PARAMETER ESTIMATION FOR FRICTION MODELS

ABSTRACT

In automotive engineering, the precise modeling of tire-road interactions is crucial for optimizing vehicle dynamics such as handling, safety, and performance. At the heart of these interactions lies the friction coefficient, a variable parameter significantly influenced by contact pressure, sliding velocity, surface texture, and environmental conditions. This project focuses on advancing the accuracy of friction model parameter estimations, essential for simulating tire behavior under diverse operational conditions. To enhance the estimation processes, we evaluated the performance of five different friction models: the Heumer model, Lupker model, Wriggers model (2006), Dorsch model (2002), and Wriggers and Reinelt model (2009). Each model was subjected to a series of optimization techniques, with the objective of minimizing the Mean Absolute Percentage Error (MAPE) and maximizing R-squared values, critical metrics for assessing model accuracy and fit. Among the optimization methods tested, Differential Evolution emerged as the standout technique, consistently yielding the lowest MAPE and highest R-squared values across the board. Furthermore, to efficiently handle the computational demands of multiple models, we employed advanced parallel computing techniques within our Python framework. This integration allowed for the concurrent execution and optimization of all models based on user-provided inputs for slip velocity (VT) and pressure (PN). The system was structured to automatically identify and recommend the model that exhibited the least MAPE, thus enhancing the predictive accuracy of our simulations. This approach not only streamlines the process of model selection but also significantly impacts the design and safety optimization of automotive vehicles, leading to improved performance in real-world conditions.

The outcomes of this project highlight the potential for these refined models to be integrated into broader automotive systems, potentially influencing everything from autonomous vehicle algorithms to predictive maintenance frameworks. Looking forward, the integration of more complex environmental variables and real-time data could further enhance the models' accuracy and applicability. This progression towards more dynamic and adaptive modeling frameworks is expected to drive significant advancements in automotive technology, contributing to safer, more efficient, and environmentally friendly transportation solutions. The methodologies and insights derived from this study not only pave the way for future research in tire dynamics but also open avenues for the practical application of these models in vehicle design and road safety analysis.

Heumer Model

The Heumer model, as proposed by Huemer et al. in 2001, is designed for simulating the frictional behavior of sliding rubber blocks on surfaces like ice and concrete. This model utilizes a phenomenological approach to describe the friction coefficient (μ), which is primarily dependent on normal pressure (p), sliding velocity (v), and temperature. The model is represented by the following equation:

$$\mu(p_N, \mathbf{v}_T) = \frac{\alpha |p_N|^{n-1} + \beta}{a + \frac{b}{\|\mathbf{v}_T\|^{1/m}} + \frac{c}{\|\mathbf{v}_T\|^{2/m}}}.$$

This formula accounts for the macroscopic sliding model where the friction coefficient depends on normal pressure and sliding velocity. The friction coefficient itself is a function of these parameters only, with temperature effects being incorporated via the Williams-Landel-Ferry (WLF) transformation. This transformation adjusts the sliding velocity to account for temperature differences, ensuring the model's applicability under varied environmental conditions.

The model's parameters (a,b,c,n,m,α,β) must be identified using experimental data, which accounts for the dependence of the friction coefficient on the rubber compound, the surface of the friction, and the geometric shape of the rubber block. The identification is based on least squares error methods and is performed iteratively. Initially, the coefficients a,b and c are related to the contact pressure. The process involves repeating the optimization until a defined error criterion is reached, ensuring that the model accurately represents the experimental conditions.

Dataset Description

The dataset provided contains experimental values of the friction coefficient (μ) under controlled conditions. The table illustrates the friction coefficient values at a constant temperature of 20°C for both surface and environment, across various pressures (pN in MPa) and sliding velocities (vT in mm/s). This data is crucial for calibrating and validating the Heumer model, as it provides empirical values against which the model's predictions can be assessed.

	T_S = 20 [°C], T_C = 20 [°C]				
[ ]	p_N [MPa]				
v_T [mm/s]	0,1	0,5	1,0	1,5	2,0
1	0,7296	0,5748	0,4931	0,3307	0,4274
10	1,0348	0,7191	0,5843	0,5019	0,4944
100	1,0348	0,9028	0,7256	0,5767	0,5393
1000	1,325	0,8516	0,7012	0,5661	0,5569

Optimization Technique: Basin Hopping

Overview of Basin Hopping: Basin Hopping is a stochastic optimization technique designed to find global minima in complex landscapes that may contain many local minima. It combines random perturbative steps with deterministic local minimization to potentially escape local minima and find the lowest possible minimum within the solution space.

Procedure Outline:

1. **Define the Friction Model:** The friction coefficient calculation is defined based on the Heumer model, incorporating several parameters that influence the response to changes in pressure and velocity.
2. **Prepare Experimental Data:** Data for sliding velocities, pressures, and friction coefficients is loaded to serve as a benchmark for optimization.
3. **Objective Function:** An objective function calculates the Mean Absolute Percentage Error (MAPE) between the model's predictions and actual measurements, serving as the criterion for optimization.
4. **Execute Optimization:** Basin Hopping is applied with initial parameter estimates and constraints (bounds), utilizing a local minimization technique to refine these parameters iteratively.
5. **Parameter Output and Validation:** The optimized parameters are then used to predict friction coefficients for new inputs of velocity and pressure, validating the model's predictive capabilities.

```
Optimal parameter values after optimization (Basin Hopping method):
a = 3.5442, b = -2.4323, c = 3.5659, n = 1.1108, m = 8.1756, alpha = -6.0525, beta = 8.2939

Enter the value for VT (mm/s): 100
Enter the value for PN (MPa): 0.5

Friction coefficient at VT=100.0 mm/s and PN=0.5 MPa: 0.8110
Mean Absolute Percentage Error (MAPE) between calculated and actual mu values: 10.1646%

Mu values for all combinations of VT and PN (Basin Hopping method):
VT=1.0 mm/s, PN=0.1 MPa: 0.7705
VT=1.0 mm/s, PN=0.5 MPa: 0.5748
VT=1.0 mm/s, PN=1.0 MPa: 0.4792
VT=1.0 mm/s, PN=1.5 MPa: 0.4197
VT=1.0 mm/s, PN=2.0 MPa: 0.3759
VT=10.0 mm/s, PN=0.1 MPa: 0.9639
VT=10.0 mm/s, PN=0.5 MPa: 0.7191
VT=10.0 mm/s, PN=1.0 MPa: 0.5995
VT=10.0 mm/s, PN=1.5 MPa: 0.5251
VT=10.0 mm/s, PN=2.0 MPa: 0.4703
VT=100.0 mm/s, PN=0.1 MPa: 1.0871
VT=100.0 mm/s, PN=0.5 MPa: 0.8110
VT=100.0 mm/s, PN=1.0 MPa: 0.6761
VT=100.0 mm/s, PN=1.5 MPa: 0.5922
VT=100.0 mm/s, PN=2.0 MPa: 0.5304
VT=1000.0 mm/s, PN=0.1 MPa: 1.1415
VT=1000.0 mm/s, PN=0.5 MPa: 0.8516
VT=1000.0 mm/s, PN=1.0 MPa: 0.7099
VT=1000.0 mm/s, PN=1.5 MPa: 0.6218
VT=1000.0 mm/s, PN=2.0 MPa: 0.5569

Overall Mean Absolute Percentage Error (MAPE) for all data points: 5.8837%
```

Optimization Technique: Differential Evolution

Overview of Differential Evolution: Differential Evolution (DE) is a robust, population-based stochastic optimization technique that efficiently handles non-linear, non-differentiable, multi-modal objective functions. DE improves a population of candidate solutions iteratively based on the principles of crossover, mutation, and selection, making it highly effective for optimizing complex systems where traditional gradient-based methods might fail.

