An Extended Generalized Disjunctive Paraconsistent Data Model for Disjunctive Information

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Abstract. This paper presents an extension of generalized disjunctive paraconsistent relational data model in which pure disjunctive positive and negative information as well as mixed disjunctive positive and negative information can be represented explicitly and manipulated. We consider explicit mixed disjunctive information in the context of disjunctive databases as there is an interesting interplay between these two types of information. *Extended generalized disjunctive paraconsistent relation* is introduced as the main structure in this model. The relational algebra is appropriately generalized to work on extended generalized disjunctive paraconsistent relations and their correctness is established.

1 Introduction

Relational data model was proposed by Ted Codd's pioneering paper [1]. Relational data model is value-oriented and has rich set of simple, but powerful algebraic operators. Moreover, a strong theoretical foundation for the model is provided by the classical first-order logic [2]. This combination of a respectable theoretical platform, ease of implementation and the practicality of the model resulted in its immediate success, and the model has enjoyed being used by many database management systems.

One limitation of the relational data model, however, is its lack of applicability to nonclassical situations. These are situations involving incomplete or even inconsistent information.

Several types of incomplete information have been extensively studied in the past such as *null* values [3,4,5], *partial* values [6], *fuzzy* and *uncertain* values [7,8], and *disjunctive* information [9,10,11].

In this paper, we present an extension of generalized disjunctive paraconsistent data model[12]. Our model is capable of representing and manipulating pure disjunctive positive facts and disjunctive negative facts as well as mixed disjunctive positive and negative facts. We introduce *extended generalized disjunctive paraconsistent relations*, which are the fundamental structures underlying our model. These structures are extension of *generalized disjunctive paraconsistent* *relations* which are capable of representing pure disjunctive positive and negative facts. The motivation of this paper is that in the context of disjunctive database, we should consider not only the pure disjunctive information but also the mixed disjunctive information. The data model represented in this paper can be applied to calculate the fixed point semantics of extended disjunctive logic programs [13] algebraically. An extended generalized disjunctive paraconsistent relation essentially consists of three kinds of information: positive tuple sets representing exclusive disjunctive positive facts (one of which belongs to the relation), negative tuple sets representing exclusive disjunctive negated facts (one of which does not belong to the relation) and mixed tuple sets representing exclusive disjunctive facts (one of which belongs to the relation or one of which does not belong to the relation). Extended generalized disjunctive paraconsistent relations are strictly more general than generalized disjunctive paraconsistent relations in that for any generalized disjunctive paraconsistent relation, there is an extended generalized disjunctive paraconsistent relation with the same information content, but not *vice versa*. We extend the representation provided in [12] by introducing mixed disjunctive facts. There is an interesting interplay among these three kinds of information. After introducing extended generalized disjunctive paraconsistent relations, we present operators to remove redundancies and inconsistencies. We also extend the standard relational algebra to operate on extended generalized disjunctive paraconsistent relations. The information content of extended generalized disjunctive paraconsistent relations is characterized in terms of generalized disjucntive paraconsistent relations which we briefly present in the next section.

The rest of the paper is organized as follows. Section 2 provides the brief overview of generalized disjunctive paraconsistent relations and the associated algebraic operators. Section 3 introduces extended generalized disjunctive paraconsistent relations as structures that are capable of representing mixed disjunctive facts. Section 4 presents the notion of *precise generalization* of algebraic operators and defines precise generalizations of several useful algebraic operators. These operators can be used for specifying queries for database systems built on such extended generalized disjunctive paraconsistent relations. Finally, Section 5 contains some concluding remarks and directions for future work.

2 Generalized Disjunctive Paraconsistent Relations

In this section, we present a brief overview of definition of generalized disjunctive paraconsistent relations. For a more detailed description, refer to [12].

Let a *relation scheme* (or just *scheme*) Σ be a finite set of *attribute names*, where for any attribute name $A \in \Sigma$, $dom(A)$ is a non-empty *domain* of values for A. A *tuple* on Σ is any map $t : \Sigma \to \cup_{A \in \Sigma} dom(A)$, such that $t(A) \in dom(A)$, for each $A \in \Sigma$. Let $\tau(\Sigma)$ denote the set of all tuples on Σ.

