

Difference in difference: Extensions

INFO/STSCI/ILRST 3900: Causal Inference

2 Nov 2023

Logistics

Logistics

- ▶ Problem set 5 extended to Sunday Nov 5 at 5pm
- ▶ Problem set 6 will be due Nov 16
- ▶ Final project writeup due Nov 21 5pm
 - ▶ summarize what the authors have done
 - ▶ propose a new quantity to estimate
 - ▶ 1,500-2,000 words total
- ▶ Final project presentations Nov 29 in discussion

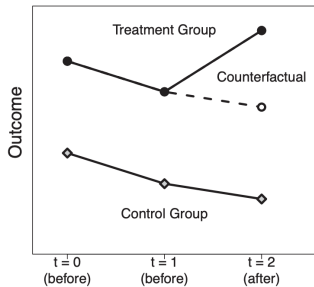
Learning goals for today

At the end of class, you will be able to:

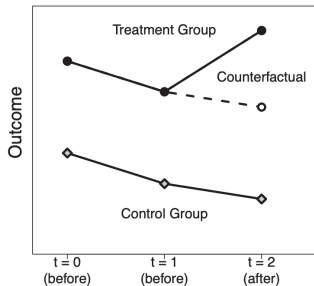
1. Use pre-treatment periods to
 - ▶ assess underlying assumptions
 - ▶ improve estimation accuracy
 - ▶ allow for a more flexible parallel trends assumption
2. and recognize that the parallel assumption remains untestable

Egami, N., & Yamauchi, S. (2023).
Using multiple pretreatment periods to improve
difference-in-differences and staggered adoption designs.
Political Analysis, 31(2), 195-212.

Difference in difference



Difference in difference

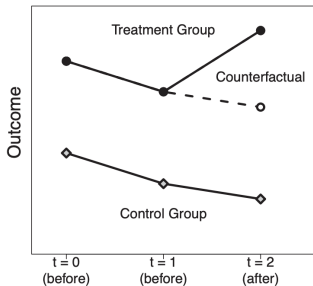


Notation

$Y_{(\text{unit})(\text{time})}^{\text{treatment value}}$

Example: Y_{i1}^0
is unit i at time 1
under treatment 0

Difference in difference



Notation

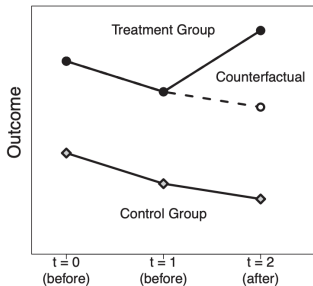
$Y_{(\text{unit})(\text{time})}^{\text{treatment value}}$

Example: Y_{i1}^0
is unit i at time 1
under treatment 0

Parallel Trends Assumption
(untestable)

$$\begin{aligned} E(Y_{\text{Treated},2}^0 - Y_{\text{Treated},1}^0) \\ = \\ E(Y_{\text{Control},2}^0 - Y_{\text{Control},1}^0) \end{aligned}$$

Difference in difference



Notation

$Y_{(\text{unit})(\text{time})}^{\text{treatment value}}$

Example: Y_{i1}^0
is unit i at time 1
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Parallel Trends Assumption (untestable)

$$\begin{aligned} E(Y_{\text{Treated},2}^0 - Y_{\text{Treated},1}^0) \\ = \\ E(Y_{\text{Control},2}^0 - Y_{\text{Control},1}^0) \end{aligned}$$

Extended Parallel Trends (testable)

$$\begin{aligned} E(Y_{\text{Treated},1}^0 - Y_{\text{Treated},0}^0) \\ = \\ E(Y_{\text{Control},1}^0 - Y_{\text{Control},0}^0) \end{aligned}$$

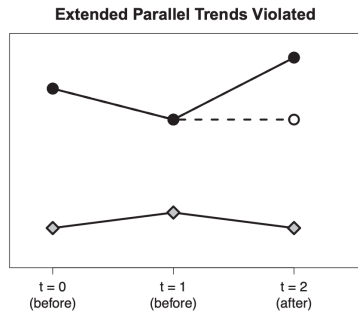
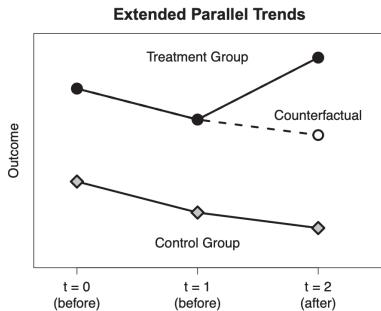


