



# Optimization Problems

- ▶ Filling a knapsack is an optimization problem
- ▶ Optimization applications are among the most difficult problems in computer science
  - NP-Completeness
- ▶ Yet, they are everywhere
  - supply chains, sport scheduling, logistics, electrical power system, manufacturing, ..

# NP-Completeness

- ▶ Complexity class for decision problems
  - e.g., can I fill a knapsack for a value above K?
- ▶ Informally, NP-Complete problems have two properties
  - We can check a solution quickly, i.e., in polynomial time
  - If we can solve one NP-Complete quickly, we can solve them all
- ▶ Widely believed to take exponential time in the worst case

# NP-Completeness

- ▶ Yet, they are everywhere

# Logistics



# Energy

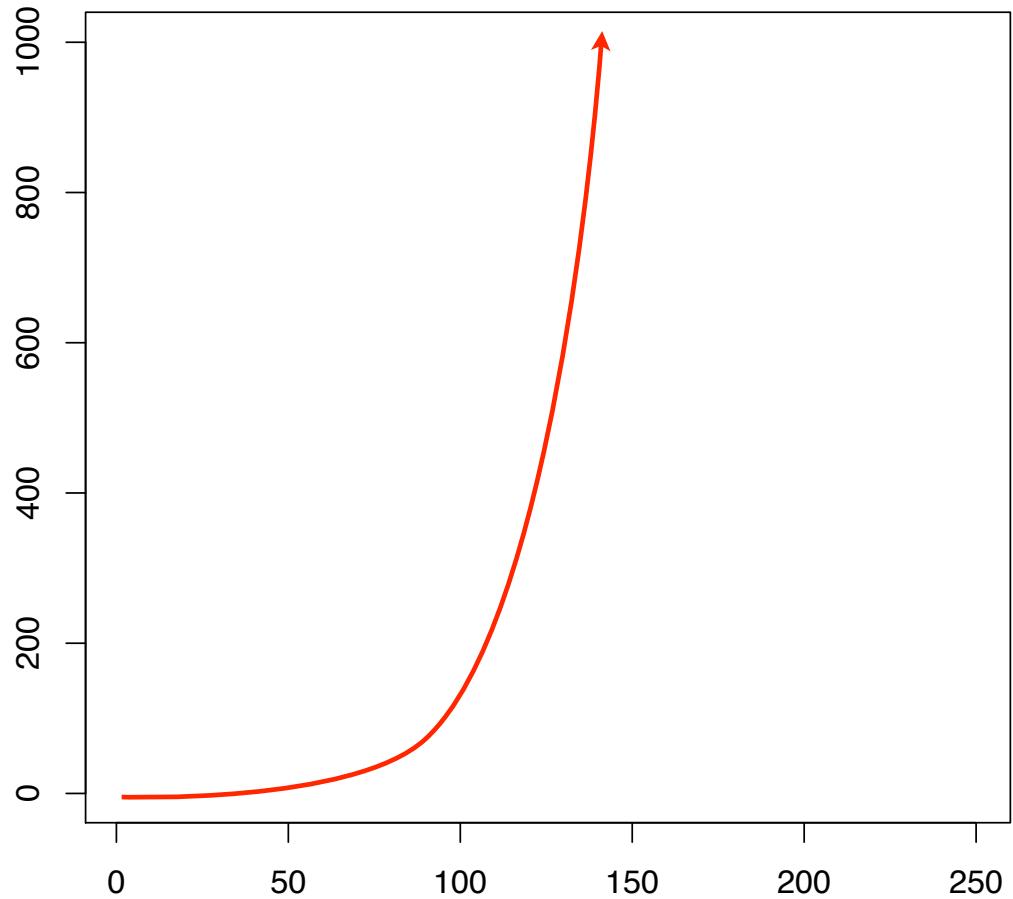


# Sport Scheduling

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
ARI	CAR	WAS	SEA	NYG	MIN	---	PIT	BAL	STL	PHI	SF	STL	DAL	SF	CLE	CIN	SEA
ATL	CHI	PHI	TB	SEA	GB	CAR	DET	---	IND	NO	TEN	MIN	HOU	CAR	JAX	NO	TB
BAL	PIT	TEN	STL	NYJ	---	HOU	JAX	ARI	PIT	SEA	CIN	SF	CLE	IND	SD	CLE	CIN
BUF	KC	OAK	NE	CIN	PHI	NYG	---	WAS	NYJ	DAL	MIA	NYJ	TEN	SD	MIA	DEN	NE
CAR	ARI	GB	JAX	CHI	NO	ATL	WAS	MIN	---	TEN	DET	IND	TB	ATL	HOU	TB	NO
CHI	ATL	NO	GB	CAR	DET	MIN	TB	---	PHI	DET	SD	OAK	KC	DEN	SEA	GB	MIN
CIN	CLE	DEN	SF	BUF	JAX	IND	---	SEA	TEN	PIT	BAL	CLE	PIT	HOU	STL	ARI	BAL
CLE	CIN	IND	MIA	TEN	---	OAK	SEA	SF	HOU	STL	JAX	CIN	BAL	PIT	ARI	BAL	PIT
DAL	NYJ	SF	WAS	DET	---	NE	STL	PHI	SEA	BUF	WAS	MIA	ARI	NYG	TB	PHI	NYG
DEN	OAK	CIN	TEN	GB	SD	---	MIA	DET	OAK	KC	NYJ	SD	MIN	CHI	NE	BUF	KC
DET	TB	KC	MIN	DAL	CHI	SF	ATL	DEN	---	CHI	CAR	GB	NO	MIN	OAK	SD	GB
GB	NO	CAR	CHI	DEN	ATL	STL	MIN	---	SD	MIN	TB	DET	NYG	OAK	KC	CHI	DET
