

Proofs of Zero Knowledge

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Abstract

We present a protocol for verification of “no such entry” replies from databases. We introduce a new cryptographic primitive as the underlying structure, the keyed hash tree, which is an extension of Merkle’s hash tree. We compare our scheme to Buldas et al.’s Undeniable Attesters and Micali et al.’s Zero Knowledge Sets.

In the following, the term *database* refers to a system supplying the simplest form of databases, a table of $(key, value)$ pairs with functions for keyed insertion and retrieval.

1 Motivation: Untrusted Databases

There are situations where the users of a database system cannot trust the database. For example, the database might be outside the users’ security perimeter, or might not follow the same security policies as the users. The users must therefore assume that the database could be taken over by an attacker.

The entities writing the database entries want to be sure that the database delivers their entries on requests unchanged. The database maintainer wants to be able to defend itself against accusations of misbehavior.

2 Threat scenario

How can an attacker or malicious database maintainer deceptively influence the database’s replies? The following list explains the attacks:

1. **Changing existing entries:** The attacker can change the value of an existing entry.
2. **Creating false entries:** The attacker can create $(key, value)$ pairs himself.
3. **Re-labeling existing entries:** The attacker can insert existing values under different keys.

4. **Returning old entries:** If the value of a $(key, value_1)$ pair is overwritten by a new $value_2$, the attacker can return the old $value_1$ on request for key .
5. **Denying existing entries:** On a *search* request, the attacker can make the database deny that a *value* exists for the given *key* in the table.

This is not only of theoretical interest. Real-life examples are web-hosting solutions, where the changing parts of all web pages are kept in a central relational database and the pages are constructed from templates. Another example is the Internet's Domain Name System (DNS), which is basically a partially distributed lookup-table. Attacks on DNS as listed above are described in (1), remedies are suggested in the DNS Security Extensions(2) and a draft by Bellovin (3).

2.1 Detecting Attacks

The readers should be able to detect if the database is lying about entries. The database should be able to prove the correctness of its answers as long as the writers are honest. For most of the listed attacks, this can easily be achieved if public key cryptography is employed. It is not necessary to assume the existence of a global Public Key Infrastructure for this purpose. It is sufficient if the readers of the database have access to all the public keys of the writers (presumably a smaller set than the readers). The keys must be stored outside the database. With this information at hand, and routines for signature creation (S_k) and verification (V_k), readers and writers can make attacks 1 – 3 detectable under standard cryptographic assumptions. Here is how:

1. **Changing existing entries:** Writers sign the values of their entries and include the signature in the *value* of the table. Readers check the signatures and detect changes by the database.

$$(key, value) = (key, data || S_k(data))$$

2. **Creating false entries:** The same applies here. Since the database has no access to the writer's secret keys, it cannot produce deceptive entries.
3. **Re-labeling existing entries:** This calls for an extension: the writers now include the *key* in the data they sign:

$$(key, value) = (key, data || S_k(key || data))$$

The readers can now check if the entry belonged to the *key* they gave in their *search* request.

Points 4 and 5, however, pose a harder problem. If there is no information about existing entries anywhere outside the database, then these attacks cannot be detected.

As for point 4, from the algebraic properties of standard digital signatures it follows that a signature on an old entry will still be valid after the entry has been

replaced. There are signature schemes where the signer’s cooperation is necessary for verification, called *undeniable signatures*. The concept was presented first by Chaum (4) in 1989. Such a scheme in our scenario would introduce much more communication than in the standard writer–database–reader setup. A writer who substitutes an existing entry would have to reliably notify the writer of the previous version, so that the previous author retracts the signature (e.g. by sending a new “signature outdated” message in the verification protocol). Readers would need to communicate with writers who might not be available every time someone wants to validate a reply. We would like to avoid such a complicated and error-prone setup.

The problem with point 5 is that from the perspective of the database, it would mean proving that it does *not know* the table entry, an impossible feat in this scenario. Since there is no *value* returned, there can be no signature, unless the writers supply a special “no entry” value tied to *every possible and unused* key.

There are ways around this, however, if we allow the writers and readers access to another service besides the database. The service is required to supply a single value on request; authenticated writers must be able to overwrite that value. We will call this service the *announcement service*;

The writers store a signed, condensed description of the database’s state. This value must be updated after every write to the database. The readers can use this description to validate the database’s replies. We will call this summary of database state a *state credential*. The validation protocol consists of two parts:

- A writer builds and publishes a new state credential from the previous one.
- Readers check the validity of a database reply. This requires interaction with the announcement service and the database.

3 State Credentials for Databases

Our requirements for the state credential (to be short and uniquely bound to the database state) suggest the application of a cryptographic hash function. We will now describe several possibilities to build state credentials with hash functions.

