Hypothetical revision and matter-of-fact supposition

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Abstract

The recent literature offers several models of the notion of *matter-of-fact supposition*[1] revealed in the acceptance of the so-called indicative conditionals. Some of those models are qualitative (Collins 1990), (Levi 1996), (Stalnaker 1984). Other probabilistic models appeal either to *infinitesimal probability* or *two place probability functions* ((van Fraassen 1995), (Joyce 1999), (McGee 1994)). Recent work has made possible to understand which is the exact qualitative counterpart of the latter probabilistic models ((van Fraassen 1995), (McGee 1994), (Arló-Costa & Thomason 1996), (Arló-Costa & Parikh 1999), (Arló-Costa 1997). In this article we show that the qualitative notion of change that thus arises is *hypothetical revision*, a notion previously axiomatized in (Arló-Costa 1997) and (Arló-Costa & Thomason 1996).

This notion is incompatible with AGM as well as with other standard methods of theory change (like Mendelzon and Katsuno's UPDATE). The way in which matter-of-fact supposition is modeled via hypothetical revision is illustrated via examples. The model is compared with other qualitative models of indicative supposition (like the one offered in (Levi 1996) and (Stalnaker 1984)), with models of subjunctive supposition, as well as with some of the well know models of learning. Applications in the theory of games and decisions are considered.

Introduction

The notion of *supposition* has a central role in probability theory and the theory of games and decisions. The notion is usually formalized by appealing to conditional probability. At each point in time the internal epistemic state of a Bayesian agent is formalized by an unconditional measure encoding its degree of belief and by conditional measures encoding the coherent degree of belief of the agent conditional on suppositions. The notion of supposition is usually carefully separated form the notion of learning. Brian Skyrms has stated this distinction as follows in (Skyrms 1987):

> Updating subjective belief to assimilate a given piece of information and supposing what the world would be like were that bit of information true, are distinct mental processes for which different rules are appropriate...

Minimizing divergence with respect to a given constraint is the hallmark of supposition, not of learning (Joyce 1999), (Diaconis & Zabell 1982). Imposing a constraint is tantamount to fix important features of the final state to which the current state P will be mapped and then minimizing some function measuring divergence from P and any of the possible states exhibiting the selected feature.

We will not focus here in the probabilistic side of the literature distinguishing learning and supposition. Reviewing this literature is well beyond the space limitations of this abstract. Nevertheless the curious reader can consult (Joyce 1999), (Levi 1996), (Levi 1980), (Skyrms 1987). Let me just mention in passing that the distinction between learning and supposing can be obviously made in a purely qualitative setting. For example, some authors doubt the universality of the so-called axiom of *success* establishing that when we revise the current state K with any input A, this input should be part of the revised output K*A. After all, when an agent is faced with certain evidence, he might decide to reject it. While this might be true for learning, this is hardly true for supposing. A moment of reflection will convince the reader that the success axiom is *constitutive* of any reasonable notion of supposition. Something similar happens with the general justification of the operation of contraction. Why any agent might be persuaded to loose information that at the moment he fully believes? This has led some researchers to propose that contraction cannot be one of the fundamental operations of theory change. Again, this is not a problem for the notion of supposition. An agent can perfectly well engage in a hypothetical exercise that requires contracting his view, while he continues to fully believe what he fully believes. Considerations of this type can be adduced in order to propose a priori reasonable axioms

*This work was partially supported by a grant from NSF.

[1] I am using here the terminology used by James Joyce in (Joyce 1999), pages 182-83.