# Binomial multichannel algorithm

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It is devoted to MVA anniversary (70+40)

**Abstract**: The binomial multichannel algorithm is proposed. Some its properties are discussed.

#### Introduction

The algorithm of universal compression (so-called MV2 algorithm) was firstly proposed in [1-2]. It is based on bit (binary) recording of file and represented an expressive example of multichannel crypto-algorithms. The theory of binomial radix with binary alphabet was stated in [3-4]. In this report the abovementioned ideas are used to construct the binomial multichannel algorithm and to discuss some its properties. This continuation of my works [5-6] in direction generalization of procedure of bit recording on other radix.

#### **Definitions**

At the beginning I shall give some definitions.

**Bit** (short for **b**inary digit - abbreviated **b**) - quantity which has equally only two values: a zero or a one.

**N-digit tuple** – a bit's sequence consisting of N bit.

**Binary binomial numbers** - the numerical function of a *n*-digit *k*-binomial radix with binary alphabet.

**N-digit alphabet** ( $A_N$ ) - set of all various N-digit tuple.

**n-digit k-binomial alphabet**  $(B_{k,n})$  - set of all various binary binomial numbers with parameters (n, k).

**Basic file** - a sequence of the N-digit alphabet ( $A_N$ ) without recurrence.

**Basic set** -a set of independent classes X and Y for given N.

**Length of a file** – amount of bit used for representation of a file.

**MV2** bit recording of N-digit alphabet  $(A_N)$  - isomorphism between a set of the N-digit alphabet  $(A_N)$  with constant length of a tuple N and finite set of alphabets  $A_K$  with the constant length of a tuple  $1 \le K \le N-1$ .

**Binomial bit recording of N-digit alphabet (** $A_N$ **) -** isomorphism between a set of the N-digit alphabet ( $A_N$ ) with constant length of a tuple N and finite set of alphabets  $B_{k,n}$  with the **variable** length of a tuple  $1 \le K \le N - 1$ .

The coefficient of compression  $k_{\min} = \frac{L_2}{L_1}$  - the ratio of lengths for the basic file after  $L_2$  and before  $L_1$  bit recording of the N-digit alphabet ( $A_N$ )

# MV2-algorithm and its clones

My discussion will begin with the following equation:

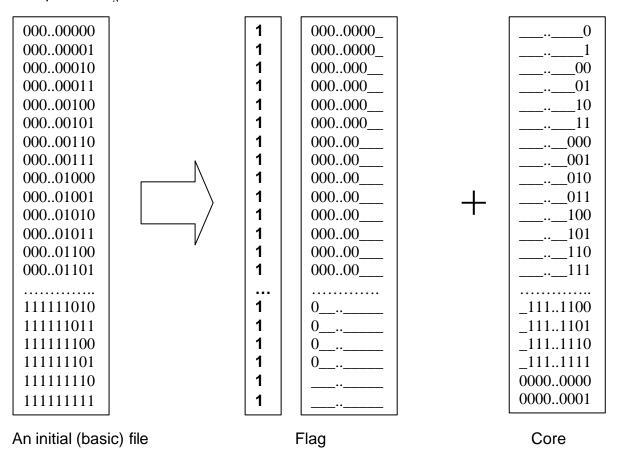
$$2^{N} = \sum_{K=1}^{K=N-1} 2_{N}^{K} + 2 = 2^{1} + ... + 2^{N-1} + 2.$$
 (1)

It shows, that N-digit alphabet  $(A_N)$  can be divided into finite set of alphabets  $(A_K)$  with constant length of a tuple  $1 \le K \le N-1$  plus two any elements from initial N-digit alphabet  $(A_N)$ , i.e.

$$A_N = \sum_{K=1}^{K=N-1} A_K + a_N^1 + a_N^2 , \qquad (2)$$

where  $a_K^i \in A_K$  and  $i \in [1;2^K]$ .

According to this MV2 bit recording of N-digit alphabet ( $A_N$ ) the initial file is divided on two parts and any of them has no neither statistical and nor semantic links with an initial file. The first part names a core (kernel) and arises from an initial file by bit recording (see equation (2)). The second part (so-called flag) gives us the information on the length of a tuple K for each element. This information is the information on number of the used alphabet also. Because of obvious compression of an initial file in comparison with the core, all procedure can be repeated some times. These are a key moments of MV2 algorithm. For the best understanding its realization is displayed for the basic file of the alphabet  $A_N$  on scheme N1.