FIGURE 2. Map of Treatment Provinces and National-Level Cities

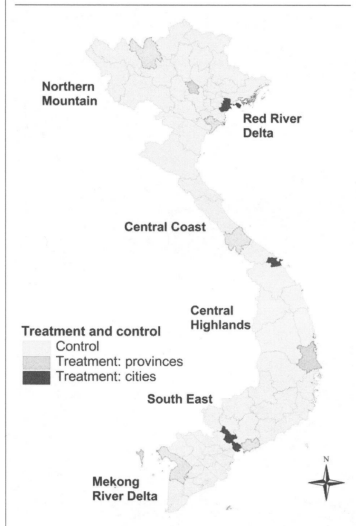
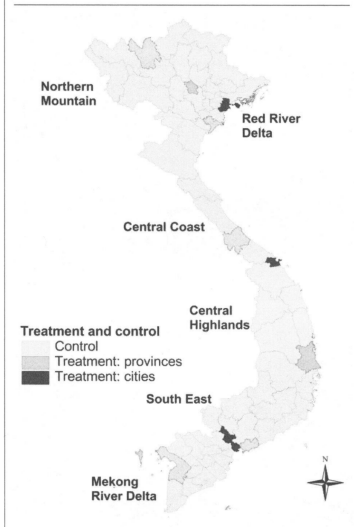


FIGURE 2. Map of Treatment Provinces and National-Level Cities



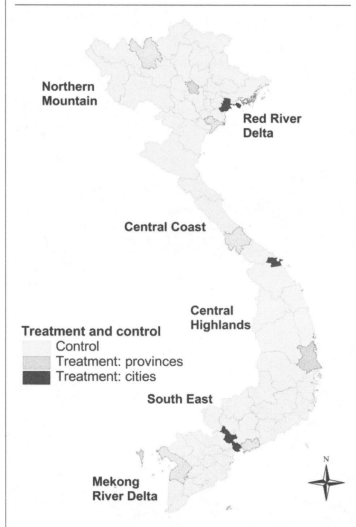
Outcome 1

Education and cultural programs

Is there the following project in the commune?

Investment on culture and education

FIGURE 2. Map of Treatment Provinces and National-Level Cities



Outcome 2

Tap water

Is there the following project in the commune?

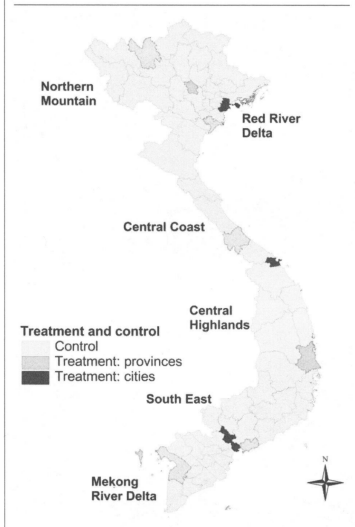
Coded 1

Indoor private piped water
Outdoor private piped water
Public piped water

Coded 0

Well water
Well with protection walls
Well without protection walls
Stream water with protection
Stream water without protection
Rainwater
Bottled water
Water brought by pedicab
Tank water
river lake pond

FIGURE 2. Map of Treatment Provinces and National-Level Cities

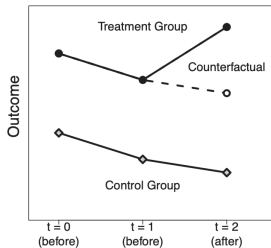


Outcome 3

Agricultural center

Is there any agriculture extension center in this commune?

