Two Representations for
Iterative Non-prioritized Change

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Abstract

We address a general representation problem for belief change, and describe two interrelated representations for iterative non-prioritized change: a logical representation in terms of persistent epistemic states, and a constructive representation in terms of flocks of bases.

1 Introduction

Two main kinds of representation for belief change have been suggested in the literature. In the AGM theory [Alchourrón *et al.*, 1985, Gärdenfors, 1988], belief change is considered as a change of *belief sets* that are taken to be deductively closed theories. According to the so-called base approach, however, a belief set should be seen as generated by some (finite) *base* (see, e.g., [Hansson, 1999]). Consequently, revisions of belief sets are determined on this approach by revisions of their underlying bases. This drastically reduces the set of alternatives and hence makes this approach constructive and computationally feasible.

The AGM theory and base approach constitute two seemingly incomparable representations for belief change, each with advantages of its own. Still, the framework of epistemic states suggested in [Bochman, 2001a, Bochman, 2001b] allows to formulate both as different species of a single general representation. The reformulation makes it possible, in particular, to reveal common constitutive principles behind them. These common principles embody, however, also common shortcomings that lead to a loss of information in iterated changes. This suggests epistemic states in their full generality as a justified representational alternative.

Epistemic states could be primarily seen as a generalization of the AGM models, namely, they form an abstract logical representation for belief change. Though the base-oriented models can be translated into epistemic states, the translation loses some important advantages of such models, namely their inherent finiteness and constructivity. Hence it would be desirable to have a constructive representation for epistemic states that would preserve these features of base models while avoiding their problems. Fortunately, such an alternative representation has already been suggested in [Fagin *et al.*, 1986], and it amounts to using sets, or *flocks*, of bases. As we will show, a certain modification of the original flock models will provide us with a constructive representation for an important class of epistemic states and belief change processes in them.

Remark. The theory described below, is different from that suggested in [Bochman, 1999, Bochman, 2000], since the latter has turned out to be flawed in its treatment of expansions. The shortcoming has created the need for shifting the level of representation from pure epistemic states used in the above papers to more general persistent epistemic states (see below).

2 Epistemic states

A common feature of the AGM and base representations is a preference structure on certain subsets of the belief set. In the case of the AGM paradigm, it is a preference structure on the maximal subtheories of the belief set, while for the base representation it is a preference structure on subtheories that are generated by the subsets of the base. The notion of an epistemic state, defined below, reflects and generalizes this common structure of the two representations.

Definition 2.1. An *epistemic state* $\mathbb{E}$ is a triple $(\mathcal{S}, l, \prec)$, where $\mathcal{S}$ is a set of *admissible belief states*, $\prec$ is a preference relation on $\mathcal{S}$, while l is a labelling function assigning a deductively closed theory (called

an admissible belief set) to every admissible belief state from $\mathcal{S}$.

Any epistemic state determines a unique set of propositions that are *believed* in it, namely, the set a propositions that hold in all maximally preferred admissible belief states. Thus, even if an epistemic state contains conflicting preferred belief states, we can still believe in propositions that hold in all of them. Still, the belief set associated with an epistemic state does not always constitute an admissible belief set by itself. The latter will hold, however, in an ideal situation when an epistemic state contains a unique most preferred admissible belief state. Such epistemic states will be called *determinate*. Both AGM and base models correspond to determinate epistemic states. Nevertheless, non-determinate epistemic states have turned to be essential for an adequate representation of both nonmonotonic reasoning and belief change. In particular, the necessity of accepting non-determination arises most clearly in the analysis of contractions (see below).

In what follows, we will concentrate on a special kind of persistent epistemic states that will provide a representation for a non-prioritized belief change.

Definition 2.2. An epistemic state will be called *persistent* if it satisfies

Persistence   If $s \prec t$, then $l(s) \subseteq l(t)$.

