Difference in difference: Extensions

INFO/STSCI/ILRST 3900: Causal Inference

6 Nov 2025

Logistics

- PSET 5 Due Nov 11
- Peer Review 5 and Quiz 5 on Nov 25
- Project Part 2 Instructions posted: Due Dec 8
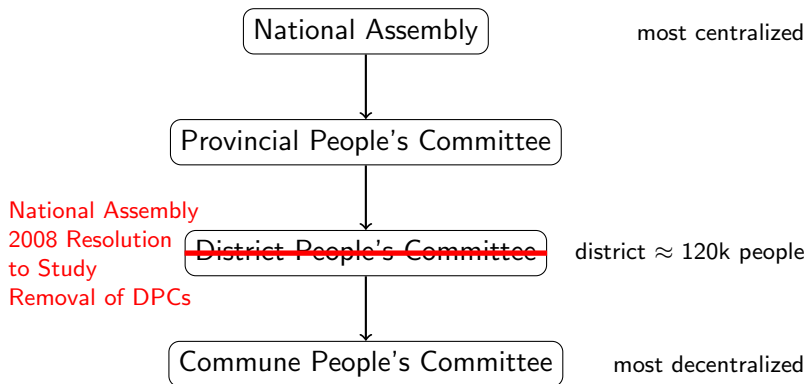
Learning goals for today

At the end of class, you will be able to:

1. Use pre-treatment periods to
   - ▶ assess underlying assumptions
   - ▶ improve estimation accuracy
2. and recognize that the parallel assumption remains untestable

Malesky, E. J., Nguyen, C. V., & Tran, A. (2014).
The impact of recentralization on public services:
A difference-in-differences analysis of the abolition of elected councils in Vietnam.
American Political Science Review, 108(1), 144-168.

Does government work better when it is centralized or decentralized?

Vietnam setting: A study of recentralization

National Assembly — most centralized

Provincial People's Committee

National Assembly 2008 Resolution to Study Removal of DPCs

District People's Committee — district ≈ 120k people

Commune People's Committee — most decentralized

Vietnam setting: A study of recentralization

Input from social scientists

Vietnam setting: A study of recentralization

Input from social scientists
1. Enough treated units to study

Vietnam setting: A study of recentralization

Input from social scientists

1. Enough treated units to study
2. Sampling stratified by region

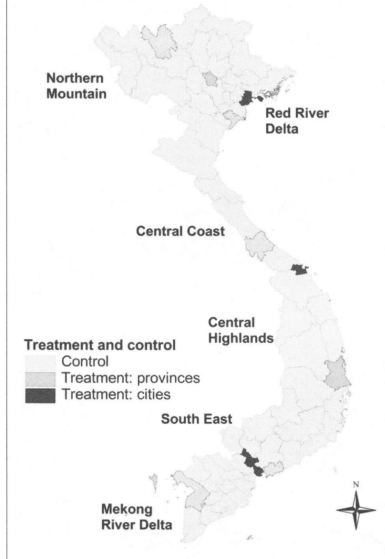# Vietnam setting: A study of recentralization

Input from social scientists

1. Enough treated units to study
2. Sampling stratified by region
3. Sampling stratified by
   - ▶ city versus rural
   - ▶ lowland versus highland
   - ▶ midland versus inter-nationally bordered land

Vietnam setting: A study of recentralization

Input from social scientists

1. Enough treated units to study
2. Sampling stratified by region
3. Sampling stratified by
   - city versus rural
   - lowland versus highland
   - midland versus inter-nationally bordered land
4. Sampling stratified by socioeconomic and public administration performance

FIGURE 2. Map of Treatment Provinces and National-Level Cities

Northern Mountain

Red River Delta

Central Coast

Central Highlands

Treatment and control
Control
Treatment: provinces
Treatment: cities

South East

Mekong River Delta

N

Vietnam Household Living Standards Survey
Reports by each local commune by commune leaders

- ▶ 2006 and 2008: Before DPC abolition
- ▶ 2010: After DPC abolition

One outcome we will examine:
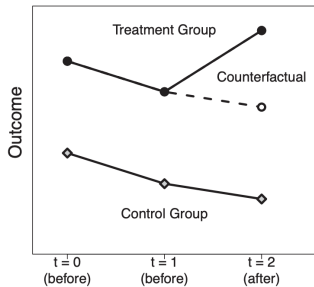Is there the following project in the commune?
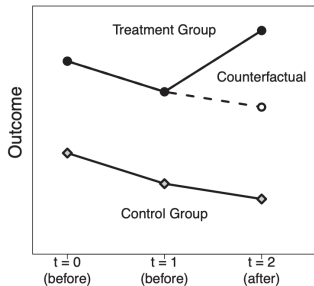
- ▶ Investment on culture and education

