A Directed -Threshold Multi-Signature Scheme

Sunder Lal * and Manoj Kumar **

* Dept of Mathematics, IBS Khandari.Dr. B.R.A.University Agra.
Sunder_lal2@rediffmail.com.in.

** Dept of Mathematics, HCST, Farah – Mathura, [U. P.] – 281122.
Yamu_balyan@yahoo.co.in

Abstract. In this paper, we propose a **Directed Threshold Multi-Signature Scheme.** In this threshold signature scheme, any malicious set of signers cannot impersonate any other set of signers to forge the signatures. In case of forgery, it is possible to trace the signing set. This threshold signature scheme is applicable when the message is sensitive to the signature receiver; and the signatures are generated by the cooperation of a number of people from a given group of senders.

Key words. Digital signature, Directed signatures, Multi-signatures, Threshold-signatures, Lagrange interpolation, discrete logarithm problems.

1. Introduction

Digital signature is a cryptographic tool to authenticate electronic communications. Digital signature scheme allows a user with a public key and a corresponding private key to sign a document in such a way that anyone can verify the signature on the document (using her/his public key), but no one can forge the signature on any other document. This self-authentication is required for some applications of digital signatures such as certification by some authority.

In most situations, the signer is generally a single person. However, in some cases the message is sent by one organization and requires the approval or consent of several people. In these cases, the signature generation is done by more than one consenting person. A common example of this policy is a large bank transaction, by one organization, which requires the signature of more than one partner. Such a policy could be implemented by having a separate digital signature for every required signer, but this solution increases the effort to verify the message linearly with the number of signer. To solve this problems, Multisignature schemes [10,11,13,17,18] and threshold signature schemes [5,6,7,9,12,20] are used where more than one signers share the responsibility of signing messages

Threshold signatures are closely related to the concept of threshold cryptography, first introduced by Desmedt [5, 6, 7]. In 1991, Desmedt and Frankel [6] proposed the first (t, n) threshold digital signature scheme based on the RSA system.

In (t, n) threshold signature scheme, any subgroup of t or more—shareholders of the designated group can generate a valid group signature in such a way that the verifier can check the validity of the signature without identifying the identities of the signers. In threshold schemes, when any t or more shareholders

act in collusion, they can impersonate any other set of shareholders to forge the signatures. In this case, the malicious set of signers does not have any responsibility for the signatures and it is impossible to trace the signers. Unfortunately, with threshold schemes proposed so far, this problem cannot be solved.

In multisignature schemes, the signers of a multisignature are identified in the beginning and the validity of the multisignature has to be verified with the help of identities of the signers. For multisignatures, it is indeed unnecessary to put a threshold value to restrict the number of signers. Consider the situation, where a group of anonymous members would have to generate a multisignature. The members of this group use pseudonyms as their identities in the public directory. What concerns the verifier most is that a message is signed by at least *t* members and they indeed come from that group. Nevertheless, the verifier has no way to verify whether a user is in fact a member of that group because of the anonymity of the membership. In this case, the multisignature schemes cannot solve this problems, however, the threshold signature schemes do.

On the other hands, there are so many situations, when the signed message is sensitive to the signature receiver. Signatures on medical records, tax information and most personal/business transactions are such situations. Signatures used in such situations are called directed signatures [1, 2, 3, 14, 15, 19, 23, 24]. In directed signature scheme, the signature receiver has full control over the signature verification process and can prove the validity of the signature to any third party, whenever necessary. Nobody can check the validity of signature without his cooperation.

Combining these ideas, we propose a digital signature scheme named as Directed - threshold multi-signature scheme. The proposed scheme is based on Shamir's threshold signature scheme [21] and Schnorr signature scheme [22]. These basic tools are briefly described in the next section. This paper is organized as follows: -

The section-2 presents some basic tools. Section-3 proposes a Directed - threshold multi - signature scheme. Section-4 discusses the security of the Scheme. An illustration is discussed in section-5. Remarks are in section-6.

2. Preliminaries

Throughout this paper, we will use the following system setting.

