

A primer on Answer Set Programming [★]

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Syntax

The following definitions describe the language DATALOG⁺ as well as logic programs with no function symbols.

Assume a language of constants and predicate constants. Assume also that terms and atoms are built as in the corresponding first-order language. Unlike classical logic and standard logic programming, no function symbols are allowed. A rule is an expression of the form:

$$\rho : A_0 \leftarrow A_1, \dots, A_m, \text{not } A_{m+1}, \dots, \text{not } A_n \quad (1)$$

where A_0, \dots, A_n are atoms and *not* is a logical connective called *negation as failure*. Also, for every rule let us define $\text{head}(\rho) = A_0$, $\text{pos}(\rho) = A_1, \dots, A_m$, $\text{neg}(\rho) = A_{m+1}, \dots, A_n$ and $\text{body}(\rho) = \text{pos}(\rho) \cup \text{neg}(\rho)$. The head of rules is never empty, while if $\text{body}(\rho) = \emptyset$ we refer to ρ as a *fact*.

A logic program is defined as a collection of rules. Rules with variables are taken as shorthand for the sets of all their ground instantiations and the set of all ground atoms in the language of a program Π will be denoted by \mathbb{B}_Π .

Queries and constraints are expressions with the same structure of rules but with empty head.

Semantics

Intuitively, a *stable model*, also called *answer set*, is a possible view of the world that is *compatible* with the rules of the program. Rules are therefore seen as constraints on these views of the world.

[★] Several portions of this document reproduce definitions given in [GelLif88] and elsewhere. This work was supported by the Information Society Technologies programme of the European Commission, Future and Emerging Technologies under the IST-2001-37004 WASP project.

Let us start defining stable models/answer sets of the subclass of positive programs, i.e. those where, for every rule ρ , $neg(\rho) = \emptyset$.

Definition 1. (*Stable model of positive programs*)

The stable model $a(\Pi)$ of a positive program Π is the smallest subset of B_Π such that for any rule (1) in Π :

$$A_1, \dots, A_m \in a(\Pi) \Rightarrow A_0 \in a(\Pi) \quad (2)$$

Clearly, positive programs have a unique stable model, which coincides with that obtained applying other semantics; in other words positive programs are unambiguous. Moreover, the stable model of positive programs can be obtained as the fixpoint of the *immediate consequence operator* T_Π iterated from \emptyset on.

Definition 2. (*Stable models of programs*)

Let Π be a logic program. For any set S of atoms, let $\Gamma(\Pi, S)$ be a program obtained from Π by deleting

- (i) each rule that has a formula “not A ” in its body with $A \in S$;
- (ii) all formulae of the form “not A ” in the bodies of the remaining rules.

Clearly, $\Gamma(\Pi, S)$ does not contain not, so that its stable model is already defined. If this stable model coincides with S , then we say that S is a stable model of Π . In other words, a stable model of Π is characterized by the equation:

$$S = a(\Gamma(\Pi, S)). \quad (3)$$

Programs which have a unique stable model are called categorical.

Let us define entailment in the stable models semantics. A ground atom α is *true in S* if $\alpha \in S$, otherwise α is *false*, i.e., by abuse of notation, $\neg\alpha$ is *true in S* . This definition can be extended to arbitrary first-order formulae in the standard way.

We will say that Π *entails a formula ϕ* (written $\Pi \models \phi$) if ϕ is true in *all* the stable models of Π . We will say that the answer to a ground query γ is

yes if γ is true in all stable models of Π , i.e. $\Pi \models \gamma$;

no if $\neg\gamma$ is true in all stable models of Π , i.e. $\Pi \models \neg\gamma$;

unknown otherwise.

It is easy to see that logic programs are *nonmonotonic*, i.e. adding new information to the program may force a reasoner associated with it to withdraw its previous conclusions.

Corollary 1. (*Gelfond and Lifschitz [GelLif91]*)

If an extended logic program has an inconsistent Answer set, this is unique.

□

For programs without explicit negation stable models and answer sets coincide, so that in the following we will refer to [consistent]answer sets or stable models indifferently.

