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Отчёт

**Солвер BICGSTAB для СЛАУ**

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Описание задания и программной реализации

Описание задания

Реализовать решатель СЛАУ BICGSTAB для разреженных матриц в формате CSR. Применить библиотеку OpenMP для распараллеливания решателя, а также применить другие методы оптимизации.

Описание программной реализации

В рамках поставленной задачи был реализован решатель СЛАУ BICGSTAB. Для этого были реализованы следующие модели данных:

класc Matrix – класс для хранения разреженной матрицы в формате CSR и выполнения операций с матрицей. В приватной области хранится структура матрицы: double \*A – не нулевые элементы матрицы, int \*JA – номера столбцов не нулевых элементов, int \*IA – позиция начала данных для строки i, int sizeIA – размер вектора IA, int sizeA – размер векторов A и JA. Были реализованы различные конструкторы (Matrix(int Nx, int Ny, int Nz), Matrix(const Matrix &mat), Matrix(const Matrix &mat, int k)). Последний из них служит генератором диагональной матрицы, элементами которой являются обратные элементы диагонали входящей матрицы. Также были реализованы функции вывода.

Класс Vector – класс для хранения вектора и выполнения операций над ним. В приватной области также хранится структура вектора: double \*A – элементы вектора, int size – размер вектора. Были реализованы различные конструкторы (Vector(int s), Vector(const Vector &vec), Vector(int s, double c), Vector(int s, double \*vec)). Первый из них заполняет вектор синусами в цикле (sin(i)).

Были реализованы три базовые операции:

friend double dot(const Vector &vec1, const Vector &vec2) – скалярное произведение двух векторов.

friend int axpby(Vector &vec1, const Vector &vec2, double a, double b) – линейная комбинация двух векторов с коэффициентами a, b.

friend int SpMV(const Matrix &mat, const Vector &vec, Vector &res) – умножение матрицы на вектор (Ax).

int solve(int N, Matrix &A, Vector &BB, double tol, int maxit, int debug) – решатель СЛАУ Ax = BB методом BICGSTAB. N – размерность матрицы, tol – невязка системы, maxit – максимальное количество итераций решателя, debug – флаг отладки.

int testFunc(int Nx, int Ny, int Nz, int N) – функция, проводящая комплексное тестирование алгоритма и функций: тестирование на разных размерах и разном количестве потоков).

Были применены различные методы оптимизации, например, более оптимальный алгоритм для умножения разреженной матрицы на вектор, развертка циклов, многопоточность.

Исследования производительности

Характеристики вычислительной системы

Тестирование проводилось на кластере вмк IMB Polus. Данная система содержит 5 вычислительных узлов, один из которых выполняет функцию фронтэнда. Основные характеристики узла можно посмотреть на сайте <http://hpc.cmc.msu.ru/polus>.

Пиковая производительность кластера 55.84 Tflop/s. lscpu даёт следующий вывод:

Architecture: ppc64le

Byte Order: Little Endian

CPU(s): 160

On-line CPU(s) list: 0-159

Thread(s) per core: 8

Core(s) per socket: 10

Socket(s): 2

NUMA node(s): 2

Model: 1.0 (pvr 004c 0100)

Model name: POWER8NVL (raw), altivec supported

CPU max MHz: 4023.0000

CPU min MHz: 2061.0000

Hypervisor vendor: (null)

Virtualization type: full

L1d cache: 64K

L1i cache: 32K

L2 cache: 512K

L3 cache: 8192K

NUMA node0 CPU(s): 0-79

NUMA node1 CPU(s): 80-159

Из информации из википедии (https://ru.wikipedia.org/wiki/POWER8 ) известно, что процессор POWER 8 имеет производительность 290 Gflop/s и 580 Gflop/s при обработке чисел двойной точности и одинарной точности соответсвенно. Максимальная пропускная способность памяти 230 GB/s.

Компиляция программы проводилась на фронтэнд узле командой g++ -std=c++11 -g -O3 -fopenmp matrix.cpp -o matrix, а постановка в очередь проводилась командой mpisubmit.pl -t 20 matrix -- 100 100 100 0.0000001 1000 8 1. В данном случае, все параметры кроме последнего не имею смысла, так как флаг тестирования (последний параметр) равен одному. Вывод данных при таком запуске будет в приложении.