- A prime modulous p, where $2^{511} ;$
- A prime modulous q, where $2^{159} < q < 2^{160}$ and q is a divisor of p-1;
- A number g, where $g \equiv k^{(p-1)/q} \mod p$, k is random integer with $1 \le k \le p-1$ such that g >1; (g is a generator of order q in $\mathbb{Z}p^*$).
- A collision free one-way hash function *h* [25];

The parameters p, q, g and h are publicly known. Every user has two keys one private and one public. We assume that a user A chooses a random $x_A \in \mathbb{Z}q$ and computes $y_A = g^{x_A} \mod p$. He keeps x_A as his private key and publishes y_A as his public key.

In **Schnorr's signature scheme** the signature of the user A on the message m is given by (r_A, S_A) , where,

$$r_A = h(g^{k_A} \mod p, m)$$
, and $S_A = k_A - x_A \cdot r_A \mod p$.

The signature are verified by checking the equality

$$r_A = h \left(g^{S_A} y^{r_A} \mod p, m \right).$$

Threshold secret sharing scheme is a scheme to distribute a secret key K into n users in such a way that any t users can cooperate to reconstruct K but a collusion of t-1 or less users reveal nothing about the secret. There are many realization of this scheme, we shall use Shamir's scheme. This scheme is based on Lagrange's interpolation in a field. To implement it, a polynomial f of degree t-1 is randomly chosen in $\mathbb{Z}q$ such that $f(0) = \mathbb{K}$. Each user i is given a public identity u_i and a secret share $f(u_i)$. Now any subset of t shareholders out of t shareholders can reconstruct the secret t is given and using

$$f(0) = \sum_{i=1}^{t} f(u_i) \prod_{j=1, j \neq i}^{t} \frac{-u_j}{u_i - u_j} \operatorname{mod} q$$

Here we assume that the authorized subset of t users consists of shareholders i for i = 1, 2, 3...t.

3. Directed - threshold multi - signature scheme

The signer of the conventional digital signature schemes is usually a single person. But when the message m (sensitive to the signature receiver R) is transmitted by an organization S to a person R and the message requires the approval of more than one person then the responsibility of signing the messages needs to be shared by a subset H_S of t or more signer from a designated group G_S of n users belonging to the organization S.

From the point of view of the signing group, what the group concerns is the tracability of the signing subgroup and no one other than the signature receiver can check the validity of the signatures. On the other hand, from the verifier point of view, whether the signature is indeed from that group and signed by at least t members (not the membership of the members in that group) is the most important. The signature receiver R can verify the signature and that R can prove its validity to any third party C, whenever necessary. Nobody can check the validity of the signature without

the help of R. Both the (t, n) threshold directed signature schemes and the multisignature schemes couldn't solve this problem independently.

In this paper, we are going to combine the idea of (t, n) threshold signature schemes and multisignature schemes with directed signature scheme and propose a threshold signature scheme, called the **Directed - threshold— multisignature scheme**, to solve the above problem.

For our construction, we assume that there is a trusted share distribution center (SDC) that determines the group secret keys, all shareholders's secret shares, and a designated combiner DC who takes the responsibility to collect and verify each partial signature and then produce a group signature. This scheme consists of the following steps: -

3.1.Group Secret Key and Secret Shares Generation for the organization S

(a). SDC selects the group public parameters p, q, g and a collision free one way hash function h [25]. SDC also selects a polynomial

$$f_S(x) = a_0 + a_1 x + \dots + a_{t-1} x^{t-1} \mod q$$
, with $a_0 = x_S = f_S(0)$.

(b). SDC computes the group public key, y_s , as,

$$y_S = g^{f_S(0)} \bmod p.$$

(c). SDC randomly selects $K \in \mathbb{Z}q$ and computes a public value

$$W = g^{-K} \bmod p.$$

(d). **SDC** randomly selects
$$K_i \in \mathbb{Z}q$$
 and computes
$$l_i = [K_i + f_S(u_{S_i})] \mod q.$$

Here u_{S_i} is the public value associated with each user i in the group G_{S_i} .

