Cell growth and proliferation: mathematical modeling and simulation

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Cell growth division and proliferation

Let the cell size s grow exponentially according to the equation

$$\frac{ds}{dt} = \alpha s,\ s(0) = s_0,$$

where α is the growth rate, s_0 is the initial cell size, and t is the experiment time. Also, let the cell cycle time τ be the time it takes for a cell to grow and divide. The cell cycle progress is represented by the equation

$$\frac{d\tau}{dt} = 1,\ \tau(0) = 0.$$

A division event occurs when a cell splits into two daughter cells. This event resets both the cell size to half and the cell cycle time to zero, that is,

$$s \mapsto s/2, \quad \tau \mapsto 0,$$

marking the end of one cycle and the start of a new one. Let $P(\tau)$ define when division happens as per

$$P(\tau) = U(\tau - \bar{\tau}) = \begin{cases} 1, & \text{if } \tau > \bar{\tau} \\ 0, & \text{otherwise,} \end{cases}$$

where $U(\tau - \bar{\tau})$ is the unit step function and $\bar{\tau}$ is the time to division since the start of the cell cycle. Cell performs a division event if $P(\tau) = 1$.

The division rate can be defined as

$$\begin{aligned} p(\tau) &= \frac{dP}{d\tau} \\ &= \delta(\tau - \bar{\tau}), \end{aligned}$$

where $\delta(\tau)$ is the *Delta Dirac* function. The above description is summarized using graph or automata notation

Cell size s at a given experimental time t can be written as the combination of sizes in individual cell cycles, that is,

$$s(t) = \sum_{n=0}^{\infty} s_n(t) P_n(t),$$

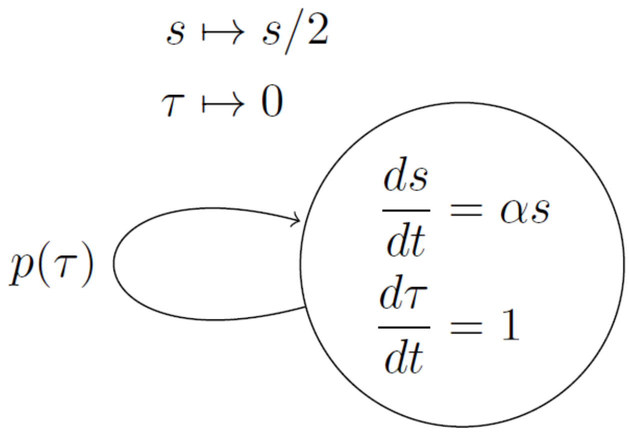

Figure 1: Growth and cell division representation using automata notation.

where

$$
\begin{aligned}
s_n(t) =& s_0 \left[\prod_{i=1}^{n} \frac{e^{\alpha(t_i - t_{i-1})}}{2} \right] e^{\alpha(t-t_n)} \\
=& \frac{s_0}{2^n} e^{\alpha t}
\end{aligned}
$$

is the cell size s at experimental time t given cell performed n divisions. The sequence of experimental times at which division happens is t_i with $t_i - t_{i-1} = \bar{\tau}$. The cell cycle time after n divisions is given by $\tau = t - t_n$.

$$
P_n(t) = \begin{cases} 1, \text{ if } \quad n\,\bar{\tau} < t \leq (n+1)\bar{\tau} \\ 0, \text{ otherwise} \end{cases}
$$

dictates the number of cell cycles n performed by the cell at time t. Cell size $s(t)$ is a periodic function in time, with a period $\bar{\tau}$ and a repeating dynamics represented by exponential growth $s_0 e^{\alpha t}$ in the cell cycle interval $\tau \in [0, \bar{\tau}]$.

1 Introduction

This is a book created from markdown and executable code.

See Knuth (1984) for additional discussion of literate programming.

```
1 + 1
```

```
[1] 2
```

2 Summary

In summary, this book has no content whatsoever.

```
1 + 1
```

```
[1] 2
```

References

Knuth, Donald E. 1984. "Literate Programming." *Comput. J.* 27 (2): 97–111. https://doi.org/10.1093/comjnl/27.2.97.