

# Exchangeability (cont) and conditionally randomized experiments

INFO/STSCI/ILRST 3900: Causal Inference

29 Aug 2023

# Learning goals for today

At the end of class, you will be able to:

1. Explain conditionally randomized experiments
2. Identify the “idealized experiment” as a goal

# Logistics

- ▶ Problem Set 1 is due on Today at 5pm on Canvas
- ▶ Ch 2.1 and 2.2 in Hernan and Robins 2023

# Exchangeability

**Definition:** Exchangeability means that the potential outcomes,  $Y^{a=1}$  and  $Y^{a=0}$ , are independent of the observed treatment ( $A$ )

# Exchangeability

**Definition:** Exchangeability means that the potential outcomes,  $Y^{a=1}$  and  $Y^{a=0}$ , are independent of the observed treatment ( $A$ )

In mathematical notation,

$$\underbrace{Y^{a=1}, Y^{a=0}}_{\text{potential outcomes}} \perp\!\!\!\perp \underbrace{A}_{\text{observed treatment}}$$

**Exchangeability:** By an experimental procedure

**Exchangeability:** By an experimental procedure

Flip a coin

Two groups of people: heads and tails

**Exchangeability**: By an experimental procedure

Flip a coin

Two groups of people: heads and tails



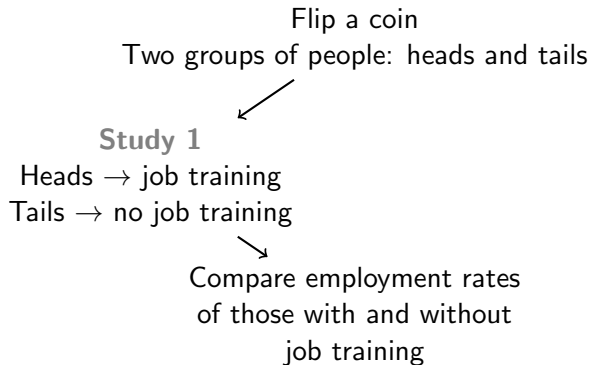
**Study 1**

Heads → job training

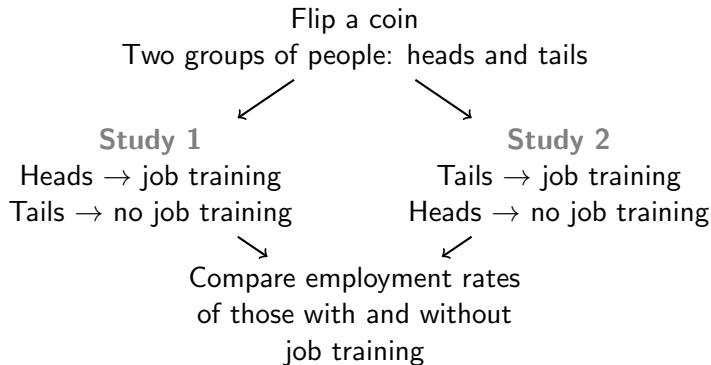
Tails → no job training



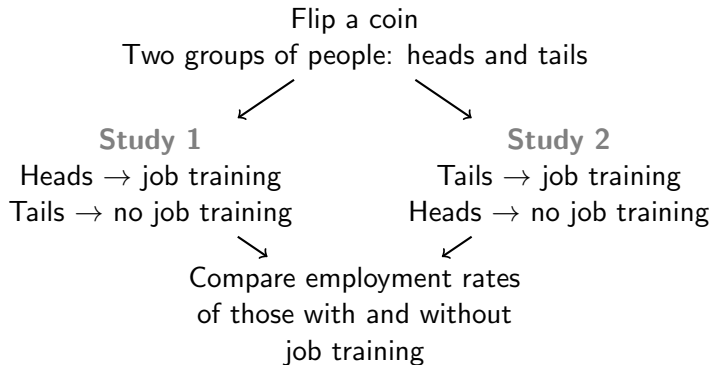
**Exchangeability:** By an experimental procedure



**Exchangeability:** By an experimental procedure

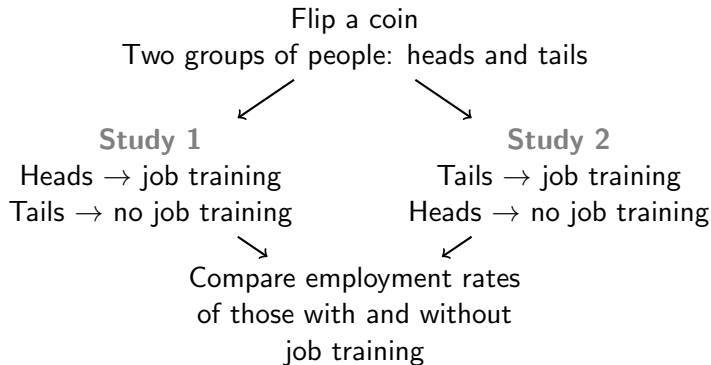


**Exchangeability:** By an experimental procedure



**Question:** Are both studies valid?

**Exchangeability:** By an experimental procedure



**Question:** Are both studies valid?

Yes. The (H/T) groups are **exchangeable**.

