Calculating the Speed of Light

1 Introduction & Rationale

The speed of light, denoted as c, is a fundamental quantity in electrodynamics following from Maxwell's Equations. In this experiment, we modify Foucault's rotating mirror experiment to measure the speed of light at a relatively high precision, making use of basic optics principles. Code used for analysis of data is written in Python and can be found in Appendix.

2 Experimental Set-up & Equipment

In our set-up, a beam from a laser is directed to the center of a rotating mirror. The rotating mirror is attached to a motor that allows us to manually adjust the constant frequency of rotation, f. This beam is referred to as the forward ray. The forward ray is reflected all around, and is directed toward an intermediate mirror, which then directs the forward ray to the convex lens with a focal length, $F = 2$m, then to the return mirror, before being reflected back along the same path from the return mirror, to the convex lens, to the intermediate mirror, then to the rotating mirror.

The sum of the distances between the convex lens and the intermediate mirror, and between the rotating mirror and the intermediate mirror is equal to the focal length of the convex lens (i.e. the rotating mirror is placed at the focal point of the convex lens). This ensures that the reflected ray from the return mirror travels along the same path back to the rotating mirror as the forward ray. This is illustrated in Figure 1 for three different orientations of the rotating mirror. No matter what the initial orientation of the rotating mirror is, the return ray will always come back to the rotating mirror through exactly the same path as it was going from the rotating mirror to the return mirror.

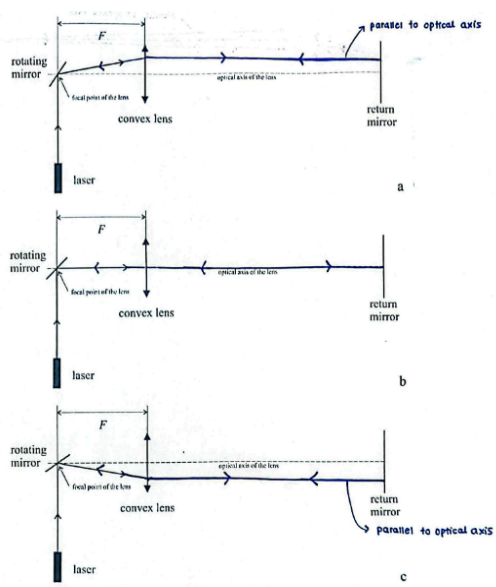

Figure 1: In these 3 simplified diagrams, the invariability of the path taken by the laser beam reflected off the rotating mirror placed at the focal length of the convex lens is shown

While the beam is traveling from the rotating mirror to the return mirror and back along the same path (and therefore the same path length), the rotating mirror changes its orientation by a small angle θ, causing the return ray reflected off the rotating mirror to miss the laser aperture by some small distance x. The displacement x of the return ray is so small that any device that we try to use to measure it naturally blocks the laser aperture. To solve this problem, we redirect this return ray to the CCD camera sensor using the beam splitter placed between the rotating mirror and the laser.

The labeled experimental set-up is shown in Figure 2 below. We indicate our measured distances in Figure 2 and Table 1 as well.

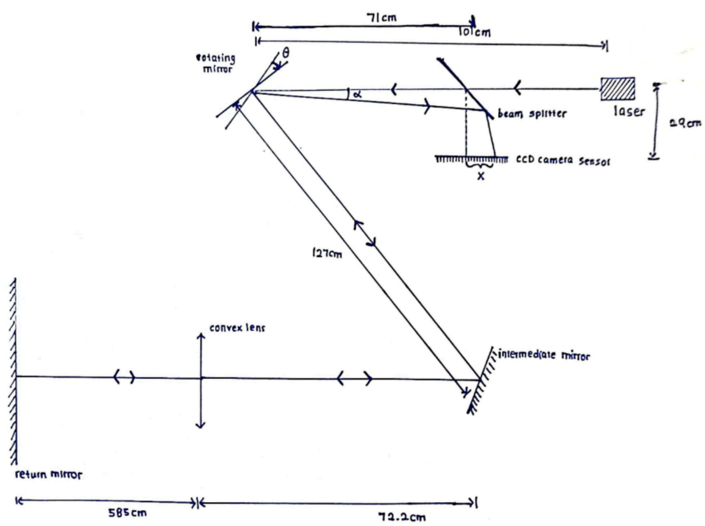

Figure 2: Labeled diagram of the experimental set-up

Distance	value	uncertainty
Laser to rotating mirror	1010 mm	+/- 5 mm
Rotating mirror to intermediate mirror	1270 mm	+/- 7.5 mm
Intermediate mirror to lens	722 mm	+/- 10 mm
Lens to return mirror	5850 mm	+/- 10 mm
Rotating mirror to beam splitter	710 mm	+/- 5 mm
Beam splitter to camera	290 mm	+/- 5 mm

