Estimating temperature of a bulb filament by spectral analysis of light intensity at various wavelengths

1 Introduction and Rationale

A blackbody is an object that completely absorbs all incident light and radiates all wavelengths of light, according to Planck's law in (1), which gives emitted spectral intensity density per unit of wavelength.

$$I_{emitted}(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

where:

- h: Planck's constant,
- c: Speed of light,
- λ: Wavelength,
- k_B: Boltzmann constant,
- T: Temperature.

(1)

Although no object is an ideal blackbody, in many circumstances, the measured radiation spectrum of an object can be used to approximate its temperature. In this experiment, we detect light emitted by an incandescent bulb filament at three different central wavelengths and use Python to analyze the ratio of intensities of the light inside these bands in order to estimate the filament temperature.

2 Experimental Set-up & Equipment

The experimental setup is shown in Figure 1 below.

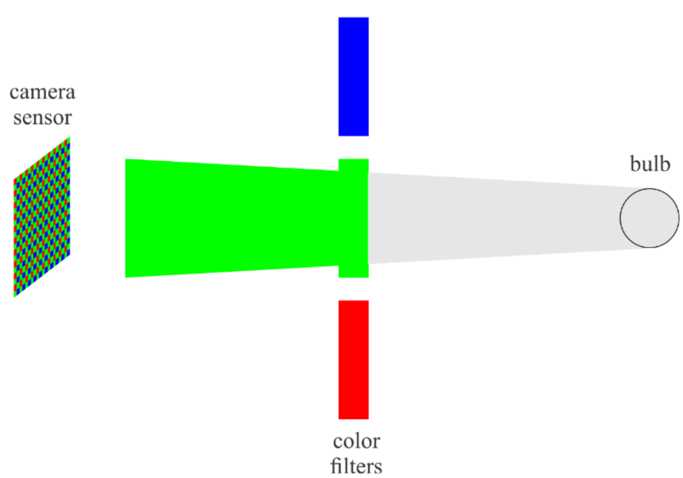

Figure 1: Diagram of experimental set-up. Three different bandpass color filters (blue, green, red) are used to measure intensity of visible light at three different wavelengths

It consists of an incandescent bulb with 3.2 V applied, three separate bandpass color filters, and a Basler acA1300-200uc camera sensor as a light detector. The camera sensor of this camera has three color filters: red, green, and blue.

Light from the incandescent bulb passes through the bandpass filter, then falls onto the camera sensor. The bandpass color filters characteristics are shown in Table 1 below.

Filter name	Central wavelength CW, nm	Full width half maximum FWHM, nm
FB450-40	450±8 nm	40±8 nm
FB550-40	550±8 nm	40±8 nm
FB650-40	650±8 nm	40±8 nm

Table 1: Characteristics of bandpass color filters. FB450-40 is the blue filter, FB550-40 is the green filter, FB650-40 is the red filter

The output of the camera sensor is proportional to the total energy of the light received. By normalizing the measured signal from the camera sensor by the exposure time (which is explained below), we obtain a signal proportional to the intensity of the light and use it to estimate the temperature of the bulb filament.

3 Estimation of temperature of bulb filament from measured signal

The three images obtained from the camera sensor with the bulb turned on are shown below.

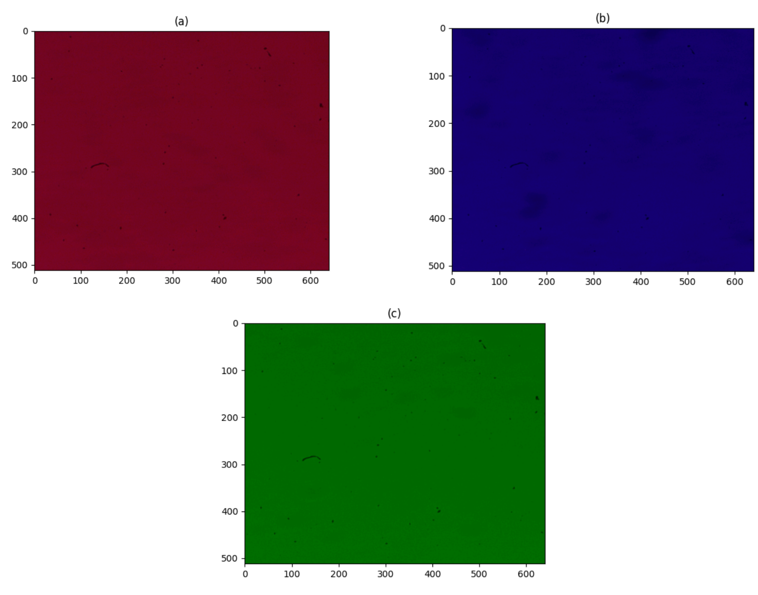

Figure 2(a): *redv01.raw* taken through red filter with bulb on, Figure 2(b): *bluev01.raw* taken through blue filter with bulb on, Figure 2(c): *greenv01.raw* taken through green filter with bulb on

Similarly, the three images obtained from the camera sensor with the bulb turned off are shown below.

