A Sampling Algebra for Aggregate Estimation

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ABSTRACT

As of 2005, sampling has been incorporated in all major database systems. While efficient sampling techniques are realizable, determining the accuracy of an estimate obtained from the sample is still an unresolved problem. In this paper, we present a theoretical framework that allows an elegant treatment of the problem. We base our work on generalized uniform sampling (GUS), a class of sampling methods that subsumes a wide variety of sampling techniques. We introduce a key notion of equivalence that allows GUS sampling operators to *commute* with selection and join, and derivation of confidence intervals. We illustrate the theory through extensive examples and give indications on how to use it to provide meaningful estimations in database systems.

1. INTRODUCTION

Sampling has long been used by database practitioners to speed up query evaluation, especially over very large data sets. For many years it was common to see SQL code of the form "`WHERE RAND() > 0.99`". Widespread use of this sort of code lead to the inclusion of the `TABLESAMPLE` clause in the SQL-2003 standard [?]. Since then, all major databases have incorporated native support for sampling over relations. One such query, using the TPC-H schema, is:

```
SELECT SUM(l_discount*(1.0-l_tax))
FROM lineitem TABLESAMPLE (10 PERCENT),
     orders TABLESAMPLE (1000 ROWS)
WHERE l_orderkey  = o_orderkey AND
      l_extendedprice > 100.0;
```

The result of this query is obtained by taking a Bernoulli sample with $p = .1$ over `lineitem` and joining it with a sample obtained without replacement (WOR) of size 1000 from `orders` and *evaluating* the `SUM` aggregate.

In practice, there are two main reasons practitioners write such code. One is that sampling is useful for debugging expensive queries. The query can be quickly evaluated over a sample as a sanity check, before it is unleashed upon the full database.

The second reason is that the practitioner is interested in obtaining an idea as to what the actual answer to the query would be, in less time than would be required to run the query over the entire database. This might be useful as a prelude to running the query "for real"—the user might want to see if the result is potentially interesting—or else the estimate might be used in place of the actual answer. Often, this situation arises when the query in question performs an aggregation, since it is fairly intuitive to most users that sampling can be used to obtain a number that is a reasonable approximation of the actual answer.

The problem we consider in this paper comes from the desire to use sampling as an approximation methodology. In this case, the user is not actually interested in computing an aggregate such as "`SUM(l_discount`*(1.0-`l_tax`))" over a sample of the database. Rather, s/he is interested in estimating the *answer* to such a query over the entire database using the sample. This presents two obvious problems:

- First, what SQL code should the practitioner write in order to compute an estimate for a particular aggregate?
- Second, how does the practitioner have any idea how accurate that estimate is?

Ideally, a database system would have built-in mechanisms that automatically provide estimators for user-supplied aggregate queries, and that automatically provide users with accuracy guarantees. Along those lines, in this paper we study how to automatically support SQL of the form:

```
CREATE VIEW APPROX (lo, hi) AS
SELECT QUANTILE(SUM(l_discount*(1.0-l_tax)), 0.05)
       QUANTILE(SUM(l_discount*(1.0-l_tax)), 0.95)
FROM lineitem TABLESAMPLE (10 PERCENT),
       orders TABLESAMPLE(1000 ROWS)
WHERE l_orderkey  = o_orderkey AND
        l_extendedprice > 100.0;
```

Presented with such a query, the database engine will use the user-specified sampling to automatically compute two values `lo` and `hi` that can be used as a $[0.05, 0.95]$ confidence bound on the true answer to the query. That is, the user has asked the system to compute values `lo` and `hi` such that there is a 5% chance that the true answer is less than `lo`, and there is a 95% chance that the true answer is less than `hi`. In the general case, the user should be able to specify any aggregate over any number of sampled base tables using any sampling scheme, and the system would automatically figure out how to compute an estimate of the desired quantile. A database practitioner need have no idea how to compute an estimate for the answer, nor does s/he need to have any idea how to compute confidence bounds; the user only specifies the desired quantiles, and the system does the rest.

Existing Work on Database Sampling. This is not an easy problem to solve. While there has been a lot of research on implementing efficient sampling algorithms [**?**, **?**], providing confidence intervals for the sample estimate is understood only for a few restricted cases. The simplest is when only a single relation is sampled. A slightly more complicated case was handled by the AQUA system developed at Bell labs [**?**, **?**, **?**, **?**]. AQUA considered *correlated sampling* where a fact table in a star schema is sampled. These cases are relatively simple because when a single table is sampled, classical sampling theory applies with a few easy modifications. Simultaneous work on ripple joins and online aggregation [**?**, **?**, **?**, **?**, **?**] extended the class of queries amenable to analysis to include those queries where multiple tables are sampled with replacement and then joined.

Unfortunately, the extension to other types of sampling is not straightforward, and to date new formulas have been derived every time a new sampling is considered (for example, two-table without-replacement sampling [**?**]). Our goal is to provide a simple theory that makes it possible to handle very general types of queries over virtually any uniform sampling scheme: with replacement sampling, fixed-size without replacement sampling, Bernoulli sampling, or whatever other sampling scheme is used. The ability to easily handle arbitrary types of sampling is especially important given that the current SQL standard allows for a somewhat mysterious `SYSTEM` sampling specification, whose exact implementation (and hence its statistical properties) are left up to the database designers. Ideally, it should be easy for a database designer to apply our theory to an arbitrary `SYSTEM` sampling implementation.

Generalized Uniform Sampling. One major reason that new theory and derivations were previously required for each new type of sampling is that the usual analysis is tuple-based, where the inclusion probability of each tuple in the output set is used as the basic building block; computing expected values and variances requires intricate algebraic manipulations of complicated summations. In previous work, we defined a notion that we called *Generalized Uniform Sampling* (GUS) [**?**] that subsumes many different sampling schemes (including all of the aforementioned ones, as well as block-based variants thereof). In this paper, we develop an algebra over many common relational operators, as well as the GUS operator. This makes it possible to take any query plan that contains one or more GUS operators and the supported relational operators, and perform a statistical analysis of the accuracy of the result in an algebraic fashion, working from the leaves up to the top of the plan.

No complicated algebraic manipulations over nested summations are required. This algebra can form the basis for a lightweight tool for providing estimates and quantiles, that should be easily integrable into any database system. The database need only feed the tool the user-specified quantiles, the set of tuples returned by the query, some simple lineage information over those result tuples, and the query plan, and the tool can automatically compute the desired quantiles.

Our Contributions. The specific contributions we make in this paper are:

- We define the notion of Second Order Analytical (SOA)-equivalence, a key equivalence relationship between query plans that is strong enough to allow quantile analysis but weak enough to ensure commutativity of sampling and relational operators.

- We define the GUS operator that emulates a wide class of sampling methods. This operator commutes with most relational operators under SOA-equivalence.

- We develop an algebra over GUS and relational operators that allows derivation of SOA-equivalent plans. These plans easily allow moment calculations that can be used to estimate quantiles.

- We describe how our theory can be used to add estimation capabilities to existing databases so that the required changes to the query optimizer and execution engine are minimal. Alternatively, the estimator can be implemented as an external tool.

Our work provides a straightforward analysis for the SUM aggregate. It can be easily extended for COUNT by substituting the aggregated attribute to 1 and applying the analysis for SUM on this attribute. Though the analysis for AVERAGE presents a slightly non-linear case, the analyses for SUM and COUNT lay a foundation for it. The confidence intervals can be derived using a method for approximating probability distribution/variance such as the delta method. The analysis for MIN, MAX and DISTINCT are extremely hard problems to solve due to their non-linearity. For example DISTINCT requires an estimate of all the distinct values in the data and the number of such values. It is thus beyond the scope of this paper.

While selections and joins are the highlight of our paper, we show that SOA-equivalence allows analysis for other database operators like cross-product (compaction), intersection (concatenation) and union. Due to space constraints, we are unable to include all technical proofs, implementation details and discussions. These are available in the the extended version of this paper[].

The rest of the paper is organized as follows. In Section 2, we provide a brief overview of related work in this area. In Section 3, we introduce GUS methods and give details on how to get estimates and confidence intervals for them. In Section 4, we introduce the notion of SOA-equivalence between query plans and prove that GUS operators commute with a variety of relational operators in the SOA sense. In Section 5, we investigate interactions between GUS operators when applied to the same data and explore more possibilities in using them. In Section **??**, we provide insight on how our theory can be used to implement a separate add-on tool and how to enhance the performance of the variance estimation. In Section **??**, we test our implementation thoroughly, and provide accuracy and runtime analysis. We explore some possible applications in Section **??** and conclude with a discussion in Section 8.