HOU	IND	MIA	NO	PIT	OAK	BAL	TEN	JAX	CLE	TB	---	JAX	ATL	CIN	CAR	IND	TEN
IND	HOU	CLE	PIT	TB	KC	CIN	NO	TEN	ATL	JAX	---	CAR	NE	BAL	TEN	HOU	JAX
JAX	TEN	NYJ	CAR	NO	CIN	PIT	BAL	HOU	---	IND	CLE	HOU	SD	TB	ATL	TEN	IND
KC	BUF	DET	SD	MIN	IND	---	OAK	SD	MIA	DEN	NE	PIT	CHI	NYJ	GB	OAK	DEN
MIA	NE	HOU	CLE	SD	---	NYJ	DEN	NYG	KC	WAS	BUF	DAL	OAK	PHI	BUF	NE	NYJ
MIN	SD	TB	DET	KC	ARI	CHI	GB	CAR	---	GB	OAK	ATL	DEN	DET	NO	WAS	CHI
NE	MIA	SD	BUF	OAK	NYJ	DAL	---	PIT	NYG	NYJ	KC	PHI	IND	WAS	DEN	MIA	BUF
NO	GB	CHI	HOU	JAX	CAR	TB	IND	STL	TB	ATL	---	NYG	DET	TEN	MIN	ATL	CAR
NYG	WAS	STL	PHI	ARI	SEA	BUF	---	MIA	NE	SF	PHI	NO	GB	DAL	WAS	NYJ	DAL
NYJ	DAL	JAX	OAK	BAL	NE	MIA	SD	---	BUF	NE	DEN	BUF	WAS	KC	PHI	NYG	MA
OAK	DEN	BUF	NYJ	NE	HOU	CLE	KC	---	DEN	SD	MIN	CHI	MIA	GB	DET	KC	SD
PHI	STL	ATL	NYG	SF	BUF	WAS	---	DAL	CHI	ARI	NYG	NE	SEA	MIA	NYJ	DAL	WAS
PIT	BAL	SEA	IND	HOU	TEN	JAX	ARI	NE	BAL	CIN	---	KC	CIN	CLE	SF	STL	CLE
SD	MIN	NE	KC	MIA	DEN	---	NYJ	KC	GB	OAK	CHI	DEN	JAX	BUF	BAL	DET	OAK
SEA	SF	PIT	ARI	ATL	NYG	---	CLE	CIN	DAL	BAL	STL	WAS	PHI	STL	CHI	SF	ARI
SF	SEA	DAL	CIN	PHI	TB	DET	---	CLE	WAS	NYG	ARI	BAL	STL	ARI	PIT	SEA	STL
STL	PHI	NYG	BAL	WAS	---	GB	DAL	NO	ARI	CLE	SEA	ARI	SF	SEA	CIN	PIT	SF
TB	DET	MIN	ATL	IND	SF	NO	CHI	---	NO	HOU	GB	TEN	CAR	JAX	DAL	CAR	ATL
TEN	JAX	BAL	DEN	CLE	PIT	---	HOU	IND	CIN	CAR	ATL	TB	BUF	NO	IND	JAX	HOU
WAS	NYG	ARI	DAL	STL	---	PHI	CAR	BUF	SF	MIA	DAL	SEA	NYJ	NE	NYG	MIN	PHI



# Exponential Runtime



# Tricks of the Trade



**YOU CANNOT FAIL!**

**SO I AM LOWERING MY  
STANDARDS.**

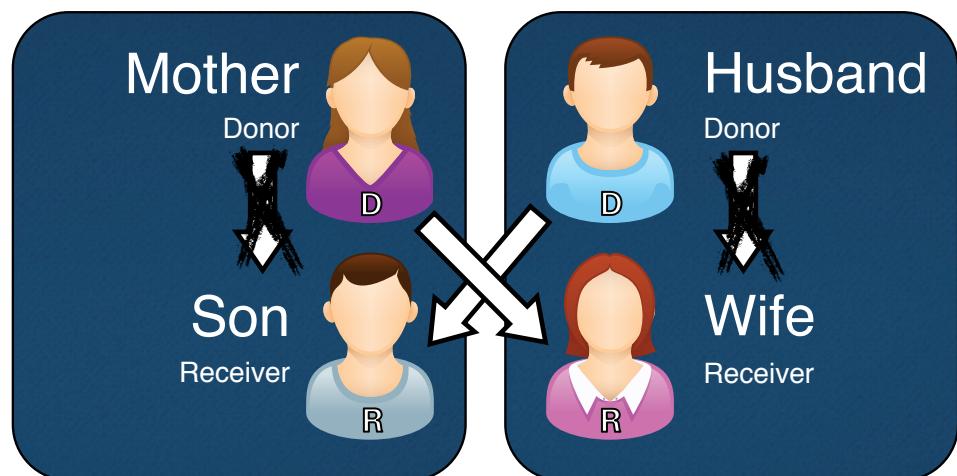
# Optimization Problems

- ▶ Optimization problems are everywhere
- ▶ They are incredibly hard to solve
- ▶ But we need to solve them
- ▶ It is fun to do so
- ▶ It is important to solve them

