# Regression Discontinuity: Extensions

INFO/STSCI/ILRST 3900: Causal Inference

30 Oct 2025

# Learning goals for today

At the end of class, you will be able to:

1. Explain how fuzzy discontinuities can be used for causal identification

2. Understand how manipulation or other discontinuities may post potential threats to identification

# The big idea

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- ▶ Assume $E(Y^{a=1} \mid R = r)$ and $E(Y^{a=0} \mid R = r)$ varies smoothly

# The big idea

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▶ Assume $E(Y^{a=1} \mid R = r)$ and $E(Y^{a=0} \mid R = r)$ varies smoothly

▶ Using observed data, estimate, $E(Y \mid R = r)$ for $r$ closer and closer to the cut-off

▶ Estimate local ATE $E(Y_i^{a=1} \mid R_i = c) - E(Y_i^{a=0} \mid X_i = c)$ by

$$\underbrace{lim_{x \to c^+} E(Y \mid X = x)}_{\text{from above the cut-off}} - \underbrace{\lim_{x \to c^-} E(Y \mid X = x)}_{\text{from below the cut-off}}$$

# What can go wrong?

- **Other discontinuity**: Something other than treatment also jumps at the threshold
- **Fuzzy RDD**: Some units are treated on either side of threshold
- **Manipulation**: Units have control over over their running variable

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# Other discontinuities

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- ▶ Jump in outcome is only due to treatment!
- ▶ **Other discontinuity**: If something else "jumps" at the cut-off, then we can't distinguish between effect of treatment and the other thing
- ▶ National Merit example:
  - ▶ Suppose students above cut-off also receive tutoring on how to write better personal statements
  - ▶ Cannot distinguish between effect of Certificate of Merit and tutoring

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  - ▶ See if RDD "detects" a causal effect of treatment on unrelated outcome
  - ▶ Ex: scores on writing sample)

# Fuzzy RDD

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- **Standard setting**: everyone with running variable above threshold is treated, everyone with running threshold is not treated
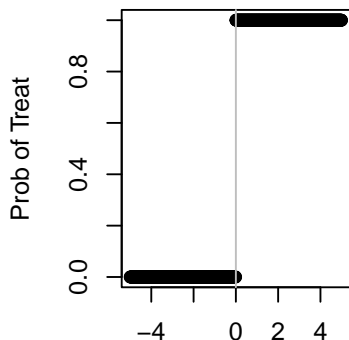
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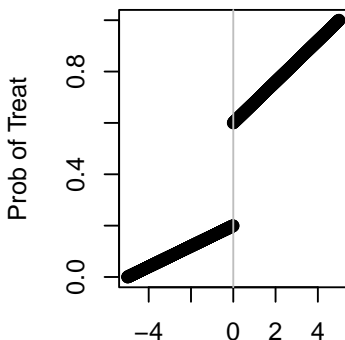- **Fuzzy setting**: Probability of receiving treatment jumps at the threshold

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**Sharp RDD**

**Fuzzy RDD**

# Fuzzy RDD

What is the effect of retirement on spending?[1]

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[1]The Retirement Consumption Puzzle: Evidence from a Regression
Discontinuity Approach. Battistin et al (2009)

# Fuzzy RDD

What is the effect of retirement on spending?[1]

- ▶ Spending habits depend on age
- ▶ Probability of retirement jumps at 65 because of benefits elegibility
- ▶ Some people retire before 65, some people continue to work

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# Fuzzy RDD

What is the effect of retirement on spending?[1]

- ▶ Spending habits depend on age
- ▶ Probability of retirement jumps at 65 because of benefits elegibility
- ▶ Some people retire before 65, some people continue to work
- ▶ Treatment: Retirement
- ▶ Outcome: Spending
- ▶ Running variable: Age, cutoff: 65

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- Effect of being above threshold on outcome

$$\lim_{r \to c^+} \mathsf{E}(Y \mid R = r) - \lim_{x \to c^-} \mathsf{E}(Y \mid R = r)$$

- Effect of being above threshold on treatment

$$\lim_{r \to c^+} \mathsf{E}(A \mid R = r) - \lim_{r \to c^-} \mathsf{E}(A \mid R = r)$$

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- ▶ Effect of being above threshold on treatment

$$\lim_{r \to c^+} E(A \mid R = r) - \lim_{r \to c^-} E(A \mid R = r)$$

- ▶ Dividing gives us the **local ATE for compliers**:

$$\frac{\lim_{r \to c^+} E(Y \mid R = r) - \lim_{x \to c^-} E(Y \mid R = r)}{\lim_{r \to c^+} E(A \mid R = r) - \lim_{r \to c^-} E(A \mid R = r)}$$

# Manipulation

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[2]Evaluating the effect of an antidiscrimination law using a regression-discontinuity design. Hahn et al. (1999)

# Manipulation

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    - Federal EEOC law prohibits discrimination and applies to firms with 15 or more employees
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    - Federal EEOC law prohibits discrimination and applies to firms with 15 or more employees
    - Firms with 14 should be essentially the same as firms with 15 employees
    - Firms have direct control over how many employees they hire
    - Those wanting to avoid EEOC law may decide to stay under 15 employees

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# Manipulation

- ▶ In many settings, units have control over their running variable to some extent

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- ▶ In many settings, units have control over their running variable to some extent
- ▶ Manipulation becomes a problem when units can choose precisely to be above/below threshold
- ▶ Units above the threshold are no longer essentially the same as units below the threshold
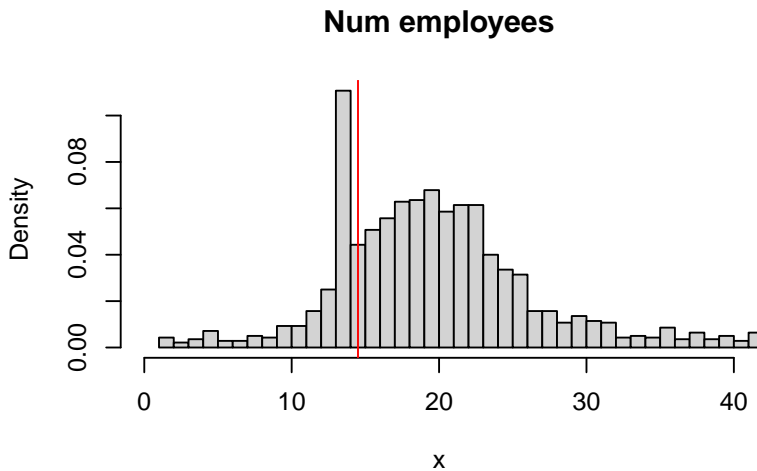
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▶ Hard to test for manipulation directly
▶ Check for balance in covariates
▶ If manipulation is occurring, we would expect to see "heaping" on one side of the cut-off



**Num employees**

# Code

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