Education and Cultural Program
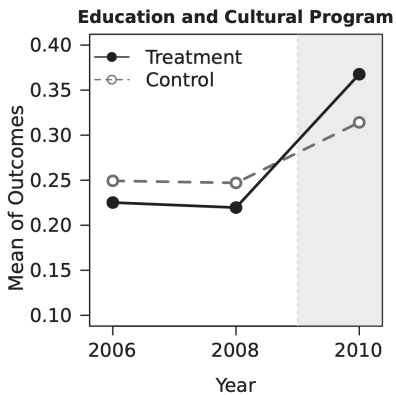
Screenshot of re-analysis by Egami & Yamauchi 2023, Fig 3

Egami, N., & Yamauchi, S. (2023).
Using multiple pretreatment periods to improve
difference-in-differences and staggered adoption designs.
Political Analysis, 31(2), 195-212.

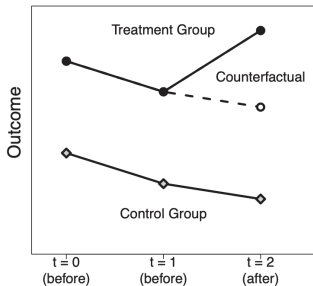# Difference in difference

Difference in difference

Notation

$$Y^{\text{treatment value}}_{(\text{unit})(\text{time})}$$

Example: Y^0_{i1}
is unit i at time 1
under treatment 0
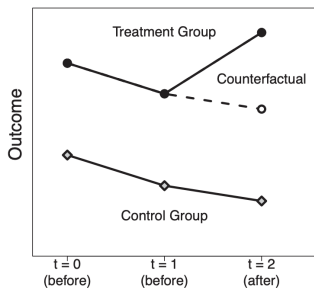
Difference in difference

Notation

$$Y^{\text{treatment value}}_{(\text{unit})(\text{time})}$$

Example: Y^0_{i1}
is unit i at time 1
under treatment 0

Parallel Trends Assumption
(untestable)

$$E(Y^0_{\text{Treated},2} - Y^0_{\text{Treated},1})$$
$$=$$
$$E(Y^0_{\text{Control},2} - Y^0_{\text{Control},1})$$

Difference in difference

Notation

$Y_{\text{(unit)(time)}}^{\text{treatment value}}$

Example: Y_{i1}^0
is unit i at time 1
under treatment 0

Parallel Trends Assumption
(untestable)

$$E(Y_{\text{Treated},2}^0 - Y_{\text{Treated},1}^0)$$
$$=$$
$$E(Y_{\text{Control},2}^0 - Y_{\text{Control},1}^0)$$

Extended Parallel Trends
(testable)

$$E(Y_{\text{Treated},1}^0 - Y_{\text{Treated},0}^0)$$
$$=$$
$$E(Y_{\text{Control},1}^0 - Y_{\text{Control},0}^0)$$

Extended Parallel Trends

Treatment Group

Counterfactual

Control Group

Extended Parallel Trends Violated

t = 0 (before) t = 1 (before) t = 2 (after)

Outcome

FIGURE 2. Map of Treatment Provinces and National-Level Cities

FIGURE 2. Map of Treatment Provinces and National-Level Cities

Northern Mountain

Red River Delta

Central Coast

Central Highlands

Treatment and control
Control
Treatment: provinces
Treatment: cities

South East

Mekong River Delta
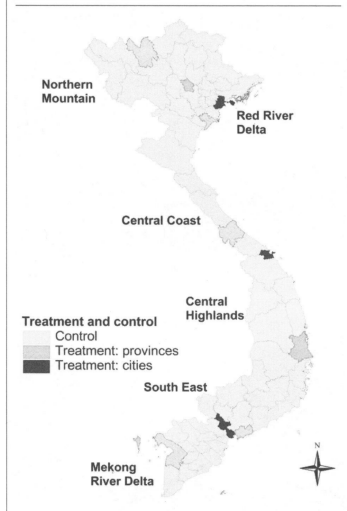
Outcome 1
Education and cultural programs