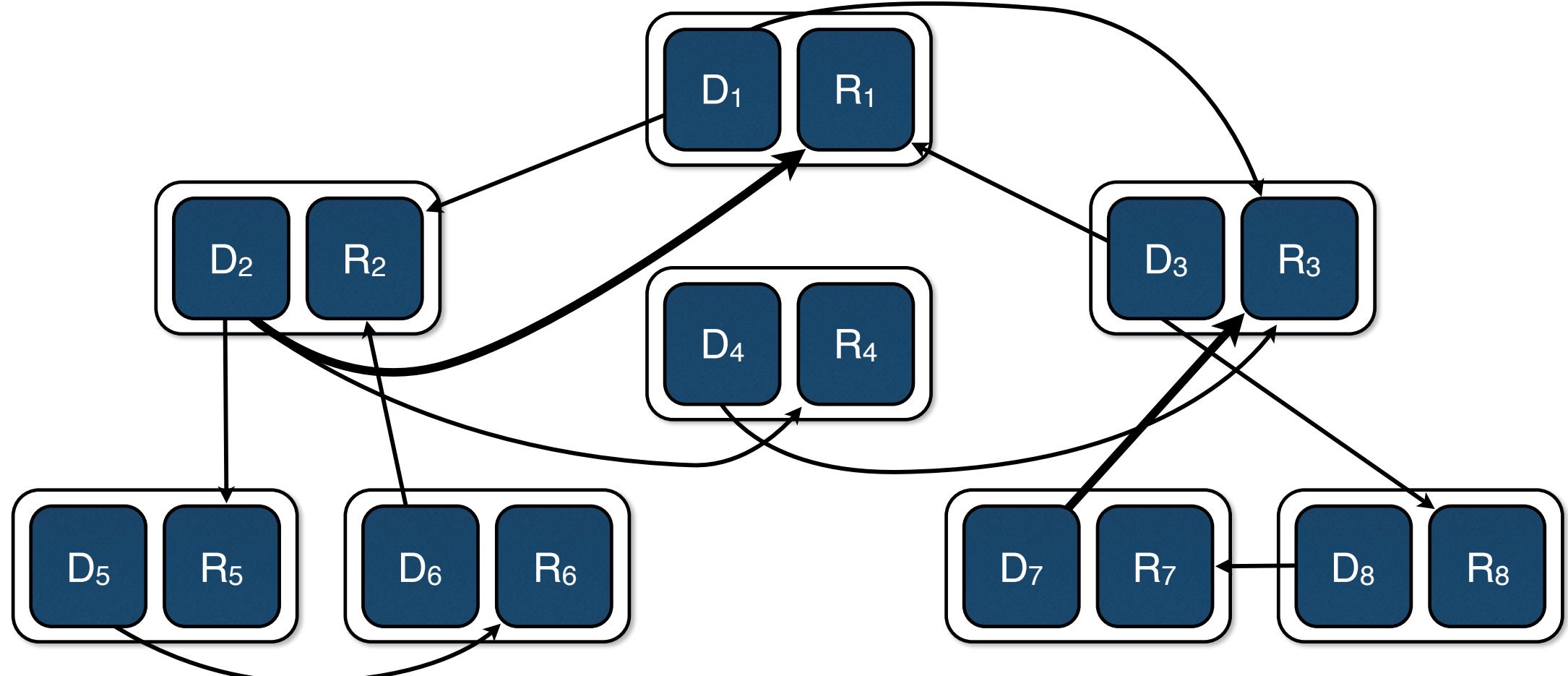
# Kidney Exchanges

## ► Basic Facts

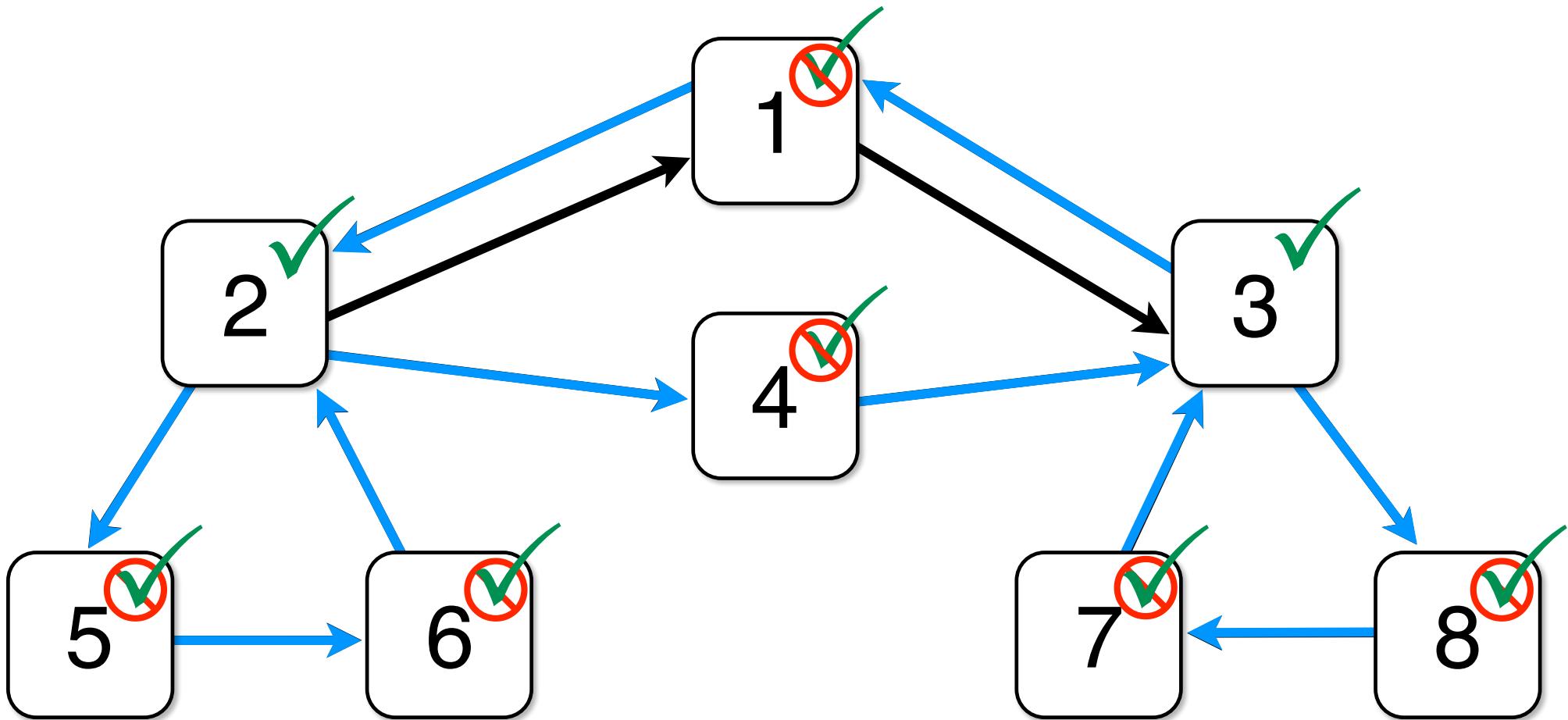
- We need only one kidney
- about 80,000 patients