Результаты измерений производительности.

Тестирование проводилось следующим образом:

для каждого размера матрицы было произведено тестирование каждой базовой операции на различном количестве ядер (1, 2, 4, 8, 10, 16). Размерности матрицы были следующими в формате (Nx, Ny, Nz): (10, 10, 10), (10, 10, 100), (10, 100, 100), (100, 100, 100), (100, 100, 1000).

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| op | Nt | (10, 10, 10) time | (10, 10, 100) time | (10, 100, 100) time | (100, 100, 100) time | (100, 100, 1000) time |
| Dot | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1.64956e-05 | | 1.58958e-05 | | 3.58e-05 | | 4.88143e-05 | | 5.36125e-05 | | 0.000119943 | | |  | | --- | | 1.98092e-05 | | 3.4418e-05 | | 4.07323e-05 | | 5.22081e-05 | | 0.000499249 | | 0.000707226 | | |  | | --- | | 0.000167701 | | 0.000151753 | | 8.65292e-05 | | 8.50409e-05 | | 7.27009e-05 | | 0.000757292 | | |  | | --- | | 0.00162745 | | 0.000851253 | | 0.000460314 | | 0.00035521 | | 0.000321552 | | 0.000289688 | | |  | | --- | | 0.015234 | | 0.00897158 | | 0.00392793 | | 0.00282883 | | 0.00287875 | | 0.00280449 | |
| Axpby | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 2.33203e-06 | | 2.61702e-06 | | 2.59466e-06 | | 4.56162e-06 | | 5.31413e-06 | | 3.7495e-06 | | |  | | --- | | 1.1174e-05 | | 2.0951e-05 | | 6.72974e-06 | | 6.43916e-06 | | 1.79671e-05 | | 1.26194e-05 | | |  | | --- | | 0.000103682 | | 5.37541e-05 | | 4.75682e-05 | | 4.25186e-05 | | 3.76701e-05 | | 3.83221e-05 | | |  | | --- | | 0.00158068 | | 0.000553669 | | 0.000470335 | | 0.000403432 | | 0.000193922 | | 0.000204971 | | |  | | --- | | 0.0108691 | | 0.00527205 | | 0.00281621 | | 0.00289013 | | 0.00319287 | | 0.00259172 | |
| SpMV | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1.66558e-05 | | 1.09859e-05 | | 1.18371e-05 | | 1.649e-05 | | 1.93529e-05 | | 2.1698e-05 | | |  | | --- | | 0.000139346 | | 0.000108816 | | 5.26477e-05 | | 4.8779e-05 | | 4.43347e-05 | | 5.99138e-05 | | |  | | --- | | 0.00131051 | | 0.000676891 | | 0.00047799 | | 0.000431422 | | 0.000393994 | | 0.000396525 | | |  | | --- | | 0.012966 | | 0.00660141 | | 0.00456148 | | 0.00257296 | | 0.00203139 | | 0.0021855 | | |  | | --- | | 0.131624 | | 0.0672737 | | 0.0362108 | | 0.0267259 | | 0.0300968 | | 0.0225217 | |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| op | Nt | (10, 10, 10) flops | (10, 10, 100) flops | (10, 100, 100) flops | (100, 100, 100) flops | (100, 100, 1000) flops |
| Dot | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 0.121245 | | 0.125819 | | 0.0558659 | | 0.0409716 | | 0.0373047 | | 0.0166746 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | |
| Axpby | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | |
| SpMV | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | | |  | | --- | | 1 | | 2 | | 4 | | 8 | | 10 | | 16 | |

Приложение

Вывод теста, запущенный на IBM Polus:

10 10 10 1000

> Threads = 1

> DOT = 499.509

> Time of DOT = 1.64956e-05

> GFLOPS = 0.121245

> SpeedUp = 1

> AXPBY L2 norm = 67.0491

> Time of AXPBY = 2.33203e-06

> GFLOPS = 1.28643

> SpeedUp = 1

> SpMV L2 norm = 48.7737

> Time of SpMV = 1.66558e-05

> TFLOPS = 0.768502

> SpeedUp = 1

> Threads = 2

> DOT = 499.509

> Time of DOT = 1.58958e-05

> GFLOPS = 0.125819

> SpeedUp = 1.03773

> AXPBY L2 norm = 67.0491

> Time of AXPBY = 2.61702e-06

> GFLOPS = 1.14634

> SpeedUp = 0.891103

> SpMV L2 norm = 48.7737

> Time of SpMV = 1.09859e-05

> TFLOPS = 1.16513

> SpeedUp = 1.51611

> Threads = 4

> DOT = 499.509

> Time of DOT = 3.58e-05

> GFLOPS = 0.0558659

> SpeedUp = 0.46077

> AXPBY L2 norm = 67.0491

> Time of AXPBY = 2.59466e-06

> GFLOPS = 1.15622

> SpeedUp = 0.89878

> SpMV L2 norm = 48.7737

> Time of SpMV = 1.18371e-05

> TFLOPS = 1.08135

> SpeedUp = 1.40708

> Threads = 8

> DOT = 499.509

> Time of DOT = 4.88143e-05

> GFLOPS = 0.0409716

> SpeedUp = 0.337925

> AXPBY L2 norm = 67.0491

> Time of AXPBY = 4.56162e-06

> GFLOPS = 0.657661

> SpeedUp = 0.511229

> SpMV L2 norm = 48.7737

> Time of SpMV = 1.649e-05

> TFLOPS = 0.776228

> SpeedUp = 1.01005

> Threads = 10

> DOT = 499.509

> Time of DOT = 5.36125e-05

> GFLOPS = 0.0373047

> SpeedUp = 0.307682

> AXPBY L2 norm = 67.0491

> Time of AXPBY = 5.31413e-06

> GFLOPS = 0.564533

> SpeedUp = 0.438836

> SpMV L2 norm = 48.7737

> Time of SpMV = 1.93529e-05

> TFLOPS = 0.6614

> SpeedUp = 0.860635

> Threads = 16

> DOT = 499.509

> Time of DOT = 0.000119943

> GFLOPS = 0.0166746

> SpeedUp = 0.137528

> AXPBY L2 norm = 67.0491

> Time of AXPBY = 3.7495e-06

> GFLOPS = 0.800106

> SpeedUp = 0.621957

> SpMV L2 norm = 48.7737

> Time of SpMV = 2.1698e-05

> TFLOPS = 0.589917

> SpeedUp = 0.76762

10 10 100 10000

> Threads = 1

> DOT = 4999.86

> Time of DOT = 1.98092e-05

> GFLOPS = 1.00963

> SpeedUp = 1

> AXPBY L2 norm = 212.129

> Time of AXPBY = 1.1174e-05

> GFLOPS = 2.6848

> SpeedUp = 1

> SpMV L2 norm = 158.553

> Time of SpMV = 0.000139346

> TFLOPS = 9.44409

> SpeedUp = 1

> Threads = 2

> DOT = 4999.86

> Time of DOT = 3.4418e-05

> GFLOPS = 0.581092

> SpeedUp = 0.575549

> AXPBY L2 norm = 212.129

> Time of AXPBY = 2.0951e-05

> GFLOPS = 1.43191

> SpeedUp = 0.533339

> SpMV L2 norm = 158.553

> Time of SpMV = 0.000108816

> TFLOPS = 12.0938

> SpeedUp = 1.28057

> Threads = 4

> DOT = 4999.86

> Time of DOT = 4.07323e-05

> GFLOPS = 0.491011

> SpeedUp = 0.486327

> AXPBY L2 norm = 212.129

> Time of AXPBY = 6.72974e-06

> GFLOPS = 4.45783

> SpeedUp = 1.66039

> SpMV L2 norm = 158.553

> Time of SpMV = 5.26477e-05

> TFLOPS = 24.9964

> SpeedUp = 2.64677

> Threads = 8

> DOT = 4999.86

> Time of DOT = 5.22081e-05

> GFLOPS = 0.383082

> SpeedUp = 0.379428

> AXPBY L2 norm = 212.129

> Time of AXPBY = 6.43916e-06

> GFLOPS = 4.65899

> SpeedUp = 1.73532

> SpMV L2 norm = 158.553

> Time of SpMV = 4.8779e-05

> TFLOPS = 26.9788

