of textures. d. Performance vs numbers of C_2 features.

3.2 Texture classification

Figs. 5-a,b,c and Fig. 6 show respectively 3 pairs of textures that were used for binary classification and a group of 10 textures that were used for multiple-class (10-class) classification, all from the Brodatz database⁴. As summarized in Table 2, the proposed algorithm achieved perfect results for binary classification and for the challenging multiple class classification its performance was comparable to the state-of-the-art methods [8, 6, 17]. Indeed the random patch extraction applied in the algorithm is ideal for classifying stationary patterns such as textures. Fig. 5 shows that stable performance is achieved with as few as 40 features, which confirms the good texture classification results and the robustness of the algorithm.

²http://www.robots.ox.ac.uk/~vgg/data3.html

³ROC accuracy: R = 1 - ((1 - p)x + p(1 - y)), where x and y are respectively the false positive rate on the negative samples and true positive rate on the positive samples, p is the proportion of the positive samples.

⁴Fig. 5-a,b,c: D4 and D84; D12 and D17; D5 and D92. Fig. 6 : D4, D9, D19, D21, D24, D28, D29, D36, D37, D38.

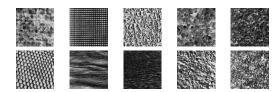


Figure 6. A group of 10 textures.

Data sets	Proposed	Avg. [17]	Best [17]	Others
Fig. 5-a	100	88.0	99.3	91.5 [6]
Fig. 5-b	100	96.2	99.7	N/A
Fig. 5-c	100	87.6	97.5	88.6 [6]
Fig. 6	82.6	52.6	67.7	83.1 [8]

Table 2. Texture classification performance. From left to right: proposed method, average and best performance of the algorithms summarized in [17], other methods. N/A means that the results were not shown.

Classifying the whole Brodatz database (111 textures) is a more challenging task. Combining C_2 coefficients with the histogram of the wavelet approximation coefficients as features, the proposed algorithm achieved 87.8% accuracy for the 111-texture classification, comparable to the 88.2% accuracy rate reported in [7] obtained with a state-of-the-art texture classification approach.

3.3 Satellite image classification

Fig. 7 displays 4 classes of satellite images at $0.5\ m$ resolution: urban areas, rural areas, forests and sea. Since access to images at other resolutions is restricted, we simulated the images at resolutions $1\ m$ and $2\ m$ by Gaussian convolution and sub-sampling.

The first experiment tested the multi-class classification of mono-resolution images shown in Fig. 7. 100% classification accuracy was achieved for images of all the 4 classes. The second experiment validated the scale invariance of the proposed algorithm. Images at resolution 0.5 m were used to train the classifier while the classification was tested on images at resolutions 1 m and 2 m. Again the classification accuracy was 100%, same as reported in a recent work [11] and significantly higher than earlier methods [5] referenced therein. In addition, image resolution is assumed to be known in [11], whereas the proposed algorithm does not need this information, thanks to its scale invariance.









Figure 7. Satellite images. From left to right: forest, urban areas, rural areas, sea.

3.4 Language identification

Language identification aims to determine the underlying language of a document in an imaged format, and is often carried out as a preprocessing of optical character recognition (OCR). Based on principles totally different from traditional approaches [10], the proposed algorithm achieved 100% success rate in a 8-language identification task, as shown in Fig 8.

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Figure 8. From top to bottom, left to right: document texts in Arabic, Chinese, English, Greek, Hebrew, Japanese, Korean, Russian.

3.5 Sound Classification

The main idea is to directly extend the above algorithm to sound applications is to view time-frequency representations of sound as textures. Preliminary experiments suggest this may be a fruitful direction of research.

Fig. 9 illustrates 5 types of sounds and samples of their log-spectrograms. 2 minutes excerpts of each sound were collected. The spectrograms were segmented (in time) into segments of 5 seconds. Half were used for training and the rest for test. A direct application of the proposed algorithm using the spectrograms as the visual patterns resulted in 100% accuracy in the 5-sound classification.

3.6 Feedback: multiple-object scenes

Recognition performance tends to degrade when multiple stimuli are presented in the receptive field. Fig. 10-a shows an example of a multiple-object scene in which one searched an object, say an airplane, through a binary classification against a background image. Due to the perturbation from the coexisting stimuli, the feedforward recognition accuracy is as low as 74%. The feedback procedure introduced in Subsection 2.3 improves considerably the accuracy to 98% by focusing attention on each object in turn.

