ISOC 731-Project

Network formation games

In this project we will investigate the emergence of networks as a non centralised behaviour. Let's assume that we have a set of N peoples $\{1, \ldots, n\}$ or entities, (companies, etc…) who are making connexion for benefit. We will investigate several increasingly realistic scenario in this project and look at the resulting networks properties.

Prerequisite : for doing the project we will be using graph processing library. You can use python based networkx or graphstream en java ou Boost::graph en C++. For visualising graph we will use the Gephi environment. So first step is to look at these two tools.

1- **Random graph-** let's assume that each entity v decides uniformly and randomly to connect to any other entity, *i.e.,* with a fix probability p an entity $v \in \{1, \ldots, n\}$ connects to any other of the N-1 other entities. When p is very small the resulting graph is not connected, *i.e.*, we have a large number of small disconnected clusters. With increasing p there is more connectivity and when p=1 all nodes are connected to each other (we have a clique). Can you observe a minimal value of p where all entities are connected by a graph. In order to observe this, for all value of p (from 0.01 to 0.99 with a 0.01 step) *,* generate 100 graphs over 10 000 nodes and look at the size of the largest connected component (use networkx library in python for that purpose). Plot a graph with the percentage of the nodes in the largest component. Do you observe an interesting behaviour ?
Show also the evolution of the node degree distribution. Can you guess the degree distribution ?
Find interesting visualisations for being able to show the evolution with the confidence intervals.

2-**Simple graph formation game-** let's assume that the value of an entity $v \in \{1, \ldots, n\}$*,* $N(v)$, is evaluated as the sum of the values of its direct neighbouring entities in the graph. The value of a non-connected node is 0 and the value of a node connected only to a single other node is 1. Let's assume the following game. Different entities choose each a random time T_v. At its time the entity T_v looks at all order nodes and connect to the one that will maximises its value (it choose randomly in there is egality). Could you predict in advance what will be the resulting topology ? Which entity will get the largest value ? Develop a simulator that will simulate this simple graph formation game. Compare the resulting graph, the connectivity, the minimal distance between points, *etc.* with the random graphs.

We will make this model a little more realistic by assuming that at the time of playing the connection game you cannot infer precisely the value of all nodes the value that we infer is uniformly distributed between 50% and 150% of its real value. When the connection is made the real value of neighbours revealed to both entities. What will change in the topology ? Do the topology change in between different runs ? What will be the distribution of node value at the end of the execution? You might need several run to get a stable distribution. Plot some of the resulting graph topologies.

3- **More advanced graph formation game-** Now we will make the model more interesting and more realistic. In previous part we assumed that when a link between two entities is made it is never severed. Now we will assume that entities might behave badly. They might decide to cut the link to some entities so a node can cooperate, or defect. In the first round of the game we will play the more realistic version of previous game. In the second round each entity is connected to some other entities. Now he might decide to continue to cooperate and stay connected to all previous links. Or it might decide to defect and choose another node to connect to. However this will results into bad karma. We assume that each node will have a karma value equal to the percentage of time it has defected in the past.. Design a new value evaluation that will use this karma value to decide to connect to another node. Test if your method can results into a stable connected graph, *i.e.*, the resulting graph is connecting a very large percentage of entities all the time. Make some analysis of the distribution of entity value, of its karma, etc.

4- **becoming a good king-** Could you devise a strategy that will ensure that a given entity will become among the 10 largest value entity ? Is your strategy robust toward bad entities behaviour ? Make a discussion about behaviour and perspectives in graph evolution.