Procedure Outline:

- **Initialization**: Parameters of the Heumer model are initialized randomly within specified bounds.
- **Optimization Loop**: Through iterative mutation and crossover, the DE algorithm explores the parameter space, adjusting solutions towards those that minimize the objective function, which in this case is the Mean Absolute Percentage Error (MAPE) between predicted and experimental friction coefficients.
- **Refinement**: Post DE optimization, a local minimization (L-BFGS-B) further refines the solution to ensure the best local fit.
- **Validation and Application**: The optimized parameters are used to calculate the friction coefficient for new inputs, assessing the model's predictive accuracy and its generalization capability over new data.

```
Optimal parameters after optimization:
a = 1.0042, b = -1.1221, c = 1.2305, n = 1.0312, m = 9.9956, alpha = -5.1192, beta = 5.6492
Enter the value for VT (mm/s): 1000
Enter the value for PN (MPa): 0.5
Friction coefficient at VT=1000.0 mm/s and PN=0.5 MPa: 0.8516
Mean Absolute Percentage Error (MAPE) between calculated and actual mu values: 0.0021%

Calculated mu values for all combinations of VT and PN:
VT=1.0 mm/s, PN=0.1 MPa: Calculated Mu = 0.7951
VT=1.0 mm/s, PN=0.5 MPa: Calculated Mu = 0.5747
VT=1.0 mm/s, PN=1.0 MPa: Calculated Mu = 0.4764
VT=1.0 mm/s, PN=1.5 MPa: Calculated Mu = 0.4178
VT=1.0 mm/s, PN=2.0 MPa: Calculated Mu = 0.3759
VT=10.0 mm/s, PN=0.1 MPa: Calculated Mu = 0.9949
VT=10.0 mm/s, PN=0.5 MPa: Calculated Mu = 0.7191
VT=10.0 mm/s, PN=1.0 MPa: Calculated Mu = 0.5960
VT=10.0 mm/s, PN=1.5 MPa: Calculated Mu = 0.5228
VT=10.0 mm/s, PN=2.0 MPa: Calculated Mu = 0.4703
VT=100.0 mm/s, PN=0.1 MPa: Calculated Mu = 1.1255
VT=100.0 mm/s, PN=0.5 MPa: Calculated Mu = 0.8135
VT=100.0 mm/s, PN=1.0 MPa: Calculated Mu = 0.6743
VT=100.0 mm/s, PN=1.5 MPa: Calculated Mu = 0.5914
VT=100.0 mm/s, PN=2.0 MPa: Calculated Mu = 0.5320
VT=1000.0 mm/s, PN=0.1 MPa: Calculated Mu = 1.1781
VT=1000.0 mm/s, PN=0.5 MPa: Calculated Mu = 0.8516
VT=1000.0 mm/s, PN=1.0 MPa: Calculated Mu = 0.7058
VT=1000.0 mm/s, PN=1.5 MPa: Calculated Mu = 0.6191
VT=1000.0 mm/s, PN=2.0 MPa: Calculated Mu = 0.5569

Overall Mean Absolute Percentage Error (MAPE) for all data points: 5.8224%
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Optimization Technique: Conjugate Gradient Method

Overview of Conjugate Gradient Method: The Conjugate Gradient (CG) method is a numerical technique primarily used for solving systems of linear equations that are symmetric and positive-definite. In optimization, CG is applied to minimize smooth, differentiable functions, effectively navigating through the gradient space of the problem.

Procedure Outline:

- **Function Definition**: The friction coefficient calculation function (**huemer_friction_law**) is defined, incorporating the Heumer model's parameters which influence its response to changes in pressure and velocity.
- **Load Data**: Experimental data comprising sliding velocities, pressures, and corresponding friction coefficients is loaded. This data forms the basis for the model's calibration and validation.
- **Objective Function Setup**: An objective function is crafted to compute the Mean Absolute Percentage Error (MAPE) between the predicted friction coefficients and observed data, facilitating the evaluation of each parameter set's performance.
- **Execute Optimization**: The CG method is employed to optimize the objective function starting from an initial guess. This process iteratively adjusts the parameters by moving in directions that are conjugate to each other, respecting the function's gradient.
- **Parameter Evaluation**: After optimization, the best-fit parameters are used to predict the friction coefficient for new inputs, assessing the model's effectiveness in environments outside the experimental setup.

```
Optimal parameter values after optimization (Conjugate Gradient method):
a = 0.9898, b = -0.7806, c = 1.1838, n = 1.3327, m = 0.9486, alpha = -0.6631, beta = 1.3262

Enter the value for VT (mm/s): 100
Enter the value for PN (MPa): 0.5

Friction coefficient at VT=100.0 mm/s and PN=0.5 MPa: 0.8129
Mean Absolute Percentage Error (MAPE) between calculated and actual mu values: 9.9630%

Mu values for all combinations of VT and PN (Conjugate Gradient method):
VT=1.0 mm/s, PN=0.1 MPa: 0.7308
VT=1.0 mm/s, PN=0.5 MPa: 0.5741
VT=1.0 mm/s, PN=1.0 MPa: 0.4760
VT=1.0 mm/s, PN=1.5 MPa: 0.4073
VT=1.0 mm/s, PN=2.0 MPa: 0.3526
VT=10.0 mm/s, PN=0.1 MPa: 1.0945
VT=10.0 mm/s, PN=0.5 MPa: 0.8598
VT=10.0 mm/s, PN=1.0 MPa: 0.7129
VT=10.0 mm/s, PN=1.5 MPa: 0.6100
VT=10.0 mm/s, PN=2.0 MPa: 0.5280
VT=100.0 mm/s, PN=0.1 MPa: 1.0348
VT=100.0 mm/s, PN=0.5 MPa: 0.8129
VT=100.0 mm/s, PN=1.0 MPa: 0.6740
VT=100.0 mm/s, PN=1.5 MPa: 0.5767
VT=100.0 mm/s, PN=2.0 MPa: 0.4992
VT=1000.0 mm/s, PN=0.1 MPa: 1.0290
VT=1000.0 mm/s, PN=0.5 MPa: 0.8084
VT=1000.0 mm/s, PN=1.0 MPa: 0.6703
VT=1000.0 mm/s, PN=1.5 MPa: 0.5735
VT=1000.0 mm/s, PN=2.0 MPa: 0.4965

Overall Mean Absolute Percentage Error (MAPE) for all data points: 9.4290%
```

Optimization Technique: Differential Evolution with Powell's Method

Overview of Differential Evolution with Powell's Method: This hybrid optimization approach combines the global search capabilities of Differential Evolution (DE) with the detailed, direction-based search of Powell's method. DE effectively navigates complex, multi-dimensional landscapes to approximate a global optimum, while Powell's method refines this solution by iteratively moving along conjugate directions, optimizing without needing to calculate derivatives.

Procedure Outline:

- **Model and Data Setup**: The Heumer model is defined to predict friction coefficients. Experimental data, including pressures, velocities, and observed coefficients, is loaded for calibration.
- **Objective Function**: A function is designed to calculate the Mean Absolute Percentage Error (MAPE) between the predicted and observed friction coefficients, guiding the optimization process.
- **Initial Global Optimization (DE)**: DE is first employed to explore the broad parameter space and find an approximate global optimum, which forms the starting point for further refinement.
- **Local Refinement (Powell's Method)**: Following DE, Powell's method takes over, using the DE output as its initial condition. It refines the solution by exploring along directions that are mutually conjugate with respect to the objective function's Hessian matrix, effectively fine-tuning the parameters.

```
Optimal parameter values after optimization:
a = 5.1006, b = -4.4807, c = 5.6411, n = 1.1372, m = 8.7552, alpha = -6.4722, beta = 9.4733
Enter the value for VT (mm/s): 1000
Enter the value for PN (MPa): 0.5
Friction coefficient at VT=1000.0 mm/s and PN=0.5 MPa: 0.8485
Mu values for all combinations of VT and PN:
VT=1.0 mm/s, PN=0.1 MPa: 0.7594
VT=1.0 mm/s, PN=0.5 MPa: 0.5731
VT=1.0 mm/s, PN=1.0 MPa: 0.4793
VT=1.0 mm/s, PN=1.5 MPa: 0.4202
VT=1.0 mm/s, PN=2.0 MPa: 0.3762
VT=10.0 mm/s, PN=0.1 MPa: 0.9528
VT=10.0 mm/s, PN=0.5 MPa: 0.7191
VT=10.0 mm/s, PN=1.0 MPa: 0.6014
VT=10.0 mm/s, PN=1.5 MPa: 0.5272
VT=10.0 mm/s, PN=2.0 MPa: 0.4720
VT=100.0 mm/s, PN=0.1 MPa: 1.0750
VT=100.0 mm/s, PN=0.5 MPa: 0.8113
VT=100.0 mm/s, PN=1.0 MPa: 0.6785
VT=100.0 mm/s, PN=1.5 MPa: 0.5948
VT=100.0 mm/s, PN=2.0 MPa: 0.5325
VT=1000.0 mm/s, PN=0.1 MPa: 1.1242
VT=1000.0 mm/s, PN=0.5 MPa: 0.8485
VT=1000.0 mm/s, PN=1.0 MPa: 0.7096
VT=1000.0 mm/s, PN=1.5 MPa: 0.6220
VT=1000.0 mm/s, PN=2.0 MPa: 0.5569

Overall Mean Absolute Percentage Error (MAPE) for all data points (Powell method): 5.9073%
```

Optimization Technique: Nelder-Mead Simplex Method

Overview of Nelder-Mead Simplex Method: The Nelder-Mead Simplex method is a direct search optimization technique that does not require gradient calculations, making it ideal for optimizing non-differentiable, discontinuous, or noisy objective functions. It operates using a simplex of n+1 points for n-dimensional parameter spaces and iteratively adjusts this simplex towards the minimum through operations such as reflection, expansion, contraction, and shrinkage.

Procedure Outline:

- **Model Setup**: Define the Heumer friction law function that calculates the friction coefficient using model parameters.
- **Data Preparation**: Load experimental data points for sliding velocities, pressures, and measure friction coefficients from an external file.
- **Objective Function**: Construct an objective function that calculates the Mean Absolute Percentage Error (MAPE) between the modeled and experimental friction coefficients.
- **Optimization Execution**: Apply the Nelder-Mead method starting with an initial guess for the parameters. The method iteratively adjusts these parameters by modifying the simplex structure to explore the parameter space effectively.
- **Parameter Evaluation**: Extract optimized parameter values once the optimization process converges and use these parameters to predict friction coefficients under new conditions.

