Measuring of Causal effects and Standardization

INFO/STSCI/ILRST 3900: Causal Inference

5 Sep 2023

Learning goals for today

At the end of class, you will be able to:

1. Describe different ways to quantitatively measure a causal effect
2. Estimate the average causal effect using data from a conditionally randomized experiment

Logistics

- Ch 1.3 and 2.3 in Hernan and Robins 2023

Conditional randomization

Exchangeability may not hold in every randomized experiment

- Age ≥ 55 receive vaccine with $2/3$; more likely to get COVID if treated
- Age < 55 get vaccine with probability $1/2$; less likely to get COVID if treated

Conditional randomization

Exchangeability may not hold in every randomized experiment

- Age ≥ 55 receive vaccine with $2/3$; more likely to get COVID if treated
- Age < 55 get vaccine with probability $1/2$; less likely to get COVID if treated
- Exchangeability does not hold in entire population
- Exchangeability holds within each sub-population
- Two separate experiments; both are exchangeable

Conditional randomization

- **Marginal exchangeability:** $Y^a \perp\!\!\!\perp A$ for all a
- **Conditional exchangeability:** $Y^a \perp\!\!\!\perp A \mid L$ for all a
  The potential outcomes are independent of treatment **conditional on** L
- **Stratification**: We can directly estimate causal effect within each sub-population (or stratum)
- If the treatment effect varies across sub-population, we say there is **treatment effect heterogeneity**

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- **Conditional exchangeability:** $Y^a \perp\!\!\!\perp A \mid L$ for all a
  The potential outcomes are independent of treatment **conditional on** L
- If you tell me $A_i = 1$, I learn something about $Y_i^{a=1}, Y_i^{a=0}$
- Suppose you first tell me someone's age, I learn something about $Y_i^{a=1}, Y_i^{a=0}$. Next you tell me $A_i = 1$, I don't learn anything new about $Y_i^{a=1}, Y_i^{a=0}$ (in addition to what I previously knew)

Conditional randomization

- **Stratification**: We can directly estimate causal effect within each sub-population (or stratum)

$$\begin{aligned} \mathrm{E}(Y \mid A = a, L = \ell) &\overset{\text{consis}}{=} \mathrm{E}(Y^a \mid A = a, L = \ell) \\ &\overset{\text{exchange}}{=} \mathrm{E}(Y^a \mid L = \ell) \end{aligned}$$

- If the treatment effect varies across sub-population, we say there is **treatment effect heterogeneity**

$$\begin{gathered} \mathrm{E}(Y^{a=1} \mid L = \text{Age } 55+) - \mathrm{E}(Y^{a=0} \mid L = \text{Age } 55+) \\ \neq \\ \mathrm{E}(Y^{a=1} \mid L = \text{Age} < 55) - \mathrm{E}(Y^{a=0} \mid L = \text{Age} < 55) \end{gathered}$$

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- Can be useful in designing experiments
  - If $Y^{a=1}$ has higher variability in some sub-population, assign more units to treated group

Conditional randomization

- Can be useful in designing experiments
  - If $Y^{a=1}$ has higher variability in some sub-population, assign more units to treated group
- Most useful as an idealized experiment to target with observational analysis
- Marginal exchangeability is very unlikely in observational data
- Conditional exchangeability may be more reasonable

Excercise

Suppose we have data gathered by surveying individuals in Fall of 2021

- Whether the individual was vaccinated for Covid
  $A_i = 1$ if vaccinated, $A_i = 0$ if not vaccinated
- Whether the individual tested positive for Covid in 2021
  $Y_i = 1$ if positive test, $Y_i = 0$ if no positive test
- What additional information could you gather about each individual to make conditional exchangeability might be plausible?

$$Y^{a=1}, Y^{a=0} \perp\!\!\!\perp A \mid L$$

Measures of association/causation[1]

- For binary outcomes $Pr(Y^a = 1) = \mathrm{E}(Y^a)$
- Average Causal Effect $\mathrm{E}(Y^{a=1}) - \mathrm{E}(Y^{a=0})$
- Also called average treatment effect and causal risk difference
- No average causal effect if $ACE = 0$

[1] Ch 1.2 and 1.3 of Hernan and Robins

Measures of association/causation[1]

- For binary outcomes $Pr(Y^a = 1) = \mathrm{E}(Y^a)$
- Average Causal Effect $\mathrm{E}(Y^{a=1}) - \mathrm{E}(Y^{a=0})$
- Also called average treatment effect and causal risk difference
- No average causal effect if $ACE = 0$
- Sharp null hypothesis: $Y_i^{a=1} = Y_i^{a=0} = 0$ for all i
- Sharp null hypothesis also means ACE $= 0$, but not the other way around!