Definition 1. A *paraconsistent relation* on scheme Σ is a pair $R = \langle R^+, R^- \rangle$, where R^+ and R^- are any subsets of $\tau(\Sigma)$. We let $\mathcal{P}(\Sigma)$ be the set of all paraconsistent relations on Σ. $\square$

Definition 2. A *disjunctive paraconsistent relation, R,* over the scheme Σ consists of two components $< R^+, R^- >$ where $R^+ \subseteq 2^{\tau(\Sigma)}$ and $R^- \subseteq \tau(\Sigma)$. R^+, the *positive* component, is a set of tuple sets. Each tuple set in this component represents a disjunctive positive fact. In the case where the tuple set is a singleton, we have a definite positive fact. R^-, the *negative* component consists of tuples that we refer to as definite negative tuples. Let $\mathcal{D}(\Sigma)$ represent all disjunctive paraconsistent relations over the scheme Σ. □

Definition 3. A *generalized disjunctive paraconsistent relation, R,* over the sc heme Σ consists of two components $\langle R^+, R^- \rangle$ where $R^+ \subseteq 2^{\tau(\Sigma)}$ and $R^- \subset 2^{\tau(\Sigma)}$. R^+, the *positive* component, is a set of tuple sets. Each tuple set in t his component represens a disjunctive positive fact. In the case where the tuple set is a singleton, we have a definite positive fact. R^-, the *negative* component consists of a set of tuple sets. Each tuple set in this componen t represents a disjunctive negative fact. In the case where the tuple set is a singleton, we have a definite negated fact. Let $\mathcal{GD}(\Sigma)$ rep resent all generalized disjunctive paraconsistent relatios over the scheme Σ. □

A generalized disjunctive paraconsistent relation is called *normalized* if it does not contain any inconsistencies. We let $\mathcal{GN}(\Sigma)$ denote the set of all normalized generalized disjunctive paraconsistent relations over scheme Σ.

3 Extended Generalized Disjunctive Paraconsistent Relations

In this section, we present the main structure underling our model, the *extended generalized disjunctive paraconsistent relations*. We identify several types of redundancies and inconsistencies that may appear and provide operators to remove them. Finally, we present the information content of extended generalized paraconsistent relations.

Definition 4. An*extended generalized disjunctive paraconsistent relation, R,* over the scheme Σ consists of three components $\langle R^+, R_M, R^- \rangle$ where $R^+ \subseteq 2^{\tau(\Sigma)}$, each element r_M of R_M consists of two parts $r_M^+ \in 2^{\tau(\Sigma)}$ and $r_M^- \in 2^{\tau(\Sigma)}$, and $R^- \subset 2^{\tau(\Sigma)}$. R^+, the *positive* component, is a set of tuple sets. Each tuple set in this component represens a disjunctive positive fact. In the case where the tuple set is a singleton, we have a definite positive fact. R_M, the *mixed* component, is a set of pair tuple sets. The first tuple set represents a disjunctive positive facts. The second tuple set represents a disjunctive negated facts. And the relationship between these two tuple sets is disjunctive. R^-, the *negative* component consists of a set of tuple sets. Each tuple set in this component represents a disjunctive negative fact. In the case where the tuple set is a singleton, we have a definite negated fact. Let $\mathcal{EGD}(\Sigma)$ represent all extended generalized disjunctive paraconsistent relations over the scheme Σ. □

Inconsistences can be present in an extended genearlaized disjunctive paraconsistent relation in three situations. First, if all the tuples of a tuple set of the posistive component are also present in the union of the singleton tuple set of the negative component. We deal with this inconsistency by removing both the positive tuple set and all its corresponding singleton tuple sets from the negative component. Second, if all the tuples of a tuple set of the negative component are also present in the union of the singleton tuple set of the positive component. We deal with this inconsistency by removing both the negative tuple set and all its corresponding singleton tuple sets from the positive component. Third, if all the tuples of the first tuple set of the pair tuple sets of mixed component are also present in the union of the singleton tuple set of the negative component and all the tuples of the second tuple set of the pair tuple sets of mixed component are also present in the union of the singleton tuple set of the positive component. We deal with this inconsistency by removing the pair tuple sets and its corresponding singleton tuple sets from the negative component and its corresponding singleton tuple sets from the positive component. This is done by the **eg_norm** operator defined as follows:

Definition 5. Let R be an extended generalized disjunctive paraconsistent relation over Σ. $R^+ = \{w_1, w_2, \cdots, w_n\}$, $R_M = \{< v_1, x_1 >, \cdots, < v_k, x_k >\}$ and $R^- = \{u_1, u_2, \cdots, u_m\}$. Then,
$\mathbf{eg\_norm}(R)^+ = R^+ -$
$\{w | w \in R^+ \wedge w \subseteq \cup u_i \wedge 1 \leq i \leq m \rightarrow u_i \in R^- \wedge |u_i| = 1\} -$
$\{w_i | 1 \leq i \leq n \rightarrow w_i \in R^+ \wedge |w_i| = 1 \wedge (\exists u)(u \in R^- \wedge u \subseteq \cup w_i \wedge w_i \subseteq u)\} -$
$\{w_i | 1 \leq i \leq n \rightarrow w_i \in R^+ \wedge |w_i| = 1 \wedge (\exists \langle v, x\rangle)(\langle v, x\rangle \in R_M \wedge x \subseteq \cup w_i \wedge w_i \subseteq x \wedge v \subseteq \cup u_j \wedge u_j \in R^- \wedge |u_j| = 1)\}$
$\mathbf{eg\_norm}(R)_M = R_M -$
$\{\langle v, x\rangle | \langle v, x\rangle \in R_M \wedge v \subseteq \cup u_i \wedge 1 \leq i \leq m \rightarrow u_i \in R^- \wedge |u_i| = 1 \wedge x \subseteq \cup w_j \wedge 1 \leq j \leq n \rightarrow w_j \in R^+ \wedge |w_j| = 1\}$
$\mathbf{eg\_norm}(R)^- = R^- -$
$\{u | u \in R^- \wedge u \subseteq \cup w_i \wedge 1 \leq i \leq n \rightarrow w_i \in R^+ \wedge |w_i| = 1\} -$
$\{u_i | 1 \leq i \leq m \rightarrow u_i \in R^- \wedge |u_i| = 1 \wedge (\exists w)(w \in R^+ \wedge w \subseteq \cup u_i \wedge u_i \subseteq w)\} -$
$\{u_i | 1 \leq i \leq n \rightarrow u_i \in R^- \wedge |u_i| = 1 \wedge (\exists \langle v, x\rangle)(\langle v, x\rangle \in R_M \wedge v \subseteq \cup u_i \wedge u_i \subseteq v \wedge x \subseteq \cup w_j \wedge w_j \in R^+ \wedge |w_j| = 1)\}$

□

An extended generalized disjunctive paraconsistent relation is called *normalized* if it does not contain any inconsistencies. We let $\mathcal{EGN}(\Sigma)$ denote the set of all normalized extended generalized disjunctive paraconsistent relations over scheme Σ. We now identify the following eight types of redundancies in a normalized extended generalized disjunctive paraconsistent relation R:

1. $w_1 \in R^+$, $w_2 \in R^+$, and $w_1 \subset w_2$. In this case, w_1 subsumes w_2. To eliminate this redundancy, we delete w_2 from R^+.
2. $u_1 \in R^-$, $u_2 \in R^-$, and $u_1 \subset u_2$. In this case, u_1 subsumes u_2. To eliminate this redundancy, we delete u_2 from R^-.
3. $1 \leq i \leq n$, $w_i \in R^+$, $|w_i| = 1$, $u \in R^-$, and $\cup w_i \subset u$. This redundancy is eliminated by deleting the tuple set u from R^- and adding the tuple set $u - \cup w_i$

to R^-. Since we are dealing with normalized generalized disjunctive paraconsistent relations, $u - \cup w_i$ cannot be empty.

4. $1 \le i \le m$, $u_i \in R^-$, $|u_i| = 1$, $w \in R^+$, and $\cup u_i \subset w$. This redundancy is eliminated by deleting the tuple set w from R^+ and adding the tuple set $w - \cup u_i$ to R^+. Since we are dealing with normalized generalized disjunctive paraconsistent relations, $w - \cup u_i$ cannot be empty.
5. $\langle v_1, x_1\rangle \in R_M$, $\langle v_2, x_2\rangle \in R_M$, $v_1 \subset v_2$ and $x_1 \subset x_2$. In this case, $\langle v_1, x_1\rangle$ subsumes $\langle v_2, x_2\rangle$. To eliminate this redundancy, we delete $\langle v_2, x_2\rangle$ from R_M.
6. $\langle v, x\rangle \in R_M$, $w \in R^+$, and $w \subseteq v$. In this case, w subsumes $v \vee x$. To eliminate this redundancy, we delete $\langle v, x\rangle$ from R_M.
7. $\langle v, x\rangle \in R_M$, $u \in R^-$, and $u \subseteq x$. In this case, u subsumes $v \vee x$. To eliminate this redundancy, we delete $\langle v, x\rangle$ from R_M.
8. $1 \le i \le n$, $w_i \in R^+$, $|w_i| = 1$, $1 \le j \le m$, $u_j \in R^-$, $|u_j| = 1$, $\langle v, x\rangle \in R_M$, $\cup u_j \cap v \neq \emptyset$ or $\cup w_i \cap x \neq \emptyset$. This redundancy is eliminated by deleteing the pair tuple sets $\langle v, x\rangle$ from R_M. And addes the pair tuple sets $\langle v - \cup u_j, x - \cup w_i\rangle$ to R_M if $v - \cup u_j$ and $x - \cup w_i$ are not empty. If $v - \cup u_j$ is empty then addes the tuple set $x - \cup w_i$ to R^-. If $x - \cup w_i$ is empty then adds the tuple set $v - \cup u_j$ to R^+. Since we are dealing with normalized generalized disjunctive paraconsistent relations, $x - \cup w_i$ and $v - \cup u_j$ cannot be both empty.