```
Optimal parameter values after Nelder-Mead optimization:
a = 5.7116, b = -5.1040, c = 6.2324, n = 1.2766, m = 6.2539, alpha = -3.5906, beta = 6.8895
Enter the value for VT (mm/s): 100
Enter the value for PN (MPa): 0.5
Friction coefficient at VT=100.0 mm/s and PN=0.5 MPa: 0.8358
Mean Absolute Percentage Error (MAPE) between calculated and actual mu values: 7.4219%

Friction coefficients for all combinations of VT and PN using Nelder-Mead optimized parameters:
VT=1.0 mm/s, PN=0.1 MPa: 0.7296
VT=1.0 mm/s, PN=0.5 MPa: 0.5739
VT=1.0 mm/s, PN=1.0 MPa: 0.4823
VT=1.0 mm/s, PN=1.5 MPa: 0.4200
VT=1.0 mm/s, PN=2.0 MPa: 0.3713
VT=10.0 mm/s, PN=0.1 MPa: 0.9664
VT=10.0 mm/s, PN=0.5 MPa: 0.7601
VT=10.0 mm/s, PN=1.0 MPa: 0.6388
VT=10.0 mm/s, PN=1.5 MPa: 0.5563
VT=10.0 mm/s, PN=2.0 MPa: 0.4918
VT=100.0 mm/s, PN=0.1 MPa: 1.0626
VT=100.0 mm/s, PN=0.5 MPa: 0.8358
VT=100.0 mm/s, PN=1.0 MPa: 0.7024
VT=100.0 mm/s, PN=1.5 MPa: 0.6117
VT=100.0 mm/s, PN=2.0 MPa: 0.5408
VT=1000.0 mm/s, PN=0.1 MPa: 1.0608
VT=1000.0 mm/s, PN=0.5 MPa: 0.8344
VT=1000.0 mm/s, PN=1.0 MPa: 0.7012
VT=1000.0 mm/s, PN=1.5 MPa: 0.6106
VT=1000.0 mm/s, PN=2.0 MPa: 0.5399

