Conditional Independence in DAGs

INFO/STSCI/ILRST 3900: Causal Inference

26 Sep 2024

Learning goals for today

At the end of class, you will be able to:

1. Identify whether paths in a causal diagram are open or blocked given a conditioning set
2. Understand why conditioning on colliders differs from conditioning on non-colliders

Logistics

- Ed discussion
- Ch 6.4 of Hernan and Robins

Causal Graphs

- Causal Directed Acyclic Graphs (DAG) help communicate modeling assumptions and implications
- Check (marginal) independence by looking at paths in graph

Checking Marginal Independence

$$A \to Z_1 \to Z_2 \leftarrow Z_3 \to Y$$

- Two types of nodes on a path:
  - Collider: $\to Z \leftarrow$
  - Non-colliders: $\underbrace{\to Z \to}_{\text{mediator}}$ or $\underbrace{\leftarrow Z \to}_{\text{common cause}}$
- Path is unblocked if it does **not** contain a collider
- Two variables are statistically dependant if there is an unblocked path between them

Exchangeability and DAGs

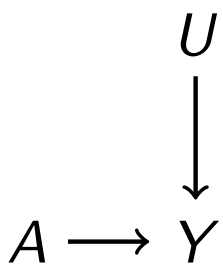
- (Marginal) Exchangeability: $Y^a \perp\!\!\!\perp A$
- **Causal path** path in which all arrows point away from the treatment toward the outcome
- Exchangeability holds if all unblocked paths are causal paths

DAGs help us reason about exchangeability

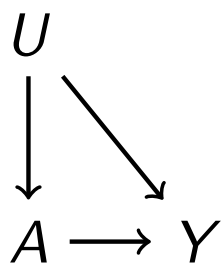
Procedure

1) List all paths between A to Y
2) Cross out the blocked paths
3) Exchangeability holds if all remaining paths are causal

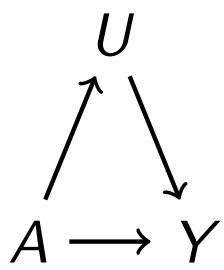
DAG 1



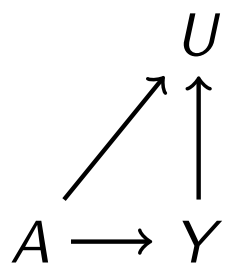
DAG 2



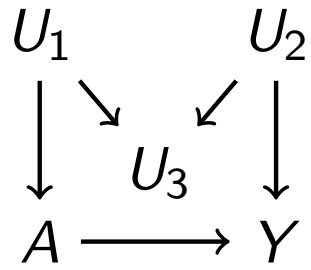
DAG 3



DAG 4



DAG 5



Open or blocked?

How do we check if a path in the DAG is open or blocked when conditioning on a set of variables L?

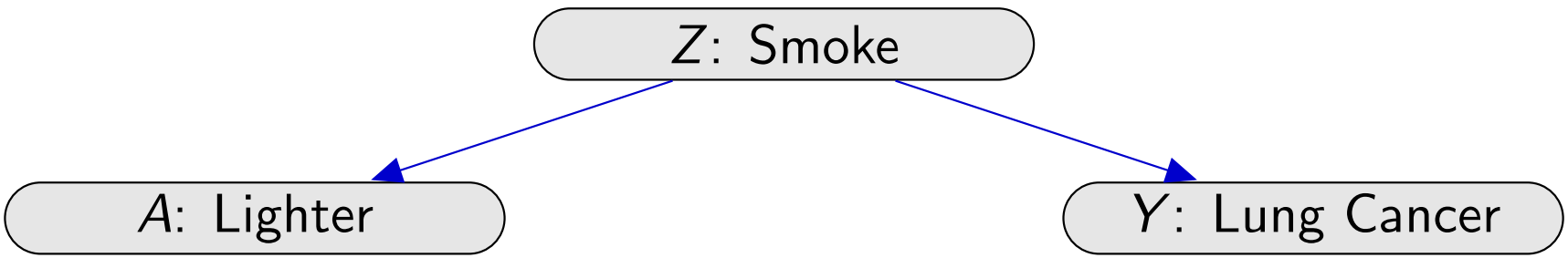
$$A \rightarrow Z_1 \rightarrow Z_2 \leftarrow Z_3 \rightarrow Y$$

