

# Sufficient adjustment sets in DAGs

INFO/STSCI/ILRST 3900: Causal Inference

1 Oct 2024

# Learning goals for today

At the end of class, you will be able to:

1. Identify a sufficient adjustment set using the backdoor criterion
2. Assess whether selection bias may hold in a gathered sample

# Logistics

- ▶ Project Task 1 due Thursday
- ▶ Read project description on [Course Project Page](#)
- ▶ Pset 3 will be posted Thursday

# Big picture

- Fundamental problem of causal inference:  
We want to know

$$E(Y^a)$$

Causal Quantity

but only observe

$$E(Y \mid A = a)$$

Observational Quantity

- When exchangeability holds, we can estimate causal quantities using observed quantities

$$A \perp\!\!\!\perp (Y^1, Y^0) \quad \Rightarrow \quad E(Y \mid A = a) = E(Y^a)$$

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$$A \perp\!\!\!\perp (Y^1, Y^0) \quad \Rightarrow \quad E(Y \mid A = a) = E(Y^a)$$

- Exchangeability typically doesn't hold in observational data, but conditional exchangeability is sometimes plausible

$$A \perp\!\!\!\perp (Y^1, Y^0) \mid L \quad \Rightarrow \quad E(Y \mid A = a, L = \ell) = E(Y^a \mid L = \ell)$$

- Use standardization or inverse probability weighting to estimate the average causal effect

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- ▶ Given a graph and a conditioning set  $L$ , we can determine if conditional exchangeability holds or not
  1. List out all paths from  $A$  to  $Y$
  2. Check if each path is open or blocked conditional on  $L$
  3. Conditional exchangeability holds if all open paths are causal paths



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  3. Conditional exchangeability holds if all open paths are causal paths

**If** we believe the causal DAG, then we have reason to believe the estimated causal effect

# Sufficient adjustment set

To conduct a causal analysis, we need to find a conditioning set of variables  $L$  which give us conditional exchangeability

- ▶ Find a set of variables  $L$  that blocks all non-causal paths from  $A$  and  $Y$
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- ▶  $L$  is called **sufficient adjustment set**
- ▶ Various ways to do this, we will talk about one criterion

# Open or blocked?

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# Open or blocked?

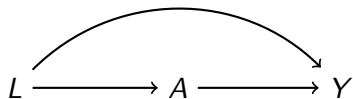
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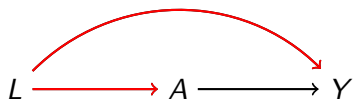
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# Backdoor criterion



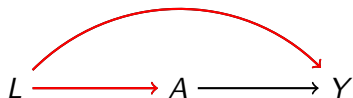


# Backdoor criterion



**Backdoor path** starts with an edge pointing in to  $A$  and ends at  $Y$

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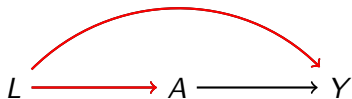


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A set of variables satisfies the backdoor criterion if

1. Blocks all backdoor paths
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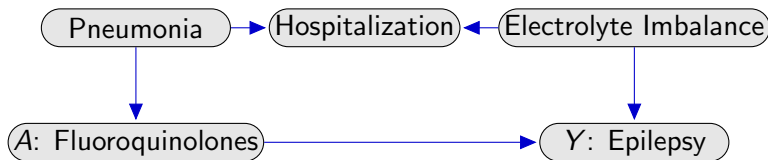
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Sets that satisfy the backdoor criterion are sufficient adjustment sets!

# Exercise

Researchers may be interested in the effect of fluoroquinolones, a class of antibiotics, on epilepsy<sup>1</sup>



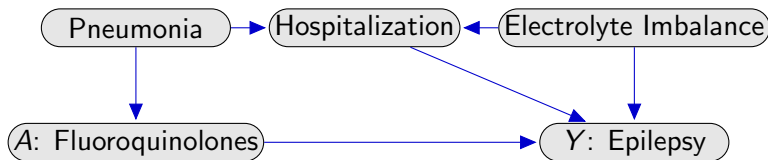
- Does a sufficient adjustment set exist? If so, what is it?