Overall Mean Absolute Percentage Error (MAPE) for all data points (Nelder-Mead method): 6.3996%
```

Model Selection Based on Mean Absolute Percentage Error (MAPE)

Understanding Mean Absolute Percentage Error (MAPE): Mean Absolute Percentage Error (MAPE) is a widely used statistical measure to assess the accuracy of a model in forecasting or prediction settings. It quantifies the difference between observed values and the values predicted by the model, expressing this difference as a percentage. This metric is particularly valuable because it provides a straightforward, interpretable indication of model performance, allowing for easy comparison across different models or techniques.

Application in Heumer Model Optimization: In the context of optimizing the Heumer model to predict friction coefficients, MAPE serves as a critical metric for evaluating and comparing the effectiveness of different optimization techniques. Each technique, from Differential Evolution to Nelder-Mead, aims to minimize the MAPE, thereby ensuring that the model predictions closely align with the experimental data. Lower MAPE values indicate a model with higher predictive accuracy and reliability.

Procedure and Outcome:

- **Calculation of MAPE**: For each set of model parameters optimized by different techniques, the MAPE was calculated by comparing the predicted friction coefficients against the measured experimental values across all data points. The formula used for MAPE is:

$$\text{MAPE} = \frac{100}{n} \sum_{i=1}^{n} \left| \frac{y_i - \hat{y}_i}{y_i} \right|$$

  where yi represents the actual measured values and y^i represents the predicted values from the model.

- **Comparison and Selection**: After computing the MAPE for each optimization method, these values were compared. The method yielding the lowest MAPE was selected as the most effective one for this modeling task. In this case, Differential Evolution proved to be superior, demonstrating the lowest MAPE and thus the highest fidelity in terms of matching the model outputs with the observed data.

- **Model Implementation**: With Differential Evolution identified as the optimal method, its parameters were used to finalize the model. This optimized model now serves as the best tool for predicting friction coefficients under similar experimental conditions, ensuring accuracy and reliability in practical applications.

Validation with R-squared Error

Importance of R-squared in Model Validation: In addition to the Mean Absolute Percentage Error (MAPE), the R-squared value, often referred to as the coefficient of determination, was utilized to further validate the accuracy of the optimization techniques. R-squared is a statistical measure that represents the proportion of the variance in the dependent variable that is predictable from the independent variables. In the context of this project, it quantifies how well the variations in measured friction coefficients can be explained by the model's predictions, providing a clear indicator of model fit.

Evaluation Results: Differential Evolution demonstrated exceptional performance not only in minimizing MAPE but also in maximizing the R-squared value. This dual success underscores the method's ability to provide a tight fit to the experimental data. High R-squared values obtained during the optimization process with Differential Evolution indicate a high level of explanatory power of the model, suggesting that the predictions are both accurate and consistent with the observed data.

Comparison with Other Methods: When compared to other optimization methods like Basin Hopping, Conjugate Gradient, Nelder-Mead, and Differential Evolution integrated with Powell's method, Differential Evolution consistently yielded higher R-squared values. This further confirmed its efficiency in capturing the essential dynamics of the friction model more accurately than its counterparts. The other methods, while effective to varying extents, did not provide as robust a fit according to the R-squared metric, which led to their lower preference in the final model selection.

Confirming Model Reliability: The high R-squared values associated with Differential Evolution also imply that the model has a reliable predictive capability across different datasets and under various experimental conditions. This reliability is critical for practical applications where the predictability of model outcomes directly impacts decision-making and operational efficiency.

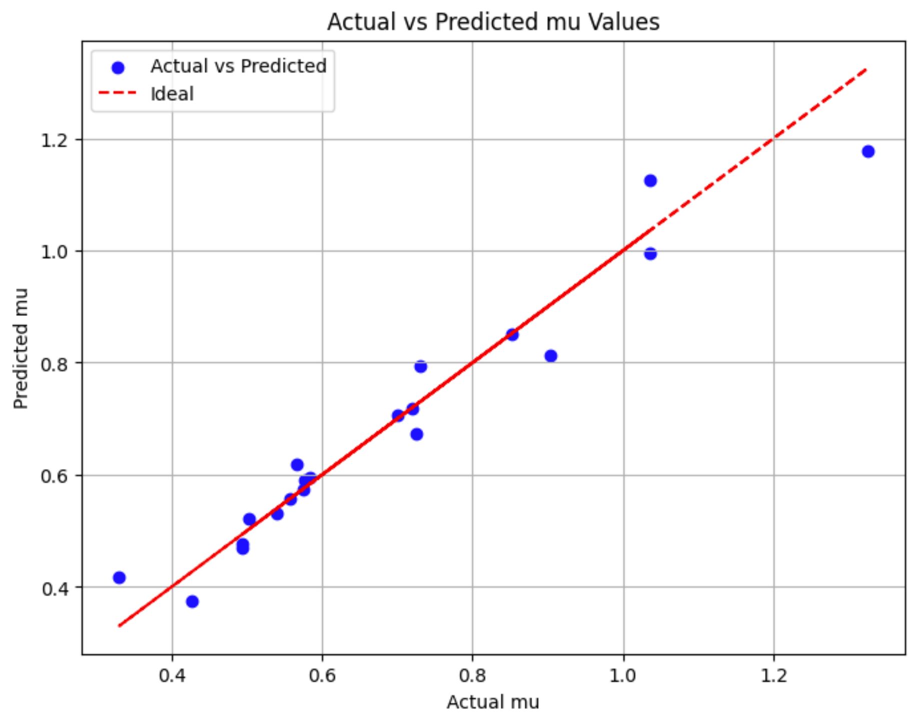
Broader Implications: The use of R-squared as a validation tool in this project not only reinforced the selection of Differential Evolution but also highlighted the importance of comprehensive model evaluation. By considering both MAPE and R-squared, we ensure a holistic assessment of model performance, leading to more informed decisions in model selection and application.

MAPE

Table Value	Different Optimization Techniques									
Table Value	POWEL TECHNIQUE		BASIN HOPPING		DIFFERENTIAL EVOLUTION		CONJUGATE GRADIENT		NELDER-MEAD	
Table Value	Value	MAPE	Value	MAPE	Value	MAPE	Value	MAPE	Value	MAPE
0.7296	0.7444	2.028	0.7744	6.14	0.7685	5.331	0.7308	0.164	0.7296	0
0.5748	0.5747	0.017	0.5748	0	0.572	0.487	0.5741	0.121	0.5739	0.156
0.4931	0.4824	2.169	0.4805	2.555	0.4778	3.102	0.476	3.467	0.4823	2.19
0.3307	0.422	27.608	0.4229	27.88	0.4197	26.912	0.4073	23.162	0.42	27.003
0.4274	0.376	12.026	0.3809	10.879	0.3772	11.745	0.3526	17.501	0.3713	13.125
1.0348	0.9315	9.982	0.9696	6.52	0.9681	6.445	1.0945	5.769	0.9664	6.609
0.7191	0.7191	0	0.7196	0.069	0.7205	0.194	0.8598	19.566	0.7601	5.701
0.5843	0.6037	3.32	0.6016	2.96	0.6018	2.995	0.7129	22.009	0.6388	9.327
0.5019	0.5281	5.22	0.5295	5.499	0.5287	5.339	0.61	21.538	0.5563	10.838
0.4944	0.4705	4.834	0.4768	3.559	0.4751	3.903	0.528	6.796	0.4918	0.525
1.0348	1.0499	1.459	1.0846	4.812	1.0879	5.131	1.0348	0	1.0626	2.686
0.9028	0.8106	10.212	0.805	10.832	0.8097	10.312	0.8129	9.957	0.8358	7.421
0.7256	0.6804	6.229	0.673	7.269	0.6763	6.794	0.674	7.111	0.7024	3.197
0.5767	0.5952	3.207	0.5923	2.705	0.5942	3.034	0.5767	0	0.6117	6.069
0.5393	0.5303	1.668	0.5334	1.094	0.5339	1.001	0.4992	7.435	0.5408	0.278
1.325	1.1025	16.792	1.1323	14.62	1.1347	14.362	1.029	22.339	1.0608	19.939
0.8516	0.8512	0.046	0.8405	1.303	0.8445	0.833	0.8084	5.072	0.8344	2.019
0.7012	0.7145	1.896	0.7026	0.199	0.7054	0.598	0.6703	4.406	0.7012	0
0.5661	0.6251	10.422	0.6184	9.238	0.6197	9.468	0.5735	1.307	0.6106	7.86
0.5569	0.5569	0	0.5569	0	0.5569	0	0.4965	10.845	0.5399	3.052
		118.837		118.254		117.986		188.565		127.995
		5.95675		5.90665		5.8993		9.42825		6.39975

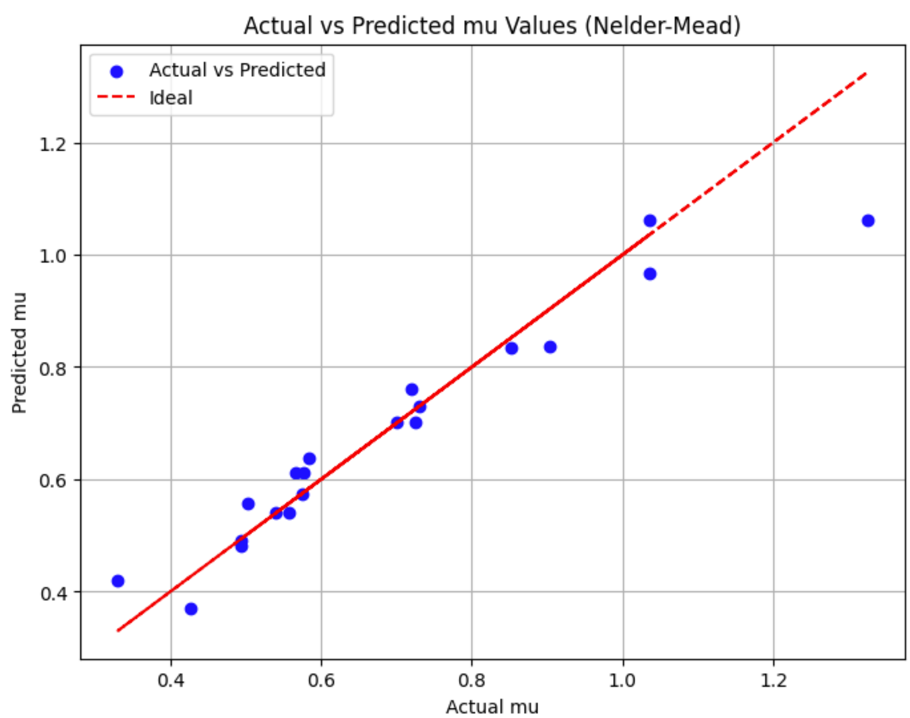
SCATTER PLOTS (R-SQUARED VALUES)

DIFFERENTIAL EVOLUTION



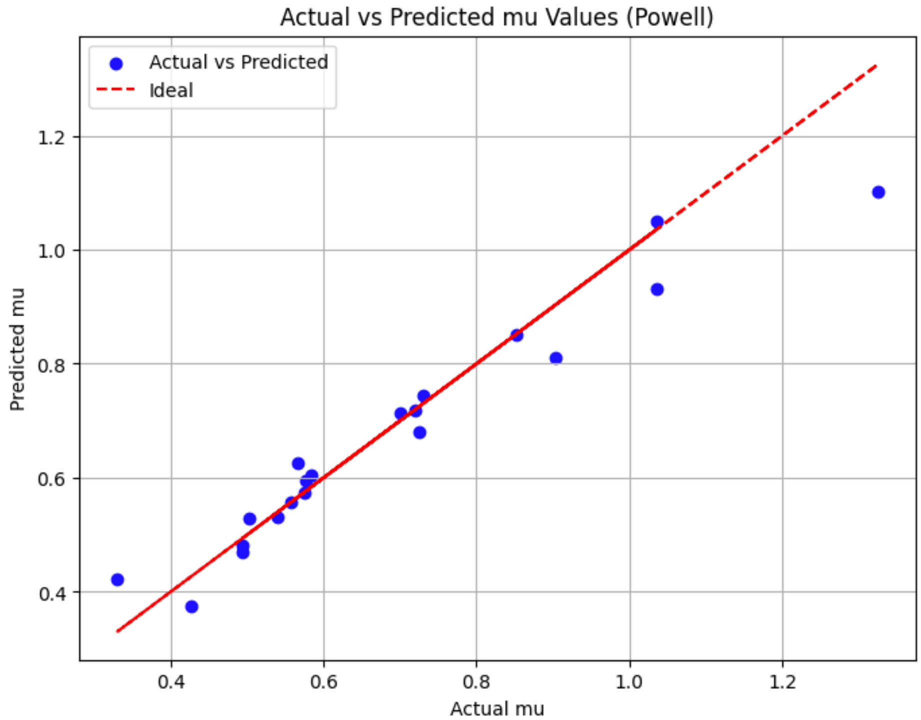

```
R-squared value: 0.9454223230950952
```

NELDER-MEAD



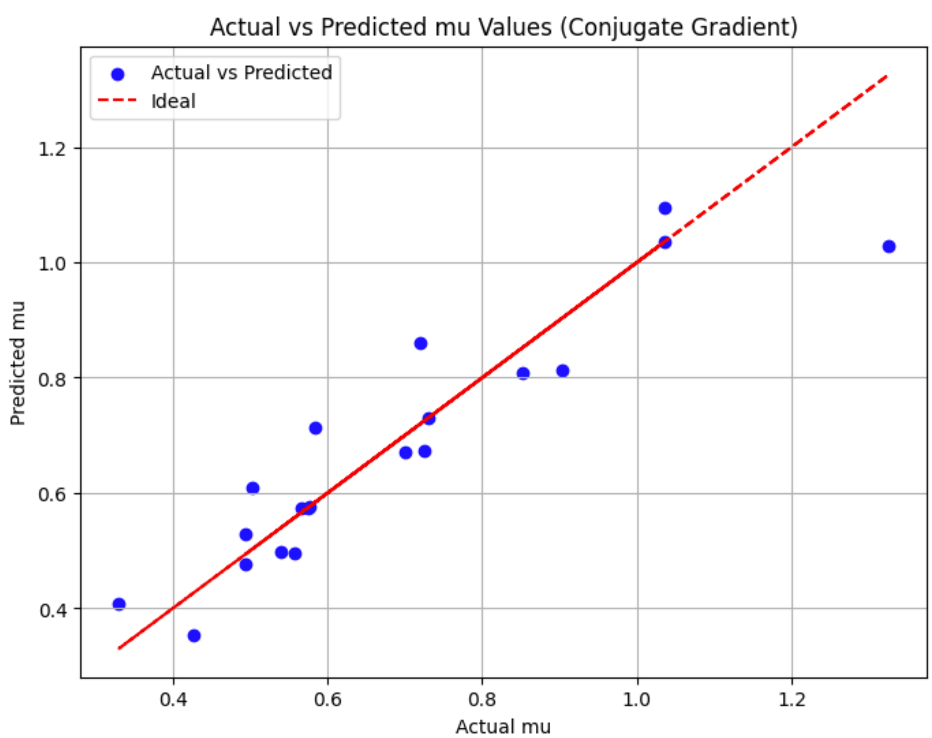