For persistent epistemic states, the informational content of admissible belief states is always preserved (persist) in transitions to more preferred states. This property reflects the idea that the informational content of an admissible state is an essential (though presumably not the only) factor in determining the place of this state in the preference structure.

Persistent epistemic states are formally similar to a number of *information models* suggested in the logical literature, such as Kripke's semantics for intuitionistic logic, Veltman's data semantics, etc. (see [Benthem, 1988] for an overview). All such models consist of a partially ordered set of informational states. The relevant partial order represents possible ways of information growth, so it is assumed to satisfy the persistence requirement by its very meaning.

As will be shown, persistent epistemic states constitute a smallest natural class of epistemic states that is closed under belief change operations that do not involve prioritization. Moreover, it is precisely this class of epistemic states that admits a constructive representation in terms of flocks of bases.

An epistemic state will be called *pure* if it satisfies

Pure Monotonicity $s \preceq t$ if and only if $l(s) \subseteq l(t)$.

Pure epistemic states is a special kind of persistent states for which the preference relation is determined solely by the informational content of admissible belief states. Accordingly, a pure epistemic state can be defined simply as a set of deductively closed theories, with the intended understanding that the relation of set inclusion among such theories plays the role of a preference relation. In other words, for pure epistemic states, preference is given to maximal theories.

Pure epistemic states have been used in [Bochman, 1999, Bochman, 2000] as a basis for a foundational approach to belief change. As we mentioned, however, the approach has turned out to be flawed, since it does not provide an adequate representation of belief expansions. The present study stems from a more general approach to representing belief change suggested in [Bochman, 2001b].

3 Bases and flocks

Assume that a belief set **B** is generated by some base Δ with respect to a certain consequence relation Th (that is, $\mathbf{B} = \text{Th}(\Delta)$). This structure is representable as an epistemic state of the form $(P(\Delta), l, \prec)$, where admissible belief states are the subsets of Δ, l assigns each such subset its deductive closure, while $\prec$ is a preference relation on $P(\Delta)$. In the simplest (non-prioritized) case, this preference relations is definable via set inclusion: $\Gamma \prec \Phi$ iff $\Gamma \subset \Phi$.

In the latter case base-generated epistemic states are equivalent to pure epistemic states consisting of theories of the form $\text{Th}(\Gamma)$, where Γ ranges over the subsets of Δ. Notice that any such epistemic state will be determinate, since it contains a most preferred theory, namely $\text{Th}(\Delta)$.

Unfortunately, we will see later that base-generated epistemic states (and bases themselves) are arguably inadequate for representing belief contractions. A generalization of bases that overcomes this shortcoming has been suggested in [Fagin *et al.*, 1986] and consists in using sets (or 'flocks') of bases.

By a *flock* we will mean an arbitrary set of sets of propositions $\mathcal{F} = \{\Delta_i\}$, for $i \in I$. Such a flock can be considered as a collection of bases Δ_i, and the following construction of the epistemic state generated by a flock can be seen as a natural generalization of base-generated epistemic states.

Any flock generates an epistemic state $\mathbb{E}_\mathcal{F} = \langle \mathcal{F}_\downarrow, l, \prec \rangle$ defined as follows:

- $\mathcal{F}_\downarrow$ is a set of all nonempty sets Γ such that $\Gamma \subseteq \Delta$, for some $\Delta \in \mathcal{F}$;

- $l(\Gamma) = \text{Th}(\Gamma)$, for each $\Gamma \in \mathcal{F}_\downarrow$;

- $\Gamma \prec \Phi$ holds iff $\Gamma \subset \Phi$.

As can be seen, flocks constitute a generalization of bases. Namely, any base Δ can be identified with a singular flock $\{\Delta\}$.

As in our study, flocks were used in [Fagin *et al.*, 1986] as a framework for belief change operations. Our subsequent results will be different, however. The main difference can be described as follows.