Table 1: Optical set-up geometric distance measurements

In reality, the laser beam is a divergent beam. To solve this divergence problem, we use the existing convex lens. We place the return mirror at the distance where the laser beam is focused by the convex lens. This is illustrated in the simplified diagram Figure 3. This distance from the lens to the return mirror is calculated using the thin lens equation, given that we ensured that the distance from the laser to the rotating mirror was approximately 1m and the sum of the distances from the rotating mirror to the intermediate mirror and from the intermediate mirror to the convex lens, f, was approximately 2m. This gives an object distance, x_1, of approximately 1m + 2m = 3m, and the distance from the lens to the return mirror, x_2, is calculated to be approximately 6m from the thin lens equation.

$$\frac{1}{f} = \frac{1}{x_1} + \frac{1}{x_2} \therefore \frac{1}{2} = \frac{1}{3} + \frac{1}{x_2} \therefore x_2 = 6$$

This is also the reason why we use an intermediate mirror. The size limitations of the room in which we conducted our experiment necessitate the use of the intermediate mirror to ensure a total image distance of 6m can be physically measured within the confined space of the room. As seen in Figure 2, the actual distances measured in our experiment were not precisely the ideal x_1, x_2, f but the laser spot was focused sufficiently to ensure negligible difference in path length traveled by the forward ray and the return ray to and from the return mirror.

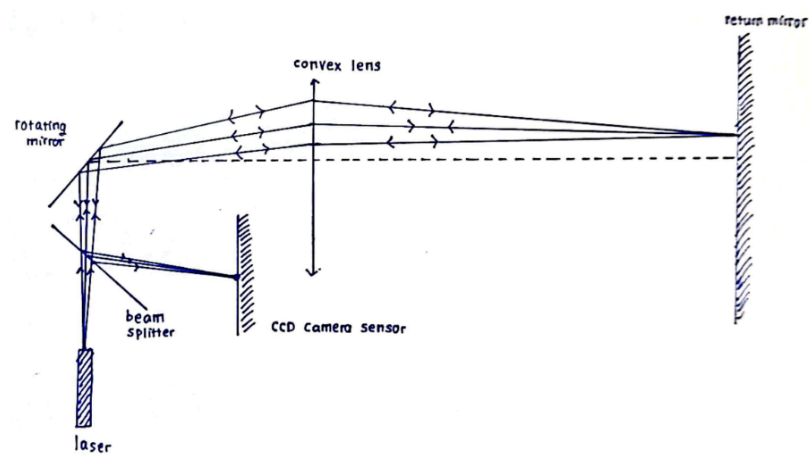

Figure 3: In this simplified diagram, the placement of the return mirror at the point at which the divergent laser beam focuses enables the focus of the return ray on the CCD camera screen

The sum of the distances from the rotating mirror to the beam splitter and from the beam splitter to the CCD camera sensor is adjusted to be approximately 1m, equal to the distance from the laser to the rotating mirror. Consequently, the return ray will be focused on the screen of the CCD camera sensor, ensuring higher clarity of the laser spot images.

We also use a 1000x neutral density filter attached to the CCD camera mount to reduce the intensity of the laser beam and prevent the camera sensor from being overexposed. Keeping the camera sensor from being overexposed is important for more precise measurement of the laser spot position. The last piece of additional equipment that we use is a photodiode. We use it to measure the frequency the rotating mirror spins around with. When the rotating mirror spins, it reflects the laser beam all around, creating a red horizontal line on the walls of the room. This is the reflection of the laser beam from the rotating mirror, when it does not go through the lens. If we place the photodiode such that this line goes through the aperture of the diode, then every time the laser beam hits the diode, the voltage at the output of the diode will spike. The frequency of these spikes is equal to the frequency of the rotating mirror, and it is measured with an oscilloscope. The frequency of the rotating mirror was measured at power percentages in the range 7-22% as shown in Table 2.

