

DAGs Review

INFO/STSCI/ILRST 3900: Causal Inference

2 Oct 2023

Agenda

- ▶ Reminders and Announcements
- ▶ Review: open/closed paths and independence in DAGs
- ▶ Example from lecture
- ▶ Practice with DAGs

Reminders and Announcements

- ▶ Peer reviews- make sure to review all you're assigned
- ▶ Task 1 is **due tomorrow**, Oct 3rd by 11:59pm
- ▶ In-class assignments:
 - ▶ Notify us in advance if you miss a class
 - ▶ Otherwise, get one free pass
 - ▶ After that, without advance notice, 0 on assignment
- ▶ You get **5** flex days, use them!
- ▶ Office hours:
 - ▶ Filippo: Monday 11am-12pm in Comstock 1187
 - ▶ Shira: Tuesday 3-4pm in Comstock 1187
- ▶ Check Ed for announcements and use for HW help!

Review

How to check if a *path* is **open** or **blocked**:

1. Traverse the path node by node
2. If any node is blocked, the entire path is blocked
3. If all nodes are open, then entire path is open

Review

How to check if a *path* is **open** or **blocked**:

1. Traverse the path node by node
2. If any node is blocked, the entire path is blocked
3. If all nodes are open, then entire path is open

How to check if a *node* is **open** or **blocked**:

- ▶ If **collider** ($\rightarrow X \leftarrow$):
 - ▶ Open if it or any of its descendants are in the conditioning set
 - ▶ Otherwise it is blocked
- ▶ If **non-collider** ($\rightarrow X \rightarrow$ or $\leftarrow X \leftarrow$ or $\leftarrow X \rightarrow$):
 - ▶ Blocked if it is in the conditioning set
 - ▶ Otherwise it is open

Review

How to check if a *path* is **open** or **blocked**:

1. Traverse the path node by node
2. If any node is blocked, the entire path is blocked
3. If all nodes are open, then entire path is open

How to check if a *node* is **open** or **blocked**:

- ▶ If **collider** ($\rightarrow X \leftarrow$):
 - ▶ Open if it or any of its descendants are in the conditioning set
 - ▶ Otherwise it is blocked
- ▶ If **non-collider** ($\rightarrow X \rightarrow$ or $\leftarrow X \leftarrow$ or $\leftarrow X \rightarrow$):
 - ▶ Blocked if it is in the conditioning set
 - ▶ Otherwise it is open

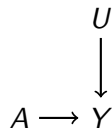
Two variables are dependent (statistically associated) if there exists an unblocked path between them. They are independent if all paths between them are blocked.

DAGs help us reason about exchangeability

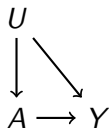
Procedure

- 1) List all paths between A to Y
- 2) Cross out the blocked paths
- 3) Exchangeability holds if all remaining paths are causal

DAG 1



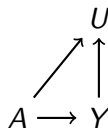
DAG 2



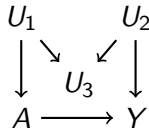
DAG 3



DAG 4



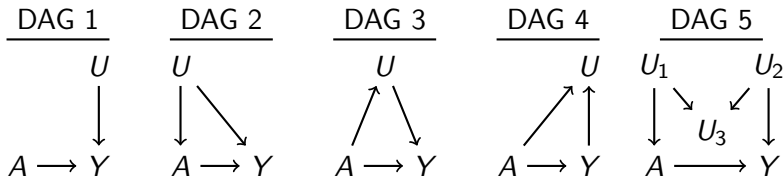
DAG 5



DAGs help us reason about exchangeability

Procedure

- 1) List all paths between A to Y
- 2) Cross out the blocked paths
- 3) Exchangeability holds if all remaining paths are causal