2. RELATED WORK

The idea of using sampling in databases for deriving estimates for a single relation was first studied by Shapiro et al. [?]. Since then, much research has focused on implementing efficient sampling algorithms in databases [?, ?]. Providing confidence intervals on estimates for SQL aggregate queries is difficult, which is why there has been limited progress in this area. Olken [?] studied the problem for specific sampling methods for a single relation. This line of work ended abruptly when Chaudhuri et al. [?, ?] proved that extracting IID samples from a join of two relations is infeasible.

Another line of research was the extension to the *correlated sampling* pioneered by the AQUA system [?, ?, ?]. AQUA is applicable to a star schema, where the goal is sampling from the fact table, and including all tuples in dimension tables that match selected fact table tuples. The AQUA type of sampling has been incorporated in DB2 [?].

The reason confidence intervals can be provided for AQUA type sampling is the fact that *independent identically distributed* (IID) samples are obtained from the set over which the aggregate is computed. A straightforward use of the *central limit theorem* readily allows computation of good estimates and confidence intervals. Indeed, it is widely believed [?, ?, ?, ?, ?, ?] that IID samples at the top of the query plan are required to provide *any* confidence interval. This idea leads to the search for a *sampling operator* that commutes with database operators. This endeavor proved to be very difficult from the beginning [?] when joins are involved. To see why this is the case, consider a tuple $t \in$ `orders` and two tuples u_1, u_2 in `lineitem` that join with t (i.e. they have the same value for `orderkey`). Random selection of tuples t, u_1, u_2 in the sample does not guarantee random selection of result tuples (t, u_1) and (t, u_2). If t is not selected, neither tuple can exist, and thus sampling is correlated. A lot of effort [?, ?] has been spent in finding practical ways to de-correlate the result tuples with only limited success.

Progress has been made using a different line of thought by Hellerstein and Hass [?] and the generalization in [?] for the special case of sampling with replacement. The problem of producing IID result samples is avoided by developing central limit theorem-like results for the combination of relation level sampling with replacement. The theory was generalized first to *sampling without replacement* for single join queries [?], then further generalized to arbitrary uniform sampling over base directions and arbitrary `SELECT-FROM-WHERE` queries without duplicate elimination in DBO [?], and finally to allow sampling across multiple relations in Turbo-DBO [?]. Even though some simplification occurred through these theoretical developments, they are mathematically heavy and hard to understand/interpret. Moreover, the theory, especially DBO and Turbo-DBO, is tightly coupled with the systems developed to exploit it.

Technically, one major problem in all the mathematics used to analyze sampling schemes is the fact the analyses use functions and summations over tuple domains, and not the operators and algebras that the database community is used to. This makes the theory hard to comprehend and apply. The fact that no database system picked up these ideas to provide a confidence interval facility is a direct testament of these difficulties.

3. GENERALIZED UNIFORM SAMPLING

Previous attempts at accommodating a sampling operator in a query plan were limited to specific sampling methods. In previous work [?], we analyzed a large class of sampling methods for which the analysis can be unified: Generalized Uniform Sampling(GUS). Sampling methods such as uniform sampling with/without replacement, Bernoulli sampling and more elaborate strategies like the chaining in [?] are members of the GUS family. Moreover, the variance of any GUS sampling can be efficiently estimated. We briefly introduce GUS sampling methods in this section and investigate them further in this paper.

DEFINITION 1 (GUS SAMPLING [?]). *A randomized selection process $\mathcal{G}_{(a,\bar{b})}$ which gives a sample $\mathcal{R}$ from $\mathbf{R} = R_1 \times R_2 \times \cdots \times R_n$ is called Generalized Uniform Sampling (GUS) method, , if, for any given tuples $t = (t_1, \ldots, t_n) \in \mathbf{R}$ $t' = (t'_1, \ldots, t'_n)$, $P(t \in \mathcal{R})$ is independent of t, and $P(t, t' \in \mathcal{R})$ depends only on $\{i : t_i = t'_i\}$. In such a case, the GUS parameters a, $\bar{b} = \{b_T | T \subset \{1 : n\}\}$ are defined as:*

$$a = P[t \in \mathcal{R}]$$
$$b_T = P[t \in \mathcal{R} \wedge t' \in \mathcal{R} | \forall i \in T, t_i = t'_i, \forall j \in T^C, t_j \neq t'_j].$$

This definition requires GUS sampling to behave like a *randomized filter*. In particular, any GUS operator can be viewed as a selection process from the underlying data, a process that can introduce correlations. The uniformity of GUS requires that the randomized filtering is performed on *lineage* of tuples and not on the content. As simple as the idea is, expressing any sampling process in the form of GUS is a non-trivial task. Example 1 shows the calculation of GUS parameters for a simple case.

EXAMPLE 1. *In this example we show how the GUS definition above can be used to characterize the estimation necessary for the query from the paper's introduction. We denote by `l_s` the Bernoulli sample with $p = 0.1$ from `lineitem` and by `o_s` the WOR sample of size 1000 from `orders`. We assume that cardinality of `orders` is 150000. Henceforth, for ease of exposition, we will denote all base relations involved by their first letters. For example, `lineitem` will be denoted by `l`.*

Applying the definition above and the independence between sampling processes, we can derive the parameters for this GUS as follows: For any tuple $t \in$ `lineitem` and tuple $u \in$ `orders`:

$$a = P[(t \in \texttt{l\_s}) \wedge (u \in \texttt{u\_s})] = 0.1 \times \frac{1000}{150000} = 6.667 \times 10^{-4}$$

since the base relations are sampled independently from each other. For any tuples $t, t' \in$ `lineitem` and $u, u' \in$ `orders`:

$$\begin{aligned} b_\varnothing &= P[(t, t' \in \texttt{l\_s}) \wedge (u, u') \in \texttt{o\_s}] \\ &= 0.1 \times 0.1 \times \frac{1000}{150000} \times \frac{999}{149999} \\ &= 4.44 \times 10^{-7}, \end{aligned}$$

and

$$\begin{aligned} b_o &= P[t \in \texttt{l\_s}] \times P[t' \in \texttt{l\_s} | t \in \texttt{l\_s}] \times P[u \in \texttt{o\_s}] \\ &= 0.1 \times 0.1 \times \frac{1000}{150000} = 6.667 \times 10^{-5}. \end{aligned}$$

Similarly,

$$\begin{aligned} b_l &= P[(t \in \texttt{l\_s}) \wedge (u, u' \in \texttt{o\_s})] \\ &= P[t \in \texttt{l\_s}] \times P[u \in \texttt{o\_s}] \times P[u' \in \texttt{o\_s} | u \in \texttt{o\_s}] \\ &= 0.1 \times \frac{1000}{150000} \times \frac{999}{149999} \\ &= 4.44 \times 10^{-6}. \end{aligned}$$

The last term is

$$b_{l,o} = P[(t \in \texttt{l\_s}) \wedge (u \in \texttt{o\_s})] = 0.1 \times \frac{1000}{150000} = 6.667 \times 10^{-4}.$$

Notice that the GUS captures *the entire estimation process, not only the two individual sampling methods. The above analysis dealt with a simple join consisting of two base relations. For more complex query plans, the derivation of GUS parameters would involve consideration of all possible interactions between participating tuples. This will make the analysis highly complex.*

The analysis of any GUS sampling method for a SUM-like aggregate is given as follows.

THEOREM 1. *[?] Let $f(t)$ be a function/property of $t \in R$, and $\mathcal{R}$ be the sample obtained by a GUS method $\mathcal{G}_{(a,\bar{b})}$. Then, the aggregate $\mathcal{A} = \sum_{\mathbf{t} \in R} f(\mathbf{t})$ and the sampling estimate $X = \frac{1}{a} \sum_{\mathbf{t} \in \mathcal{R}} f(\mathbf{t})$ have the property:*

$$E[X] = \mathcal{A}$$
$$\sigma^2(X) = \sum_{S \subset \{1:n\}} \frac{c_S}{a^2} y_S - y_\phi \qquad (1)$$

with

$$y_S = \sum_{t_i \in R_i | i \in S} \left(\sum_{t_j \in R_j | j \in S^C} f(t_i, t_j) \right)^2$$
$$c_S = \sum_{T \in \mathcal{P}(n)} (-1)^{|T|+|S|} b_T.$$

The above theorem indicates that the GUS estimates of SUM-like aggregates are unbiased and that the variance is simply a linear combination of properties of the data, terms y_S and properties of the GUS sampling method c_S. Moreover, y_S can be estimated from samples of *any* GUS [?]. This result is not asymptotic; it gives the exact analysis even for very small samples. Once the estimate and the variance are computed, confidence intervals can be readily provided using either the normality assumption or the more conservative *Chebychev* bound [?].

In the rest of the paper, we will study GUS sampling methods in detail.