We now introduce an operator called **eg_reduce** to take care of redundancies.

Definition 6. Let R be a normalized extended generalized disjunctive paraconsistent relation. Then,
$\textbf{eg\_reduce}(R)^+ = \{w' | (\exists w)(w \in R^+ \wedge w' = w - U \wedge \neg(\exists w_1)(w_1 \in R^+ \wedge (w_1 - U) \subset w'))\} \cup \{w' | (\exists \langle v, x\rangle)(\langle v, x\rangle \in R_M \wedge w' = v - U \wedge x - W = \emptyset)\}$
$\textbf{eg\_reduce}(R)_M = \{\langle v', x'\rangle | (\exists \langle v, x\rangle)(\langle v, x\rangle \in R_M \wedge \neg(\exists w)(w \in R^+ \wedge w \subseteq v) \wedge \neg(\exists u)(u \in R^- \wedge u \subseteq x) \wedge v' = v - U \wedge x' = x - W \wedge v - U \neq \emptyset \wedge x - W \neq \emptyset \wedge \neg(\exists \langle v_1, x_1\rangle)(\langle v_1, x_1\rangle \in R_M \wedge (v_1 - U) \subset v' \wedge (x_1 - W) \subset x')\}$
$\textbf{eg\_reduce}(R)^- = \{u' | (\exists u)(u \in R^- \wedge u' = u - W \wedge \neg(\exists u_1)(u_1 \in R^- \wedge (u_1 - W) \subset u'))\} \cup \{u' | (\exists \langle v, x\rangle)(\langle v, x\rangle \in R_M \wedge u' = x - W \wedge v - U = \emptyset)\}$
where, $U = \{u_i | u_i \in R^- \wedge |u_i| = 1\}$ and $W = \{w_i | w_i \in R^+ \wedge |w_i| = 1\}$. □

The information content of an extended generalized disjunctive paraconsistent relation can be defined to be a collection of generalized disjunctive paraconsistent relations. The different possible generalized disjunctive paraconsistent relations are constructed by selecting one of the tuple sets within a pair tuple sets for each pair tuple sets in the mixed component. In doing so, we may end up with non-minimal generalized disjunctive paraconsistent relations or even with inconsistent generalized disjunctive paraconsistent relations. These would have to be removed in order to obtain the exact information content of extended generalized disjunctive paraconsistent relations. The formal definitions follow:

Definition 7. Let $U \subseteq \mathcal{GD}(\Sigma)$. Then, $\textbf{eg\_normrep}_{\Sigma}(U) = \{R | R \in U \wedge \neg(\exists w)(w \in R^+ \wedge w \subseteq \cup u_i \wedge u_i \in R^- \wedge |u_i| = 1) \wedge \neg(\exists u)(u \in R^- \wedge u \subseteq \cup w_i \wedge w_i \in R^+ \wedge |w_i| = 1)\}$ □

The **eg_normrep** operator removes all inconsistent generalized disjunctive paraconsistent relations from its input.

Definition 8. Let $U \subseteq \mathcal{GD}(\Sigma)$. Then, $\mathbf{eg\_reducerep}_\Sigma(U) = \{R | R \in U \wedge \neg(\exists S)(S \in U \wedge R \neq S \wedge S^+ \subseteq R^+ \wedge S^- \subseteq R^-)\}$ □

The **eg_reducerep** operator keeps only the "minimal" generalized disjunctive paraconsistent relations and eliminates any generalized disjunctive paraconsistent relation that is "subsumed" by others.

Definition 9. The information content of extended generalized disjunctive paraconsistent relations is defined by the mapping $\mathbf{eg\_rep}_\Sigma : \mathcal{EGN}(\Sigma) \to \mathcal{GD}(\Sigma)$. Let R be a normalized extended generalized disjunctive paraconsistent relation on scheme Σ *with* $R_M = \{\langle v_1, x_1\rangle \ldots, \langle v_k, x_k\rangle\}$. Let $U = \{R^+ \cup V, R^- \cup X | V = \{v_i | 1 \le i \le k\} \wedge X = \{x_j | 1 \le j \le k\} \wedge i \neq j \wedge |V| + |X| = k\}$. Then, $\mathbf{eg\_rep}_\Sigma(R) = \mathbf{eg\_reducerep}_\Sigma(\mathbf{eg\_normrep}_\Sigma(U))$ □

Note that the information content is defined only for normalized extended generalized disjunctive paraconsistent relations.

