Defining causal effects

STSCI / INFO / ILRST 3900: Causal Inference

28 Aug 2025

Learning goals for today

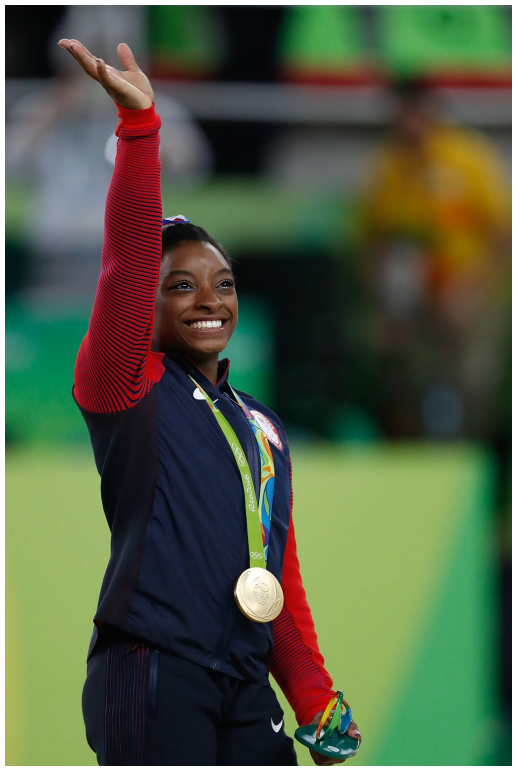
By the end of class, you will be able to

- explain the fundamental problem of causal inference and the need for causal arguments
- define potential outcomes

Logistics

- We will cover material needed for Problem Set 1 by the end of today
- Problem Set 1 due Sep 9

Causal claims hinge on arguments, not just data



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Right photo: By Agencia Brasil Fotografias - EUA levam ouro na ginástica artística feminina; Brasil fica em 8 lugar, CC BY 2.0, https://commons.wikimedia.org/w/index.php?curid=50584648

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What do we mean when we say "cause"?

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	Swing	Do not swing	of swinging
Simone Biles	Yes (1)	?	?
Sam	?	No (0)	?

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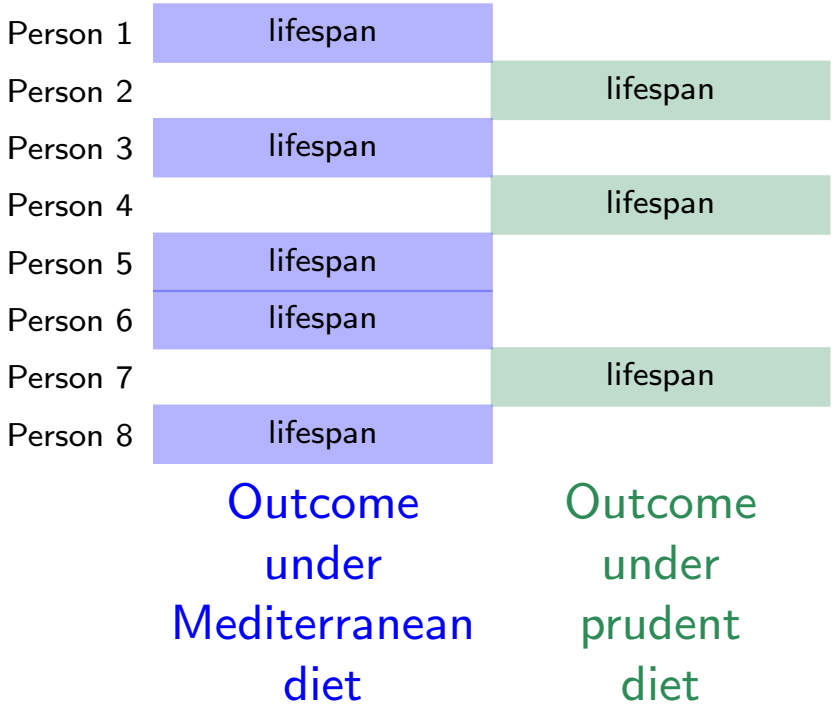
Fundamental problem of causal inference

Holland 1986

Descriptive evidence

average lifespan — average lifespan



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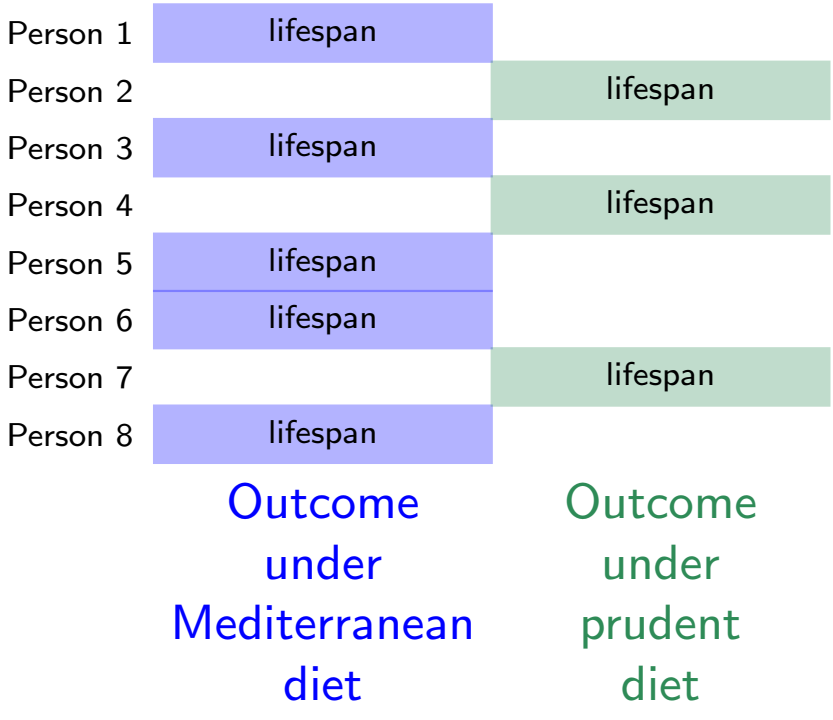

