Using the Michelson Interferometer to Measure the Index of Refraction of Air

1 Introduction & Rationale

In this experiment, we use the Michelson interferometer to measure the index of refraction of air, by varying the optical path difference between two split laser beams traveling across the same physical geometric path length. We do this by passing one of the laser beams through a vacuum cell, in which the air pressure is lowered with a vacuum pump. The phase difference we observe from the interference pattern of the two beams is then used to derive a formula for the index of refraction of air, under the assumption that the index of refraction of the air in the vacuum cell, n, is given a linear function with respect to the pressure P.

2 Experimental Set-up & Equipment

As shown in Figure 1, a Helium-Neon laser beam is projected through 2 steering mirrors, mirrors 1 and 2, and then through a beam expander that is made of two convex lenses mounted in a cage assembly.

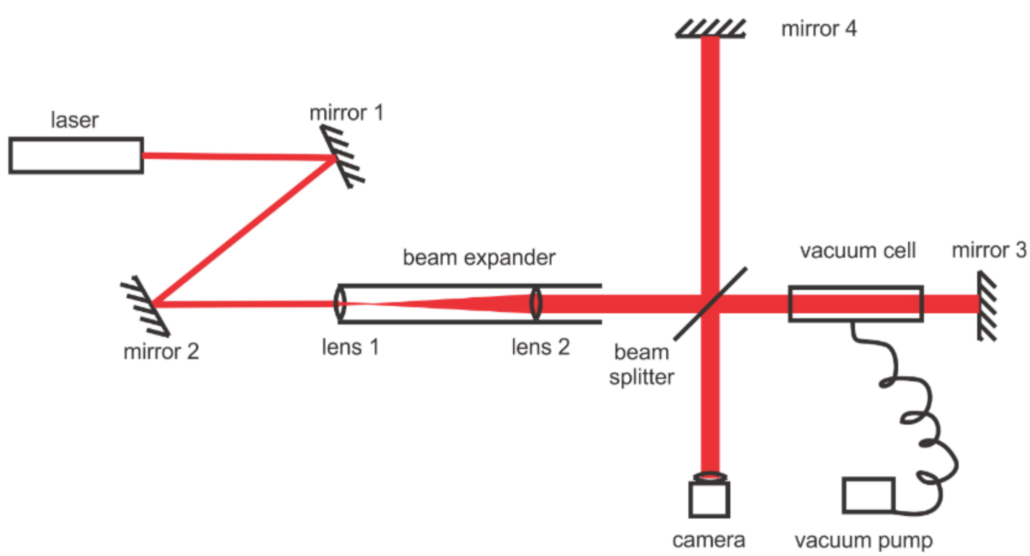

Figure 1: Diagram of modified Michelson interferometer set-up, consisting of two steering mirrors, beam expander, standard Michelson Interferometer, vacuum cell with a pump, and camera

Figure 2 shows the diagram of a beam expander, which is also known as a collimator.

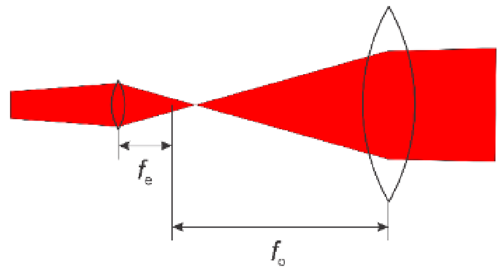

Figure 2: Diagram of beam expander, depicting propagation of real (divergent) laser beam through the beam expander. The diameter of the output beam, D_{out}, is approximately P times larger than the diameter of the input beam, D_{in}, and the divergence of the output beam θ_{out} is approximately P times smaller than divergence of the input beam θ_{in}

The beam expander is arranged such that the distance between the two lenses is equal to the sum of their focal lengths as shown above. The beam expander serves two different purposes. The first is that it increases the cross section of the beam. The ratio of the output diameter D_{out} of the laser beam to its input diameters D_{in} is approximately equal to the power of the beam expander, P, which is the ratio of the focal lengths of the lenses,

$$P = \frac{f_{\mathrm{o}}}{f_{\mathrm{e}}} \approx \frac{D_{\mathrm{out}}}{D_{\mathrm{in}}}$$

The focal length of the first lens we used was $f_e = 50$ mm and the focal length of the second lens we used was $f_o = 200$ mm, such that the distance between the two lenses in the beam expander is approximately equal to 250 mm. The power P is calculated to be 4.