3 Relationship Between Speed of Light, Experimental Measurements & Parameters

We now find the relationship between x, the speed of light c, the mirror frequency f, and the geometry of the experiment. Let d be the total distance from the rotating mirror to the return mirror (it is the sum of the distance from the convex lens to the intermediate mirror, the distance from the rotating mirror to the intermediate mirror, and the distance from the convex lens to the return mirror). Let t be the total time taken for the laser beam to travel from the rotating mirror to the return mirror and back. Let r be the distance from the rotating mirror to the laser. According to Figure 2, we have θ as the change in orientation of the rotating mirror over time t, and α as the angle of deviation from the original path of the return ray reflected off the rotating mirror.

$$\theta = 2\pi f t$$
$$t = \frac{2d}{c}$$
$$\therefore \theta = \frac{4\pi d f}{c}$$

$$\alpha = 2\theta$$
$$\tan\alpha = \tan(2\theta) \approx \frac{x}{r}$$
$$\tan 2\theta \approx 2\theta, \text{ by small angle approximation}$$
$$2\theta = \frac{x}{r}$$
$$\therefore x = 2\theta r = \frac{8\pi r d f}{c}$$

$$x = \frac{8\pi r d f}{c} \tag{1}$$

3 Experimental Results & Analysis

Figure 4: Example laser image captured by CCD camera where the white dot is the calculated center-of-mass coordinates of the laser spot

Figure 4 is a typical image of the laser spot, taken at a mirror frequency of 231.2 Hz. To extract the laser spot position from the image, we filter out the green and blue color channels from the RGB image and set an arbitrary threshold value of 100, for which pixels with values above the threshold retain their values and pixels with values below the threshold are reduced to 0. A center of mass function, where masses are the pixel values and position coordinates are the corresponding pixel coordinates, is used to find the single position (in pixels) of the laser spot, which is indicated by the white dot above.

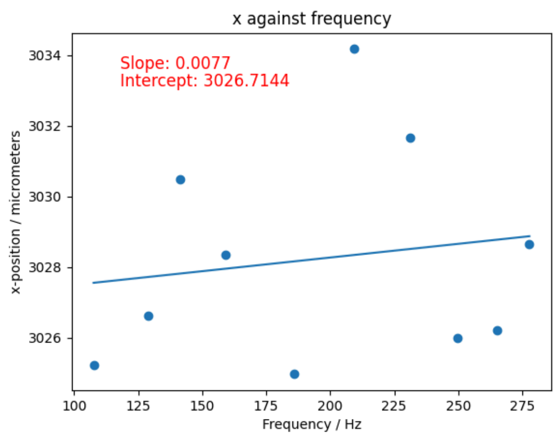

Figure 5(a): Linear fit of x-position of laser spot as a function of mirror frequency

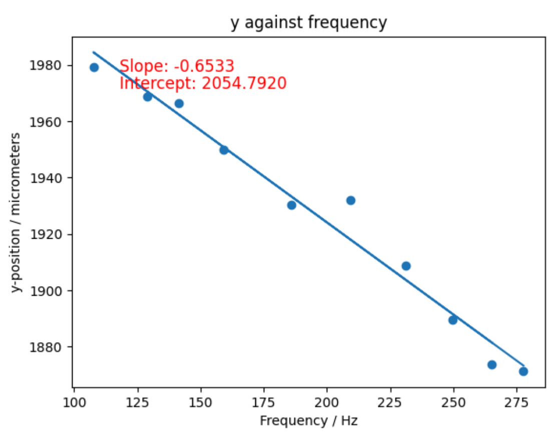

Figure 5(b): Linear fit of y-position of laser spot as a function of mirror frequency

Given that the position x of the laser spot is measured in pixels, we convert it into micrometers using the linear size of the pixel, which is 4.8 μm. Then, Figures 5(a) and 5(b) depicting the dependencies of horizontal and vertical position of the laser spot on mirror frequency are plotted. The positions of the laser spot are fit with a least-squares linear function $x = A_x f + B_x$ and $y = A_y f + B_y$. A_x, A_y, B_x and B_y, along with their uncertainties (rounded to 2 significant figures), are reported in Table 3. Numerical data for the x-positions and y-positions of the laser spot at varying mirror frequencies is reported in Table 2.

