LAW OF LARGE NUMBER

“

REFERENCES:

1) *Law of Large Numbers Chapter 8.1 of Grinstead*

2) *https://github.com/letaoZ/MAT331/blob/master/week03/inverse_transform_method.pdf*

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DISCRETE

Let $x_1, x_2, \ldots, x_n$ be a sequence of independent, identically-distributed (IID) values from a random variable X. Suppose that X has the finite mean μ. Then the average of the first n of them:

$$S_n = \frac{1}{n} \sum_{i=1}^{n} x_i,$$

converges to the mean of X μ as $n \to \infty$:

$$S_n \to \mu \, as \, n \to \infty.$$

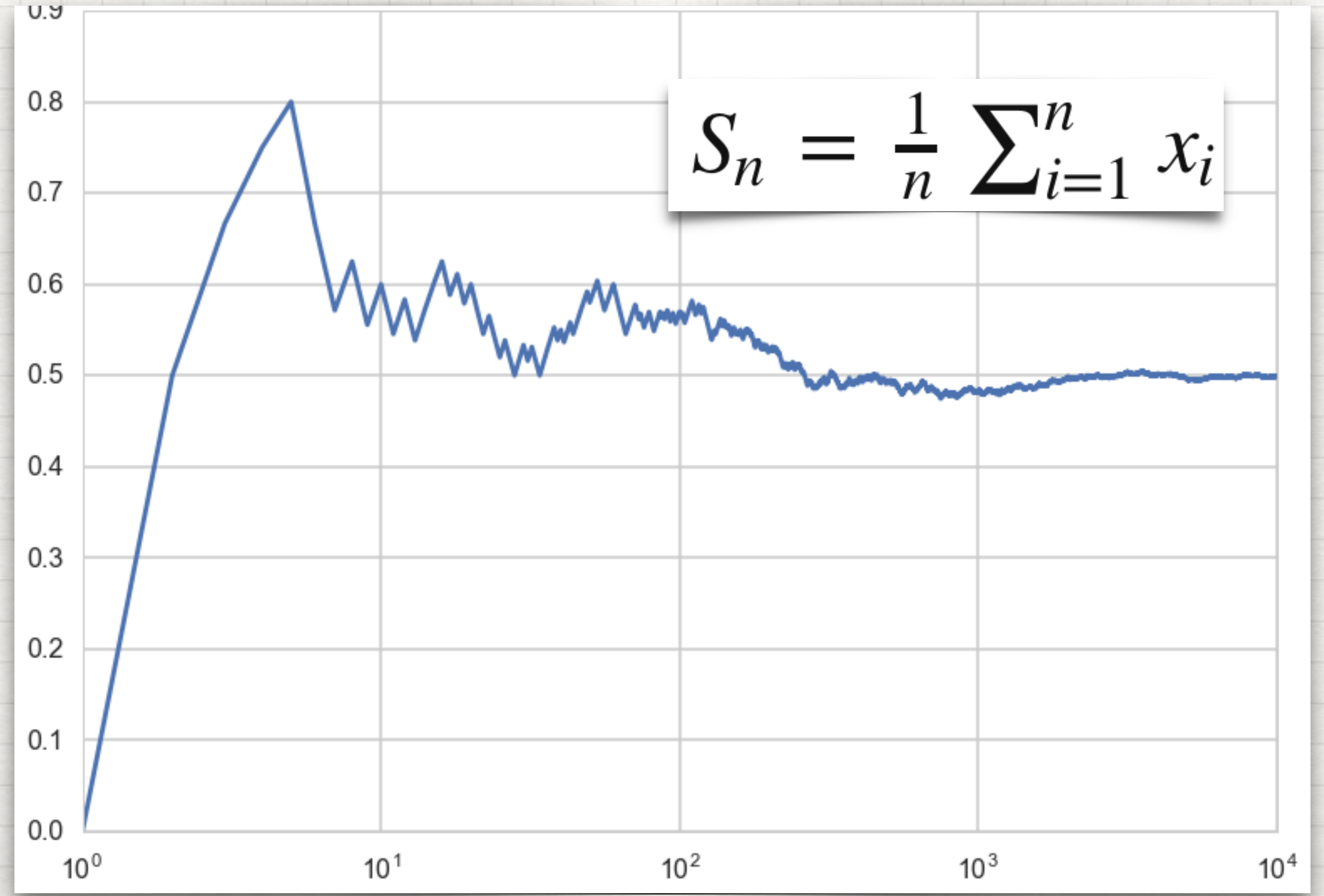
FLIPPING COIN EXAMPLE

- Imagine a sequence of length n of coin flips.
- Each coin flip follows Bernoulli(0.5)
- Lets keep increasing the length of the sequence of coin flips n
- Compute a running average Sn of the coin-flip random variables

$$S_n = \frac{1}{n} \sum_{i=1}^{n} x_i$$

We plot this running mean

It converges to the mean of the distribution from which the random variables are plucked, i.e. the Bernoulli distribution with p=0.5.