4. ANALYSIS OF SAMPLING QUERY PLANS

The high-level goal of this paper, is to introduce a tool that computes the confidence bounds of estimates based on sampling. Given a query plan with sampling operators interspersed at various points, our tool transforms it to an *analytically equivalent* query plan that has a particular structure: all relational operators except the final aggregate form a subtree that is the input to a single GUS sampling operator. The GUS operator feeds the aggregate operator that produces the final result. Note that this transformation is done solely for the purpose of computing the confidence bounds of the result; it does not provide a better alternative to the execution plan used as input. Once this transformation is accomplished, Theorem 1 readily gives the desired analysis – the equivalence ensures that the analysis for the special plan coincides with the analysis for the original plan.

A natural strategy to obtain the desired structure is to perform multiple local transformations on the original query plan. These local transformations are based on a notion of analytical equivalence, that we call Second Order Analytical (SOA) equivalence. They allow both commutativity of relational and GUS operators, and consolidation of GUS operators. Effectively, these local transformations allow a plan to be put in the special form in which there is a single GUS operator just before the aggregate.

In this section, we first define the SOA-equivalence and then use it to provide *equivalence* relationships that allow the plan transformations mentioned. A more elaborate example showcases the theory in the latter part of the section.

4.1 SOA-Equivalence

The main reason the previous attempts to design a *sampling operator* were not fully successful is the requirement to ensure IID samples at the top of the plan. Having IID samples makes the analysis easy since Central Limit Theorem readily provides confidence intervals. However it is too restrictive to allow plans with multiple joins to be dealt with. It is important to notice that the difficulty is not in *executing* query plans containing sampling but in *analyzing* such query plans.

The fundamental question we ask in this section is: What is the least restrictive requirement we can have and still produce useful estimates? Our main interest is in how the requirement can be transformed into a notion of *equivalence*. This will enable us to talk about *equivalent plans*, initially, but more usefully about *equivalent expressions*. The key insight comes from the observation that it is enough to compute the expected value and variance of any approximate query plan. Then either the conservative Chebychev bounds or the optimistic[1] normal-distribution based bounds can be used to produce confidence intervals. Note that confidence intervals are the end goal, and, preserving expected value and variance is enough to guarantee the same confidence interval using both CLT and Chebychev methods.

Thus, for our purposes, *two query plans are equivalent if their result has the same expected value and variance.* This equivalence relation between plans already allows significant progress. It is an extension of the classic plan equivalence based on obtaining the same answer to *approximate/randomized* plans. From an operational sense, though, the plan equivalence is not sufficient to provide interesting characterizations. The main problem is the fact that the equivalence exists only between complete plans that compute aggregates. It is not clear what can be said about intermediate results—the equivalent of non-aggregate relational algebra expressions.

The key to extend the equivalence of plans to equivalence of expressions is to first design such an extension for the classic relational algebra. To this end, assume that we can only

[1]While the CLT theorem does not apply due to the lack of IID samples, the distribution of most complex random variables made out of many loosely interacting parts tends to be normal.

use equality on numbers that are results of SUM-like aggregates but we cannot directly compare sets. To ensure that two expressions are equivalent, we could require that they produce the same answer using **any** SUM-aggregate. Indeed, if the expressions produce the same relation/set, they *must* agree on *any* aggregate computation using these sets since aggregates are deterministic and, more importantly, do not depend on the order in which the computation is performed. The SUM-aggregates are crucial for this definition since they form a vector space. Aggregates A_t that sums function $f_t(u) = \delta_{tu}$ are the basis of this vector space; agreement on these aggregates ensures set agreement. Extending these ideas to randomized estimation, we obtain the following.

Definition 2 (SOA-equivalence). *Given (possibly randomized) expressions $\mathcal{E}(R)$ and $\mathcal{F}(R)$, we say*

$$\mathcal{E}(R) \overset{SOA}{\Longleftrightarrow} \mathcal{F}(R)$$

*if for **any arbitrary** SUM-aggregate $\mathcal{A}_f(S) = \sum_{t \in S} f(t)$*

$$E[\mathcal{A}_f(\mathcal{E}(R))] = E[\mathcal{A}_f(\mathcal{F}(R))]$$
$$Var[\mathcal{A}_f(\mathcal{E}(R))] = Var[\mathcal{A}_f(\mathcal{F}(R))].$$

From the above discussion, it immediately follows that SOA-equivalence is a generalization and implies set equivalence for non-randomized expressions, as stated in the following proposition.

Proposition 1. *Given two relational algebra expressions $E(R)$ and $F(R)$ we have:*

$$E(R) = F(R) \Leftrightarrow E(R) \overset{SOA}{\Longleftrightarrow} F(R)$$

The next proposition establishes that SOA-equivalence is indeed an *equivalence relation* and can be manipulated like relational equivalence.

Proposition 2. *SOA-equivalence is an equivalence relation, i.e., for any expressions $\mathcal{E}, \mathcal{F}, \mathcal{H}$ and relation R:*

$$\mathcal{E}(R) \overset{SOA}{\Longleftrightarrow} \mathcal{E}(R)$$
$$\mathcal{E}(R) \overset{SOA}{\Longleftrightarrow} \mathcal{F}(R) \Rightarrow \mathcal{F}(R) \overset{SOA}{\Longleftrightarrow} \mathcal{E}(R)$$
$$\mathcal{E}(R) \overset{SOA}{\Longleftrightarrow} \mathcal{F}(R) \wedge \mathcal{F}(R) \overset{SOA}{\Longleftrightarrow} \mathcal{H}(R) \Rightarrow \mathcal{E}(R) \overset{SOA}{\Longleftrightarrow} \mathcal{H}(R).$$

SOA-equivalence subsumes relational algebra equivalence. The strength of SOA-equivalence is the fact that it does not depend on a notion of randomized set equivalence, an equivalence that would be hard to define especially if it has to preserve aggregates.

Proposition 3. *Given two relational algebra expressions $\mathcal{E}(R)$ and $\mathcal{F}(R)$ we have:*

$$\mathcal{E}(R) \overset{SOA}{\Longleftrightarrow} \mathcal{F}(R)$$
$$\Leftrightarrow$$
$$\forall t \in R, \quad P[t \in \mathcal{E}(R)] = P[t \in \mathcal{F}(R)] \quad \textit{and}$$
$$\forall t, u \in R, \quad P[t, u \in \mathcal{E}(R)] = P[t, u \in \mathcal{F}(R)]$$

Proposition 3 provides a powerful alternative to SOA-equivalence. This equivalence is in terms of first and second order probabilities, and we refer to it as *SOA-set equivalence.* Another way to interpret the result above is that SOA-set equivalence is the same as agreement on all SUM-like aggregates. More importantly for this paper, SOA-set equivalence provides an alternative proof technique to show SOA-equivalence. Often, proofs based on SOA-set equivalence are simpler and more compact.

Sampling method	GUS parameters
Bernoulli(p)	$a = p$, $b_\varnothing = p^2$, $b_R = p$
WOR (n, N)	$a = \frac{n}{N}, b_\varnothing = \frac{n(n-1)}{N(N-1)}, b_R = \frac{n}{N}$

Figure 1: GUS parameters for known sampling methods on a single relation

Section **??** contains a recipe for expected value and variance computation for a specific situation, when there is a single overall GUS sampling on top. Starting with the given query plan that contains both sampling and relational operators, if we find a SOA-equivalent plan that is equivalent and has no sampling except a GUS at the top, we readily have a way to compute the expected value and variance of the original plan. In the rest of this section we pursue this idea further and show how SOA-equivalent plans with the desired structure can be obtained from a general query plan.

4.2 GUS Quasi-Operators

Except under restrictive circumstances, the sampling operators will not commute with relational operators. This, as we mentioned is the main reason previous work made limited progress on the issue. As we will see later in this section, GUS sampling does commute in a SOA-equivalence sense with most relational operators. The reason we can commute GUS (but not specific sampling methods) is that, due to its generality, it can *capture* the correlations induced by the relational operators. The first step in our analysis has to be a *translation* from specific sampling to GUS-sampling.

Before we talk about the translation from sampling to GUS operators, we need to clarify and refine the Definition 1 of GUS sampling. As part of the definition, terms of the form $t_i = t'_i$ or $t_j \neq t'_j$ are used. The meaning of these terms is somewhat fuzzy in both [**?**] and [**?**]. Intuitively, they capture the idea that tuples (or parts) are the same or different. Since in this paper we will have multiple GUS operators involved, it is important to make the meaning of such terms very clear. We do this through a notion that proved useful in probabilistic databases (among other uses): *lineage*[**?**]. Lineage allows dissociation of the ID of a tuple from the content of the tuple, for base relation, and tracking the composition of derived tuples. With this, $t_i = t'_i$ means that the two tuples are the same – have the same ID/lineage – not that they have the same content.

