Exchangeability (cont) and conditionally randomized experiments

INFO/STSCI/ILRST 3900: Causal Inference

4 Sep 2025

Learning goals for today

At the end of class, you will be able to:

1. Explain conditionally randomized experiments
2. Identify the "idealized experiment" as a goal

Logistics

- Problem Set 1 is due on Today at 5pm on Canvas
- Ch 2.1 and 2.2 in Hernan and Robins 2023

Exchangeability

Definition: Exchangeability means that the potential outcomes, $Y^{a=1}$ and $Y^{a=0}$, are independent of the observed treatment (A)

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In mathematical notation,

$$\underbrace{Y^{a=1}, Y^{a=0}}_{\text{potential outcomes}} \perp\!\!\!\perp \underbrace{A}_{\text{observed treatment}}$$

Exchangeability: By an experimental procedure

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Flip a coin

Two groups of people: heads and tails

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Study 1

Heads $\rightarrow$ job training

Tails $\rightarrow$ no job training

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$\swarrow$

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$\searrow$

Compare employment rates
of those with and without
job training

Exchangeability: By an experimental procedure

Flip a coin

Two groups of people: heads and tails

↙ ↘

Study 1

Heads → job training

Tails → no job training

Study 2

Tails → job training

Heads → no job training

↘ ↙

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Exchangeability: By an experimental procedure

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Question: Are both studies valid?

Exchangeability: By an experimental procedure

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Two groups of people: heads and tails

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Compare employment rates
of those with and without
job training

Question: Are both studies valid?

Yes. The (H/T) groups are **exchangeable**.
Any statistical pattern between (H/T) and employment can only arise from the causal effect of job training

Why is exchangeability good?

When exchangeability is true, it implies

$$\underbrace{\mathrm{E}(Y^{a=1} \mid A = 1)}_{\text{Within treated}} = \underbrace{\mathrm{E}(Y^{a=1} \mid A = 0)}_{\text{Within not treated}} = \underbrace{\mathrm{E}(Y^{a=1})}_{\text{everyone}}$$

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This allows us to identify the average causal effect (ACE)

$$\text{ACE} = \underbrace{\mathsf{E}(Y^{a=1})}_{\text{if everyone is treated}} - \underbrace{\mathsf{E}(Y^{a=0})}_{\text{if no-one is treated}}$$

because we can plug-in

$$\underbrace{\mathsf{E}(Y^{a=1} \mid A=1)}_{\substack{\text{outcomes for people who} \\ \text{are \textbf{actually} treated}}} \quad \text{and} \quad \underbrace{\mathsf{E}(Y^{a=0} \mid A=0)}_{\substack{\text{outcomes for people who} \\ \text{are \textbf{actually} not treated}}}$$

When does exchangeability hold?

- Data does not tell us directly whether exchangeability holds
- We must know how the data was gathered
- Exchangeability holds by design in experiments

Exercise

- Exchangeability implies that $Y^a \perp\!\!\!\perp A$ for all treatment values a
- How is this different than $Y \perp\!\!\!\perp A$?
- In randomized experiments, $Y^a \perp\!\!\!\perp A$ is usually true. Is $Y \perp\!\!\!\perp A$ ever true?

Limits of experiments

Experiments may not be possible because of

- **Feasibility**: What is the causal effect on global average temperature of decreasing global CO_2 levels by 100 ppm?

Limits of experiments

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- **Cost**: What is the causal effect of giving every student a Lamborghini on traffic in Collegetown?

Limits of experiments

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- **Feasibility**: What is the causal effect on global average temperature of decreasing global CO_2 levels by 100 ppm?
- **Cost**: What is the causal effect of giving every student a Lamborghini on traffic in Collegetown?
- **Ethics**: What is the causal effect on cancer of smoking cigarettes?

Making decisions with data

- Randomized experiments are powerful tools for learning causal relationships
- Experiments may have negative effect on participants or larger population[1]
- Belmont Report[2]
  - Respect for persons: protect personal autonomy
  - Beneficence: Do no harm
  - Justice: distribute the burden/benefits fairly

[1] Mcdermott and Hatemi PNAS 2020 https://www.pnas.org/doi/10.1073/pnas.2012021117

[2] https://www.hhs.gov/ohrp/regulations-and-policy/belmont-report/read-the-belmont-report/index.html

Making decisions with data

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- Experiments may have negative effect on participants or larger population[1]
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  - Respect for persons: protect personal autonomy
  - Beneficence: Do no harm
  - Justice: distribute the burden/benefits fairly
- Causal inference with observational data is even more important
- Causal inference (at it's best) tells you what could be, not what ought to be

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Why are experiments good?

