

Difference in differences: Extensions

INFO/STSCI/ILRST 3900: Causal Inference

7 Nov 2024

Logistics

- ▶ Problem set 5 due tonight
 - ▶ Only one coding problem
 - ▶ Continuity/Smoothness assumption: **potential** outcomes are smooth/continuous **at the cutoff**
- ▶ Pset 5 peer reviews released Monday, due Friday
- ▶ Pset 6 released next Thurs, due following Thurs (no peer reviews)
- ▶ Final Project
 - ▶ Submit check-in by Sunday Nov 17th
 - ▶ Final paper due Dec 5
 - ▶ Video due Dec 18th (asynchronously)

no extensions or flex days... but talk to us if you foresee issues

* note : use your flex days , but if you don't have any left & need an extension , send us an email & we'll consider it

Learning goals for today

At the end of class, you will be able to:

1. Use pre-treatment periods to
 - ▶ assess underlying assumptions
 - ▶ improve estimation accuracy
 - ▶ allow for a more flexible parallel trends assumption
2. recognize that the parallel trends assumption remains untestable
3. and compare the differences between parallel trends, *extended* parallel trends, and parallel *trends-in-trends*

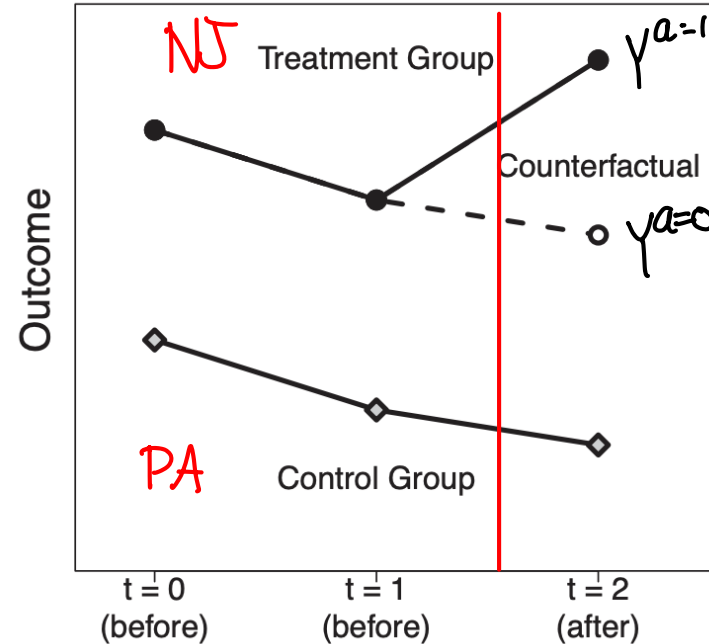
Egami, N., & Yamauchi, S. (2023). [Using multiple pretreatment periods to improve difference-in-differences and staggered adoption designs](#). *Political Analysis*, 31(2), 195-212.

PollEv: Parallel Trends Review

$$ATT = E(Y^{a=1} | \text{Treatment Group}) - E(Y^{a=0} | \text{Treatment Group})$$

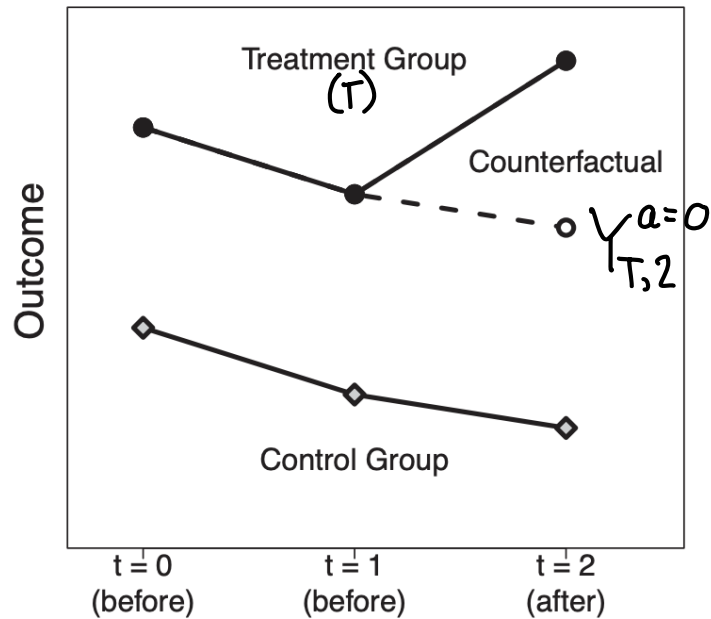
Parallel trends is...
(select all that apply)