Representing and manipulating lineage is a complex subject. In this work, since we only accommodate selection and joins the issue is significantly simpler. The selection leaves lineage unchanged, the lineage of the result of the join is the union of the lineage of the matching tuples. Thus, lineage can be represented in relational form with one attribute for each base relation participating in the expression. We can thus talk about *lineage schema* $\mathcal{L}(R)$, a synonym of the set of base relations participating in the expression of R. The lineage of a specific tuple $t \in R$ will have values for the lineage of all base relations constituting R. A particularly useful notation related to lineage is: $\mathcal{T}(t, t') = \{R_k | t_k = t_k', k \in \mathcal{L}(R)\}$, the common part of the lineage of tuples t and t', i.e. the base relations on which the lineage of t and t' agree.

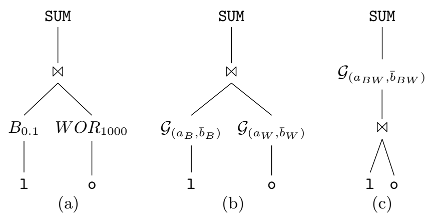

SUM
⋈
$B_{0.1}$
WOR_{1000}
l
o
(a)
SUM
⋈
$\mathcal{G}_{(a_B,\bar{b}_B)}$
$\mathcal{G}_{(a_W,\bar{b}_W)}$
l
o
(b)
SUM
$\mathcal{G}_{(a_{BW},\bar{b}_{BW})}$
⋈
l o
(c)

Figure 2: Query 1

Example 2. *The query from the introduction uses two sampling methods: Bernoulli sampling with* $p = 0.1$ *on* `lineitem` *and sampling 1000 tuples without replacement from* `orders(150,000` *tuples). These methods can be expressed in terms of GUS as* $\mathcal{G}_{(a_B,\bar{b}_B)}$ *and* $\mathcal{G}_{(a_W,\bar{b}_W)}$ *as follows: For* $\mathcal{G}_{(a_B,\bar{b}_B)}$*:* $a_B = 0.1$ *and* $\bar{b}_B = \{b_{B,\varnothing}, b_{B,\imath}; b_{B,\varnothing} = 0.01, b_{B,\imath} = 0.1\}$ *For* $\mathcal{G}_{(a_W,\bar{b}_W)}$*:* $a_W = 6.667 \times 10^{-3}$ *and* $\bar{b}_W = \{b_{W,\varnothing}, b_{W,o}; b_{W,\varnothing} = 4.44 \times 10^{-5}, b_{W,o} = 6.667 \times 10^{-3}\}$

It is important to note that the GUS is not an operator but a quasi-operator. While it corresponds to a real operator when the translation from specific sampling to GUS happens, it will not correspond to an operator after transformations. There is no need to provide or even to consider an implementation of a general GUS operator since GUS will only be used for the purpose of analysis.

In the rest of this section, we will assume that all specific sampling operators were replaced by GUS quasi-operators, thus will not be encountered by the re-writing algorithm. We designate by $\mathcal{G}_{(a,\bar{b})}(R)$, a GUS method applied to a relation R, and the resulting sample by $\mathcal{R}$. When multiple GUS methods are used, the i'th GUS method and its resulting sample will be denoted by $\mathcal{G}_{(a_i,\bar{b}_i)}$ and $\mathcal{R}_i$ respectively.

4.3 Interaction Between GUS and Rel Ops

As we stated in Section 4.1, SOA-equivalence is the key for deriving an *analyzable* plan that is *equivalent* to the one provided by the user. The results in this section provide equivalences that allow such transformations that lead to a single, top, GUS operator. The results in this section make use of the notation in Table 4.2.

Proposition 4 (Identity GUS). *The quasi-operator* $\mathcal{G}_{(1,\bar{1})}$*, i.e. a GUS operator with* $a = 1$*,* $b_T = 1$*, can be inserted at any point in a query plan without changing the result.*

Notation	Meaning
$\mathcal{R}$	Random subset of R
a	$P[t \in \mathcal{R}]$
$\mathcal{L}(R)$	Lineage schema of R
$\mathcal{L}(t)$	Lineage of tuple t
T	Subset of $\mathcal{L}(R)$
$\mathcal{T}(t,t')$	$\{R_k \vert t_k = t_k', k \in \mathcal{L}(R)\}$
b_T	$P[t,t' \in \mathcal{R} \vert T = \mathcal{T}(t,t')]$
$\bar{b}$	$\{b_T \vert T \in \mathcal{P}(n)\}$
$\mathcal{G}_{(a,\bar{b})}$	GUS method with parameters a and $\bar{b}$
$\mathcal{G}_{(a,\bar{b})}(R)$	$\mathcal{G}_{(a,\bar{b})}$ applied to relation R

Figure 3: Notation used in paper

Proof. Since $a = 1$, all input tuples are allowed with probability 1, i.e., no filtering happens. $\square$

Proposition 5 (Selection-GUS Commutativity). *For any R, selection σ_C and GUS $\mathcal{G}_{(a,\bar{b})}$,*

$$\sigma_C(\mathcal{G}_{(a,\bar{b})})(R) \overset{SOA}{\Longleftrightarrow} \mathcal{G}_{(a,\bar{b})}(\sigma_C(R)).$$

Proof. Let $R' = \sigma_C(R)$. On computing $\mathcal{R} \cap R'$ we see that

$$\forall(t \in R'), P[t \in \mathcal{R} \cap R'] = P[t \in \mathcal{R}] I_{\{t \in R'\}} = a.$$

$$\begin{aligned}\forall(t,t' \in R'), &P[t,t' \in \mathcal{R} \cap R' \vert T = \mathcal{T}(t,t')] \\ &= P[t,t' \in \mathcal{R} \vert T = \mathcal{T}(t,t')] = b_T \quad \square\end{aligned}$$

The above results are somewhat expected and have been covered for particular cases in previous literature. The following result, though, overcomes the difficulties in [?].

Proposition 6 (Join-GUS Commutativity). *For any R, S, join $\bowtie_\theta$ and GUS methods $\mathcal{G}_{(a_1,\bar{b}_1)}, \mathcal{G}_{(a_2,\bar{b}_2)}$, if $\mathcal{L}(R_1) \cap \mathcal{L}(R_2) = \varnothing$*

$$\begin{gathered}\mathcal{G}_{(a_1,\bar{b}_1)}(R_1) \bowtie_\theta \mathcal{G}_{(a_2,\bar{b}_2)}(R_2) \overset{SOA}{\Longleftrightarrow} \mathcal{G}_{(a,\bar{b})}(R_1 \bowtie_\theta R_2), \\ \textit{where}, \qquad a = a_1 a_2, \qquad b_T = b_{1,T_1} b_{2,T_2}\end{gathered}$$

with $T_1 = T \cap \mathcal{L}(R_1)$ and $T_2 = T \cap \mathcal{L}(R_2)$.

Proof. We proved in Proposition 5 that a GUS method commutes with selection. Thus, it is enough to prove commutativity of a GUS method with cross product. Let $R = R_1 \times R_2$ and $t = (t_1,t_2), t' = (t_1',t_2') \in R$. Thus, $\mathcal{L}(R) = \mathcal{L}(R_1) \cup \mathcal{L}(R_2)$. We have:

$$\begin{aligned}a = P[t \in \mathcal{R}] &= P[t_1 \in \mathcal{R}_1 \wedge t_2 \in \mathcal{R}_2] \\ &= P[t_1 \in \mathcal{R}_1 \wedge t_2 \in \mathcal{R}_2] = a_1 a_2.\end{aligned}$$

Since $\mathcal{L}(R_1) \cap \mathcal{L}(R_2) = \varnothing$, for an arbitrary $T \in \mathcal{L}(R)$, $T_1 = T \cap \mathcal{L}(R_1)$ and $T_2 = T \cap \mathcal{L}(R_2)$ we have, $T_1 \cap T_2 = \varnothing$ (disjunct lineage). With this, we first get:

$$\mathcal{T}(t,t') = T \Leftrightarrow \mathcal{T}(t_1,t_1') = T_1 \wedge \mathcal{T}(t_2,t_2') = T_2$$

and then using the above and independence of GUS methods,

$$\begin{aligned}b_T &= P[t \in \mathcal{R} \wedge t' \in \mathcal{R} \vert \mathcal{T}(t,t') = T] \\ &= P[t_1,t_1' \in \mathcal{R}_1 \wedge t_2,t_2' \in \mathcal{R}_2 \vert \mathcal{T}(t_1,t_1') = T_1 \wedge \mathcal{T}(t_2,t_2') = T_2] \\ &= P[t_1,t_1' \in \mathcal{R}_1 \vert \mathcal{T}(t_1,t_1') = T_1] P[t_2,t_2' \in \mathcal{R}_2 \vert \mathcal{T}(t_2,t_2') = T_2] \\ &= b_{1,T_1} b_{2,T_2}. \quad \square\end{aligned}$$

