Conditional Independence in DAGs

INFO/STSCI/ILRST 3900: Causal Inference

19 Sep 2023

Learning goals for today

At the end of class, you will be able to:

1. Identify whether paths in a causal diagram are open or blocked given a conditioning set
2. Explain why conditioning on colliders differs from conditioning on non-colliders

Logistics

- Ch 6.4 of Hernan and Robins

Causal Graphs

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- Check (marginal) independence by looking at paths in graph

Checking Marginal Independence

$$A \rightarrow Z_1 \rightarrow Z_2 \leftarrow Z_3 \rightarrow Y$$

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- Conditional Exchangeability: $Y^a \perp\!\!\!\perp A \mid L$
- How do we tell if a path is open or blocked when conditioning on L?

Open or blocked?

How do we check if a path in the DAG is open or blocked when conditioning on a set of variables L?

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- If **any** node on the path is blocked, then the entire path is blocked
- If all nodes on the path are open, then the entire path is open

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Conditional Exchangeability holds **given** L if all unblocked paths between A and Y are causal paths

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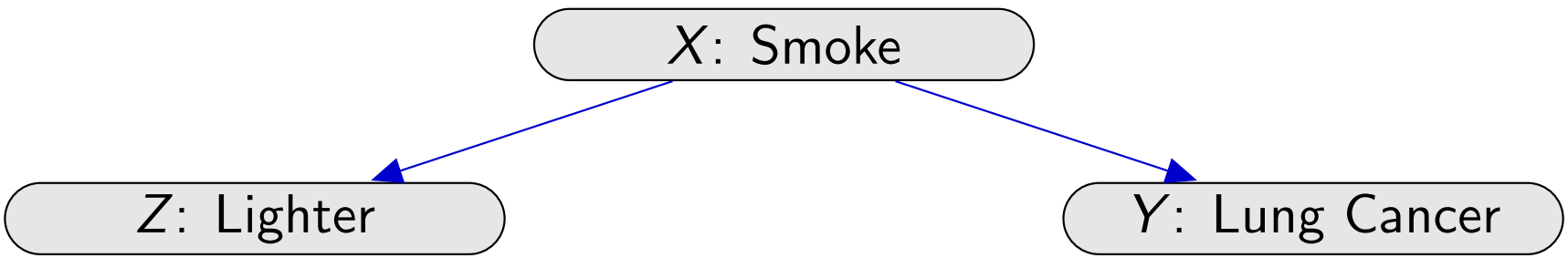
- Otherwise, Z is open

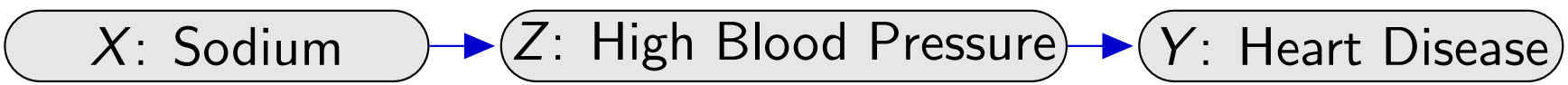
Common cause

If Z has a causal effect on both X and Y, the path is blocked when we condition on Z



Mediation

If X effects Y only via Z, the path is blocked when we condition on Z



Colliders

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When conditioning on a set of variables L

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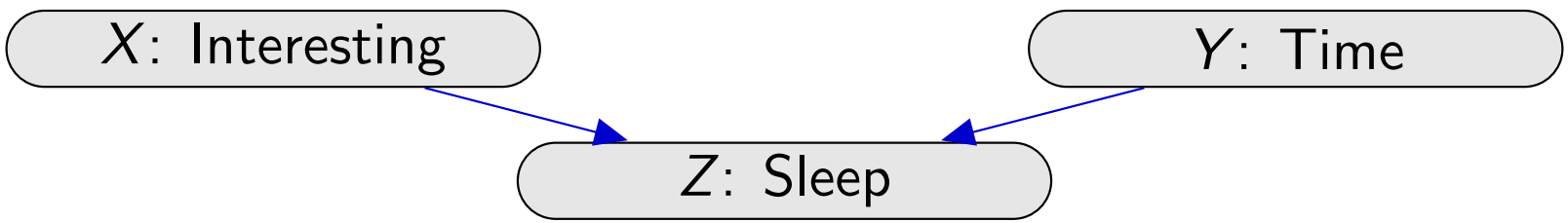
Collider

Suppose the registrar randomly schedules classes so that the time of day is not associated with whether or not a class is interesting.



