An algorithm for the k-error linear complexity of a sequence with period  $2p^n$  over GF(q)

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**Abstract--** The union cost is used, so that an efficient algorithm for computing the k-error linear complexity of a sequence with period  $2p^n$  over GF(q) is presented, where p and q are odd primes, and q is a primitive root of modulo  $p^2$ .

**Index Terms--** Periodic sequence; linear complexity; k-error linear complexity

#### I. INTRODUCTION

The linear complexity (LC) of a sequence has been used as a convenient measure of the randomness of a sequence. However, the LC has such an instability as an extreme change. The k-error LC (k-LC) of a periodic sequence was defined by Stamp and Martin in [4] as the smallest LC that can be obtained when any k or fewer of the symbols of the sequence are changed within one period. The k-LC is very effective for reducing the instability of the LC caused by symbol substitutions.

Unfortunately an effective algorithm for computing the k-LC has been known only for sequences over GF(2) with period  $2^n$  (the Stamp-Martin algorithm) in [4]. An alternative derivation of the Stamp-Martin algorithm was given in [3] for computing the k-error linear complexity of sequences over GF( $p^m$ ) with period  $p^n$ , p a prime.

This paper gives a complete description of the algorithm for the k-LC of sequences over GF(q) with period  $2p^n$ , where p and q are odd primes, and q is a primitive root of modulo  $p^2$ . The algorithm is derived from the Wei-Xiao-Chen algorithm in [5] for the linear complexity of sequences over GF(q) with period  $2p^n$  and by using the union cost different from that used in the Stamp-Martin algorithm for sequences over GF(2) with period  $2^n$ . It is shown that both of the logic of our algorithm and its description are rather simple.

# II. THE WEI-XIAO-CHEN ALGORITHM

In this paper we will consider sequences over GF(q) with period  $2p^n$ , where p and q are odd primes, and q is a primitive root of modulo  $p^2$ .

Let  $s=(a_0, a_1, \cdots)$  be a sequence with period  $N=2p^n$  over GF(q), l>0,  $A_i=(a_{(i-1)l}, a_{(i-1)l+1}, \cdots, a_{il-1})$ ,  $i=1,2,\cdots,2p$ . It is easy to prove the following lemmas.

**Lemma 1:** If 
$$\begin{cases} A_{p+1} + A_1 = A_{p+2} + A_2 = \mathbf{L} = A_{2p} + A_p \\ A_{p+1} - A_1 = (-1)^{i+1} (A_{p+i} - A_i), \ i = 1, 2, \mathbf{L}, p \end{cases}$$
, then 
$$\begin{cases} A_1 = A_3 = \mathbf{L} = A_p = A_{p+2} = \mathbf{L} = A_{2p-1} \\ A_{p+1} = A_{p+3} = \mathbf{L} = A_{2p} = A_2 = \mathbf{L} = A_{p-1} \end{cases}$$
,

hence  $(A_1, A_2) = (A_1, A_{p+1})$ .

**Lemma 2:** If  $A_{p+1} + A_1 = A_{p+2} + A_2 = \mathbf{L} = A_{2p} + A_p$ , then  $A_i - A_{i+1} = -(A_{p+i} - A_{p+i+1})$ ,  $i = 2,4,\mathbf{L}$ , p-1, hence

$$(\sum_{i=1}^{p} \ (-1)^{i+1}A_i, \sum_{i=1}^{p} \ (-1)^{i+1}A_{i+1}) = (\sum_{i=1}^{p} \ (-1)^{i+1}A_i, \sum_{i=1}^{p} \ (-1)^{i+1}A_{p+i}) \, .$$

**Lemma 3:** If 
$$A_{p+1} - A_1 = (-1)^{i+1}(A_{p+i} - A_i)$$
,  $i = 1, 2, \mathbf{L}$ ,  $p$ , then  $A_i + A_{i+1} = A_{p+i} + A_{p+i+1}$ ,  $i = 2, 4, \mathbf{L}$ ,  $p - 1$ , hence

$$(\sum_{i=1}^{p} A_i, \sum_{i=1}^{p} A_{i+1}) = (\sum_{i=1}^{p} A_i, \sum_{i=1}^{p} A_{p+i}).$$

With the above lemmas, the Wei-Xiao-Chen algorithm in [5] can be changed to the algorithm 1 in Fig.1.