The following important theorem states that information is neither lost nor gained by removing the redundancies in an extended generalized disjunctive paraconsistent relations.

Theorem 1. *Let R be an extended generalized disjunctive paraconsistent relation on scheme Σ. Then,*
$\mathbf{eg\_rep}_\Sigma(\mathbf{eg\_reduce}(R)) = \mathbf{eg\_rep}_\Sigma(R)$ □

4 Generalized Relational Algebra

In this section, we first develop the notion of *precise generalizations* of algebraic operators. This is an important property that must be satisfied by any new operator defined for extended generalized disjunctive paraconsistent relations. Then, we present several algebraic operators on extended generalized disjunctive paraconsistent relations that are precise generalizations of their counterparts on generalized disjunctive paraconsistent relations.

An n-ary *operator on generalized disjunctive paraconsistent relations with signature*
$\langle \Sigma_1, \ldots, \Sigma_{n+1}\rangle$ is a function $\Theta : \mathcal{GD}(\Sigma_1) \times \cdots \times \mathcal{GD}(\Sigma_n) \to \mathcal{GD}(\Sigma_{n+1})$, where $\Sigma_1, \ldots, \Sigma_{n+1}$ are any schemes. Similarly, an n-ary *operator on extended generalized disjunctive paraconsistent relations with signature* $\langle \Sigma_1, \ldots, \Sigma_{n+1}\rangle$ is a function: $\Psi : \mathcal{EGD}(\Sigma_1) \times \cdots \times \mathcal{EGD}(\Sigma_n) \to \mathcal{EGD}(\Sigma_{n+1})$.

We now need to extend operators on generalized disjunctive paraconsistent relations to sets of generalized disjunctive paraconsistent relations. For any operator $\Theta : \mathcal{GD}(\Sigma_1) \times \cdots \times \mathcal{GD}(\Sigma_n) \to \mathcal{GD}(\Sigma_{n+1})$ on generalized disjunctive paraconsistent relations, we let $\mathcal{S}(\Theta) : 2^{\mathcal{GD}(\Sigma_1)} \times \cdots \times 2^{\mathcal{GD}(\Sigma_n)} \to 2^{\mathcal{GD}(\Sigma_{n+1})}$ be a map on sets of generalized disjunctive paraconsistent relations defined as follows. For any sets $M_1, \ldots, M_n$ of generalized disjunctive paraconsistent relations on schemes $\Sigma_1, \ldots, \Sigma_n$, respectively,

$$\mathcal{S}(\Theta)(M_1, \ldots, M_n) = \{\Theta(R_1, \ldots, R_n) | R_i \in M_i, \text{for all } i, 1 \le i \le n\}.$$

In other words, $\mathcal{S}(\Theta)(M_1, \ldots, M_n)$ is the set of Θ-images of all tuples in the cartesian product $M_1 \times \cdots \times M_n$. We are now ready to lead up to the notion of precise operator generalization.

Definition 10. An operator Ψ on extended generalized disjunctive paraconsistent relations with signature $\langle \Sigma_1, \ldots, \Sigma_{n+1} \rangle$ is *consistency preserving* if for any normalized extended generalized disjunctive relations $R_1, \ldots, R_n$ on schemes $\Sigma_1, \ldots, \Sigma_n$, respectively, $\Psi(R_1, \ldots, R_n)$ is also normalized. $\Box$

Definition 11. A consistency preserving operator Ψ on extended generalized disjunctive paraconsistent relations with signature $\langle \Sigma_1, \ldots, \Sigma_{n+1} \rangle$ is a *precise generalization* of an operator Θ on generalized disjunctive paraconsistent relations with the same signature, if for any normalized extended generalized disjunctive paraconsistent relations $R_1, \ldots, R_n$ on schemes $\Sigma_1, \ldots, \Sigma_n$, we have
$\mathbf{eg\_rep}_{\Sigma_{n+1}}(\Psi(R_1, \ldots, R_n)) = \mathcal{S}(\Theta)(\mathbf{eg\_rep}_{\Sigma_1}(R_1), \ldots, \mathbf{g\_rep}_{\Sigma_n}(R_n))$.
$\Box$

We now present precise generalizations for the usual relation operators, such as union, join, projection. To reflect generalization, a dot is placed over an ordinary operator. For example, $\bowtie$ denotes the natural join among ordinary relations, $\dot{\bowtie}$ denotes natural join on generalized disjunctive paraconsistent relations and $\ddot{\bowtie}$ denotes natural join on extended generalized disjunctive paraconsistent relations.