Any statistical pattern between (H/T) and employment can only arise from the causal effect of job training

# Why is exchangeability good?

When exchangeability is true, it implies

$$\underbrace{E(Y^{a=1} = 1 \mid A = 1)}_{\text{Within treated}} = \underbrace{E(Y^{a=1} = 1 \mid A = 0)}_{\text{Within not treated}} = \underbrace{E(Y^{a=1} = 1)}_{\text{everyone}}$$

# Why is exchangeability good?

When exchangeability is true, it implies

$$\underbrace{E(Y^{a=1} = 1 \mid A = 1)}_{\text{Within treated}} = \underbrace{E(Y^{a=1} = 1 \mid A = 0)}_{\text{Within not treated}} = \underbrace{E(Y^{a=1} = 1)}_{\text{everyone}}$$

This allows us to identify the average causal effect (ACE)

$$\text{ATE} = \underbrace{E(Y^{a=1} = 1)}_{\text{if everyone is treated}} - \underbrace{E(Y^{a=0} = 1)}_{\text{if no-one is treated}}$$

because we can plug-in

$$\underbrace{E(Y^{a=1} = 1 \mid A = 1)}_{\text{outcomes for people who are **actually** treated}} \quad \text{and} \quad \underbrace{E(Y^{a=0} = 1 \mid A = 0)}_{\text{outcomes for people who are **actually** not treated}}$$

# When does exchangeability hold?

- ▶ Data does not tell us directly whether exchangeability holds
- ▶ We must know how the data was gathered
- ▶ Exchangeability holds by design in experiments

# Exercise

- ▶ Exchangeability implies that  $Y^a \perp\!\!\!\perp A$  for all treatment values  $a$
- ▶ How is this different than  $Y \perp\!\!\!\perp A$ ?
- ▶ In randomized experiments,  $Y^a \perp\!\!\!\perp A$  is usually true. Is  $Y \perp\!\!\!\perp A$  ever true?



# Limits of experiments

Experiments may not be possible because of

- ▶ **Feasibility:** What is the causal effect on global average temperature of decreasing global  $CO_2$  levels by 100 ppm?

# Limits of experiments

Experiments may not be possible because of

- ▶ **Feasibility:** What is the causal effect on global average temperature of decreasing global  $CO_2$  levels by 100 ppm?
- ▶ **Cost:** What is the causal effect of giving every student a Lamborghini on traffic in Collegetown?

# Limits of experiments

Experiments may not be possible because of

- ▶ **Feasibility:** What is the causal effect on global average temperature of decreasing global  $CO_2$  levels by 100 ppm?
- ▶ **Cost:** What is the causal effect of giving every student a Lamborghini on traffic in Collegetown?
- ▶ **Ethics:** What is the causal effect on cancer of smoking cigarettes?

# Making decisions with data

- ▶ Randomized experiments are powerful tools for learning causal relationships
- ▶ Experiments may have negative effect on participants or larger population<sup>1</sup>
- ▶ Belmont Report<sup>2</sup>
  - ▶ Respect for persons: protect personal autonomy
  - ▶ Beneficence: Do no harm
  - ▶ Justice: distribute the burden/benefits fairly

---

<sup>1</sup>Mcdermott and Hatemi PNAS 2020

<https://www.pnas.org/doi/10.1073/pnas.2012021117>

<sup>2</sup><https://www.hhs.gov/ohrp/regulations-and-policy/belmont-report/read-the-belmont-report/index.html>

# Making decisions with data

- ▶ Randomized experiments are powerful tools for learning causal relationships
- ▶ Experiments may have negative effect on participants or larger population<sup>1</sup>
- ▶ Belmont Report<sup>2</sup>
  - ▶ Respect for persons: protect personal autonomy
  - ▶ Beneficence: Do no harm
  - ▶ Justice: distribute the burden/benefits fairly
- ▶ Causal inference with observational data is even more important
- ▶ Causal inference (at it's best) tells you what could be, not what ought to be

---

<sup>1</sup>Mcdermott and Hatemi PNAS 2020

<https://www.pnas.org/doi/10.1073/pnas.2012021117>

<sup>2</sup><https://www.hhs.gov/ohrp/regulations-and-policy/belmont-report/read-the-belmont-report/index.html>

# Why are experiments good?

- ▶ Experiments are great because exchangeability holds by design
- ▶ To estimate the causal effect from experimental data, we can simply take the difference in observed means

# Why are experiments good?

- ▶ Experiments are great because exchangeability holds by design
- ▶ To estimate the causal effect from experimental data, we can simply take the difference in observed means
- ▶ But they are also great for other reasons

# Precise questions

Experiments allow us to answer precise questions

What is the causal effect of vaccination on covid?



