

$$\int_{\Gamma} \epsilon_{ij} + C_{ijkl} \epsilon_{kl} \delta_{ij} ds + \int_{\Gamma} t_j \delta_{ij} \delta_{kl} ds$$

$$= t \cdot \partial R - \int_{\Gamma} t_j \delta_{ij} \delta_{kl} ds$$

(常-部分)

$$+ C_{ijkl} \epsilon_{kl} = d + \sigma J + (W + \sigma + \sigma W)^T.$$

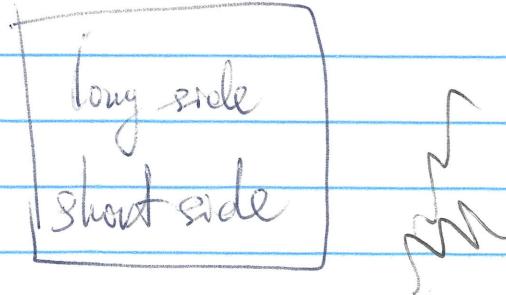
当前为

$$\begin{aligned} \text{基值 } & W = \frac{1}{2} [\nabla \vec{U} - (\nabla \vec{U})^T] & \text{修正 } & \nabla U = [A^T U \quad A^T U] \\ & \vec{e} = \frac{1}{2} [\nabla \vec{U} + (\nabla \vec{U})^T] & \text{修正 } & A^T U \quad A^T U \end{aligned}$$

$$\begin{aligned} + C_{ijkl} \epsilon_{kl} &= \left[C_1 \text{Tr}(\sigma) e + C_2 \sigma \text{Tr}(e) + C_3 \frac{\text{Tr}(\sigma e)}{\text{Tr}(\sigma)} e \right. \\ &\quad \left. + C_4 (\sigma + \sigma^*) \|e\|_1 \right] + (W + \sigma + \sigma W)^T. \end{aligned}$$

$$e = \begin{bmatrix} A^T U & \frac{1}{2}(A^T U + A^T V) \\ \frac{1}{2}(A^T V + A^T U) & A^T V \end{bmatrix}$$

$$W = \begin{bmatrix} 0 & \frac{1}{2}(A^T U - A^T V) \\ \frac{1}{2}(A^T V - A^T U) & 0 \end{bmatrix}$$



$$\delta W_f \sigma = \frac{1}{2} \begin{bmatrix} J_{21}(A_2^T U - A_1^T V) & J_{22}(A_2^T U - A_1^T V) \\ J_{11}(A_1^T V - A_2^T U) & J_{12}(A_1^T V - A_2^T U) \end{bmatrix}$$

$$\delta \sigma_f W^T = \frac{1}{2} \begin{bmatrix} J_{21}(A_2^T U - A_1^T V) & J_{11}(A_1^T V - A_2^T U) \\ J_{22}(A_1^T V - A_2^T U) & J_{21}(A_1^T V - A_1^T V) \end{bmatrix}$$

$$\Rightarrow \delta W_f \sigma + \delta \sigma_f W^T$$

$$= \begin{bmatrix} \frac{J_{21} + J_{22}}{2}(A_2^T U - A_1^T V) & \frac{J_{11} - J_{22}}{2}(A_1^T V - A_2^T U) \\ \frac{J_{11} - J_{22}}{2}(A_1^T V - A_2^T U) & \frac{J_{12} + J_{21}}{2}(A_1^T V - A_2^T U) \end{bmatrix}$$

* $C_1 \text{Tr}(\sigma) e$

$$= C_1 (J_{11} + J_{22}) \begin{bmatrix} A_1^T U & \frac{1}{2}(A_2^T U + A_1^T V) \\ \frac{1}{2}(A_1^T V + A_2^T U) & A_2^T V \end{bmatrix}$$

* $C_2 \text{Tr}(e) \sigma =$

$$= C_2 \sigma (A_1^T U + A_2^T V)$$

$$* C_3 \Rightarrow C_3 \frac{1}{J_{11} + J_{22}} \left[\sigma_1 A_1^T U + \frac{J_{12} + J_{21}}{2} (A_1^T V + A_2^T U) + \sigma_2 A_2^T V \right]$$

* σ

~~$$\star C_4 \Rightarrow C_4 (\sigma + \sigma^*) \|e\| \quad \frac{1}{2}(e + e) = \frac{1}{2}e$$

逆元(非滿性)~~

~~$$= C_4 \|e\| \left(2\sigma - \frac{1}{2} \text{tr}(\sigma) I \right)$$~~

~~$$= C_4 \|e\| \begin{pmatrix} 2\sigma_{11} - \frac{\sigma_{11} + \sigma_{22}}{2} & 2\sigma_{12} \\ 2\sigma_{21} & 2\sigma_{22} - \frac{\sigma_{11} + \sigma_{22}}{2} \end{pmatrix}$$~~