Definition 12. Let R and S be two normalized extended generalized disjunctive paraconsistent relations on scheme Σ with $R_M = \{\langle p_1, n_1 \rangle, \ldots, \langle p_k, n_k \rangle\}$ and $S_M = \{\langle u_1, v_1 \rangle, \ldots, \langle u_m, v_m \rangle\}$. Then, *$R \dot{\cup} S$ is an extended generalized disjunctive paraconsistent relation over scheme Σ given by* $R \dot{\cup} S = \mathbf{eg\_reduce}(T)$, where T is defined as follows. Let $E = \{\langle \mathbf{eg\_reduce}(R)^+ \cup P, \mathbf{eg\_reduce}(R)^- \cup N \rangle | P = \{p_i | (\forall i) \langle p_i, n_i \rangle \in \mathbf{eg\_reduce}(R)_M\} \wedge N = \{n_j | (\forall j) \langle p_j, n_j \rangle \in \mathbf{eg\_reduce}(R)_M\} \wedge i \ne j \wedge |P| + |N| = |\mathbf{eg\_reduce}(R)_M|\}$ and
$F = \{\langle \mathbf{eg\_reduce}(S)^+ \cup U, \mathbf{eg\_reduce}(S)^- \cup V \rangle | U = \{u_i | (\forall i) \langle u_i, v_i \rangle \in \mathbf{eg\_reduce}(S)_M\} \wedge V = \{v_j | (\forall j) \langle u_j, v_j \rangle \in \mathbf{eg\_reduce}(S)_M \wedge i \ne j \wedge |U| + |V| = |\mathbf{eg\_reduce}(S)_M|\}$. Let the normalized elements of E be $E_1, \ldots, E_e$ and those of F be $F_1, \ldots, F_f$ and let $A_{ij} = E_i \overline{\cup} F_j$, for $1 \le i \le e$ and $1 \le j \le f$. Let $A_1, \ldots, A_g$ be the distinct $A_{ij}s$. Then,
$T^+ = \{w | (\exists t_1) \cdots (\exists t_g)(t_1 \in A_1^+ \wedge \cdots \wedge t_g \in A_g^+ \wedge w = \{t_1, \ldots, t_g\})\}$
$T_M = \{\langle P, N \rangle | P = \{p_i | (\forall i) p_i \in A_i^+\} \wedge N = \{n_j | (\forall j) n_j \in A_j^-\} \wedge i \ne j \wedge |p| \ne 0 \wedge |N| \ne 0 \wedge |P| + |N| = g\}$
$T^- = \{u | (\exists t_1) \cdots (\exists t_g)(t_1 \in A_1^- \wedge \cdots \wedge t_g \in A_g^- \wedge u = \{t_1, \ldots, t_g\})\}$.
and $R \dot{\cap} S$ *is an extended generalized disjunctive paraconsistent relation over scheme Σ given by* $R \dot{\cap} S = \mathbf{eg\_reduce}(T)$, where T is defined as follows. Let

$B_{ij} = E_i \overline{\cap} F_j$, for $1 \leq i \leq e$ and $1 \leq j \leq f$. Let $B_1, \ldots, B_g$ be the distinct $B_{ij}s$ Then,
$T^+ = \{w | (\exists t_1) \cdots (\exists t_g)(t_1 \in B_1^+ \wedge \cdots \wedge t_g \in B_g^+ \wedge w = \{t_1, \ldots, t_g\})\}$
$T_M = \{\langle P, N\rangle | P = \{p_i | (\forall i) p_i \in B_i^+\} \wedge N = \{n_j | (\forall j) n_j \in B_j^-\} \wedge i \neq j \wedge |p| \neq 0 \wedge |N| \neq 0 \wedge |P| + |N| = g\}$
$T^- = \{u | (\exists t_1) \cdots (\exists t_g)(t_1 \in B_1^- \wedge \cdots \wedge t_g \in B_g^- \wedge u = \{t_1, \ldots, t_g\})\}$. □

The following theorem establishes the *precise generalization* property for union and intersection:

Theorem 2. *Let R and S be two normalized extended generalized disjunctive paraconsistent relations on scheme Σ. Then,*

1. $\mathbf{eg\_rep}_\Sigma(R \dot{\cup} S) = \mathbf{eg\_rep}_\Sigma(R) \mathcal{S}(\overline{\cup}) \mathbf{eg\_rep}_\Sigma(S)$.
2. $\mathbf{eg\_rep}_\Sigma(R \dot{\cap} S) = \mathbf{eg\_rep}_\Sigma(R) \mathcal{S}(\overline{\cup}) \mathbf{eg\_rep}_\Sigma(S)$. □