Let  $s=(a_0, a_1, \cdots)$  be a sequence with period  $N=2p^n$  over GF(q), where p and q are odd primes, and q is a primitive root of modulo  $p^2$ , and let  $s^N=(a_0, a_1, \cdots, a_{N-1})$  be the first period of s.

$$\begin{array}{l} \mathbf{a} = s^{N}; \ l = \mathbf{p^{n}} \ ; \ c = 0; \\ \text{while } l > 1 \ \text{do} \\ \\ l = l \ / \mathbf{p} \ ; \ \mathbf{A}_{i} = (\mathbf{a}_{(i-1)l}, \, \mathbf{a}_{(i-1)l+1}, \cdots, \, \mathbf{a}_{il-1}), \ \text{for i} = 1, 2, \cdots, 2\mathbf{p}; \\ \text{if } \ A_{p+1} + A_{1} = A_{p+2} + A_{2} = \mathbf{L} = A_{2p} + A_{p} \ \ \text{then} \\ \\ \text{if } \ A_{p+1} - A_{1} = (-1)^{i+1} (A_{p+i} - A_{i}) \ \text{for } \ i = 1, 2, \mathbf{L}, \ p \ \ \text{then} \\ \\ \ a = (A_{1}, A_{p+1}); \\ \text{else} \end{array}$$

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c=c+(p-1)l; a=(\sum_{i=1}^{p} (-1)^{i+1} A_i, \sum_{i=1}^{p} (-1)^{i+1} A_{p+i});
   else
      if A_{n+1} - A_1 = (-1)^{i+1} (A_{n+i} - A_i) for i = 1, 2, \mathbf{L}, p then
             c=c+(p-1) l; a=(\sum_{i=1}^{p} A_i, \sum_{i=1}^{p} A_{p+i});
      else
             c=c+2(p-1) l; a=(\sum_{i=1}^{p} A_{2i-1}, \sum_{i=1}^{p} A_{2i});
       end if
   end if
end while
if a \neq (0,0) then
   if a_0=a_1 then
      c=c+1
   else
      if a_0 + a_1 = 0
      else
            c=c+2
      end if
   end if
end if
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Fig.1. Algorithm 1, computing the linear complexity of a sequence with period 2p<sup>n</sup> over GF(q)

#### III. A k-ERROR LINEAR COMPLEXITY ALGORITHM

The *k*-LC of a sequence  $s=(a_0, a_1, \cdots)$  over GF(q) with period  $N=2p^n$  is defined as k-LC(s)=min{LC(s+e) | w<sub>H</sub>(e)  $\leq k$ }

where  $e=(e_0, e_1, \cdots)$  is an error sequence over GF(q) with period N and  $w_H(e)$  is the Hamming weight of the first N-tuple,  $(e_0, e_1, \cdots, e_{N-1})$ , of e, i.e., the number of nonzero  $e_j$ 's. If we have no effective algorithm for computing the k-LC, we must repeatedly apply the Wei-Xiao-Chen algorithm at the worst case

$$\sum_{i=0}^{k} (q-1)^{i} \binom{N}{i} \tag{1}$$

times to the sequences (s+e)'s with all the possible e's having Hamming weight  $\leq k$ . However, (1) becomes very large even for moderate N and k.

In order to compute the k-LC of s, we must try to force  $A_{p+1}+A_1=A_{p+2}+A_2=\mathbf{L}=A_{2p}+A_p$  and  $A_{p+1}-A_1=(-1)^{i+1}(A_{p+i}-A_i)$  for  $i=1,2,\mathbf{L}$ , p, in the Wei-Xiao-Chen algorithm under the condition that the minimum number of changes in the original  $s^N$  is less than or equal to k. This logic is the same as that used in the Stamp-Martin algorithm in [4].

In [3,4], cost[i] is intended to measure the cost-in terms of the minimum number of changes required in the original sequence s-of changing the current element  $a_i$  without disturbing the results of any previous steps. Due to the condition that  $A_{p+1} + A_1 = A_{p+2} + A_2 = \mathbf{L} = A_{2p} + A_p$  and  $A_{p+1} - A_1 = (-1)^{i+1}(A_{p+i} - A_i)$  for  $i = 1,2,\mathbf{L}$ , p, the cost of changing the element  $a_i$  and the cost of changing the element  $a_{l+i}$  are interrelated. Thus the union cost is used to measure the cost of changing  $a_i$  and  $a_{l+i}$  at the same time.