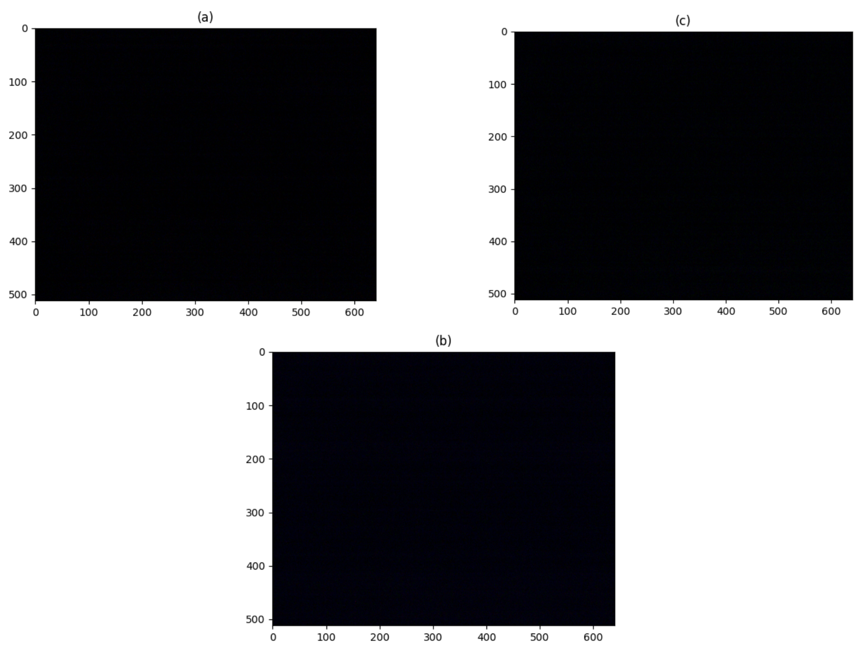

Figure 3(a): *redv02.raw* taken through red filter with bulb off, Figure 3(b): *bluev02.raw* taken through blue filter with bulb off, Figure 3(c): *greenv02.raw* taken through green filter with bulb off

In Table 2 below, we record the image file names, one with the bulb turned off, and one with the bulb turned on, and we record the exposure times for each image.

Filter	**Bulb**	**File Name**	**Exposure Time**
Red Filter	ON	redv01.raw	5000 μs
	OFF	redv02.raw	5000 μs
Green Filter	ON	greenv01.raw	20000 μs
	OFF	greenv02.raw	20000 μs
Blue Filter	ON	bluev01.raw	70000 μs
	OFF	bluev02.raw	70000 μs

Table 2: Image file names for each filter with the bulb either on or off, and corresponding exposure times

The unnormalized measured signal from the camera sensor is taken to be the total value of the corresponding color layer, as shown in Table 3 below.

Filter	Bulb	Color Average/Total Value	Exposure Time
Red Filter	ON	Avg: 117.39, Total: 3846654.00	5000 μs
	OFF	Avg: 3.09, Total: 1013515.00	5000 μs
Green Filter	ON	Avg: 107.39, Total: 35190162.00	20000 μs
	OFF	Avg: 3.84, Total: 1256784.00	20000 μs
Blue Filter	ON	Avg: 110.75, Total: 36291641.00	70000 μs
	OFF	Avg: 12.65, Total: 4145813.00	70000 μs

Table 3: Average and total color values for each filter with the bulb either on or off, and corresponding exposure times. We use the total values in our calculations

We calculate the intensity signal of the light through the red, green and blue filters as shown below, where the unnormalized signal (total value of the corresponding color layer) is calculated by subtracting the background signals attributed to the signal detected by the camera sensor when the bulb is off, from the measured signal when the bulb is on. The difference is then normalized by dividing by the corresponding exposure time as shown in (2).