(e). SDC computes a public value v_{S_i} for each member of the group G_S , as,

$$v_{S_i} = l_i \cdot y_{S_i}^{K} \mod p.$$

Here y_{S_i} is the public value associated with each user i in the group G_S .

- (f). SDC sends $\{v_{S_i}, W\}$ to each user i in the group G_S through a public channel.
- (g). SDC also needs to computes public values, m_i and n_i as,

$$m_i = g^{l_i} \mod p$$
 and $n_i = g^{K_i} \mod p$.

3.2. Partial Signature generation by any t users and verification

If any t members of the organization S out of n members agree to sign a message m for a person R. R possesses a pair (X_R, y_R) . Then the signature generation has the following steps.

(a) Each user $i \in H_S$ randomly selects K_{i_1} and $K_{i_2} \in \mathbb{Z}q$ and computes

$$u_i = g^{-K_{i_2}} \mod p, v_i = g^{K_{i_1}} \mod p \text{ and } w_i = g^{K_{i_1}} y_R^{K_{i_2}} \mod p.$$

(b) Each user makes u_i , w_i publicly and v_i secretly available to each member $i \in H_S$. Once all u_i , v_i and w_i are available, each member $i \in H_S$ computes the product U_S , V_S , W_S and a hash value R_S , as,

$$U_S = \prod_{i \in H_S} u_i \mod q, \quad V_S = \prod_{i \in H_S} v_i \mod q,$$

$$W_S = \prod_{i \in H_S} w_i \mod q \text{ and } R_S = h(V_S, m) \mod q.$$

(c) Each user $i \in H_S$ recovers his/her secret share l_i , as,

$$l_i = v_{S_i} W^{x_{S_i}} \bmod p.$$

(d) Each user $i \in H_S$ modifies his/her shadow, as,

$$MS_{S_i} = l_i \cdot \prod_{j=1, j\neq i}^t \frac{-u_{S_j}}{u_{S_i} - u_{S_j}} \mod q$$
.

(e). Each user $i \in H_S$ uses his/her modified shadow MS_{S_i} and computes a value s_i , as,

$$s_i = K_{i_1} + MS_{s_i}.R_s \mod q.$$

- (e) Each member $i \in H_S$ sends his partial signature to the **designated combiner** DC.
- (f) The DC verifies the partial signature (s_i , v_i , R_S) by the following congruence,

If this congruence holds, then the partial signature (s_i, v_i, R_S) for shareholder i is valid.

3.3. Group Signature generation

(a). DC can computes the group signature S_S by combining all the partial signature, as,

$$S_S = \sum_{i \in H_S} s_i \bmod q.$$

Here we should note that there is no secret information associated with the DC.

(b). DC sends $\{S_S, U_S, W_S, m\}$ to R as signature of the group S on the message m.

3.4. Signature verification by *R*

To verify the validity of the group signature $\{S_S, U_S, W_S, m\}$ the verifier R needs his/her secret key x_R . This sub-section consists of the following steps: -

(a) The verifier R computes a verification value E, as,

$$E \qquad = \qquad \prod_{i \in H} n_i \left(\prod_{j \in H_S, j \neq i} \frac{0 - u_{S_j}}{u_{S_i} - u_{S_j}} \right) \mod q \mod p.$$

(b). The verifier R can recovers the values R_R and R_S , as

$$R_R = W_S$$
. $U_S^{x_R} \mod p$ and $R_S = h(R_R, m)$.

(c). The verifier R uses the following congruence to check the validity of the signature

$$g^{S_s} \equiv R_R \cdot (E \cdot y_s)^{R_s} \mod p.$$

If this congruence holds, then the group signature $\{S_S, U_S, W_S, m\}$ is valid signature of the organization S on the message m.

3.5. Proof of validity by R to any third party C

- (a). R computes $\mu = U_S^{X_R} \mod p$ and $R_R = \mu . W_S \mod p$.
- (b). R sends { R_R , E, S_S , U_S , m, μ } to C.
- (c). C recovers $R_S = h(R_R, m)$ and uses the following congruence to check the validity of the signature

$$g^{S_s} \equiv R_R \cdot (E \cdot y_s)^{R_s} \mod p.$$

If this does not hold C stops the process; otherwise goes to the next step.

