Regression Discontinuity

INFO/STSCI/ILRST 3900: Causal Inference

28 Oct 2025

Learning goals for today

At the end of class, you will be able to:

1. Explain how discontinuities can be exploited for causal identification
2. Understand bias variance trade-off in selecting bandwiths

Logistics

- PSET 4 Peer Review due Nov 4
- Quiz 4 on Nov 4

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- Students who score above a state specific threshold get **Certificate of Merit**
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- Data contains 5,126 Certificate of Merit winners and 2,848 letters of commendation winners

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What is the causal effect of the CoM on various attributes?

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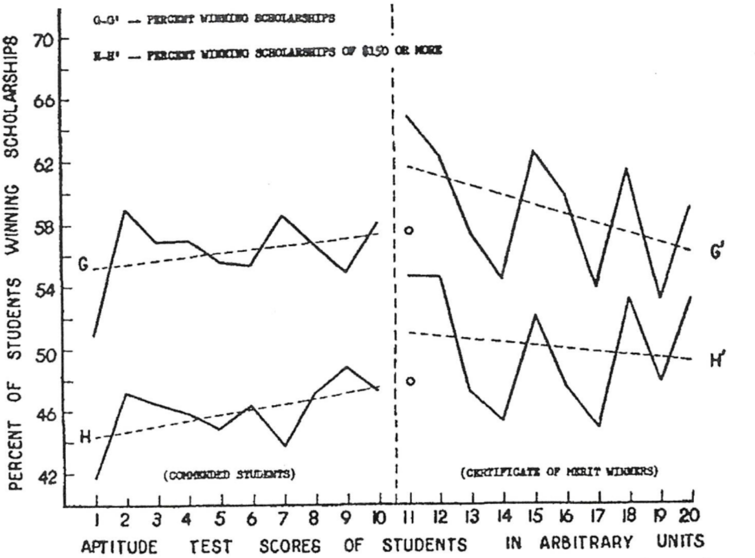

FIG. 2. Regression of success in winning scholarships on exposure determiner.

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SQT $\qquad\qquad$ CoM $\longrightarrow$ Other Scholarships

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  - Matching:
    Match each person who received Certificate of Merit with a "similar" person who received Letter of Recommendation
- In both cases, $P(\mathit{CoM} = 1 \mid \mathit{Score}) = 0$ for some scores
- Violates Positivity Assumption

$$P(A = a \mid L = \ell) > 0 \quad \text{for each } a \text{ and } \ell$$

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- Average Treatment effect for individuals at the cut-off

$$\textbf{Local } ATE = \mathrm{E}(Y_i^{a=1} \mid Score = c) - \mathrm{E}(Y_i^{a=0} \mid Score = c_0)$$

- Does not tell us about treatment effect for everyone!

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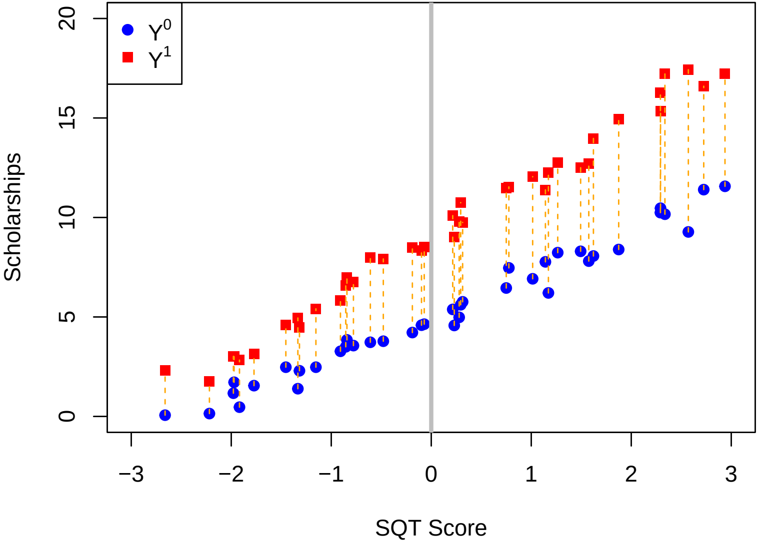
  The average **potential outcomes** for score $= 99.9$ is very close to the average for score $= 100.1$
- Using observed data, estimate, $\mathrm{E}(Y \mid R = r)$ for r closer and closer to the cut-off
- Estimate local ATE $\mathrm{E}(Y_i^{a=1} \mid R_i = c) - \mathrm{E}(Y_i^{a=0} \mid X_i = c)$ by