Example 3. *Applying the above results to the GUS coefficients obtained in Example 2, we can derive the following co-efficients for $\mathcal{G}_{(a,\bar{b})}$ in Fig 4.2:*

$$a = a_1 a_2 = 0.1 \times 6.667 \times 10^{-3} = 6.667 \times 10^{-4}.$$

$$b_\varnothing = b_{1,\varnothing} b_{2,\varnothing} = 0.01 \times 4.44 \times 10^{-5} = 4.44 \times 10^{-7}.$$

$$b_o = b_{1,\varnothing} b_{2,o} = 0.01 \times 6.667 \times 10^{-3} = 6.667 \times 10^{-5}.$$

$$b_\imath = b_{1,\imath} b_{2,\varnothing} = 0.1 \times 4.44 \times 10^{-5} = 4.44 \times 10^{-6}.$$

$$b_{\imath o} = b_{1,\imath} b_{2,o} = 0.1 \times 6.667 \times 10^{-3} = 6.667 \times 10^{-4}.$$

Example 4. *In this example we provide a complete walk-through for a larger query plan. The input is the query plan in Figure 4.a that contains 3 sampling operators, 3 joins and refers to relations* `lineitem, orders, customers` *and* `part`. *To analyze such a query, the first step is to re-write the sampling operators as GUS quasi-operators* $\mathcal{G}_{(a_1,\bar{b}_1)}$, $\mathcal{G}_{(a_2,\bar{b}_2)}$, $\mathcal{G}_{(a_3,\bar{b}_3)}$ *as in Figure 4.b. The second step, shown in Figure 4.c is to apply Proposition 6 to commute* $\mathcal{G}_{(a_1,\bar{b}_1)}$ *and* $\mathcal{G}_{(a_2,\bar{b}_2)}$ *with the join resulting in* $\mathcal{G}_{(a_{12},\bar{b}_{12})}$ *. This step also shows the application of Proposition 4 above* `customers`. *The next step in Figure 4.d again uses Proposition 6 to commute* $\mathcal{G}_{(a_{12},\bar{b}_{12})}$ *and* $\mathcal{G}_{(1,\bar{1})}$ *resulting in* $\mathcal{G}_{(a_{121},\bar{b}_{121})}$. *Figure 4.e shows the final transformation that uses the same proposition to get an overall GUS method* $\mathcal{G}_{(a_{123},\bar{b}_{123})}$ *just below the aggregate and on the top of the rest of the plan. Theorem 1 can now be used to obtain expected value and variance of the estimate. Using this and either the normal approximation or the Chebychev bounds, we obtain confidence intervals for the estimate.*

The computed coefficients for the GUS methods involved are depicted in Figure 4

5. PROPERTIES OF GUS OPERATORS

In the previous section we explored the interaction between GUS operators and relational algebra operators. In this section, we investigate interactions between GUS operators when applied to the same data. Intuitively, this will open up avenues for design of sampling operators, since it will indicate how to compute GUS quasi-operators that correspond to complex sampling schemes.

Proposition 7 (GUS Union). *For any expression R and GUS methods* $\mathcal{G}_{(a_1,\bar{b}_1)}$, $\mathcal{G}_{(a_2,\bar{b}_2)}$,

$$\mathcal{G}_{(a_1,\bar{b}_1)}(R) \cup \mathcal{G}_{(a_2,\bar{b}_2)}(R) \overset{SOA}{\Longleftrightarrow} \mathcal{G}_{(a,\bar{b})}(R)$$
$$\text{where,} \qquad a = a_1 + a_2 - a_1 a_2$$
$$b_T = 2a - 1 + (1 - 2a_1 + b_{1T})(1 - 2a_2 + b_{2T})$$

Union of GUS methods can be very useful when samples are expensive to acquire, thus there is value in reusing them. If two separate samples from relation R are available, Proposition 7 provides a way to combine them.

Proposition 8 (GUS Compaction). *For any expression R, and GUS methods* $\mathcal{G}_{(a_1,\bar{b}_1)}, \mathcal{G}_{(a_2,\bar{b}_2)}$,

$$\mathcal{G}_{(a_1,\bar{b}_1)}\left(\mathcal{G}_{(a_2,\bar{b}_2)}(R)\right) \overset{SOA}{\Longleftrightarrow} \mathcal{G}_{(a,\bar{b})}(R),$$
$$\text{where,} \qquad a = a_1 a_2, \qquad b_T = b_{1,T_1} b_{2,T_2}$$

Compaction can be also viewed as intersection. It allows sampling methods to be *stacked* on top of each other to obtain smaller samples. We will make use of this in the next section.

Interestingly, union behaves like + with the null element $\mathcal{G}_{(0,\bar{0})}$ (the sampling method that blocks everything), the compaction/intersection behaves like $*$ with the null element $\mathcal{G}_{(1,\bar{1})}$. Overall, the algebraic structure formed is that of a *semi-ring*, as stated in the following.

Theorem 2. *The GUS operators over any expression R, form a* semiring *structure with respect to the union and compaction operations with* $\mathcal{G}_{(0,\bar{0})}$ *and* $\mathcal{G}_{(1,\bar{1})}$ *as the null elements, respectively.*

The semi-ring structure of GUS methods can be exploited to design sampling operators from ingredients.

Proposition 9 (GUS Composition). *For any expressions* R_1, R_2 *and* $\mathcal{G}_{(a_1,\bar{b}_1)}$, $\mathcal{G}_{(a_2,\bar{b}_2)}$,

$$\mathcal{G}_{(a_1,\bar{b}_1)}(R_1) \circ \mathcal{G}_{(a_2,\bar{b}_2)}(R_2) \overset{SOA}{\Longleftrightarrow} \mathcal{G}_{(a,\bar{b})}(R)$$
$$a_= a_1 a_2, \qquad b_T = b_{1,T} b_{2,T}$$

GUS concatenation is very useful for design of multi-dimensional sampling operators. We use it here to design a bi-dimensional Bernoulli.

Example 5. *Suppose that we designed a bi-dimensional sampling operator* $B_{0.2,0.3}(\mathtt{l,o})$ *that combines Bernoulli sampling operators* $B_{0.2}(\mathtt{l})$ *and* $B_{0.3}(\mathtt{o})$. *Using the above result, the GUS operator* $\mathcal{G}_{(a,\bar{b})}$ *corresponding to the bi-dimensional Bernoulli is* $\mathcal{G}_{(a_1,\bar{b}_1)}(\mathtt{l}) \circ \mathcal{G}_{(a_2,\bar{b}_2)}\mathtt{o}$, *where* $\mathcal{G}_{(a_1,\bar{b}_1)}$ *is the GUS of* $B_{0.2}(\mathtt{l})$ *and* $\mathcal{G}_{(a_2,\bar{b}_2)}$ *is the GUS of* $B_{0.3}(\mathtt{o})$. *Working out the coefficients using Proposition 9 - the process is similar to the process in Example 3 we get:* $a_3 = 0.06$, $b_{3,\varnothing} = 0.0036$, $b_{3,\mathtt{o}} = 0.012$, $b_{3,\mathtt{l}} = 0.018$, $b_{3,\mathtt{lo}} = 0.06$

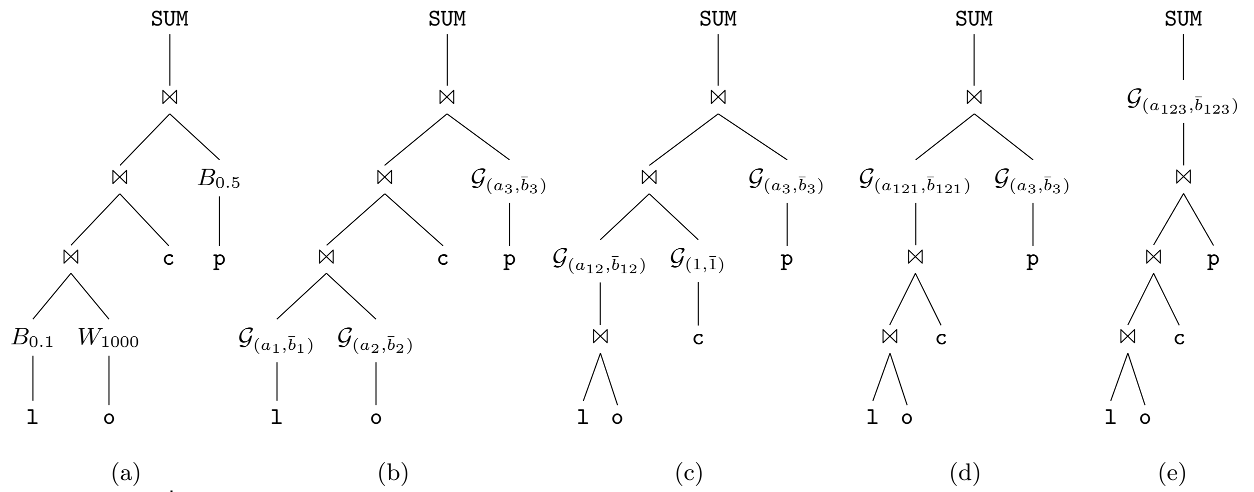
6. IMPLEMENTATION CONSIDERATIONS

In this section we carefully investigate how the theoretical ideas in the previous section can be used to add confidence interval capabilities to existing and future database systems for aggregate `'SELECT-FROM-WHERE'` queries. The main pitfall we are trying to avoid is the need to *re-design* the query processing engine. This is the main reason the *online aggregation* type of work (ripple joins[**?**], DBO[**?**]) did not have much industry impact.