- Check each node on the path
- If **any** node on the path is blocked, then the entire path is blocked
- If all nodes on the path are open, then the entire path is open

Two variables are dependent conditional on L if there is an unblocked path (when conditioning on L) between them

Conditional Exchangeability holds **given** L if all unblocked paths between A and Y are causal paths

Common cause



If Z has a causal effect on both A and Y, the path is blocked when we condition on Z

Mediation

If A effects Y through Z, the path is blocked when we condition on Z

Types of paths

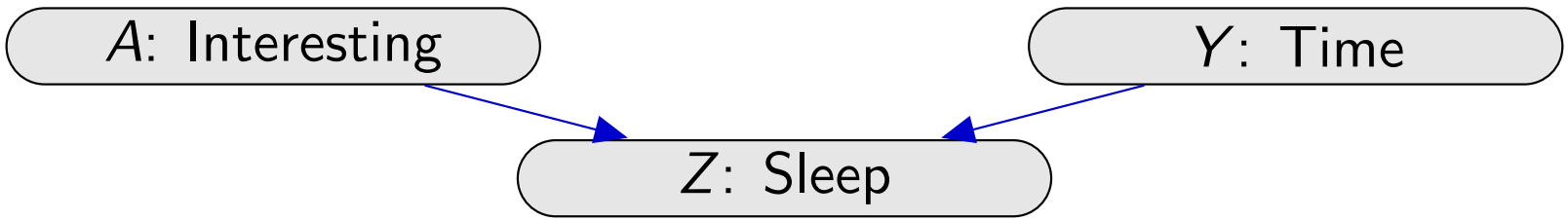
For non-colliders

- Mediators: $\rightarrow Z \rightarrow$ or $\leftarrow Z \leftarrow$
- Common causes: $\leftarrow Z \rightarrow$

- If Z is in the conditioning set, then Z is blocked
- Otherwise, Z is open

$$A \rightarrow Z_1 \rightarrow Z_2 \leftarrow Z_3 \rightarrow Y$$

Collider

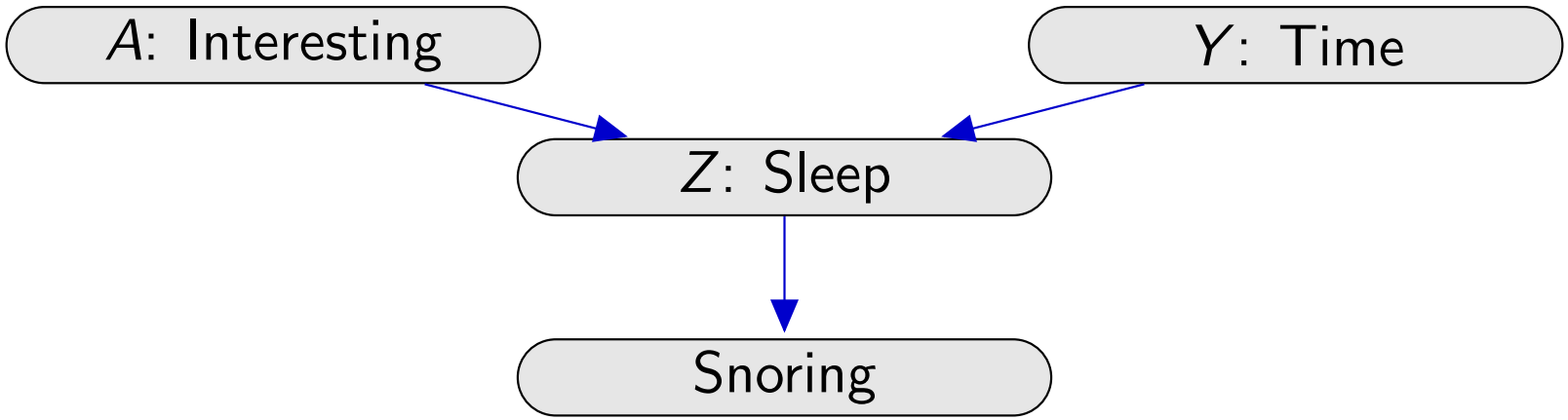


Mathematically,

$$Z = X + Y$$

If we keep Z fixed, but increase X, then to preserve the equation, Y must decrease