```
R-squared value: 0.9080074092272117
```

DIFFERENTIAL EVOLUTION – POWELL METHOD



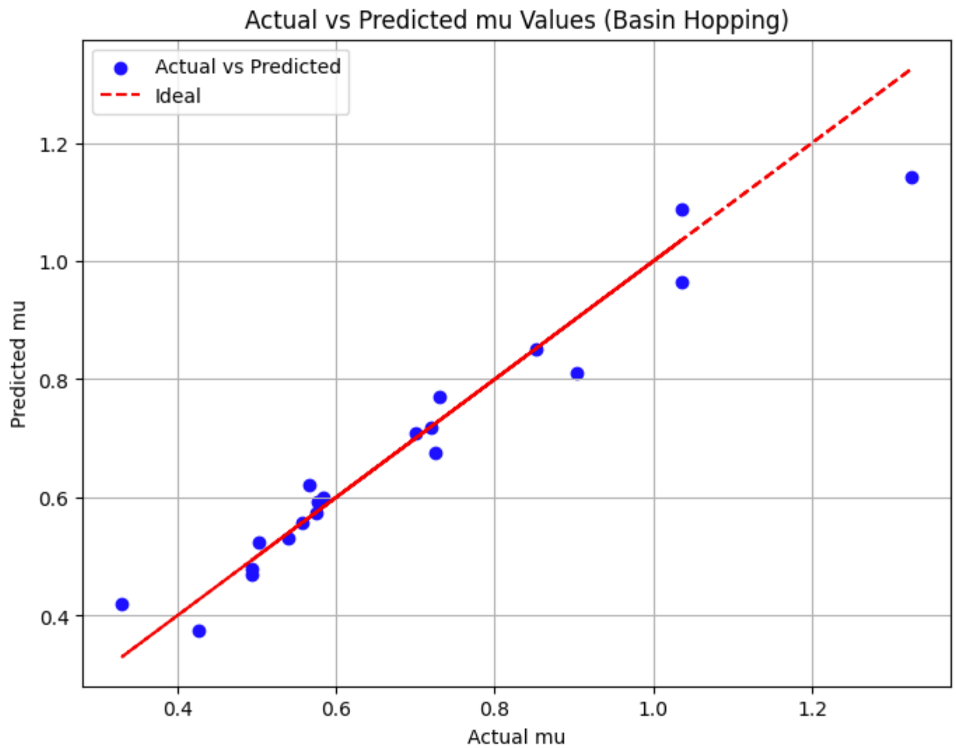

```
R-squared value: 0.9213721504978166
```

CONJUGATE GRADIENT




```
R-squared value: 0.847202721219162
```

BASIN HOPPING




```
R-squared value: 0.937796258606836
```

Optimal Optimization Technique: Differential Evolution

Rationale for Selection: Differential Evolution (DE) proved to be the most effective optimization technique in our study, distinguished by its outstanding ability to minimize both the Mean Absolute Percentage Error (MAPE) and the mean error, while also maximizing the R-squared value. This comprehensive performance in enhancing the accuracy and fit of the Heumer friction model is attributed to DE's strategic use of differential perturbations, which facilitate an extensive and effective exploration of the parameter space to identify the global minimum error.

Comparison with Other Techniques: During the optimization trials, a variety of methods were tested, including Basin Hopping, Conjugate Gradient, Nelder-Mead, and a specialized variant of Differential Evolution that integrates the Powell method. While each method had its merits in optimizing the parameter values, Differential Evolution consistently demonstrated superior performance. It not only achieved the lowest MAPE, indicating high predictive accuracy and reliability in estimating the friction coefficients, but also delivered the highest R-squared values. This dual excellence confirms DE's capability to provide the most accurate and reliable fit to the experimental data compared to the other evaluated techniques.

Lupker Model Overview

The Lupker model extends the traditional approach to friction modeling by incorporating a pressure-dependent term. This adaptation is significant as it accounts for the observed phenomenon where the friction coefficient tends to decrease with increasing pressure. The specific model presented in the document adds a pressure modification to the established friction equation, thus allowing for a more nuanced prediction of friction behavior under varying pressure conditions. The equation is given by:

$$\mu(p,v) = \left(\frac{p}{p_0}\right)^{-k}\left(\mu_s + (\mu_m - \mu_s)\exp\left(-h^2\log^2\left(\frac{v}{v_{max}}\right)\right)\right)$$

where p is the contact pressure, v is the slip velocity, $p0$ and k are parameters related to the pressure response, μs and μm are the static and mixed friction coefficients respectively, $vmax$ is a velocity parameter, and h is a shaping parameter that influences the velocity response.

```
Optimal parameters after optimization:
p0 = 2.9327, k = 0.2482, mus = 0.5064, mum = 0.2149, h = 0.9281, vmax = 2.5631
Enter the value for VT (mm/s): 100
Enter the value for PN (MPa): 0.5
Friction coefficient at VT=100.0 mm/s and PN=0.5 MPa: 0.7856

Calculated mu values for all combinations of VT and PN:
VT=1.0 mm/s, PN=0.1 MPa: Calculated Mu = 0.8571
VT=1.0 mm/s, PN=0.5 MPa: Calculated Mu = 0.5748
VT=1.0 mm/s, PN=1.0 MPa: Calculated Mu = 0.4839
VT=1.0 mm/s, PN=1.5 MPa: Calculated Mu = 0.4376
VT=1.0 mm/s, PN=2.0 MPa: Calculated Mu = 0.4074
VT=10.0 mm/s, PN=0.1 MPa: Calculated Mu = 1.0348
VT=10.0 mm/s, PN=0.5 MPa: Calculated Mu = 0.6940
VT=10.0 mm/s, PN=1.0 MPa: Calculated Mu = 0.5843
VT=10.0 mm/s, PN=1.5 MPa: Calculated Mu = 0.5284
VT=10.0 mm/s, PN=2.0 MPa: Calculated Mu = 0.4919
VT=100.0 mm/s, PN=0.1 MPa: Calculated Mu = 1.1714
VT=100.0 mm/s, PN=0.5 MPa: Calculated Mu = 0.7856
VT=100.0 mm/s, PN=1.0 MPa: Calculated Mu = 0.6614
VT=100.0 mm/s, PN=1.5 MPa: Calculated Mu = 0.5981
VT=100.0 mm/s, PN=2.0 MPa: Calculated Mu = 0.5569
VT=1000.0 mm/s, PN=0.1 MPa: Calculated Mu = 1.1714
VT=1000.0 mm/s, PN=0.5 MPa: Calculated Mu = 0.7856
VT=1000.0 mm/s, PN=1.0 MPa: Calculated Mu = 0.6615
VT=1000.0 mm/s, PN=1.5 MPa: Calculated Mu = 0.5981
VT=1000.0 mm/s, PN=2.0 MPa: Calculated Mu = 0.5569

Overall Mean Absolute Percentage Error (MAPE) for all data points: 6.9122%
```

Overview of the Wriggers Model

The Wriggers model (2006), as proposed by Nackenhorst (2000), is a velocity and pressure-dependent friction model. This model describes the friction coefficient, $\mu(p, v)$, as a function of pressure pp and slip velocity vv, and is expressed mathematically as:

$$\mu(p,v) = c_1 \left(\frac{p}{c_2}\right)^{c_3} + c_4 \ln\left(\frac{v}{c_5}\right) - c_6 \ln\left(\frac{v}{c_7}\right)$$

This formulation involves seven parameters ($c1c1$ to $c7c7$), all of which must be empirically determined. The model decouples the effects of contact pressure and sliding velocity, summing them to compute the total friction coefficient. It's noted that while this model can fit experimental data accurately, the parameters it uses do not have straightforward physical interpretations, indicating that the model's basis is more statistical than physical.