Definition 13. Let R be normalized extended generalized disjunctive paraconsistent relation on scheme Σ. Then, $\dot{-}R$ is an extended generalized disjunctive paraconsistent relation over scheme Σ given by $(\dot{-}R)^+ = \mathbf{eg\_reduce}(R)^-$, $(\dot{-}R)_M = \{\langle p_i, n_i\rangle | (\forall i)\langle n_i, p_i\rangle \in \mathbf{eg\_reduce}(R)_M\}$ and $(\dot{-}R)^- = \mathbf{eg\_reduce}(R)^+$. □

Definition 14. Let R be a normalized extended generalized disjunctive paraconsistent relation on scheme Σ with $R_M = \{\langle p_1, n_1\rangle, \ldots, \langle p_k, n_k\rangle\}$, and let F be any logic formula involving attribute names in Σ, constant symbols (denoting values in the attribute domains), equality symbol $=$, negation symbol $\neg$, and connectives $\vee$ and $\wedge$. Then, the *selection of R by F, denoted $\dot{\sigma}_F(R)$,* is an extended generalized disjunctive paraconsistent relation on scheme Σ, given by $\dot{\sigma}_F(R) = \mathbf{eg\_reduce}(T)$, where T is defined as follows.
Let $E = \{\langle \mathbf{eg\_reduce}(R)^+ \cup P, \mathbf{eg\_reduce}(R)^- \cup N\rangle | P = \{p_i | (\forall i)\langle p_i, n_i\rangle \in \mathbf{eg\_reduce}(R)_M\} \wedge N = \{n_j | (\forall j)\langle p_j, n_j\rangle \in \mathbf{eg\_reduce}(R)_M\} \wedge i \neq j \wedge |P| + |N| = |\mathbf{eg\_reduce}(R)_M|\}$.
Let the normalized elements of E be $E_1, \ldots, E_e$ and let $A_i = \overline{\sigma}_F(E_i)$, for $1 \leq i \leq e$. Let $A_1, \ldots, A_g$ be the distinct A_is. Then,
$T^+ = \{w | (\exists t_1) \cdots (\exists t_g)(t_1 \in A_1^+ \wedge \cdots \wedge t_g \in A_g^+ \wedge w = \{t_1, \ldots, t_g\})\}$
$T_M = \{\langle P, N\rangle | P = \{p_i | (\forall i) p_i \in A_i^+\} \wedge N = \{n_j | (\forall j) n_j \in A_j^-\} \wedge i \neq j \wedge |p| \neq 0 \wedge |N| \neq 0 \wedge |P| + |N| = g\}$
$T^- = \{u | (\exists t_1) \cdots (\exists t_g)(t_1 \in A_1^- \wedge \cdots \wedge t_g \in A_g^- \wedge u = \{t_1, \ldots, t_g\})\}$. □

Definition 15. Let R be a normalized extended generalized disjunctive paraconsistent relation on scheme Σ with $R_M = \{\langle p_1, n_1\rangle, \ldots, \langle p_k, n_k\rangle\}$, and $\Delta \subseteq \Sigma$. Then, the *projection of R onto Δ, denoted $\dot{\pi}_\Delta(R)$,* is a generalized extended disjunctive paraconsistent relation on scheme Δ, given by $\dot{\pi}_\Delta(R) = \mathbf{eg\_reduce}(T)$, where T is defined as follows. Let $E = \{\langle \mathbf{eg\_reduce}(R)^+ \cup P, \mathbf{eg\_reduce}(R)^- \cup N\rangle | P = \{p_i | (\forall i)\langle p_i, n_i\rangle \in \mathbf{eg\_reduce}(R)_M\} \wedge N = \{n_j | (\forall j)\langle p_j, n_j\rangle \in \mathbf{eg\_reduce}(R)_M\} \wedge i \neq j \wedge |P| + |N| = |\mathbf{eg\_reduce}(R)_M|\}$.
Let the normalized elements of E be $E_1, \ldots, E_e$ and let $A_i = \overline{\pi}_\Delta(E_i)$, for

$1 \leq i \leq e$. Let $A_1, \ldots, A_g$ be the distinct $A_i s$. Then,
$T^+ = \{w | (\exists t_1) \cdots (\exists t_g)(t_1 \in A_1^+ \wedge \cdots \wedge t_g \in A_g^+ \wedge w = \{t_1, \ldots, t_g\})\}$
$T_M = \{\langle P, N \rangle | P = \{p_i | (\forall i) p_i \in A_i^+\} \wedge N = \{n_j | (\forall j) n_j \in A_j^-\} \wedge i \neq j \wedge |p| \neq 0 \wedge |N| \neq 0 \wedge |P| + |N| = g\}$
$T^- = \{u | (\exists t_1) \cdots (\exists t_g)(t_1 \in A_1^- \wedge \cdots \wedge t_g \in A_g^- \wedge u = \{t_1, \ldots, t_g\})\}$.