Scheme N1. Realization of MV2 algorithm for alphabets  $A_N$  [1].

There is an opportunity to realize clones of MV2 algorithm - for it is necessary to use instead of the equation (2) the following equations:

$$A_{N} = \sum_{K=1}^{K=N} \sum_{i=1}^{i=M} a_{K}^{i} ;$$
 (3a)

$$\sum_{K=1}^{K=N} M_K = 2^N , (3b)$$

where  $M_K$  is quantity of elements of the alphabet ( $A_K$ ).

It means, that some elements of the initial N-digit alphabet  $(A_N)$  with constant length N (see equation (3a)) are replaced on the elements of alphabets  $(A_K)$  with a constant, but smaller length  $1 \le K \le N - 1$  at one restriction: the total of elements remains to constant (see equation (3b)). Each such clone is characterized by the coefficient of the compression  $k_{--}^{clone}$  [5-6]:

$$k_{\min}^{clone} = \frac{\sum_{K=1}^{K=N} K * M_K}{N * 2^N}.$$

It has the smallest value for MV2 algorithm:

$$k_{\min}^{MV2} = \frac{2N + 2\sum_{l=0}^{l=N-2} (l+1) \cdot 2^{l}}{N \cdot 2^{N}}.$$

### **Binomial radix**

The binomial radix is the radix with binomial weights, i.e. the numerical function of a n-digit k-binomial radix defining a quantitative equivalent of considered numbers  $B_j = b_1 b_2 \dots b_r$  by following expression

$$B_j = b_1 b_2 \dots b_j \dots b_r = \sum_{j=1}^{j=r} b_j C_{n-j}^{k-q_j},$$
 (4)

where  $q_j = \sum_{t=1}^{t=j-1} b_t$ ; n, k - parameters of a radix;  $j = [1, C_n^k]$ .

Further the binary alphabet will be used, i.e.  $b_r \in B = \{0;1\}$ . The binary binomial numbers are characterized by parameters (n, k); they have variable length r ( $1 \le r \le n-1$ ), contain or k units and thus come to an end on 1, or have (n - k) zero and thus come to an end on 0, i.e. there are following restrictions:

$$\begin{cases} k \le r \le n-1 \\ q = k \\ b_n = 1 \end{cases}$$
 (5a)

and

$$\begin{cases} n-k = r - q \\ 0 \le q \le k - 1 \\ b_r = 0 \end{cases}$$
 (5b)

According to [3-4] above-stated restrictions (5a, 5b) divide binary binomial numbers  $B_j \in B_{k,n}$  into two not crossed classes  $\textbf{\textit{X}}$  and  $\textbf{\textit{Y}}$ ;  $B_{k,n} = X \cup Y$ ;  $X \cap Y = \emptyset$ . The first class  $\textbf{\textit{X}}$  contains k units and l zero,  $0 \le l \le n-k-1$ , and the second class  $\textbf{\textit{Y}} - (n-k)$  zero and q units,  $0 \le q \le k-1$ . The range of the binomial radix represents binomial factor. As the concrete example the n-digit k-binomial alphabet  $(B_{k,n})$  with some values of parameters (n,k) is shown in table N1.

This information also is all necessary for the further consideration.

### Binomial multichannel algorithm and its properties

Let's recollect that my discussion began with the equation (1):

$$2^{N} = \sum_{K=1}^{K=N-1} 2_{N}^{K} + 2 = 2^{1} + ... + 2^{N-1} + 2.$$

However this equation can be transformed to other, combinatorial form:

$$2^{N} = (1+1)^{N} = \sum_{k=0}^{k=N} C_{N}^{K} = C_{N}^{0} + C_{N}^{1} + \dots + C_{N}^{N-1} + C_{N}^{N}.$$
 (6)

It shows that the N-digit alphabet  $(A_n)$  also can be divided into finite set of N-digit K-binomial alphabets  $(B_{K,N})$ , i.e.

$$A_{N} = \sum_{k=1}^{K=N} \mathbf{B}_{K,N} \equiv \sum_{k=1}^{K=N-1} (X_{k} + Y_{k}) + \mathbf{B}_{N,N} ;$$
 (7a)

$$B_{N,N} \equiv \{0;1\}.$$
 (7b)

Here entered the N-digit N-binomial alphabet  $(B_{N,N})$  has only two values (see equation (7b)) and represents the single element from  $C_N^0$  plus the single element  $C_N^N$  according to equation (6). Similarly **MV2 bit recording of N-digit alphabet** ( $A_n$ ) (see equation (2)) transformation (7a, 7b) will

name as **binomial bit recording of N-digit alphabet** ( $A_n$ ). After binomial bit recording the initial file

is divided by similar way on two parts and any of them has no neither statistical and nor semantic links with an initial file. However the second part (flag) gives us only the information concerning number of the used alphabet here, but not concerning the length of a tuple K for each element. Let's remind that the elements of N-digit K-binomial alphabets ( $B_{K,N}$ ) have the variable length. Because of obvious compression of an initial file in comparison with the core, all procedure can be repeated some times.