- about 4,000 die each year in the US
- Compatibility issues



# Kidney Exchanges



# Kidney Exchanges

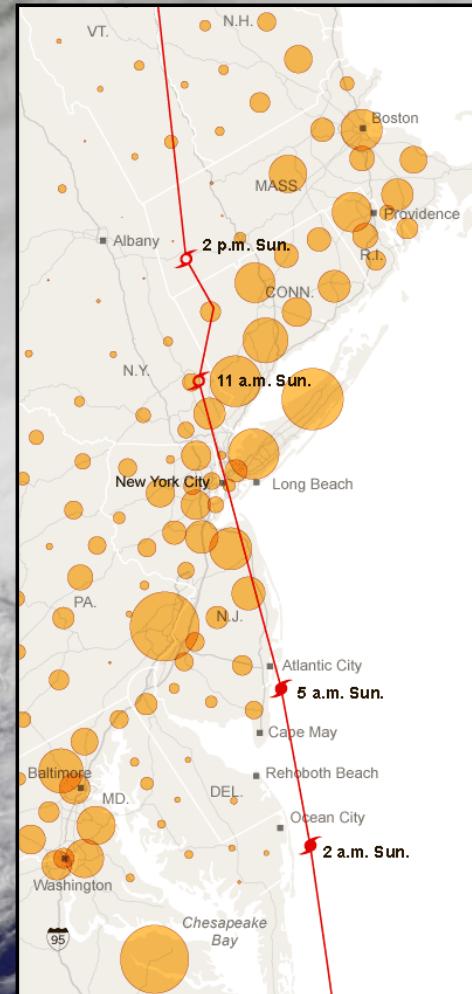
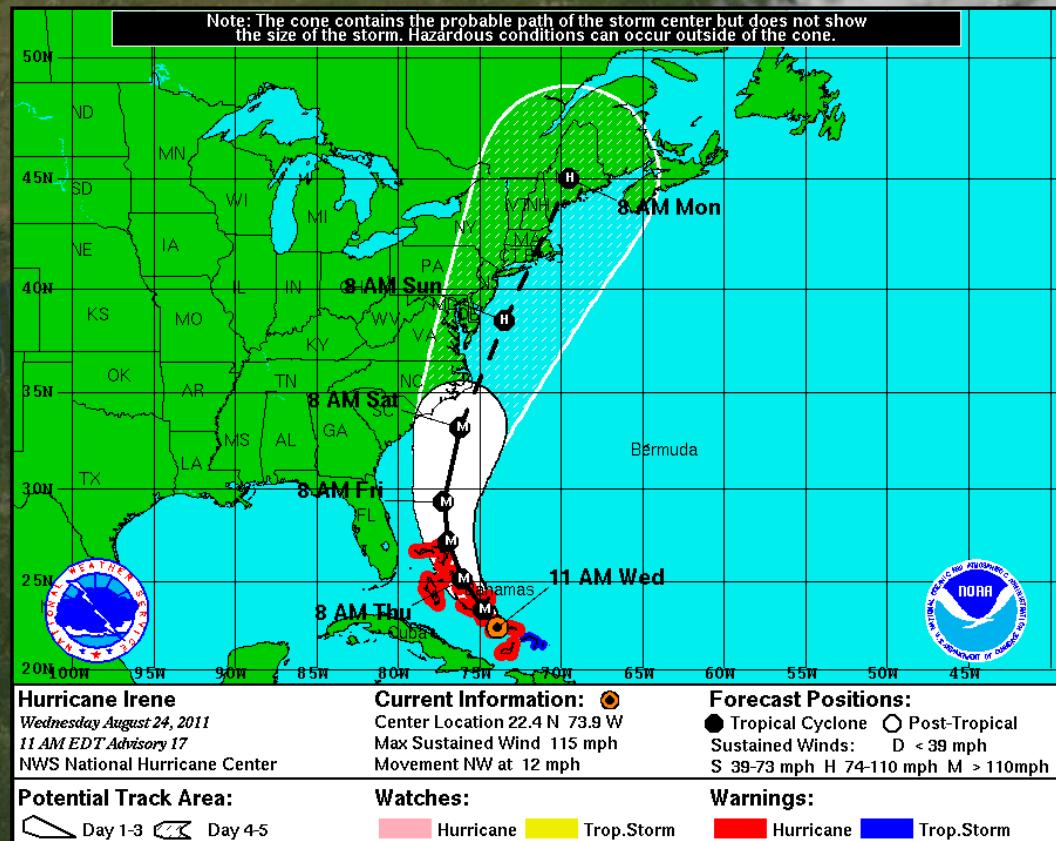