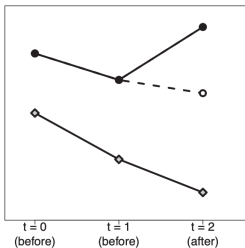
Extended Parallel Trends



Trend of Treatment Group
 $(-2, -1)$

Trend of Control Group
 $(-2, -1)$

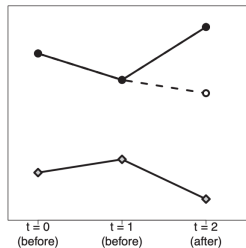
Parallel Trends-in-Trends



Trend of Treatment Group
 $(-2, -1)$

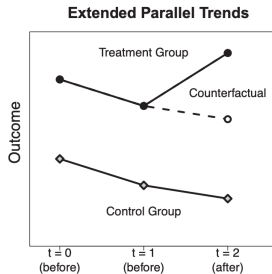
Trend of Control Group
 $(-3.5, -2.5)$

Both are Violated



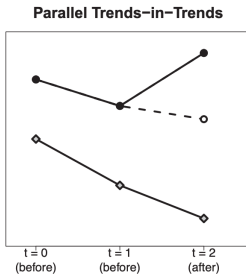
Trend of Treatment Group
 $(-2, -1)$

Trend of Control Group
 $(1, -3)$



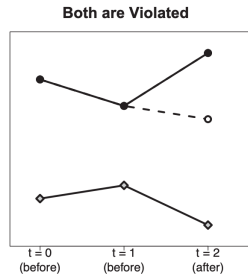
Trend of Treatment Group
(-2, -1)

Trend of Control Group
(-2, -1)



Trend of Treatment Group
(-2, -1)

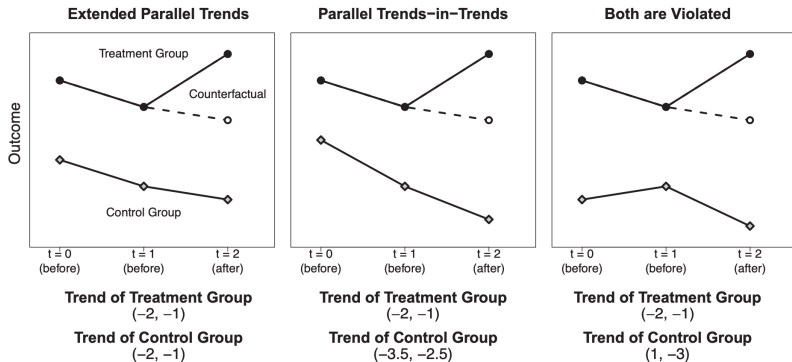
Trend of Control Group
(-3.5, -2.5)



Trend of Treatment Group
(-2, -1)

Trend of Control Group
(1, -3)

In each case, do you believe parallel trends?



In each case, do you believe parallel trends?

Table 2. Assessing underlying assumptions using the pretreatment outcomes.

	Estimate	Std. error	p-value	95% Std. equivalence CI
Education and cultural program	-0.007	0.096	0.940	[-0.166, 0.166]
Tap water	0.166	0.083	0.045	[-0.302, 0.302]
Agricultural center	0.198	0.082	0.015	[-0.332, 0.332]

Benefit 1: Assessing assumptions

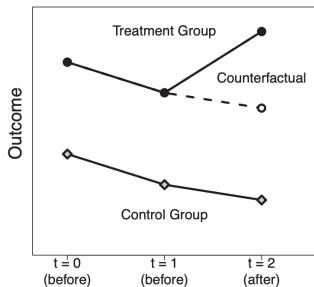
Pre-treatment periods enable us to
assess underlying assumptions

Parallel trends is untestable, but being parallel
in the pre-treatment period builds confidence

Benefit 2: Improving efficiency

Pre-treatment periods also enable us to
improve estimation accuracy
when parallel trends holds

Benefit 2: Improving efficiency



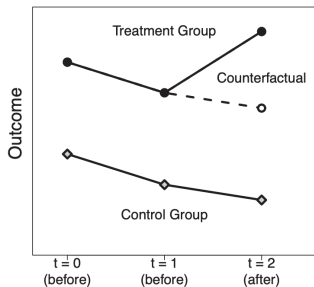
Estimator 1

Estimator 2

Notation

$y_{(\text{unit})(\text{time})}^{\text{treatment value}}$

Benefit 2: Improving efficiency



Estimator 1

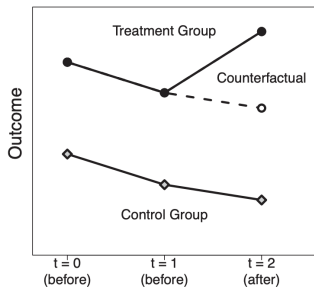
$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T1}^0)}_{\text{Treatment Group Time 2 - Time 1}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C1}^0)}_{\text{Control Group Time 2 - Time 1}}$$