From equality  $C_n^k = C_n^{n-k}$  following communication between the classes **X** and **Y** in various alphabets  $B_{k,n}$  can be received by operation P:

$$X_k = PY_{n-k} = \overline{Y_{n-k}} , \qquad (8)$$

where  $0 = P1 = \bar{1}, 1 = P0 = \bar{0}$ .

This fact allows to change equation (7a) to next form:

$$A_{N} = \sum_{k=1}^{K=N} \mathbf{B}_{K,N} \equiv \sum_{k=1}^{K=N-1} (X_{k} + Y_{k}) + \mathbf{B}_{N,N} = \sum_{k=1}^{K=N-1} (X_{k} + \overline{X_{k}}) + \mathbf{B}_{N,N};$$
 (9)

and to introduce naturally such additional attribute as value of operation P for each element  $X_k$ . In this case the third additional, also not having semantic loading of the initial text a file - a flag N2 appears. The given realization is similar to the realization of second clone for MV2 algorithm in [6].

The coefficient of the compression  $k_{min}^{bin}$  for binomial algorithm is equaled:

$$k_{\min}^{bin} = \frac{2 + \sum_{l=0}^{l=N-2} (l+1) * C_N^{l+2}}{N * 2^N}.$$

Also for the best understanding the realization of my algorithm is displayed for the basic file on schemes N2-7 and the values of the coefficient of the compression  $k_{\min}^{bin}$  ( $k_{\min}^{MV2}$ ) are given in table N2 only for  $n \le 8$ . There is very big size of schemes for other values n and because of the lack of a place they are not shown. Therefore only the basic set showed on the schemes 7;8 for a case  $n \equiv 7$ ;8. For comparison with MV2 algorithm its core is given extreme by right column after a black line on all schemes.

There is no a direct opportunity to realize clones of my algorithm – the limited set of the elements of the binomial alphabets  $B_{k,n}$  with variable length of a tuple exists. But there is the possibility to realize the mixed clones or a clones as with elements of the constant length of a tuple, belonging alphabets ( $A_K$ ), and with elements of the variable length of a tuple, belonging binomial alphabets  $B_{K,N}$ .

#### Conclusion

We have proposed the binomial multichannel algorithm and have given obvious examples of its realization for  $n \le 8$ . The generalization of considered algorithm on any values n is simple. Two forms of flag are stated. The remark about symmetry  $C_n^k = C_n^{n-k}$  or the operation P allows to add a new attribute as new channel. In difference from MV2 algorithm in my item the core is less and the flag shows only the number of a considered class. The coefficient of the compression  $k_{\min}^{bin}$  for binomial algorithm and the mixed clones are discussed.

