Chosen-Plaintext Cryptanalysis of a Clipped-Neural-Network-Based Chaotic Cipher*

Chengqing Li[1], Shujun Li[2a]**, Dan Zhang[3] and Guanrong Chen[2b]

[1] Department of Mathematics, Zhejiang University, Hangzhou 310027, China
swiftsheep@hotmail.com

[2] Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong, China
hooklee@mail.com[2a], eegchen@cityu.edu.hk[2b]

[3] College of Computer Science, Zhejiang University, Hangzhou 310027, China
zhangdan@etang.com

Abstract. In ISNN'04, a novel symmetric cipher was proposed, by combining a chaotic signal and a clipped neural network (CNN) for encryption. The present paper analyzes the security of this chaotic cipher against chosen-plaintext attacks, and points out that this cipher can be broken by a chosen-plaintext attack. Experimental analyses are given to support the feasibility of the proposed attack.

1 Introduction

Since the 1990s, the study of using chaotic systems to design new ciphers has become intensive [1]. In particular, the idea of combining chaos and neural networks has been developed [2], [3], [4], [5] and has been adopted for image and video encryption [6], [7]. In our recent work [8], it has been shown that the chaotic ciphers designed in [2], [3], [4], [6], [7] are not sufficiently secure from a cryptographical point of view.

This paper focuses on the security of a clipped-neural-network-based chaotic cipher proposed in ISNN'04 [5]. This chaotic cipher employs a chaotic pseudo-random signal and the output of a 8-cell clipped neural network to mask the plaintext, along with modulus additions and XOR operations. Also, the evolution of the neural network is controlled by the chaotic signal. With such a complicated combination, it was hoped that the chaotic cipher can resist chosen-plaintext attacks. Unfortunately, our analysis shows that it is still not secure against chosen-plaintext attacks. By choosing only two plaintexts, an attacker can derive an equivalent key to break the cipher. This paper reports our analyses and simulation results.

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** The corresponding author, personal web site: http://www.hooklee.com.

The rest of the paper is organized as follows. Section 2 is a brief introduction to the chaotic cipher under study. The proposed chosen-plaintext attack is described in detail in Sec. 3, with some experimental results. The last section concludes the paper.

2 The CNN-Based Chaotic Cipher

First, the CNN employed in the chaotic cipher is introduced. The neural network contains 8 neural cells, denoted by $S_0, \cdots, S_7 \in \{1, -1\}$, and each cell is connected with other cells via eight synaptic weights $w_{ij} \in \{1, 0, -1\}$, among which only three are non-zeros. The synaptic weights between two connected cells are identical: $\forall\, i, j = 0 \sim 7$, $w_{ij} = w_{ji}$. The neural network evolves according to the following rule: $\forall\, i = 0 \sim 7$,

$$f(S_i) = \mathrm{sign}\left(\widetilde{S}_i\right) = \begin{cases} 1, & \widetilde{S}_i > 0 \ , \\ -1, & \widetilde{S}_i < 0 \ , \end{cases} \tag{1}$$

where $\widetilde{S}_i = \sum_{j=0}^{7} w_{ij} S_j$. Note that $\widetilde{S}_i \neq 0$ holds at all times.

Now, let us see how the chaotic cipher works with the above CNN. Without loss of generality, assume that $f = \{f(i)\}_{i=0}^{N-1}$ is the plaintext signal, where $f(i)$ denotes the i-th plain-byte and N is the plaintext size in byte. Accordingly, denote the ciphertext by $f' = \{f'(i)\}_{i=0}^{N-1}$, where $f'(i)$ is a double-precision floating-point number corresponding to the plain-byte $f(i)$. The encryption procedure can be briefly depicted as follows[1].

- *The secret key* includes the initial states of the 8 neural cells in the CNN, $S_0(0), \cdots, S_7(0)$, the initial condition $x(0)$, and the control parameter r of the following chaotic tent map:

$$T(x) = \begin{cases} rx, & 0 < x \leq 0.5 \ , \\ r(1-x), & 0.5 < x < 1 \ , \end{cases} \tag{2}$$

  where r should be very close to 2 to ensure the chaoticity of the tent map.
- *The initial procedure*: 1) in double-precision floating-point arithmetic, run the tent map from $x(0)$ for 128 times before the encryption starts; 2) run the CNN for $128/8 = 16$ times (under the control of the tent map, as discussed below in the last step of the encryption procedure); 3) set $x(0)$ and $S_0(0), \cdots, S_7(0)$ to be the new states of the tent map and the CNN.
- *The encryption procedure*: for the i-th plain-byte $f(i)$, perform the following steps to get the ciphertext $f'(i)$:
  - evolve the CNN for one step to get its new states: $S_0(i), \cdots, S_7(i)$;
  - in double-precision floating-point arithmetic, run the chaotic tent map for 8 times to get 8 chaotic states: $x(8i+0), \cdots, x(8i+7)$;

[1] Note that some original notations used in [5] have been changed in order to provide a better description.