Let us say that two flocks are *identical* if they generate the same epistemic state. Now, let $\mathcal{F}$ be a flock and Δ_0 a set of propositions such that $\Delta_0 \subseteq \Delta$, for some $\Delta \in \mathcal{F}$. Then it is easy to see that flocks $\mathcal{F}$ and $\mathcal{F} \cup \{\Delta_0\}$ produce the same epistemic state, and consequently they are identical in the above sense. This shows that a flock is determined, in effect, by its inclusion-maximal elements. According to [Fagin *et al.*, 1986], however, the above two flocks are distinct, and hence the validity of propositions with respect to a flock is determined, in effect, by *minimal* sets belonging to the flock. This makes the resulting theory less plausible and more complex than it could be.

The above feature, though plausible by itself, gives rise, however, to high sensitivity of flocks with respect to the syntactic form of the propositions occurring in them.

Example 3.1. Let us consider the flock $\mathcal{F} = \{\{A'\}, \{A, B\}\}$, where A' is logically equivalent to A. Replacing A' with A, we obtain a different flock $\mathcal{F}' = \{\{A\}, \{A, B\}\}$, which is identical to $\{\{A, B\}\}$. Note that $A \wedge B$ is believed in the epistemic state generated by the latter flock, though only A is believed in the epistemic state generated by the source flock $\mathcal{F}$.

The above example shows that flocks do not admit replacements of logically equivalent propositions, at least in cases when such a replacement leads to identification of propositions with other propositions appearing elsewhere in the flock. It should be kept in mind, however, that the epistemic state generated by a flock is a syntax-independent object, though purely syntactic differences in flocks may lead to significant differences in epistemic states generated by them.

Flock-generated epistemic states are always persistent; this follows immediately from the fact that $\Gamma \prec \Phi$ holds in such a state if and only if $\Gamma \subset \Phi$, and hence $\text{Th}(\Gamma) \subseteq \text{Th}(\Phi)$. Moreover, it has been shown in [Bochman, 2001b] that any finitary persistent epis-

temic state is representable by some flock. This means that flocks constitute an adequate syntactic formalism for representing persistent epistemic states. Unfortunately, this also shows that flocks are not representable by pure epistemic states, as has been suggested in [Bochman, 1999].

4 Changing epistemic states

Since belief sets are uniquely determined by epistemic states, operations on epistemic states will also determine corresponding operations on belief sets. Two kinds of operations immediately suggest themselves as most fundamental, namely removal and addition of information to epistemic states.

4.1 Contractions

Admissible belief states of an epistemic state constitute all potential alternatives that are considered as 'serious possibilities' by the agent. In accordance with this, the contraction of a proposition A from an epistemic state $\mathbb{E}$ is defined as an operation that removes all admissible belief states from $\mathbb{E}$ that support A. We will denote the resulting epistemic state by $\mathbb{E} - A$.

The contraction operation has quite regular properties, most important of which being *commutativity*: a sequence of contractions can be performed in any order yielding the same resulting epistemic state.

Let us compare the above contraction with base contraction. According to [Hansson, 1993], the first step in performing a base contraction of A consists in finding preferred (selected) subsets of the base that do not imply A. So far, this fits well with our construction, since the latter subsets exactly correspond to preferred admissible theories of the contracted epistemic state. Then our definition says, in effect, that the contracted belief set should be equal to the intersection of these preferred theories. Unfortunately, such a solution is unacceptable for the base paradigm, since we need to obtain a unique contracted base; only the latter will determine the resulting belief set. Accordingly, [Hansson, 1993] defines first the contracted base as the intersection of all preferred subsets of the base ('partial meet base contraction'), and then the contracted belief set is defined as the set of all propositions that are implied by the new base.

The problems arising in this approach are best illustrated by the following example (adapted from [Hansson, 1992]).