3.1 Simple Hash

This is the simplest form of a state credential. The writer queries the database for all entries and inserts the *key* of his/her new entry. The writer then sorts all keys in a pre-defined order and computes the hash value over all of them. This value is signed and supplied through the announcement service. Formally:

$$c = h\left(\sum_{i=0}^n k_i\right)$$

where n is the number of entries in the database, k_i is the i th *key* in the particular order and addition denotes string concatenation after adding a special “end of key” marker to the parameters. The simple hash is not a solution, because to check a database reply against this, a reader would have to download the whole database as well.

3.2 Hash Chain

Application of hash chains make the computation of the credential cheaper. The writer pulls the latest credential from the announcement service and “adds” the new entry’s *key*. Formally:

$$\begin{aligned} c_0 &= h() \\ c_i &= h(c_{i-1} + k_i) \end{aligned}$$

where c_i is the credential after adding the i th entry to the database and $h()$ is the hash of the empty string. Addition is defined as above.

To validate a database reply, a reader would have to pull all keys up to the requested key in their order of addition. In the interesting case of a non-existing key, the reader has to pull the whole database. So this is still not satisfactory.

3.3 Hash Trees

Merkle’s (5) hash trees allow fast (Log-Time) verification of digests over many entries. Hashes over pairs of entries (the keys in our scenario) are computed and the resulting hashes are again paired and new hashes computed and so on. The number of entries has to be a power of 2. Formally:

$$\begin{aligned} H(l, j) &= h(k_j) \\ H(i, j) &= h(H(i + 1, \lceil \frac{i + j - 1}{2} \rceil).H(i + 1, \lceil \frac{i + j + 1}{2} \rceil)) \end{aligned}$$

where $l = \log_2(\#entries)$, by “.” we denote the concatenation of strings and k_j is the j th key in an arbitrary order ($H(1, 1)$ is the root of the tree).

The state credential at the announcement service must be updated for each new entry and a hash tree root can only be computed if the number of entries is a power of two. So we must find a form of state credential that allows instant updating but still can use hash trees.

The solution is to split the number of entries into powers of 2 and to generate hash trees of appropriate height for all of those powers. For this reason, the state credential will consist of a number of hash tree roots, at most $\log_2(n - 1)$ where n is the number of entries, together with the height of the trees. When an entry is added to the non-empty database, and if the number of entries is odd after the addition, then a new hash tree of height zero (simply the hash of the key) is appended to the state credential. If the number is even, then the two most recent tree roots are combined (hashed together) to form the root of a higher

tree. This is done recursively from the end. Note that we must start at the zeroth entry, since even an empty database needs a state credential.

So if the bit representation is

$$n - 1 = \sum_{i=0}^{\lfloor \log_2(n-1) \rfloor} b_i \cdot 2^i$$

and $t = \sum b_i$ (the Hamming-weight of $n - 1$), then the credential consists of the roots of t hash trees of height i for every $b_i = 1$.

The database in turn keeps a counter of entries. It can thus deduce in which of the trees in the current state credential an entry resides. This is achieved by comparing the entry's number against the current set of trees and their heights.

If a reader wishes to validate the databases' reply for a certain key k , the database has to supply a path in the tree leading to the leaf with k , as well as all pairs of hashes along that path. The reader can now verify that each pair's concatenation hashes to the corresponding hash in the next pair, up to the root of the tree in the state credential. Communication with the database for validation is bounded at worst by $\log_2(n) \cdot 2 \cdot |h()|$ bytes, where $|h()|$ is the byte-length of the hash's output.

The drawback of this scheme is that validation of a negative reply ("no such entry") still requires downloading the whole database.

4 Keyed Hash Trees

Our contribution is the introduction of *keyed hash trees*. In these, the position of a leaf in the tree is dependent on the key corresponding to the leaf. Our construction uses a tree of height $|h()|$, where h is a cryptographic hash function. For every $(key, value)$ in the database, a leaf is inserted in the tree, where the path to the leaf from the root is defined by $h(key)$ and the data in the leaf is $h(value)$. Leaves without a corresponding entry are implicitly set to the hash of the empty string $h()$.

We call a sub-tree *empty* if there is no path leading through the sub-tree's root that leads to a leaf with a non-empty value. All leaves of an empty sub-tree have the value $h()$. The nodes in the layer above the leaves all have value $h(h(), h())$, and so on. Note that all nodes in a layer have the same value, which in turn is derived from the hash of the empty word. It is easy to pre-compute these values. It allows identification of a sub-tree as being empty by a single table lookup, since all empty sub-trees of equal height have the same pre-computed root hash.