$$\underbrace{lim_{x \to c^+} \mathrm{E}(Y \mid X = x)}_{\text{from above the cut-off}} - \underbrace{\lim_{x \to c^-} \mathrm{E}(Y \mid X = x)}_{\text{from below the cut-off}}$$

Local average treatment effect



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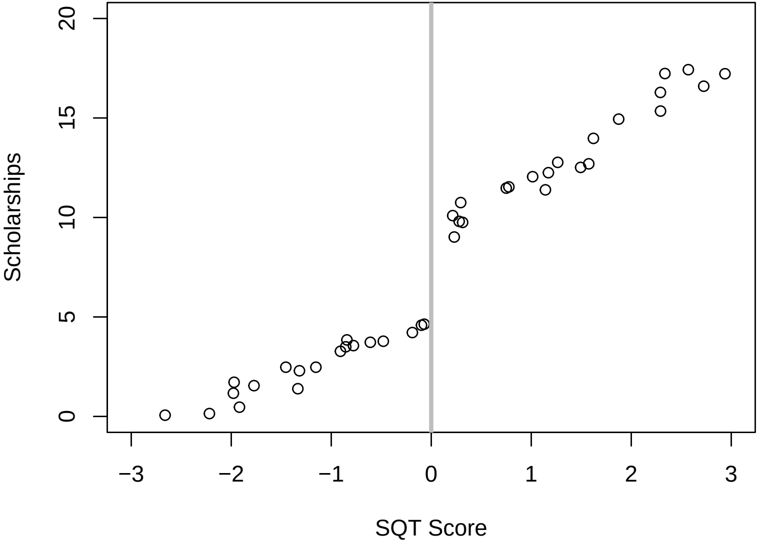

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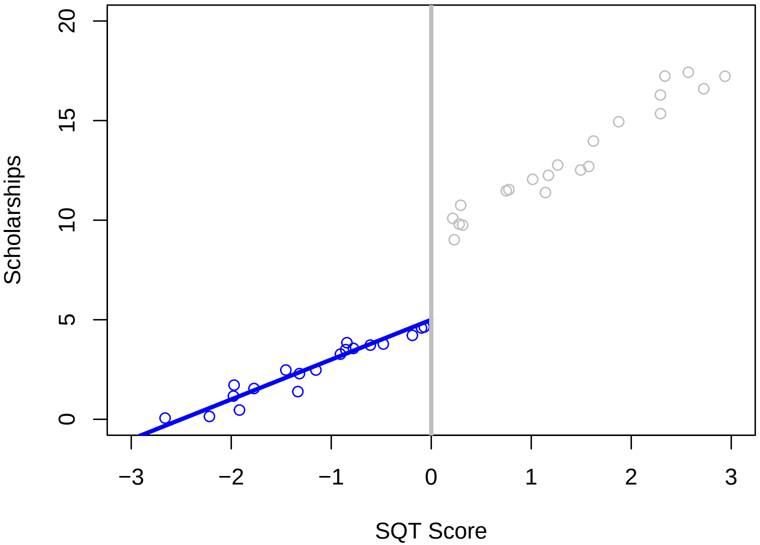

