Calculating the lifetime of muon decay

1 Introduction

When a charged particle, such as a muon, passes through a plastic scintillator, it loses some of its kinetic energy through ionization and the excitation of the detector's molecular structure. As these molecules relax back to their ground state, they emit photons, which are subsequently detected by a photomultiplier tube (PMT). The PMT converts these photons into voltage pulses, which are then transformed into TTL pulses by a discriminator and directly measured by a laboratory computer. The time intervals between pulse detections in the scintillator are assumed to be random and independent for each event, and this behavior is typically modeled using a Poisson distribution. However, at shorter time intervals between pulses, the Poisson model becomes limited due to muon decay within the detector. This results in additional detected pulses beyond what the Poisson model predicts. These extra counts can be described using a similar Poisson process for the time intervals between these events. This approach enables the calculation of the muon decay lifetime.

2 Results and Discussion

We measured a total time period of 17.62 hours, during which N_0 = 1594447 pulses were detected by the lab machine, such that the average rate of arrival was measured to be 25.14 pulses / s. By using a Poisson distribution with the parameter, μ = 25.14, the time intervals between pulses (t) is modeled by:

$$f_\mu(t) = N_0 \mu e^{-\mu t} \qquad (1)$$

The detected experimental time intervals are plotted as a histogram with the modeled expected time intervals overlaid in Figure 1 below:

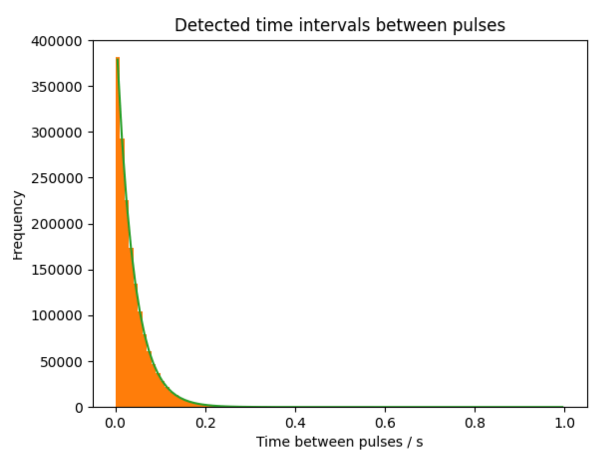

Figure 1: Histogram of experimentally measured time intervals between pulses and modeled theoretical distribution of time intervals

Visually, the fit of the model with respect to the experimental data is convincing, but because of the large quantity of detected pulses and the large range of experimental time intervals, the time intervals are further examined by looking at a log-log histogram of measured time intervals and a corresponding log-log fit, as shown in Figure 2 below:

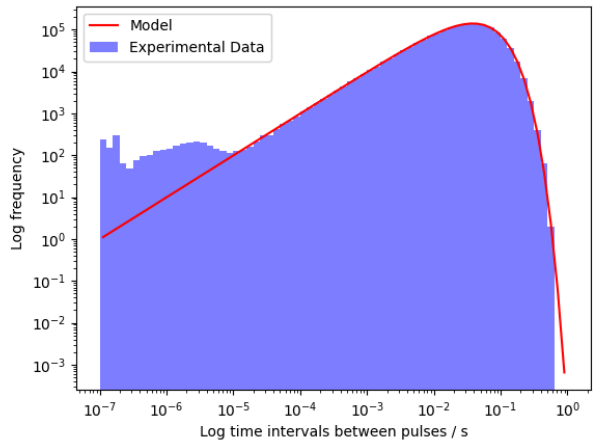

Figure 2: Histogram of experimental measured time intervals between pulses and modeled theoretical distribution of time intervals on log-log axes

The model is a visually good fit at longer times in between pulses on a log scale. However, when the time in between the pulses on the log scale is shorter than $t_c = 10^{-5}$ seconds, the model evidently deviates from the experimental counts on a log scale. This is further shown in Figure 3 below, where the model's fit at times shorter than t_c on a normal scale deviates significantly from the experimental counts.