Estimator 2

Notation

$Y^{\text{treatment value}}_{(\text{unit})(\text{time})}$

Benefit 2: Improving efficiency



Estimator 1

$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T1}^0)}_{\text{Treatment Group Time 2 - Time 1}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C1}^0)}_{\text{Control Group Time 2 - Time 1}}$$

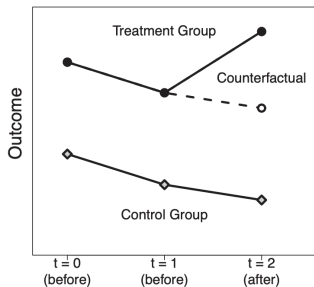
Estimator 2

$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T0}^0)}_{\text{Treatment Group Time 2 - Time 0}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C0}^0)}_{\text{Control Group Time 2 - Time 0}}$$

Notation

$Y^{\text{treatment value}}_{(\text{unit})(\text{time})}$

Benefit 2: Improving efficiency



Notation

$Y^{\text{treatment value}}_{(\text{unit})(\text{time})}$

Estimator 1

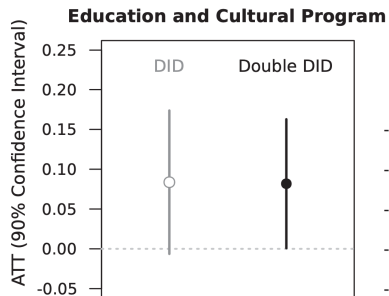
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Estimator 2

$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T0}^0)}_{\text{Treatment Group Time 2 - Time 0}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C0}^0)}_{\text{Control Group Time 2 - Time 0}}$$

Pooled estimator:
Average the two!

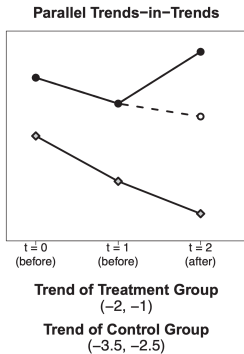
Benefit 2: Improving efficiency



Benefit 3: A more flexible assumption

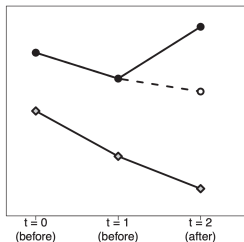
Pre-treatment periods make it possible to
allow for a more flexible parallel trends assumption

Benefit 3: A more flexible assumption



Benefit 3: A more flexible assumption

Parallel Trends-in-Trends



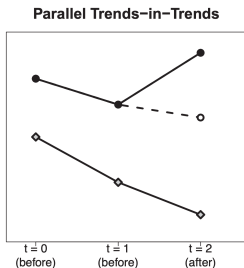
Trend of Treatment Group
(-2, -1)

Trend of Control Group
(-3.5, -2.5)

ASSUMPTION 3 (Parallel Trends-in-Trends)

$$\begin{aligned}
 & \underbrace{\{E[Y_{i2}(0) | G_i = 1] - E[Y_{i1}(0) | G_i = 1]\}}_{\text{Trend of the treatment group from } t=1 \text{ to } t=2} - \underbrace{\{E[Y_{i1}(0) | G_i = 1] - E[Y_{i0}(0) | G_i = 1]\}}_{\text{Trend of the treatment group from } t=0 \text{ to } t=1} \\
 &= \underbrace{\{E[Y_{i2}(0) | G_i = 0] - E[Y_{i1}(0) | G_i = 0]\}}_{\text{Trend of the control group from } t=1 \text{ to } t=2} - \underbrace{\{E[Y_{i1}(0) | G_i = 0] - E[Y_{i0}(0) | G_i = 0]\}}_{\text{Trend of the control group from } t=0 \text{ to } t=1}.
 \end{aligned}$$