In the Stamp-Martin algorithm, only the cost of changing the current element is measured. In fact, the cost of maintaining the current element unchanged is sometimes not zero. In our algorithm,  $\cos[i,i+l,h_0,h_1]^{(2l)}$  is the minimum number of changes required in the original sequence s to change the current element  $a_i$  to  $h_0$  and the current element  $a_{l+i}$  to  $h_1$ , where  $h_0=0,1,\cdots,q-1$ ,  $h_1=0,1,\cdots,q-1$ , and 2l is the number of current elements. When  $l=p^m$ , the initial value of  $\cos[i,i+l,a_i,a_{l+i}]^{(2l)}$  is 0, the initial value of both  $\cos[i,i+l,a_i,a_{l+i}+b]^{(2l)}$  and  $\cos[i,i+l,a_i+a,a_{l+i}]^{(2l)}$  is 1, the initial value of  $\cos[i,i+l,a_i+a,a_{l+i}]^{(2l)}$ 

 $a_i + a_i$ ,  $a_{l+i} + b_i$ ] (2) is 2, where  $i = 0, 1, \dots, l-1$ , and  $i = 1, 2, \dots, q-1$ ,  $i = 1, 2, \dots, q-1$ .

Based on algorithm 1, our algorithm for computing the k-error linear complexity is written in Fig.2.

Let  $s=(a_0, a_1, \cdots)$  be a sequence with period  $N=2p^n$  over GF(q), where p and q are odd primes, and q is a primitive root of modulo  $p^2$ , and let  $s^N=(a_0, a_1, \cdots, a_{N-1})$  be the first period of s.

$$a = s^{N}; l = p^{n}; c = 0;$$

while l > 1 do

$$l = l / p$$
;  $A_i = (a_{(i-1)l}, a_{(i-1)l+1}, \dots, a_{il-1})$ , for  $i=1,2,\dots,2p$ ;

$$(B_1, B_2, \dots, B_n) = (A_1 + A_{n+1}, A_2 + A_{n+2}, \dots, A_n + A_{2n})$$

bcost[i, h]  $^{(pl)}$ =min{cost[i, i+ pl, d<sub>1</sub>, d<sub>2</sub>] $^{(2pl)}$ |d<sub>1</sub>+d<sub>2</sub>=h}, for h=0,1,···,q-1, i=0,1,···, pl-1;

bcost[i, h] 
$$^{(l)} = \sum_{i=0}^{p-1}$$
 bcost[i+jl, h]  $^{(pl)}$ , for h=0,1,...,q-1, i=0,1,..., l-1;

$$T_{B}=\sum_{i=0}^{l-1} \quad \min_{0 \leq h < q} \{b cost[i, h]^{(l)}\};$$

if  $T_{\rm R} \leq k$  then

$$cost[i, i+l, h_0, h_1]^{(2l)} = \sum_{i=0}^{(p-1)/2} cost[i+2lj, i+pl +2lj, h_0, h_1]^{(2pl)}$$

$$+\sum_{i=0}^{(p-1)/2-1} \operatorname{cost}[i+(2j+1)l,\ i+pl+(2j+1)l,\ h_1,h_0]^{(2pl)},\ \text{for}\ h_0=0,1,\cdots,q-1,\ h_1=0,1,\cdots,q-1,\ i=0,1,\cdots,l-1;$$

$$T_{C} = \sum_{i=0}^{l-1} \quad \min_{0 \le h_{0} < q, \, 0 \le h_{1} < q} \left\{ cost[i, \, i+l, \, h_{0}, \, h_{1}]^{(2l)} \right\};$$