```
Optimal parameter values after Differential Evolution optimization:
c1 = 0.7701, c2 = 11.9627, c3 = -0.1367, c4 = 0.1665, c5 = 14.8054, c6 = 0.1357, c7 = 0.3582

Enter the value for VT (mm/s): 100
Enter the value for PN (MPa): 0.5

Friction coefficient at VT=100.0 mm/s and PN=0.5 MPa: 0.7424

Friction coefficients for all combinations of VT and PN:
VT=1.0 mm/s, PN=0.1 MPa: 0.8929
VT=1.0 mm/s, PN=0.5 MPa: 0.6005
VT=1.0 mm/s, PN=1.0 MPa: 0.4931
VT=1.0 mm/s, PN=1.5 MPa: 0.4348
VT=1.0 mm/s, PN=2.0 MPa: 0.3954
VT=10.0 mm/s, PN=0.1 MPa: 0.9639
VT=10.0 mm/s, PN=0.5 MPa: 0.6715
VT=10.0 mm/s, PN=1.0 MPa: 0.5640
VT=10.0 mm/s, PN=1.5 MPa: 0.5058
VT=10.0 mm/s, PN=2.0 MPa: 0.4663
VT=100.0 mm/s, PN=0.1 MPa: 1.0348
VT=100.0 mm/s, PN=0.5 MPa: 0.7424
VT=100.0 mm/s, PN=1.0 MPa: 0.6350
VT=100.0 mm/s, PN=1.5 MPa: 0.5767
VT=100.0 mm/s, PN=2.0 MPa: 0.5373
VT=1000.0 mm/s, PN=0.1 MPa: 1.1057
VT=1000.0 mm/s, PN=0.5 MPa: 0.8134
VT=1000.0 mm/s, PN=1.0 MPa: 0.7059
VT=1000.0 mm/s, PN=1.5 MPa: 0.6477
VT=1000.0 mm/s, PN=2.0 MPa: 0.6082

Overall Mean Absolute Percentage Error (MAPE) for all data points: 8.2598%
```

Overview of the Dorsch Model

The Dorsch model (2002) proposes phenomenological friction models that are entirely empirical and assume dependencies on velocity v and pressure p for the friction coefficient. The model describes the friction coefficient using two formulations:

1. A power law model: $\mu(p, v) = c_1 p^{c_2} v^{c_3}$
2. A linear approximation: $\mu(p, v) = c_1 p + c_2 p^2 + c_3 v + c_4 v^2 + c_5 pv$

These models are designed to fit experimental data accurately through parameters $c1$, $c2$, ..., $c5$ which need to be experimentally determined. Despite their simplicity, these models require extensive experimental data for accurate parameter determination.

```
Optimal parameter values after Differential Evolution optimization with Simulated Annealing:
C1 = 0.4934, C2 = -0.2660, C3 = 0.0543

Enter the value for VT (mm/s): 100
Enter the value for PN (MPa): 0.5

Friction coefficient at VT=100.0 mm/s and PN=0.5 MPa: 0.7619

Calculated mu values for all combinations of VT and PN:
VT=1.0 mm/s, PN=0.1 MPa: Calculated Mu = 0.9104
VT=1.0 mm/s, PN=0.5 MPa: Calculated Mu = 0.5933
VT=1.0 mm/s, PN=1.0 MPa: Calculated Mu = 0.4934
VT=1.0 mm/s, PN=1.5 MPa: Calculated Mu = 0.4429
VT=1.0 mm/s, PN=2.0 MPa: Calculated Mu = 0.4103
VT=10.0 mm/s, PN=0.1 MPa: Calculated Mu = 1.0317
VT=10.0 mm/s, PN=0.5 MPa: Calculated Mu = 0.6723
VT=10.0 mm/s, PN=1.0 MPa: Calculated Mu = 0.5591
VT=10.0 mm/s, PN=1.5 MPa: Calculated Mu = 0.5019
VT=10.0 mm/s, PN=2.0 MPa: Calculated Mu = 0.4650
VT=100.0 mm/s, PN=0.1 MPa: Calculated Mu = 1.1691
VT=100.0 mm/s, PN=0.5 MPa: Calculated Mu = 0.7619
VT=100.0 mm/s, PN=1.0 MPa: Calculated Mu = 0.6336
VT=100.0 mm/s, PN=1.5 MPa: Calculated Mu = 0.5688
VT=100.0 mm/s, PN=2.0 MPa: Calculated Mu = 0.5269
VT=1000.0 mm/s, PN=0.1 MPa: Calculated Mu = 1.3248
VT=1000.0 mm/s, PN=0.5 MPa: Calculated Mu = 0.8634
VT=1000.0 mm/s, PN=1.0 MPa: Calculated Mu = 0.7180
VT=1000.0 mm/s, PN=1.5 MPa: Calculated Mu = 0.6446
VT=1000.0 mm/s, PN=2.0 MPa: Calculated Mu = 0.5971

Overall Mean Absolute Percentage Error (MAPE) for all data points: 7.6441%
```

Overview of the Wriggers and Reinelt Model

The Wriggers and Reinelt model (2009) proposes a multi-scale approach for modeling frictional contact, which relies on analytical and numerical simulations to describe friction coefficients as a function of sliding velocity and pressure. The model captures the maximum friction coefficient μmax using a formula that combines the effects of velocity and pressure, as follows:

$$\mu(v,p) = \left(\frac{2v\bar{a}p}{v^2 + (\bar{a}p)^2}\right)^{\bar{c}} \mu_{\max}$$

$$\mu_{\max} = \frac{\bar{b}}{p} \arctan(\bar{d}p)$$

Here, $\bar{a}$, $\bar{b}$, $\bar{c}$, and $\bar{d}$ are parameters that influence the behavior of the friction coefficient with changes in pressure and velocity. The model effectively decouples the contact pressure and sliding velocity, integrating them into a unified framework that models the peak of the friction curve.

```
Optimal parameter values after Differential Evolution optimization:
a = 0.2043, b = 0.5853, c = -0.0750, d = 1.0000
Mean Absolute Percentage Error (MAPE) with optimized parameters: 0.0916%

Enter the value for VT (mm/s): 100
Enter the value for PN (MPa): 0.5

Friction coefficient at VT=100.0 mm/s and PN=0.5 MPa: 0.8637

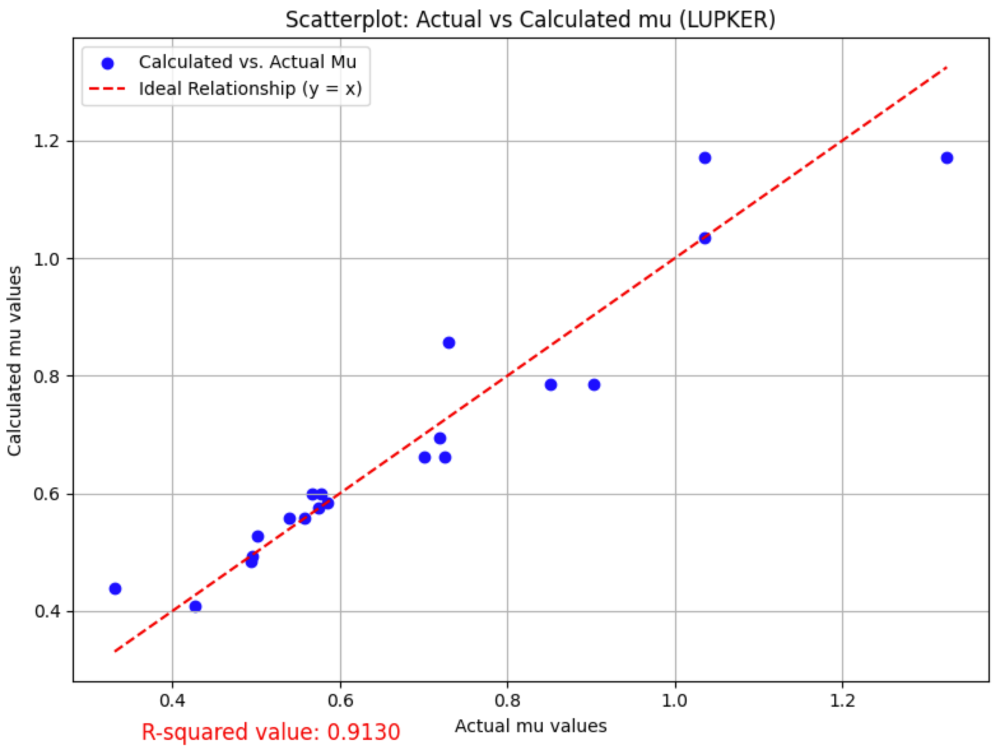
Corresponding mu values for all combinations of VT and PN:
VT=1.0 mm/s, PN=0.1 MPa: 0.7415
VT=1.0 mm/s, PN=0.5 MPa: 0.6119
VT=1.0 mm/s, PN=1.0 MPa: 0.4931
VT=1.0 mm/s, PN=1.5 MPa: 0.4005
VT=1.0 mm/s, PN=2.0 MPa: 0.3328
VT=10.0 mm/s, PN=0.1 MPa: 0.8813
VT=10.0 mm/s, PN=0.5 MPa: 0.7267
VT=10.0 mm/s, PN=1.0 MPa: 0.5843
VT=10.0 mm/s, PN=1.5 MPa: 0.4729
VT=10.0 mm/s, PN=2.0 MPa: 0.3910
VT=100.0 mm/s, PN=0.1 MPa: 1.0474
VT=100.0 mm/s, PN=0.5 MPa: 0.8637
VT=100.0 mm/s, PN=1.0 MPa: 0.6945
VT=100.0 mm/s, PN=1.5 MPa: 0.5620
VT=100.0 mm/s, PN=2.0 MPa: 0.4647
VT=1000.0 mm/s, PN=0.1 MPa: 1.2449
VT=1000.0 mm/s, PN=0.5 MPa: 1.0265
VT=1000.0 mm/s, PN=1.0 MPa: 0.8254
VT=1000.0 mm/s, PN=1.5 MPa: 0.6679
VT=1000.0 mm/s, PN=2.0 MPa: 0.5523