> SpeedUp = 2.85669

> Threads = 10

> DOT = 4999.86

> Time of DOT = 0.000499249

> GFLOPS = 0.0400602

> SpeedUp = 0.0396781

> AXPBY L2 norm = 212.129

> Time of AXPBY = 1.79671e-05

> GFLOPS = 1.66972

> SpeedUp = 0.621916

> SpMV L2 norm = 158.553

> Time of SpMV = 4.43347e-05

> TFLOPS = 29.6833

> SpeedUp = 3.14306

> Threads = 16

> DOT = 4999.86

> Time of DOT = 0.000707226

> GFLOPS = 0.0282795

> SpeedUp = 0.0280098

> AXPBY L2 norm = 212.129

> Time of AXPBY = 1.26194e-05

> GFLOPS = 2.37729

> SpeedUp = 0.885461

> SpMV L2 norm = 158.553

> Time of SpMV = 5.99138e-05

> TFLOPS = 21.9649

> SpeedUp = 2.32578

10 100 100 100000

> Threads = 1

> DOT = 50000

> Time of DOT = 0.000167701

> GFLOPS = 1.1926

> SpeedUp = 1

> AXPBY L2 norm = 670.82

> Time of AXPBY = 0.000103682

> GFLOPS = 2.89345

> SpeedUp = 1

> SpMV L2 norm = 511.255

> Time of SpMV = 0.00131051

> TFLOPS = 103.166

> SpeedUp = 1

> Threads = 2

> DOT = 50000

> Time of DOT = 0.000151753

> GFLOPS = 1.31793

> SpeedUp = 1.10509

> AXPBY L2 norm = 670.82

> Time of AXPBY = 5.37541e-05

> GFLOPS = 5.58097

> SpeedUp = 1.92883

> SpMV L2 norm = 511.255

> Time of SpMV = 0.000676891

> TFLOPS = 199.737

> SpeedUp = 1.93608

> Threads = 4

> DOT = 50000

> Time of DOT = 8.65292e-05

> GFLOPS = 2.31136

> SpeedUp = 1.93809

> AXPBY L2 norm = 670.82

> Time of AXPBY = 4.75682e-05

> GFLOPS = 6.30673

> SpeedUp = 2.17965

> SpMV L2 norm = 511.255

> Time of SpMV = 0.00047799

> TFLOPS = 282.851

> SpeedUp = 2.74172

> Threads = 8

> DOT = 50000

> Time of DOT = 8.50409e-05

> GFLOPS = 2.35181

> SpeedUp = 1.97201

> AXPBY L2 norm = 670.82

> Time of AXPBY = 4.25186e-05

> GFLOPS = 7.05574

> SpeedUp = 2.43852

> SpMV L2 norm = 511.255

> Time of SpMV = 0.000431422

> TFLOPS = 313.382

> SpeedUp = 3.03766

> Threads = 10

> DOT = 50000

> Time of DOT = 7.27009e-05

> GFLOPS = 2.751

> SpeedUp = 2.30673

> AXPBY L2 norm = 670.82

> Time of AXPBY = 3.76701e-05

> GFLOPS = 7.96387

> SpeedUp = 2.75237

> SpMV L2 norm = 511.255

> Time of SpMV = 0.000393994

> TFLOPS = 343.152

> SpeedUp = 3.32623

> Threads = 16

> DOT = 50000

> Time of DOT = 0.000757292

> GFLOPS = 0.264099

> SpeedUp = 0.221449

> AXPBY L2 norm = 670.82

> Time of AXPBY = 3.83221e-05

> GFLOPS = 7.82839

> SpeedUp = 2.70555

> SpMV L2 norm = 511.255

> Time of SpMV = 0.000396525

> TFLOPS = 340.962

> SpeedUp = 3.30499

100 100 100 1000000

> Threads = 1

> DOT = 500000

> Time of DOT = 0.00162745

> GFLOPS = 1.22892

> SpeedUp = 1

> AXPBY L2 norm = 2121.32

> Time of AXPBY = 0.00158068

> GFLOPS = 1.89792

> SpeedUp = 1

> SpMV L2 norm = 1712.5

> Time of SpMV = 0.012966

> TFLOPS = 1070.49

> SpeedUp = 1

> Threads = 2

> DOT = 500000

> Time of DOT = 0.000851253

> GFLOPS = 2.34948

> SpeedUp = 1.91183

> AXPBY L2 norm = 2121.32

> Time of AXPBY = 0.000553669

> GFLOPS = 5.4184

> SpeedUp = 2.85491

> SpMV L2 norm = 1712.5

> Time of SpMV = 0.00660141

> TFLOPS = 2102.58