- ✓ 1. an assumption in the post-treatment period
- ✓ 2. an assumption about the treatment group
- ✓ 3. an assumption about a counterfactual
- ✓ 4. untestable



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Difference in difference



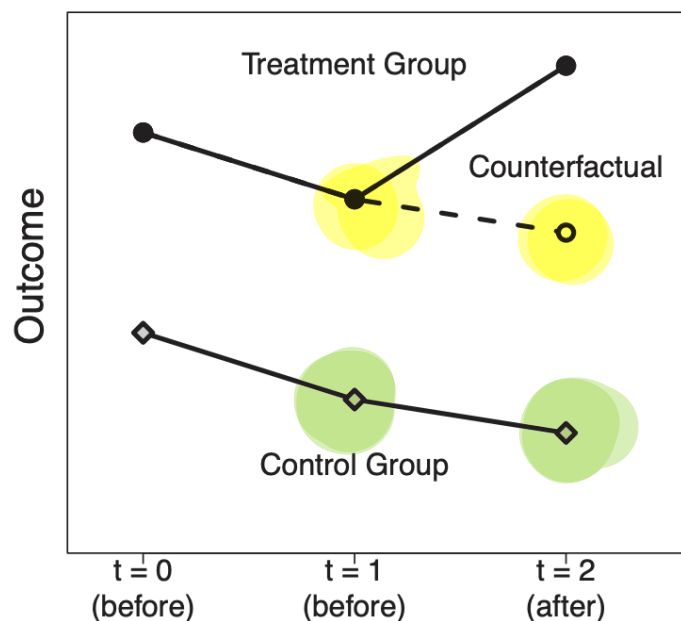
Notation

$Y^{\text{treatment value}}_{(\text{group}), (\text{time})}$

$Y^a_{i,t}$

Example: $Y^0_{Treated,1}$
 is outcome of treated group
 at time 1 under treatment 0

Difference in difference



Notation

$Y_{(\text{group}),(\text{time})}^{\text{treatment value}}$

Example: $Y_{Treated,1}^0$
is outcome of treated group
at time 1 under treatment 0

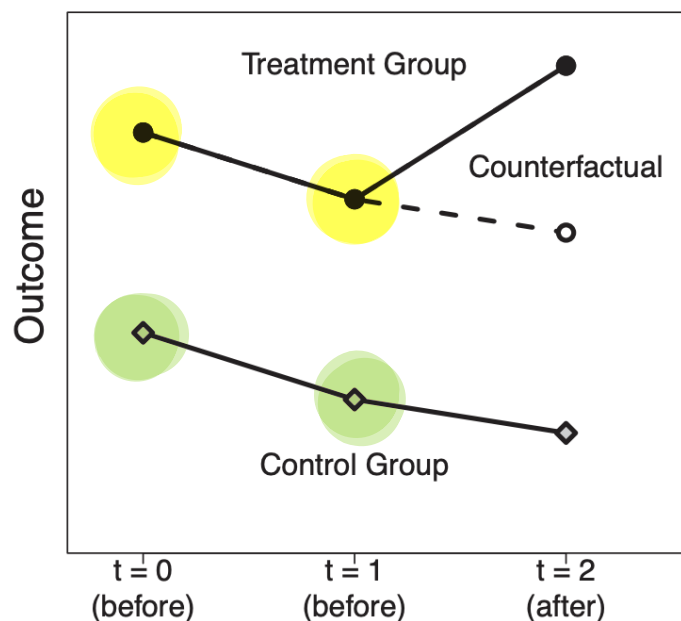
Parallel Trends Assumption
(untestable)

$$E(Y_{Treated,2}^0 - Y_{Treated,1}^0)$$

=

$$E(Y_{Control,2}^0 - Y_{Control,1}^0)$$

Difference in difference



Notation

$Y^{\text{treatment value}}_{(\text{group}),(\text{time})}$

Example: $Y^0_{Treated,1}$
is outcome of treated group
at time 1 under treatment 0

