Matching Intro

INFO/STSCI/ILRST 3900: Causal Inference

9 Oct 2025

Learning goals for today

At the end of class, you will be able to:

1. Explain how matching can be used to estimate causal effects
2. Explain bias variance trade-off in various matching procedures

Causal effect

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- **Average Treatment Effect on the Treated** (ATT)

$$E(Y^{a=1} \mid A = 1) - E(Y^{a=0} \mid A = 1)$$

Matching: The big idea

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Goal: $\mathrm{E}(Y^{a=1} \mid A=1) - \mathrm{E}(Y^{a=0} \mid A=1)$ **ATT**

$$\mathrm{E}(Y^{a=1} \mid A=1) \approx \frac{1}{n_t} \sum_{i:A_i=1} Y_i^{a=1} = \frac{1}{n_t} \sum_{i:A_i=1} Y_i$$

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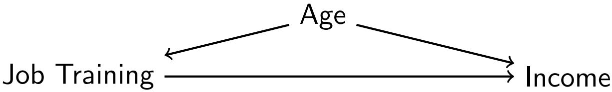
$$\frac{1}{n_m} \sum_{i \in \mathcal{M}} Y_i = \frac{1}{n_m} \sum_{i \in \mathcal{M}} Y_i^{a=0} \approx \frac{1}{n_t} \sum_{i:A_i=1} Y_i^{a=0}$$

Example



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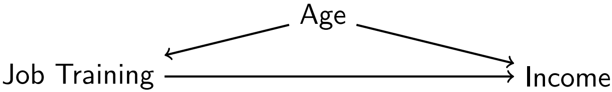

- Conditional exchangeability holds when conditioning on Age!

$$E(Y^{a=0} \mid A = 1, \text{Age} = \ell) = E(Y^{a=0} \mid A = 0, \text{Age} = \ell)$$

- Estimate

$$E(Y^{a=0} \mid A = 1) = \underbrace{\sum_{\ell} Pr(\text{Age} = \ell \mid A = 1) E(Y^{a=0} \mid A = 1, \text{Age} = \ell)}_{\text{Weighted average of averages}}$$

Example

$$\begin{tikzcd}
 & \text{Age} \arrow[dl] \arrow[dr] & \\
\text{Job Training} \arrow[rr] & & \text{Income}
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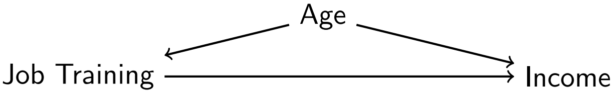

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&= \textstyle\sum_{\ell} Pr(\mathrm{Age} = \ell \mid \mathcal{M})\mathrm{E}(Y^{a=0} \mid A = 0, \mathrm{Age} = \ell)
\end{aligned}$$

- If we can make $Pr(\mathrm{Age} = \ell \mid \mathcal{M}) \approx Pr(\mathrm{Age} = \ell \mid A = 1)$, the two quantities should be the same

Matching: The big idea

Goal: Sample Average Treatment Effect on the Treated

$$E(Y^{a=1} \mid A = 1) - E(Y^{a=0} \mid A = 1)$$

Potential Solution: Create a group of untreated individuals, $\mathcal{M}$, which have a **similar distribution of** L to the treated group

$$\frac{1}{n_m} \sum_{i \in \mathcal{M}} Y_i \approx \frac{1}{n_t} \sum_{i : A_i = 1} Y_i^{a=0} \approx E(Y^{a=0} \mid A = 1)$$

Detail: How?

Example

Job training

Ind	Age	Y^{Train}	Y^{NoTrain}
1	20	19	?
2	25	63	?
3	38	65	?
4	38	43	?