Example 4.1. Two equally good and reliable friends of a student say to her, respectively, that Niamey is

a Nigerian town, and that Niamey has a university. Our student should subsequently retract her acquired belief that Niamey is a university town in Nigeria.

Let A and B denote, respectively, propositions "Niamey is a town in Nigeria", and "There is a university in Niamey". Then the above situation can be described as a contraction of $A \wedge B$ from the (belief set generated by the) base $\{A, B\}$. As has been noted in [Gärdenfors and Rott, 1995], this small example constitutes a mayor stumbling block for the base approach to belief change. Actually, we will see that none of the current approaches handles satisfactorily this example.

To begin with, it seems reasonable to expect that the contracted belief set in the above situation should contain $A \vee B$, since each of the acceptable alternatives support this belief. This result is also naturally sanctioned by the AGM theory. Using the base contraction, however, we should retreat first to the two subbases $\{A\}$ and $\{B\}$ that do not imply $A \wedge B$, and then form their intersection which happens to be empty! In other words, we have lost all the information contained in the initial base, so all subsequent changes should start from a scratch.

Next, it seems also reasonable to require that if subsequent evidence rules out A, for example, we should believe that B. In other words, contracting first $A \wedge B$ and then A from the initial belief state should make us believe in B. This time the AGM theory cannot produce this result. The reason is that the first contraction gives the logical closure of $A \vee B$ as the contracted belief set, and hence the subsequent contraction of A will not have any effect on the corresponding belief state. Notice that this information loss is not 'seen' in one-step changes; it is revealed, however, in subsequent changes.

As we see it, the source of the above problem is that traditional approaches to belief change force us to choose in situations we have no grounds for choice. And our suggested solution here amounts to retaining all the preferred alternatives as parts of the new epistemic state, instead of transforming them into a single 'combined' solution. This means, in particular, that we should allow our epistemic states to be non-determinate. This will not prevent us from determining each time a unique current set of beliefs; but we should remember more than that.

Returning to the example, our contraction operation results in a new belief set $\mathrm{Th}(A \vee B)$, as well as a new epistemic state $\{\mathrm{Th}(A), \mathrm{Th}(B)\}$ consisting of two theories. This latter epistemic state, however, is not base-generated, though it will be generated by a flock

$\{\{A\}, \{B\}\}$.

4.1.1 Contraction of flocks

Flocks emerge as a nearest counterpart of bases that will already be closed with respect to our contraction operation. Actually, the latter will correspond to the operation of deletion on flocks suggested in [Fagin *et al.*, 1986].

Definition 4.1. A *contraction of a flock* $\mathcal{F} = \{\Delta_i\}$ with respect to A (notation $\mathcal{F} - A$) is a flock consisting of all maximal subsets of each Δ_i that do not imply A.

The following result confirms that the above operation on flocks corresponds to contraction of associated epistemic states.

Lemma 4.1. *If $\mathbb{E}_{\mathcal{F}}$ is an epistemic state generated by a flock $\mathcal{F}$, then, for any A, $\mathbb{E}_{\mathcal{F}} - A = \mathbb{E}_{(\mathcal{F}-A)}$.*

Despite the similarity of the above definition of contraction with that given in [Fagin *et al.*, 1986], the resulting contraction operation will behave differently in our framework. The following example illustrates this.

Example 4.2. Contraction of the flock $\mathcal{F} = \{\{A\}, \{B\}\}$ with respect to A is a flock $\{\emptyset, \{B\}\}$, which is identical to $\{\{B\}\}$ according to our definition of identity. Consequently, B will be believed in the resulting epistemic state. This behavior seems also to agree with our intuitions, since eliminating A as an option will leave us with B as a single solution. In the representation of [Fagin *et al.*, 1986], however, $\{\emptyset, \{B\}\}$ is reducible to $\{\emptyset\}$. Consequently, nothing will be believed in the resulting epistemic state. Furthermore, this also makes the corresponding operation of deletion *non-commutative*: whereas deletion of $A \wedge B$ and then A from the base $\{A, B\}$ results in $\{\emptyset, \{B\}\}$, deletion of A first and then $A \wedge B$ will give a different flock, namely $\{\{B\}\}$.