Causal claim

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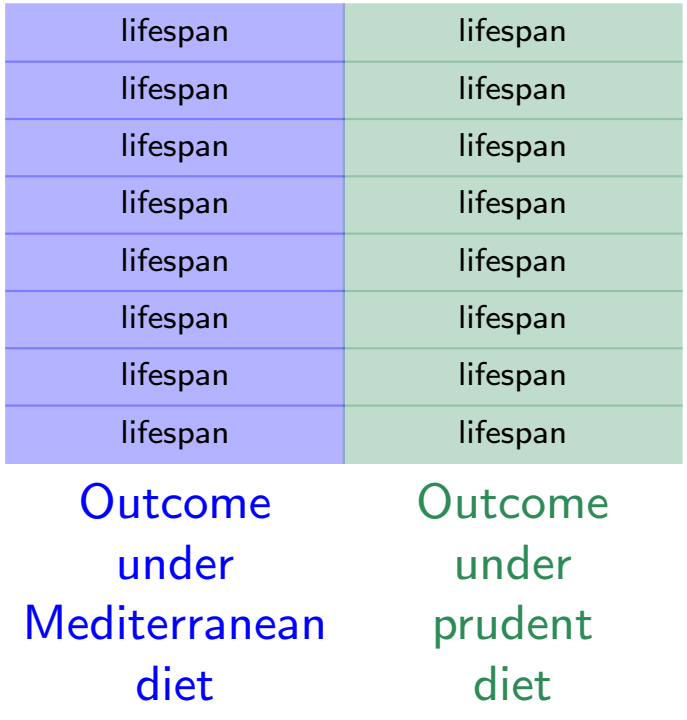

Fundamental problem of causal inference

Holland 1986

Descriptive evidence

average lifespan − average lifespan

Person 1	lifespan	missing
Person 2	missing	lifespan
Person 3	lifespan	missing
Person 4	missing	lifespan
Person 5	lifespan	missing
Person 6	lifespan	missing
Person 7	missing	lifespan
Person 8	lifespan	missing
	Outcome under Mediterranean diet	Outcome under prudent diet

Causal claim

average lifespan − average lifespan

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Causal claim

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Causal inference is a **missing data** problem

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Mathematical notation: Potential outcomes[1]

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Y_i Outcome — Whether person i survived

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$Y_{\text{Sam}} = \texttt{survived}$ We observe that Sam survived

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$Y_{\text{Sam}}^{\text{MedDiet}} = \texttt{survived}$	If Sam had been assigned a Mediterranean diet he would have survived
$Y_{\text{Sam}}^{\text{PruDiet}} = \texttt{died}$	If Sam had been assigned a prudent diet he would have died

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Practice

Using the slip of paper you received and the diet you follow, what is

- Y_i
- A_i
- Y_i^{MedDiet}
- Y_i^{PruDiet}

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- Consistency would not hold if "forcing" someone to eat a Mediterranean diet results in a different lifespan than what we observed for someone who chose the Mediterranean diet

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- Consistency implies that observed outcome for an individual who "chose" a specific treatment is same as what we would have observed if the individual was "assigned" that treatment
- Consistency would not hold if "forcing" someone to eat a Mediterranean diet results in a different lifespan than what we observed for someone who chose the Mediterranean diet
- Consistency implies treatments don't have hidden types
- Consistency would not hold if Italian olive oil resulted in a different lifespan than Greek olive oil

Practice: How would you say this in English?

We might wonder how a person's earnings relate to whether they hold a college degree

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2. $\mathsf{E}(\text{Earnings}^{\text{Degree}=\texttt{TRUE}}) > \mathsf{E}(\text{Earnings}^{\text{Degree}=\texttt{FALSE}})$

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    - Average earnings are higher among those with college degrees
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    - Average earnings are higher among those with college degrees

2. $\text{E}(\text{Earnings}^{\text{Degree}=\texttt{TRUE}}) > \text{E}(\text{Earnings}^{\text{Degree}=\texttt{FALSE}})$
    - The average earning if everyone was assigned to get a degree is higher than the average earnings if everyone was assigned to not get a degree
    - On average, getting a degree causes higher earnings

Practice:

1. On average, students who do the homework learn more than those who don't

2. On average, doing the homework causes more learning

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1. On average, students who do the homework learn more than those who don't

$$\mathsf{E}(\text{Learning} \mid \text{HW} = \texttt{TRUE}) > \mathsf{E}(\text{Learning} \mid \text{HW} = \texttt{FALSE})$$

2. On average, doing the homework causes more learning

$$\mathsf{E}(\text{Learning}^{\text{HW}=\texttt{TRUE}}) > \mathsf{E}(\text{Learning}^{\text{HW}=\texttt{FALSE}})$$

Learning goals for today

By the end of class, you will be able to

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You can now

- Read Chapter 1 of Hernán and Robins 2020
- Start Problem Set 1 (due Sep 9)