If  $T_C \leq k$  then

$$a = (A_1, A_{n+1});$$

else

c=c+(p-1)l; a=(
$$\sum_{i=1}^{p} (-1)^{i+1} A_i, \sum_{i=1}^{p} (-1)^{i+1} A_{p+i}$$
);

$$cost[i,\,i+\textit{l},\,h_0,\,h_1]^{(2\textit{l})} = min\{\,\sum_{\textit{i}=0}^{\textit{p}-1} \ cost[i+\textit{l}j,\,i+\textit{p}\textit{l}\,+\textit{l}j,\,h_{\textit{i}+\textit{l}j}^{\,0}\,,\,h_{\textit{i}+\textit{l}j}^{\,1}\,]^{(2\textit{p}\textit{l})}$$

$$|\mathbf{h}_{i+lj}^{0} + \mathbf{h}_{i+lj}^{1} = \mathbf{h}_{i}^{0} + \mathbf{h}_{i}^{1}, \text{ for } j=1,2,\cdots,p-1, \sum_{i=0}^{p-1} (-1)^{j} h_{i+lj}^{0} = \mathbf{h}_{0}, \sum_{i=0}^{p-1} (-1)^{j} h_{i+lj}^{1} = \mathbf{h}_{1} \},$$

for 
$$h_0=0,1,\dots,q-1, h_1=0,1,\dots,q-1, i=0,1,\dots, l-1;$$

end if

else

$$(D_1, D_2, \dots, D_p) = (A_{p+1} - A_1, -(A_{p+2} - A_2), \dots, A_{2p} - A_p);$$

$$\operatorname{dcost}[i+jl, h] \stackrel{(pl)}{=} \min\{\operatorname{cost}[i+jl, i+jl+pl, h \stackrel{0}{_{i+lj}}, h \stackrel{1}{_{i+lj}}]^{(2pl)} | (-1)^{-j} (h \stackrel{1}{_{i+lj}} - h \stackrel{0}{_{i+lj}}) = h\},$$

for  $h=0,1,\dots,q-1$ ,  $i=0,1,\dots,l-1$ ,  $j=0,1,\dots,p-1$ ;

dcost[i, h] 
$$^{(l)} = \sum_{j=0}^{p-1} \text{dcost}[i+jl, h]^{(pl)}$$
, for h=0,1,...,q-1, i=0,1,..., l-1;

$$T_D = \sum_{i=0}^{l-1} \min_{0 \le h < q} \{ dcost[i, h]^{(l)} \};$$

if  $T_D \leq k$  then

c=c+(p-1)
$$l$$
; a= $(\sum_{i=1}^{p} A_i, \sum_{i=1}^{p} A_{p+i})$ ;

$$\begin{split} & \operatorname{cost}[i,i+l,h_0,h_1]^{(2l)} = \min\{\sum_{j=0}^{p-1} & \operatorname{cost}[i+lj,i+pl+lj,h_0^0,h_{i+lj}^0,h_{i+lj}^0]^{(2p)} \\ & \hspace{0.5cm} | (-1)^{-j} \left(h_{1+lj}^1 - h_{i+lj}^0\right) = h_i^1 - h_i^0, \text{ for } j = 1,2,\cdots,p-1, \sum_{j=0}^{p-1} h_{i+lj}^0 = h_0, \sum_{j=0}^{p-1} h_{i+lj}^1 = h_1 \}, \\ & \hspace{0.5cm} \operatorname{for } h_0 = 0,1,\cdots,q-1, h_1 = 0,1,\cdots,q-1, i = 0,1,\cdots,l-1; \\ & \hspace{0.5cm} \operatorname{else} \\ & \hspace{0.5cm} \operatorname{cost}[i,i+l,h_0,h_1]^{(2l)} = \min\{\sum_{i=1}^{p-1} A_{2i-1}, \sum_{i=1}^{p} A_{2i}\}; \\ & \hspace{0.5cm} \operatorname{cost}[i,i+l,h_0,h_1]^{(2l)} = \min\{\sum_{j=0}^{(p-1)/2} \operatorname{cost}[i+2lj,i+pl+2lj,h_{i+2lj}^0,h_{i+2lj}^0,h_{i+2lj}^1]^{(2pl)} \\ & \hspace{0.5cm} + \sum_{j=0}^{(p-1)/2-1} \operatorname{cost}[i+(2j+1)l,i+pl+(2j+1)l,h_{i+(2j+1)l}^1,h_{i+(2j+1)l}^0]^{(2pl)} \sum_{j=0}^{p-1} h_{i+lj}^0 = h_0, \sum_{j=0}^{p-1} h_{i+lj}^1 = h_1 \}, \\ & \hspace{0.5cm} \operatorname{for } h_0 = 0,1,\cdots,q-1,h_1 = 0,1,\cdots,q-1,i=0,1,\cdots,l-1; \\ & \hspace{0.5cm} \operatorname{end if } \\ \operatorname{end if } \\ \operatorname{end while } \\ \operatorname{if } \min_{0 \leq h \leq q} \{ \operatorname{cost}[0,1,h,h]^{(2)} \} \leqslant k \text{ then } \\ & \hspace{0.5cm} \operatorname{ccc} + 1 \\ \operatorname{else } \\ & \hspace{0.5cm} \operatorname{ccc} + 1 \\ \operatorname{else } \\ & \hspace{0.5cm} \operatorname{ccc} + 2 \\ \operatorname{end if } \\ \end{array}$$