SAMPLING

INVERSE TRANSFORM METHOD

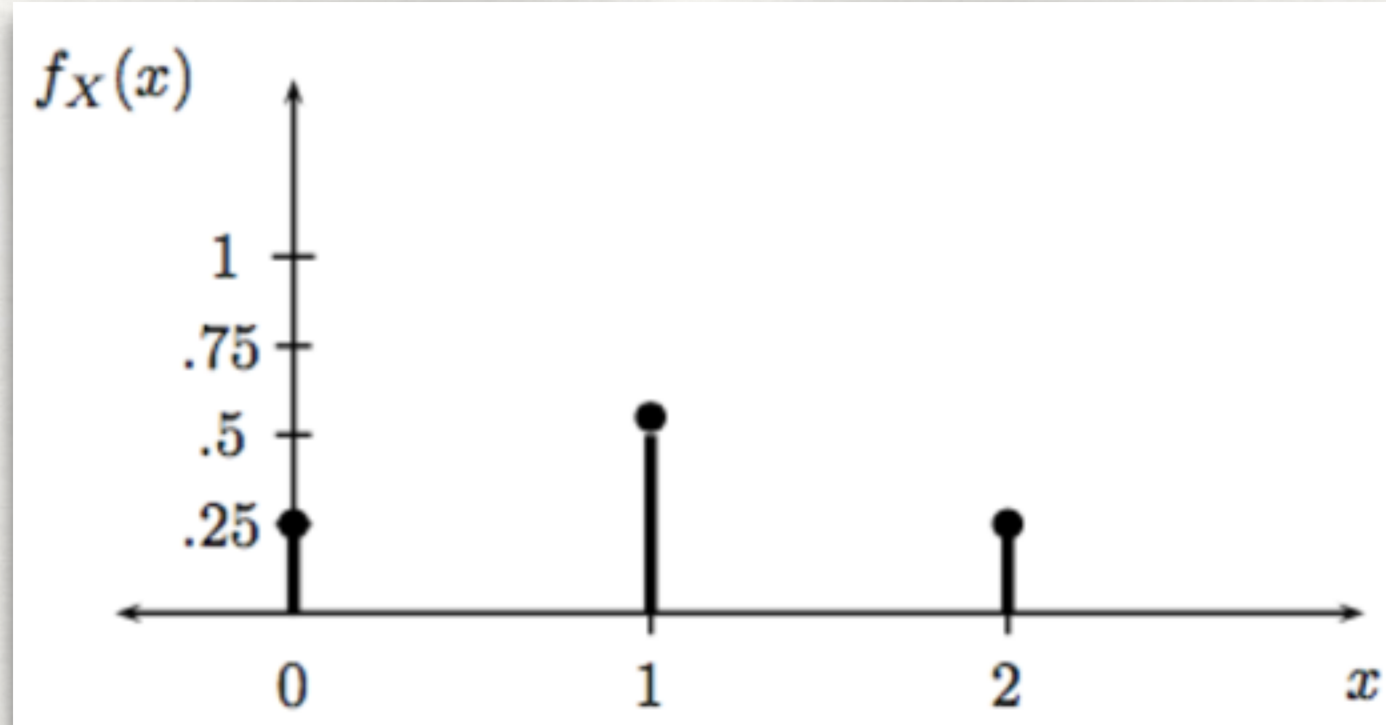
- Assume you have a random number generator
- This generator can draw a number out of [0,1] equally likely
i.e. it samples from a uniform distribution on [0,1]
- Question:
How can we use this generator to sample from another distribution?
- Recall that a uniform random variable U on [0, 1] has cumulative distribution function