# Acknowledgments

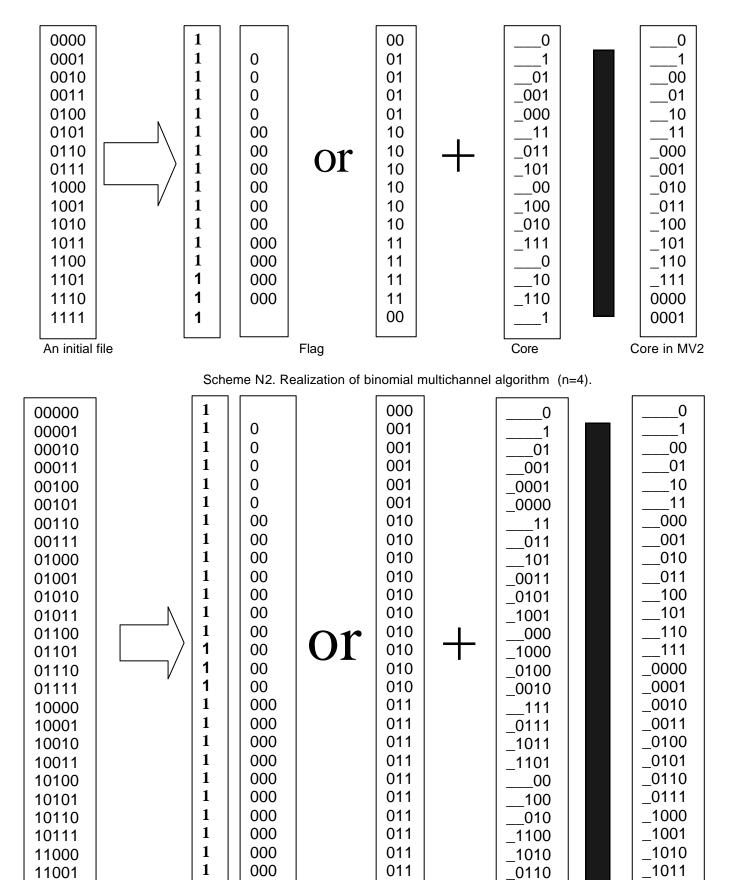
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N	K	Class $X_k \in X$	Class $Y_k \in Y$
2	1	$X_1 = \overline{Y_1} = \{1\}$	$Y_1 = \{0\}$
3	1	$X_1 = \overline{Y_2} = \{1;01\}$	$Y_1 = \{00\}$
	2	$X_2 = \overline{Y_1} = \{11\}$	$Y_2 = \{0,10\}$
4	1	$X_1 = \overline{Y_3} = \{1; 01; 001\}$	$Y_1 = \{000\}$
	2	$X_2 = \overline{Y_2} = \{11;011;101\}$	$Y_2 = \{00;100;010\}$
	3	$X_3 = \overline{Y_1} = \{111\}$	$Y_3 = \{0; 10; 110\}$
5	1	$X_1 = \overline{Y_4} = \{1; 01; 001; 0001\}$	$Y_1 = \{0000\}$
	2	$X_2 = \overline{Y_3} = \{11;011;101;0011;0101;1001\}$	$Y_2 = \{000; 1000; 0100; 0010\}$
	3	$X_3 = \overline{Y_2} = \{111;0111;1011;1101\}$	$Y_3 = \{00;100;010;1100;1010;0110\}$
	4	$X_4 = \overline{Y_1} = \{1111\}$	$Y_4 = \{0;10;110;1110\}$
	1	$X_1 = \overline{Y_5} = \{1; 01; 001; 0001; 00001\}$	$Y_1 = \{00000\}$
	2	$X_2 = \overline{Y_4} = \begin{cases} 11;011;101;0011;0101;1001;\\ 00011;00101;01001;10001 \end{cases}$	$Y_2 = \{0000;10000;01000;00100;00010\}$
6	3	$X_3 = \overline{Y_3} = \begin{cases} 111;0111;1011;1101;00111;\\ 01011;01101;10011;10101;11001 \end{cases}$	$Y_3 = \begin{cases} 000;1000;0100;0010;11000;\\ 10100;10010;01100;01010;00110 \end{cases}$
	4	$X_4 = \overline{Y_2} = \{1111;01111;10111;11011\}$	$Y_4 = \begin{cases} 00;100;010;11100;1010;0110;\\ 11100;11010;10110;01110 \end{cases}$
	5	$X_5 = \overline{Y_1} = \{11111\}$	$Y_5 = \{0;10;110;1110;11110\}$
	1	$X_1 = \overline{Y_6} = \{1; 01; 001; 0001; 00001; 000001\}$	$Y_1 = \{000000\}$
	2	$X_2 = \overline{Y_5} = \begin{cases} 11;011;101;0011;0101;1001;100001;\\ 00011;00101;01001;10001;\\ 000011;000101;001001;010001; \end{cases}$	$Y_2 = \begin{cases} 00000;100000;010000;\\ 001000;000100;000010 \end{cases}$
7	3	$X_3 = \overline{Y_4} = \begin{cases} 111;011;1011;101;00111;01011;\\ 01101;10011;10101;11001;000111;\\ 001011;010011;100011;001101;\\ 010101;011001;100101;101001;110001 \end{cases}$	$Y_3 = \begin{cases} 0000;10000;01000;00100;00010;\\ 110000;101000;100100;100010;011000;\\ 010100;001100;010010;001010;000110 \end{cases}$
	4	$X_4 = \overline{Y_3} = \begin{cases} 1111;01111;10111;11011;11101;\\ 001111;010111;011011;011101;100111;\\ 101011;110011;101101;110101;111001 \end{cases}$	$Y_4 = \begin{cases} 000; 1000; 0100; 0010; 11000; 10100; \\ 10010; 01100; 01010; 00110; 111000; \\ 110100; 101100; 011100; 110010; \\ 101010; 100110; 011010; 010110; 001110 \end{cases}$
	5	$X_{5} = \overline{Y_{2}} = \begin{cases} 11111;011111;1011111; \\ 110111;111011;111101 \end{cases}$	$Y_5 = \begin{cases} 00;100;010;1100;1010;0110;011110;\\ 11100;11010;10110;01110;\\ 111100;111010;110110;101110 \end{cases}$
	6	$X_6 = \overline{Y_1} = \{111111\}$	$Y_6 = \{0;10;110;1110;11110;111110\}$