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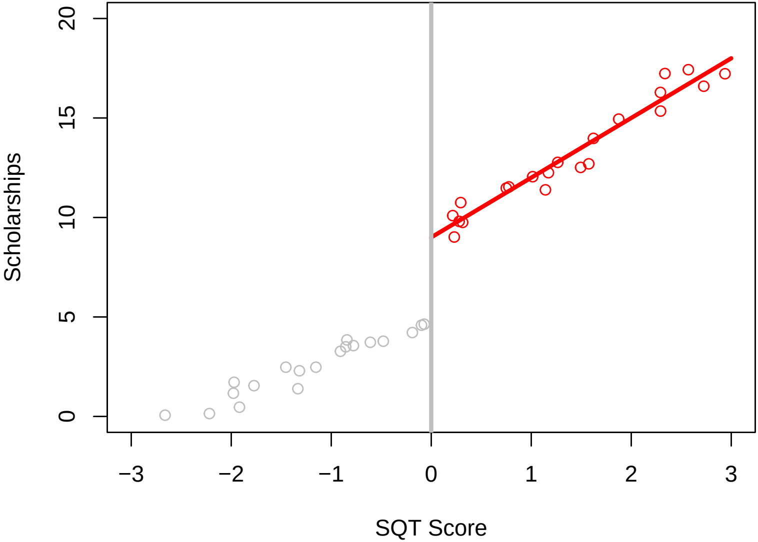

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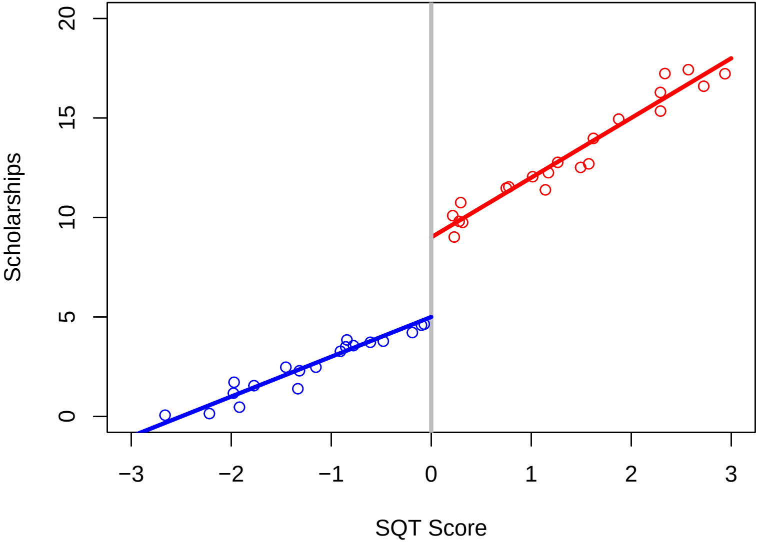

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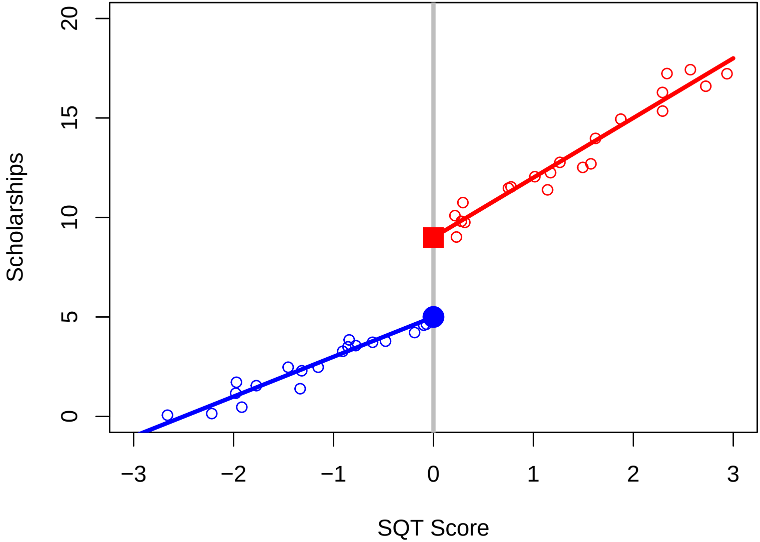