Power percentage / %	Mirror frequency, f / Hz	Frequency uncertainty, σ_f / Hz	x-position of laser spot / μm	y-position of laser spot / μm
7	107.6	0.2	3025.21165135	1979.14448343
8	128.8	0.2	3026.61190391	1968.87703752
9	141.2	0.2	3030.49036696	1966.38181598
10	159.1	0.2	3028.35355429	1949.86781224
12	185.9	0.2	3024.96189921	1930.4282929
14	209.2	0.2	3034.16897905	1932.05573202
16	231.2	0.2	3031.6594307	1908.74793674
18	249.8	0.2	3025.98815072	1889.47622851
20	265.3	0.2	3026.2174147	1873.878016
22	277.8	0.2	3028.63897024	1871.25860361

Table 2: Power percentages and corresponding mirror frequencies and their uncertainties, corresponding x-positions and y-positions of the laser spot

Parameters	x-position against frequency	y-position against frequency
Slope ($A_{x/y}$) / μm/Hz	0.0077	-0.6533
Slope ($A_{x/y}$) uncertainty / μm/Hz	0.2	0.036
Intercept ($B_{x/y}$) / μm	3027	2055
Intercept ($B_{x/y}$) uncertainty / μm	3.6	7.4

Table 3: Fitting parameters and uncertainties for x-position and y-position of the laser spot

We note that our CCD camera sensor was rotated 90° anticlockwise from its intended orientation, and we therefore expect the *y*-position of the laser spot to decrease with increasing mirror frequency and the *x*-position of the laser spot to remain relatively constant with increasing mirror frequency. Therefore, the correlation between *y*-position of the laser spot and mirror frequency that we observe in Figure 5(b) is in accordance with our predictions. From Figures 5(a) and 5(b), we see that the *y*-position of the laser spot decreases with increasing mirror frequency, whereas the *x*-position of the laser spot remains relatively constant. We conclude that this is so because under the assumption that the fluctuations in the *x*-position of the laser spot obey a Gaussian distribution and using the standard deviation as the uncertainty, the probability that the true mean of the x-position is within one true standard deviation from the measured value of 0.0077 μm/Hz is 68%. The slope of the fitted line for the *x*-position of the laser spot is 0.0077 μm/Hz and therefore, the probability that the true slope is 0 μm/Hz is significant. On the other hand, under similar assumptions for the *y*-position of the laser spot, the probability that the true slope is 0 is insignificant, since 0 μm/Hz lies more than 5 standard deviations away from the measured value of -0.6533 μm/Hz. Therefore, the *y*-position of the laser spot has significant correlation with the mirror frequency.

From our previously defined variables, we calculate the value of *c* as follows:

Given values:

$$r = 1.01\,\mathrm{m}$$

$$d = 1.27\,\mathrm{m} + 0.722\,\mathrm{m} + 5.85\,\mathrm{m}$$

$$b = -0.6533 \cdot 10^{-6}$$

From (1),

$$x = \frac{8\pi r d f}{c}$$

Thus:

$$\therefore b = \frac{8\pi r d}{c} \qquad \therefore c = \frac{8\pi r d}{b} = \frac{8\pi(1.01)(1.27 + 0.722 + 5.85)}{-0.6533 \cdot 10^{-6}} = -3.04702076 \cdot 10^{8}\,\mathrm{m/s}$$

We can ignore the negative sign and simply take the absolute value as our value of *c* since it is trivially due to the orientation of our CCD camera and choice of direction. Therefore, our final value of *c*, 3.04702076×10^8 m/s, is consistent with the accepted value of approximately 3.0×10^8 m/s. We calculate our uncertainty in *c* via the error propagation formula as follows:

The formula for the uncertainty in c, denoted as σ_c^2, is:

$$\sigma_c^2 = \left(\frac{\partial c}{\partial r}\sigma_r\right)^2 + \left(\frac{\partial c}{\partial d}\sigma_d\right)^2 + \left(\frac{\partial c}{\partial b}\sigma_b\right)^2 \tag{2}$$

Substituting the partial derivatives:

$$\sigma_c^2 = \left(\frac{8\pi d}{b} \cdot \sigma_r\right)^2 + \left(\frac{8\pi r}{b} \cdot \sigma_d\right)^2 + \left(-\frac{8\pi r d}{b^2} \cdot \sigma_b\right)^2$$