As we will see in this section and the further refinement in Section 7, the solution we propose (a) will work with existing and future sampling methods/operators - the only requirement is that they are expressible as GUS operators, (b) the analysis is easy to integrate with existing query optimizers or as a separate tool, (c) there is no significant restriction on the query plan - the optimizer is not hindered in the search for a good execution plan, (d) the analysis needs minimal extra information, and (e) the estimation process can be confined to a single module that works like a *black box*.

Our solution, exemplified for Query 1 in Example 1, is depicted in Figure **??**. All the work is performed by the statistical estimator, denoted as the SBox component, that is interspersed between the main query plan and the aggregate computation. The only information the SBox needs is the *lineage* and the value of the aggregate for each tuple consumed by the aggregate. Since the SBox needs to perform the aggregation in any case, the aggregate operator can be omitted; the SBox will provide the entire result. An enhancement of this solution, that removes the need to funnel all tuples to the SBox, is explored in Section 7. We use Query 1 in Example 1 to make the entire process concrete; this allows us to express the required computations as SQL statements, which are more familiar and easier to understand. There is nothing special about Query 1. This approach works for *any* query supported by our theory with the appropriate changes to the computation.

There are three tasks that need to be performed by the SBox: use the query plan and transformations in Section 4 to compute the coefficients of the top GUS operator, estimate the coefficients y_S from the samples and perform the


SUM
⋈
$B_{0.5}$
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W_{1000}
l
o
c
p
$\mathcal{G}_{(a_1,\bar{b}_1)}$
$\mathcal{G}_{(a_2,\bar{b}_2)}$
$\mathcal{G}_{(a_3,\bar{b}_3)}$
$\mathcal{G}_{(a_{12},\bar{b}_{12})}$
$\mathcal{G}_{(1,\bar{1})}$
$\mathcal{G}_{(a_{121},\bar{b}_{121})}$
$\mathcal{G}_{(a_{123},\bar{b}_{123})}$
(a)
(b)
(c)
(d)
(e)

GUS method	Parameters
$\mathcal{G}_{(a_1,\bar{b}_1)}$	$a_1 = 0.1, b_{1,\varnothing} = 0.01, b_{1,\texttt{l}} = 0.1$
$\mathcal{G}_{(a_2,\bar{b}_2)}$	$a_2 = 6.667 \times 10^{-3}, b_{2,\varnothing} = 4.44 \times 10^{-5}, b_{2,\texttt{o}} = 6.667 \times 10^{-3}$
$\mathcal{G}_{(a_3,\bar{b}_3)}$	$a_3 = 0.5, b_{3,\varnothing} = 0.25, b_{3,\texttt{p}} = 0.5$
$\mathcal{G}_{(a_{12},\bar{b}_{12})}$	$a_{12} = 6.667\times10^{-4}, b_{12,\varnothing} = 4.44\times10^{-7}, b_{12,\texttt{o}} = 6.667\times10^{-5}, b_{12,\texttt{l}} = 4.44\times10^{-6}, b_{12,\texttt{lo}} = 6.667\times10^{-4}$
$\mathcal{G}_{(a_{121},\bar{b}_{121})}$	$a_{121} = 6.667 \times 10^{-4}, b_{121,\varnothing} = 4.44 \times 10^{-7}, b_{121,\texttt{c}} = 4.44 \times 10^{-7}, b_{121,\texttt{o}} = 6.667 \times 10^{-5}, b_{121,\texttt{oc}} = 6.667\times10^{-5}, b_{121,\texttt{l}} = 4.44\times10^{-6}, b_{121,\texttt{lc}} = 4.44\times10^{-6}, b_{121,\texttt{lo}} = 6.667\times10^{-4}, b_{121,\texttt{loc}} = 6.667\times10^{-4}$
$\mathcal{G}_{(a_{123},\bar{b}_{123})}$	$a_{123} = 3.334 \times 10^{-4}, b_{123,\varnothing} = 1.11 \times 10^{-7}, b_{123,\texttt{p}} = 2.22 \times 10^{-7}, b_{123,\texttt{c}} = 1.11 \times 10^{-7}, b_{123,\texttt{cp}} = 2.22 \times 10^{-7}, b_{123,\texttt{o}} = 1.667 \times 10^{-5}, b_{123,\texttt{op}} = 3.335 \times 10^{-5}, b_{123,\texttt{oc}} = 1.667 \times 10^{-5}, b_{123,\texttt{ocp}} = 3.335 \times 10^{-5}, b_{123,\texttt{l}} = 1.11 \times 10^{-6}, b_{123,\texttt{lp}} = 2.22 \times 10^{-6}, b_{123,\texttt{lc}} = 1.11 \times 10^{-6}, b_{123,\texttt{lcp}} = 2.22 \times 10^{-6}, b_{123,\texttt{lo}} = 1.667 \times 10^{-4}, b_{123,\texttt{lop}} = 3.334 \times 10^{-4}, b_{123,\texttt{loc}} = 1.667 \times 10^{-4}, b_{123,\texttt{locp}} = 3.334 \times 10^{-4}$

Figure 4: Transformation of the query plan to allow analysis

final expected value and variance estimate, and confidence interval computation. We discuss each part below.

6.1 Computation of the SOA-equivalent plan

Given a description of the plan the execution engine will run, and the plan that uses sampling operators, the theory in Section 4 is used to compute a SOA-equivalent plan that has a single GUS operator below the aggregate. The goal of this step is to compute the coefficients of this GUS operator. No other information is needed for other parts. This process starts by computing the GUS operators that correspond to the sampling operators using the technique in Section 4.2 – this is a simple instantiation process using Table 4.2. Then, the GUS operators are *pushed up* the query tree using the transformation rules in Section 4.3. With careful implementation, this process need not take more than a few milliseconds even for plans involving 10 relations. At the end of the process, $\mathcal{G}_{(a,\bar{b})}$ is computed. From its coefficients, using the formula in Theorem 1, the coefficients c_S are computed with the formula:

$$c_S = \sum_{T \in \mathcal{P}(n)} (-1)^{|T|+|S|}\, b_T$$

6.2 Lineage information

As mentioned in Section 4.2 the GUS operators require *lineage* information to express the computation. In the estimation process, the lineage needs to be made available to the SBox. In general, adding lineage to databases is a nontrivial issue [?]. Luckily, for GUS operators, we need only a restrictive version: the lineage of each tuple in a base table is an ID, the lineage of an intermediate tuple, is the list of IDs for each base relation tuple that participated. Since we can only accommodate selection and joins in this work, the lineage of the result of a join is the concatenation of the lineage of the arguments. In practice, all there is needed is to carry IDs of tuples through the query plan and make them available, together with the aggregate, to the SBox. For our running example, Query 1, this means that the SBox gets the result of the SQL query:

```
CREATE TABLE samples AS
SELECT l_orderkey*10+l_linenumber as l,
   o_orderkey as o, l_discount*(1.0-l_tax) as f
FROM lineitem TABLESAMPLE (10 PERCENT),
   orders TABLESAMPLE(1000 ROWS)
WHERE l_orderkey  = o_orderkey AND
   l_extendedprice > 100.0;
```

The IDs of tuples need to be unique for each tuple in a base relation. If the database engine maintains row ids internally, they can be used. If not, as is the case here, the attributes forming the primary key can be used to compute an ID, either through some computation – this is the case for `lineitem` above, or through the application of a hash function with a large domain. As required by the theory, the only operation the SBox is allowed to perform is comparison of IDs, thus any one-to-one mapping suffices.

In some systems, the extra lineage information might add significant overhead. Section 7 deals with the issue and allows further improvement.

