Exchangeability (cont) and conditionally randomized experiments

INFO/STSCI/ILRST 3900: Causal Inference

4 Sep 2025

Learning goals for today

At the end of class, you will be able to:

1. Explain conditionally randomized experiments
2. Identify the "idealized experiment" as a goal

Logistics

- Problem Set 1 is due Tuesday at 11pm on Canvas
- After class, read Ch 2.1 and 2.2 in Hernan and Robins 2023

Exchangeability

Definition: Exchangeability means that the potential outcomes, $Y^{a=1}$ and $Y^{a=0}$, are independent of the observed treatment (A)

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In mathematical notation,

$$\underbrace{Y^{a=1}, Y^{a=0}}_{\text{potential outcomes}} \perp\!\!\!\perp \underbrace{A}_{\text{observed treatment}}$$

Exchangeability: By an experimental procedure

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Flip a coin

Two groups of people: heads and tails

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$\swarrow$

Study 1

Heads $\rightarrow$ job training

Tails $\rightarrow$ no job training

Exchangeability: By an experimental procedure

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Compare employment rates
of those with and without
job training

Exchangeability: By an experimental procedure

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$\swarrow$ $\searrow$

Study 1	**Study 2**
Heads $\rightarrow$ job training	Tails $\rightarrow$ job training
Tails $\rightarrow$ no job training	Heads $\rightarrow$ no job training

$\searrow$ $\swarrow$

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Exchangeability: By an experimental procedure

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↙ ↘

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Heads → job training

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Study 2

Tails → job training

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Compare employment rates of those with and without job training

Question: Are both studies valid?

Exchangeability: By an experimental procedure

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Compare employment rates of those with and without job training

Question: Are both studies valid?

Yes. The (H/T) groups are **exchangeable**.
Any statistical pattern between (H/T) and employment can only arise from the causal effect of job training

Why is exchangeability good?

When exchangeability is true, it implies

$$\underbrace{\mathsf{E}(Y^{a=1} \mid A = 1)}_{\text{Within treated}} = \underbrace{\mathsf{E}(Y^{a=1} \mid A = 0)}_{\text{Within not treated}} = \underbrace{\mathsf{E}(Y^{a=1})}_{\text{everyone}}$$

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This allows us to identify the average causal effect (ACE)

$$\mathrm{ACE} = \underbrace{\mathrm{E}(Y^{a=1})}_{\text{if everyone is treated}} - \underbrace{\mathrm{E}(Y^{a=0})}_{\text{if no-one is treated}}$$

because we can plug-in

$$\underbrace{\mathrm{E}(Y \mid A=1)}_{\substack{\text{observed outcomes} \\ \text{for } \textbf{actually} \text{ treated}}} = \underbrace{\mathrm{E}(Y^{a=1} \mid A=1)}_{\substack{\text{potential outcomes} \\ \text{for } \textbf{actually} \text{ treated}}} = \underbrace{\mathrm{E}(Y^{a=1})}_{\substack{\text{potential outcomes} \\ \text{for everyone}}}$$

When does exchangeability hold?

- Data does not tell us directly whether exchangeability holds
- We must know how the data was gathered
- Exchangeability holds by design in experiments

Exercise

- Exchangeability implies that $Y^a \perp\!\!\!\perp A$ for all treatment values a
- How is this different than $Y \perp\!\!\!\perp A$?
- In randomized experiments, $Y^a \perp\!\!\!\perp A$ is usually true. Is $Y \perp\!\!\!\perp A$ ever true?

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- Experiments are great because exchangeability holds by design
- To estimate the causal effect from experimental data, we can simply take the difference in observed means
- But they are also great for other reasons

Precise questions

Experiments allow us to answer precise questions

What is the causal effect of vaccination on covid?

Precise questions

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What is the causal effect of Two shots of the Pfizer vaccine 21 days apart on Covid?

Precise questions

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What is the causal effect of Two shots of the Pfizer vaccine 21 days apart on a positive Covid test within 14 weeks of vaccination in 2020?

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- Experiments allow us to (more easily) specify precise treatments and outcomes

Limits of experiments

Experiments may not be possible because of

- **Feasibility**: What is the causal effect on global average temperature of decreasing global CO_2 levels by 100 ppm?

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- **Cost**: What is the causal effect of giving every student a Lamborghini on traffic in Collegetown?

