Exchangeability and Consistency

STSCI/INFO/ILRST 3900: Causal Inference

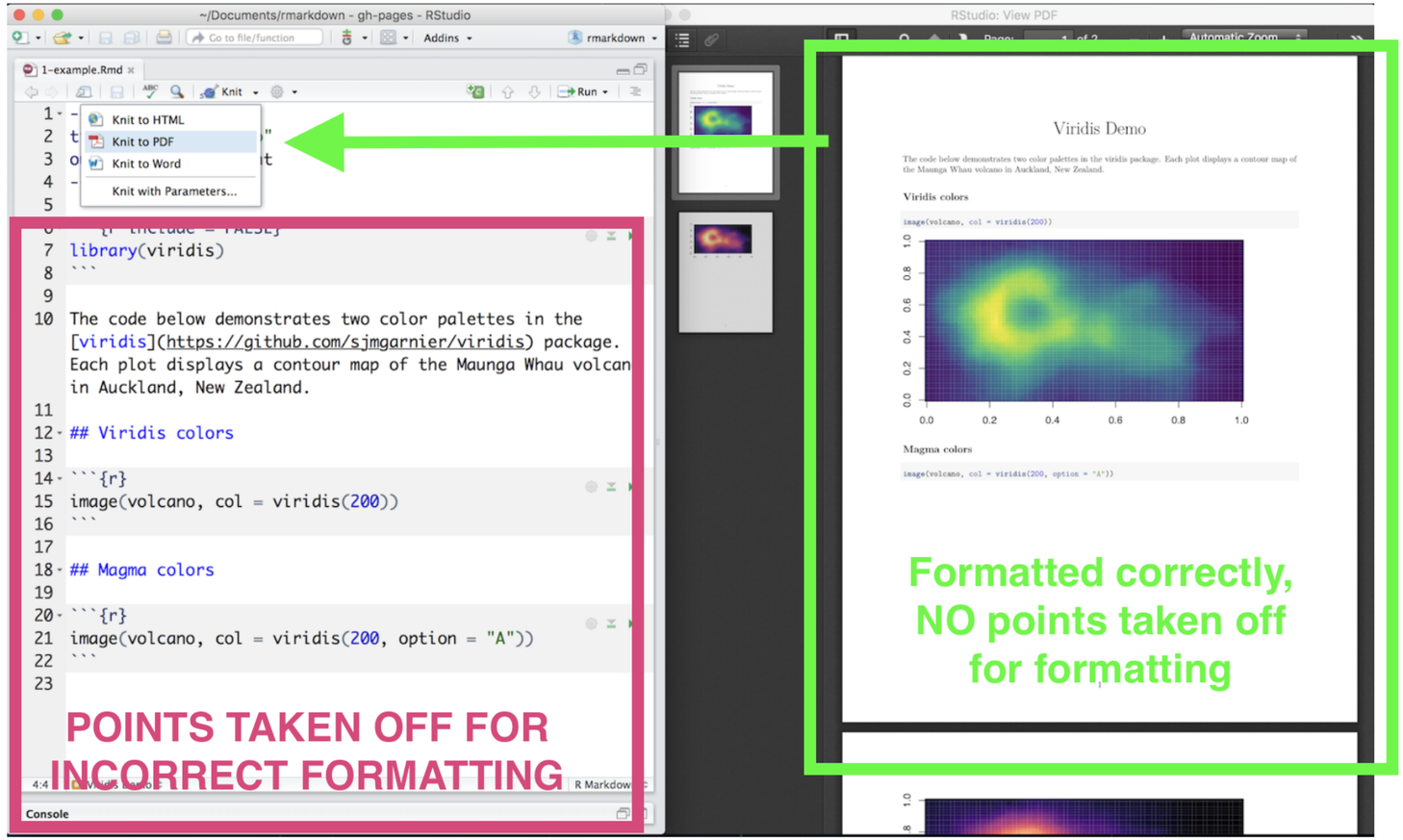
September 18, 2024

Agenda

- Reminders and Announcements
- Class activity
- Homework Check-in and Questions

Reminders and Announcements

- HW 2 due Tuesday (September 24) by 5pm
- Submit the PDF file



Reminders and Announcements

- Office Hours
  - **Filippo:** Monday 11am-12pm in Comstock 1187
  - **Shira:** Wednesday 5:30-6:30pm in in Comstock 1187
  - See Ed Discussion for Zoom links/info

Consistency and Exchangeability

- **Consistency-** the observed outcome equals the potential outcome that corresponds to the given treatment

$$Y_i = \begin{cases} Y_i^{a=1} & A_i = 1 \\ Y_i^{a=0} & A_i = 0 \end{cases}$$

- **Exchangeability**- the treatment assignment is independent of the potential outcomes

$$P(Y^a = 1 \,|\, A = 1) = P(Y^a = 1 \,|\, A = 0)$$

Class Activity

1. Fill in the blanks such that **exchangeability** holds
2. Fill in the observed outcome (Y) such that **consistency** holds
3. Calculate the quantities at the bottom (without simplifying)

Note:

$$E[Y^{a=1}] \overset{\text{exchangeability}}{=} E[Y^{a=1}|A=1] \overset{\text{consistency}}{=} E[Y|A=1]$$

	i	$Y^{a=0}$	$Y^{a=1}$	A	Y
1	Rheia	0	1	0	
2	Kronos	1	0	1	
3	Demeter	0	0	0	
4	Hades	1	0	0	
5	Hestia	0	0	1	
6	Poseidon	1	0		