$$
\begin{aligned}
I_{\text{red}} &= \frac{\sum \text{Red}_{\text{ON}}}{\text{exposure time}} - \frac{\sum \text{Red}_{\text{OFF}}}{\text{exposure time}} \\
I_{\text{green}} &= \frac{\sum \text{Green}_{\text{ON}}}{\text{exposure time}} - \frac{\sum \text{Green}_{\text{OFF}}}{\text{exposure time}} \\
I_{\text{blue}} &= \frac{\sum \text{Blue}_{\text{ON}}}{\text{exposure time}} - \frac{\sum \text{Blue}_{\text{OFF}}}{\text{exposure time}}
\end{aligned} \tag{2}
$$

The calculated total intensity signals to 2 decimal places are recorded in Table 4 below:

Color	Color Average/Total Intensity
Red	Total Intensity: 7490.43, Average: 0.02
Green	Total Intensity: 1696.67, Average: 0.01
Blue	Total Intensity: 459.23, Average: 0.00

Table 4: Calculated total intensity signals for 3 different colored filters, normalized by dividing total color value of images from the camera sensor by exposure time

Using the calculated total intensity signals, we calculate the intensity ratios in (3).

$$
\begin{aligned}
R_{\text{red/green}} &= \frac{I_{\text{red}}}{I_{\text{green}}} = 4.41\,(2\,\text{d.p.}) \\
R_{\text{green/blue}} &= \frac{I_{\text{green}}}{I_{\text{blue}}} = 3.70\,(2\,\text{d.p.}) \\
R_{\text{red/blue}} &= \frac{I_{\text{red}}}{I_{\text{blue}}} = 16.31\,(2\,\text{d.p.})
\end{aligned} \tag{3}
$$

We model the bulb filament as an ideal blackbody and therefore, its emitted spectral intensity density per unit of wavelength as given by (1). The transmission coefficient $Tr_{camera}(\lambda)$ of the filters has a complex wavelength dependence. The additional color filters are assumed to have very narrow bandwidth, and the transmission $Tr_{filter}(\lambda)$ of the filters also has complex wavelength dependence. The transmission coefficients of the external colored filters and the camera filters are given below in Tables 5 and 6.

Filter name	Transmission coefficient
FB550-40 (green filter)	$Tr_{\text{filter, green}}(550\text{ nm}) = 6.7064 \times 10^{-1}$
FB450-40 (blue filter)	$Tr_{\text{filter, blue}}(450\text{ nm}) = 7.1581 \times 10^{-1}$
FB650-40 (red filter)	$Tr_{\text{filter, red}}(650\text{ nm}) = 6.9453 \times 10^{-1}$

Table 5: Transmission coefficients of 3 different colored filters at central wavelengths (550 nm for FB550-40, 450 nm for FB450-40, 650 nm for FB650-40)

Central wavelength / nm	Transmission coefficient
550	$Tr_{\text{camera, green}}(550\text{ nm}) = 8.3253 \times 10^{-1}$
450	$Tr_{\text{camera, blue}}(450\text{ nm}) = 5.8647 \times 10^{-1}$
650	$Tr_{\text{camera, red}}(650\text{ nm}) = 9.3389 \times 10^{-1}$

Table 6: Transmission coefficients of camera filter at central wavelengths (550 nm for FB550-40, 450 nm for FB450-40, 650 nm for FB650-40)

The effective transmission for the light reaching the camera sensor is the product of the corresponding transmissions as shown in (4).

$$Tr(\lambda) = Tr_{camera}(\lambda) \cdot Tr_{filter}(\lambda) \tag{4}$$

Furthermore, the received intensity signals are known to depend on the emitted spectral intensity and also on the geometry of the experiment. We group these geometric factors into a proportionality constant *A* that is consistent throughout the experiment for all colors. Therefore, the expected received intensity density by the camera sensor for different wavelengths as a function of temperature can be modeled as in (5).

$$\begin{gathered} I_{\text{red}}(T) = A \times Tr_{camera,red} \times Tr_{filter,red} \times I_{\text{emitted}}(T, \lambda = 650nm) \\ I_{\text{blue}}(T) = A \times Tr_{camera,blue} \times Tr_{filter,blue} \times I_{\text{emitted}}(T, \lambda = 450nm) \\ I_{\text{green}}(T) = A \times Tr_{camera,green} \times Tr_{filter,green} \times I_{\text{emitted}}(T, \lambda = 550nm) \end{gathered} \tag{5}$$

Therefore, the theoretical ratios of intensities of radiation at different wavelengths are calculated as in (6).

$$R_{\text{red/green}} = \frac{I_{\text{red}}}{I_{\text{green}}}$$
$$R_{\text{green/blue}} = \frac{I_{\text{green}}}{I_{\text{blue}}}$$
$$R_{\text{red/blue}} = \frac{I_{\text{red}}}{I_{\text{blue}}} \tag{6}$$

The ratios of intensities of radiation at different wavelengths can be used to distinguish blackbody spectra corresponding to different temperatures, as shown in Figure 4.

