Comments on "A new combination of evidence based on compromise" by K. Yamada

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In a previous issue of this journal, Yamada (2008) presents an interesting new model of combination of evidence called *combination by compromise* based on previous works published in Yamada (2006b,a).

In the theory of belief functions, one of the most important problems is the one of reassigning the conflicting belief mass, as highlighted by the famous Zadeh's example, see Zadeh (1979). To date, many combination rules have been developed, proposing a solution to this problem. Yamada (2008) recalls some of them in his paper.

Basically, most combination rules are mainly based on a conjunctive operator and propose a specific way of redistributing the global (or the partial) conflicting belief mass among some elements of the power-set (or the hyper-power set) of the frame of discernment. Using the conjunctive rule means that if the experts agree (*i.e.* their testimonies have a non-empty intersection), we consider them as reliable and if they are in conflict (empty intersection), at least one of the experts is considered unreliable see Dubois and Prade (1988). Then, the disjunctive combination rule can be employed instead see Dubois and Prade (1986). However the disjunctive rule is generally not used because it deteriorates the specificity of the expert's responses, *i.e.* the combined mass is usually less specific after the disjunctive fusion than the mass of each source taken separately. If the reliability of the experts is unknown, there are different techniques to estimate it Elouedi et al. (2004); Martin et al. (2008).

The first idea published in Yamada (2006b,a) is based on the assumption that we do not have information on the reliability of the experts. So in order to define a compromise between the conjunctive and the disjunctive rules, Yamada proposes to transfer the basic belief assignments/masses $m_1(X).m_2(Y)$ given

by two experts to $X \cap Y$, $X \cap Y^c$ and $X^c \cap Y$, even if X and Y do not conflict. The repartition is made proportionally to the basic belief assignment with a weight given by a ratio of cardinalities. In the more recent paper, Yamada (2008) proposes a general form of repartition, with thresholds instead of the weights based on cardinalities.

After a review of criticisms against Dempster's rule and its advantages, Yamada (2008) concludes on three possible ideas "to combine two even hypotheses with the same reliability into one" p 1698:

(1) Believe the common part of hypotheses (combination by exclusion, CBE).
(2) Believe the disjunction of hypotheses ("united part" in Yamada (2008)) (combination by union, CBU).
(3) Believe the common part strongly and the other part weakly (combination by compromise CBC).

The CBE is similar to Dempster's rule. The CBU is similar to the disjunctive combination rule. Yamada (2008) "proposes the third approach, CBC as a natural consensus. The basic idea is to share the mass $m_1(X)m_2(Y)$ among subsets included in $X \cup Y$." Yamada (2008) examines "three ways of sharing":

(1) The mass is shared between X and Y.
(2) The mass is shared between $X \cap Y$ and $X \cup Y$.
(3) The mass is shared among $X \cap Y = C$, $X \cap Y^c = X_Y$ and $X^c \cap Y = Y_X$.

For the first way of sharing, Yamada (2008) proposes the equations (16) and (17) p 1698 given by:

$$m(X) = \frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} \qquad \text{and} \qquad m(Y) = \frac{m_1(X)m_2(Y)^2}{m_1(X) + m_2(Y)}$$

For the second way of sharing, Yamada (2008) proposes the equations (18) and (19) p 1699 given by:

$$m(X \cap Y) = \delta m_1(X)m_2(Y) \qquad \text{and} \qquad m(X \cup Y) = (1 - \delta)m_1(X)m_2(Y)$$

with $1 \leq \delta \leq 1$. "The value of δ could be chosen as the degree of overlapping between X and Y, i.e. $\delta |X \cap Y|/|X \cup Y|$, where $| \bullet |$ means cardinality."

For the third way of sharing, Yamada (2008) proposes the equations (20), (21) and (22) p 1699 given by:

$$m(C) = \lambda_1 m_1(X)m_2(Y)$$
$$m(X_Y) = \lambda_2 m_1(X)m_2(Y)$$
$$m(Y_X) = \lambda_3 m_1(X)m_2(Y)$$

where $0 \leq \lambda_1, \lambda_2, \lambda_3 \leq 1$ and $\lambda_1 + \lambda_2 + \lambda_3 = 1$. The mass sharing of the CBC is proposed in the section 4.3. in Yamada (2008). When $C = \emptyset$, the whole mass of $m_1(X)m_2(Y)$ is distributed to X and Y according to the first way of sharing. In Yamada (2008), the paper focuses on this third approach, but "does not deny the qualification of the other subsets completely."