From the collision resistance for cryptographic hash functions, it follows that

$$\forall a, b : a \neq b : h(a.b) \neq h(b.a).$$

The value of the tree's root thus depends on all the leaves' values and all the paths. In contrast to the schemes above, non-existing entries *do* have leaves and paths and can thus be validated.

Storing and working on a binary tree of height 160 (for $h = \text{SHA1}$ for example) seems daunting at first, since there would be $2^{160+1} - 1$ entries, only a few orders of magnitude less than the number of hydrogen atoms in the whole universe. But all except n of the leaves are empty, where n is the number of entries in the database. We will show in Section 5 that it is sufficient to store only the non-empty leaves and the branches leading to them from the root.

Before inserting a new entry ($key, value$) in the database, a writer requests all pairs of hashes on the path to the new entry. The writer then validates the hashes against the current state credential, which is the root of the keyed hash tree. The writer substitutes the hash in the respective leaf by the hash of the *value*, and computes a new root hash. He then signs this root hash and puts it on the announcement service after inserting the new entry in the database.

To validate a database reply, the reader requests all pairs of hashes on the path derived from the key. For positive replies, the reader hashes over the returned value, compares that with the hash in the corresponding leaf and proceeds as with a standard hash tree.

If the reply was negative (“no such entry”), then the path must lead to an empty leaf and — as in the hash tree scheme above — the hash values can be checked recursively up to the root. The root must be the same as the signed value received from the announcement service.

5 Sparse Hash Tree Algorithms

Our database should be able to respond quickly to requests for pairs of hashes along a given path. To do this, we need an internal representation of the keyed hash tree corresponding to the current database table.

We are helped by the keyed hash tree’s property that all empty sub-trees of equal height are identical (an algorithm for creating a list of all empty sub-trees’ roots is given in figure 1).

Instead of storing $2^{161} - 1$ nodes, we need to store only those nodes that are part of a path to an existing entry in the database, i.e. the hash of at least one key in the database describes a path leading through the node. To reduce the space further, we store only one node for every sub-tree that contains exactly one entry, i.e. the first node from the root down which lies on one single path is used to describe the whole sub-tree containing the corresponding leaf. We call the resulting data structure a *sparse hash tree*.

The main problem is that searching in the tree is done from the root down to the leaf, while the hash values in the nodes are computed from the leaves up.

Each stored node in our hash tree representation is a **struct** defined in listing 2

The **b[2]** array contains pointers to the left and right children, or **NULL** if there is no respective child. **flag** indicates whether this node represents a **BRANCH** or a **LEAF**. **hash** is the hash of the tree rooted in this node. Only in leaf nodes is **e** not **NULL** and contains a pointer to a **struct entry** defined in listing 2. Note that leaf nodes do represent the leaves of the tree, but only non-empty

Listing 1: Pre-Computation of all empty sub-trees

```
list empty_init(void){
    i = 0;
    Empty[i] = h();
    for (;i<159;i++) {
        Empty[i] = h(Empty[i-1] . Empty[i-1]);
    }
    return Empty;
}
```

Listing 2: Structures `node` and `entry`

```
struct node {
    struct node *b[2];
#define LEAF 1
#define BRANCH 2
    int flag;
    struct entry *e;
    unsigned char* hash;
};

struct entry {
    unsigned char* path;
    unsigned char* value;
};
```

leaves have leaf nodes. The nodes contain data structures that allow dynamic computation of all node-values of the hash tree below their `path`.

`entry` holds the information necessary to compute all nodes below the node pointing to it (see algorithm `leafnode` for details). `path` is a bit-string of 0s and 1s and describes the path to the actual leaf from the leaf node, i.e. the part of the path for which we don't store nodes, but generate them as needed. `value` contains the hash of the entry at the actual leaf.

5.1 Computing Pairs along a Path

The function `rootpath` defined in listing 3 on page 7 returns a list of pairs of hashes along a given path. The first two `If` statements handle the special cases that there is at most one entry in the sparse tree. `rootpath` calls the auxiliary functions `nullnode`, `leafnode` and `branchnode`.

`nullnode` simply builds pairs of empty sub-tree roots in ascending order.