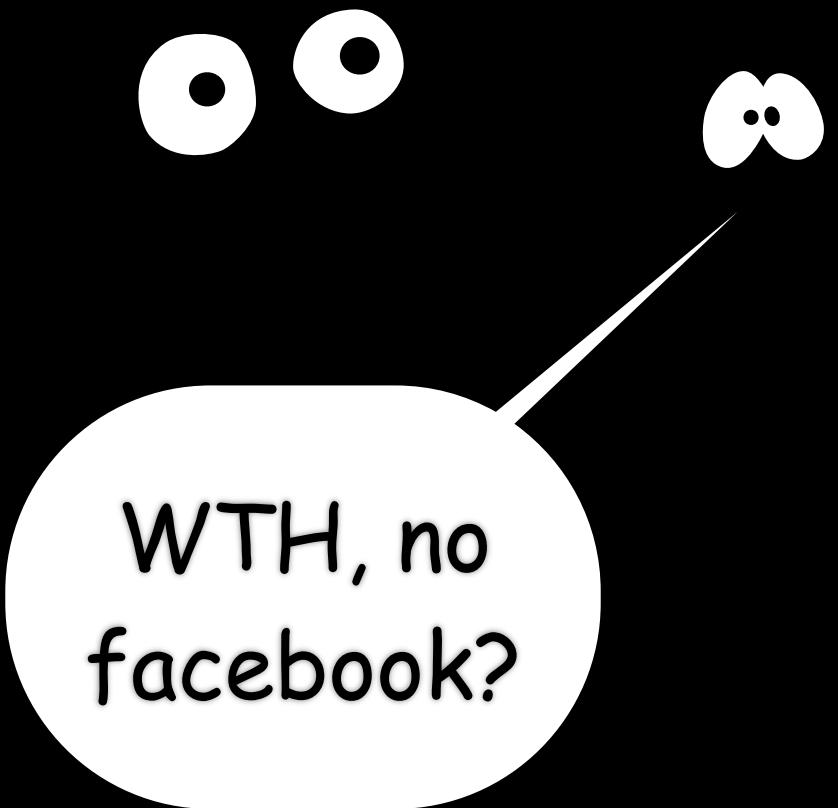
# Disaster Management

- ▶ Category 3
  - August 21-28, 2011
- ▶ Fatalities
  - 49 direct (+7)
- ▶ Damages
  - \$15 Billion USD