- generate 8 bits by extracting the 4-th bits of the 8 chaotic states: $b(8i+0), \cdots, b(8i+7)$, and then $\forall\, j = 0 \sim 7$, set $E_j = 2 \cdot b(8i+j) - 1$;
- encrypt $f(i)$ as follows[2]:

$$f'(i) = \left(\left(\frac{f(i) \oplus B(i)}{256} + x(8i+7) \right) \bmod 1 \right) , \tag{3}$$

  where $B(i) = \sum_{j=0}^{7} \left(\frac{S_j(i)+1}{2} \right) \cdot 2^{7-j}$;
- $\forall\, i = 0 \sim 7$, if $S_i \neq E_i$, update all the three non-zero weights of the i-th neural cell and the three mirror weights as follows: $w_{ij} = -w_{ij}$, $w_{ji} = -w_{ji}$.

– *The decryption procedure* is similar to the above one with the following decryption formula:

$$f(i) = (256 \cdot ((f'(i) - x(8i+7)) \bmod 1)) \oplus B(i) . \tag{4}$$

3 The Chosen-Plaintext Attack

In chosen-plaintext attacks, it is assumed that the attacker can intentionally choose a number of plaintexts to try to break the secret key or its equivalent [9]. Although it was claimed that the chaotic cipher under study can resist this kind of attacks [5, Sec. 4], our cryptanalysis shows that such a claim is not true. By choosing two plaintexts, f_1 and f_2, satisfying $\forall\, i = 0 \sim N-1$, $f_1(i) = \overline{f_2(i)}$, one can derive two masking sequences as equivalent keys for decryption.

Before introducing the chosen-plaintext attack, three lemmas are given, which are useful in the following discussions.

Lemma 1. *$\forall\, a, b, c \in \mathbb{R}, c \neq 0$ and $n \in \mathbb{Z}^+$, if $a = (b \mod c)$, one has $a \cdot n = ((b \cdot n) \bmod (c \cdot n))$.*

Proof. From $a = (b \mod c)$, one knows that $\exists\, k \in \mathbb{Z}$, $b = c \cdot k + a$ and $0 \leq a < c$. Thus, $\forall\, n \in \mathbb{Z}^+$, $b \cdot n = c \cdot n \cdot k + a \cdot n$ and $0 \leq a \cdot n < c \cdot n$, which immediately leads to $a \cdot n = ((b \cdot n) \bmod (c \cdot n))$ and completes the proof of this lemma. □

Lemma 2. *$\forall\, a, b, c, n \in \mathbb{R}$ and $0 \leq a, b < n$, if $c = ((a-b) \mod n)$, one has $a - b \in \{c, c-n\}$.*

Proof. This lemma can be proved under two conditions. i) When $a \geq b$, it is obvious that $((a-b) \bmod n) = a - b = c$. ii) When $a < b$, $((a-b) \bmod n) = ((n+a-b) \bmod n)$. Since $-n < a - b < 0$, one has $0 < n + a - b < n$, which means that $((a-b) \bmod n) = n + a - b = c$. That is, $a - b = c - n$. Combining the two conditions, this lemma is thus proved. □

Lemma 3. *Assume that a, b are both 8-bit integers. If $a = b \oplus 128$, then $a \equiv (b + 128) \pmod{256}$.*

[2] In [5], $x(8i+7)$ was mistaken as $x(8)$.