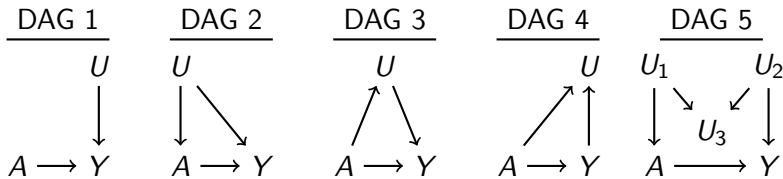
1)

$$A \rightarrow Y \quad \left| \quad A \rightarrow Y \right. \quad \left| \quad A \rightarrow Y \right. \quad \left| \quad A \rightarrow Y \right. \quad \left| \quad A \rightarrow Y \right.$$
$$A \leftarrow U \rightarrow Y \quad \left| \quad A \rightarrow U \rightarrow Y \right. \quad \left| \quad A \rightarrow U \leftarrow Y \right. \quad \left| \quad A \leftarrow U_1 \rightarrow U_3 \leftarrow U_2 \rightarrow Y \right.$$

DAGs help us reason about exchangeability

Procedure

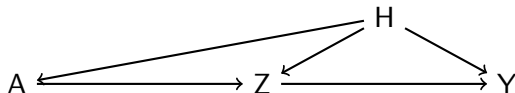
- 1) List all paths between A to Y
- 2) Cross out the blocked paths
- 3) Exchangeability holds if all remaining paths are causal



- 1)
- $$A \rightarrow Y \quad \left| \quad A \rightarrow Y \quad \left| \quad A \rightarrow Y \quad \left| \quad A \rightarrow Y \quad \left| \quad A \rightarrow Y \right. \right. \right. \\ \left. \left. \left. \begin{array}{l} A \leftarrow U \rightarrow Y \\ A \rightarrow U \rightarrow Y \\ A \rightarrow U \leftarrow Y \\ A \leftarrow U_1 \rightarrow U_3 \leftarrow U_2 \rightarrow Y \end{array} \right. \right. \right.$$
- 2)
- $$A \rightarrow Y \quad \left| \quad A \rightarrow Y \quad \left| \quad A \rightarrow Y \quad \left| \quad A \rightarrow Y \quad \left| \quad A \rightarrow Y \right. \right. \right. \\ \left. \left. \left. \begin{array}{l} A \leftarrow U \rightarrow Y \\ A \rightarrow U \rightarrow Y \end{array} \right. \right. \right.$$

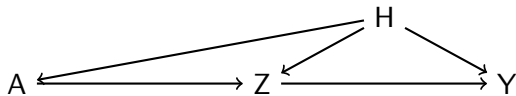
Practice Identifying Paths

Go through this exercise in groups.

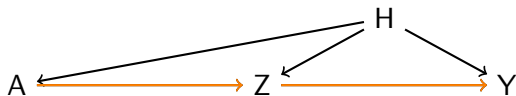


- ▶ What are the paths from A to Y ?
- ▶ Determine if each of the paths is causal or non-causal
- ▶ Determine whether each node on each path is a collider or non-collider

Practice



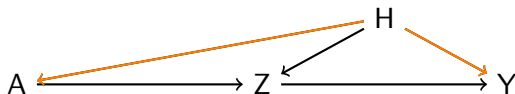
Practice



► $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$

causal path

Practice



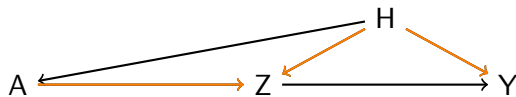
$$\blacktriangleright A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$$

causal path

$$\blacktriangleright A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$$

non-causal

Practice



$$\blacktriangleright A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$$

causal path

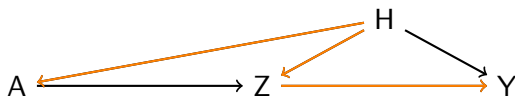
$$\blacktriangleright A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$$

non-causal

$$\blacktriangleright A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$$

non-causal

Practice



$$\blacktriangleright A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$$

causal path

$$\blacktriangleright A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$$

non-causal

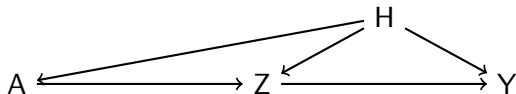
$$\blacktriangleright A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$$

non-causal

$$\blacktriangleright A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$$

non-causal

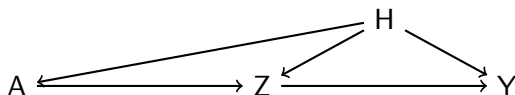
Practice: Is the path open or blocked?



If we condition on $L = \emptyset$, which paths are open? Which paths are blocked?