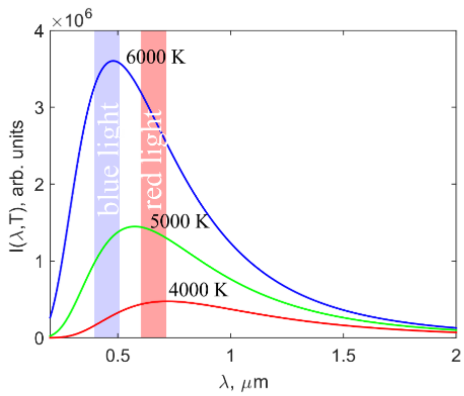

Figure 4: Blackbody spectral intensity density $I(\lambda, T)$ for three different temperatures. As temperature increases, the peak of the spectral density increases and moves to shorter wavelengths. In the above example with blue light and red light bands, $R_{\text{red/blue}}$ is expected to decrease with increasing temperature

Figures 5(a), 5(b) and 5(c) show the three graphs obtained, one for each theoretical ratio: $R_{\text{red/green}}$, $R_{\text{green/blue}}$ and $R_{\text{red/blue}}$ respectively, as a function of temperature.

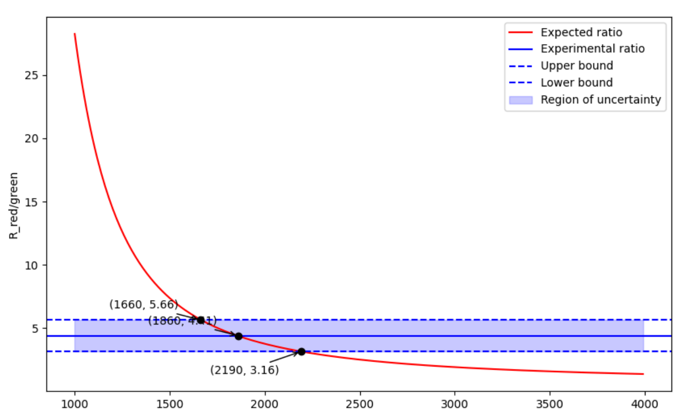

Figure 5(a): Graph of expected ratio of intensity of radiation at 650 nm to intensity of radiation at 550 nm as a function of temperature, with the experimental ratio used to interpolate and obtain the temperature of the bulb filament

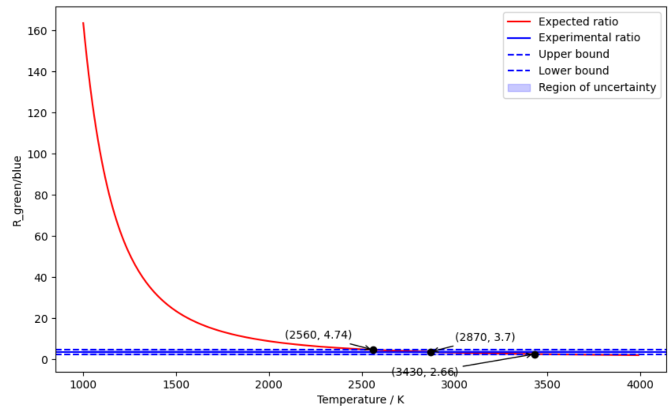

Figure 5(b): Graph of expected ratio of intensity of radiation at 550 nm to intensity of radiation at 450 nm as a function of temperature, with the experimental ratio used to interpolate and obtain the temperature of the bulb filament