Table N1. The n-digit k-binomial alphabet (  $\mathbf{B}_{k,n}$  ).



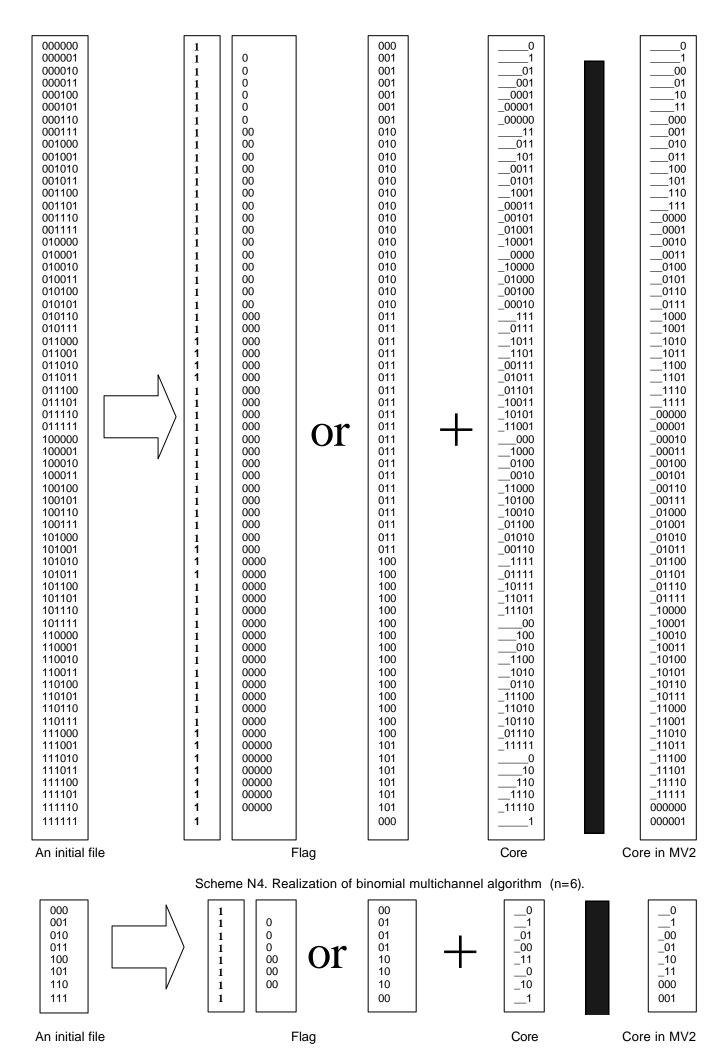
An initial file Flag Core Core in MV2

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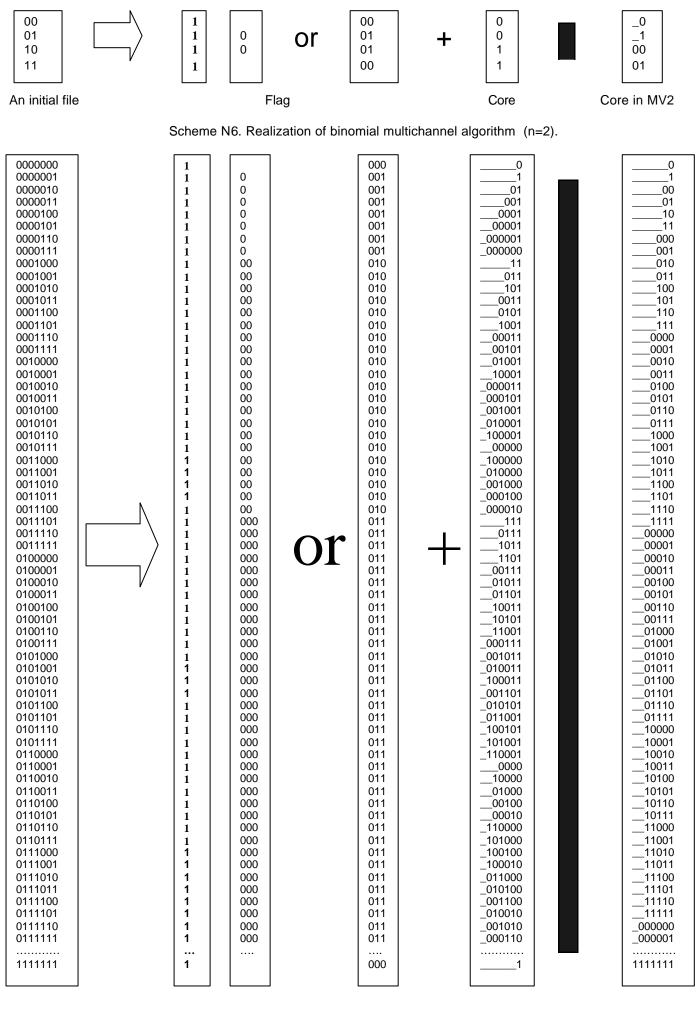
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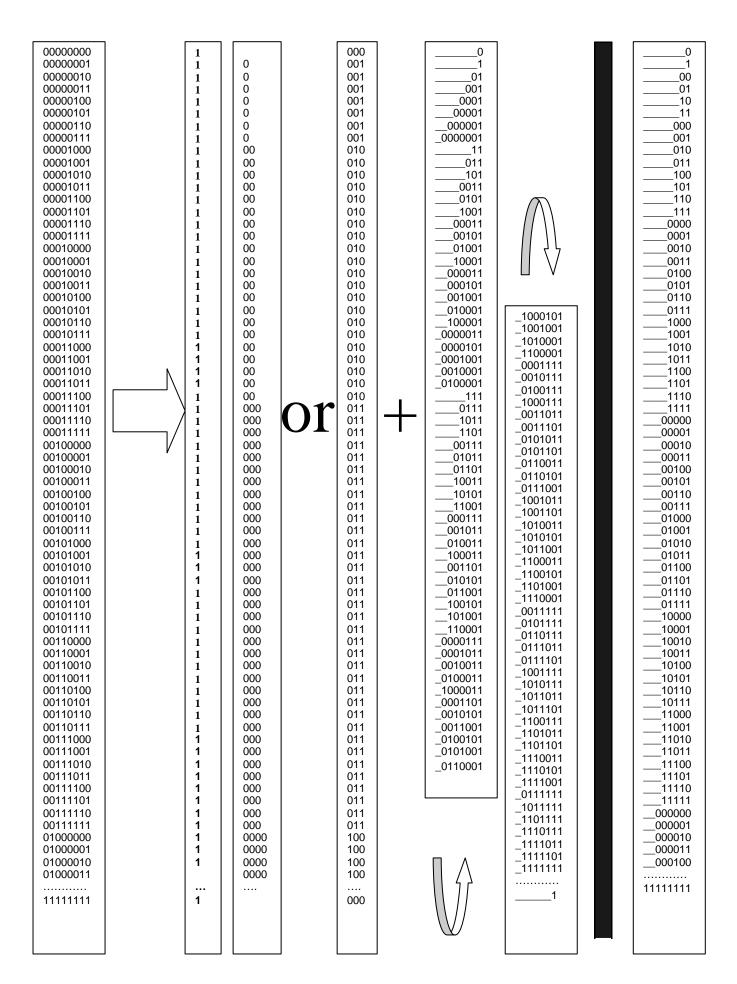
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Scheme N5. Realization of binomial multichannel algorithm (n=3).



An initial file Flag Core Core in MV2



N	$k_{_{ m min}}^{bin}$	$k_{_{ m min}}^{~MV~2}$
2	0.5	0.75
3	0.5	0.6(6)
4	0.5625	0.65625
5	0.625	0.675
6	0.677083(3)	0.703125
7	0.71875	0,73214285714285714285714285714286
8	0,751953125	0,7587890625

Table N2. The coefficient of the compression  $k_{_{\min}}^{_{bin}}$  ( $k_{_{\min}}^{^{MV2}}$ ) for binomial (MV2) algorithm.