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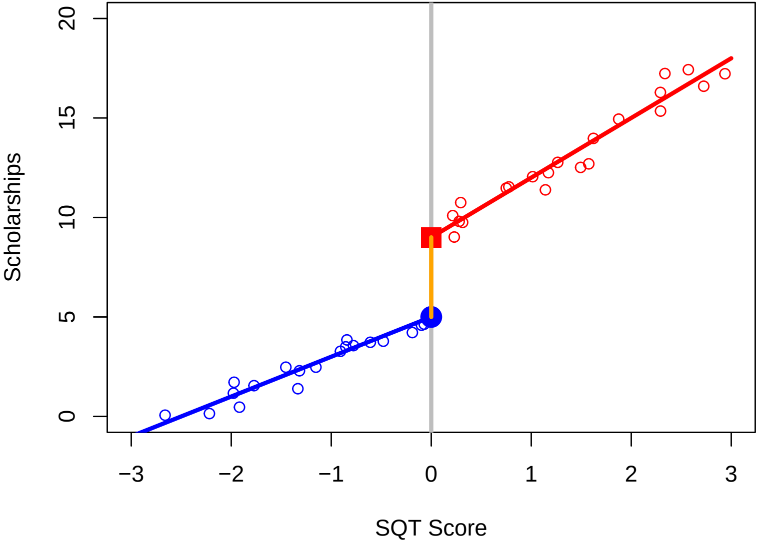

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- Ending up above or below the threshold is more or less chance
  Scoring 100.1 vs 99.9 is essentially random
- Conditional exchangeability holds for people very close to the cut-off
- Conditional exchangeability does not hold for people further from the cut-off

Discontinuities in the wild

Discontinuities turn up in lots of places...

- Enrollment in flagship state universities may require certain test scores[1]

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- Medicare eligibility requires age to be 65+[3]
- ...

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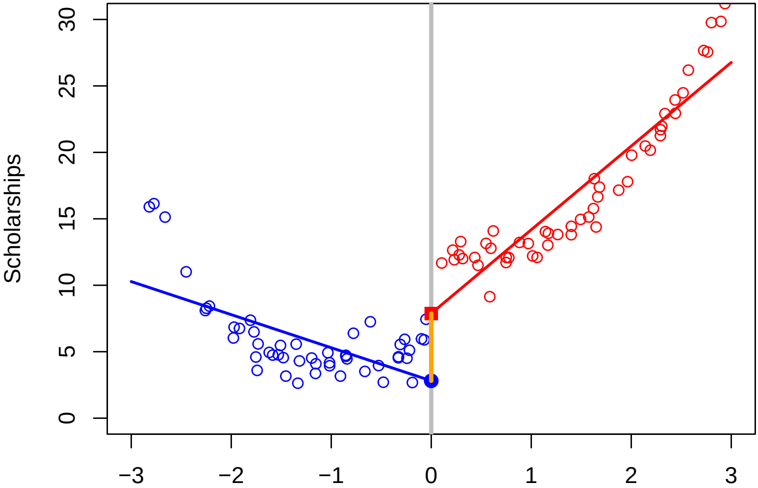

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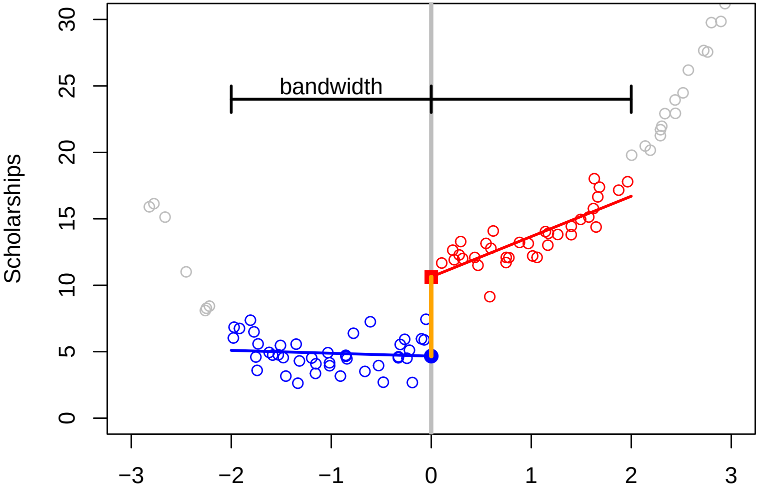