Example

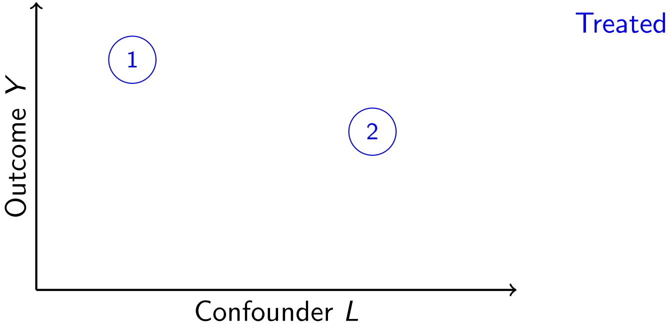
Job training

Ind	Age	Y^{Train}	Y^{NoTrain}
1	20	19	?
2	25	63	?
3	38	65	?
4	38	43	?

No job training

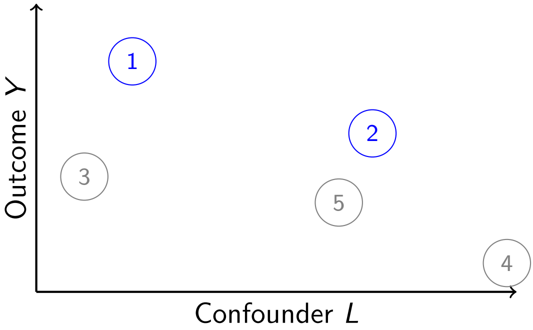
Ind	Age	Y^{NoTrain}
1	19	82
2	18	39
3	20	49
4	20	56
5	24	33
6	26	82
7	26	35
8	38	35
9	28	83
10	30	79
11	24	63
12	32	52
13	34	58
14	34	70
15	35	47
16	37	42
17	37	83
18	38	33
19	39	37
20	39	60

Matching: The big idea



You have a some treated units.

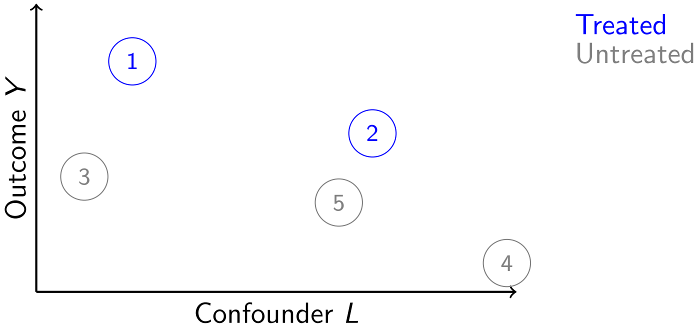
Matching: The big idea



Treated
Untreated

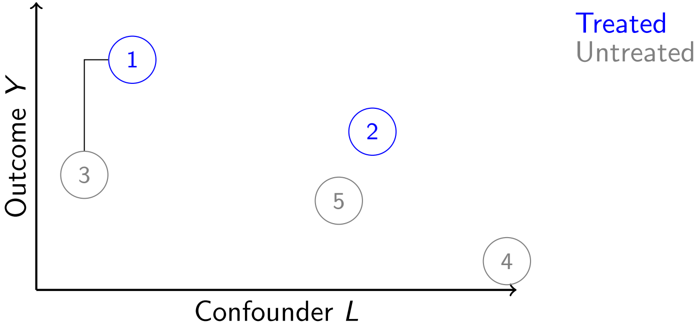
You go find some untreated units.

Matching: The big idea



You find the closest matches along L

Matching: The big idea



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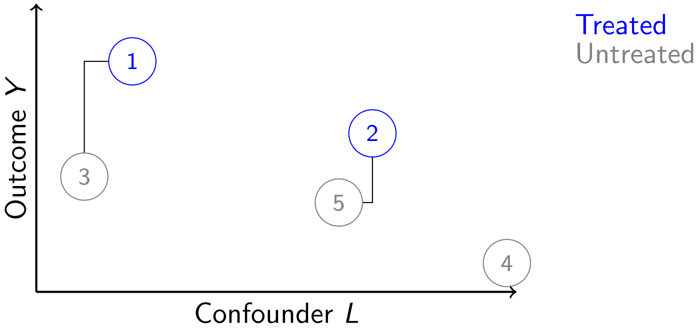