Simplifying: $\sigma_c^2 = \frac{(8\pi d)^2}{b^2}\sigma_r^2 + \frac{(8\pi r)^2}{b^2}\sigma_d^2 + \frac{(8\pi r d)^2}{b^4}\sigma_b^2 =$

$$\left(\tfrac{5}{1000}\right)^2 \left(\frac{8\pi(1.27+0.722+5.85)}{-0.6533 \cdot 10^{-6}}\right)^2 + \left(0.036 \cdot 10^{-6}\right)^2 \left(\frac{8\pi(1.01)(1.27+0.722+5.85)}{(0.6533 \cdot 10^{-6})^2}\right)^2 +$$
$$\left(\left(\tfrac{7.5}{1000}\right)^2 + \left(\tfrac{10}{1000}\right)^2 + \left(\tfrac{10}{1000}\right)^2\right)\left(\frac{8\pi(1.01)}{-0.6533 \cdot 10^{-6}}\right)^2 = 2.84585218 \times 10^{14}$$

$\sigma_c = \sqrt{2.84585218 \times 10^{14}} \approx 1.69 \times 10^7$

With all values rounded to 2 significant figures, the contribution from r is $|\frac{8\pi d}{b} \cdot \sigma_r| = 1.5 \times 10^6$. The contribution from d is $\frac{8\pi r}{b} \cdot \sigma_d = 6.2 \times 10^5$. The contribution from b is $|-\frac{8\pi r d}{b^2} \cdot \sigma_b| = 1.68 \times 10^7$.

It is evident that the major contributor to the uncertainty in the speed of light *c* is *b*. We note that the actual uncertainty in *b*, σ_b, is not in fact large. What causes the contribution of *b* to be large is the nature of the expression of the contribution by *b*, as well as the order of magnitude of *b* itself. *b* indicates the relationship between the *y*-position of the laser spot and mirror frequency. As indicated above, the contribution by *b* is given as

$$\left|-\frac{8\pi r d}{b^2} \cdot \sigma_b\right|$$

b is relatively small as it is calculated in μm/Hz. Furthermore, its squared value is in the denominator of the above expression, a small value of *b* contributes to the large magnitude of the uncertainty of *b*, and consequently the uncertainty in the speed of light. In order to reduce the uncertainty in *b*, we can use longer geometric distances in our experimental set-up, such that we increase the distance between the beam splitter and the CCD camera sensor, as well as the distance between the rotating mirror and the beam splitter. This allows the return ray reflected off the rotating mirror to travel a longer distance and therefore deviate further from the forward beam before hitting the CCD camera sensor. This will increase the order of magnitude of the *y*-position of the laser spot and *b* will consequently be larger in magnitude, thus reducing the contribution to the uncertainty in the speed of light by *b*. To reduce the uncertainty in *b*, it is also possible to check for any outliers in our data and remove them, although we did not find any in this experiment.

We now analyze if the uncertainty in the slope comes solely from uncertainty in the mirror frequency. We first estimate the expected uncertainty in the y-position of the laser spot σ_y the only source of the. Applying (2) to the function $y = A_y f + B_y$, the linear function used to fit the data, we obtain:

$$\delta_y^2 = (A_y \cdot \delta_f)^2$$

δ_y is the uncertainty in the y-position of the laser spot due to only the uncertainty in the mirror frequency. δ_f is the characteristic uncertainty in the mirror frequency.

From Table 2, $\delta_f = 0.2$ Hz. From Table 3, $A_y = -0.6533\ \mu m/\text{Hz}$.

$$\delta_y^2 = (0.2)^2 (0.6533)^2 = 0.0170720356\ \mu m^2$$

$$\therefore\ \delta_y = 0.13\ \mu m \text{ (rounded to 2 significant figures)}$$

We calculate the observed standard deviation of the data with respect to the linear function that is used to fit our data, $y = A_y f + B_y$, using (3):

$$B_y = 2055\ \mu m$$

$$\sigma_y^2 = \frac{1}{N-2} \sum_{i=1}^{N} (y_i - (A_y \cdot f_i + B_y))^2 \tag{3}$$

, where f_i are the mirror frequencies in Table 2, and N is the number of frequency measurements (10 in our case), and we obtain $\sigma_y = 6.5\ \mu m$ (rounded to 2 significant figures).

The observed standard deviation of the data is larger than the uncertainty in the *y*-position of the laser spot due to only the uncertainty in the mirror frequency calculated using (3). As such, the observed fluctuations in the *y*-position of the laser spot, σ_y, are not solely the result of fluctuations in mirror frequency.