4.2 Merge and expansion

The operation of expansion consists in adding information to epistemic states. In the AGM theory, this is achieved through a straightforward addition of the new proposition to the belief set, while in the base approach the new proposition is added directly to the base.

The framework of epistemic states drastically changes, however, the form and content of expansion operations. This stems already from the fact that adding a proposition to an epistemic state is no longer reducible to adding it to the belief set; it should determine also the place of the newly added proposition in

the structure of the expanded epistemic state. This will establish the degree of firmness with which we should believe the new proposition, as well as its dependence and justification relations with other beliefs. As a way of modelling this additional information, we suggest to treat the latter as a special case of merging the source epistemic state with another epistemic state that will represent the added information. Accordingly, we will describe first some merge operation on epistemic states. Then an expansion will be defined roughly as a merge of $\mathbb{E}$ with a rudimentary epistemic state that is generated by the base $\{A\}$.

4.2.1 Merging epistemic states

Merge is a procedure of combining a number of epistemic states into a single epistemic state, in which we seek to combine information that is supported by the source epistemic states. It turns out that this notion of merging can be captured using a well-known algebraic construction of *product*. Roughly, a merge of two epistemic states $\mathbb{E}_1$ and $\mathbb{E}_2$ is an epistemic state in which the admissible states are all *pairs* of admissible states from $\mathbb{E}_1$ and $\mathbb{E}_2$, the labelling function assigns each such pair the deductive closure of the union of their corresponding labels, while the resulting preference relation agrees with the 'component' preferences (see [Bochman, 2001b] for a formal description).

Since the primary subject of this study is non-prioritized change, we will consider below only one kind of merge operations, namely a pure merge that treats the source epistemic states as having an equal 'weight'.

Definition 4.2. A *pure merge* of epistemic states $\mathbb{E}_1 = (\mathcal{S}_1, l_1, \prec_1)$ and $\mathbb{E}_2 = (\mathcal{S}_2, l_2, \prec_2)$ is an epistemic state $\mathbb{E}_1 \times \mathbb{E}_2 = (\mathcal{S}_1 \times \mathcal{S}_2, l, \prec)$ such that

- $l(s_1, s_2) = \text{Th}(l_1(s_1) \cup l_2(s_2))$, for any $(s_1, s_2) \in \mathcal{S}_1 \times \mathcal{S}_2$;

- $(s_1, s_2) \preceq (t_1, t_2)$ iff $s_1 \preceq_1 t_1$ and $s_2 \preceq_2 t_2$.

A pure merge is a merge operation that treats the two component epistemic states as two equally reliable sources. It is easy to see, in particular, that it is a commutative operation.

Being applied to base-generated epistemic states, pure merge corresponds to a straightforward union of two bases:

Lemma 4.2. *If $\mathbb{E}_1$ and $\mathbb{E}_2$ are epistemic states generated, respectively, by bases Δ_1 and Δ_2, then $\mathbb{E}_1 \times \mathbb{E}_2$ is equivalent to an epistemic state generated by $\Delta_1 \cup \Delta_2$.*

4.2.2 Merging flocks

To begin with, the following result shows that pure merge preserves persistence of epistemic states.