Collider



- If there is a causal path $X \to \ldots \to Z$, then Z is a descendant of X

Colliders

For Colliders $\rightarrow Z \leftarrow$

- If Z (or any descendant of Z) is in the conditioning set, then Z is open
- Otherwise Z is blocked

$$A \rightarrow Z_1 \rightarrow Z_2 \leftarrow Z_3 \rightarrow Y$$

Open or blocked?

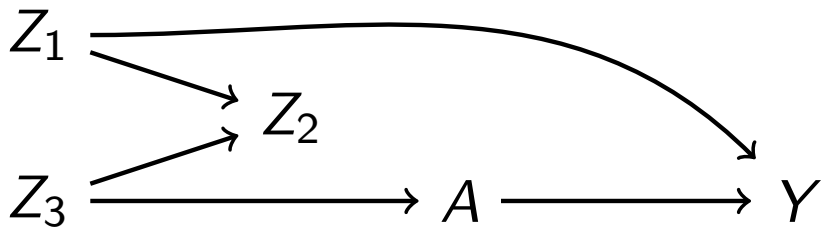
How to check if a path is open or blocked:

1. Traverse the path node by node
2. If any node is blocked, the entire path is blocked
3. If all nodes are open, then entire path is open

How to check if a node is open or blocked:

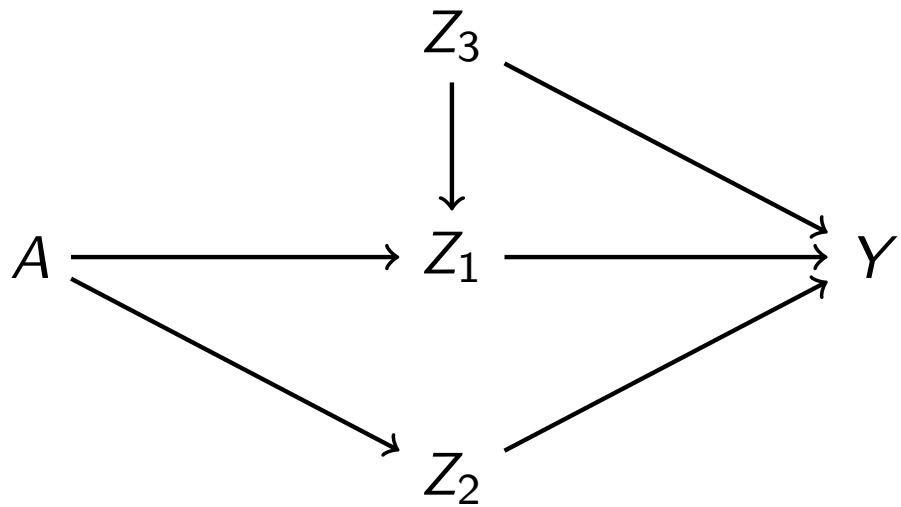
- If non-collider:
  - Open if it is not in the conditioning set
  - Blocked if it is in the conditioning set
- If collider:
  - Open if it or any of its descendants are in the conditioning set
  - Otherwise it is blocked

Exercise



- What are the paths from A to Y?
- When conditioning on $L = \{Z_1\}$ are those paths open or blocked?
- When conditioning on $L = \{Z_2\}$ are those paths open or blocked?
- When conditioning $L = \{Z_1, Z_2\}$ are those paths open or blocked?

Exercise



- What are the paths from A to Y?
- When conditioning on $L = \{Z_2\}$ are those paths open or blocked?
- When conditioning $L = \{Z_1, Z_2\}$ are those paths open or blocked?

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1. Identify whether paths in a causal diagram are open or blocked given a conditioning set
2. Understand why conditioning on colliders differs from conditioning on non-colliders