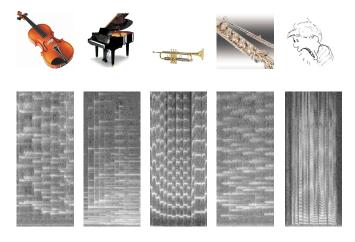


Figure 9. From left to right: violin, piano, trumpet, flute, speech. The figures in the second row are spectrograms of the sounds illustrated in the first row.



Figure 10. a. A 4-object scene. b. C_2 coefficients clustering.

4 Conclusion and future work

Inspired by the biologically motivated work of [20], we have described a wavelet-based algorithm which can compete with the state-of-the-art methods for fast and robust object recognition, texture and satellite image classification, language recognition and sound classification. A feedback procedure has been introduced to improve recognition performance in multiple-object scenes.

Potential applications also include video archiving (semantic video analysis), video surveillance, high-throughput drug development, texture retrieval, and robotic learning by imitation.

To further improve and extend the algorithm, a key aspect will be a more refined use of feedback between different levels. Such feedback will naturally involve stability and convergence questions, which will in turn both guide the design of the algorithm and shape its performance. In addition, contrary to the nervous system, the algorithm need not be constrained by information transmission delays between different levels. Preliminary ideas in this direction are briefly discussed in the appendix.

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on invariant image recognition.

APPENDIX

A Dynamic System Perspective

A.0.1 Basic algorithm

The first step towards introducing a dynamic systems perspective aimed at further development of feedback mechanisms is simply to rewrite the algorithm in terms of differential equations, which puts it in a form more suitable to subsequent analysis of stability and convergence.

Let \mathbf{x}_1 be the output of the S_1/C_1 layer, and \mathbf{x}_2 be the output of the S_2/C_2 layer (in our static implementation above, we simply have $\mathbf{x}_1=C_1$ and $\mathbf{x}_2=C_2$).

For a single object, the basic algorithm can be trivially computed by a dynamic system of the form

$$\dot{\mathbf{x}}_1 = -k_1(\mathbf{x}_1 - C_1)$$
$$\dot{\mathbf{x}}_2 = -k_2(\mathbf{x}_2 - C_2)$$

For multiple objects, the clustering process described in section 2.3 can be implemented by introducing a scalar state x_3 , which spikes for each object in sequence

$$\dot{x}_3 = \rho(x_3, C_2)$$

with spike amplitude equal to 1 — the function $\rho(x_3,C_2)$ is discussed later in this section and in a companion paper. The dynamics of \mathbf{x}_2 can be modified in turn so that states corresponding to each object appear in sequence according to the state x_3

$$\dot{\mathbf{x}}_1 = -k_1(\mathbf{x}_1 - C_1)
\dot{\mathbf{x}}_2 = -k_2(\mathbf{x}_2 - x_3 \, \mathbf{k}(C_{2A}))
\dot{x}_3 = \rho(x_3, C_2)$$

where, componentwise, $\mathbf{k}(C_{2A})=C_{2A}$ where C_{2A} is active and and $\mathbf{k}(C_{2A})=0$ otherwise. Note that x_3 smoothly transitions between 0 and 1 according to the attended object. The positive gain k_2 is chosen such that $k_2T\gg 1$, where T is the spike duration, itself a fraction of the interspike period.

The above equations simply implement the basic algorithm and display objects in sequence, without introducing any new feature at this point.

Techniques for globally stable spike-based clustering are described in a companion paper, based on modified FitzHugh-Nagumo neural oscillators [3, 15], similar to [1],

$$\dot{v}_i = 3v_i - v_i^3 - v_i^7 + 2 - w_i + I_i
\dot{w}_i = c[\alpha(1 + \tanh(\beta v_i)) - w_i]$$
(7)

where v_i is the membrane potential of the oscillator, w_i is an internal state variable representing gate voltage, I_i represents the external current input, and α , β and c are strictly positive constants. Using a diagonal metric transformation $\Theta = \mathrm{diag}(\sqrt{c\alpha\beta},1)$, one easily shows, similarly to [23], that

$$\Theta J \Theta^{-1} < \operatorname{diag}(3 + \frac{\alpha \beta}{4}, 0)$$

where J is the Jacobian matrix of (7), leading to simple global stability conditions based on [16] (section 2.2).

A.1 Generalized diffusive connections

One of the most immediate additional feedback mechanisms to be explored is that of generalized diffusive connections ([16], section 3.1.2). In a feedback hierarchy, these correspond to achieving consensus between multiple processes of different dimensions.

A.2 Tracking of time-varying images

Similarly to [9], composite variables for dynamic tracking can be used at every level, based on both top-down an bottom-up information. This allows one to implicitly introduce time-derivatives of signals in the differential equations, without having to measure or compute these terms explicitly.

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