Lemma 4.3. *A pure merge of two persistent epistemic states is also a persistent epistemic state.*

Since finitary persistent epistemic states are representable by flocks, a pure merge gives rise to a certain operation on flocks. This operation can be described as follows:

Let us consider two flocks $\mathcal{F}_1$ and $\mathcal{F}_2$ that have no propositions in common. Then a *merge* of $\mathcal{F}_1$ and $\mathcal{F}_2$ will be a flock

$$\mathcal{F}_1 \times \mathcal{F}_2 = \{\Delta_i \cup \Delta_j \mid \Delta_i \in \mathcal{F}_1, \Delta_j \in \mathcal{F}_2\}$$

Thus, the merge of two disjoint flocks is obtained by a pairwise combination of bases belonging to each flock. Note, however, that the assumption of disjointness turns out to be essential for establishing the correspondence between merge of flocks and pure merge of associated epistemic states. Still, this requirement is a purely syntactic constraint that can be easily met by replacing some of the propositions with logically equivalent, though syntactically different propositions. A suitable example will be given later when we will consider expansions of flocks that are based on the above notion of merge.

The following result shows that merge of flocks corresponds exactly to a pure merge of associated epistemic states.

Theorem 4.4. *If $\mathbb{E}_{\mathcal{F}}$ and $\mathbb{E}_{\mathcal{G}}$ are epistemic states generated, respectively, by disjoint flocks $\mathcal{F}$ and $\mathcal{G}$, then $\mathbb{E}_{\mathcal{F}} \times \mathbb{E}_{\mathcal{G}}$ is isomorphic to $\mathbb{E}_{\mathcal{F} \times \mathcal{G}}$.*

4.2.3 Pure expansions

For any proposition A, we will denote by $\mathbb{E}_A$ the epistemic state generated by a singular base $\{A\}$. This pure epistemic state consists of just two theories, namely $\text{Th}(\emptyset)$ and $\text{Th}(A)$. Accordingly, it gives a most 'pure' expression of the belief in A. Now the idea behind the definition below is that an expansion of an epistemic state with respect to A amounts to merging it with $\mathbb{E}_A$.

Definition 4.3. A *pure expansion* of an epistemic state $\mathbb{E}$ with respect to a proposition A (notation $\mathbb{E} + A$) is a pure merge of $\mathbb{E}$ and the epistemic state $\mathbb{E}_A$ that is generated by a base $\{A\}$.

In a pure expansion the new proposition is added as an independent piece of information, that is, as a proposi-

tion that is not related to others with respect to priority. Being applied to base-generated epistemic states, pure expansion corresponds to a straightforward addition of a new proposition to the base.

Corollary 4.5. *If $\mathbb{E}$ is generated by a base Δ, then $\mathbb{E} + A$ is isomorphic to an epistemic state generated by $\Delta \cup \{A\}$.*

Since pure merge is a commutative operation, pure expansions will also be commutative.

As any other kind of change in epistemic states, expansions generate corresponding changes in belief sets of epistemic states. It turns out that expansions generate in this sense precisely AGM belief expansion functions:

Lemma 4.6. *If $\mathbf{B}$ is a belief set of $\mathbb{E}$, then the belief set of $\mathbb{E} + A$ coincides with $\mathrm{Th}(\mathbf{B}, A)$.*

Thus, belief expansions generated by expansions of epistemic states behave just as AGM expansions: the underlying epistemic state plays no role in determining the resulting expanded belief set, since the latter can be obtained by a direct addition of new propositions to the source belief set. It should be kept in mind, however, that identical expansions of belief sets can be produced by expansions of different epistemic states, and even by different expansions of the same epistemic state. These differences will be revealed in subsequent contractions and revisions of the expanded belief set.

4.2.4 Expansions of flocks

As we have seen earlier, pure merge generates a corresponding merge operation on flocks. Consequently, pure expansion corresponds in this sense to a certain expansion operation on flocks.

Definition 4.4. An *expansion of a flock* $\mathcal{F} = \{\Delta_i \mid i \in I\}$ with respect to a proposition A that does not appear in $\mathcal{F}$ is a flock $\mathcal{F} + A = \{\Delta_i \cup \{A\} \mid i \in I\}$.