Proof. This lemma can be proved under two conditions. i) When $0 \leq a < 128$: $b = a \oplus 128 = a + 128$, so $a \equiv (b + 128) \pmod{256}$. ii) When $128 \leq a \leq 255$: $b = a \oplus 128 = a - 128$, so $a \equiv (b - 128) \equiv (b + 128) \pmod{256}$. $\square$

From Lemma 1, one can rewrite the encryption formula Eq. (3) as follows:

$$256 \cdot f'(i) = (((f(i) \oplus B(i)) + 256 \cdot x(8i + 7)) \bmod 256) \ . \tag{5}$$

Given two plain-bytes $f_1(i) \neq f_2(i)$ and the corresponding cipher-blocks $f_1'(i), f_2'(i)$, one has $256 \cdot (f_1'(i) - f_2'(i)) \equiv ((f_1(i) \oplus B(i)) - (f_2(i) \oplus B(i))) \pmod{256}$. Without loss of generality, assume that $f_1'(i) > f_2'(i)$ and that $\Delta_{f_{1,2}} = 256 \cdot (f_1'(i) - f_2'(i))$. It is true that $0 < \Delta_{f_{1,2}} < 256$. Thus, one has

$$\Delta_{f_{1,2}} = (((f_1(i) \oplus B(i)) - (f_2(i) \oplus B(i))) \bmod 256) \ . \tag{6}$$

Because $f_1(i) \oplus B(i)$ and $f_2(i) \oplus B(i)$ are 8-bit integers and $\Delta_{f_{1,2}} \neq 0$, from Lemma 2, one of the following facts is true:

1. $(f_1(i) \oplus B(i)) - (f_2(i) \oplus B(i)) = \Delta_{f_{1,2}} \in \{1, \cdots, 255\}$; (7a)
2. $(f_2(i) \oplus B(i)) - (f_1(i) \oplus B(i)) = \left(256 - \Delta_{f_{1,2}}\right) \in \{1, \cdots, 255\}$. (7b)

For the above two equations, when $f_1(i) = \overline{f_2(i)}$ is satisfied, two possible values of $B(i)$ can be uniquely derived according to the following theorem.

Theorem 1. *Assume that a, b, c, x are all 8-bit integers, and $c > 0$. If $a = \bar{b}$, then the equation $(a \oplus x) - (b \oplus x) = c$ has an unique solution $x = a \oplus (1, c_7, \cdots, c_1)_2$, where $c = (c_7, \cdots, c_0)_2 = \sum_{i=0}^{7} c_i \cdot 2^i$.*

Proof. Since $a = \bar{b}$, one has $b \oplus x = \overline{a \oplus x}$. Thus, by substituting $y = a \oplus x$ and $\bar{y} = \overline{a \oplus x} = b \oplus x$ into $(a \oplus x) - (b \oplus x) = c$, one can get $y - \bar{y} = c$, which is equivalent to $y = \bar{y} + c$. Let $y = \sum_{i=0}^{7} y_i \cdot 2^i$, and consider the following three conditions, respectively.

1) When $i = 0$, from $y_0 \equiv (\bar{y}_0 + c_0) \pmod{2}$, one can immediately get $c_0 = 1$. Note the following two facts: i) when $y_0 = 0$, $\bar{y}_0 + c_0 = 2$, a carry bit is generated for the next bit, so $y_1 \equiv (\bar{y}_1 + c_1 + 1) \pmod{2}$ and $c_1 = 0$; ii) when $y_0 = 1$, $\overline{y_0} + c_0 = 1$, no carry bit is generated, so $y_1 \equiv (\bar{y}_1 + c_1) \pmod{2}$ and $c_1 = 1$. Apparently, it is always true that $y_0 = c_1$. Also, a carry bit is generated if $c_1 = 0$ is observed.

2) When $i = 1$, if there exists a carry bit, set $c_1' = c_1 + 1 \in \{1, 2\}$; otherwise, set $c_1' = c_1 \in \{0, 1\}$. From $y_1 \equiv (\bar{y}_1 + c_1') \pmod{2}$, one can immediately get $c_1' = 1$. Then, using the same method shown in the first condition, one has $y_1 = c_2$ and knows whether or not a carry bit is generated for $i = 2$. Repeat the above procedure for $i = 2 \sim 6$, one can uniquely determine that $y_i = c_{i+1}$.

3) When $i = 7$, it is always true that the carry bit does not occur, so $c_7' = 1$, and $y_7 \equiv 1$.

Combining the above three conditions, one can get $y = (1, c_7, \cdots, c_1)_2$, which results in $x = a \oplus (1, c_7, \cdots, c_1)_2$. $\square$

Assume that the two values of $B(i)$ derived from Eqs. (7a) and (7b) are $B_1(i)$ and $B_2(i)$, respectively. The following corollary shows that the two values have a deterministic relation: $B_2(i) = B_1(i) \oplus 128$.