Listing 3: `rootpath()`: Generating all pairs of hashes along `path`

```
list rootpath(node root, bitstring path) {
    if(root == NULL){
        /* empty sub-tree */
        return nullnode(path);
    }
    If(root->flag == LEAF){
        /* exactly one leaf in subtree */
        return leafnode(root, path);
    }
    if(root->flag == BRANCH){
        /* non-unique path */
        init (List);
        return branchnode(path, root, List);
    }
}
```

Listing 4: `nullnode()`

```
list nullnode(bitstring path){
    init (List);
    for(i=0; i < length(path); i++){
        push List, (Empty[i], Empty[i]);
    }
    return(List);
}
```

Listing 5: `singlepath()`: Computing pairs of hashes in a sub-tree with exactly one given (path, value)

```
list singlepath(bitstring path, char value[160]) {
    revpath = reverse (path);
    init (List);
    init (pair);
    i=1;
    bit = shift revpath;
    pair[bit] = value;
    pair[not bit] = Empty[0];
    push (List, pair);
    while(bit = shift revpath) {
        pair[bit] = h(List[0][0] . List[0][1]);
        pair[not bit] = Empty[i++];
        push (List, pair);
    }
    return List;
}
```

leafnode calls **singlepath** to generate a temporary list of all non-empty sub-trees below the leaf node. It then compares the supplied path against the path to the leaf and selects the pairs from the temporary list for the maximum left-match of the paths. For the rest of the path, empty sub-tree roots are appended to the selected pairs. This list is returned.

branchnode walks recursively down the tree as long as the path runs through BRANCH nodes. At each step it appends the hashes in the children of the current node to the list. At the point where the path changes to an undefined or LEAF node, **branchnode** calls **nullnode** or **leafnode** respectively, to complete the list.

5.2 Inserting an Entry

insert (algorithm 8 on page 18) recursively walks down the tree along the given path, putting the traversed nodes on the stack. If the path runs into an empty subtree, a LEAF node with the given (path, value) is created. If the path runs through a LEAF node, the **entry** in the node is moved one step along its path down the tree, the node is converted to a BRANCH, and **insert** is called again on it. On the way up to the root, all the hashes on the path are adjusted.

5.3 Deleting an Entry

delete (algorithm 9 on page 19) recursively walks down the tree until it reaches the LEAF node with the entry to delete. On the way up, it checks if the current node has only one child, which is a LEAF. If so, the **entry** in the leaf is attached

Listing 6: `leafnode()`: Generating pairs of hashes along a **path** below a leaf node

```
list leafnode (struct node *node, bitstring path) {
    init (TmpList);
    init (List);
    init (pair);
    i=0;
    /* create list of non-empty nodes' hashes */
    TmpList = singlepath(node->e->path, node->e->value);

    /* compare given path against path of leaf */
    while(path[i] == node->e->path[i]) {
        append (List, TmpList[i]);
        i++;
    }
    height = length(path) - i;

    /* if the two diverge, return empty nodes' hashes */
    while(height > 0) {
        pair = (Empty[height], Empty[height]);
        append (List, pair);
        height--;
    }
    return List;
}
```

Listing 7: `branchnode()`: Generating pairs of hashes along a **path** below a branch node

```
list branchnode (struct node *n, bitstring path) {
    init (pair);
    /* get hashes of children */
    for(b = 0, 1){
        if (n->b[b] != NULL){
            pair[b] = n->b[b]->hash;
        } else {
            pair[b] = Empty[length(path)];
        }
    }
    append (List, pair);
    bit = shift path;

    if (n->b[bit] != NULL && n->b[bit]->flag == BRANCH){
        /* walk down */
        append (List, branchnode(path, n->b[bit], List));
    } else {
        if (n->n[bit] != NULL && n->n[bit]->flag == LEAF){
            /* we hit a leaf */
            append (List, leafnode(path, n->n[bit]));
        } else {
            /* walked into an empty sub-tree */
            append (List, nullnode(path));
        }
    }
    return List;
}
```

to the parent, and the child deleted. This is to handle situations where two leaves hang at the end of a stalk. If one of the leaves is removed, the stalk would be a series of roots of subtrees with exactly one entry. To keep storage minimal, this should be avoided. `delete` attaches the lonely leaf at the uppermost branch of the stalk.

5.4 Properties

We assume that the hash function h behaves as an ideal hash function would, an assumption often used in cryptography called the Random Oracle Model. It implies that the output of h is indistinguishable from the output of a random function. Under this assumption, our algorithms have the following properties:

The sparse tree in memory is nearly balanced. If h 's output behaves as randomness, the paths in the tree will be random walks starting at the root. For a large number n of entries, $\frac{1}{2^i}$ paths will lead through each node at the i th layer in the tree, and the average path length is $\log_2(n)$.

Space for the sparse tree is bounded by twice the number of entries in the database. We store n entries at n leaves, and each pair of nodes has one parent-node. This means that

$$\begin{aligned} \text{maxnodes}(n) &= \sum_{i=0}^{\lceil \log_2(n) \rceil} 2^i \\ &= 2^{\lceil \log_2(n) \rceil + 1} - 1 \\ &< 2 \cdot n \end{aligned}$$

Pairs along paths to a non-existing key can be computed quickly. If the key is not in the table, then the path will enter an empty subtree after $\log_2(n)$ steps, in the mean. For one million entries, for example, this means that after traversal of 20 nodes on average, `nullnode` will be called, and all that remains to be done is table lookups.