- (d) In a zero knowledge fashion R proves to C that $\log_{U_s} \mu = \log_g y_R$ as follows:-
- C chooses random $u, v \in \mathbb{Z}p$ computes $w = (U_S)^{\mathcal{U}} \cdot g^{\mathcal{V}} \mod p$ and sends w to R.
- R chooses random $\alpha \in \mathbb{Z}p$ computes $\beta = w$. $g \mid \alpha \mod p$, $\gamma = \beta^{x_R} \mod p$ and sends β , γ to C.
- C sends u, v to R, by which R can verify that $w = (U_S)^{u}$. $g^{v} \mod p$.
- R sends α to C, by which she can verify that

$$\beta = (U_S)^u \cdot g^{v+\alpha} \mod p$$
, and $\gamma = (\mu)^u y_R^{v+\alpha} \mod p$.

4. Security discussions

In this sub-section, we discuss the security aspects of proposed scheme. Here we discuss several possible attacks. But show that, none of these can successfully break our system.

(a). Can any one retrieve the secret keys $x_s = f_s$ (0) from the group public key y_s ?

This is as difficult as solving discrete logarithm problem. No one can get the secret key x_S , since f_S is a randomly and secretly selected polynomial. On the other hand, by using the public keys y_S no one can get the secret keys x_S because this is as difficult as solving discrete logarithm problem.

(b). Can one retrieve the secret shares, f_S (u_{S_i}) $i \in G_S$, from the equation

$$v_{S_i}$$
 = $f_S(u_{S_i})$. $y_{S_i}^K \mod p$?

No because f_S is a randomly and secretly selected polynomial and K is also a randomly and secretly selected integer by the SDC.

(c). Can one retrieve the secret shares, f_S (u_{S_i}) $i \in G_S$, from the equation

$$f_S(u_{S_i}) = v_{S_i} W^{x_{S_i}} \mod p$$
?

Only the user i can recovers his secret shares, $f_S(u_{S_i})$, because f_S is a randomly and secretly selected polynomial and x_{S_i} is secret key of the user $i \in G_S$.

(d). Can one retrieve the modified shadow MS_{S_i} , integer K_{i_1} , R_S and partial signature s_i , $i \in G_S$ from the equation

$$s_i = K_{i_1} + MS_{S_i}.R_S \mod q ?$$

It is computationally infeasible for a forger to collect the MS_{S_i} , integer K_{i_1} , R_S and partial signature $s_i, i \in G_S$.

(e). Can the designated combiner DC retrieve the any partial information from the equation,

$$S_S = \sum_{i \in H_S} s_i \bmod q ?$$

Obviously, this is computationally infeasible for *DC*.

(f). Can one impersonate a user $i \in H$?

A forger may try to impersonate a user $i \in H_S$, by randomly selecting two integers K_{i_1} and $K_{i_2} \in \mathbb{Z}q$ and broadcasting u_i , v_i and w_i . But without knowing the secret shares, $f_S(u_{S_i})$ and R_S , it is difficult to generate a valid partial signature s_i to satisfy the verification equations,

$$\mathbf{g}^{S_i} \qquad \qquad \stackrel{?}{\equiv} \qquad \qquad v_i \cdot m_i \left(\prod_{j \in H_{S,j} \neq i} \frac{-u_{S_j}}{u_{S_i} - u_{S_j}} \bmod q \right)^{R_S} \bmod p.$$

(g). Can one forge a signature $\{S_S, U_S, W_S, m\}$ by the following equation, $g^{S_S} \equiv R_R \cdot (E \cdot y_S)^{R_S} \mod p ?$

A forger may randomly selects an integer R_R and then computes the hash value R_S such that $R_S = h(R_R, m) \mod q$.