> SpeedUp = 1.96413

> Threads = 4

> DOT = 500000

> Time of DOT = 0.000460314

> GFLOPS = 4.34486

> SpeedUp = 3.53552

> AXPBY L2 norm = 2121.32

> Time of AXPBY = 0.000470335

> GFLOPS = 6.37844

> SpeedUp = 3.36075

> SpMV L2 norm = 1712.5

> Time of SpMV = 0.00456148

> TFLOPS = 3042.87

> SpeedUp = 2.8425

> Threads = 8

> DOT = 500000

> Time of DOT = 0.00035521

> GFLOPS = 5.63047

> SpeedUp = 4.58165

> AXPBY L2 norm = 2121.32

> Time of AXPBY = 0.000403432

> GFLOPS = 7.43619

> SpeedUp = 3.91807

> SpMV L2 norm = 1712.5

> Time of SpMV = 0.00257296

> TFLOPS = 5394.56

> SpeedUp = 5.03933

> Threads = 10

> DOT = 500000

> Time of DOT = 0.000321552

> GFLOPS = 6.21983

> SpeedUp = 5.06122

> AXPBY L2 norm = 2121.32

> Time of AXPBY = 0.000193922

> GFLOPS = 15.4701

> SpeedUp = 8.1511

> SpMV L2 norm = 1712.5

> Time of SpMV = 0.00203139

> TFLOPS = 6832.76

> SpeedUp = 6.38283

> Threads = 16

> DOT = 500000

> Time of DOT = 0.000289688

> GFLOPS = 6.90398

> SpeedUp = 5.61793

> AXPBY L2 norm = 2121.32

> Time of AXPBY = 0.000204971

> GFLOPS = 14.6362

> SpeedUp = 7.7117

> SpMV L2 norm = 1712.5

> Time of SpMV = 0.0021855

> TFLOPS = 6350.96

> SpeedUp = 5.93275

100 100 1000 10000000

> Threads = 1

> DOT = 5e+06

> Time of DOT = 0.015234

> GFLOPS = 1.31286

> SpeedUp = 1

> AXPBY L2 norm = 6708.2

> Time of AXPBY = 0.0108691

> GFLOPS = 2.76013

> SpeedUp = 1

> SpMV L2 norm = 5431.56

> Time of SpMV = 0.131624

> TFLOPS = 10572.5

> SpeedUp = 1

> Threads = 2

> DOT = 5e+06

> Time of DOT = 0.00897158

> GFLOPS = 2.22926

> SpeedUp = 1.69802

> AXPBY L2 norm = 6708.2

> Time of AXPBY = 0.00527205

> GFLOPS = 5.69039

> SpeedUp = 2.06164

> SpMV L2 norm = 5431.56

> Time of SpMV = 0.0672737

> TFLOPS = 20685.6

> SpeedUp = 1.95655

> Threads = 4

> DOT = 5e+06

> Time of DOT = 0.00392793

> GFLOPS = 5.09173

> SpeedUp = 3.87837

> AXPBY L2 norm = 6708.2

> Time of AXPBY = 0.00281621

> GFLOPS = 10.6526

> SpeedUp = 3.85946

> SpMV L2 norm = 5431.56

> Time of SpMV = 0.0362108

> TFLOPS = 38430.5

> SpeedUp = 3.63494

> Threads = 8

> DOT = 5e+06

> Time of DOT = 0.00282883

> GFLOPS = 7.07005

> SpeedUp = 5.38525

> AXPBY L2 norm = 6708.2

> Time of AXPBY = 0.00289013

> GFLOPS = 10.3802

> SpeedUp = 3.76075

> SpMV L2 norm = 5431.56

> Time of SpMV = 0.0267259

> TFLOPS = 52069.4

> SpeedUp = 4.92497

> Threads = 10

> DOT = 5e+06

> Time of DOT = 0.00287875

> GFLOPS = 6.94746

> SpeedUp = 5.29187

> AXPBY L2 norm = 6708.2

> Time of AXPBY = 0.00319287

> GFLOPS = 9.39595

> SpeedUp = 3.40417

> SpMV L2 norm = 5431.56

> Time of SpMV = 0.0300968

> TFLOPS = 46237.5

> SpeedUp = 4.37336

> Threads = 16

> DOT = 5e+06

> Time of DOT = 0.00280449

> GFLOPS = 7.13142

> SpeedUp = 5.43199

> AXPBY L2 norm = 6708.2

> Time of AXPBY = 0.00259172

> GFLOPS = 11.5753

> SpeedUp = 4.19376

> SpMV L2 norm = 5431.56

> Time of SpMV = 0.0225217

> TFLOPS = 61789.4

> SpeedUp = 5.84433

> Solver test...