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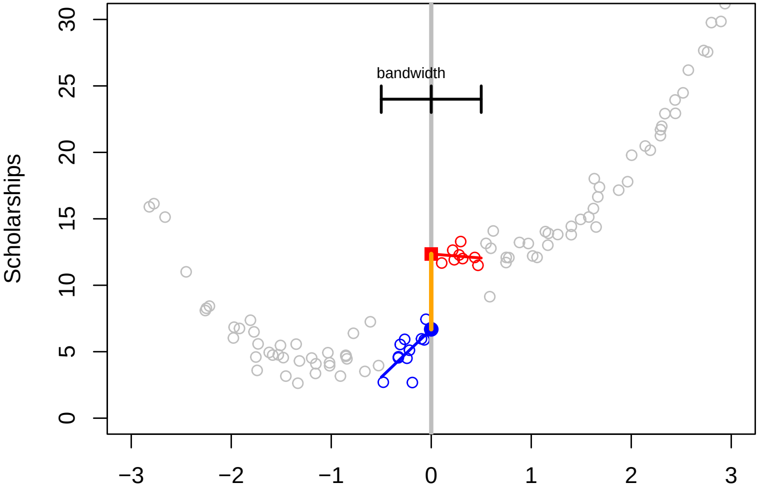

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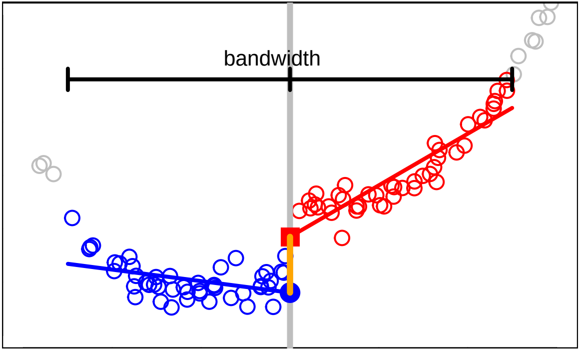

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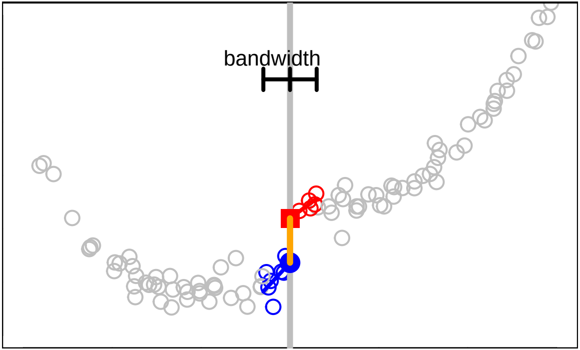

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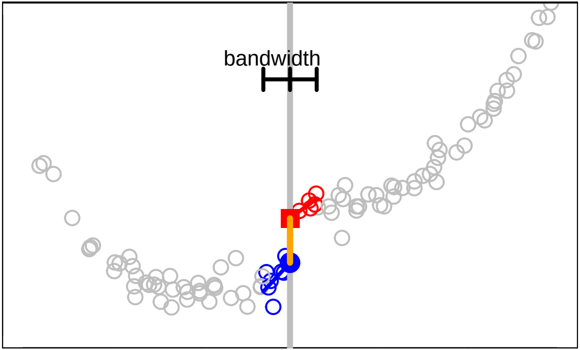

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- Roughly speaking, bandwidth should be smaller when your data set is larger

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- Very few assumptions required
- Plausible in many real applications

Cons:

- Can only estimate local ATE, does not generalize well
- Results depend on picking a bandwidth

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