□

Definition 16. Let R and S be normalized extended generalized disjunctive paraconsistent relations on schemes Σ and Δ, respectively with
$R_M = \{\langle p_1, n_1 \rangle, \ldots, \langle p_k, n_k \rangle\}$ and $S_M = \{\langle u_1, v_1 \rangle, \ldots, \langle u_m, v_m \rangle\}$. Then, the *natural join of* R and S, denoted $R \dot{\bowtie} S$, is a generalized extended disjunctive paraconsistent relation on scheme $\Sigma \cup \Delta$, given by $R \dot{\bowtie} S = \mathbf{g\_reduce}(T)$, where T is defined as follows. Let $E = \{\langle \mathbf{eg\_reduce}(R)^+ \cup P, \mathbf{eg\_reduce}(R)^- \cup N \rangle | P = \{p_i | (\forall i) \langle p_i, n_i \rangle \in \mathbf{eg\_reduce}(R)_M\} \wedge N = \{n_j | (\forall j) \langle p_j, n_j \rangle \in \mathbf{eg\_reduce}(R)_M\} \wedge i \neq j \wedge |P| + |N| = |\mathbf{eg\_reduce}(R)_M|\}$ and
$F = \{\langle \mathbf{eg\_reduce}(S)^+ \cup U, \mathbf{eg\_reduce}(S)^- \cup V \rangle | U = \{u_i | (\forall i) \langle u_i, v_i \rangle \in \mathbf{eg\_reduce}(S)_M\} \wedge V = \{v_j | (\forall j) \langle u_j, v_j \rangle \in \mathbf{eg\_reduce}(S)_M \wedge i \neq j \wedge |U| + |V| = |\mathbf{eg\_reduce}(S)_M|\}$. Let the norma lized elements of E be $E_1, \ldots, E_e$ and those of F be $F_1, \ldots, F_f$ and let $A_{ij} = E_i \overline{\bowtie} F_j$, for $1 \leq i \leq e$ and $1 \leq j \leq f$. Let $A_1, \ldots, A_g$ be the distinct $A_{ij} s$. Then,
$T^+ = \{w | (\exists t_1) \cdots (\exists t_g)(t_1 \in A_1^+ \wedge \cdots \wedge t_g \in A_g^+ \wedge w = \{t_1, \ldots, t_g\})\}$
$T_M = \{\langle P, N \rangle | P = \{p_i | (\forall i) p_i \in A_i^+\} \wedge N = \{n_j | (\forall j) n_j \in A_j^-\} \wedge i \neq j \wedge |p| \neq 0 \wedge |N| \neq 0 \wedge |P| + |N| = g\}$
$T^- = \{u | (\exists t_1) \cdots (\exists t_g)(t_1 \in A_1^- \wedge \cdots \wedge t_g \in A_g^- \wedge u = \{t_1, \ldots, t_g\})\}$.

□

Theorem 3. *Let R and S be two normalized extended generalized disjunctive paraconsistent relations on scheme Σ_1 and Σ_2. Also let F be a selection formula on scheme Σ_1 and $\Delta \subseteq \Sigma_1$. Then,*
1. $\mathbf{eg\_rep}_{\Sigma_1}(\dot{\sigma}_F(R)) = \mathcal{S}(\overline{\sigma}_F)(\mathbf{eg\_rep}_{\Sigma_1}(R))$.
2. $\mathbf{eg\_rep}_{\Sigma_1}(\dot{\pi}_\Delta(R)) = \mathcal{S}(\overline{\pi}_\Delta)(\mathbf{eg\_rep}_{\Sigma_1}(R))$.
3. $\mathbf{eg\_rep}_{\Sigma_1 \cup \Sigma_2}(R \dot{\bowtie} S) = \mathbf{eg\_rep}_{\Sigma_1}(R) \mathcal{S}(\overline{\bowtie}) \mathbf{eg\_rep}_{\Sigma_2}(S)$.

□

5 Conclusions and Future Work

We have presented a framework for relational databases under which disjunctive positive facts, explicit disjunctive negative facts and mixed disjunctive facts can be represented and manipulated. It is the generalization of generalized disjunctive paraconsistent relation in [12]. The direction for future work would be to find applications of the model presented in this paper. There has been some interest in studying extended disjunctive logic programs in which the head of clauses can have one or more literals [14]. This leads to two notions of negation: *implicit* negation (corresponding to negative literals in the body) and *explicit* negation (corresponding to negative literals in the head). The model presented in this paper could provide a framework under which the semantics of extended logic programs could be constructed in a bottom-up manner.

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