Conditional Independence in DAGs

INFO/STSCI/ILRST 3900: Causal Inference

23 Sep 2025

Learning goals for today

At the end of class, you will be able to:

1. Identify whether paths in a causal diagram are open or blocked given a conditioning set
2. Understand why conditioning on colliders differs from conditioning on non-colliders

Logistics

- ▶ Peer Review 2 on Sep 26
- ▶ Quiz 2 on Sep 30
- ▶ Other tentative dates on website

Causal Graphs

- ▶ Causal Directed Acyclic Graphs (DAG) help communicate modeling assumptions and implications

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- ▶ Check (marginal) independence by looking at paths in graph

Checking Marginal Independence

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- ▶ Two types of nodes on a path:
  - ▶ Collider: $\to Z \leftarrow$

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    - ▶ Non-colliders: $\underbrace{\to Z \to}_{\text{mediator}}$ or $\underbrace{\leftarrow Z \to}_{\text{common cause}}$
- ▶ Path is unblocked if it does **not** contain a collider
- ▶ Two variables are statistically dependant if there is an unblocked path between them

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- ▶ **Causal path** path in which all arrows point away from the treatment toward the outcome
- ▶ Exchangeability holds if all unblocked paths are causal paths

DAGs help us reason about exchangeability
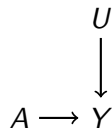
DAGs help us reason about exchangeability

Procedure
1) List all paths between A to Y
2) Cross out the blocked paths
3) Exchangeability holds if all remaining paths are causal
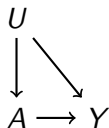
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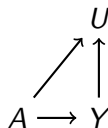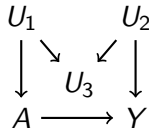DAG 1   DAG 2   DAG 3   DAG 4   DAG 5

$$
\begin{array}{ccccc}
U & U & U & U & U_1 \quad\quad U_2 \\
\downarrow & \downarrow\searrow & \nearrow\searrow & \nearrow\uparrow & \downarrow\searrow\;\;\swarrow\downarrow \\
 & & & & U_3 \\
A \to Y & A \to Y & A \to Y & A \to Y & A \longrightarrow Y
\end{array}
$$

Open or blocked?

How do we check if a path in the DAG is open or blocked when conditioning on a set of variables L?

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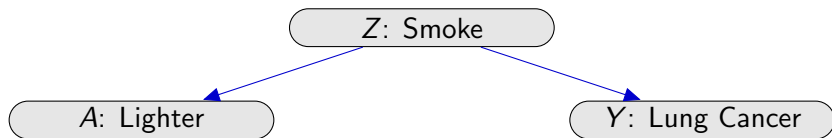- ▶ If all nodes on the path are open, then the entire path is open

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Conditional Exchangeability holds **given** L if all unblocked paths between A and Y are causal paths

Common cause
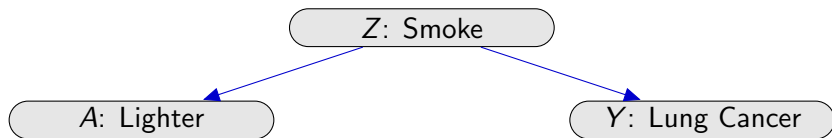
Common cause

If Z has a causal effect on both A and Y, the path is blocked when we condition on Z

Mediation

A: Sodium $\rightarrow$ Z: High Blood Pressure $\rightarrow$ Y: Heart Disease

Mediation

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If A effects Y through Z, the path is blocked when we condition on Z

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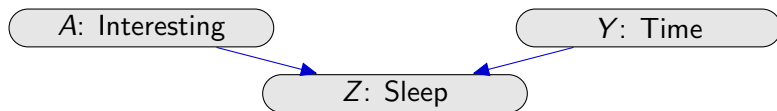- ▶ If Z is in the conditioning set, then Z is blocked
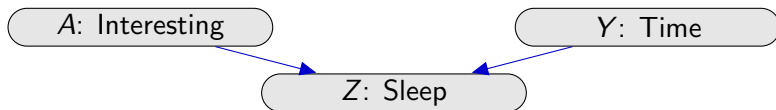
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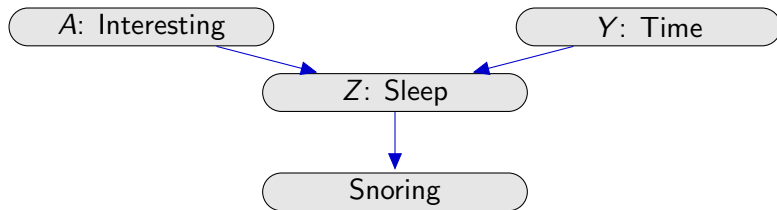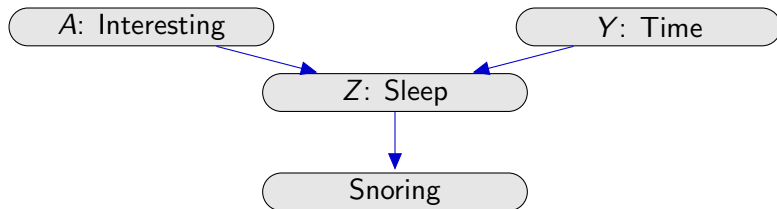
Collider

Collider

Mathematically,

$$Z = X + Y$$

If we keep Z fixed, but increase X, then to preserve the equation, Y must decrease

Collider

Collider

- If there is a causal path $X \to \ldots \to Z$, then Z is a descendant of X

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For Colliders $\rightarrow Z \leftarrow$

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How to check if a node is open or blocked:

- ▶ If non-collider:
    - ▶ Open if it is not in the conditioning set
    - ▶ Blocked if it is in the conditioning set

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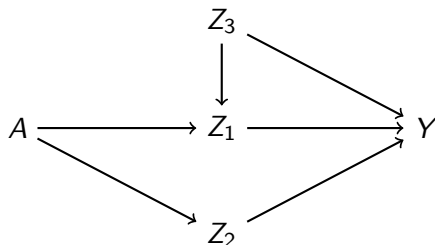 - Otherwise it is blocked

Exercise
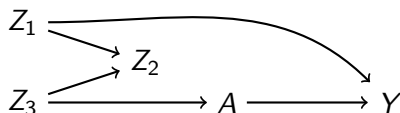
- ▶ What are the paths from A to Y?
- ▶ When conditioning on $L = \{Z_1\}$ are those paths open or blocked?
- ▶ When conditioning on $L = \{Z_2\}$ are those paths open or blocked?
- ▶ When conditioning $L = \{Z_1, Z_2\}$ are those paths open or blocked?

Exercise

- ▶ What are the paths from A to Y?
- ▶ When conditioning on $L = \{Z_2\}$ are those paths open or blocked?
- ▶ When conditioning $L = \{Z_1, Z_2\}$ are those paths open or blocked?

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