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<sup>1</sup>Example from “Using Causal Diagrams to Improve the Design and Interpretation of Medical Research” (Etminan et. al. 2020, Chest)

# Exercise

Researchers may be interested in the effect of fluoroquinolones, a class of antibiotics, on epilepsy<sup>2</sup>



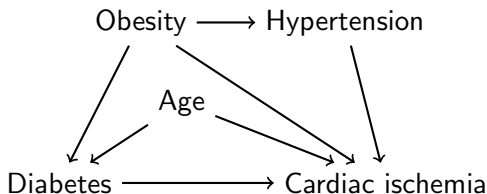
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# Exercise

Researchers may be interested in the effect of diabetes on cardiac Ischemia<sup>3</sup>



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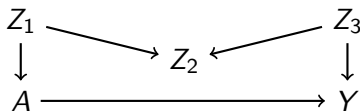
<sup>3</sup>Example from “Using Causal Diagrams for Biomedical Research”  
(Kyriacou et. al. 2023, Annals of Emergency Medicine )

# Selection Bias

- ▶ Sufficient adjustment set to close backdoor paths

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- ▶ Sufficient adjustment set to close backdoor paths
- ▶ Does not always mean conditioning on more things





# Selection Bias

In some settings, certain variables may already be “conditioned on”

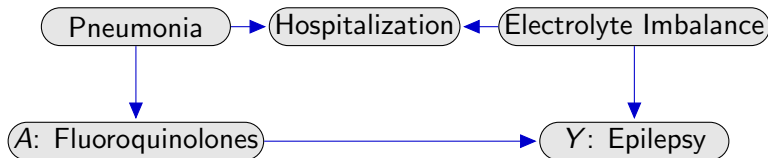
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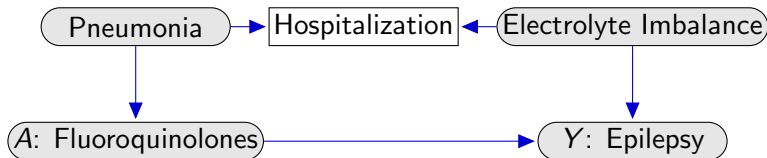
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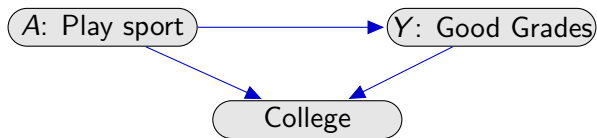
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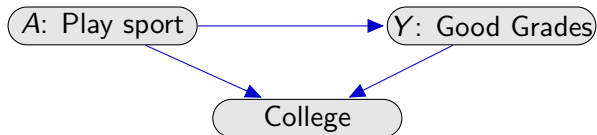
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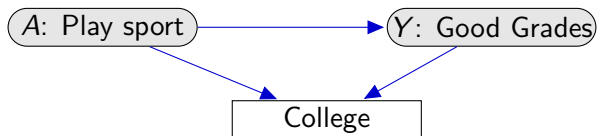


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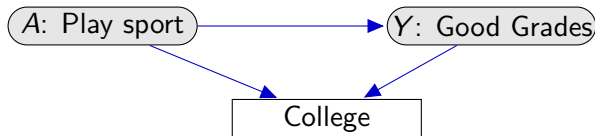
$$A \perp\!\!\!\perp Y^a$$

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$$A \not\perp Y^a \mid \text{College}$$

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- ▶ May open non-causal paths

# Takeaways

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- ▶ Clear rules for what to adjust for **if the DAG is true**
  - ▶ **Sufficient adjustment set** blocks all non-causal paths
  - ▶ If it exists, use standardization or IPW to estimate causal effect
  - ▶ If it does not exist, consider gathering more variables
- ▶ Carefully consider the data gathering process

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  - ▶ If it does not exist, consider gathering more variables
- ▶ Carefully consider the data gathering process
- ▶ Causal claims come from assumptions + data

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