Defining causal effects

Cornell STSCI / INFO / ILRST 3900: Causal Inference
Fall 2023

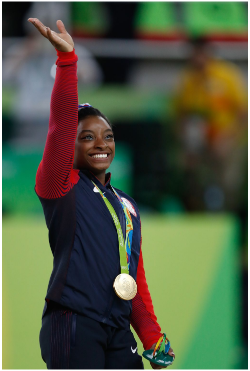
24 Aug 2022

Learning goals for today

By the end of class, you will be able to

- explain the fundamental problem of causal inference and the need for causal arguments
- define potential outcomes
- recall mathematical concepts from probability
  - random variables
  - expectation
  - conditional expectation

Causal claims hinge on arguments, not on data



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	Swing	Do not swing	of swinging
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Fundamental problem of causal inference

Holland 1986

Descriptive evidence

average lifespan − average lifespan

	Outcome under Mediterranean diet	Outcome under standard diet
Person 1	lifespan	
Person 2		lifespan
Person 3	lifespan	
Person 4		lifespan
Person 5	lifespan	
Person 6	lifespan	
Person 7		lifespan
Person 8	lifespan	

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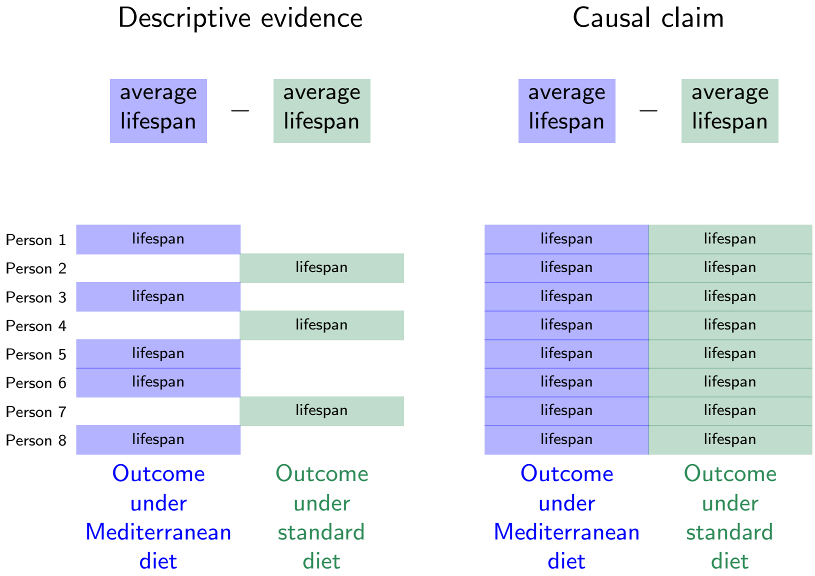

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	Outcome under Mediterranean diet	Outcome under standard diet
Person 1	lifespan	missing
Person 2	missing	lifespan
Person 3	lifespan	missing
Person 4	missing	lifespan
Person 5	lifespan	missing
Person 6	lifespan	missing
Person 7	missing	lifespan
Person 8	lifespan	missing

Causal claim

average lifespan − average lifespan

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Fundamental problem of causal inference

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Causal claim

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Causal inference is a **missing data** problem

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Examples:

$Y_{\text{Ian}} = \texttt{survived}$	Ian survived

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$Y_{\text{Ian}}^{\text{MediterraneanDiet}} = \texttt{survived}$	Ian would survive on a Mediterranean diet

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Discuss.

Which potential outcome is observed?

Which is counterfactual?

The consistency assumption

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$Y_i^{\text{MediterraneanDiet}}$

$Y_i^{\text{StandardDiet}}$

Potential Outcomes

The consistency assumption

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Potential Outcomes

Y_i

Factual Outcomes

The consistency assumption

Consistency Assumption

$$Y_i^{A_i} = Y_i$$

$Y_i^{\text{MediterraneanDiet}}$

$Y_i^{\text{StandardDiet}}$

Potential Outcomes

Y_i

Factual Outcomes

Mathematical notation: Potential outcomes are fixed

A person's potential outcome is a **fixed quantity**

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- Draw a random person from the population

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- The outcome $Y^{\text{MediterraneanDiet}}$ is a random variable:
  - takes the value `survived` if we randomly sample some people
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Check for understanding:

Does it make sense to write $\text{V}(Y_i^a)$? How about $\text{V}(Y^a)$

Notation: Expectation operator

The **expectation operator** E() denotes the population mean

$$\mathrm{E}(Y^a) = \frac{1}{n}\sum_{i=1}^{n} Y_i^a$$

The quantity Y^a inside the expectation must be a random variable

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A **conditional expectation** is denoted with a vertical bar

$$\text{E}(Y \mid A = a) = \frac{1}{n_a}\sum_{i:A_i=a} Y_i$$

Practice: How would you say this in English?

We might wonder how a person's earnings relate to whether they hold a college degree

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2. $\text{E}(\text{Earnings}^{\text{Degree}=\texttt{TRUE}}) > \text{E}(\text{Earnings}^{\text{Degree}=\texttt{FALSE}})$
    - On average, a degree causes higher earnings

Practice: How would you write this in math?

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