[1] Ch 1.2 and 1.3 of Hernan and Robins

Measures of association/causation[2]

- Causal Risk Ratio:

$$\frac{E(Y^{a=1})}{E(Y^{a=0})}$$

- Causal Odds Ratio:

$$\frac{Pr(Y^{a=1}=1)/Pr(Y^{a=1}=0)}{Pr(Y^{a=0}=1)/Pr(Y^{a=0}=0)}$$

[2] Ch 1.2 and 1.3 of Hernan and Robins

Measures of association/causation[2]

- Causal Risk Ratio:

$$\frac{\mathrm{E}(Y^{a=1})}{\mathrm{E}(Y^{a=0})}$$

- Causal Odds Ratio:

$$\frac{Pr(Y^{a=1}=1)/Pr(Y^{a=1}=0)}{Pr(Y^{a=0}=1)/Pr(Y^{a=0}=0)}$$

- No average causal effect if $CRR = COR = 1$

[2] Ch 1.2 and 1.3 of Hernan and Robins

Measures of association/causation[3]

- All measures will agree if $E(Y^{a=1} = 1) = E(Y^{a=0} = 1)$
- If $E(Y^{a=1} = 1) \neq E(Y^{a=0} = 1)$, the different measures may be easier/harder to interpret
- What is the ACE and CRR if
  - $E(Y^{a=1} = 1) = .5; E(Y^{a=0} = 1) = .25$
  - $E(Y^{a=1} = 1) = .001; E(Y^{a=0} = 1) = .0005$

[3] Ch 1.2 and 1.3 of Hernan and Robins

Standardization

- Under conditional exchangeability, we can directly estimate the average causal effect for each sub-population
- **Standardization** allows us to estimate the ACE by combining estimates from each sub-population

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- For $L = 0, 1$

$$
\begin{aligned}
\mathrm{E}(Y^a) &= Pr(L=1)\mathrm{E}(Y^a = 1 \mid L = 1) \\
&\quad + Pr(L=0)\mathrm{E}(Y^a = 1 \mid L = 0) \\
&= Pr(L=1)\mathrm{E}(Y = 1 \mid L = 1, A = a) \\
&\quad + Pr(L=0)\mathrm{E}(Y^a = 1 \mid L = 0, A = a)
\end{aligned}
$$

Standardization

- Under conditional exchangeability, we can directly estimate the average causal effect for each sub-population
- **Standardization** allows us to estimate the ACE by combining estimates from each sub-population
- For $L = 0, 1$

$$\begin{aligned}
E(Y^a) &= Pr(L = 1)E(Y^a = 1 \mid L = 1) \\
&\quad + Pr(L = 0)E(Y^a = 1 \mid L = 0) \\
&= Pr(L = 1)E(Y = 1 \mid L = 1, A = a) \\
&\quad + Pr(L = 0)E(Y^a = 1 \mid L = 0, A = a)
\end{aligned}$$

- More generally,

$$Pr(Y^a) = \sum_{l} Pr(L = l)E(Y \mid L = l, A = a)$$

Standardization

$$Pr(Y^a) = \sum_l \mathrm{E}(Y \mid L = l, A = a) Pr(L = l)$$

Excercise

$$E(Y^a) = \sum_l E(Y \mid L = l, A = a) Pr(L = l)$$

		L	A	Y
1	Rheia	0	0	0
2	Kronos	0	0	1
3	Demeter	0	0	0
4	Hades	0	0	0
5	Hestia	0	1	0
6	Poseidon	0	1	0
7	Hera	0	1	0
8	Zeus	0	1	1

		L	A	Y
9	Artemis	1	0	1
10	Apollo	1	0	1
11	Leto	1	0	0
12	Ares	1	1	1
13	Athena	1	1	1
14	Hephaestus	1	1	1
15	Aphrodite	1	1	1
16	Polyphemus	1	1	1
17	Persephone	1	1	1
18	Hermes	1	1	0
19	Hebe	1	1	0
20	Dionysus	1	1	0

Excercise

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17	Persephone	1	1	1
18	Hermes	1	1	0
19	Hebe	1	1	0
20	Dionysus	1	1	0

$$E(Y^a) = Pr(L=1)E(Y=1 \mid L=1, A=a)$$

$$+ Pr(L=0)E(Y^a=1 \mid L=0, A=a)$$

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