The second is that the beam expander is used to decrease the divergence of the output beam θ_{out} compared to the divergence of the input laser beam θ_{in} of the laser by a factor of approximately P such that

$$\frac{\theta_{out}}{\theta in} \approx \frac{1}{P}$$

We need to decrease the divergence as increased divergence can lead to lower intensity and precision at the camera and interference patterns that are not sharp, leading to increased uncertainty in the

After passing through the beam expander, the expanded laser beam enters the standard Michelson interferometer that has a vacuum cell in one of its arms. Changing air pressure in the vacuum cell with a vacuum pump changes the optical path of the laser beam that travels through the corresponding arm of the interferometer, consequently causing a change in the optical path difference between two arms of the interferometer. The beam splitter directs two separate beams, that can be considered to be from two coherent sources, to mirrors 3 and 4 respectively, and the two beams arriving at the camera from the two separate arms of the interferometer produce an interference pattern due to the optical path difference induced by the vacuum cell. The camera is equipped with a neutral density filter to reduce the intensity of the laser beam and reduce background noise. The observed interference pattern is used to retroactively calculate the change in the optical path and thus the index of refraction of the room air (n_0) by counting the number of fringe shifts during the time that it takes for the vacuum cell to pump out the existing air pressure.

3 Derivation of index of refraction of air

In the bright regions of the pattern, the crests of the waves of the two beams arrive together. In the dark areas the crest of one wave arrives at the same time as the trough of the other. Due to the optical path difference as a result in the change in the air pressure of the vacuum cell, a phase difference between the two beams results in an interference pattern captured by the camera, as shown in Figure 3.

Figure 3: Image of interference pattern with multiple observable fringes (bright regions) against dark regions

The dependence of the phase difference is dependent on the optical path difference and the wavelength of the light in the vacuum

$$\Delta\phi = \frac{\Delta L_{opt}}{\lambda_{vacuum}}$$

If the optical path length of one beam changes by one wavelength, the interference pattern is shifted by one fringe. The optical path length is given by:

$$L_{opt} = nL \tag{1}$$

, where n is the Index of refraction and L is the geometric path length, which we consider to be the length of the vacuum cell. The optical path length can be varied by changing n, which occurs when the air pressure in the vacuum cell changes. In this experiment, the beam passes through the vacuum cell twice, so the optical path length of the beam is $2nL$. The other beam passes through the same length of air, but without passing through a vacuum cell, the pressure will remain constant.

In this experiment, the index of refraction of the air in the vacuum cell, n, is modeled by a linear function with respect to the pressure P:

$$n(P) = 1 + (n_0 - 1)P \tag{2}$$

, where n_0 is the initial index of refraction of the air in the room. We take $n = n_0$ at $P = 1$ i.e. we assume the air pressure at $n = n_0$ to be 1 bar. Therefore, $n - n_0 = (n_0 - 1)(P - 1)$, where we have:

$$\triangle P = |P - 1|$$

We have the dependence of the optical path difference on the change in refractive index, and change in pressure, given by

$$\triangle L_{opt} = 2nL - 2n_0L = 2(n - n_0)L = 2\triangle nL$$

$$= 2L(n_0 - 1)\triangle P \tag{3}$$

As air is sucked out from the vacuum cell, the observed interference pattern will shift by 1 fringe each time the refractive index changes by:

$$\triangle n = \frac{\lambda}{2L}$$

Therefore, if there is a shift of *k* fringes,

$$\triangle n = \frac{k\lambda}{2L}$$

Consequently, equating this to the expression in (3), we obtain that

$$k\lambda = 2L(n_0 - 1)\triangle P$$

, which finally gives us:

$$n_0 = \frac{k\lambda}{2L\triangle P} + 1 \tag{4}$$

The index of refraction of air is modeled in terms of the length of the vacuum cell, *L*, the number of fringes, *k*, the wavelength of the Helium-Neon laser beam, and the change in pressure, ΔP, as shown in (4).