Obviously, to compute the integer S_S is equivalent to solving the discrete logarithm problem. On the other hand, the forger can randomly select R_S and S_S first, then try to determine a value R_R that satisfy the equation

$$g^{S_S} \equiv R_R \cdot (E \cdot y_S)^{R_S} \mod p.$$

However, according to the property of the hash function h, it is quite impossible. Thus, this attack will not be successful.

(h). Can t or more shareholders act in collusion to reconstruct the polynomial $f_S(x)$?

According to the equation,

$$f_S(x) = \sum_{i=1}^t f(u_{S_i}) \prod_{j=1, j \neq i}^t \frac{x - u_{S_j}}{u_{S_i} - u_{S_j}} \mod q,$$

the secret polynomial f_S can be reconstructed with the knowledge of any t secret shares, f_S (u_{S_i}) $i \in G_S$.

But in our proposed scheme, the secret shares l_i , $i \in G_S$, contains the integer K_i which only the trusted SDC knows and has to be removed first before reconstructing the polynomial $f_S(x)$. A malicious shareholder i may try to retrieve the integer K_i from the public key n_i . However, the difficulty is same as solving the discrete logarithm problem. Thus, any t or more shareholders cannot conspire to reconstruct the polynomial $f_S(x)$ by providing their own secret shares.

So, if in an organization the shareholders are known to each other, even than they cannot reconstruct the polynomial $f_S(x)$.

5. Illustration

The following illustration is supporting our scheme for practical implementation. Suppose $|G_S| = 7$, $|H_S| = 5$, p = 47, q = 23, g = 25.

5.1. Group Secret Key and Secret Shares Generation for the organization S

- (a). **SDC** selects a polynomial $f_S(x) = 13 + 18x^4 \mod 23$, so $x_S = 13$ and $y_S = 16$.
- (b). SDC randomly selects K = 14 and computes a public value W = 2.

(c). **SDC** computes the secret key, public key, secret share and public value for each member of the group G_S , as shown in the following table.

VALUE	Secret	Public	Secret	Secret	Secret	Public	Public	Public
USER	X_i	y_i	K_{i}	$f_{S}(u_{S_{i}})$	l_i	m_{i}	n_{i}	v_i
User–S ₁	13	16	8	2	10	3	17	41
User–S ₂	18	4	22	21	20	9	32	29
User–S ₃	19	6	10	16	3	21	3	1
User–S ₄	20	9	17	3	20	9	34	19
User–S ₅	17	34	14	14	5	12	24	38
User–S ₆	22	32	16	14	7	27	7	14
User–S ₇	15	36	21	22	20	9	37	44

5.2Signature generation by any t users

If any five users S_2 , S_4 , S_5 , S_6 , and S_7 out of seven members agree to sign a message m, for a person R possessing $X_R = 9$, $Y_R = 2$ then the signature generation has the following steps.

- (a). S_2 randomly selects $K_{2_1} = 18$, $K_{2_2} = 17$ and computes $u_2 = 18$, $v_2 = 4$ and $w_2 = 3$.
- (b). S_4 randomly selects $K_{4_1} = 17$, $K_{4_2} = 19$ and computes $u_4 = 8$, $v_4 = 34$ and $w_4 = 8$.
- (c). S₅ randomly selects $K_{5_1} = 14$, $K_{5_2} = 13$ and computes $u_5 = 3$, $v_5 = 24$ and $w_4 = 7$.
- (d). S₆ randomly selects $K_{6_1} = 19$, $K_{6_2} = 21$ and computes $u_6 = 14$, $v_6 = 6$ and $w_6 = 25$.
- (e). S_7 randomly selects $K_{7_1} = 16$, $K_{7_2} = 18$ and computes $u_7 = 12$, $v_7 = 7$ and $w_7 = 34$.
- (f). Each user computes the product $U_S = 8, V_S = 36, W_S = 14$ and $R_S = 9$ (let).
- (g). S_2 recovers his/her secret share $l_2 = 20$ and computes $MS_{S_2} = 10$, $s_2 = 16$.
- (h). S_4 recovers his/her secret share $l_4 = 20$ and computes $MS_{S_4} = 16$, $s_4 = 0$.
- (i). S_5 recovers his/her secret share $l_5 = 5$ and computes $MS_{S_5} = 6$, $s_5 = 22$.
- (j). S_6 recovers his/her secret share $l_6 = 7$ and computes $MS_{S_6} = 5$, $s_6 = 18$.
- (k). S_7 recovers his/her secret share $l_7 = 20$ and computes $MS_{S_7} = 5$, $s_7 = 15$.