For example: $E[Y^{a=0}] = 1 \times P(Y^{a=0}=1) + 0 \times P(Y^{a=0}=0) = \frac{3}{6}$

$$E[Y^{a=0}|A=0] = \ldots = \begin{cases} \frac{1}{3} & \text{if } A_6 = 1 \\ \frac{1+1}{3+1} & \text{if } A_6 = 0 \end{cases}$$

* This data is slightly different than the one in the assignment where $N = 20$

Class Activity

Table

	i	$Y^{a=0}$	$Y^{a=1}$	A	Y
1	Rheia	0	1	0	
2	Kronos	1	0	1	
3	Demeter	0	0	0	
4	Hades	0	0	0	
5	Hestia	0	0	1	
6	Poseidon	1	0	0	
7	Hera	0	0	1	
8	Zeus	0	1	1	
9	Artemis	1	1	0	
10	Apollo	1	0	0	
11	Leto	0	1		
12	Ares	1	1		
13	Athena	1	1	0	
14	Hephaestus	0	0	1	
15	Aphrodite	0	0	1	
16	Polyphemus	0	1	1	
17	Persephone	1	1	1	
18	Hermes	1	0	1	
19	Hebe	1	1	1	
20	Dionysus	1	1	1	

Class Activity

Solutions

- $E[Y^{a=0}] = 1 \times P(Y^{a=0} = 1) + 0 \times P(Y^{a=0} = 0) = 1 \times \frac{10}{20} + 0 \times \frac{10}{20} = \frac{10}{20}$
- $$E[Y^{a=0}|A = 0] = \begin{cases} \frac{4}{7} & \text{if } A_{11} = 1 \text{ and } A_{12} = 1 \\ \frac{4}{8} & \text{if } A_{11} = 0 \text{ and } A_{12} = 1 \\ \frac{5}{8} & \text{if } A_{11} = 1 \text{ and } A_{12} = 0 \\ \frac{5}{9} & \text{if } A_{11} = 0 \text{ and } A_{12} = 0 \end{cases}$$
- $A_{11} = 1$
- $A_{12} = 0$

IPW

- Standardization: constructs an estimate of $E(Y^a)$ through a weighted average
- Inverse probability weighted (IPW) estimator is equivalent to standardization
- Estimator for the population expected potential outcome

$$E(Y^a) = \frac{1}{N} \sum_{i:A_i=a} \frac{Y_i}{\pi_i^a}$$

- $\pi_i^a = P(A_i = a \mid L = \ell_i)$ is the probability of the observed treatment conditioning on confounders
- N is the total number of observations

$$\widehat{ATE}_{\text{IPW}} = \frac{1}{N} \sum_{i:A_i=1} \frac{Y_i}{\pi_i^1} - \frac{1}{N} \sum_{i:A_i=0} \frac{Y_i}{\pi_i^0}$$

IPW

Exercise

$$E(Y^{a=1}) = \frac{1}{N} \sum_{i:A_i=1} Y_i/\pi_i^1 \qquad \pi_i^1 = Pr(A_i = 1 \mid L = l_i)$$

$$E(Y^{a=0}) = \frac{1}{N} \sum_{i:A_i=0} Y_i/\pi_i^0 \qquad \pi_i^0 = Pr(A_i = 0 \mid L = l_i)$$

Name	L	A	Y
Rheia	0	0	0
Kronos	0	0	1
Demeter	0	0	0
Hades	0	0	0
Hestia	0	1	0
Poseidon	0	1	0
Hera	0	1	0
Zeus	0	1	1

Name	L	A	Y
Artemis	1	0	1
Apollo	1	0	1
Leto	1	0	0
Ares	1	1	1
Athena	1	1	1
Hephaestus	1	1	1
Aphrodite	1	1	1
Polyphemus	1	1	1
Persephone	1	1	1
Hermes	1	1	0
Hebe	1	1	0
Dionysus	1	1	0

IPW

Exercise Answer

$$\pi_i^1 = Pr(A_i = 1 \mid L = l_i) = \begin{cases} \frac{1}{2} & l_i = 0 \\ \frac{3}{4} & l_i = 1 \end{cases}$$

$$\pi_i^0 = Pr(A_i = 0 \mid L = l_i) = \begin{cases} \frac{1}{2} & l_i = 0 \\ \frac{1}{4} & l_i = 1 \end{cases}$$

Name	L	A	Y
Rheia	0	0	0
Kronos	0	0	1
Demeter	0	0	0
Hades	0	0	0
Hestia	0	1	0
Poseidon	0	1	0
Hera	0	1	0
Zeus	0	1	1

Name	L	A	Y
Artemis	1	0	1
Apollo	1	0	1
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Hephaestus	1	1	1
Aphrodite	1	1	1
Polyphemus	1	1	1
Persephone	1	1	1
Hermes	1	1	0
Hebe	1	1	0
Dionysus	1	1	0

IPW

Exercise Answer

$$E(Y^{a=1}) = \frac{1}{N} \sum_{i:A_i=1} Y_i/\pi_i^1 = \frac{1}{20}\left(1 \cdot \frac{2}{1} + 6 \cdot \frac{4}{3}\right) = \frac{1}{2}$$

$$E(Y^{a=0}) = \frac{1}{N} \sum_{i:A_i=0} Y_i/\pi_i^0 = \frac{1}{20}\left(1 \cdot \frac{2}{1} + 2 \cdot \frac{4}{1}\right) = \frac{1}{2}$$

We estimate **no** causal effect:

$$E(Y^{a=1}) - E(Y^{a=0}) = \frac{1}{2} - \frac{1}{2} = 0$$

Questions about the HW?