Sufficient adjustment sets in DAGs

STSCI / INFO / ILRST 3900: Causal Inference

Sep 25 2025

Learning goals for today

At the end of class, you will be able to:

1. Identify a sufficient adjustment set using the backdoor criterion
2. Assess whether selection bias may hold in a gathered sample

Logistics

- Peer Review 2 Sep 26
- Quiz 2 Sep 30

Big picture

- Fundamental problem of causal inference:
  We want to know

$$\mathrm{E}(Y^a) \qquad \text{Causal Quantity}$$

  but only observe

$$\mathrm{E}(Y \mid A = a) \qquad \text{Observational Quantity}$$

- When exchangeability holds, we can estimate causal quantities using observed quantities

$$A \perp\!\!\!\perp (Y^1, Y^0) \qquad \Rightarrow \qquad \mathrm{E}(Y \mid A = a) = \mathrm{E}(Y^a)$$

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  $$A \perp\!\!\!\perp (Y^1, Y^0) \qquad \Rightarrow \qquad \mathrm{E}(Y \mid A = a) = \mathrm{E}(Y^a)$$

- Exchangeability typically doesn't hold in observational data, but conditional exchangeability is somtimes plausible

  $$A \perp\!\!\!\perp (Y^1, Y^0) \mid L \quad \Rightarrow \quad \mathrm{E}(Y \mid A = a, L = \ell) = \mathrm{E}(Y^a \mid L = \ell)$$

- Use standardization or inverse probability weighting to estimate the average causal effect

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- Given a graph and a conditioning set L, we can determine if conditional exchangeability holds or not
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  2. Check if each path is open or blocked conditional on L
  3. Conditional exchangeability holds if all open paths are causal paths

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If we believe the causal DAG, then we have reason to believe the estimated causal effect

Sufficient adjustment set

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- Various ways to do this, we will talk about one criterion

Open or blocked?

How to check if a path is open or blocked:

1. Traverse the path node by node
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Backdoor criterion

$$
\begin{tikzcd}
L \arrow[r] \arrow[rr, bend left] & A \arrow[r] & Y
\end{tikzcd}
$$

Backdoor criterion

$$\begin{tikzcd} L \arrow[r, red] \arrow[rr, red, bend left] & A \arrow[r] & Y \end{tikzcd}$$

Backdoor path starts with an edge pointing in to A and ends at Y

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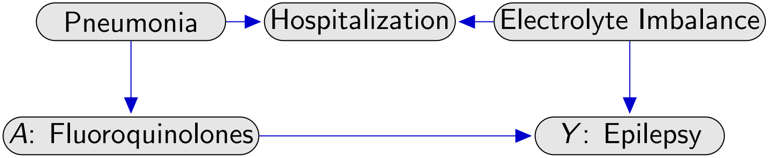
Sets that satisfy the backdoor criterion are sufficient adjustment sets!

Exercise

Researchers may be interested in the effect of fluoroquinolones, a class of antibiotics, on epilepsy[1]



- Does a sufficient adjustment set exist? If so, what is it?

[1] Example from "Using Causal Diagrams to Improve the Design and Interpretation of Medical Research" (Etminan et. al. 2020, Chest)

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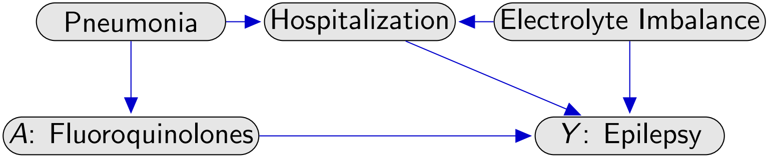

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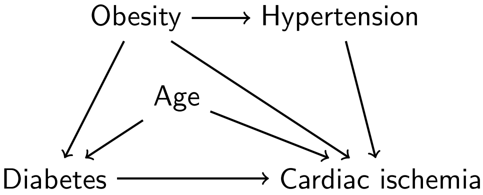
Exercise

Researchers may be interested in the effect of diabetes on cardiac Ischemia[3]



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[3] Example from "Using Causal Diagrams for Biomedical Research" (Kyriacou et. al. 2023, Annals of Emergency Medicine)

Selection Bias

- Sufficient adjustment set to close backdoor paths

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- Sufficient adjustment set to close backdoor paths
- Does not always mean conditioning on more things

$$
\begin{tikzcd}
Z_1 \arrow[dr] \arrow[dd] & & Z_3 \arrow[dl] \arrow[dd] \\
& Z_2 & \\
A \arrow[rr] & & Y
\end{tikzcd}
$$

Selection Bias

In some settings, certain variables may already be "conditioned on"

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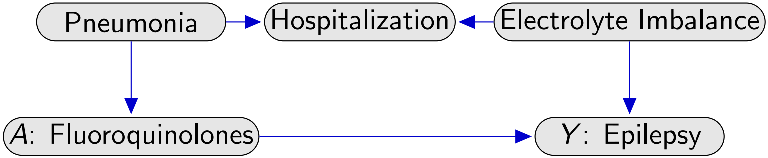

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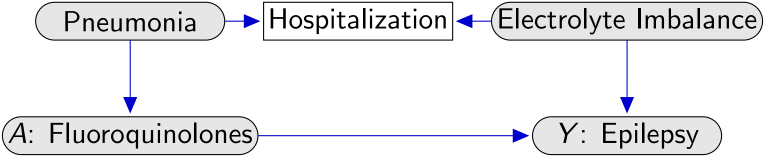

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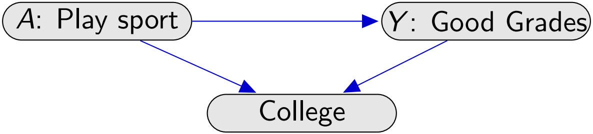

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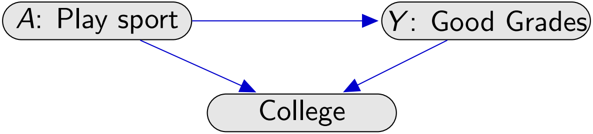
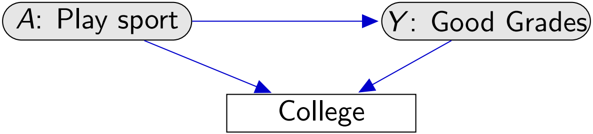


$$A \perp\!\!\!\perp Y^a$$

Selection Bias



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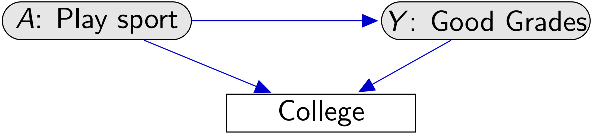

$$A \not\perp\!\!\!\perp Y^a \mid \text{College}$$

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- This may implicitly condition on a variable
- May open non-causal paths

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  - If it exists, use standardization or IPW to estimate causal effect
  - If it does not exist, consider gathering more variables
- Carefully consider the data gathering process

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- Carefully consider the data gathering process
- Causal claims come from assumptions + data

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