— 3 的內積形式 —

~~$$\delta e = \begin{bmatrix} A_1^T \delta U, \frac{1}{2}(A_2^T \delta U + A_1^T \delta V) \\ \frac{1}{2}(A_1^T \delta V + A_2^T \delta U) & A_2^T \delta V \end{bmatrix}$$~~

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~~$$\begin{aligned} \delta U^T A_1 \left(\frac{\sigma_{11} + \sigma_{12}}{2} \right) (A_2^T U - A_1^T V) + \frac{1}{2} (\delta U^T A_2 + \delta V^T A_1) \frac{\sigma_{11} - \sigma_{22}}{2} (A_1^T V - A_2^T U) \\ + \frac{1}{2} (\delta V^T A_1 + \delta U^T A_2) \frac{\sigma_{11} - \sigma_{22}}{2} (A_1^T V - A_2^T U) \\ + \delta V^T A_2 \left(\frac{\sigma_{21} + \sigma_{22}}{2} \right) (A_1^T V - A_2^T U) \end{aligned}$$~~

~~$$= \delta U^T \left[\left(\frac{\sigma_{21} + \sigma_{12}}{2} \right) [A_1 A_2^T U - A_1 A_1^T V] + \left(\frac{\sigma_{11} - \sigma_{22}}{2} \right) [A_2 A_1^T V - A_2 A_2^T U] \right]$$~~

~~$$+ \delta V^T \left[\left(\frac{\sigma_{11} - \sigma_{22}}{2} \right) [A_1 A_1^T V - A_1 A_2^T U] + \cancel{\delta U^T} \left(\frac{\sigma_{21} + \sigma_{12}}{2} \right) [A_2 A_1^T V - A_2 A_2^T U] \right]$$~~

$$= \delta U^T \left[\frac{\sigma_{11} + \sigma_{22}}{2} A_1 A_1^T - \frac{\sigma_{11} - \sigma_{22}}{2} A_2 A_2^T \right] U$$

$$\delta U^T \left[-\frac{\sigma_{11} - \sigma_{22}}{2} A_2 A_2^T - \frac{\sigma_{11} + \sigma_{22}}{2} A_1 A_1^T \right] V$$

$$\delta V^T \left[-\frac{\sigma_{11} - \sigma_{22}}{2} A_1 A_1^T - \frac{\sigma_{11} + \sigma_{22}}{2} A_2 A_2^T \right] U$$

$$\delta V^T \left[+ \frac{\sigma_{11} - \sigma_{22}}{2} A_1 A_1^T + \left(\frac{\sigma_{11} + \sigma_{22}}{2} \right) A_2 A_2^T \right] V$$

* $C_1 \Rightarrow$

$$C_1(\sigma_{11} + \sigma_{22}) \left[\delta U^T A_1 A_1^T U + \frac{1}{2} (\delta U^T A_2^T + \delta V^T A_1^T) (A_2^T U + A_1^T V) \right. \\ \left. + \delta V^T A_2 A_2^T V \right]$$

$$= \delta U^T \left[C_1(\sigma_{11} + \sigma_{22}) \left[A_1 A_1^T + \frac{1}{2} A_2 A_2^T \right] \right] U$$

$$\delta U^T \left[C_1(\sigma_{11} + \sigma_{22}) \frac{1}{2} A_2 A_2^T \right] V$$

$$\delta V^T \left[C_1(\sigma_{11} + \sigma_{22}) \frac{1}{2} A_1 A_1^T \right] U$$

$$\delta V^T \left[C_1(\sigma_{11} + \sigma_{22}) \left[\frac{1}{2} A_1 A_1^T + A_2 A_2^T \right] \right] V$$

* $C_2 \Rightarrow$

$(AU + AV)$

$$C_2 \left[\sigma_{11} \delta U^T A_1 + \frac{\sigma_{12} + \sigma_{21}}{2} (A_1^T \delta V + A_2^T \delta U) + \sigma_{22} A_2^T \delta V \right]$$

$$= \mathcal{S}U^T \left[C_2 \sigma_{11} A_1 A_1^T + C_2 \frac{\sigma_{12} + \sigma_{21}}{2} A_2 A_1^T \right] U$$

$$\mathcal{S}U^T \left[C_2 \sigma_{11} A_1 A_2^T + C_2 \frac{\sigma_{12} + \sigma_{21}}{2} A_2 A_2^T \right] V$$

$$\mathcal{S}V^T \left[C_2 \frac{\sigma_{12} + \sigma_{21}}{2} A_1 A_1^T + C_2 \sigma_{22} A_2 A_1^T \right] U$$

$$\mathcal{S}V^T \left[C_2 \frac{\sigma_{12} + \sigma_{21}}{2} A_1 A_2^T + C_2 \sigma_{22} A_2 A_2^T \right] V$$