6.3 Estimating terms y_S

The computation of the variance of the sampling estimator in Theorem 1 uses the coefficients y_S defined as:

$$y_S = \sum_{t_i \in R_i | i \in S} \left(\sum_{t_j \in R_j | j \in S^C} f(t_i, t_j) \right)^2 .$$

The terms y_S essentially requires a *group by lineage* followed by a specific computation. This is better understood through an example – Query 1 – and equivalent expressions in SQL:

```
CREATE TABLE unagg AS
SELECT  l_orderkey*10+l_linenumber as l,
   o_orderkey as o, l_discount*(1.0-l_tax) as f
FROM lineitem TABLESAMPLE (10 PERCENT),
   orders TABLESAMPLE(1000 ROWS)
WHERE l_orderkey  = o_orderkey AND
   l_extendedprice > 100.0;

SELECT sum(f)^2 as y_empy FROM unagg;

SELECT sum(F*F) as y_l FROM ( SELECT sum(f) as F
       FROM unagg GROUP BY l);

SELECT sum(F*F) as y_o FROM ( SELECT sum(f) as F
       FROM unagg GROUP BY o);

SELECT sum(f*f) as y_lo FROM unagg;
```

The computation of the y_S terms using the above code is harder than the evaluation of the exact query, thus resulting in an impractical solution. We can use the sample provided to the SBox to *estimate these terms* by essentially replacing `unagg` above with `samples`. These estimates, Y_S can be used to obtain unbiased estimates $\hat{Y}_S$ of terms y_S using the formula[**?**]

$$\hat{Y}_S = \frac{1}{c_{S,\emptyset}} \left(Y_S - \sum_{T \subset S^C, T \neq \emptyset} c_{S,T} \hat{Y}_{S \cup T} \right)$$

where

$$c_{S,T} = \sum_{U \subset T} (-1)^{|U|+|S|} b_{S \cup U}.$$

Note that the major effort is in evaluating Y_S terms over the sample - the rest of the computation only depends on the number of the relations.

6.4 Confidence interval computation

Once the $\hat{Y}_S$ estimates of y_S are computed, the variance formula in Theorem 1 can be used. In particular, the estimate of variance of the sampling estimate is:

$$\hat{\sigma^2} = \sum_{S \subset \{1:n\}} \frac{c_S}{a^2} \hat{Y}_S - \hat{Y}_\phi.$$

To produce actual confidence intervals, we can use one of the following techniques:

Optimistic confidence intervals In most circumstances the distribution of the sampling estimate is very close to *normal distribution*. The techniques in this paper allow the computation of estimates of the expected value $\hat{\mu}$ and variance $\hat{\sigma^2}$. The concrete formula for a 95 percent confidence interval is:

$$[\hat{\mu} - 1.96\hat{\sigma}, \hat{\mu} + 1.96\hat{\sigma}]$$

Pessimistic Chebychev confidence intervals If the normality of the distribution of sampling estimate is doubtful, the Chebychev bound can be used to provide 95 percent confidence interval using:

$$[\hat{\mu} - 4.47\hat{\sigma}, \hat{\mu} + 4.47\hat{\sigma}]$$

The Chebychev confidence intervals are correct for any distribution, at the expense of a factor of 2 in width.

7. EFFICIENT VARIANCE ESTIMATOR

As we explained in Section 6, the estimator of the true result, the expected value of the sampling estimator, does not require any lineage information. It is simply a scaled version of the result of the query containing sampling. When it comes to the variance estimate, there are two main concerns when the number of result tuples before aggregation is large: (a) the number of terms to be evaluated is 2^n where n is the number of base relations, and (b) ea6ch term consists of a `GROUP BY` query that is possibly expensive. In this section we address these problems using the extension of the base theory in Section 5.

We start by making an observation about the computation of the variance of the sampling estimator: it depends, in orthogonal ways, on properties of the data through terms y_S and on properties of the sampling through c_S. The base theory does not require any particular way to compute/estimate terms y_S. *Using the available sample for estimating y_S terms is only one of the possibilities.* While many ways to estimate terms y_S can be explored, a particularly interesting one in this context is to use *another sampling method* for the purpose. More specifically, we could use a sample of the available sample for estimation of the terms y_S and the full sample for the estimation of the true value. This process is depicted in Figure **??**.

To understand what benefits we can get from this idea, we observe that we do not need very precise estimates of the terms y_S. Should we make a mistake, it will only affect the confidence interval by a small constant factor but will still allow the shrinking of the confidence interval with the increase of the sample. Based on the experience in DBO and TurboDBO, using 10000 result tuples for the estimation of y_S terms suffices. This means that the 2^n y_S terms are evaluated, as explained in Section 6 only on datasets of size at around 10000. Moreover, only for these 10000 samples the system needs to provide lineage information; samples used for evaluation of the expected value need no lineage.

There are two alternatives when it comes to reducing the number of samples used for estimation of terms y_S: select a more restrictive sampling method, depicted in Figure **??**, or further sample from the provided sample. The later approach can be applied when needed in case in which the size of the sample is overwhelming for the computation of terms y_S. Specifically, we can use a multi-dimensional Bernoulli GUS on top of the existing query plan for result tuples. This can be obtained by applying Proposition 9 until desired dimension is reached. The extra results in Section 5 together with the core results in Section 4 provide the means to analyze this modified sampling process. Example 6 and the accompanying Figure 5 provide such analysis for Query 1 and exemplifies how the extra Bernoulli sampling can be dealt with.

EXAMPLE 6. *This example shows how the query plan for Query 1 can be sampled further to efficiently obtain y_S terms. Figure 5.a shows the original query plan. Figure 5.b shows the sampling in terms of a GUS quasi-operator. Figure 5.c shows the placement of a bi-dimensional Bernoulli sampling method. Figures 5.d, 5.e, 5.f make use of propositions in Section 4 to obtain a SOA-equivalent plan, suitable for analysis.*

For the estimation process to be correct, the Bernoulli sampling out of the sample computed by the query plan needs some care in implementation. The main issue is the fact that it has to be a GUS method - if it decides to eliminate a tuple from a base relation, it has to do so in all result tuples in which it appears. This can be easily achieved efficiently and with little space using pseudo-random functions that combine seeds and lineage to provide a [0,1] number. The pseudo-randomness ensures that the value of the function will return the same value for the same tuple, thus providing the same decision. The only memory required to run such a sub-sampling algorithm is minimal: one seed per base relation. The process is also very efficient since it only requires evaluation of simple functions.

8. CONCLUSIONS AND FUTURE WORK

While technically challenging to create, the theory in this paper is in essence easy to use. Sampling is treated as a quasi-operator. In order to incorporate sampling based approximations, such operators are introduced in the query plans and the mechanisms described in Section 4 are used to analyze the estimators. We have already seen an example of use of the theory: the sub-sampling technique in Section **??**. With very little effort (introducing a final Bernoulli sampling quasy operator), we dealt with a seemingly hard problem: how to use a subsample to predict the behavior of the main sample. The straightforwardness of this process encourages us to suggest that the theory presented here will allow significant progress in a number of hard to solve problems explored in the approximate query processing literature. We briefly mention such potential in the remaining of this section.

Database as a sample. By viewing the database itself as a sample, *robustness analysis* is possible. In particular, if we assume that 1% of the tuples are mistakenly lost and we wish to predict the impact on the query results we can view the database as a 99% Bernoulli sample. A large variance will indicate that the query results are sensitive to such perturbations and thus not robust.

Choosing sampling parameters. By using the unbiased y_S estimates from a single sampling instance, the theory allows for plugging in co-efficients for different sampling strategies to predict the respective variances. This can give the user insight on comparing diffrent sampling strategies and parameters to suit his/her needs.

Estimating the size of intermediate relations. Query execution engines maintain a sample of the data and evaluate aggregates on it to predict the size of the intermediate relations. Our theory allows for the evaluation of the precision of these, thereby preventing the selection of *inferior* plans.

Data Streaming and Load Shedding. An interesting problem in load shedding is determing a sampling rate so that the the system can keep up with fast-rate incoming data while minimizing the error[**?**]. While such analysis was done for single relations, our theory provides for similar analyis with multiple relations.