- ▶ $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$
- ▶ $A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$

Practice: Is the path open or blocked?



If we condition on $L = \emptyset$, which paths are open? Which paths are blocked?

$$\blacktriangleright A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$$

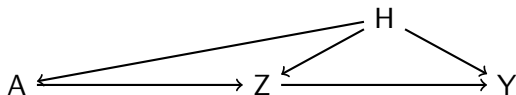
Open

$$\blacktriangleright A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$$

$$\blacktriangleright A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$$

$$\blacktriangleright A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$$

Practice: Is the path open or blocked?



If we condition on $L = \emptyset$, which paths are open? Which paths are blocked?

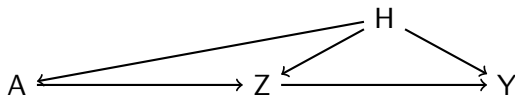
► $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$ Open

► $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$ Open

► $A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$

► $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$

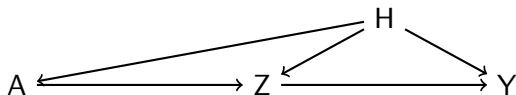
Practice: Is the path open or blocked?



If we condition on $L = \emptyset$, which paths are open? Which paths are blocked?

- ▶ $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$ Open
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$ Open
- ▶ $A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$ Blocked
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$

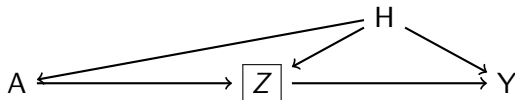
Practice: Is the path open or blocked?



If we condition on $L = \emptyset$, which paths are open? Which paths are blocked?

- ▶ $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$ Open
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$ Open
- ▶ $A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$ Blocked
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$ Open

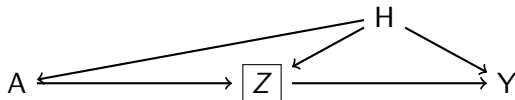
Practice: Is the path open or blocked?



If we condition on $L = \{Z\}$, which paths are open? Which paths are blocked?

- ▶ $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$
- ▶ $A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$

Practice: Is the path open or blocked?



If we condition on $L = \{Z\}$, which paths are open? Which paths are blocked?

$$\blacktriangleright A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$$

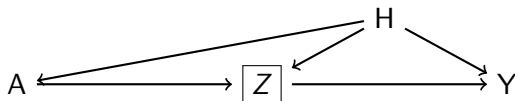
Blocked

$$\blacktriangleright A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$$

$$\blacktriangleright A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$$

$$\blacktriangleright A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$$

Practice: Is the path open or blocked?



If we condition on $L = \{Z\}$, which paths are open? Which paths are blocked?

$$\blacktriangleright A \rightarrow \underbrace{\boxed{Z}}_{\text{NC}} \rightarrow Y$$

Blocked

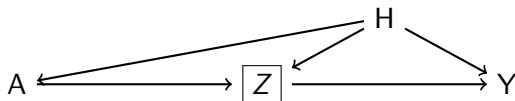
$$\blacktriangleright A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$$

Open

$$\blacktriangleright A \rightarrow \underbrace{\boxed{Z}}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$$

$$\blacktriangleright A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow \underbrace{\boxed{Z}}_{\text{NC}} \rightarrow Y$$

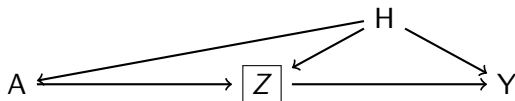
Practice: Is the path open or blocked?



If we condition on $L = \{Z\}$, which paths are open? Which paths are blocked?

- ▶ $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$ Blocked
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$ Open
- ▶ $A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$ Open
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$

Practice: Is the path open or blocked?



If we condition on $L = \{Z\}$, which paths are open? Which paths are blocked?

- ▶ $A \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$ Blocked
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$ Open
- ▶ $A \rightarrow \underbrace{Z}_{\text{Col}} \leftarrow \underbrace{H}_{\text{NC}} \rightarrow Y$ Open
- ▶ $A \leftarrow \underbrace{H}_{\text{NC}} \rightarrow \underbrace{Z}_{\text{NC}} \rightarrow Y$ Blocked