It is likely that there are other significant sources of uncertainty due to experimental error affecting the position apart from the uncertainty in the *y*-position of the laser spot due to only the uncertainty in the mirror frequency calculated using (3). One possible source of error could be our slightly inaccurate measurements of the geometric distances, as reported in Table 1 and different from our theoretically calculated values using the thin lens equation, in our experimental set-up. The difference in the distance at which we observed the laser spot to be focused on the return mirror could have been affected by slight wear and tear or inappropriate manual handling of the convex lens, and this may have contributed to additional uncertainty not due to the uncertainty in the mirror frequency. Our usage of an intermediate mirror due to the size constraints of the room in which the experiment was conducted could have also been an additional source of uncertainty. As *d* was measured in three separate intervals (from the rotating mirror to the intermediate mirror, from the convex lens to the intermediate mirror, and from the convex lens to the return mirror), measuring the entire length as a whole would have reduced the uncertainty in *d*. As such the experiment could have been carried out in a larger room with a width larger than 6m to eliminate the need for an intermediate mirror.

It is also plausible to carry out the experiment in a dark room to reduce the effect of interference from external light sources that may have increased the uncertainty in the measured *y*-position of the laser beam. Our usage of a center-of-mass function to calculate the *y*-position of the laser spot involved the use of a relatively low threshold of a pixel R channel value of 100 and this might not have eliminated sufficiently dim regions of the captured images, resulting in the calculated center of mass being slightly inaccurate. A higher threshold could have been used.

4 Conclusion

In this experiment, we successfully determine a value for the speed of light in air that is close to the generally accepted value of 3.0×10^8 m/s.

5 Appendix

Code

```
import numpy as np
from PIL import Image
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
from scipy.ndimage import center_of_mass
import os

freq_list = [277.8, 107.6, 128.8, 141.2, 159.1, 185.9, 209.2, 231.2,
249.8, 265.3]

positions = []
image_directory = 'C:/Users/Admin/Downloads/newimages'
for image_filename in os.listdir(image_directory):
    image_path = os.path.join(image_directory, image_filename)
    LaserSpotImage = Image.open(image_path)
    LaserSpotData = np.array(LaserSpotImage)
    data = LaserSpotData[:, :, 0]
    threshold = 100
    bit_mask = np.where(data > threshold, data, 0)
    position = center_of_mass(bit_mask)

    '''
    image = mpimg.imread(image_path)
    plt.scatter(position[0], position[1])
    plt.imshow(image)
    plt.show()
    '''

    position = tuple((pos * 4.8) for pos in position)
    positions.append(position)

positions = np.array(positions)
print(positions)
plt.scatter(positions[:, 1], positions[:, 0])
plt.title("Test")
plt.show()
```

```
plt.scatter(freq_list, positions[:, 1])
coeffs_x, covmat_x = np.polyfit(freq_list, positions[:, 1], 1, cov=True)
squared_std_slope_x = covmat_x[0, 0]
squared_std_intercept_x = covmat_x[1, 1]
print("Uncertainty squared, slope and intercept, x: ",
squared_std_slope_x, squared_std_intercept_x)
plt.plot(np.unique(freq_list), np.poly1d(np.polyfit(freq_list,
positions[:, 1], 1))(np.unique(freq_list)))
plt.title("x against frequency")
plt.text(0.1, 0.9, f'Slope: {coeffs_x[0]:.4f}',
transform=plt.gca().transAxes, color='red', fontsize=12)
plt.text(0.1, 0.85, f'Intercept: {coeffs_x[1]:.4f}',
transform=plt.gca().transAxes, color='red', fontsize=12)
plt.xlabel("Frequency / Hz")
plt.ylabel("x-position / micrometers")
plt.show()

plt.scatter(freq_list, positions[:, 0])
coeffs_y, covmat_y = np.polyfit(freq_list, positions[:, 0], 1, cov=True)
squared_std_slope_y = covmat_y[0, 0]
squared_std_intercept_y = covmat_y[1, 1]
print("Uncertainty squared, slope and intercept, y: ",
squared_std_slope_y, squared_std_intercept_y)
plt.plot(freq_list, np.poly1d(np.polyfit(freq_list, positions[:, 0],
1))(freq_list))
plt.text(0.1, 0.9, f'Slope: {coeffs_y[0]:.4f}',
transform=plt.gca().transAxes, color='red', fontsize=12)
plt.text(0.1, 0.85, f'Intercept: {coeffs_y[1]:.4f}',
transform=plt.gca().transAxes, color='red', fontsize=12)
plt.xlabel("Frequency / Hz")
plt.ylabel("y-position / micrometers")
plt.title("y against frequency")
plt.show()
```