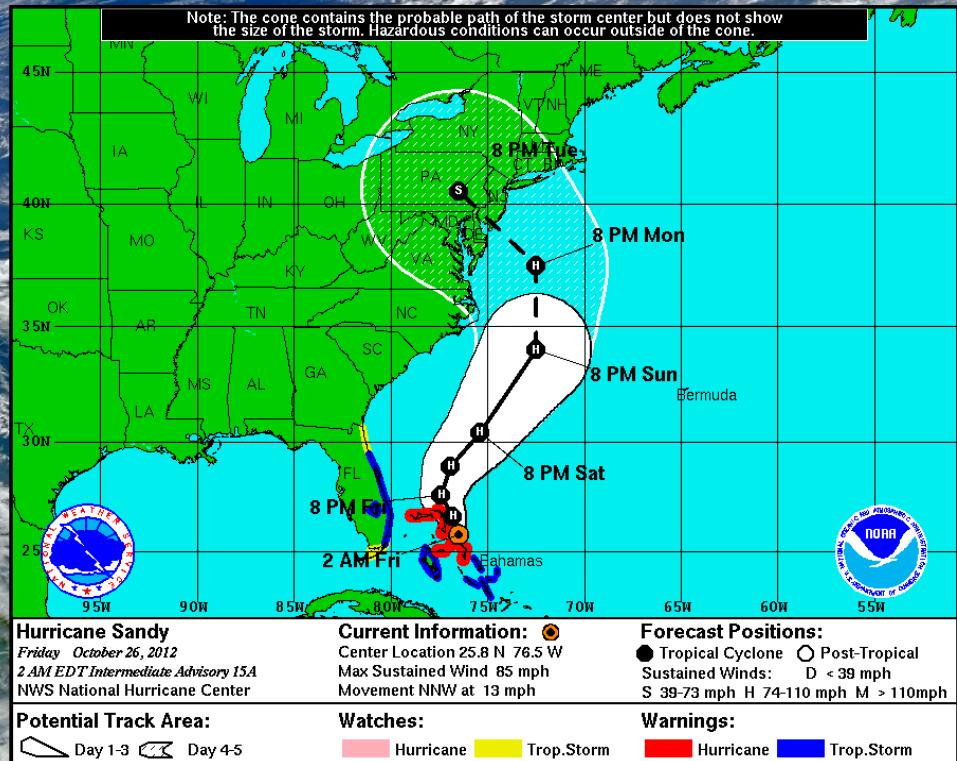


# Hurricane Irene, 2011





# Hurricane Sandy, 2012



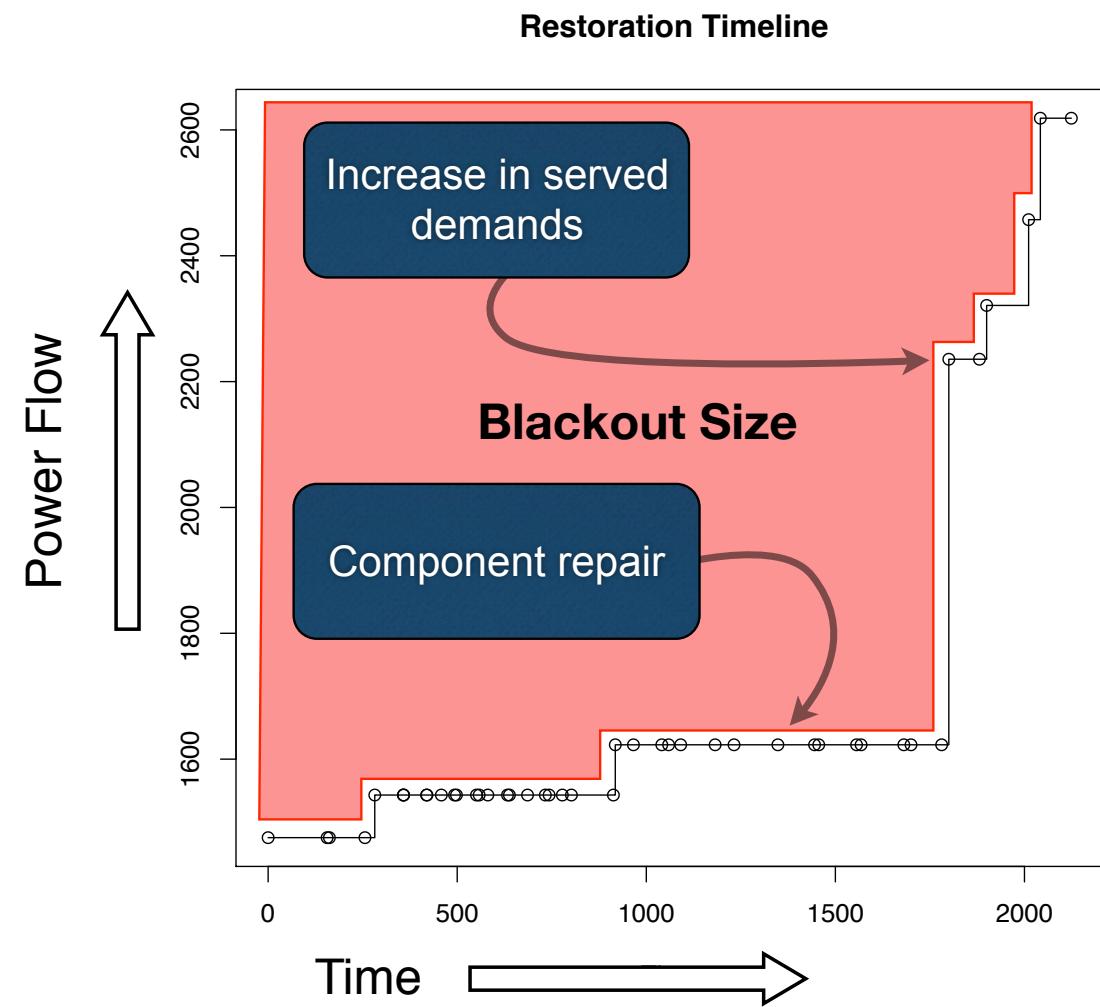
# Sandy Blackout NYC 2012



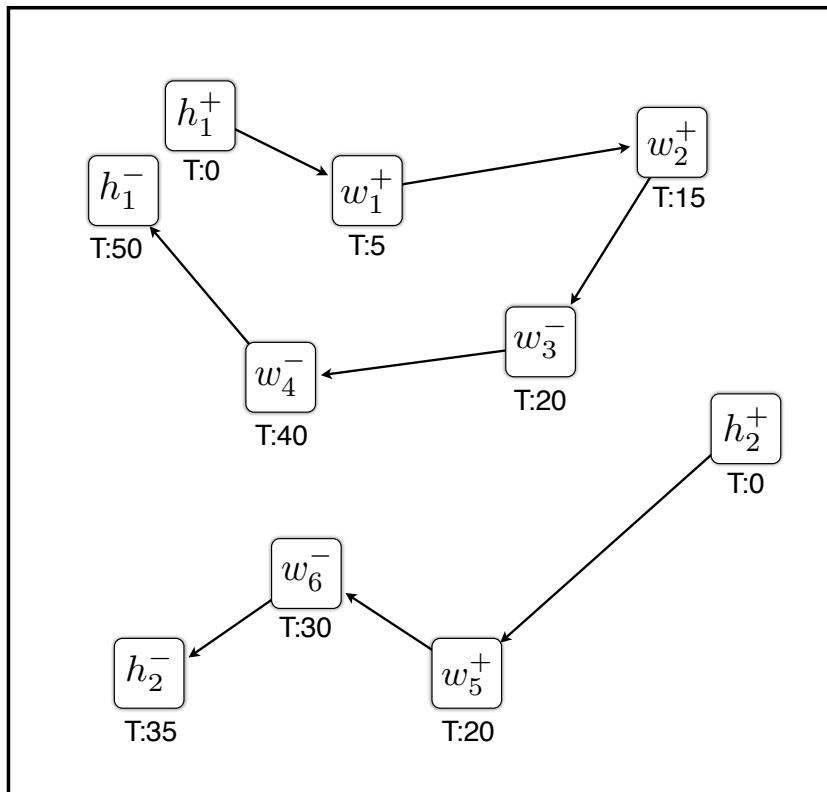
# Power Restoration

- ▶ How do we schedule a fleet of repair crews to repair the grid and minimize the overall size of the blackout after a disaster?