Overall Mean Absolute Percentage Error (MAPE) between actual and calculated mu values: 9.1634%
```

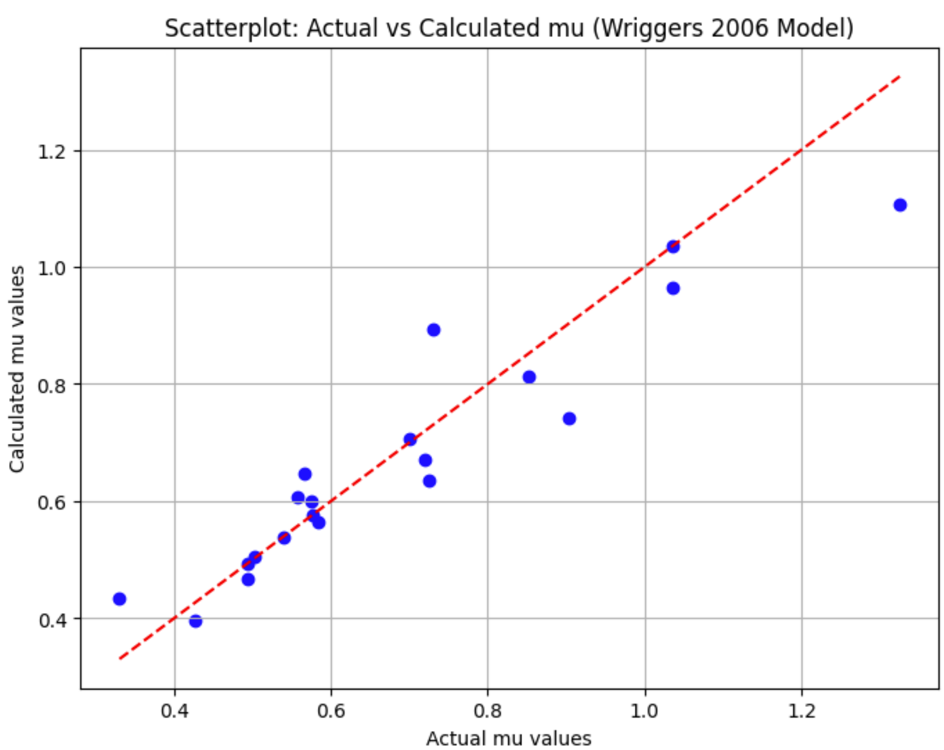
MAPE

VT (mm/s)	PN (Mpa)	Table Value		Different Model and their MAPE													
VT (mm/s)	PN (Mpa)	Table Value		LUPKER MODEL			WRIGGER 2006 MODEL			DORCH MODEL			HUEMER MODEL			WRIGGERS AND REINELT MODEL	
VT (mm/s)	PN (Mpa)	Table Value		Value	MAPE		Value	MAPE		Value	MAPE		Value	MAPE		Value	MAPE
1	0.1	0.7296		1.0348	41.83%		0.8929	22.38%		0.9106	24.81%		0.7685	5.34%		0.7415	1.63%
1	0.5	0.5748		0.6959	21.06%		0.6005	6.86%		0.5935	3.25%		0.572	0.48%		0.6114	6.37%
1	1	0.4931		0.5865	18.95%		0.4931	0.00%		0.4936	0.10%		0.4778	3.11%		0.4931	0.00%
1	1.5	0.3307		0.5307	60.49%		0.4348	16.55%		0.4431	33.99%		0.4197	26.93%		0.4011	21.30%
1	2	0.4274		0.4944	15.68%		0.3954	4.47%		0.4105	3.96%		0.3772	11.75%		0.334	21.85%
10	0.1	1.0348		1.0348	0.00%		0.9639	6.62%		1.0319	0.28%		0.9681	6.45%		0.8828	14.69%
10	0.5	0.7191		0.6959	3.23%		0.6715	17.77%		0.6725	6.48%		0.7205	0.20%		0.727	1.10%
10	1	0.5843		0.5865	0.38%		0.564	4.49%		0.5593	4.28%		0.6018	3.00%		0.5843	0.00%
10	1.5	0.5019		0.5307	5.75%		0.5019	0.00%		0.5021	0.04%		0.5287	5.34%		0.4727	5.81%
10	2	0.4944		0.4944	0.00%		0.4663	3.47%		0.4651	5.92%		0.4751	3.90%		0.3908	20.95%
100	0.1	1.0348		1.0348	0.00%		1.0348	12.49%		1.1693	13.00%		1.0879	5.13%		1.051	1.56%
100	0.5	0.9028		0.6959	22.92%		0.7424	0.68%		0.7621	15.59%		0.8097	10.31%		0.8656	4.12%
100	1	0.7256		0.5865	19.16%		0.635	31.49%		0.6338	12.66%		0.6763	6.79%		0.6956	4.13%
100	1.5	0.5767		0.5307	7.97%		0.5767	0.77%		0.569	1.34%		0.5942	3.03%		0.5627	2.42%
100	2	0.5393		0.4944	8.33%		0.5393	0.00%		0.5271	2.27%		0.5339	1.00%		0.4652	13.74%
1000	0.1	1.325		1.0348	21.90%		1.1057	14.41%		1.325	0.00%		1.1347	14.37%		1.2513	5.56%
1000	0.5	0.8516		0.6959	18.29%		0.8134	7.49%		0.8636	1.40%		0.8445	0.83%		1.0305	21.01%
1000	1	0.7012		0.5865	16.35%		0.7059	5.68%		0.7182	2.42%		0.7054	0.60%		0.8282	18.11%
1000	1.5	0.5661		0.5307	6.25%		0.6477	0.37%		0.6447	13.89%		0.6197	9.47%		0.67	18.35%
1000	2	0.5569		0.4944	11.22%		0.6082	9.22%		0.5972	7.24%		0.556	0.00%		0.5539	0.54%
Total MAPE					14.99%			8.26%			7.64%			5.96%			9.16%

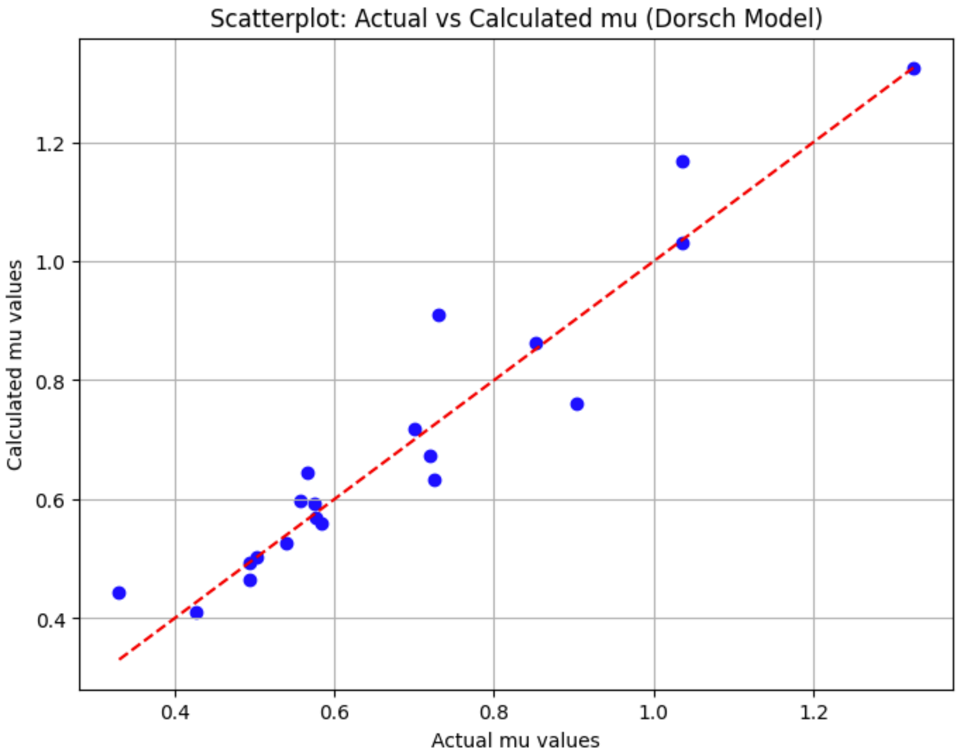
SCATTER PLOTS (R-SQUARED VALUES)



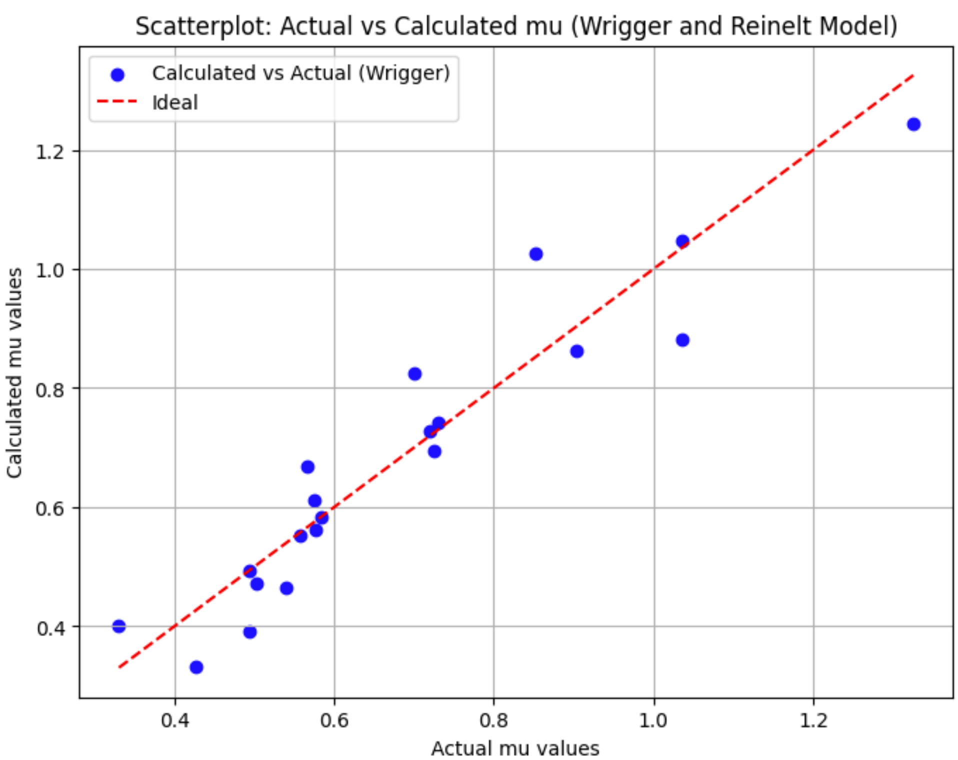
R-squared value: 0.9130



R-squared value: 0.8744



R-squared value: 0.9067



R-squared value for Wrigger model: 0.8912

Efficacy of Mean Absolute Percentage Error (MAPE) for Performance Estimation

Mean Absolute Percentage Error (MAPE) is recognized as an effective metric for assessing model performance, particularly suitable for applications involving diverse magnitudes of data values, such as in the estimation of friction coefficients. The suitability of MAPE is attributed to several key features:

1. **Scale Independence**: MAPE is beneficial in that it expresses errors as a percentage, thereby rendering it independent of the scale of the data. This characteristic is crucial when comparing the efficacy of models across different measurement units or scales.

2. **Interpretability**: The percentage-based error measure of MAPE facilitates an intuitive understanding of the error magnitude. For example, a MAPE of 5% indicates that, on average, the predictions of the model deviate from actual values by 5%, offering straightforward interpretability for stakeholders.

3. **Equal Weighting**: By normalizing the absolute errors relative to the actual values, MAPE ensures that all data points, irrespective of their absolute magnitude, contribute equally to the overall error metric. This normalization prevents the overshadowing of smaller values by larger ones, which could occur if absolute errors were utilized.

4. **Robustness to Outliers**: Unlike other error metrics that may disproportionately amplify the effects of outliers, MAPE tends to be less sensitive to extreme values. This is because the relative error measurement mitigates the impact of large deviations, which can be especially useful in datasets where outliers are present but do not necessarily represent common conditions.
5. **Easy Comparison Between Models**: MAPE enables straightforward comparisons between different models or forecasting methods. It provides a clear, quantifiable measure that can be universally applied, regardless of the model's complexity or the nature of the data being analyzed. This universal applicability makes it easier for decision-makers to evaluate the relative performance of various models.

Given its scale independence, interpretability, equal weighting, robustness to outliers, and ease of comparison between models, MAPE emerges as an exceptionally suitable metric for assessing the performance of models, especially in the context of friction coefficient estimation. These characteristics ensure that the error measurement is both fair and indicative of actual model performance under varied operational conditions. Therefore, MAPE is selected as the preferred error metric for optimizing the friction models in this project, ensuring that the most accurate and reliable model is identified based on quantifiable and understandable criteria.

Overview of the Implementation of Parallel Computing of all the models

The Python script developed for this project orchestrates the process of concurrently optimizing and evaluating multiple friction models. The approach leverages differential evolution and multiprocessing to efficiently determine the most accurate model based on MAPE. Below is an outline of the implemented process:

1. **Data Preparation**:
    - The **read_data** function retrieves experimental data for slip velocities, pressures, and friction coefficients from a text file, facilitating the subsequent modeling steps.
2. **Model Definitions**:
    - Five distinct friction models—Lupker, Wriggers, Dorsh, Huemer, and Wriggers and Reinelt—are defined. Each model incorporates a specific function to compute the friction coefficient using various parameter sets.
3. **Optimization Setup**:
    - Each model is paired with an **objective_function** designed to compute the MAPE between the model's predicted friction coefficients and the experimental data. This function is crucial for the differential evolution algorithm, which seeks to identify the parameter set that minimizes the MAPE.
4. **Concurrent Model Execution**:
    - Utilizing **ProcessPoolExecutor**, the project script concurrently executes the optimization for each model. This parallel processing approach enhances computational efficiency, making it feasible to manage complex models and extensive datasets effectively.
5. **Model Comparison and Selection**:
    - Upon completion of the optimizations and evaluations, the results from all models are collated and analyzed to determine which model yields the lowest MAPE and, consequently, the most accurate friction coefficient predictions. This comparative analysis is critical for selecting the optimal model for practical applications.