Readers recognize non-existing keys after $\log_2(n)$ steps from the root, in the mean. The pre-computation shown in 1 consists of 160 calls to the hash function. After this, a reader simply checks the returned pairs from the database against the pre-computed table `Empty`.

Maximum message size per validation is $2 \cdot |h()| \cdot |h()|$ bits. Two pairs of $|h()|$ bit hashes per step lie on a path with $|h()|$ steps. For SHA-1 as h , this would mean 6400 bytes per validation. If stronger collision resistance is required, a cryptographic hash function with longer output may be chosen. While the resistance grows exponentially with the hash size, the messages grow only linearly.

The database can be distributed over several machines. The `insert`, `delete` and `rootpath` functions are independent of the actual height of the stored tree. As long as writers and readers know how to compute the $2^{d+1} - 1$ hashes at the root of the tree, they can use d independent databases. This would also allow d concurrent writes, by locking at the branches.

6 Related Work

There are other schemes with different objectives related to the one just presented. We will discuss the differences to our proposal.

6.1 Undeniable Attesters

Ahto Buldas et al. presented schemes for accountable certificate management in 2000 ((6) and (7)). In their scenario, the database is a part of a certificate authority (CA), i.e. the list of valid certificates. Their goal is to make sure that there is never an ambiguity about the state of a certificate. On request for a key k , the database sends an *attester* $p = P(k, S)$, which is a cryptographic statement about the presence or absence of the key k in the table S . The database also returns a *digest* $d = D(S)$ which is a summary of the database's table S . Any party can then call a verification algorithm $V(k, d, p)$ which returns "Accept" if the statement p about k was correct for table summary d or else "Reject". Buldas et al. call an attester *undeniable* if a CA can produce two contradicting attesters for the same key with only negligible probability. Formally, this is defined as follows:

Let \mathcal{EA} be the class of probabilistic algorithms of polynomial runtime. Let k be the security parameter (in our context the bitlength of the hash function's output). The attester \mathcal{A} is given by the tuple of algorithms $\mathcal{A} = (\mathcal{G}, \mathcal{P}, \mathcal{D}, \mathcal{V})$ ¹. Its resilience against an attacker A of class \mathcal{EA} is defined as

$$\text{UN}_{\mathcal{A}, \parallel}(A) = \Pr[(x, d, p, \bar{p}) \leftarrow A(1^k) : V(x, d, p) = \text{Accept} \wedge V(x, d, \bar{p}) = \text{Reject}].$$

If k can be chosen as to minimize $\text{UN}_{\mathcal{A}, \parallel}$ below any given ϵ , then \mathcal{A} is called undeniable.

The paper examines different schemes for attesters and concludes with an efficient, undeniable attester based on search trees.

6.1.1 Search Trees

A search tree (see for example Knuth (8)) is a binary tree with the additional property that there is a comparison relation $<$ and each node contains a value v for which the following holds:

$$v_l < v < v_r,$$

¹ G serves only to select a hash function for a given security parameter k .

where v_l and v_r are the values in the left and right child of the node, respectively. If one or both children do not exist, the empty string is used instead. Buldas et al. add another field to each node, which holds the hash $h_v = h(h_{v_l}.v.h_{v_r})$. The digest d — the root of the search tree — is signed by the CA and published through an untrusted Publication Authority (PA). The digest corresponds to our state credential and the PA to our announcement service.

A proof p output by P for a key k is a list $p = (V_0, \dots, V_m)$ of values $V_i = (h_L, k_i, h_R)$, where k_i is the value of a node and h_L, h_R are the hashes stored in the left and right child of the node, respectively, or the empty string if there is no such node. The values are selected such that $k_0 = k$ if k is in the database. If k is not in the database, then the tree contains two keys j and l such that l is the smallest value larger than k and j the largest value smaller than k . By construction of the tree, there is a path from the node of j to the root leading through l or vice versa. The key of j and l which is lower in the tree is used as k_0 in that case. If k is larger or smaller than all keys in the search tree, then l or j is set to the largest or smallest key in the tree, respectively.

p together with the published digest d allows to prove the absence of a key. The verifier can check the hashes from k_0 to the root and verify the strict order of child nodes. If a key k is not in the tree, then p will contain j, l such that $j < k < l \wedge \neg \exists k_i \in p : j < k_i < k \vee k < k_i < l$. By the strict order and lack of right children of j and left children of l , k cannot be a node's value in the tree.