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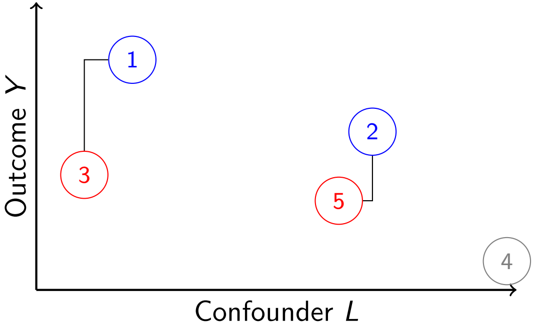
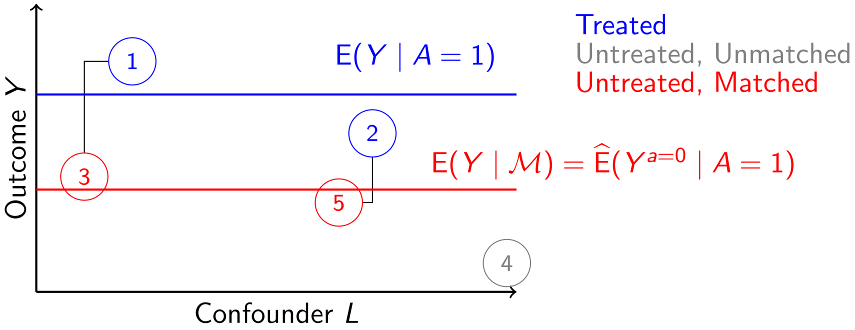


Treated
Untreated, Unmatched
Untreated, Matched

Compare the averages

Matching: The big idea



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4. Model-free*
   - * but you have to define what makes a match "good"

Bias vs variance

The idea of matching is straightforward, but the details matter!

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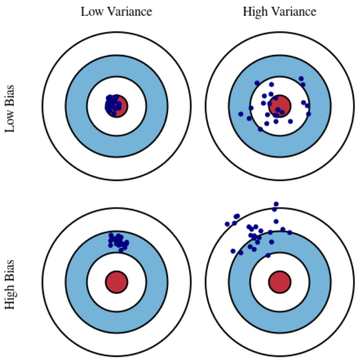
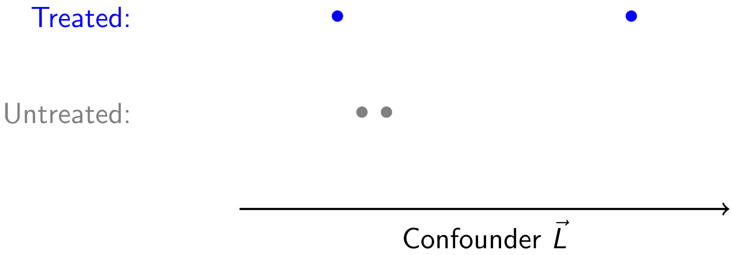


Matching in univariate settings: Algorithms

- Caliper or no caliper
- 1:1 vs k:1
- With replacement vs without replacement
- Greedy vs optimal

Caliper or no caliper matching



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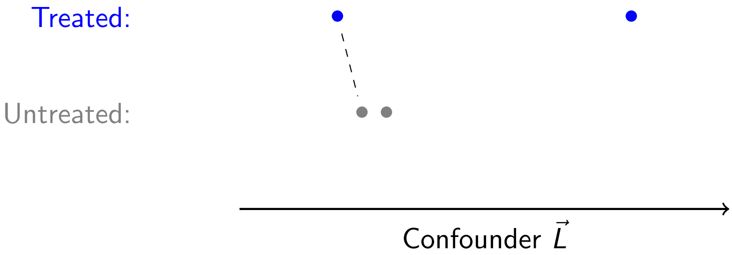

