

Defining causal effects

STSCI / INFO / ILRST 3900: Causal Inference

28 Aug 2025

Learning goals for today

By the end of class, you will be able to

- ▶ explain the fundamental problem of causal inference and the need for causal arguments
- ▶ define potential outcomes

Logistics

- ▶ We will cover material needed for Problem Set 1 by the end of today
- ▶ Problem Set 1 due Sep 9

Causal claims hinge on arguments, not just data



Left photo: By Fernando Frazão/Agência Brasil - http://agenciabrasil.ebc.com.br/sites/_agenciabrasil2013/files/fotos/1035034-_mg_0802_04.08.16.jpg, CC BY 3.0, <https://commons.wikimedia.org/w/index.php?curid=50548410>
Right photo: By Agencia Brasil Fotografias - EUA levam ouro na ginástica artística feminina; Brasil fica em 8 lugar, CC BY 2.0, <https://commons.wikimedia.org/w/index.php?curid=50584648>

Causal claims hinge on arguments, not just data

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- ▶ Swinging on the uneven bars causes a person to win a gold medal.

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What do we mean when we say “cause”?

Causal claims hinge on arguments, not just data

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		Swing	Do not swing	of swinging
Simone Biles	Yes (1)		?	?
Sam	?		No (0)	?

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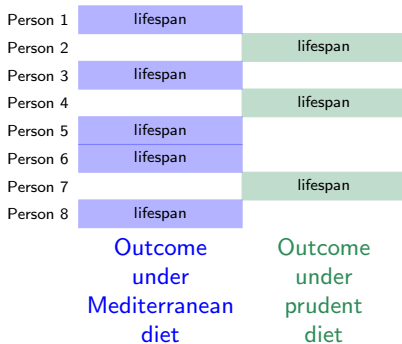
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Fundamental problem of causal inference

Holland 1986

Descriptive evidence



Fundamental problem of causal inference

Holland 1986

Descriptive evidence



Person 1	lifespan	
Person 2		lifespan
Person 3	lifespan	
Person 4		lifespan
Person 5	lifespan	
Person 6	lifespan	
Person 7		lifespan
Person 8	lifespan	

Outcome
under
Mediterranean
diet

Outcome
under
prudent
diet

Causal claim



lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
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Fundamental problem of causal inference

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Descriptive evidence



Causal claim



Person 1	lifespan	missing
Person 2	missing	lifespan
Person 3	lifespan	missing
Person 4	missing	lifespan
Person 5	lifespan	missing
Person 6	lifespan	missing
Person 7	missing	lifespan
Person 8	lifespan	missing

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lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
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Causal claim



Causal inference is a **missing data** problem

Person 1	lifespan	missing
Person 2	missing	lifespan
Person 3	lifespan	missing
Person 4	missing	lifespan
Person 5	lifespan	missing
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Mathematical notation: Potential outcomes¹

¹Capital letters and lowercase letters mean different things!

Mathematical notation: Potential outcomes¹

Y_i Outcome

Whether person i survived

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Mathematical notation: Potential outcomes¹

Y_i	Outcome	Whether person i survived
A_i	Treatment	Whether person i ate a Mediterranean diet

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Mathematical notation: Potential outcomes¹

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Examples:

If assigned prudent diet	If assigned mediterranean diet
Died	Survived

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$Y_{\text{Sam}} = \text{survived}$ We observe that Sam survived

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he would have survived

$Y_{\text{Sam}}^{\text{PruDiet}} = \text{died}$ If Sam had been assigned a prudent diet
he would have died

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Practice

Using the slip of paper you received and the diet you follow, what is

- ▶ Y_i
- ▶ A_i
- ▶ Y_i^{MedDiet}
- ▶ Y_i^{PruDiet}

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- ▶ Consistency would not hold if “forcing” someone to eat a Mediterranean diet results in a different lifespan than what we observed for someone who chose the Mediterranean diet

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- ▶ Consistency would not hold if “forcing” someone to eat a Mediterranean diet results in a different lifespan than what we observed for someone who chose the Mediterranean diet
- ▶ Consistency implies treatments don't have hidden types
- ▶ Consistency would not hold if: Italian olive oil resulted in a different lifespan than Greek olive oil

Practice: How would you say this in English?

We might wonder how a person's earnings relate to whether they hold a college degree

1. $E(\text{Earnings} \mid \text{Degree} = \text{TRUE}) > E(\text{Earnings} \mid \text{Degree} = \text{FALSE})$

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► Average earnings are higher among those with college degrees

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- ▶ Average earnings are higher among those with college degrees

2. $E(\text{Earnings}^{\text{Degree}=\text{TRUE}}) > E(\text{Earnings}^{\text{Degree}=\text{FALSE}})$

- ▶ The average earning if everyone was assigned to get a degree is higher than the average earnings if everyone was assigned to not get a degree
- ▶ On average, getting a degree causes higher earnings

Practice:

1. On average, students who do the homework learn more than those who don't
2. On average, doing the homework causes more learning

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You can now

- ▶ Read Chapter 1 of [Hernán and Robins 2020](#)
- ▶ Start Problem Set 1 (due Sep 9)