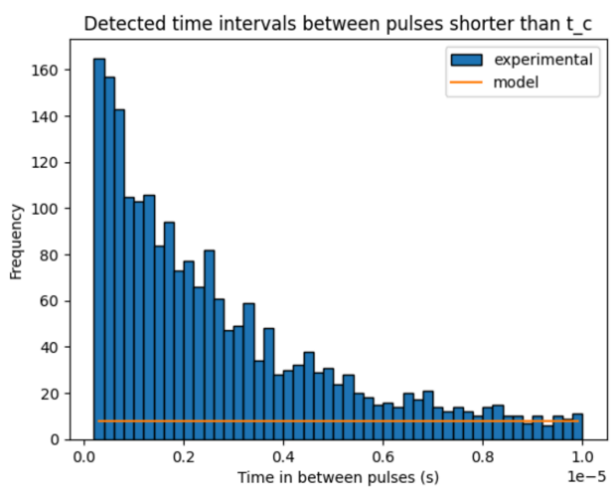

Figure 3: Histogram of experimental pulses shorter than t_c compared to the modeled distribution

The experimental pulses shorter than t_c have a significantly larger count than anticipated from the model. The total number of time intervals between pulses shorter than t_c as expected from the model was 401, whereas the observed total number of intervals shorter than t_c was 2098. A possible contributor to this deviation could be the fact that the detector is sensitive to extra counts via ripples in the pulse whenever the initial pulse is so large that the second ripple can cross the threshold of detection. However, it is important to note that the detector does not detect solely muons. Rather, it detects any charged particle that can excite and ionize the molecules of the plastic scintillator that emits a photon after the plastic molecules have relaxed to their ground state. Limitations in the earlier Poisson model could have resulted in this deviation between the model and experimental counts since we assumed that the detection of a particle is independent of any other particle. However, if a muon loses so much kinetic energy when it enters the detector that it stops inside the detector, it will decay into a muon neutrino, electron antineutrino, and an electron, which can then be detected shortly after the original muon was detected. Thus, this decay of a muon inside the detector could be a cause of the extra counts of the experimental time intervals with respect to the earlier model. This muon decay can be modeled using a similar Poisson process as before where

$$f_{extra}(t) = N_{extra} v e^{-vt} \tag{2}$$

As shown in Figure 4 below, this model of the muon decay fits the extra counts of the experimental time intervals as $f_{extra}(t) = Ae^{-Bt}$ where $A = N_{extra}v$ and $B = v$. The fitting parameters and their respective uncertainties are shown in Table 1.

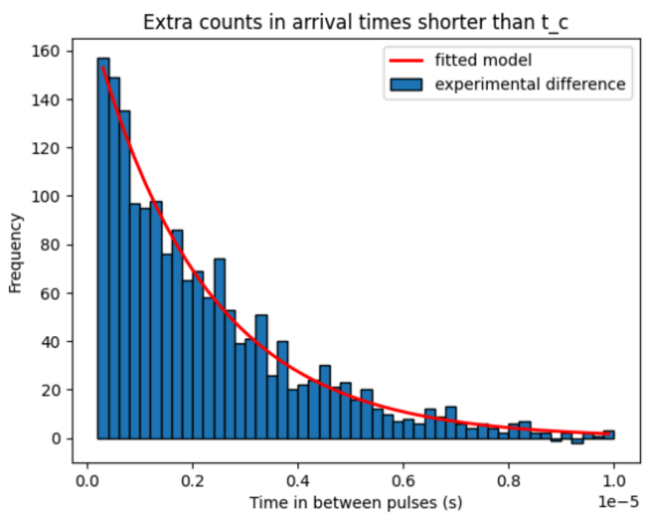

Figure 4: Histogram of the difference between experimental counts and the modeled counts compared to the fitted model of muon decay

Fitting Parameter	Value and Uncertainty
A	176 +/- 5
B	464000 +/- 16000

Table 1: The fitting parameters of the modeled muon decay

From the fit of the extra counts, we obtain that $\nu = 1/\tau = 464000 \pm 16000$. Thus, the experimental lifetime of muon decay is $(2.16 \pm 0.07) \times 10^{-6}$ seconds. Since the known muon lifetime of 2.20×10^{-6} is within the width of the uncertainty, it is likely that our model of muon decay accounts for the extra counts between the experimental times in between pulses and the original modeled counts.

3 Appendix

Calculation of uncertainty in muon lifetime

$$\delta\tau = \sqrt{\frac{d\tau}{d\nu}\delta\nu} = \frac{1}{\nu^2} \cdot \delta\nu \approx 7 \times 10^{-8} \text{ seconds}$$