Prototype: Attractor Reconstruction of a Dark-Matter Field

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Abstract

We propose a fixed-point (attractor) reconstruction of the projected dark-matter field $\psi(x)$ from combined gravitational lensing and stellar-stream data. The method alternates short gravitational relaxation steps under the Vlasov–Poisson equations with observational pull-backs, producing a candidate attractor ψ^* whose stability and uniqueness can be directly tested.[1]

If a candidate prior (cold, warm/fuzzy, or nonlocal) both fits the data and successfully cross-predicts independent observations, it provides a concrete discriminator among dark-matter models. The framework is offered not as a claim of discovery, but as a testable hypothesis designed to guide further analysis and simulation.

1 Objective

Reconstruct the projected dark matter field $\psi(x)$ as a fixed point of an operator F that alternates between: (i) gravitational relaxation under Vlasov–Poisson, and (ii) pullbacks from data (lensing + stellar streams). Converged solutions are attractors; stability and uniqueness are testable.

2 Mathematical Formulation

We define an energy functional:

$$E[\psi] = \tfrac{1}{2}\langle \delta, K^{-1}\delta\rangle + \chi^2_{\text{lens}}[\psi] + \chi^2_{\text{stream}}[\psi] + \lambda\|\nabla\psi\|^2, \tag{1}$$

with $\delta = e^{\psi} - 1$. This is equivalent to adopting the common transformation $\psi = \log(1+\delta)$, which guarantees $\delta > -1$ and is widely used in cosmology to Gaussianize the density field, handling both overdense and underdense regions.

The kernel K encodes dark-matter microphysics: parameterized families spanning CDM (baseline), WDM/fuzzy (cutoff), or DIN/plasma-like non-local kernels. Attractor condition: $\psi^* = F(\psi^*)$. Stability: spectral radius $\rho(J) < 1$ of the Jacobian J at ψ^*.

[1]This work was inspired by Geoffrey Hinton's public remarks that AI may be especially suited to connecting seemingly unrelated concepts: *Geoffrey Hinton reveals the surprising truth about AI's limits and potential*, YouTube (2024). Link: https://youtu.be/n4IQOBka8bc

3 Assumptions

We hypothesize that the operator F admits stable fixed points, motivated by analogies to attractor dynamics in Hopfield networks and the relaxation properties of collisionless N-body systems. Convergence of F is not guaranteed for arbitrary priors or data; safeguards include adaptive step sizes, multiple initialization strategies, and explicit convergence diagnostics beyond relative change.

The solution may not be unique given the inverse problem's ill-posed nature; characterizing the solution manifold, quantifying Bayesian uncertainty, and exploring multiple attractors are important follow-ups.

4 Inputs

- Weak/strong lensing κ-maps of well-studied fields (e.g. HST Frontier Fields).
- One stellar stream line-density profile from Gaia DR3 (e.g. GD-1, Pal 5).
- Optional: rotation curves or velocity dispersions.

Relative weights w_L and w_S between lensing and streams can be adaptively tuned to reflect observational depth and signal-to-noise.

5 Operator F

Each iteration consists of:

1. **Gravity step:** compute $\delta = e^{\psi} - 1$, solve $\nabla^2 \phi = 4\pi G \bar{\rho}\, \delta$.
2. **Data pullback:** forward lensing projection $\kappa(\psi)$; integrate stellar orbits in ϕ; update misfit gradients.
3. **Prior/proximal step:**

$$\psi \leftarrow \arg\min \left\{ \tfrac{1}{2}\langle \delta, K^{-1}\delta \rangle + \lambda \|\nabla \psi\|^2 + \tfrac{\alpha}{2}\|\psi - \tilde{\psi}\|^2 \right\}.$$

Repeat until convergence.

6 Implementation Sketch (pseudocode)

```
for t in range(T):
    delta = np.exp(psi) - 1
    phi = poisson_solve(delta)
    grad_lens   = lensing_adjoint(lensing_project(psi) - kappa_obs)
    grad_stream = stream_adjoint(stream_line_density(phi), stream_obs_power)
    grad_data = wL*grad_lens + wS*grad_stream
    psi_tilde = psi - eta*grad_data
    psi = proximal_fourier(psi_tilde, K, alpha, lambda_reg)
    if rel_change(psi, prev) < eps: break
```

7 Nonlocal Priors

Candidate priors may include Gaussian kernels, Yukawa-type falloffs, or convolutional operators derived from N-body simulations. Rather than discrete kernel choices, parameterized families can be tested, spanning CDM, WDM/fuzzy, and plasma-like nonlocality. Validation relies on cross-prediction: fit lensing and predict streams, or the reverse. Priors that succeed in both channels provide evidence for the correct microphysics.

8 Toy Example

A minimal synthetic test can illustrate feasibility. For instance, generate a Gaussian clump $\psi(x)$, apply one iteration of F (gravity step + lensing misfit), and verify convergence toward a stable solution. A 2D toy model, matching the dimensionality of lensing and stream observables, provides the natural proof-of-concept to demonstrate the operator's behavior and test stability and uniqueness.

9 Computational Cost

Full convergence on high-resolution cosmological fields may be expensive. Reduced-dimension toy models (1D streams, 2D lensing patches) allow validation before scaling. FFT Poisson solvers, sparse representations, and GPU acceleration mitigate costs.

10 Outputs

- Convergence history.
- Stability: spectral radius $\rho(J)$ at ψ^*, and optionally Lyapunov exponents.
- Basin robustness: convergence from random seeds.
- Cross-prediction between lensing and streams.

11 Why It Matters

Even null results are informative: convergence under CDM alone is nontrivial. If a non-local prior improves joint fit and stability, that suggests hidden-layer physics. If both datasets fit individually but only one cross-predicts, the method discriminates among dark-matter models. Given possible non-uniqueness, families of attractors may exist; characterizing these and quantifying uncertainty represent essential next steps.

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