Thus, an expansion of a flock is obtained simply by adding the new proposition to each base from the flock. Our earlier results immediately imply that this operation exactly corresponds to a pure expansion of the associated epistemic state with A:

Corollary 4.7. *If $\mathbb{E}_{\mathcal{F}}$ is an epistemic state generated by the flock $\mathcal{F}$, and A does not appear in $\mathcal{F}$, then $\mathbb{E}_{\mathcal{F}} + A$ is isomorphic to $\mathbb{E}_{\mathcal{F}+A}$.*

The above operation is quite similar in spirit to the operation of insertion into flocks used in [Fagin *et al.*, 1986], though the latter was intended to preserve consistency of the component bases, so they

defined, in effect, the corresponding revision operation based on contraction and expansion in our sense. Note, however, that our flock expansion is defined only when the added proposition does not appear in the flock. The need for the restriction is illustrated by the following example.

Example 4.3. Let us return to the flock $\mathcal{F} = \{\{A\}, \{B\}\}$, where A and B denote, respectively, propositions "Niamey is a town in Nigeria", and "There is a university in Niamey". Recall that this flock is obtainable by contracting $A \wedge B$ from the base $\{A, B\}$. In other words, it reflects an informational situation in which we have reasons to believe in each of these propositions, but cannot believe in both.

Now let us expand the epistemic state $\mathbb{E}_{\mathcal{F}}$ with B. This expansion can be modeled by expanding $\mathcal{F}$ with some proposition B' that is logically equivalent to B. In other words, the epistemic state generated by the flock $\mathcal{F} + B' = \{\{B', A\}, \{B', B\}\}$ will be equivalent to the expansion of $\mathbb{E}_{\mathcal{F}}$ with B. This flock sanctions belief in B in full accordance with our intuitions. Actually, it can be shown that the latter flock is reducible to a flock $\{\{B', A\}, \{B\}\}$ in the sense that the latter flock will produce an equivalent epistemic state. However, B' cannot be replaced with B in these flocks: the flock $\{\{B\}, \{B, A\}\}$ is already reducible to a single base $\{\{A, B\}\}$ in which both A and B are believed, contrary to our intuitions about the relevant situation: receiving a new support for believing that there is a university in Niamey should not force us to believe also that Niamey is a town in Nigeria. This also shows most vividly that a straightforward addition of B to each base in the flock $\{\{A\}, \{B\}\}$ does not produce intuitively satisfactory results[1].

An additional interesting aspect of the above representation is that, though we fully believe in B in the flock $\{\{B\}, \{B', A\}\}$, the option A has not been forgotten; if we will contract now B from the latter flock, we will obtain the flock $\{\{A\}\}$ which supports belief in A. A little reflection shows that this is exactly what would be reasonable to believe in this situation.

5 Conclusions

The purpose of this study was to give a formal representation for iterative non-prioritized change. As has been shown, such a representation can be achieved in the framework of persistent epistemic states, with flocks providing the corresponding constructive representation. Moreover, these representations overcome

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[1] Exactly this was a by-product of the representation suggested in [Bochman, 1999].

shortcomings of both the AGM and base-oriented models that incur loss of information in iterative changes.

Contraction and expansions are two basic operations on epistemic states that allow to define the majority of derived belief changes. Thus, a *revision* of an epistemic state is definable via Levi identity (on the level of epistemic states), namely as a combination of contraction and expansion. As can be shown, the resulting operation will be sufficiently expressive to capture any relational belief revision function in the sense of AGM.

To conclude the paper, we want to mention an interesting problem concerning expressivity of our belief change operations vis-a-vis flocks. Namely, it has been shown in [Bochman, 2001b] that there are flocks that are not constructible from simple primitive flocks using the contraction and expansion operations (a simplest example being the flock $\{\{p\}, \{p \wedge q\}\}$). This apparently suggests that our stock of belief change operations is not complete and need to be extended with other operations that would provide functional completeness with respect to constructibility of flocks. The resulting theory would give then a truly complete constructive representation of non-prioritized belief change.

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