Is there the following project in the commune?
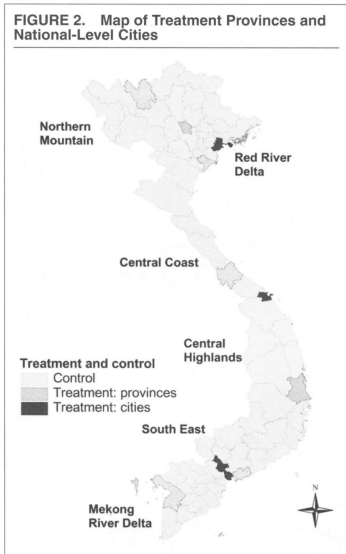
  Investment on culture and education

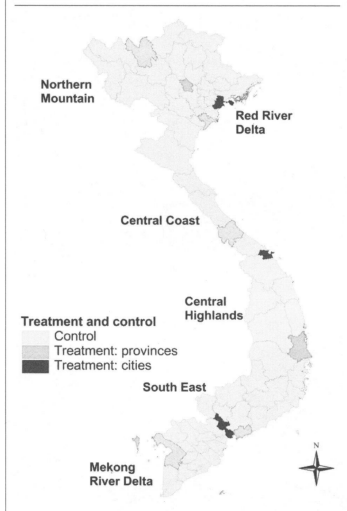FIGURE 2. Map of Treatment Provinces and National-Level Cities

Northern Mountain

Red River Delta

Central Coast

Central Highlands

Treatment and control
- Control
- Treatment: provinces
- Treatment: cities

South East

Mekong River Delta

Outcome 2
Tap water

Is there the following project in the commune?

Coded 1
Indoor private piped water
Outdoor private piped water
Public piped water

Coded 0
Well water
Well with protection walls
Well without protection walls
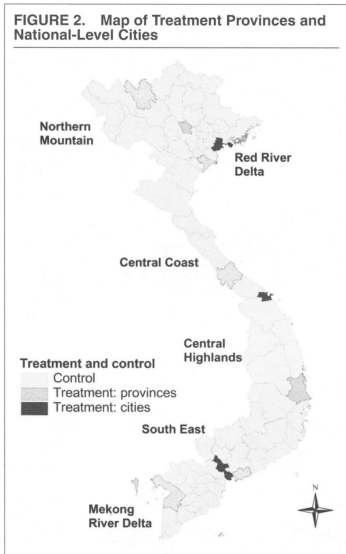Stream water with protection
Stream water without protection
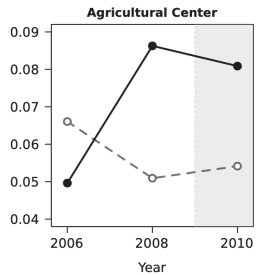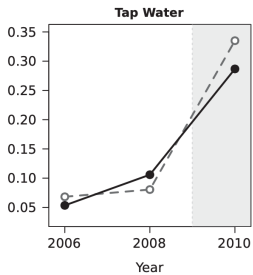Rainwater
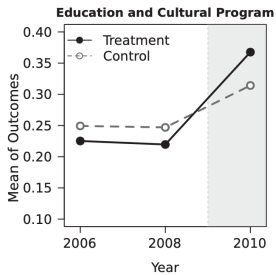Bottled water
Water brought by pedicab
Tank water
river lake pond

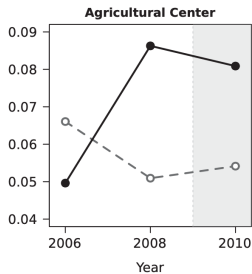FIGURE 2. Map of Treatment Provinces and National-Level Cities

Outcome 3
Agricultural center

Is there any agriculture extension center in this commune?

Education and Cultural Program — Tap Water — **Agricultural Center**

In each case, do you believe parallel trends?

Education and Cultural Program / Tap Water / Agricultural Center
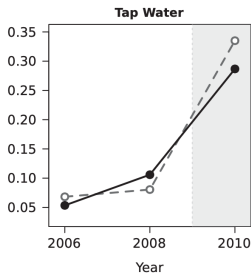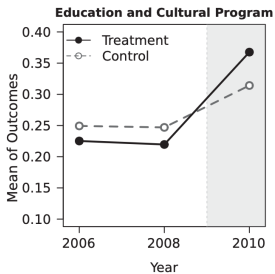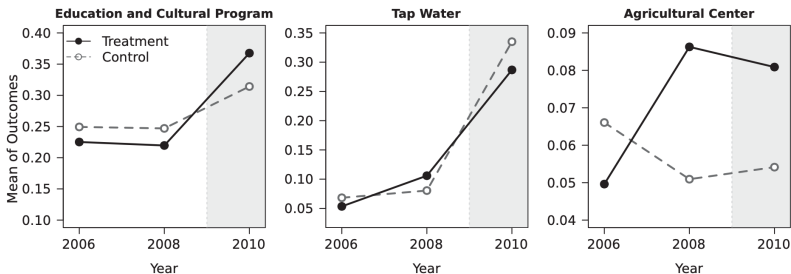
In each case, do you believe parallel trends?