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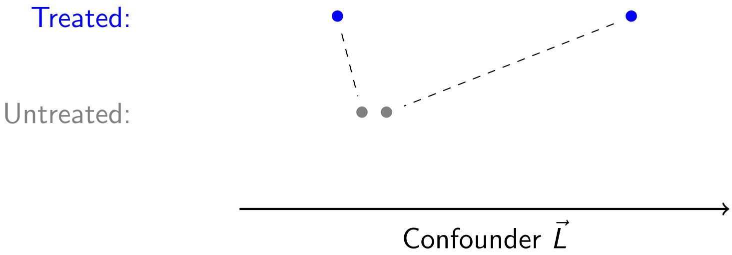

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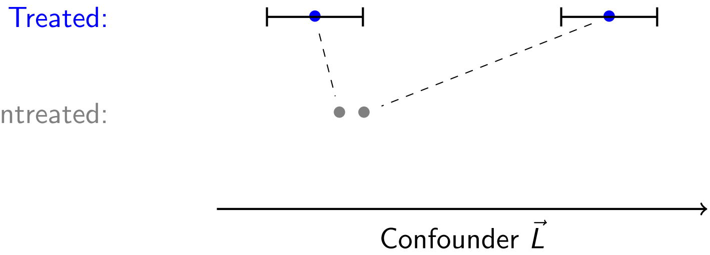

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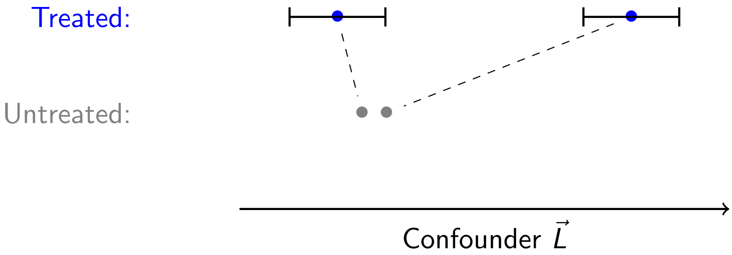

- Caliper: A radius around a treated unit such that we would rather drop the unit than make a match beyond that radius

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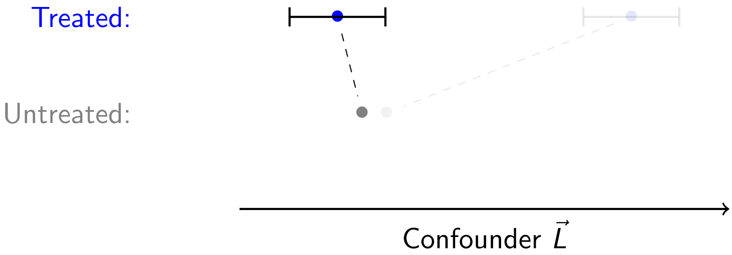

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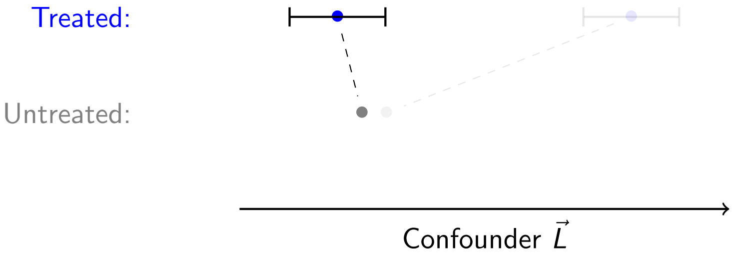
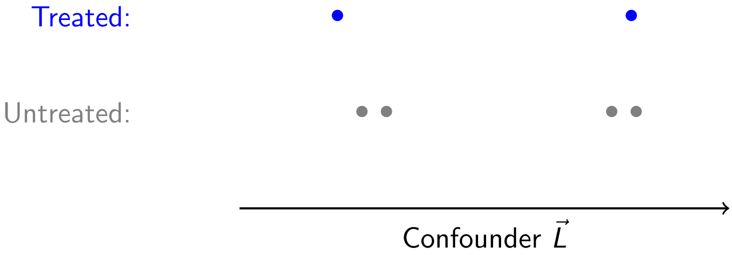