```
Enter the value for VT (mm/s): 100
Enter the value for PN (MPa): 0.5
The best model is Wriggers and Reinelt Model with a MAPE of 4.12% and a calculated mu of 0.8656
Optimal parameters for the best model: [ 0.25566289  0.59386405 -0.07576211  1.00001994]
```

CONCLUSION

This project on **Parameter Estimation for friction models** was conducted within the Google Colab environment, leveraging its collaborative features and computational resources. The primary objective was to estimate parameters for various friction models and assess their performance using metrics such as Mean Absolute Percentage Error (MAPE) and R-squared. The analysis involved curve fitting, optimization techniques, and the integration of multiple models into a Python code. Through rigorous analysis and experimentation, valuable insights were gained into the parameter estimation process for friction models. The successful integration of optimization techniques, along with the incorporation of multiple models into a unified Python code, underscores the project's practical applicability in engineering and scientific domains. Moving forward, the methodologies and findings from this project serve as a foundation for further research and application in the field of friction modeling and analysis.

SOURCE CODE

HEUMER MODEL ESTIMATIONS:

https://colab.research.google.com/drive/1d0KGcJOIC-cLf6EDOnBBpLClcpvv3u3f?usp=sharing

PARAMETER ESTIMATION OF ALL MODELS:

https://colab.research.google.com/drive/1NF_DmVe_yd8dydxsL72o6PMlDH7KndHr?usp=sharing

SCATTER PLOTS FOR ALL THE MODELS:

https://colab.research.google.com/drive/1R2IT5fikXd3Eid-TGrIn2OXHXH2SnsTu?usp=sharing

PARALLEL PROCESSING:

https://colab.research.google.com/drive/1dHecjuJ6DzvMgnf2qwlx8o7igonwNQZg?usp=sharing