Benefit 3: A more flexible assumption



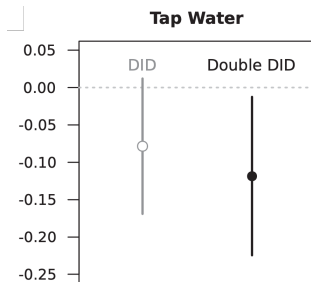
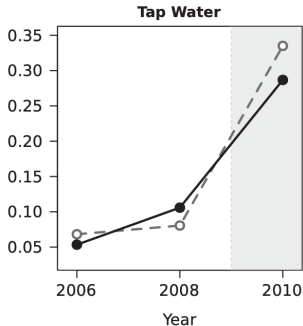
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 &= \underbrace{\{E[Y_{i2}(0) | G_i = 0] - E[Y_{i1}(0) | G_i = 0]\}}_{\text{Trend of the control group from } t=1 \text{ to } t=2} - \underbrace{\{E[Y_{i1}(0) | G_i = 0] - E[Y_{i0}(0) | G_i = 0]\}}_{\text{Trend of the control group from } t=0 \text{ to } t=1}.
 \end{aligned}$$

Sequential DID Estimator

$$\begin{aligned}
 \hat{\tau}_{\text{s-DID}} = & \left\{ \left(\frac{\sum_{i: G_i=1} Y_{i2}}{n_{12}} - \frac{\sum_{i: G_i=1} Y_{i1}}{n_{11}} \right) - \left(\frac{\sum_{i: G_i=0} Y_{i2}}{n_{02}} - \frac{\sum_{i: G_i=0} Y_{i1}}{n_{01}} \right) \right\} \\
 & - \left\{ \left(\frac{\sum_{i: G_i=1} Y_{i1}}{n_{11}} - \frac{\sum_{i: G_i=1} Y_{i0}}{n_{10}} \right) - \left(\frac{\sum_{i: G_i=0} Y_{i1}}{n_{01}} - \frac{\sum_{i: G_i=0} Y_{i0}}{n_{00}} \right) \right\},
 \end{aligned}$$

Benefit 3: A more flexible assumption

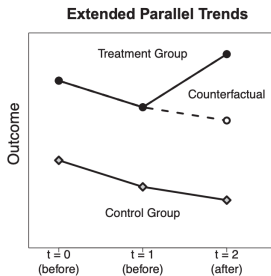


Benefits of multiple pre-treatment periods

1. assess underlying assumptions
2. improve estimation accuracy
3. allow for a more flexible parallel trends assumption

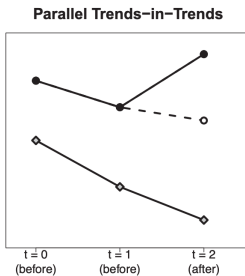
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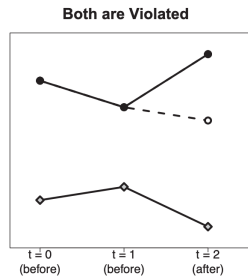
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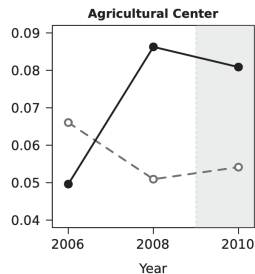
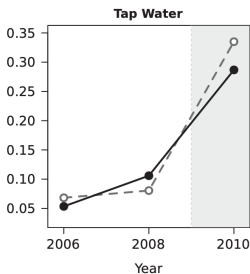
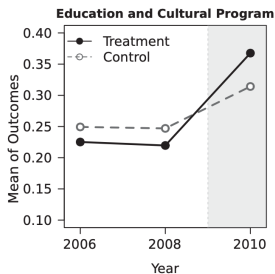


Trend of Treatment Group
(-2, -1)

Trend of Control Group
(1, -3)

Benefits of multiple pre-treatment periods

1. assess underlying assumptions
2. improve estimation accuracy
3. allow for a more flexible parallel trends assumption



Learning goals for today

At the end of class, you will be able to:

1. Use pre-treatment periods to
 - ▶ assess underlying assumptions
 - ▶ improve estimation accuracy
 - ▶ allow for a more flexible parallel trends assumption
2. and recognize that the parallel assumption remains untestable