The verifier $V(x, d, p)$ returns “Error” if any of the following fails:

- Checking the order of keys in p .
- Computing and comparing the hashes along the path given in p up to the root.
- Comparing d against the root hash in p .

If x is the first key in p , V returns “Accept”, else “Reject”.

6.1.2 Differences to our Proposal

The attester states absence or presence in a list, exclusively. A state credential includes the *value* of the entry under the *key*, so that changes of an entry can be expressed.

Buldas et al. assume that the database is reigned by a single entity which is trustworthy at the moment when an attester is issued, but may turn untrusted or unavailable later. That the database signs the digest itself establishes a different scenario than in our setting.

To show that the database cheated, a user has to find another attester contradicting the attester he/she received. In our setting, a reader can prove that the database cheated immediately and without contacting other readers.

6.1.3 Reduction of State Credentials to Attesters

We can build undeniable attesters for keys from signed state credentials. Since values of entries are irrelevant for attesters, we will substitute the $(key, value)$

pairs in our algorithms by (key, key) pairs. Since the empty word might be an entry's key in the generality of the proof, we will use the reserved key τ instead in that case. The digest d is the state credential, but is supplied by the database instead of the announcement service. Our (P, D, V) are defined as follows:

$P(x, S)$ is the list of pairs of hashes along the path given by $h(x)$ in the keyed hash tree generated from all entries in S .

$D(S)$ is the root of said keyed hash tree.

$V(x, d, p)$ outputs “Error” if any of the following fails:

- Comparing the hash at the leaf corresponding to x against $h(x)$ or $h()$.
- Hashing the pairs of hashes up along the path $h(x)$.
- Comparing the resulting root hash against d .

If the leaf value at the end of path $h(x)$ is the empty hash, then V returns “Reject”. If the leaf value is $h(x)$, then “Accept”.

6.1.4 Proof of Undeniability

Assume that some $A \in \mathcal{EA}$ outputs (x, d, p, \bar{p}) with probability ϵ , such that $V(x, d, p) = \text{Accept}$ and $V(x, d, \bar{p}) = \text{Reject}$. Since V did not return “Error”, all the pairs of hashes in p and in \bar{p} resolved up to d . This means that A produced a collision in the hash h with probability ϵ , and the values $x_1, x_2, y_1, y_2 : (x_1.x_2) \neq (y_1.y_2) : h(x_1.x_2) = h(y_1.y_2)$ are members of p and \bar{p} . Thus we have reduced the security of our attester to the collision resistance of the hash h .

6.1.5 Considerations about the underlying Data Structures

Bulda's et al. use the search keys as they are, because the proofs of non-membership in the table rely on the greater-than relation between the keys. This has the disadvantage that the tree becomes unbalanced if the inserted entries are not uniformly distributed or if the first entry (the subsequent root) is not near the median of the set of entries. Unbalanced search trees cause longer searches, as more than $\log_2(n)$ nodes need to be traversed for some keys. The paper does not explain whether there is any re-balancing (see for example (8)) done on the search tree. Re-balancing this particular tree type would change all the hash-entries on the path from the previous root to the new one. Our sparse hash tree in memory is always balanced for large n .

Using the keys as they are makes the attester's size unpredictable. With state credentials, the size is fixed (and quite small).

6.2 Zero-Knowledge Sets

Unknown to this author, S. Micali, M. Rabin and J. Kilian presented a more general scheme, called Zero-Knowledge Sets (9) in 2003. Their goal was to prove set-membership non-interactively and that without leaking any other information about the set. The data structure underlying their scheme is almost identical to the keyed hash trees defined above.

Micali et al. use a commitment scheme to bind the database to its previous statements about its contents.

6.2.1 Pedersen's Commitment Scheme

In commitment schemes, a prover P *commits* to a message m by making public a commitment string c and computing a proof r . Before P publishes m or r , nothing should be inducible about the message m from c . After P sends message m , a verifier V can check whether this was the message committed to by c . For this, P supplies V with r . A commitment scheme is secure if P can produce contradicting proofs r, r' only with negligible probability.

Pedersen's scheme requires four public parameters, i.e., two primes p, q such that $q|p-1$, and two generators g, h , where g and h generate $G \subset \mathbb{F}_p^*$, the subgroup of order q in \mathbb{F}_p^* . P commits to m by publishing $c = g^m h^r \bmod p$. r is the corresponding proof, a randomly chosen value $\leq q$. To verify the commitment, V re-computes c for herself and compares.