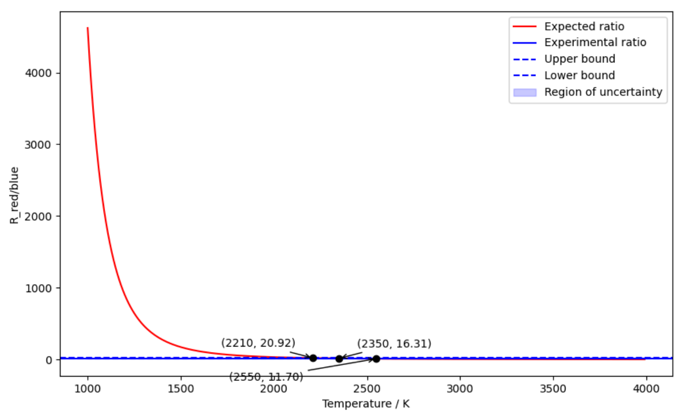

Figure 5(c): Graph of expected ratio of intensity of radiation at 650 nm to intensity of radiation at 450 nm as a function of temperature, with the experimental ratio used to interpolate and obtain the temperature of the bulb filament

As shown above, from the calculated experimental ratios, we use interpolation on the graphs to estimate the temperature of the bulb filament. We obtain 3 estimates as shown in Table 7 below. The uncertainties are reported in Table 7 and calculations can be found in Appendix.

Expected ratio	Estimated temperature / K	Estimated temperature lower bound / K	Estimated temperature upper bound / K	Uncertainty in expected ratio
$R_{red/green}$ = 4.41	1860	1660	2190	+/- 1.25
$R_{green/blue}$ = 3.70	2870	2560	3430	+/- 1.04

$R_{red/blue}$ = 16.31	2350	2210	2550	+/- 4.61

Table 7: Expected ratio of intensities and corresponding uncertainties, estimated temperature obtained by interpolation

4 Discussion

We note that the three estimated temperatures vary largely from one another. The highest estimated temperature is 2870 K from interpolating on the graph of $R_{green/blue}$ as a function of temperature, and the lowest estimated temperature is 1860 K from interpolating on the graph of $R_{red/green}$ as a function of temperature, yielding a discrepancy of 1010 K.

Firstly, when calculating the experimental intensity ratios, we did not consider a wavelength band for each filter and performed numerical integration. and we only considered the central wavelength of the band for each filter. This could have led to inaccuracy in our calculated intensities and therefore, the intensity ratios.

The proportionality constant due to the geometry of the setup, *A*, in (5), was also assumed to cancel out when dividing one received intensity signal by another to obtain their intensity ratio. Different components and arrangements of the setup could have affected radiation of different wavelengths to different extents, and this could have contributed to the large discrepancy in estimated temperature of the bulb filament.

Lastly, the bulb filament is not a perfect blackbody, and as our expected intensity ratio as a function of temperature was calculated according to Planck's Law, which assumes the source to be a perfect blackbody, this could have contributed to the large discrepancy in the estimated temperature of the bulb filament as well.

5 Conclusion

In this experiment, we detected light emitted by an incandescent bulb filament at three different central wavelengths and analyzed the ratio of intensities of the light of these wavelengths in order to estimate the temperature of the bulb filament. We obtain three different estimated temperatures, 1860 K, 2350 K and 2870 K, of large discrepancy, from experimentally determined intensity ratios of $R_{red/green}$ = 4.41 +/- 1.25, $R_{green/blue}$ = 3.70 +/- 1.04, $R_{red/blue}$ = 16.31 +/- 4.61.

6 Appendix

The only known sources of error in this experiment were the uncertainty in the external color filter width (40 ± 8 nm for all filters) and uncertainty in the position of the central frequency of the filter (650 ± 8 nm for the red filter, 550 ± 8 nm for the green filter, and 450 ± 8 nm for the blue filter). The intensity detected for each color spectrum is directly proportional to the amount of light transmitted through the filter, which is dependent on the filter width, and not on the central wavelength of the filter. Therefore, in our calculation of the uncertainty in the intensity ratios through error propagation done in this experiment, only the uncertainty in the filter width is accounted for.

$$R_{x/y} = \frac{I_x}{I_y}$$

$$\delta_{R_{x/y}} = \sqrt{(\frac{1}{I_y}\delta I_x)^2 + (\frac{-I_x}{I_y^2}\delta I_y)^2}$$

, where

$$\delta I_x = \frac{8}{40} \times I_x$$

We obtain the following uncertainties for the intensity ratios.