Parallel Trends Assumption
(untestable) *post-treatment*

$$E(Y^0_{Treated,2} - Y^0_{Treated,1})$$

=

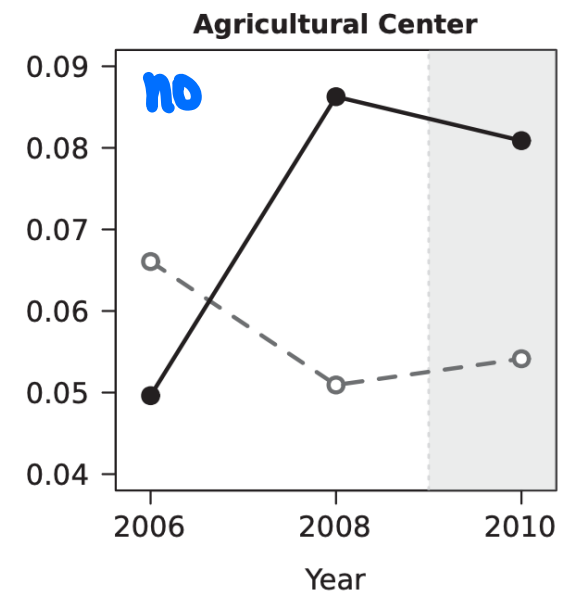
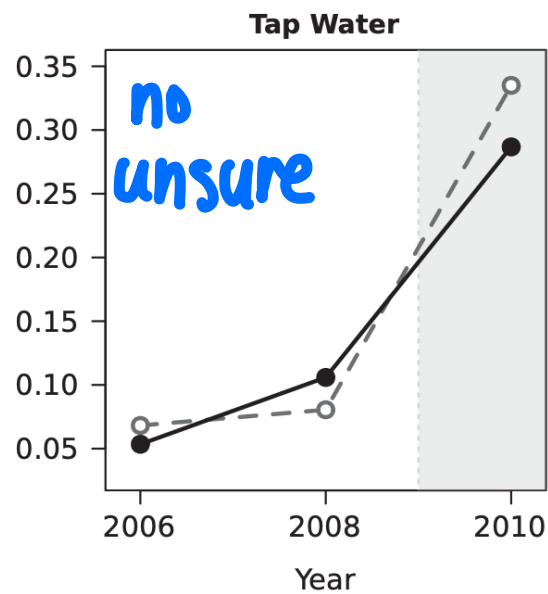
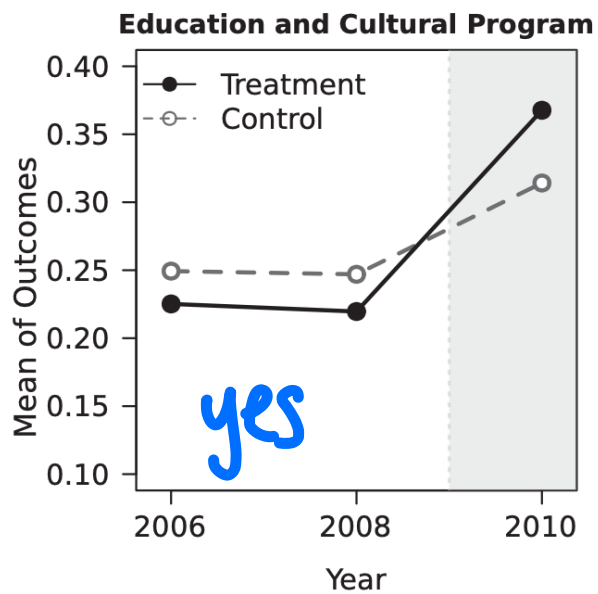
$$E(Y^0_{Control,2} - Y^0_{Control,1})$$

observed outcomes
Extended Parallel Trends
(testable) *pre-treatment*

$$E(Y^0_{Treated,1} - Y^0_{Treated,0})$$

=

$$E(Y^0_{Control,1} - Y^0_{Control,0})$$



extended parallel trends
pre-treatment (observed outcomes)
testable

In each case, do you believe
parallel trends?