To produce two proofs r, r' for differing messages m, m' , P would have to find r, r' such that $g^m h^r = g^{m'} h^{r'} \bmod p$. Assume she succeeded. She then could compute $g^{m-m'} = h^{r'-r} \bmod p$ and from this $(g^{m-m'})^{r'-r} = (h^{r'-r})^{r-r'} = h \bmod p$ and thus get $\log_g(h) \bmod p$, breaking the discrete logarithm mod p . It follows that the security of the Pedersen commitment is reducible to the discrete log problem. It also follows that if $\log_g(h)$ is known to the prover, she can fake arbitrary commitments.

Micali et al. use this scheme to construct a hash function $H : \{0, 1\}^* \rightarrow \{0, 1\}^k$ where k is the bitlength of prime p . They use this hash function to process a data-structure very much like our keyed hash tree from section 4.

Their prover builds the tree by first inserting all leaves derived from the database's entries, such that the value in the leaf is $H(D(x))$, where $D(x)$ is the value stored in the database under key x , and the leaf's position in the tree is determined by the path described by $H(x)$. The prover adds all nodes on those paths to the root, their values remain undefined until later. She adds those nodes whose parents are now in the tree, but does not repeat this recursively. These nodes correspond to the empty subtree's roots in our construction.

In the next step, the prover generates a random exponent e_v for every node v in the structure, and stores $h_v = h^{e_v}$ in it if v is a non-empty leaf or the ancestor of one, and $h_v = g^{e_v}$ for the empty leaves. Thus the prover knows $\log_g h_v$ for these nodes, a property exploited later.

The prover now computes commitments for the leaves by computing $c_v = g^{m_v} h_v^{r_v}$, where $m_v = H(D(x))$ or 0, h_v the value stored in the last step, and r_v a random value chosen per leaf.

In the last step, the internal nodes of the tree get their commitment values generated. The process runs from the leaves upwards as in Merkle’s scheme. Each internal node u is associated a value $m_u = H(c_{u0}, h_{u0}, c_{u1}, h_{u1})$, where $u0, u1$ are the left and right children of node u . From this and the value h_u stored already in u , c_u is computed as $g^{m_u} h_u^{r_u}$ for a randomly chosen $r_u \in \mathbb{F}_p^*$. The commitment value for the whole database is the commitment of the tree’s root c_D . This value is published before any queries are answered.

To prove that the value stored under key x is y in the database, the prover provides the tuples (m_v, e_v, h_v, r_v) for every node on the path through the tree given by $H(x)$, together with the values $c_{v0}, h_{v0}, c_{v1}, h_{v1}$ for v ’s siblings.

To check this proof, verifier V

1. compares the leaf’s m_v against $H(y)$.
2. checks recursively that $m_v = H(c_{v0}, h_{v0}, c_{v1}, h_{v1})$
3. checks for every v on the path that $h_v = h^{e_v}$ and $c_v = g^{m_v} h_v^{r_v}$.

For a proof of the non-existence of key x in the database, the prover provides all the nodes’ values along the path given by $H(x)$ as long as these nodes are in the generated tree. The last of these nodes, u , will have $m_u = 0$. If P would supply this to V , then the verifier would learn that there are no non-empty nodes in this sub-tree. To disguise this fact, P generates a branch of non-empty nodes down to a virtual, empty leaf (the computation goes bottom-up however). Node u becomes the branch’s root for this proof, the value $m_u = 0$ is substituted by the value in the uppermost node of the freshly generated branch.

The prover can do this convincingly because she can re-commit to any (m_u, r_u) in node u . This is caused by the special construction of h_u for empty leaves, where P knows the value of $\log_g(h_u)$ by design. This allows to “glue” the generated branch of nodes, complete with their commitments, to node u .

The verification runs as above, except that V checks for $m_{H(x)} = 0$.

6.2.2 Differences to our proposal

In our proposal, the writers are different entities from the database, but they compute the commitment string. They are continuously updating and changing entries. In Micali et al.’s terminology, our writers *commit* to the history of their changes to the database, while the database later *proves* that it did reply in accordance to it state after the latest update. Zero-knowledge was no requirement in our design. Since the motivations and requirements are different, the mechanisms behind Micali et al.’s scheme and ours differ in several points:

Multiple, mutually trusting authors in our design, multiple database authors (writers) were a prerequisite. It is not obvious, how zero knowledge sets could be adapted for multiple writers. Two writers would have to

communicate at least the secret e_v, r_v for each node to be able to insert entries. This would require a secure authenticated channel between all writers.

Zero knowledge the zero-knowledge requirement is violated in our scheme, because a proof about any key will leak information about the existence or non-existence of other keys.