$$\delta_{R_{\mathbf{red/green}}} = \sqrt{\left(\frac{1}{1696.67} \cdot (7490.43 \cdot 0.2)\right)^2 + \left(\frac{7490.43}{1696.67^2} \cdot (1696.67 \cdot 0.2)\right)^2} = 1.25(2d.p.)$$

$$\delta_{R_{\mathbf{green/blue}}} = \sqrt{\left(\frac{1}{459.23} \cdot (1696.67 \cdot 0.2)\right)^2 + \left(\frac{1696.67}{459.23^2} \cdot (459.23 \cdot 0.2)\right)^2} = 1.04(2d.p.)$$

$$\delta_{R_{\mathbf{red/blue}}} = \sqrt{\left(\frac{1}{459.23} \cdot (7490.43 \cdot 0.2)\right)^2 + \left(\frac{7490.43}{459.23^2} \cdot (459.23 \cdot 0.2)\right)^2} = 4.61(2d.p.)$$

, where I_{red}, I_{blue}, I_{green}, are obtained as the total intensities in Table 4.

Code to obtain Figures 5(a), 5(b), and 5(c)

```
import numpy as np
import matplotlib.pyplot as plt

def blackbody_spectrum(lambdas, T):
    h = 6.626e-34
    c = 3.00e8
    k_B = 1.381e-23
    I_func = ((2 * np.pi * h * c**2) / (lambdas**5)) / (np.exp((h * c) /
(lambdas * k_B * T)) - 1)
    return I_func

if __name__ == "__main__":
```

```
    trans_camera_green = 8.3253431e-01
    trans_camera_red = 9.3389338e-01
    trans_camera_blue = 5.8647080e-01
    trans_video_red = 6.9452750e-01
    trans_video_green = 6.7064220e-01
    trans_video_blue = 7.1580720e-01

    temp_range = np.arange(1000, 4000, 10)

    green_spectrum = blackbody_spectrum(550e-9, temp_range) *
trans_camera_green * trans_video_green
    blue_spectrum = blackbody_spectrum(450e-9, temp_range) *
trans_camera_blue * trans_video_blue

    gb_ratio = green_spectrum / blue_spectrum

    experimental_gb_ratio = 3.7
    uncertainty_gb = 1.04
    upper_bound_ratio = experimental_gb_ratio + uncertainty_gb
    lower_bound_ratio = experimental_gb_ratio - uncertainty_gb
    diffs_nominal = np.abs(gb_ratio - experimental_gb_ratio)
    diffs_upper = np.abs(gb_ratio - upper_bound_ratio)
    diffs_lower = np.abs(gb_ratio - lower_bound_ratio)

    temp_nominal = temp_range[np.argmin(diffs_nominal)]
    temp_upper = temp_range[np.argmin(diffs_upper)]
    temp_lower = temp_range[np.argmin(diffs_lower)]

    plt.figure(figsize=(10, 6))
    plt.plot(temp_range, gb_ratio, label='Expected ratio', color='red')
    plt.axhline(y=experimental_gb_ratio, color='blue', linestyle='-',
label="Experimental ratio")
    plt.axhline(y=upper_bound_ratio, color='blue', linestyle='--',
label="Upper bound")
    plt.axhline(y=lower_bound_ratio, color='blue', linestyle='--',
label="Lower bound")
    plt.fill_between(temp_range,
                     experimental_gb_ratio - uncertainty_gb,
                     experimental_gb_ratio + uncertainty_gb,
```

```
                     color='blue', alpha=0.2, label="Region of
uncertainty")
    plt.scatter(temp_nominal, experimental_gb_ratio, color='black',
zorder=5)
    plt.annotate(f"({temp_nominal}, {experimental_gb_ratio})",
                 (temp_nominal, experimental_gb_ratio),
                 textcoords="offset points", xytext=(50, 10), ha='center',
                 arrowprops=dict(arrowstyle="->", color='black'))
    plt.scatter(temp_upper, upper_bound_ratio, color='black', zorder=5)
    plt.annotate(f"({temp_upper}, {upper_bound_ratio:.2f})",
                 (temp_upper, upper_bound_ratio),
                 textcoords="offset points", xytext=(-50, 10),
ha='center',
                 arrowprops=dict(arrowstyle="->", color='black'))
    plt.scatter(temp_lower, lower_bound_ratio, color='black', zorder=5)
    plt.annotate(f"({temp_lower}, {lower_bound_ratio:.2f})",
                 (temp_lower, lower_bound_ratio),
                 textcoords="offset points", xytext=(-100, -20),
ha='center',
                 arrowprops=dict(arrowstyle="->", color='black'))
    plt.xlabel("Temperature / K")
    plt.ylabel("R_green/blue")
    plt.legend()
    plt.show()
```