- Experiments are great because exchangeability holds by design
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Why are experiments good?

- Experiments are great because exchangeability holds by design
- To estimate the causal effect from experimental data, we can simply take the difference in observed means
- But they are also great for other reasons

Precise questions

Experiments allow us to answer precise questions

What is the causal effect of vaccination on covid?

Precise questions

Experiments allow us to answer precise questions

What is the causal effect of Two shots of the Pfizer vaccine 21 days apart on Covid?

Precise questions

Experiments allow us to answer precise questions

What is the causal effect of Two shots of the Pfizer vaccine 21 days apart on a positive Covid test within 14 weeks of vaccination in 2020?

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- Experiments allow us to (more easily) specify precise treatments and outcomes

Idealized experiment as goal

- Formulate a precise causal question
  - Treatment
  - Outcome and timeframe
  - Population of interest
- Experiments are "gold standard" for estimating causal effects
- Imagine the "ideal experiment" to answer

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- Formulate a precise causal question
  - Treatment
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- Experiments are "gold standard" for estimating causal effects
- Imagine the "ideal experiment" to answer
- Try to replicate the "ideal experiment" with an observational analysis

Conditional randomization

Exchangeability may not hold in every randomized experiment

- "With the use of an interactive Web-based system, participants in the trial were randomly assigned in a 1:1 ratio to receive 30 μg of BNT162b2 (0.3 ml volume per dose) or saline placebo."

Conditional randomization

Exchangeability may not hold in every randomized experiment

- "With the use of an interactive Web-based system, participants in the trial were randomly assigned in a 1:1 ratio to receive 30 μg of BNT162b2 (0.3 ml volume per dose) or saline placebo."
- Suppose the researchers instead split the group into two groups
  - Age ≥ 55: Vaccine with probability $2/3$
  - Age < 55: Vaccine with probability $1/2$
- Does exchangeability still hold?

Conditional randomization

Exchangeability may not hold in every randomized experiment

- Age ≥ 55 more likely to get vaccine; more likely to get COVID if treated
- Age ≥ 55 less likely to get vaccine; less likely to get COVID if treated

Conditional randomization

Exchangeability may not hold in every randomized experiment

- Age ≥ 55 more likely to get vaccine; more likely to get COVID if treated
- Age ≥ 55 less likely to get vaccine; less likely to get COVID if treated
- Vaccinated individuals have $Y^{a=1}$ that are more likely to be 1 than unvaccinated individuals
- Exchangeability does not hold in entire population
- Exchangeability holds within each sub-population
- Two separate experiments; both are exchangeable

Conditional randomization

- **Marginal exchangeability:** $Y^a \perp\!\!\!\perp A$ for all a
- **Conditional exchangeability:** $Y^a \perp\!\!\!\perp A \mid L$ for all a
  The potential outcomes are independent of treatment **conditional on** L

Conditional randomization

- **Marginal exchangeability:** $Y^a \perp\!\!\!\perp A$ for all a
- **Conditional exchangeability:** $Y^a \perp\!\!\!\perp A \mid L$ for all a
  The potential outcomes are independent of treatment **conditional on** L
- If you tell me $A_i = 1$, I learn something about $Y_i^{a=1}, Y_i^{a=0}$
- Suppose you first tell me someone's age, I learn something about $Y_i^{a=1}, Y_i^{a=0}$. Next you tell me $A_i = 1$, I don't learn anything new about $Y_i^{a=1}, Y_i^{a=0}$ (in addition to what I previously knew)

Conditional randomization

- **Stratification**: We can directly estimate causal effect within each sub-population (or stratum)
- If the treatment effect varies across sub-population, we say there is **treatment effect heterogeneity**

Conditional randomization

- Most useful as an idealized experiment to target with observational analysis
- Marginal exchangeability is very unlikely in observational data
- Conditional exchangeability may be more reasonable

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