- Caliper: A radius around a treated unit such that we would rather drop the unit than make a match beyond that radius
- Feasible Sample Average Treatment Effect on the Treated (FSATT): Average among treated units for whom an acceptable match exists

1:1 vs k:1 matching



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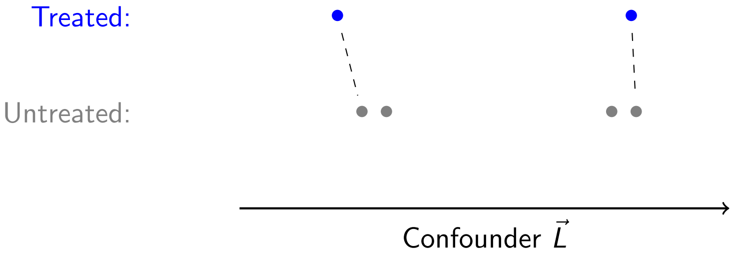

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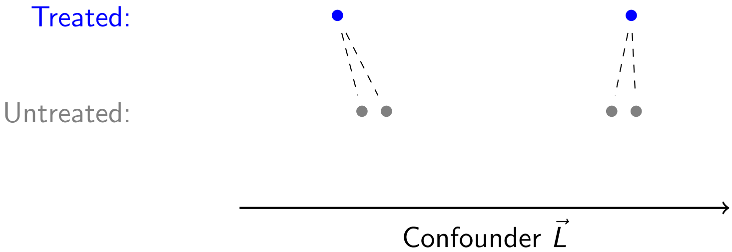

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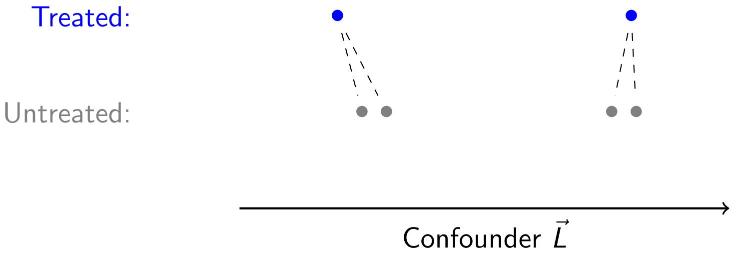
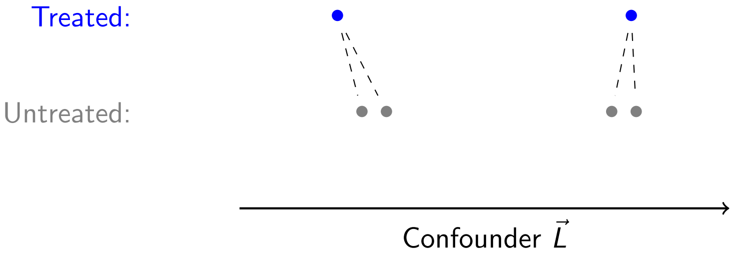


- Benefit of 2:1 matching
- Benefit of 1:1 matching

1:1 vs k:1 matching



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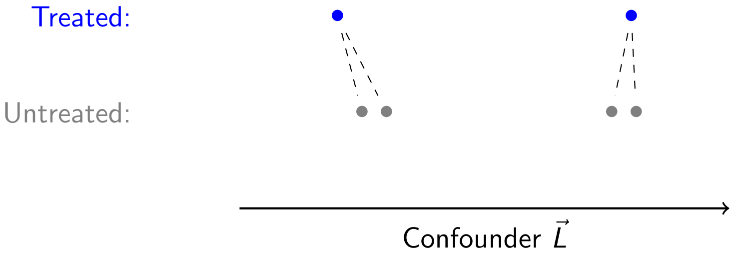
  - Lower variance. Averaging over more cases.
- Benefit of 1:1 matching