post-treatment
untestable (because of
counterfactual)



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Benefit 1: Assessing assumptions

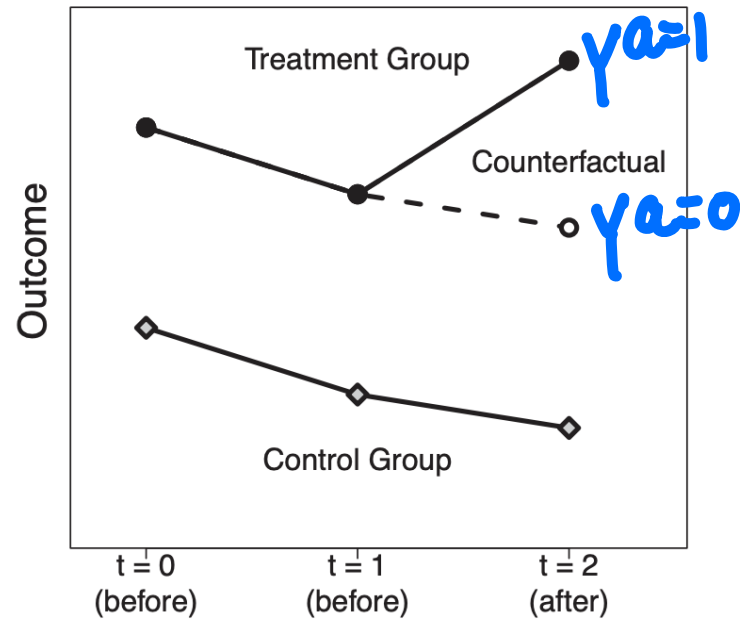
Pre-treatment periods enable us to
assess underlying assumptions

Parallel trends is untestable, but being parallel
in the pre-treatment period builds confidence (not
called "extended parallel-trends" definitive
proof)

Benefit 2: Improving efficiency

Pre-treatment periods also enable us to
improve estimation accuracy
when parallel trends holds

Benefit 2: Improving efficiency



Estimator 1

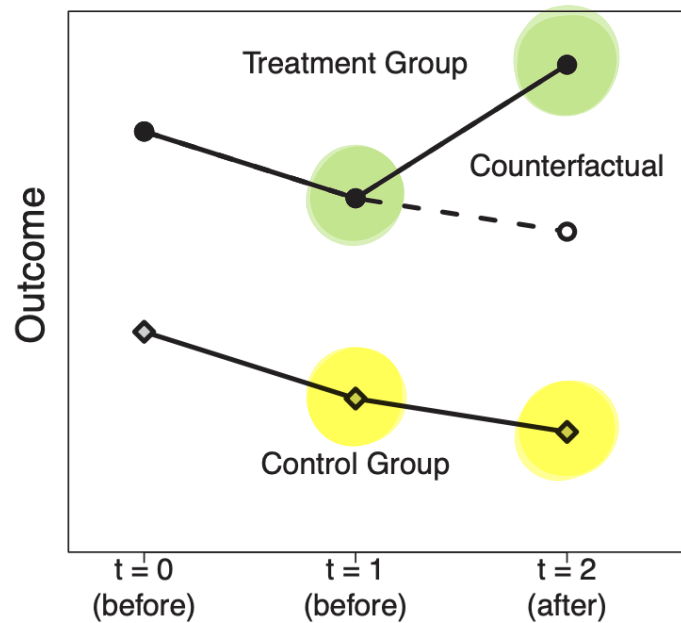
Estimator 2

Notation

$y^{\text{treatment value}}$
(unit)(time)