Database modification one of our requirements was that the database can be modified by the writers who can re-compute the credential/attester/-commitment for the database. The zero-knowledge property of Micali et al.'s scheme is lost if the database is allowed to change/add/remove entries. This is because the commitment values change along the path to a modified node and thus give away information about the minimum number of modified entries. This makes their construction extremely static and not suitable for our application. Micali et al. mention this in their open problems section.

Speed because Micali et al.'s scheme must satisfy additional constraints, it is much slower (five modular exponentiations per node versus one application of an optimized hash function).

7 Summary

We have presented an efficient mechanism that allows readers to verify the replies of a database with the help of the writers. To achieve this, we introduced a new cryptographic primitive, an extension of Merkle's hash trees. A verification protocol between reader and database validates a reply with a low amount of data traffic. This extends even to those replies where the database denies the existence of the requested entry. The mechanism thus allows a database to prove that it does not know about such an entry. We examined the relationship to work in the area of certificate management, and showed how our proposal can be applied there as well. In comparison to the more general Zero-knowledge sets by Micali et al., our scheme has the advantage that it is faster and allows subsequent modifications by multiple writers.

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Listing 8: `insert()`: Insertion of a new entry

```

struct node *insert(bitstring path, char value[160], struct node *node) {
    if (node == NULL){
        /* virgin sub-tree: create new leaf */
        init (List);
        init (newnode);
        init (entry);
        newnode->flag = LEAF;
        List = leafnode(path, value);
        entry->path = path;
        entry->value = value;
        newnode->e = entry;
        newnode->hash = h(List[0][0] . List[0][1]);
        free(List);
        return(newnode);
    }
    if(node->flag == BRANCH){
        init (newnode);
        bit = shift path;
        /* walk down */
        newnode = insert(path, value, node->b[bit]);
        /* update the hash value on the way back */
        init (pair);
        pair[bit] = newnode->hash;
        if (node->b[not bit] == NULL) {
            pair[not bit] = Empty[length(path)];
        } else {
            pair[not bit] = node->b[not bit]->hash;
        }
        node->hash = h(pair[0] . pair[1]);
        return(node);
    }
    if(node->flag == LEAF){
        /* change LEAF to BRANCH */
        init (tmppath);
        init (tmpval);
        tmppath = node->e->path;
        tmpval = node->e->value;
        node->flag = BRANCH;
        /* move leaf's content down along its own path */
        free node->entry;
        node->entry = NULL;
        node = insert(tmppath, tmpval, node);
        /* insert new (path, value) in the now BRANCH */
        bit = shift path;
        init (newnode);
        newnode = insert(path, value, node->b[bit]);
        /* update hash on the way back */
        node->b[bit] = newnode;
        init (pair);
        pair[bit] = newnode->hash;
        if (node->b[not bit] == NULL) {
            pair[not bit] = Empty[length(path)];
        } else {
            pair[not bit] = node->b[not bit]->hash;
        }
        node->hash = h(pair[0] . pair[1]);
        return(node);
    }
}

```

Listing 9: delete(): Deleting the entry at the end of a given path

```

struct node *delete(bitstring path, struct node *node){
    if(node->flag == LEAF) {
        /* remove LEAF, stop recursion */
        free (node->entry);
        free (node);
        return NULL;
    }
    /* we're on the way to the leaf still */
    init (pair);
    /* remember the bit */
    bit = shift path;

    /* walk down */
    node->b[bit] = delete(path, node->b[bit]);

    /* reconstruct the path */
    path = prepend bit, path;

    /* how many non-children ? */
    for (i = 0, 1) {
        if (node->b[i] == NULL) {
            emp = i;
            numemp++;
        }
    }
    if (numemp == 2){
        /* node was a stalk with a single leaf, delete */
        free (node);
        return NULL;
    }
    if (numemp == 1){
        if (node->b[not emp]->flag == LEAF){
            /* one child, a leaf, move it up */
            node->e = node->b[not emp]->e;
            node->e->path = node->e->path;
            /* correct the path */
            pop node->e->path;
            node->flag = LEAF;
            pair[emp] = Empty[length(path)];
            pair[not emp] = node->b[not emp]->hash;
            node->hash = h(pair[0] . pair[1]);
            free (node->b[not emp]->e);
            free (node->b[not emp]);
            return node;
        } else {
            /* branch node leading to at least two leafs */
            /* update the hash */
            pair[emp] = Empty[length(path)];
            pair[not emp] = node->b[not emp]->hash;
            node->hash = h(pair[0] . pair[1]);
            return node;
        }
    }
    if (numemp == 0){
        /* branch node with at least two leafs below */
        /* update the hash */
        pair[0] = node->b[0]->hash;
        pair[1] = node->b[1]->hash;
        node->hash = h(pair[0] . pair[1]);
        return node;
    }
}

```