1:1 vs k:1 matching



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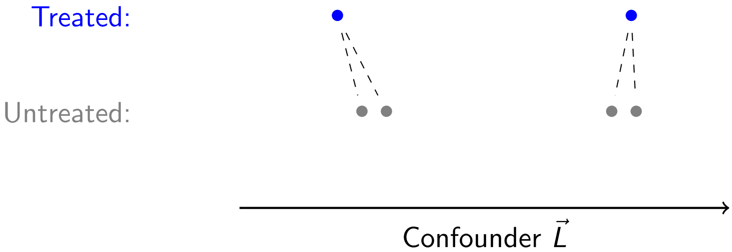

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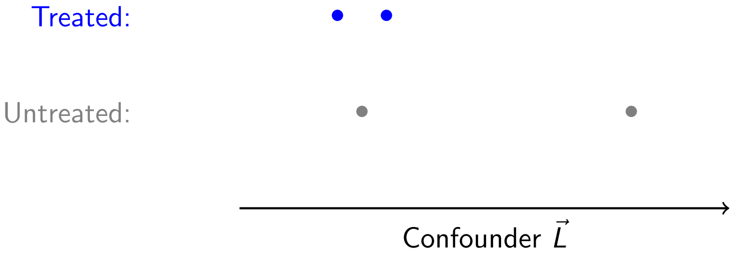
- Benefit of 1:1 matching
  - Lower bias. Only the best matches.
- Greater $k \to$ lower variance, higher bias

With replacement vs without replacement matching



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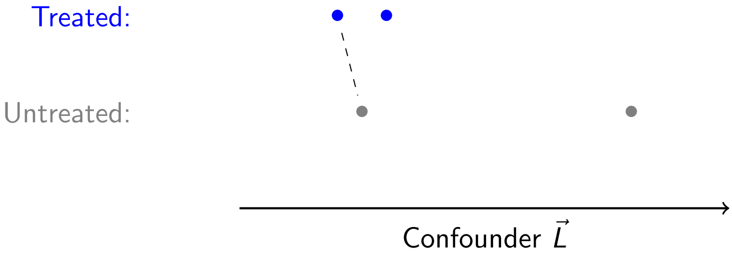

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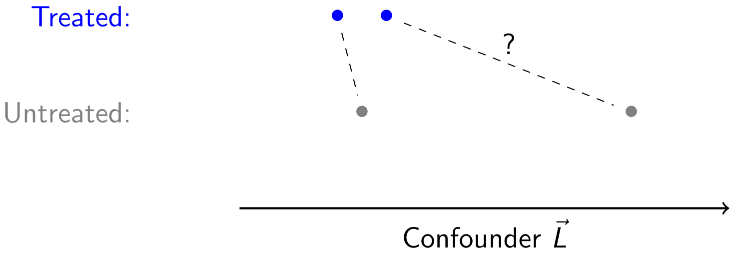

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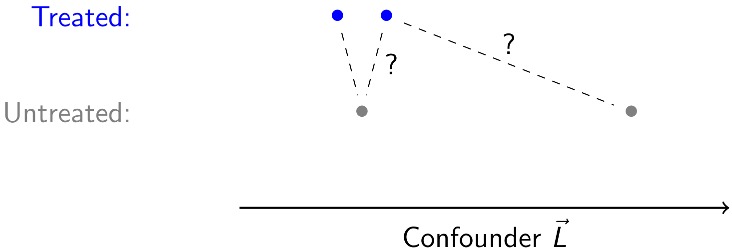
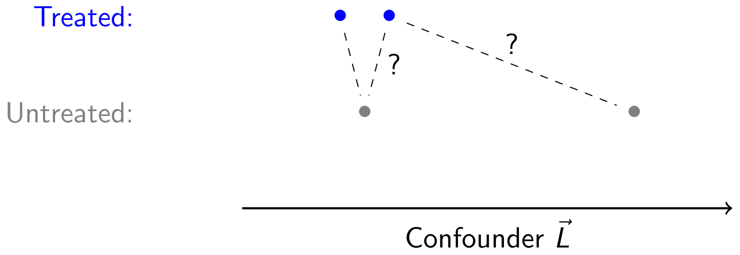