However, we would like to recall some similarities between Yamada's two first ways of sharing the mass, and previously published combination rules not reported in the references of Yamada's paper.

Whenever $X \cap Y = \emptyset$ and the mass that should be assigned to $X \cap Y$ is redistributed to X and Y proportionally to original masses $m_1(X)$ and $m_2(Y)$, Yamada's first way of sharing in Yamada (2008) (equations (16) and (17) p 1698) is equivalent to the principle of sharing of the Proportional Conflict Redistribution rules #5 (PCR5) and #6 (PCR6) for two experts[1] published in Smarandache and Dezert (2004, 2005, 2006); Martin and Osswald (2006).

The PCR5 and the PCR6 for two experts are given for two basic belief assignments m_1 and m_2 and for all $X \in 2^\Theta$, $X \neq \emptyset$ by:

$$m_{\text{PCR5}}(X) = m_{\text{Conj}}(X) + \sum_{Y \in 2^\Theta, \, X \cap Y = \emptyset} \left(\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)} \right),$$

where $m_{\text{Conj}}(.)$ is the conjunctive rule. In the case of two experts, the Yamada's rule and the PCR5-PCR6 will be the same if all pairs of X_i^1 and X_j^2 chosen respectively from the expert 1 and 2 have the empty intersection $(X_i^1 \cap X_j^2 = \emptyset, \forall i, j)$. For M experts, if any pair of focal elements chosen from M focal elements $X_i^1, \cdots, X_j^M$ have an empty intersection $(X_i^{k_1} \cap X_j^{k_2} = \emptyset, \forall k_1, k_2 = 1, \cdots, M, k_1 \neq k_2$ and $\forall i, j)$ the Yamada's rule (equations (38), (39) and (40) p 1701 in Yamada (2008)) is the PCR6 given explicitly for $X \in 2^\Theta$, $X \neq \emptyset$ by:

$$m_{\text{PCR6}}(X) = m_{\text{Conj}}(X) + \sum_{i=1}^{M} m_i(X)^2 \sum_{\substack{\bigcap_{k=1}^{M-1} Y_{\sigma_i(k)} \cap X = \emptyset \\ (X_{\sigma_i(1)}, \ldots, X_{\sigma_i(M-1)}) \in (2^\Theta)^{M-1}}} \left(\frac{\prod_{j=1}^{M-1} m_{\sigma_i(j)}(X_{\sigma_i(j)})}{m_i(X) + \sum_{j=1}^{M-1} m_{\sigma_i(j)}(X_{\sigma_i(j)})} \right),$$

where $X_k \in 2^\Theta$ is the response of the expert k, $m_k(X_k)$ the associated belief function and σ_i counts from 1 to M avoiding i:

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[1] The PCR5 and PCR6 rules coincide for the two experts case.

$$\begin{cases} \sigma_i(j) = j & \text{if } j < i, \\ \sigma_i(j) = j + 1 & \text{if } j \geq i, \end{cases}$$

Indeed, in this special case, the equation (40) p 1701 in Yamada (2008) is given by:

$$m_k(X_k) = \frac{m_k(X_k)}{\sum\limits_{k=1}^{M} m_k(X_k)} \prod\limits_{k=1}^{M} m_k(X_k).$$

A presentation of PCR rules with many examples are proposed in Smarandache and Dezert (2004, 2005, 2006); Martin and Osswald (2006).

The second way of sharing proposed in Yamada (2008) (equations (18) and (19)) is exactly the same as the mixed rule proposed in Martin and Osswald (2007), equation (17), that also transfers the basic belief assignment $m_1(X) \cdot m_2(Y)$ even if X and Y do not conflict. One of the proposed values of δ in Martin and Osswald (2007) is also given by $|X \cap Y|/|X \cup Y|$. In case of conflict $X \cap Y = \emptyset$, all the mass is transferred to $X \cup Y$. The mixed rule proposed in Martin and Osswald (2007) was inspired by Dubois and Prade (1988); Florea et al. (2006). The recent paper Florea et al. (2008) presents more details on a robust combination rule.

To conclude, the proposed approach by Yamada can be an interesting alternative to the disjunctive rule and reconsiders the conjunctive rule. In very recent years many rules of combination have been proposed in the theory of belief functions. The choice of a rule is usually difficult and must be guided by the prior and the exogenous information (if available) related to the application one has to deal with.

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