Benefit 2: Improving efficiency

$$E(\gamma^a) \approx \bar{\gamma}^a$$



Estimator 1

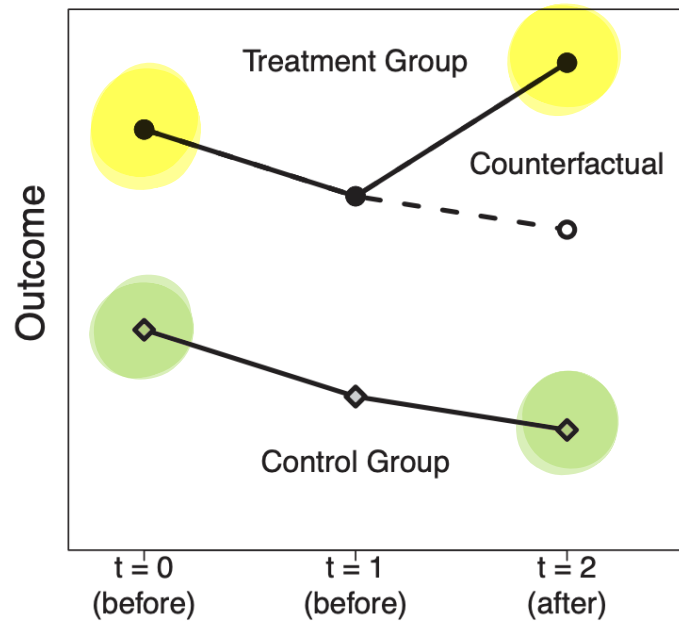
$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T1}^0)}_{\text{Treatment Group Time 2 - Time 1}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C1}^0)}_{\text{Control Group Time 2 - Time 1}}$$

Estimator 2

Notation

$\gamma^{\text{treatment value}}_{(\text{unit})(\text{time})}$

Benefit 2: Improving efficiency



Estimator 1

$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T1}^0)}_{\text{Treatment Group Time 2 - Time 1}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C1}^0)}_{\text{Control Group Time 2 - Time 1}}$$

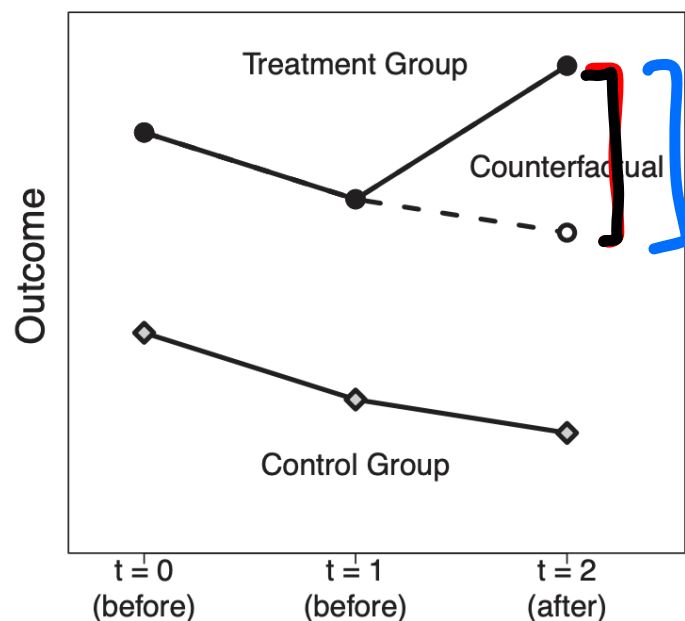
Estimator 2

$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T0}^0)}_{\text{Treatment Group Time 2 - Time 0}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C0}^0)}_{\text{Control Group Time 2 - Time 0}}$$

Notation

$y^{\text{treatment value}}_{(\text{unit})(\text{time})}$

Benefit 2: Improving efficiency



Notation

$y^{\text{treatment value}}_{(\text{unit})(\text{time})}$

Estimator 1

$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T1}^0)}_{\text{Treatment Group Time 2 - Time 1}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C1}^0)}_{\text{Control Group Time 2 - Time 1}}$$