> Number of threads = 1

It = 0; res = 2236.07; tol = 1

It = 1; res = 837.045; tol = 0.374338

It = 2; res = 90.2838; tol = 0.0403762

It = 3; res = 19.6953; tol = 0.00880802

It = 4; res = 5.82113; tol = 0.00260329

It = 5; res = 1.54234; tol = 0.000689753

It = 6; res = 0.447256; tol = 0.000200019

It = 7; res = 0.106756; tol = 4.77425e-05

It = 8; res = 0.0284587; tol = 1.27271e-05

It = 9; res = 0.0075798; tol = 3.38979e-06

It = 10; res = 0.00235927; tol = 1.0551e-06

It = 11; res = 0.000887506; tol = 3.96905e-07

It = 12; res = 0.000252407; tol = 1.1288e-07

It = 13; res = 7.53324e-05; tol = 3.36897e-08

It = 14; res = 2.48026e-05; tol = 1.10921e-08

It = 15; res = 7.72044e-06; tol = 3.45269e-09

It = 16; res = 2.70016e-06; tol = 1.20755e-09

It = 17; res = 9.65432e-07; tol = 4.31754e-10

> Final discrepancy = 9.65432e-07

> Numder of iters = 17

> Final time of computation = 9.47466

> Number of threads = 2

It = 0; res = 2236.07; tol = 1

It = 1; res = 837.045; tol = 0.374338

It = 2; res = 90.2838; tol = 0.0403762

It = 3; res = 19.6953; tol = 0.00880802

It = 4; res = 5.82113; tol = 0.00260329

It = 5; res = 1.54234; tol = 0.000689753

It = 6; res = 0.447256; tol = 0.000200019

It = 7; res = 0.106756; tol = 4.77425e-05

It = 8; res = 0.0284587; tol = 1.27271e-05

It = 9; res = 0.0075798; tol = 3.38979e-06

It = 10; res = 0.00235927; tol = 1.0551e-06

It = 11; res = 0.000887506; tol = 3.96905e-07

It = 12; res = 0.000252407; tol = 1.1288e-07

It = 13; res = 7.53324e-05; tol = 3.36897e-08

It = 14; res = 2.48026e-05; tol = 1.10921e-08

It = 15; res = 7.72044e-06; tol = 3.45269e-09

It = 16; res = 2.70016e-06; tol = 1.20755e-09

It = 17; res = 9.65432e-07; tol = 4.31754e-10

> Final discrepancy = 9.65432e-07

> Numder of iters = 17

> Final time of computation = 4.98321

> Number of threads = 4

It = 0; res = 2236.07; tol = 1

It = 1; res = 837.045; tol = 0.374338

It = 2; res = 90.2838; tol = 0.0403762

It = 3; res = 19.6953; tol = 0.00880802

It = 4; res = 5.82113; tol = 0.00260329

It = 5; res = 1.54234; tol = 0.000689753

It = 6; res = 0.447256; tol = 0.000200019

It = 7; res = 0.106756; tol = 4.77425e-05

It = 8; res = 0.0284587; tol = 1.27271e-05

It = 9; res = 0.0075798; tol = 3.38979e-06

It = 10; res = 0.00235927; tol = 1.0551e-06

It = 11; res = 0.000887506; tol = 3.96905e-07

It = 12; res = 0.000252407; tol = 1.1288e-07

It = 13; res = 7.53324e-05; tol = 3.36897e-08

It = 14; res = 2.48026e-05; tol = 1.10921e-08

It = 15; res = 7.72044e-06; tol = 3.45269e-09

It = 16; res = 2.70016e-06; tol = 1.20755e-09

It = 17; res = 9.65432e-07; tol = 4.31754e-10

> Final discrepancy = 9.65432e-07

> Numder of iters = 17

> Final time of computation = 2.87267

