Defining causal effects

STSCI / INFO / ILRST 3900: Causal Inference
Fall 2024

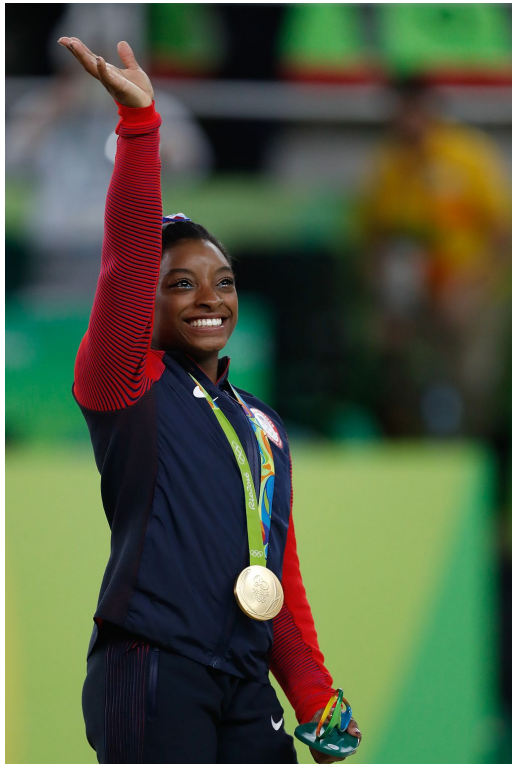
29 Aug 2024

Learning goals for today

By the end of class, you will be able to

- explain the fundamental problem of causal inference and the need for causal arguments
- define potential outcomes
- recall mathematical concepts from probability
  - random variables
  - expectation
  - conditional expectation

Causal claims hinge on arguments, not just data



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Right photo: By Agencia Brasil Fotografias - EUA levam ouro na ginástica artística feminina; Brasil fica em 8 lugar, CC BY 2.0, https://commons.wikimedia.org/w/index.php?curid=50584648

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What do we mean when we say "cause"?

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Simone Biles	Yes (1)	?	?
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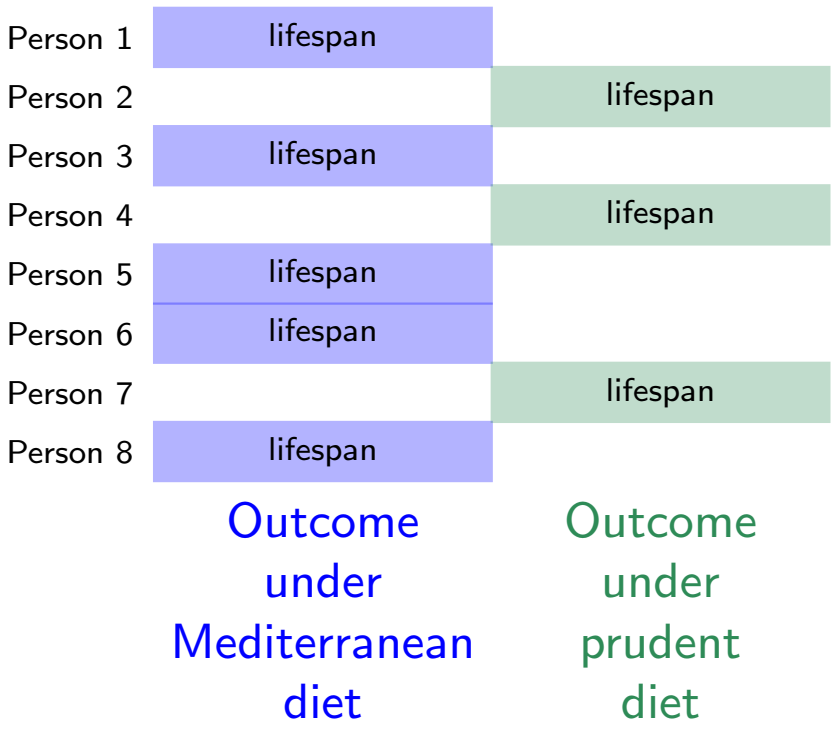
Fundamental problem of causal inference

Holland 1986

Descriptive evidence

average lifespan — average lifespan



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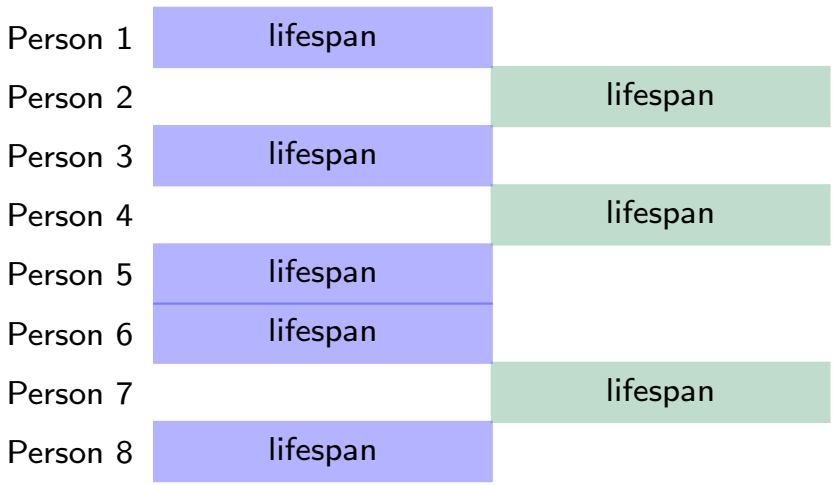