Table 2. Assessing underlying assumptions using the pretreatment outcomes.

|	Estimate	Std. error	*p*-value	95% Std. equivalence CI
Education and cultural program	−0.007	0.096	0.940	$[-0.166, 0.166]$
Tap water	0.166	0.083	0.045	$[-0.302, 0.302]$
Agricultural center	0.198	0.082	0.015	$[-0.332, 0.332]$

Benefit 1: Assessing assumptions

Pre-treatment periods enable us to
assess underlying ssumptions

Parallel trends is untestable, but being parallel
in the pre-treatment period builds confidence

Benefit 2: Improving efficiency

Pre-treatment periods also enable us to
improve estimation accuracy
when parallel trends holds
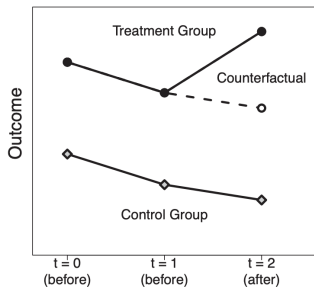
Benefit 2: Improving efficiency
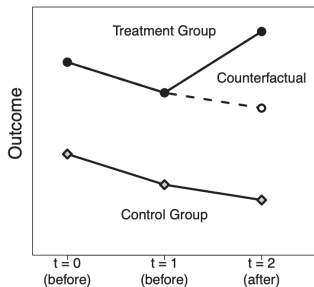
Estimator 1

Estimator 2

Notation

$Y_{(unit)(time)}^{treatment\ value}$
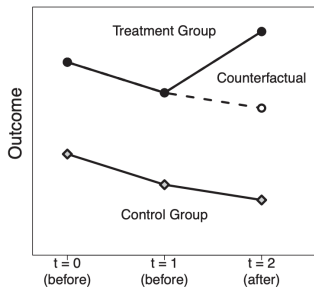
Benefit 2: Improving efficiency

Estimator 1

$$\underbrace{(\bar{Y}^1_{T2} - \bar{Y}^0_{T1})}_{\substack{\text{Treatment Group} \\ \text{Time 2 - Time 1}}} - \underbrace{(\bar{Y}^0_{C2} - \bar{Y}^0_{C1})}_{\substack{\text{Control Group} \\ \text{Time 2 - Time 1}}}$$

Estimator 2

Notation

$Y^{\text{treatment value}}_{(\text{unit})(\text{time})}$

Benefit 2: Improving efficiency

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Estimator 2

$$\underbrace{(\bar{Y}^1_{T2} - \bar{Y}^0_{T0})}_{\substack{\text{Treatment Group} \\ \text{Time 2 - Time 0}}} - \underbrace{(\bar{Y}^0_{C2} - \bar{Y}^0_{C0})}_{\substack{\text{Control Group} \\ \text{Time 2 - Time 0}}}$$

Notation

$$Y^{\text{treatment value}}_{\text{(unit)(time)}}$$

Benefit 2: Improving efficiency

Estimator 1

$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T1}^0)}_{\substack{\text{Treatment Group} \\ \text{Time 2 - Time 1}}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C1}^0)}_{\substack{\text{Control Group} \\ \text{Time 2 - Time 1}}}$$
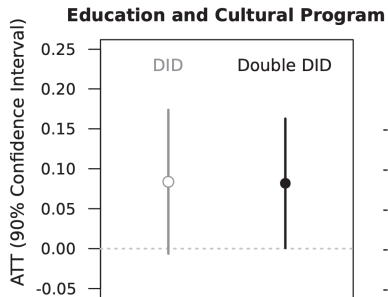
Estimator 2

$$\underbrace{(\bar{Y}_{T2}^1 - \bar{Y}_{T0}^0)}_{\substack{\text{Treatment Group} \\ \text{Time 2 - Time 0}}} - \underbrace{(\bar{Y}_{C2}^0 - \bar{Y}_{C0}^0)}_{\substack{\text{Control Group} \\ \text{Time 2 - Time 0}}}$$

Notation

$$Y_{\text{(unit)(time)}}^{\text{treatment value}}$$

Pooled estimator:
Average the two!

Benefit 2: Improving efficiency

Education and Cultural Program

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