- Benefit of matching without replacement

- Benefit of matching with replacement

With replacement vs without replacement matching



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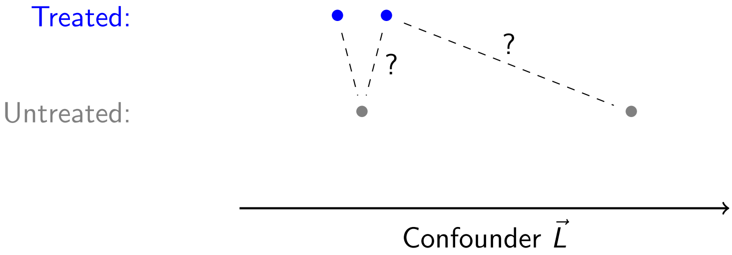
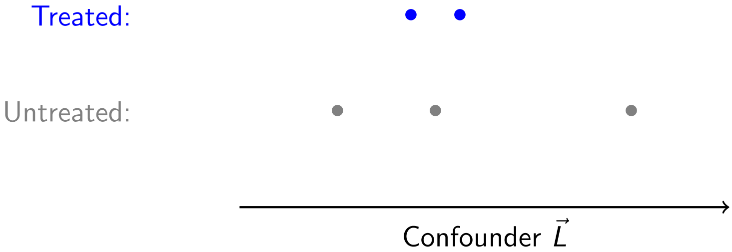


- Benefit of matching without replacement
  - Lower variance. Averaging over more cases.
- Benefit of matching with replacement
  - Lower bias. Better matches.

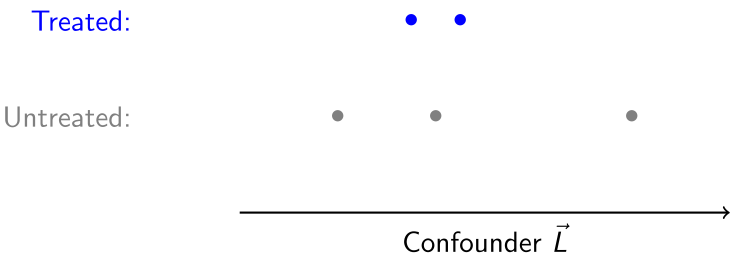
Greedy vs optimal matching[1]



[1] Gu, X. S., & Rosenbaum, P. R. (1993). Comparison of multivariate matching methods: Structures, distances, and algorithms. Journal of Computational and Graphical Statistics, 2(4), 405-420.

Greedy vs optimal matching[1]

Greedy Matching:
Match sequentially



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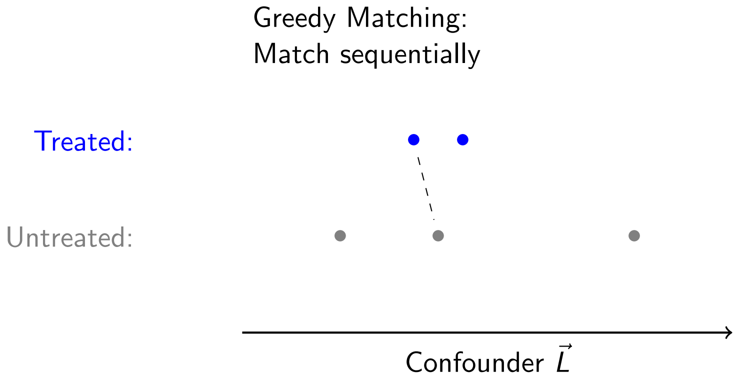