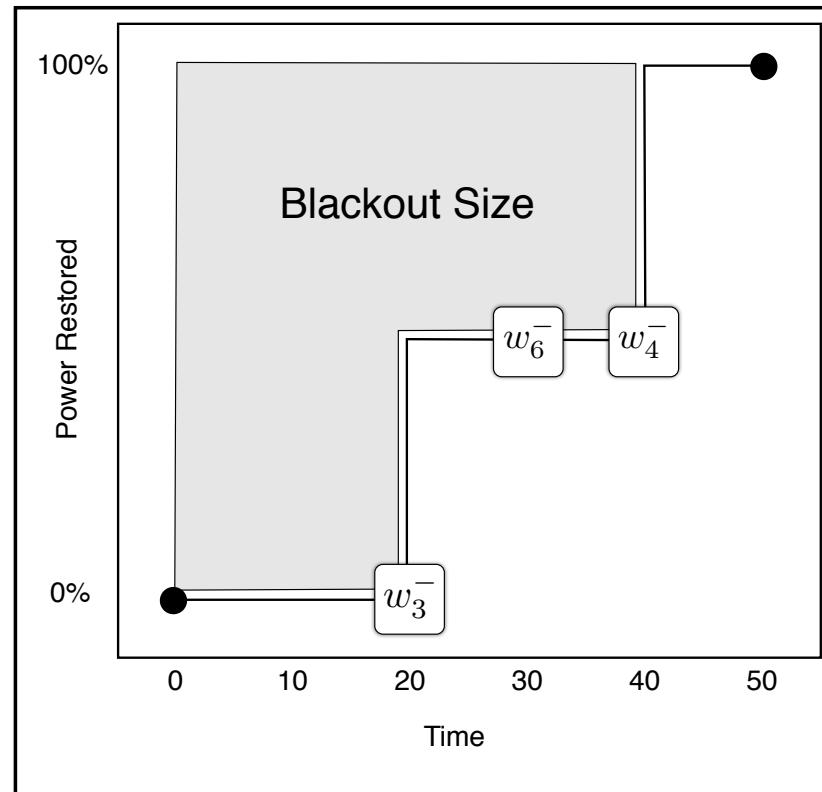
# Power Restoration



# Power Restoration



Routing Aspect



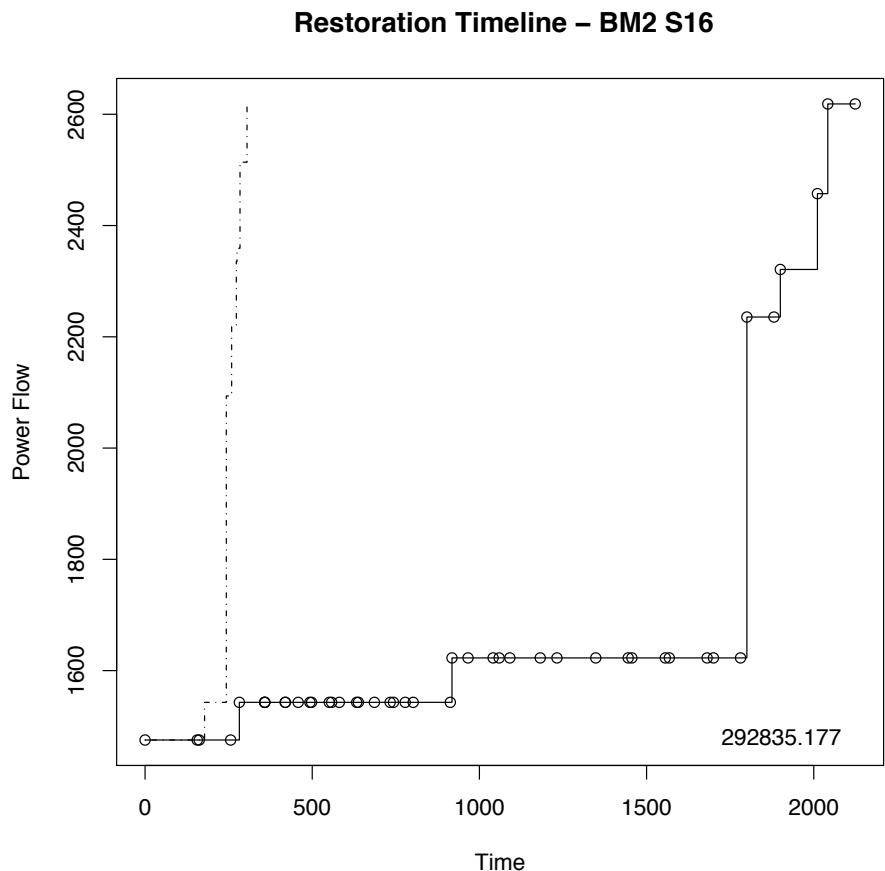
Power Flow Aspect

# Power Restoration

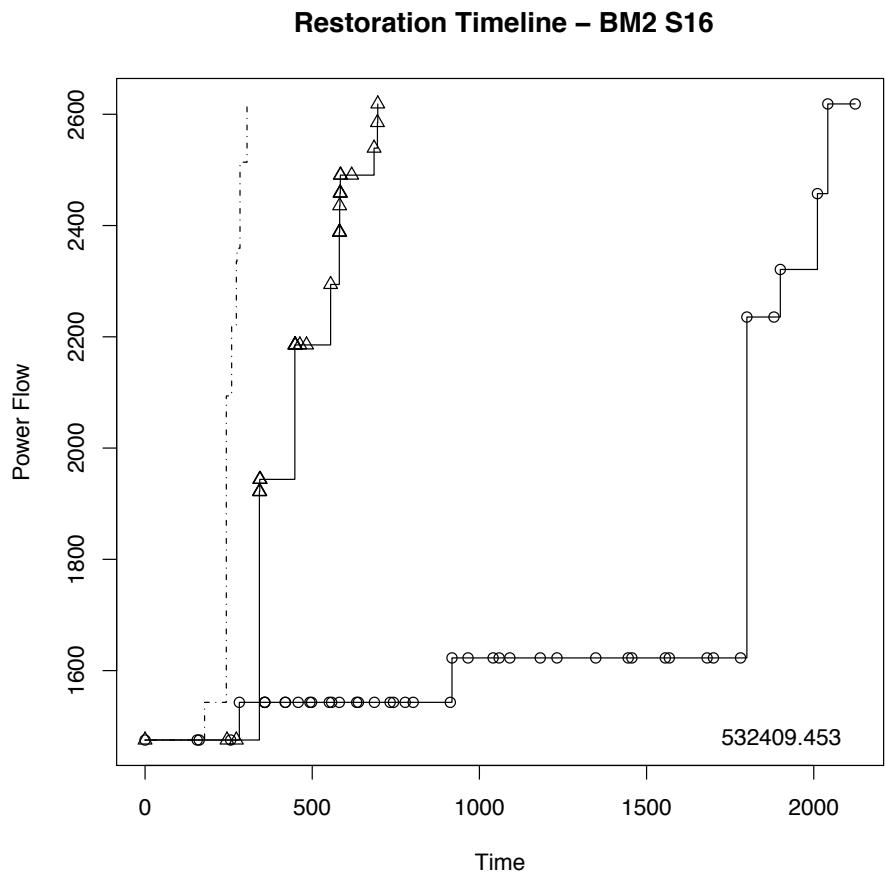
$$p_{nm} = |V_n|^2 g_{nm} - |V_n| |V_m| (g_{nm} \cos(\theta_n - \theta_m) + b_{nm} \sin(\theta_n - \theta_m))$$

$$q_{nm} = -|V_n|^2 b_{nm} - |V_n| |V_m| (g_{nm} \sin(\theta_n - \theta_m) - b_{nm} \cos(\theta_n - \theta_m))$$

# Power Restoration



# Power Restoration



# Welcome to Discrete Optimization

# Citations

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Data Source: The Fantasy Shrink: 2011 NFL Schedule Grid <http://www.fantasyshrink.com/2011/04/20/2011-nfl-schedule-grid/> accessed March, 18

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NHC\_irene1.GIF [http://www.nhc.noaa.gov/archive/2011/graphics/al09/loop\\_5W.shtml](http://www.nhc.noaa.gov/archive/2011/graphics/al09/loop_5W.shtml)

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"Flooding, Power Failures, Rainfall and Damage From Hurricane Irene" interactive infographic (New England and 'Customers without power on Sunday afternoon, by county' overlay) by Joe Burgess, Amanda Cox, Alicia Parlapiano, Archie Tse, Lisa Waananen And Tim Wallace. The New York Times, August 31, 2011. Web. 22 Mar. 2013. <<http://www.nytimes.com/interactive/2011/08/27/us/preparations-for-hurricane-irene-and-reports-of-damage.html>>.

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