Fig. 2. Algorithm 2, computing the k-error linear complexity of a sequence with period  $2p^n$  over GF(q)

This new algorithm reduces to the Wei-Xiao-Chen algorithm [5] in the case k = 0. The validity of our algorithm can be shown by using the following propositions.

**Proposition 1.** At the jth( $j \le n$ ) step, we may prevent  $(p-1)p^{n-j}$  or  $2(p-1)p^{n-j}$  from being added to c, and the total of all remaining possible additions is only  $2p^{n-j}$ .

**Proof:** 
$$\begin{bmatrix} (p-1)p^{n-1} & (p-1)p^{n-2} & \mathbf{L} & (p-1)p & p-1 & 1 \\ (p-1)p^{n-1} & (p-1)p^{n-2} & \mathbf{L} & (p-1)p & p-1 & 1 \end{bmatrix}$$

Here the jth column of the matrix represents the possible additions to c at the jth step.

The total of all remaining possible additions is  $2(p-1)p^{m-j-1}+2(p-1)p^{m-j-2}+\cdots+2(p-1)+2=2p^{m-j}$ .

**Proposition 2.** 
$$\sum_{l=0}^{l-1} \min_{0 \le h_0 < q, 0 \le h_0 < q} \{ \operatorname{cost}[i, i+l, h_0, h_1]^{(2l)} \} \le k, \quad l = p^n, \dots, p, 1$$

**Proof:** When  $l = p^n$ , it is easy to show that  $\sum_{i=0}^{l-1} \min_{0 \le h_0 < q, 0 \le h_1 < q} \{ \text{cost}[i, i+l, h_0, h_1]^{(2l)} \} = 0.$ 

If 
$$T_C \le k$$
 at the jth step, we have  $\sum_{i=0}^{l-1} \min_{0 \le h_0 < q, 0 \le h_1 < q} \{ \text{cost[i, i+}l, h_0, h_1]^{(2l)} \} = T_C \le k.$ 

If 
$$T_B \le k$$
 and  $T_C > k$  at the jth step, we have 
$$\sum_{i=0}^{l-1} \min_{0 \le h_0 < q, 0 \le h_i < q} \{ \text{cost[i, i+}l, h_0, h_1]^{(2l)} \} = T_B \le k.$$

If 
$$T_D \leq k$$
 and  $T_B > k$  at the jth step, we have  $\min_{0 \leq h_0 < q, 0 \leq h_1 < q} \{ \operatorname{cost}[i, i+l, h_0, h_1]^{(2l)} \} = T_D \leq k$ .

If  $T_D > k$  and  $T_B > k$  at the jth step, we have

$$\sum_{i=0}^{l-1} \quad \min_{0 \leq h_0 < q, \, 0 \leq h_1 < q} \; \{ \; \operatorname{cost}[\mathrm{i}, \, \mathrm{i} + l, \, h_0, \, h_1]^{(2l)} \} = \\ \sum_{i=0}^{pl-1} \quad \min_{0 \leq h_0 < q, \, 0 \leq h_1 < q} \; \{ \; \operatorname{cost}[\mathrm{i}, \, \mathrm{i} + pl, \, h_0, \, h_1]^{(2pl)} \} \leqslant k. \; \text{The proof is completed.}$$

# IV. NUMERICAL EXAMPLE

**Example 1.** Let s be a sequence over GF(3) with period  $N=2 \cdot 5^2$  whose one period is **Initial values:**  $a = s^N$ ;  $l = 5^2$ ; c = 0;

where the ith( $0 \le i < l$ ) column of the matrix represents cost[i,i+l,h<sub>0</sub>,h<sub>1</sub>]<sup>(2l)</sup>, h<sub>0</sub>=0,1,···,q-1, h<sub>1</sub>=0,1,···,q-1.