Causal claim

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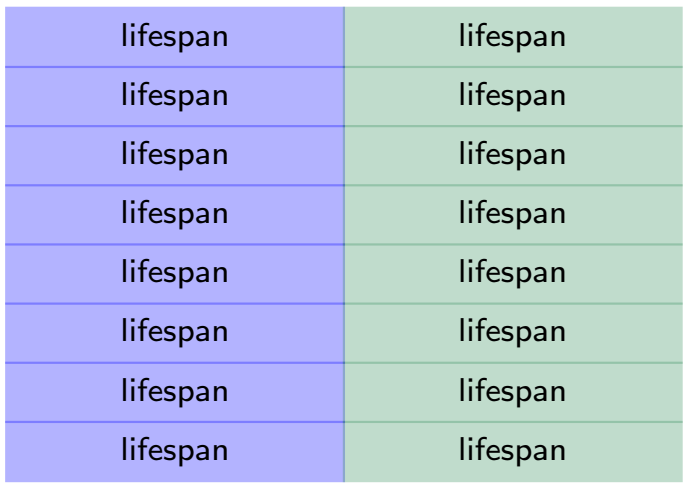

Fundamental problem of causal inference

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	Outcome under Mediterranean diet	Outcome under prudent diet
Person 1	lifespan	missing
Person 2	missing	lifespan
Person 3	lifespan	missing
Person 4	missing	lifespan
Person 5	lifespan	missing
Person 6	lifespan	missing
Person 7	missing	lifespan
Person 8	lifespan	missing

Causal claim

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Fundamental problem of causal inference

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Causal claim

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Causal inference is a **missing data** problem

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Mathematical notation: Potential outcomes[1]

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If assigned prudent diet	If assigned mediterranean diet
Died	Survived

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$Y_{\text{Sam}}^{\text{PruDiet}} = \texttt{died}$	If Sam had been assigned a prudent diet he would have died

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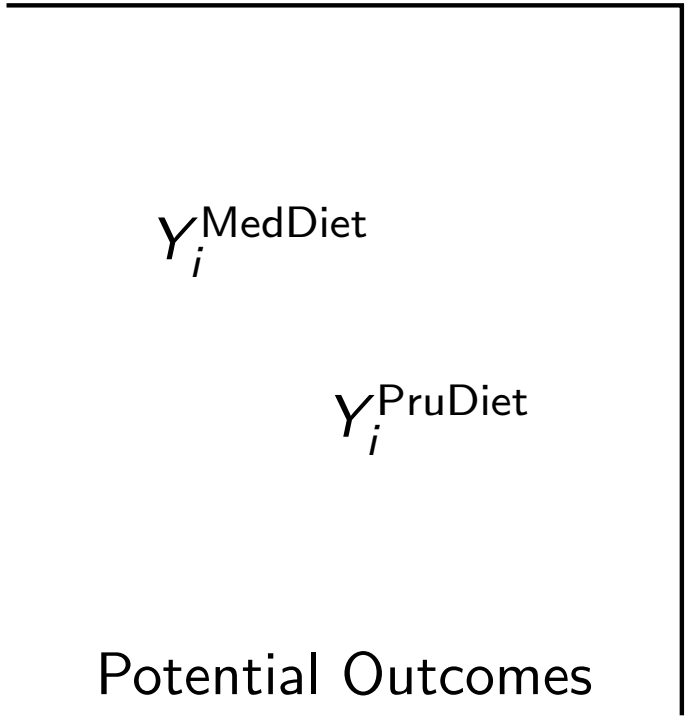
Practice

Using the slip of paper you received and the diet you follow, what is

- Y_i
- A_i
- Y_i^{MedDiet}
- Y_i^{PruDiet}

The consistency assumption

The consistency assumption



The consistency assumption

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Consistency Assumption

$$Y_i^{A_i} = Y_i$$

Notation: Expectation operator

A **random variable**[2] Y is a process which assigns outcomes to different cases by chance

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The **expectation operator** $\mathrm{E}(\cdot)$ denotes the population mean

- The average if we had an infinite amount of data
- If the population of interest has n individuals

$$\mathrm{E}(Y^a) = \frac{1}{n}\sum_{i=1}^{n} Y_i^a = (Y_1^a + Y_2^a + \ldots Y_n^a)/n$$

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A **conditional expectation** is denoted with a vertical bar

$$\mathrm{E}(Y \mid A = a) = \frac{1}{n_a}\sum_{i:A_i=a} Y_i$$

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Practice: How would you say this in English?

We might wonder how a person's earnings relate to whether they hold a college degree

1. $\mathrm{E}(\text{Earnings} \mid \text{Degree} = \texttt{TRUE}) > \mathrm{E}(\text{Earnings} \mid \text{Degree} = \texttt{FALSE})$

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2. $\mathsf{E}(\text{Earnings}^{\text{Degree}=\texttt{TRUE}}) > \mathsf{E}(\text{Earnings}^{\text{Degree}=\texttt{FALSE}})$
    - On average, a degree causes higher earnings

Practice:

1. On average, individuals who eat a Mediterranean diet survive more/less than those who eat a prudent diet:

2. On average, eating a Mediterranean diet causes people to survive more/less

Practice:

1. On average, students who do the homework learn more than those who don't

2. On average, doing the homework causes more learning

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You can now

- Read Chapter 1 of Hernán and Robins 2020
- Complete survey on canvas