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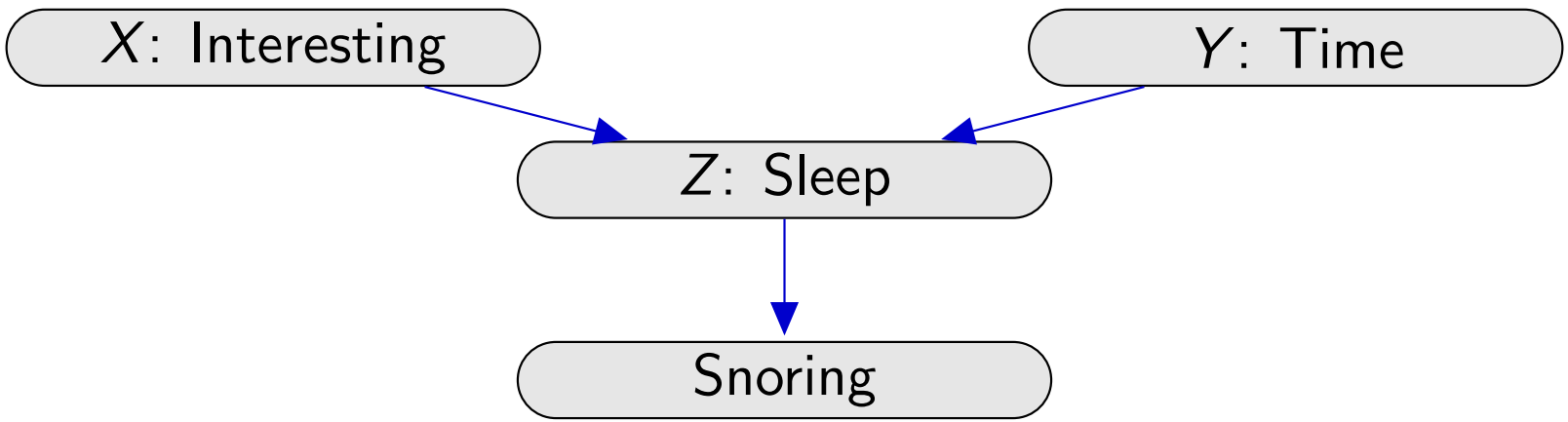

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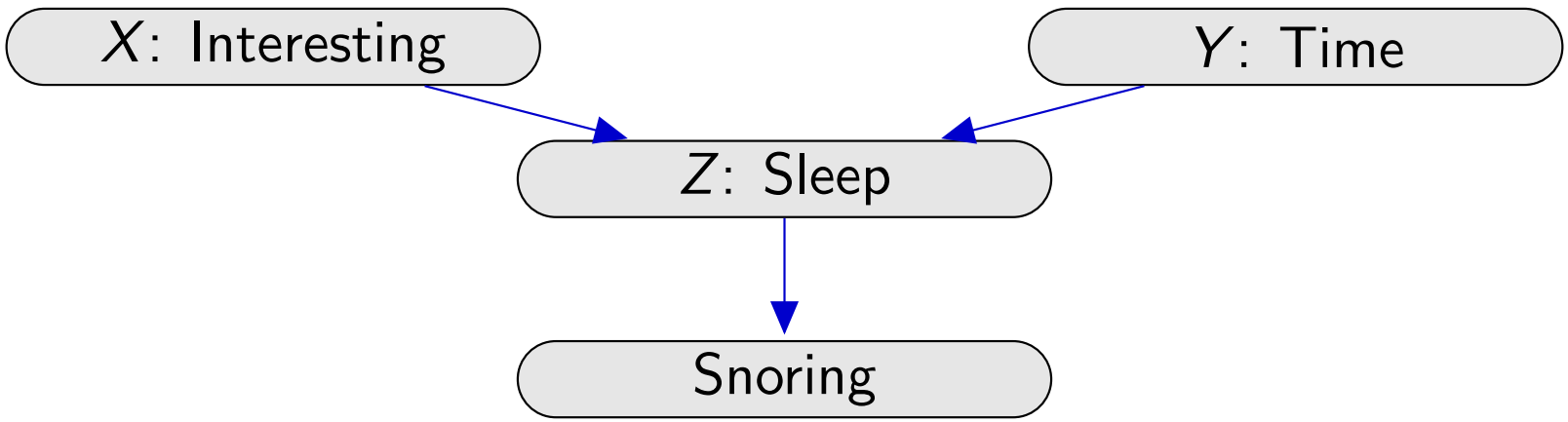

Mathematically,

$$Z = X + Y$$

If we keep Z fixed, but increase X, then to preserve the equation, Y must decrease

Colliders

For Colliders $\rightarrow Z \leftarrow$

Colliders

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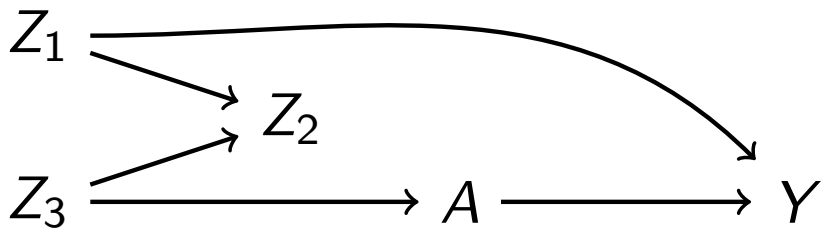
When conditioning on a set of variables L

- If Z (or any descendant of Z) is in the conditioning set L, then Z is open
- Otherwise Z is blocked

Exercise



- What are the paths from A to Y?
- When conditioning on $L = \{Z_1\}$ are those paths open or blocked?
- When conditioning on $L = \{Z_2\}$ are those paths open or blocked?
- When conditioning $L = \{Z_1, Z_2\}$ are those paths open or blocked?

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