Limits of experiments

Experiments may not be possible because of

- **Feasibility**: What is the causal effect on global average temperature of decreasing global CO_2 levels by 100 ppm?
- **Cost**: What is the causal effect of giving every student a Lamborghini on traffic in Collegetown?
- **Ethics**: What is the causal effect on cancer of smoking cigarettes?

Making decisions with data

- Randomized experiments are powerful tools for learning causal relationships
- Experiments may have negative effect on participants or larger population[1]
- Belmont Report[2]
  - Respect for persons: protect personal autonomy
  - Beneficence: Do no harm
  - Justice: distribute the burden/benefits fairly

[1] Mcdermott and Hatemi PNAS 2020 https://www.pnas.org/doi/10.1073/pnas.2012021117

[2] https://www.hhs.gov/ohrp/regulations-and-policy/belmont-report/read-the-belmont-report/index.html

Making decisions with data

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- Experiments may have negative effect on participants or larger population[1]
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  - Respect for persons: protect personal autonomy
  - Beneficence: Do no harm
  - Justice: distribute the burden/benefits fairly
- Causal inference with observational data is even more important
- Causal inference (at it's best) tells you what could be, not what ought to be

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Idealized experiment as goal

- Formulate a precise causal question
  - Treatment
  - Outcome and timeframe
  - Population of interest
- Experiments are "gold standard" for estimating causal effects
- Imagine the "ideal experiment" to answer

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- Formulate a precise causal question
  - Treatment
  - Outcome and timeframe
  - Population of interest
- Experiments are "gold standard" for estimating causal effects
- Imagine the "ideal experiment" to answer
- Try to replicate the "ideal experiment" with an observational analysis

Conditional randomization

Exchangeability may not hold in every randomized experiment

- "With the use of an interactive Web-based system, participants in the trial were randomly assigned in a 1:1 ratio to receive 30 μg of BNT162b2 (0.3 ml volume per dose) or saline placebo."

Conditional randomization

Exchangeability may not hold in every randomized experiment

- "With the use of an interactive Web-based system, participants in the trial were randomly assigned in a 1:1 ratio to receive 30 μg of BNT162b2 (0.3 ml volume per dose) or saline placebo."
- Suppose the researchers instead split the group into two groups
  - Age ≥ 55: Vaccine with probability $2/3$
  - Age < 55: Vaccine with probability $1/2$
- Does exchangeability still hold?

Conditional randomization

Exchangeability may not hold in every randomized experiment

- Age ≥ 55 more likely to get vaccine; more likely to get COVID even if treated
- Age ≥ 55 less likely to get vaccine; less likely to get COVID even if treated

Conditional randomization

Exchangeability may not hold in every randomized experiment

- Age ≥ 55 more likely to get vaccine; more likely to get COVID even if treated
- Age ≥ 55 less likely to get vaccine; less likely to get COVID even if treated
- Vaccinated individuals have $Y^{a=1}$ that are more likely to be 1 than unvaccinated individuals
- Exchangeability does not hold in entire population
- Exchangeability holds within each sub-population
- Two separate experiments; both are exchangeable

Conditional randomization

- **Marginal exchangeability:** $Y^a \perp\!\!\!\perp A$ for all a
- **Conditional exchangeability:** $Y^a \perp\!\!\!\perp A \mid L$ for all a
  The potential outcomes are independent of treatment **conditional on** L

Conditional randomization

- **Marginal exchangeability:** $Y^a \perp\!\!\!\perp A$ for all a
- **Conditional exchangeability:** $Y^a \perp\!\!\!\perp A \mid L$ for all a
  The potential outcomes are independent of treatment **conditional on** L
- If you tell me $A_i = 1$, I learn something about $Y_i^{a=1}, Y_i^{a=0}$
- Suppose you first tell me someone's age, I learn something about $Y_i^{a=1}, Y_i^{a=0}$. Next you tell me $A_i = 1$, I don't learn anything new about $Y_i^{a=1}, Y_i^{a=0}$ (in addition to what I previously knew)

Conditional randomization

- **Stratification**: We can directly estimate causal effect within each sub-population (or stratum)
- Estimate causal effect for 55$+$, estimate causal effect for ≤ 55
- If the treatment effect varies across sub-population, we say there is **treatment effect heterogeneity**

Conditional randomization

- Most useful as an idealized experiment to target with observational analysis
- Marginal exchangeability is very unlikely in observational data
- Conditional exchangeability may be more reasonable

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