# Precise questions

Experiments allow us to answer precise questions

What is the causal effect of **Two shots of the Pfizer vaccine 21 days apart** on Covid?

# Precise questions

Experiments allow us to answer precise questions

What is the causal effect of **Two shots of the Pfizer vaccine 21 days apart** on a **positive Covid test within 14 weeks of vaccination in 2020**?

# Precise questions

Experiments allow us to answer precise questions

What is the causal effect of **Two shots of the Pfizer vaccine 21 days apart** on a **positive Covid test within 14 weeks of vaccination in 2020**?

- Experiments allow us to (more easily) specify precise treatments and outcomes

# Idealized experiment as goal

- ▶ Formulate a precise causal question
  - ▶ Treatment
  - ▶ Outcome and timeframe
  - ▶ Population of interest
- ▶ Experiments are “gold standard” for estimating causal effects
- ▶ Imagine the “ideal experiment” to answer

# Idealized experiment as goal

- ▶ Formulate a precise causal question
  - ▶ Treatment
  - ▶ Outcome and timeframe
  - ▶ Population of interest
- ▶ Experiments are “gold standard” for estimating causal effects
- ▶ Imagine the “ideal experiment” to answer
- ▶ Try to replicate the “ideal experiment” with an observational analysis

# Conditional randomization

Exchangeability may not hold in every randomized experiment

- ▶ “With the use of an interactive Web-based system, participants in the trial were randomly assigned in a 1:1 ratio to receive 30  $\mu g$  of BNT162b2 (0.3 ml volume per dose) or saline placebo.”

# Conditional randomization

Exchangeability may not hold in every randomized experiment

- ▶ “With the use of an interactive Web-based system, participants in the trial were randomly assigned in a 1:1 ratio to receive 30  $\mu\text{g}$  of BNT162b2 (0.3 ml volume per dose) or saline placebo.”
- ▶ Suppose the researchers instead split the group into two groups
  - ▶ Age  $\geq 55$ : Vaccine with probability  $2/3$
  - ▶ Age  $< 55$ : Vaccine with probability  $1/2$
- ▶ Does exchangeability still hold?

# Conditional randomization

Exchangeability may not hold in every randomized experiment

- ▶ Age  $\geq 55$  more likely to get vaccine; more likely to get COVID if treated
- ▶ Age  $\geq 55$  less likely to get vaccine; less likely to get COVID if treated



# Conditional randomization

Exchangeability may not hold in every randomized experiment

- ▶ Age  $\geq 55$  more likely to get vaccine; more likely to get COVID if treated
- ▶ Age  $\geq 55$  less likely to get vaccine; less likely to get COVID if treated
- ▶ Vaccinated individuals have  $Y^{a=1}$  that are more likely to be 1 than unvaccinated individuals
- ▶ Exchangeability does not hold in entire population
- ▶ Exchangeability holds within each sub-population
- ▶ Two separate experiments; both are exchangeable

# Conditional randomization

- ▶ **Marginal exchangeability:**  $Y^a \perp\!\!\!\perp A$  for all  $a$
- ▶ **Conditional exchangeability:**  $Y^a \perp\!\!\!\perp A \mid L$  for all  $a$   
The potential outcomes are independent of treatment  
**conditional on  $L$**

# Conditional randomization

- ▶ **Marginal exchangeability:**  $Y^a \perp\!\!\!\perp A$  for all  $a$
- ▶ **Conditional exchangeability:**  $Y^a \perp\!\!\!\perp A \mid L$  for all  $a$   
The potential outcomes are independent of treatment **conditional on  $L$**
- ▶ If you tell me  $A_i = 1$ , I learn something about  $Y_i^{a=1}, Y_i^{a=0}$
- ▶ Suppose you first tell me someone's age, I learn something about  $Y_i^{a=1}, Y_i^{a=0}$ . Next you tell me  $A_i = 1$ , I don't learn anything new about  $Y_i^{a=1}, Y_i^{a=0}$  (in addition to what I previously knew)

# Conditional randomization

- ▶ **Stratification:** We can directly estimate causal effect within each sub-population (or stratum)
- ▶ If the treatment effect varies across sub-population, we say there is **treatment effect heterogeneity**

# Conditional randomization

- ▶ Can be useful in designing experiments
  - ▶ If  $Y^{a=1}$  has higher variability in some sub-population, assign more units to treated group

# Conditional randomization

- ▶ Can be useful in designing experiments
  - ▶ If  $Y^{a=1}$  has higher variability in some sub-population, assign more units to treated group
- ▶ Most useful as an idealized experiment to target with observational analysis
- ▶ Marginal exchangeability is very unlikely in observational data
- ▶ Conditional exchangeability may be more reasonable

# Learning goals for today

At the end of class, you will be able to:

1. Explain conditionally randomized experiments
2. Identify the “idealized experiment” as a goal