* $C_3 \Rightarrow$

$$C_3 \left[\sigma_{11} \mathcal{S}U^T A_1 + \frac{\sigma_{12} + \sigma_{21}}{2} (A_1 \mathcal{S}V + A_2 \mathcal{S}U) + \sigma_{22} A_2^T \mathcal{S}V \right]$$

$$\frac{1}{\sigma_{11} + \sigma_{22}} \left[\sigma_{11} A_1 \mathcal{S}U + \frac{\sigma_{12} + \sigma_{21}}{2} (A_1 V + A_2 U) + \sigma_{22} A_2^T V \right]$$

$$= C_3 \left[\mathcal{S}U^T \left[\sigma_{11} A_1 + \frac{\sigma_{12} + \sigma_{21}}{2} A_2 \right] + \mathcal{S}V^T \left[\frac{\sigma_{12} + \sigma_{21}}{2} A_1 + \sigma_{22} A_2 \right] \right]$$

$$\left[\left[\frac{\sigma_{11} A_1^T + \frac{\sigma_{12} + \sigma_{21}}{2}}{\sigma_{11} + \sigma_{22}} A_2^T \right] U + \left[\frac{\sigma_{12} + \sigma_{21}}{2(\sigma_{11} + \sigma_{22})} A_1^T + \frac{\sigma_{22}}{\sigma_{11} + \sigma_{22}} A_2^T \right] V \right]$$

$$= \cancel{\mathcal{S}U^T} C_3 \left[\sigma_{11} A_1 + \frac{\sigma_{12} + \sigma_{21}}{2} A_2 \right]^T \left[\frac{\sigma_{12} + \sigma_{21}}{2(\sigma_{11} + \sigma_{22})} A_1^T + \frac{\sigma_{22}}{\sigma_{11} + \sigma_{22}} A_2^T \right]$$

$$= \mathcal{S}U^T C_3 \left[\sigma_{11} A_1 + \frac{\sigma_{12} + \sigma_{21}}{2} A_2 \right] \left[\frac{\sigma_{11} A_1^T}{\sigma_{11} + \sigma_{22}} + \frac{\sigma_{12} + \sigma_{21}}{2(\sigma_{11} + \sigma_{22})} A_2^T \right] U$$

$$\mathcal{S}U^T C_3 \left[\sigma_{11} A_1 + \frac{\sigma_{12} + \sigma_{21}}{2} A_2 \right] \left[\frac{\sigma_{12} + \sigma_{21}}{2(\sigma_{11} + \sigma_{22})} A_1^T + \frac{\sigma_{22}}{\sigma_{11} + \sigma_{22}} A_2^T \right] V$$

$$\mathcal{S}V^T C_3 \left[\frac{\sigma_{12} + \sigma_{21}}{2} A_1 + \sigma_{22} A_2 \right] \left[\frac{\sigma_{11} A_1^T}{\sigma_{11} + \sigma_{22}} + \frac{\sigma_{12} + \sigma_{21}}{2(\sigma_{11} + \sigma_{22})} A_2^T \right] U$$

$$\mathcal{S}V^T C_3 \left[\frac{\sigma_{12} + \sigma_{21}}{2} A_1 + \sigma_{22} A_2 \right] \left[\frac{\sigma_{12} + \sigma_{21}}{2(\sigma_{11} + \sigma_{22})} A_1^T + \frac{\sigma_{22}}{\sigma_{11} + \sigma_{22}} A_2^T \right] V$$

C₄ ⇒

$$\text{C}_4 \text{ left} \left\{ \begin{array}{l} \delta U^T A_1 \left[2\sigma_{11} - \frac{\sigma_{11} + \sigma_{22}}{2} \right] + (\delta U^T A_2 + \delta V^T A_1) \sigma_{12} \\ + [\delta V^T A_1 + \delta U^T A_2] \sigma_{21} + \delta V^T A_2 \left(\sigma_{22} - \frac{\sigma_{11} + \sigma_{22}}{2} \right) \end{array} \right\}$$

$$= \delta U^T \left\{ \text{C}_4 \text{ left} \left(A_1 \left[2\sigma_{11} - \frac{\sigma_{11} + \sigma_{22}}{2} \right] + [\sigma_{12} A_2 + \sigma_{21} A_2] \right) \right\}$$

$$\delta V^T \left\{ \text{C}_4 \text{ left} \left([A_{12} A_1 + \sigma_{11} A_1] + A_2 \left[2\sigma_{22} - \frac{\sigma_{11} + \sigma_{22}}{2} \right] \right) \right\}$$

$$\int_{\Gamma} \partial_{\nu} \varphi \nu_i ds + \int_{\Gamma} t^k \partial_{\nu} \varphi \nu_i ds$$