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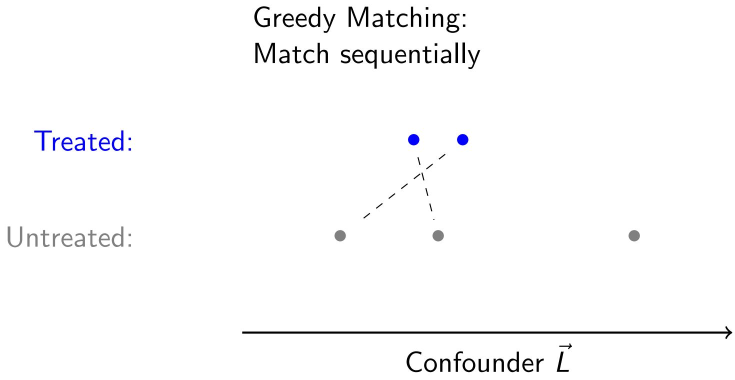

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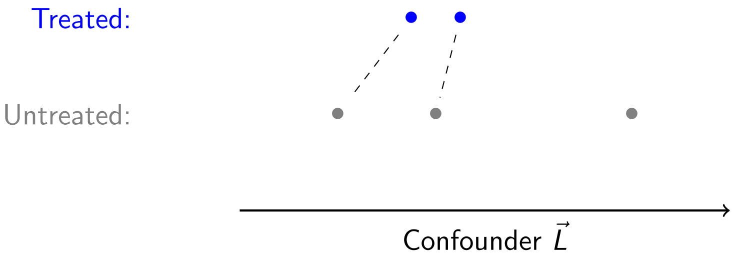
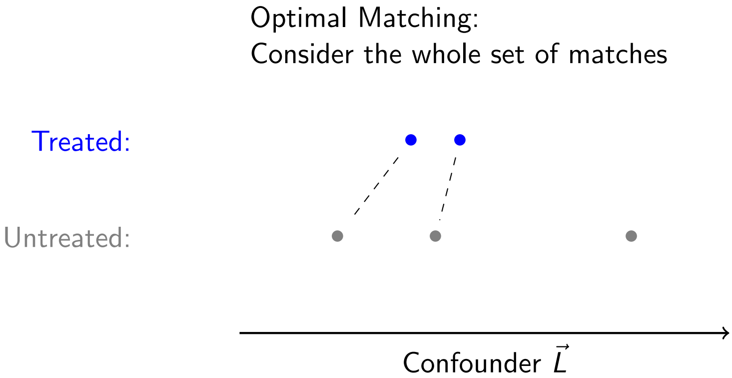
Optimal Matching:
Consider the whole set of matches



[1]Gu, X. S., & Rosenbaum, P. R. (1993). Comparison of multivariate matching methods: Structures, distances, and algorithms. Journal of Computational and Graphical Statistics, 2(4), 405-420.

Greedy vs optimal matching[1]



- Optimal is better. Just computationally harder.

[1] Gu, X. S., & Rosenbaum, P. R. (1993). Comparison of multivariate matching methods: Structures, distances, and algorithms. Journal of Computational and Graphical Statistics, 2(4), 405-420.

Matching in univariate settings: Algorithms

- Caliper or no caliper
- 1:1 vs k:1
- With replacement vs without replacement
- Greedy vs optimal

Matching in univariate settings: Algorithms

- Caliper or no caliper
- 1:1 vs k:1
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- Greedy vs optimal

Many reasonable choices, good choices depend on the data you have

Learning goals for today

At the end of class, you will be able to:

1. Explain how matching can be used to estimate causal effects
2. Explain bias variance trade-off in various matching procedures