Corollary 1. *Assume that a, b, c, x are all 8-bit integers, $a = \bar{b}$ and $c > 0$. Given two equations, $(a \oplus x) - (b \oplus x) = c$ and $(b \oplus x') - (a \oplus x') = c'$, if $c' = 256 - c$, then $x' = x \oplus 128$.*

Proof. Since $c + \bar{c} = 255$, one has $c' = 256 - c = \bar{c} + 1$. Let $c = \sum_{i=0}^{7} c_i \cdot 2^i$, and observe the first condition of the proof of Theorem 1. One can see that $c_0 = 1$, so $c'_0 = \bar{c}_0 + 1 = 1$. Since there is no carry bit, one can deduce that $\forall\, i = 1 \sim 7$, $c'_i = \bar{c}_i$. Applying Theorem 1 for $(a \oplus x) - (b \oplus x) = c$, one can uniquely get $x = a \oplus (1, c_7, \cdots, c_1)_2$. Then, applying Theorem 1 for $(b \oplus x') - (a \oplus x') = c'$, one has $x' = b \oplus (1, c'_7, \cdots, c'_1)_2 = \bar{a} \oplus (1, \bar{c}_7, \cdots, \bar{c}_1)_2 = (a_7, \bar{a}_6 \oplus \bar{c}_7, \cdots, \bar{a}_0 \oplus \bar{c}_1)_2 = (a_7, a_6 \oplus c_7, \cdots, a_0 \oplus c_1)_2 = a \oplus (1, c_7, \cdots, c_1)_2 \oplus (1, 0, \cdots, 0)_2 = x \oplus 128$. Thus, this corollary is proved. □

For any one of the two candidate values of $B(i)$, one can further get an equivalent chaotic state $\hat{x}(8i+7)$ from $B(i)$, $f(i)$ and $f'(i)$ as follows:

$$\hat{x}(8i+7) = 256 \cdot f'(i) - (f(i) \oplus B(i)) \equiv 256 \cdot x(8i+7) \pmod{256} \ . \tag{8}$$

With $B(i)$ and $\hat{x}(8i+7)$, the encryption formula Eq. (3) becomes

$$f'(i) = \frac{((f(i) \oplus B(i)) + \hat{x}(8i+7)) \bmod 256}{256} \ , \tag{9}$$

and the decryption formula Eq. (4) becomes

$$f(i) = ((256 \cdot f'(i) - \hat{x}(8i+7)) \bmod 256) \oplus B(i) \ . \tag{10}$$

Assume that $\hat{x}_1(8i+7)$ and $\hat{x}_2(8i+7)$ are calculated by Eq. (8), from $B_1(i)$ and $B_2(i)$, respectively. Then, we have the following proposition.

Proposition 1. *$(B_1(i), \hat{x}_1(8i+7))$ and $(B_2(i), \hat{x}_2(8i+7))$ are equivalent for the above encryption procedure Eq. (9), though only one corresponds to the correct value generated from the secret key. That is,*

$$((f(i) \oplus B_1(i)) + \hat{x}_1(8i+7)) \equiv ((f(i) \oplus B_2(i)) + \hat{x}_2(8i+7)) \pmod{256} \ .$$

Proof. From $B_1(i) = B_2(i) \oplus 128$, one has $f(i) \oplus B_1(i) = (f(i) \oplus B_2(i) \oplus 128)$. Then, following Lemma 3, it is true that $(f(i) \oplus B_1(i)) \equiv ((f(i) \oplus B_2(i)) + 128) \pmod{256}$. As a result, $\hat{x}_1(8i+7) = (256 \cdot f'(i) - (f(i) \oplus B_1(i))) \equiv (256 \cdot f'(i) - ((f(i) \oplus B_2(i)) - 128)) \pmod{256} \equiv (\hat{x}_2(8i+7) + 128) \pmod{256}$, which immediately leads to the following fact: $((f(i) \oplus B_1(i)) + \hat{x}_1(8i+7)) \equiv ((f(i) \oplus B_2(i)) + \hat{x}_2(8i+7)) \pmod{256}$. Thus, this proposition is proved. □

Considering the symmetry of the encryption and decryption procedures, the above proposition immediately leads to a conclusion that $(B_1(i), \hat{x}_1(8i+7))$ and $(B_2(i), \hat{x}_2(8i+7))$ are also equivalent for the decryption procedure Eq. (10).

From the above analyses, with two chosen plaintexts f_1 and $f_2 = \bar{f}_1$, one can get the following two sequences: $\{B_1(i), \hat{x}_1(8i+7)\}_{i=0}^{N-1}$ and $\{B_2(i), \hat{x}_2(8i+7)\}_{i=0}^{N-1}$. Given a ciphertext $f' = \{f'(i)\}_{i=0}^{N-1}$, $\forall\, i = 0 \sim N-1$, one can use either $(B_1(i), \hat{x}_1(8i+7))$ or $(B_2(i), \hat{x}_2(8i+7))$ as an equivalent of the secret key to decrypt the i-th plain-byte $f(i)$, following Eq. (10). This means that the chaotic cipher under study is not sufficiently secure against the chosen-plaintext attack.