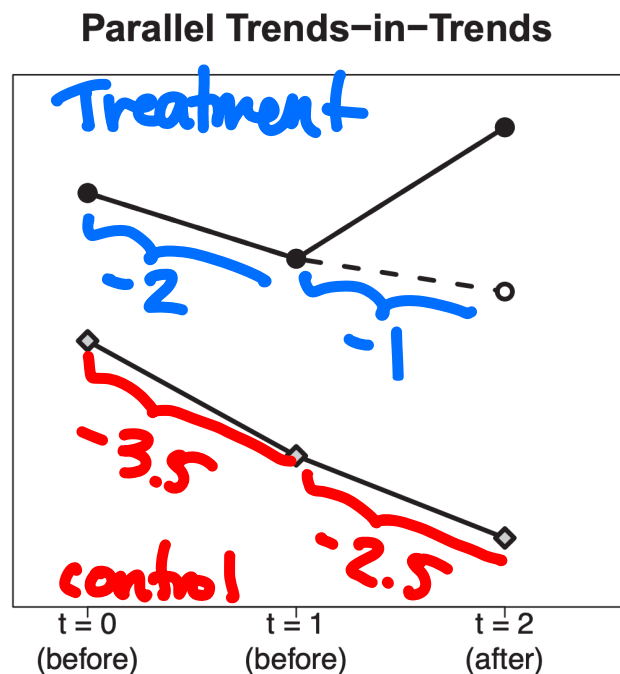
Estimator 2

$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T0}^0)}_{\text{Treatment Group Time 2 - Time 0}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C0}^0)}_{\text{Control Group Time 2 - Time 0}}$$

**Double DID Estimator:
Average the two!**

Benefit 3: A more flexible assumption

Pre-treatment periods make it possible to
allow for a more flexible parallel trends assumption



$$\uparrow -2 + 1 = -1$$

$$\uparrow 1$$

$$-3.5 + 1 = -2.5$$

Trend of Treatment Group
(-2, -1)

Trend of Control Group
(-3.5, -2.5)

Benefit 3: A more flexible assumption

Trends in pre-treatment period are parallel (testable)

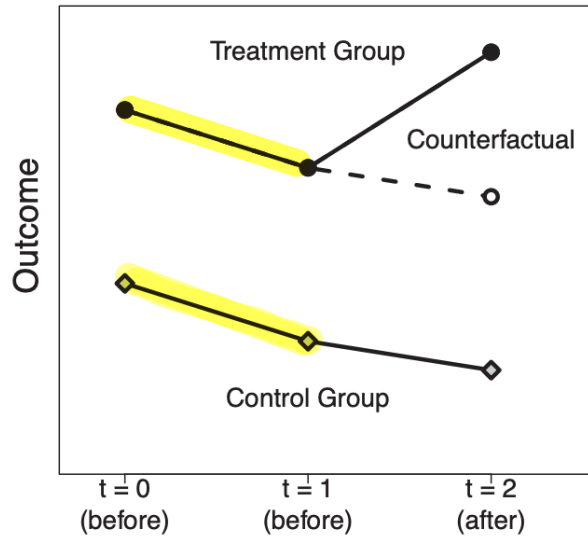
trends are not parallel but the way the slopes change is the same (untestable)

trends are not parallel pre-treatment & even the way the slopes change are different