9. DISCUSSION FUTURE DIRECTIONS

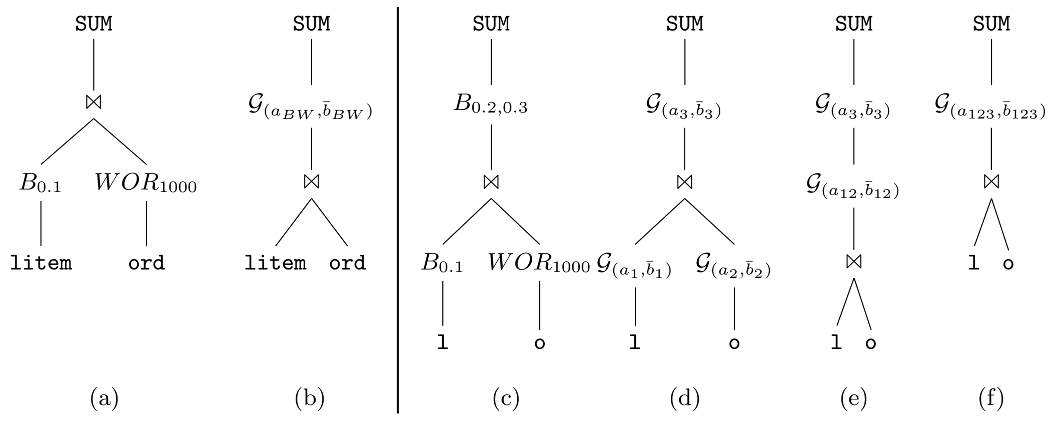
In this section we briefly mention possible theoretical developments that use the current work as a starting point.

Random sets. Proposition 3 establishes a connection between SOA-equivalence and an equivalence relation on random sets: two random sets are equivalent if they agree on probabilities of inclusions of tuples and pairs of tuples. This equivalence is a relaxed version of the equivalence relationship that would require the *same distribution*. When SUM-like aggregates are computed over random sets, this relationship between random sets is the same as SOA-equivalence, as stated in Proposition 3. An interesting question to explore is what other *properties* are preserved by this equivalence. This might prove to be the key to tackling non-aggregate queries in the presence of randomization.

Extending randomized filtering.. In Section 3 we saw that GUS is required to be a *filter*. This means that sampling methods that produce duplicates, like sampling with replacement, are not accommodated. More importantly, the filter behavior required the removal of duplicates in Proposition 7, which is potentially costly. We believe the current theory can be extended naturally to allow and capture such duplicates and still maintain the current elegance.

Average and non-linear combinations of SUM-like aggregates. . Our theory works only for SUM-like aggregates. For non-linear combinations of such aggregates such as `AVERAGE`, it is not possible to compute exactly the moment but there is a good chance that good quality approximations can be provided, using for example the *delta method*[**?**]. An interesting question is whether the structure in this paper can be extended for such approximation, especially since the Taylor expansions used in the delta method have the same linear structure.

Dealing with Self-Joins:. Proposition 6 requires no overlap in the lineage of the two arguments of the join. This effectively bans self-joins. Self-joins introduce difficulties for probabilistic analysis - this happens for probabilistic databases- since the probabilistic event specifying the presence of a tuple is used twice in the estimation. This creates extra dependencies that are not fully captured by inclusion probabilities of tuples and pairs of tuples as is the case for GUS. An interesting question is whether more but a finite extra inclusion probabilities are enough to deal with the problem. We conjecture that inclusion probabilities of combinations of 4 tuples is sufficient.



SUM
⋈
$B_{0.1}$
WOR_{1000}
litem
ord
$\mathcal{G}_{(a_{BW},\bar{b}_{BW})}$
$B_{0.2,0.3}$
$\mathcal{G}_{(a_3,\bar{b}_3)}$
$\mathcal{G}_{(a_{12},\bar{b}_{12})}$
$\mathcal{G}_{(a_{123},\bar{b}_{123})}$
$\mathcal{G}_{(a_1,\bar{b}_1)}$
$\mathcal{G}_{(a_2,\bar{b}_2)}$
l
o
(a)
(b)
(c)
(d)
(e)
(f)

GUS method	Parameters
$\mathcal{G}_{(a_1,\bar{b}_1)}$	$a_1 = 0.1, b_{1,\varnothing} = 0.01, b_{1,\mathtt{l}} = 0.1$
$\mathcal{G}_{(a_2,\bar{b}_2)}$	$a_2 = 6.667 \times 10^{-3}, b_{2,\varnothing} = 4.44 \times 10^{-5}, b_{2,\mathtt{o}} = 6.667 \times 10^{-3}$
$\mathcal{G}_{(a_3,\bar{b}_3)}$	$a_3 = 0.06, b_{3,\varnothing} = 0.0036, b_{3,\mathtt{o}} = 0.012, b_{3,\mathtt{l}} = 0.018, b_{3,\mathtt{lo}} = 0.06$
$\mathcal{G}_{(a_{12},\bar{b}_{12})}$	$a_{12} = 6.667 \times 10^{-4}, b_{12,\varnothing} = 4.44 \times 10^{-7}, b_{12,\mathtt{o}} = 6.667 \times 10^{-5}, b_{12,\mathtt{l}} = 4.44 \times 10^{-6}, b_{12,\mathtt{lo}} = 6.667 \times 10^{-4}$
$\mathcal{G}_{(a_{123},\bar{b}_{123})}$	$a_{123} = 4 \times 10^{-5}, b_{123,\varnothing} = 1.598 \times 10^{-9}, b_{123,\mathtt{o}} = 8 \times 10^{-7}, b_{123,\mathtt{l}} = 7.992 \times 10^{-8}, b_{123,\mathtt{lo}} = 4 \times 10^{-5}$

Figure 5: Transformation of the query plan to allow analysis

Dealing with `DISTINCT`. The GUS family of sampling methods is not general enough to commute with distinct – counter examples can be readily build. The main problem is the fact that the DISTINCT needs more information than the interaction between two tuples, even for the computation of expected value. We believe that there is a deep connection between this problem and the *safe plans* in probabilistic databases. An interesting future development would be identifying a more general sampling class then GUS and establishing a connection with the safe plans.

APPENDIX

A. SOA-EQUIVALENCE: PROOF

Proof Suppose $\mathcal{E}(R) \overset{\text{SOA}}{\Longleftrightarrow} \mathcal{F}(R)$. For every $t \in R$, define the function f_t as $f_t(s) = 1_{\{s=t\}}$. Hence, $\mathcal{A}_{f_t}(S) = 1_{\{t \in S\}}$. It follows that

$$\begin{aligned} P(t \in \mathcal{E}(R)) &= E\left[\mathcal{A}_{f_t}(\mathcal{E}(R))\right] \\ &= E\left[\mathcal{A}_{f_t}(\mathcal{F}(R))\right] \\ &= P(t \in \mathcal{F}(R)). \end{aligned}$$

Now, for every $t, t' inR$, define the function $f_{t,t'}$ as $f_{t,t'}(s) = 1_{\{s=t\}} + 1_{\{s=t'\}}$. It follows that

$$\begin{aligned} & E\left[\mathcal{A}_{f_t}(\mathcal{E}(R))^2\right] \\ = \; & E\left[\left(1_{\{t\in\mathcal{E}(R)\}} + 1_{\{t'\in\mathcal{E}(R)\}}\right)^2\right] \\ = \; & E\left[1_{\{t\in\mathcal{E}(R)\}} + 1_{\{t'\in\mathcal{E}(R)\}} + 21_{\{t,t'\in\mathcal{E}(R)\}}\right] \\ = \; & P(t \in \mathcal{E}(R)) + P(t' \in \mathcal{E}(R)) + 2P(t, t' \in \mathcal{E}(R)). \end{aligned}$$

Similarly,

$$\begin{aligned} & E\left[\mathcal{A}_{f_t}(\mathcal{F}(R))^2\right] \\ = \; & P(t \in \mathcal{F}(R)) + P(t' \in \mathcal{F}(R)) + 2P(t, t' \in \mathcal{F}(R)). \end{aligned}$$

Note that

$$E\left[\mathcal{A}_{f_t}(\mathcal{E}(R))^2\right] = E\left[\mathcal{A}_{f_t}(\mathcal{F}(R))^2\right].$$

It follows by (2) that

$$P(t, t' \in \mathcal{E}(R)) = P(t, t' \in \mathcal{F}(R)).$$

Hence, one direction of the equivalence is proved. Let us now assume that

$$P(t \in \mathcal{E}(R)) = P(t \in \mathcal{F}(R)) \; \forall t \in R,$$

and

$$P(t, t' \in \mathcal{E}(R)) = P(t, t' \in \mathcal{F}(R)) \; \forall t, t' \in R.$$

The SOA-equivalence of $\mathcal{E}(R)$ and $\mathcal{F}(R)$ immediately follows by noting that for an arbitray function f on R, and an arbitrary (possibly randomized) expression $S(R)$

$$E\left[\mathcal{A}_f(S(R))\right] = \sum_{t \in R} P(t \in S(R) f(t),$$

and

$$E\left[\mathcal{A}_f(S(R))^2\right] = \sum_{t,t' \in R} P(t, t' \in S(R) f(t) f(t').$$

□