**Step 1.** l=5,  $A_1=12101$ ,  $A_2=00012$ ,  $A_3=12021$ ,  $A_4=02202$ ,  $A_5=11221$ ,  $A_6=12121$ ,  $A_7=21210$ ,  $A_8=12101$ ,  $A_9=21021$ ,  $A_9=21021$ ,  $A_{10}=12101$ ,  $A_{11}=12101$ ,  $A_{12}=12101$ ,  $A_{13}=12101$ ,  $A_{14}=12101$ ,  $A_{15}=12101$ ,  $A_{15}=12$  $A_{10} = 10100$ 

$$(bcost[i,h]^{(l)}) = \begin{pmatrix} 5 & 4 & 4 & 5 & 4 \\ 5 & 1 & 4 & 5 & 4 \\ 0 & 5 & 2 & 0 & 2 \end{pmatrix}, \quad T_B = 5 = k. \quad (cost[i, i+l, h_0, h_1]^{(2l)}) = \begin{pmatrix} 8 & 8 & 7 & 6 & 8 \\ 7 & 10 & 5 & 8 & 7 \\ 10 & 7 & 7 & 8 & 7 \\ 5 & 5 & 8 & 6 & 5 \\ 4 & 7 & 6 & 8 & 4 \\ 7 & 4 & 8 & 8 & 4 \\ 6 & 6 & 7 & 4 & 9 \\ 5 & 8 & 5 & 6 & 8 \\ 8 & 5 & 7 & 6 & 8 \end{pmatrix}$$

$$T_{C}=21>k, \text{ so c}=20. \qquad (cost[i, i+l, h_{0}, h_{1}]^{(2l)})= \begin{pmatrix} 5 & 4 & 4 & 5 & 4 \\ 5 & 1 & 4 & 5 & 4 \\ 0 & 5 & 2 & 0 & 2 \\ 5 & 2 & 4 & 5 & 4 \\ 2 & 5 & 2 & 2 & 2 \\ 5 & 4 & 4 & 5 & 4 \\ 2 & 5 & 2 & 2 & 2 \\ 5 & 4 & 4 & 5 & 4 \\ 5 & 1 & 4 & 5 & 4 \end{pmatrix}$$

$$(\operatorname{dcost}[i,h]^{(ph)}) = \begin{pmatrix} 2 & 1 & 2 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 \\ 0 & 1 & 2 & 2 & 2 \end{pmatrix}, (\operatorname{dcost}[i,h]^{(h)}) = \begin{pmatrix} 9 \\ 8 \\ 7 \end{pmatrix}. T_{D} = 7 > k, \text{ so } c = 20 + 8 = 28. (\operatorname{cost}[i, i+l, h_0, h_1]^{(2h)}) = \begin{pmatrix} 7 \\ 7 \\ 7 \\ 5 \\ 7 \\ 5 \\ 7 \end{pmatrix}$$

**Step 3.** a=(1,1). Since  $cost[0, 1, 0, 0]^{(2)}=5=k$ , therefore c=28. Finally the 5-error linear complexity is 28.

#### V. CONCLUSION

First, we optimize the structure of the Wei-Xiao-Chen algorithm in [5] for the linear complexity of sequences over GF(q) with period  $N = 2p^n$ , where p and q are odd primes, and q is a primitive root (mod  $p^2$ ).

Second, we presented an algorithm for determining the k-error linear complexity of a sequence with period  $N = 2p^n$  over GF(q), where p and q are odd primes, and q is a primitive root (mod  $p^2$ ). The algorithm is derived from the Wei-Xiao-Chen algorithm for the linear complexity of sequences over GF(q) with period  $2p^n$  and by using the union cost different from that used in the Stamp-Martin algorithm for sequences over GF(2) with period  $2^n$ . The algorithm reduces to the Wei-Xiao-Chen algorithm in the case k = 0.

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