To demonstrate the feasibility of the proposed attack, some experiments have been performed for image encryption, with secret key $r = 1.99$, $x(0) = 0.41$ and $[S_0(0), \cdots, S_7(0)] = [1, -1, 1, -1, 1, -1, 1, -1]$. One plain-image "Lenna" of size 256×256 is chosen as f_1 and another plain-image is manually generated as follows: $f_2 = \bar{f}_1$. The two plain-images and their cipher-images are shown in Fig. 1. With the two chosen plain-images, two sequences, $\{B_1(i), \hat{x}_1(8i+7)\}_{i=0}^{256\times256-1}$ and $\{B_2(i), \hat{x}_2(8i+7)\}_{i=0}^{256\times256-1}$, are generated by using the above-mentioned algorithm. The first ten elements of the two sequences are given in Table 1. $\forall\, i = 0 \sim (256\times256-1)$, either $(B_1(i), \hat{x}_1(8i+7))$ or $(B_2(i), \hat{x}_2(8i+7))$ can be used to recover the plain-byte $f(i)$. As a result, the whole plain-image ("Peppers" in this test) can be recovered as shown in Fig. 1f.

Table 1. The first ten elements of $\{B_1(i), \hat{x}_1(8i+7)\}_{i=0}^{256\times256-1}$ and $\{B_2(i), \hat{x}_2(8i+7)\}_{i=0}^{256\times256-1}$

i	0	1	2	3	4	5	6	7	8	9
$B_1(i)$	146	231	54	202	59	243	166	173	233	82
$B_2(i)$	18	103	182	74	187	115	38	45	105	210
$\hat{x}_1(8i+7)$	242.40	38.63	242.62	222.09	81.03	214.73	240.91	203.59	138.20	9.33
$\hat{x}_2(8i+7)$	114.40	166.63	114.62	94.09	209.03	86.73	112.91	75.59	10.20	137.33

4 Conclusion

In this paper, the security of a chaotic cipher based on clipped neural network has been analyzed in detail. It is found that the scheme can be effectively broken with only two chosen plain-images. Both theoretical and experimental analyses have been given to support the proposed attack. Therefore, this scheme is not suggested for applications that requires a high level of security.

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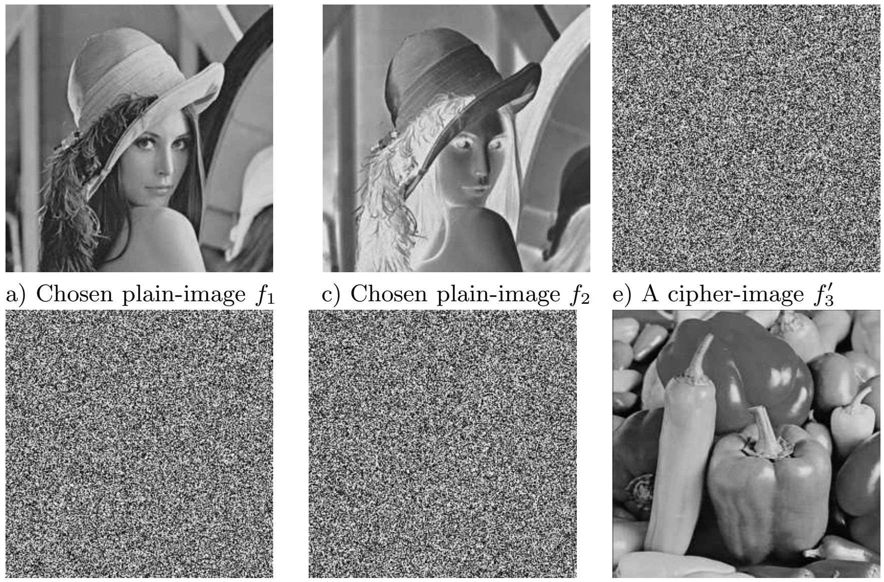

a) Chosen plain-image f_1 c) Chosen plain-image f_2 e) A cipher-image f_3'

b) Cipher-image f_1' d) Cipher-image f_2' f) Recovered image f_3

Fig. 1. The proposed chosen-plaintext attack

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