Extended Parallel Trends

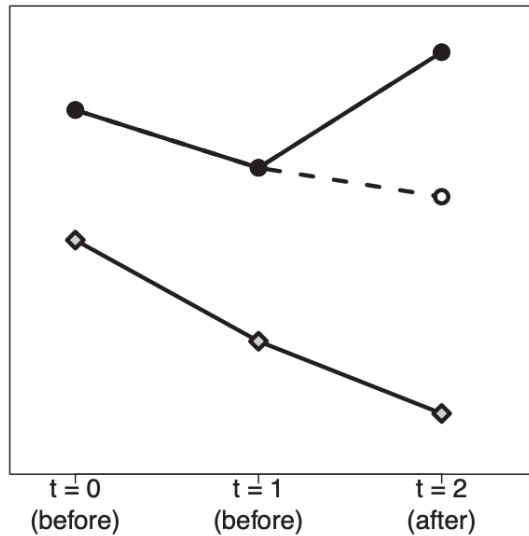
Parallel Trends-in-Trends

Both are Violated



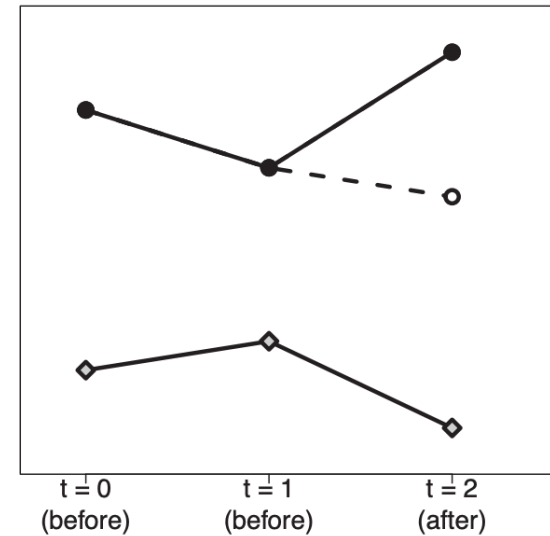
Trend of Treatment Group
(-2, -1)

Trend of Control Group
(-2, -1)



Trend of Treatment Group
(-2, -1) ↑ 1

Trend of Control Group
(-3.5, -2.5) ↑ 1



Trend of Treatment Group
(-2, -1) ↑ 1

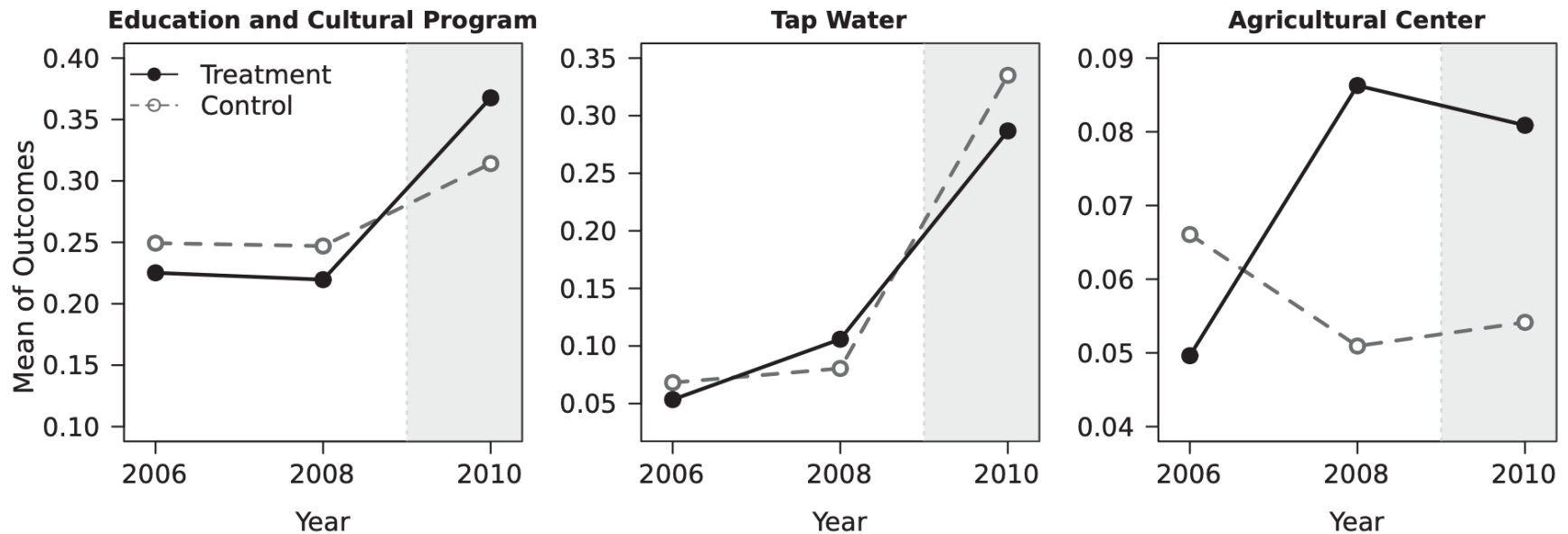
Trend of Control Group
(1, -3) ↓ 4

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(recap) Main DiD assumption: parallel trends

Benefits of multiple pre-treatment periods

1. assess underlying assumptions
2. improve estimation accuracy
3. allow for a more flexible parallel trends assumption



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