5.3. Partial signature verification and Signature generation by DC

(a) DC verifies each partial signature. For example for user S_2 ,

$$s_2 = 16$$
, $v_2 = 4$, $m_2 = 9$, $R_S = 9$,
$$\prod_{j=4,5,6,7,i=2} \frac{-u_{S_j}}{u_{S_i} - u_{S_j}} \mod 23 = 12.$$
He checks $25^{16} \stackrel{?}{\equiv} 4 \cdot (9^{12})^9 \mod 47$. This holds.

Similarly, he checks other partial signatures.

(b). DC computes a group value $S_S = 2$ and sends $\{2, 14, 8, m\}$ as signature of the group S for the message m.

5.4. Signature verification by the person R

- (a). The verifier R computes a verification value E = 18.
- (b). The verifier R can recovers the values $R_R = 36$ and $R_S = 9$.
- (c). The verifier R uses the following congruence to check the validity of the signature

$$25^2$$
 \equiv $36. (18.16)^9 \mod 47.$

This congruence holds, so the group signature $\{2,14,8,m\}$ is valid signature of the group S on the message m for the person R.

5.5. Proof of validity by R to any third party C

- (a). R computes $\mu = 16$, $R_R = 36$ and sends $\{36, 18, 2, 8, m, 16\}$ to C.
- (b). C recovers $R_S = 9$ and check the concurrence $25^2 \equiv 36 \cdot (18.16)^9$ for the validity of the signature. This holds; so, C goes to the next steps.
- (c). R in a zero knowledge fashion proves to C that $\log_8 16 = \log_{25} 2$ as follows:-
- C chooses random u = 9, v = 11 and computes w = 25 and sends w to R.
- R chooses random $\alpha = 37$ computes $\beta = 36$, $\gamma = 9$ and sends β , γ to C.
- C sends u, v to R, by which R can verify that w = 25.
- R sends α to C, by which she can verify that $\beta = 36$ and $\gamma = 9$.

6. Remarks

In this paper, we have proposed a **Directed –Threshold Multi - signature Scheme.** There is a **trusted SDC** that determine the group secret keys, all shareholders's secret shares, and a **designated combiner** *DC* who takes the responsibility to collect and verify each partial signature and then produce a group signature, **but no secret information is associated with the** *DC*.

In this threshold signature scheme, any malicious set of signers cannot impersonate any other set of signers to forge the signatures. In case of forgery, it is possible to trace the signing set. Any t or more shareholders acting in collusion cannot conspire to reconstruct the polynomial $f_S(x)$ by providing their own secret shares and hence they cannot recover the group secret key.

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Manoj Kumar received the B.Sc. degree in mathematics from Meerut University Meerut, in 1993; the M. Sc. in Mathematics (Goldmedalist) from C.C.S.University Meerut, in 1995; the M.Phil. (Goldmedalist) in *Cryptography*, from Dr. B.R.A. University Agra, in 1996; submitted the Ph.D. thesis in *Cryptography*, in 2003. He also taught applied Mathematics at DAV College, Muzaffarnagar, India from Sep, 1999 to March, 2001; at S.D. College of Engineering & Technology, Muzaffarnagar, and U.P., India from March, 2001 to Nov, 2001; at Hindustan College of Science &

Technology, Farah, Mathura, continue since Nov, 2001. He also qualified the *National Eligibility Test* (NET), conducted by *Council of Scientific and Industrial Research* (CSIR), New Delhi- India, in 2000. He is a member of Indian Mathematical Society, Indian Society of Mathematics and Mathematical Science, Ramanujan Mathematical society, and Cryptography Research Society of India. His current research interests include Cryptography, Numerical analysis, Pure and Applied Mathematics.