> Number of threads = 8

It = 0; res = 2236.07; tol = 1

It = 1; res = 837.045; tol = 0.374338

It = 2; res = 90.2838; tol = 0.0403762

It = 3; res = 19.6953; tol = 0.00880802

It = 4; res = 5.82113; tol = 0.00260329

It = 5; res = 1.54234; tol = 0.000689753

It = 6; res = 0.447256; tol = 0.000200019

It = 7; res = 0.106756; tol = 4.77425e-05

It = 8; res = 0.0284587; tol = 1.27271e-05

It = 9; res = 0.0075798; tol = 3.38979e-06

It = 10; res = 0.00235927; tol = 1.0551e-06

It = 11; res = 0.000887506; tol = 3.96905e-07

It = 12; res = 0.000252407; tol = 1.1288e-07

It = 13; res = 7.53324e-05; tol = 3.36897e-08

It = 14; res = 2.48026e-05; tol = 1.10921e-08

It = 15; res = 7.72044e-06; tol = 3.45269e-09

It = 16; res = 2.70016e-06; tol = 1.20755e-09

It = 17; res = 9.65432e-07; tol = 4.31754e-10

> Final discrepancy = 9.65432e-07

> Numder of iters = 17

> Final time of computation = 2.00667

> Number of threads = 10

It = 0; res = 2236.07; tol = 1

It = 1; res = 837.045; tol = 0.374338

It = 2; res = 90.2838; tol = 0.0403762

It = 3; res = 19.6953; tol = 0.00880802

It = 4; res = 5.82113; tol = 0.00260329

It = 5; res = 1.54234; tol = 0.000689753

It = 6; res = 0.447256; tol = 0.000200019

It = 7; res = 0.106756; tol = 4.77425e-05

It = 8; res = 0.0284587; tol = 1.27271e-05

It = 9; res = 0.0075798; tol = 3.38979e-06

It = 10; res = 0.00235927; tol = 1.0551e-06

It = 11; res = 0.000887506; tol = 3.96905e-07

It = 12; res = 0.000252407; tol = 1.1288e-07

It = 13; res = 7.53324e-05; tol = 3.36897e-08

It = 14; res = 2.48026e-05; tol = 1.10921e-08

It = 15; res = 7.72044e-06; tol = 3.45269e-09

It = 16; res = 2.70016e-06; tol = 1.20755e-09

It = 17; res = 9.65432e-07; tol = 4.31754e-10

> Final discrepancy = 9.65432e-07

> Numder of iters = 17

> Final time of computation = 1.99185

> Number of threads = 16

It = 0; res = 2236.07; tol = 1

It = 1; res = 837.045; tol = 0.374338

It = 2; res = 90.2838; tol = 0.0403762

It = 3; res = 19.6953; tol = 0.00880802

It = 4; res = 5.82113; tol = 0.00260329

It = 5; res = 1.54234; tol = 0.000689753

It = 6; res = 0.447256; tol = 0.000200019

It = 7; res = 0.106756; tol = 4.77425e-05

It = 8; res = 0.0284587; tol = 1.27271e-05

It = 9; res = 0.0075798; tol = 3.38979e-06

It = 10; res = 0.00235927; tol = 1.0551e-06

It = 11; res = 0.000887506; tol = 3.96905e-07

It = 12; res = 0.000252407; tol = 1.1288e-07

It = 13; res = 7.53324e-05; tol = 3.36897e-08

It = 14; res = 2.48026e-05; tol = 1.10921e-08

It = 15; res = 7.72044e-06; tol = 3.45269e-09

It = 16; res = 2.70016e-06; tol = 1.20755e-09

It = 17; res = 9.65432e-07; tol = 4.31754e-10

> Final discrepancy = 9.65432e-07

> Numder of iters = 17

> Final time of computation = 1.80727