The camera is used to record a video of a fixed section of the interference pattern over time. A fixed point is selected arbitrarily and the number of fringes that move past the fixed point as the vacuum cell is evacuated to its minimum pressure is counted manually.

4 Calculation of results

As air is pumped out of the vacuum cell, we observe 54 instances of a fringe changing from constructive to destructive back to constructive interference with an uncertainty of about 0.2 fringes. The wavelength of the Helium-Neon is 632.8 nm. The length of the vacuum cell is 64.5 mm, and the change in pressure was taken to be approximately 1 bar. The refractive index of air is calculated to 6 decimal places according to (4) as:

$$n_0 = 1.000265$$

The only source of uncertainty is the uncertainty in *k*, the number of fringes counted,

$$\sigma_k = 0.5$$

Therefore, we calculate the uncertainty in the index of refraction n_0 as

$$\sigma_{n_0} = \sqrt{(\frac{\partial \sigma_{n_0}}{\partial k}\sigma_k)^2} = 0.5 \times \frac{\lambda}{2L\triangle P}$$

$$= 2.45271318 \times 10^{-6}$$

$$= 0.000002 \text{ (6 decimal places)}$$

We note that the generally accepted index of refraction of air is:

$$n_0 = 1.000293$$

And our calculated value

$$n_0 = 1.000261 \pm 0.000002$$

, disagrees with the generally accepted value above as it is > 14 standard deviations away from the generally accepted value.

5 Discussion

In our experiment, we assumed the total pressure change to be 1 bar, by assuming that the air pressure in the vacuum cell was decreased to 0 bar, a complete vacuum. However, this ideal scenario is not achievable in a practical setting due to limitations of the vacuum pump used. As such, the change in pressure used to calculate n_0 may not have been accurate. For example, taking the minimum pressure to be 0.1 bar and consequently, the change in pressure to be 0.9 bars, we obtain $n_0 = 1.000294$, which agrees with the generally accepted value for the index of refraction of air. The pressure in the room in which the experiment was carried out could have also not been exactly 1 bar. By the barometric formula, 1 bar is roughly the atmospheric pressure on Earth at an altitude of 111 m at 15 °C. The temperature in the room was definitely higher than 15 °C, and this would have resulted in lower air pressure and consequently a smaller change in pressure. To ensure that an accurate change in pressure is used for the calculation of the index of refraction of air, the initial and final pressures in the vacuum cell could have been measured to a higher accuracy using a pressure gauge, and the air pressure in the room could have been measured using a barometer during the experiment. Other atmospheric factors like the humidity of the air in the room could have also resulted in the air pressure being different from 1 bar.

According to (2), we modeled the refractive index as a linear function of pressure, and this could have been an oversimplified model. The relationship between the refractive index of air and pressure could have been non-linear, contributing to an inaccurate formula for the index of refraction of air. Another source of error could have been the inaccurate wavelength of the Helium-Neon laser beam, it could have been not exactly 632.8 nm. However, noting that the order of magnitude of the wavelength is significantly smaller than other factors like the change in pressure, this might not have been a significant factor contributing to the inaccurate calculation of n_0.

6 Conclusion

In conclusion, we calculated the value for the index of refraction of air to be:

$$n_0 = 1.000261 \pm 0.000002$$

, which is in disagreement with the generally accepted value of 1.000293 as it is > 14 standard deviations away from the generally accepted value. However, we have substantial reason to suspect that inaccuracies in measurement of the change in pressure due to lab apparatus limitations and inaccurate assumptions about air conditions in the room like humidity and temperature could have contributed to the discrepancy.