Study Notes

Yangyang Li
yangyang.li@northwestern.edu

Update on August 11, 2023

Contents

CONTENTS

List of Figures

List of Theorems

List of Definitions

LIST OF DEFINITIONS

Acronyms

MAP Maximum A Posteriori Estimation 13
MLE Maximum Likelihood Estimation 13, 14

pdf Probability Density Function 14

Acronyms

Preface

Contents

0.1 Features of this template

> *TeX, stylized within the system as LaTeX, is a typesetting system which was designed and written by Donald Knuth and first released in 1978. TeX is a popular means of typesetting complex mathematical formulae; it has been noted as one of the most sophisticated digital typographical systems.*
>
> \- Wikipedia

0.1.1 crossref

different styles of clickable definitions and theorems

- nameref: Gaussian distribution
- autoref: Definition A.1, **??**
- cref: Definition A.1,
- hyperref: Gaussian,

0.1.2 ToC (Table of Content)

- mini toc of sections at the beginning of each chapter
- list of theorems, definitions, figures
- the chapter titles are bi-directional linked

0.1.3 header and footer

fancyhdr

- right header: section name and link to the beginning of the section
- left header: chapter title and link to the beginning of the chapter
- footer: page number linked to ToC of the whole document

0.1.4 bib

- titles of reference is linked to the publisher webpage e.g., [Kit+02]
- backref (go to the page where the reference is cited) e.g., [Chi09]
- customized video entry in reference like in [Bab16]

0.1.5 preface, index, quote (epigraph) and appendix

index page at the end of this document...

Part I

Machine Learning

Chapter 1

Probability

Contents

1.1 Maximum Likelihood Estimation

$$\boldsymbol{X} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N)^{\mathrm{T}}, \boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{ip})^{\mathrm{T}} \tag{1.1}$$

in which N is the number of samples, p is the number of features. The data is sampled from a distribution $p(\boldsymbol{x} \mid \theta)$, where θ is the parameter of the distribution.

For N i.i.d. samples, the likelihood function is $p(\boldsymbol{X} \mid \theta) = \prod_{i=1}^{N} p(\boldsymbol{x}_i \mid \theta))$

In order to get θ, we use Maximum Likelihood Estimation (MLE) to maximize the likelihood function.

$$\theta_{\texttt{MLE}} = \underset{\theta}{\operatorname{argmax}} \log p(\boldsymbol{X} \mid \theta) = \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^{N} \log p(\boldsymbol{x}_i \mid \theta) \tag{1.2}$$

1.2 Maximum A Posteriori Estimation

In Bayes' theorem, the θ is not a constant value, but $\theta \sim p(\theta)$. Hence,

$$p(\theta \mid \boldsymbol{X}) = \frac{p(\boldsymbol{X} \mid \theta)p(\theta)}{p(\boldsymbol{X})} = \frac{p(\boldsymbol{X} \mid \theta)p(\theta)}{\int_{\theta} p(\boldsymbol{X} \mid \theta)p(\theta)d\theta} \tag{1.3}$$

In order to get θ, we use Maximum A Posteriori Estimation (MAP) to maximize the posterior function.

$$\theta_{\texttt{MAP}} = \underset{\theta}{\operatorname{argmax}}\, p(\theta \mid \boldsymbol{X}) = \underset{\theta}{\operatorname{argmax}} \frac{p(\boldsymbol{X} \mid \theta)p(\theta)}{p(\boldsymbol{X})} \tag{1.4}$$

After θ is estimated, then calculating $\frac{p(\boldsymbol{X} \mid \theta) \cdot p(\theta)}{\int_{\theta} p(\boldsymbol{X} \mid \theta)p(\theta)d\theta}$ to get the posterior distribution. We can use the posterior distribution to predict the probability of a new sample $\boldsymbol{x}$.

$$p(x_{\texttt{new}} \mid \boldsymbol{X}) = \int_{\theta} p(x_{\texttt{new}} \mid \theta) \cdot p(\theta \mid \boldsymbol{X})d\theta \tag{1.5}$$

1.3 Gaussian Distribution

Gaussian distribution is also called normal distribution.

$$\theta = (\mu, \sigma^2), \quad \mu = \frac{1}{N}\sum_{i=1}^{N} x_i, \quad \sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2 \tag{1.6}$$

For MLE,

$$\theta = (\mu, \Sigma) = (\mu, \sigma^2), \quad \theta_{\texttt{MLE}} = \operatorname*{argmax}_{\theta} \log p(\boldsymbol{X} \mid \theta) = \operatorname*{argmax}_{\theta} \sum_{i=1}^{N} \log p(\boldsymbol{x}_i \mid \theta) \tag{1.7}$$

Generally, the Probability Density Function (pdf) of a Gaussian distribution is:

$$p(x \mid \mu, \Sigma) = \frac{1}{\sqrt{(2\pi)^p \det(\Sigma)}} \exp\left(-\frac{1}{2}(\boldsymbol{x}-\mu)^{\mathrm{T}}\Sigma^{-1}(\boldsymbol{x}-\mu)\right) \tag{1.8}$$

in which μ is the mean vector, Σ is the covariance matrix, $\det$ is the determinant of matrix. $\det$ is the product of all eigenvalues of a matrix.

Hence,

$$\log p(\boldsymbol{X} \mid \theta) = \sum_{i=1}^{N} \log p(x_i \mid \theta) = \sum_{i=1}^{N} \log \frac{1}{\sqrt{(2\pi)^p \det(\Sigma)}} \exp\left(-\frac{1}{2}(\boldsymbol{x}-\mu)^{\mathrm{T}}\Sigma^{-1}(\boldsymbol{x}-\mu)\right) \tag{1.9}$$

Let's only consider 1 dimension case for brevity, then

$$\log p(\boldsymbol{X} \mid \theta) = \sum_{i=1}^{N} \log p(x_i \mid \theta) = \sum_{i=1}^{N} \log \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right) \tag{1.10}$$

Let's get the optimal value for μ,

$$\mu_{\texttt{MLE}} = \operatorname*{argmax}_{\mu} \log p(\boldsymbol{X} \mid \theta) = \operatorname*{argmax}_{\mu} \sum_{i=1}^{N} (x_i - \mu)^2 \tag{1.11}$$

So,

$$\frac{\partial \log p(\boldsymbol{X} \mid \theta)}{\partial \mu} = \sum_{i=1}^{N} -2\,(x_i - \mu) = 0 \rightarrow \mu_{\texttt{MLE}} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{1.12}$$

Part II

Algorithm and Data Structure

Chapter 2

Algorithm

Contents

2.1 Graph

Part III

Programming

Chapter 3

C++

Chapter 4

Rust

Part IV

Research

Chapter 5

Paper Reading

Appendix A

Formulas

A.1 Gaussian distribution

Definition A.1 (Gaussian distribution)**.** *Gaussian distribution*

Theorem A.1 (Central limit theorem)**.**

Bibliography

[Bab16] László Babai. "Graph Isomorphism in Quasipolynomial Time". Jan. 19, 2016. arXiv: `1512.03547 [cs, math]` (cit. on p. 10). Online video

[Chi09] Andrew M. Childs. *Universal Computation by Quantum Walk*. Physical Review Letters 102.18 (May 4, 2009), p. 180501. arXiv: `0806.1972` (cit. on p. 10).

[Kit+02] Alexei Yu Kitaev et al. *Classical and quantum computation*. 47. American Mathematical Soc., 2002 (cit. on p. 10).

BIBLIOGRAPHY

Alphabetical Index