$$= \text{int } \partial R - \int_{\Gamma} t^k \partial_{\nu} \varphi \nu_i ds$$

第二部分

$$\eta = \frac{1}{2} \frac{\partial u_k}{\partial x_i} \frac{\partial u_k}{\partial x_j}$$

$$\partial \eta = \frac{1}{2} \frac{\partial^2 u_k}{\partial x_i \partial x_j} + \frac{1}{2} \frac{\partial \eta}{\partial x_i} \frac{\partial \eta}{\partial x_j}$$

$$\delta \eta_{11} = \frac{1}{2} (U^T A_1 A_1^T U + V^T A_1 A_1^T V)$$

$$\eta_{12} = \frac{1}{2} (U^T A_1 A_2^T U + V^T A_1 A_2^T V)$$

$$\eta_{21} = \frac{1}{2} (U^T A_2 A_1^T U + V^T A_2 A_1^T V)$$

$$\eta_{22} = \frac{1}{2} (U^T A_2 A_2^T U + V^T A_2 A_2^T V)$$

$$\delta \eta_{11} = \delta U^T A_1 A_1^T U + \delta V^T A_1 A_1^T V$$

$$\delta \eta_{12} = \frac{1}{2} (\delta U^T A_1 A_2^T U + \delta U^T A_2 A_1^T U) \\ \delta V^T A_1 A_2^T V + \delta V^T A_2 A_1^T V$$

$$\delta \eta_{21} = \frac{1}{2} (\delta U^T A_2 A_1^T U + \delta U^T A_1 A_2^T U) \\ \delta V^T A_2 A_1^T V + \delta V^T A_1 A_2^T V$$

$$\delta \eta_{22} = \delta U^T A_2 A_2^T U + \delta V^T A_2 A_2^T V$$

~~$\partial_{ij} \delta^T \ln j$~~

$$= \sigma_{11} \delta U^T A_1 A_1^T U + \sigma_{11} \delta V^T A_1 A_1^T V$$

$$+ \frac{\sigma_{12} + \sigma_{21}}{2} \left[\begin{array}{l} \delta U^T A_1 A_2^T U + \delta U^T A_2 A_1^T U \\ \delta V^T A_1 A_2^T V + \delta V^T A_2 A_1^T V \end{array} \right]$$

$$+ \sigma_{22} \delta U^T A_2 A_2^T U + \sigma_{22} \delta V^T A_2 A_2^T V$$

$$= \delta U^T \left[\sigma_{11} A_1 A_1^T + \left(\frac{\sigma_{12} + \sigma_{21}}{2} \right) (A_1 A_2^T + A_2 A_1^T) + \sigma_{22} A_2 A_2^T \right] U$$

$$\delta V^T \left[\sigma_{11} A_1 A_1^T + \left(\frac{\sigma_{12} + \sigma_{21}}{2} \right) (A_1 A_2^T + A_2 A_1^T) + \sigma_{22} A_2 A_2^T \right] V$$

$$\int_{t_0}^{t_1} \delta \phi_{ij} \partial_n \delta \phi_{ij} dt + \int_{t_0}^{t_1} \delta \phi_j^T \partial_{ij} \delta \phi_i \delta \phi_{ij} dt$$

$$= \overset{t=t_0}{\cancel{\int R}} - \int_{t_0}^{t_1} \delta \phi_j^T \partial_{ij} \delta \phi_i \delta \phi_{ij} dt$$

第二部分

$$e = \begin{bmatrix} A^T U & \frac{1}{2}(A^T U + A^T V) \\ \frac{1}{2}(A^T V + A^T U) & A^T V \end{bmatrix}$$

$$\delta e = \begin{bmatrix} \delta U^T A_1 & \frac{1}{2} \delta U^T A_2 + \frac{1}{2} \delta V^T A_1 \\ \frac{1}{2} \delta V^T A_1 + \frac{1}{2} \delta U^T A_2 & \delta V^T A_2 \end{bmatrix}$$

$$[\delta \delta e] = \delta U^T [\delta A_1 + \left(\frac{\delta U + \delta V}{2}\right) \cancel{\delta A_2}]$$

都去掉

$$+ \delta V^T \left[\left(\frac{\delta U + \delta V}{2}\right) A_1 + \delta A_2 \right]$$

$$\int f_{ts} + \text{C}_1 n + \text{C}_2 \delta + \text{C}_3 dts + \int f_{ts} + \text{C}_4 \delta + \text{C}_5 dts$$

$$= t \text{tot} \partial R - \int f_{ts} + \text{C}_4 \delta + \text{C}_5 dts$$

2) $\partial R =$

~~$$\frac{1}{2} (\partial W_1 + \partial W_2) J_1 \times d$$~~