1 Reasoning with Answer Sets

In the following we report a basic result from Marek and Subramanian which -together with its corollaries- will be used in proofs about logic programs.

The result is slightly more general than the original, as it refers to answer sets and it is given a simple proof based on minimality.

Lemma 1 (Marek and Subramanian). *The following result on answer sets is due to Marek and Subramanian, originally for general logic programs.*

For any answer set A of an extended logic program Π :

– *For any ground instance of a rule of the type:*

$$L_0 \leftarrow L_1, \dots, L_m, \text{not } L_{m+1}, \dots, \text{not } L_n \quad (4)$$

from Π , if

$$\{L_1, \dots, L_m\} \subseteq A \text{ and } \{L_{m+1}, \dots, L_n\} \cap A = \emptyset$$

then $L_0 \in A$.

- If A is a consistent Answer set of Π and $L_0 \in A$, then there exists a ground instance rule of type 4 from Π such that:

$$\{L_1, \dots, L_m\} \subseteq A \text{ and } \{L_{m+1}, \dots, L_n\} \cap A = \emptyset.$$

□

Corollary 2. If $\{L \leftarrow\} \in \Pi$ then L belongs to every Answer set of Π . It follows directly from Lemma 1.

□

Definition 3. We will say that an axiom r supports a literal L if the head of r matches with L . Moreover, we say that L is supported only by r if there is no other ground rule with head L .

Definition 4. We will say that a rule r justifies a literal L w.r.t. an answer set A if

- a) r supports L ;
- b) r satisfies the conditions set forth in the first half of Lemma 1 w.r.t. A : the atoms occurring positively in the body being in A while those occurring negatively being not.

Clearly, justified literals belong to A .

Corollary 3. If A is a consistent answer set of Π , $L_0 \in A$, and L_0 is supported only by an axiom r of type (4) from Π then:

$$\{L_1, \dots, L_m\} \subseteq A \text{ and } \{L_{m+1}, \dots, L_n\} \cap A = \emptyset.$$

It follows directly from Lemma 1.

□

2 Examples

Example 1. $\pi_1 =$
 $happy \leftarrow not\ sad.$
 $sad \leftarrow not\ happy.$

has two answer sets: $\{happy\}$ and $\{sad\}$.

Example 2. $\pi_2 =$
 $happy \leftarrow not\ sad.$
 $sad \leftarrow not\ soandso.$
 $soandso \leftarrow not\ happy.$

has no answer set.

Example 3. $\pi_3 =$
 $drinks \leftarrow happy.$
 $drinks \leftarrow sad.$
 $happy \leftarrow not\ sad.$
 $sad \leftarrow not\ happy.$

has two answer sets: $\{drinks, happy\}$ and $\{drinks, sad\}$.

Example 4. $\pi_4 =$
 $soandso \leftarrow not\ sad, not\ happy.$
 $happy \leftarrow not\ sad, not\ soandso.$
 $sad \leftarrow not\ happy, not\ soandso.$

has three answer sets: $\{happy\}$ and $\{sad\}$ and $\{soandso\}$.

Example 5. $\pi_5 =$
 $f \leftarrow not\ f, not\ a.$
 $a \leftarrow not\ b.$
 $b \leftarrow not\ a.$

has only one answer set: $\{a\}$.

Example 6. $\pi_6 =$
 $f \leftarrow not\ f, a.$
 $a \leftarrow not\ b.$
 $b \leftarrow not\ a.$

has only one answer set: $\{b\}$.

Exercise 1. $\pi_x =$
 $f \leftarrow b.$
 $c \leftarrow a.$
 $a \leftarrow d.$
 $d \leftarrow not\ b.$
 $b \leftarrow not\ a.$

2.1 Examples with explicit negation

Example 7. $\pi_7 =$

$\neg a \leftarrow \text{not } a.$

$b \leftarrow \neg a.$

has only one answer set: $\{b, \neg a\}$.

3 Sources

Several ASP solvers are now available and can be downloaded from [Solvers].

A textbook on Answer Set Programming is now available [Bar03], and exercises can be downloaded from there.

References

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