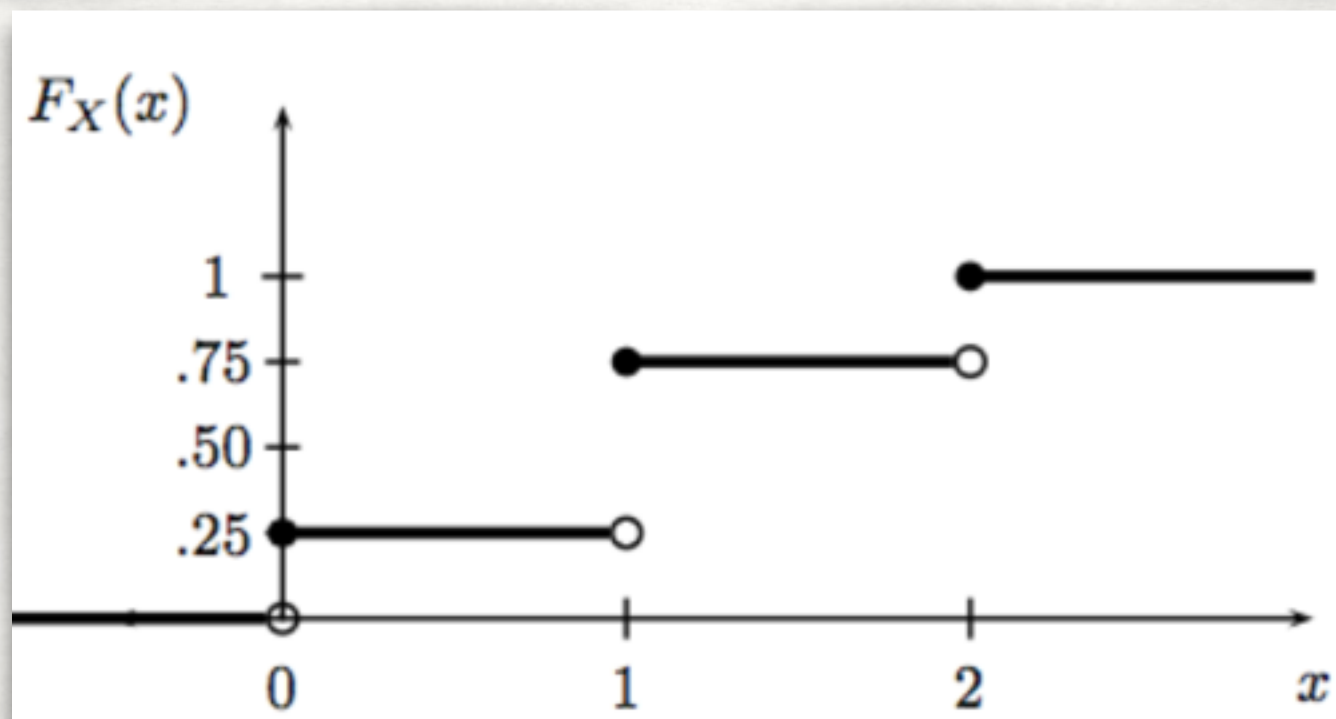
$$F_U(x) = P(U \leq x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1. \end{cases}$$

INVERSE TRANSFORM METHOD

DISCRETE

- f(x) pmf for discrete random variable X representing the number of heads in two coin tosses.
- F(X) the cumulative density function for X

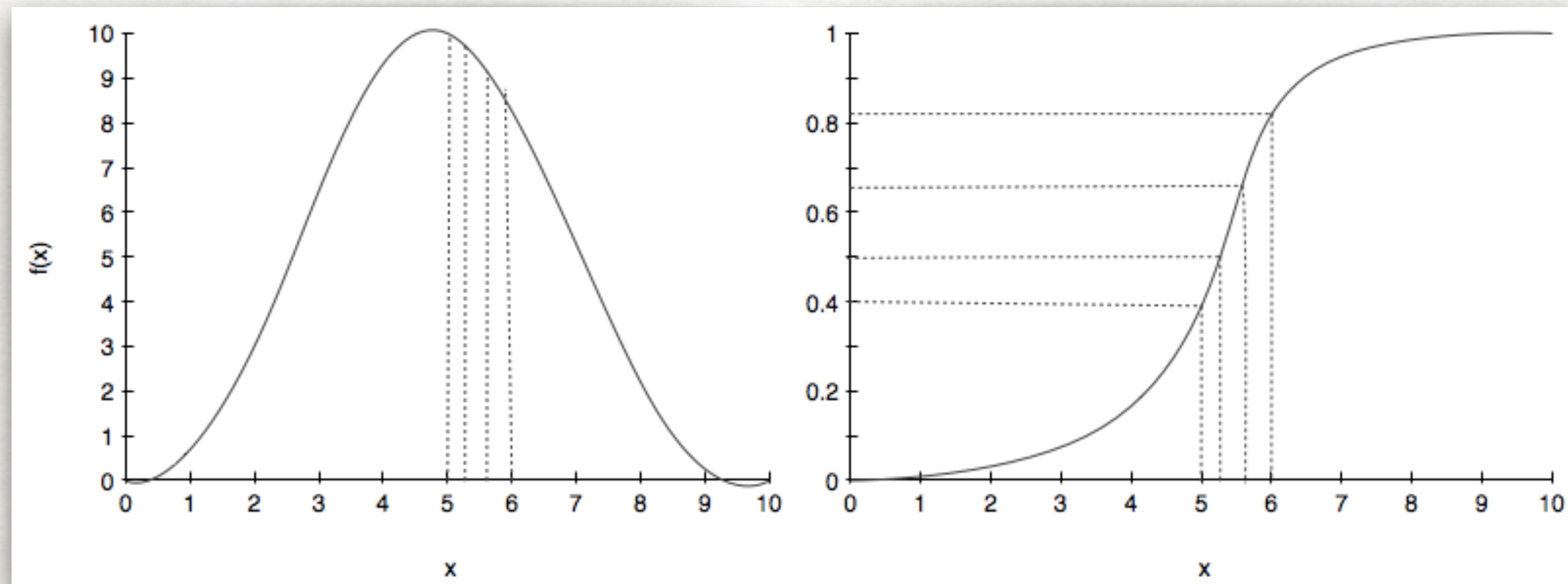




INVERSE TRANSFORM METHOD

CONTINUOUS RANDOM VARIABLE

- pdf: area below f(x), 5<x<6 CDF: F(x) = P(5<x<6)

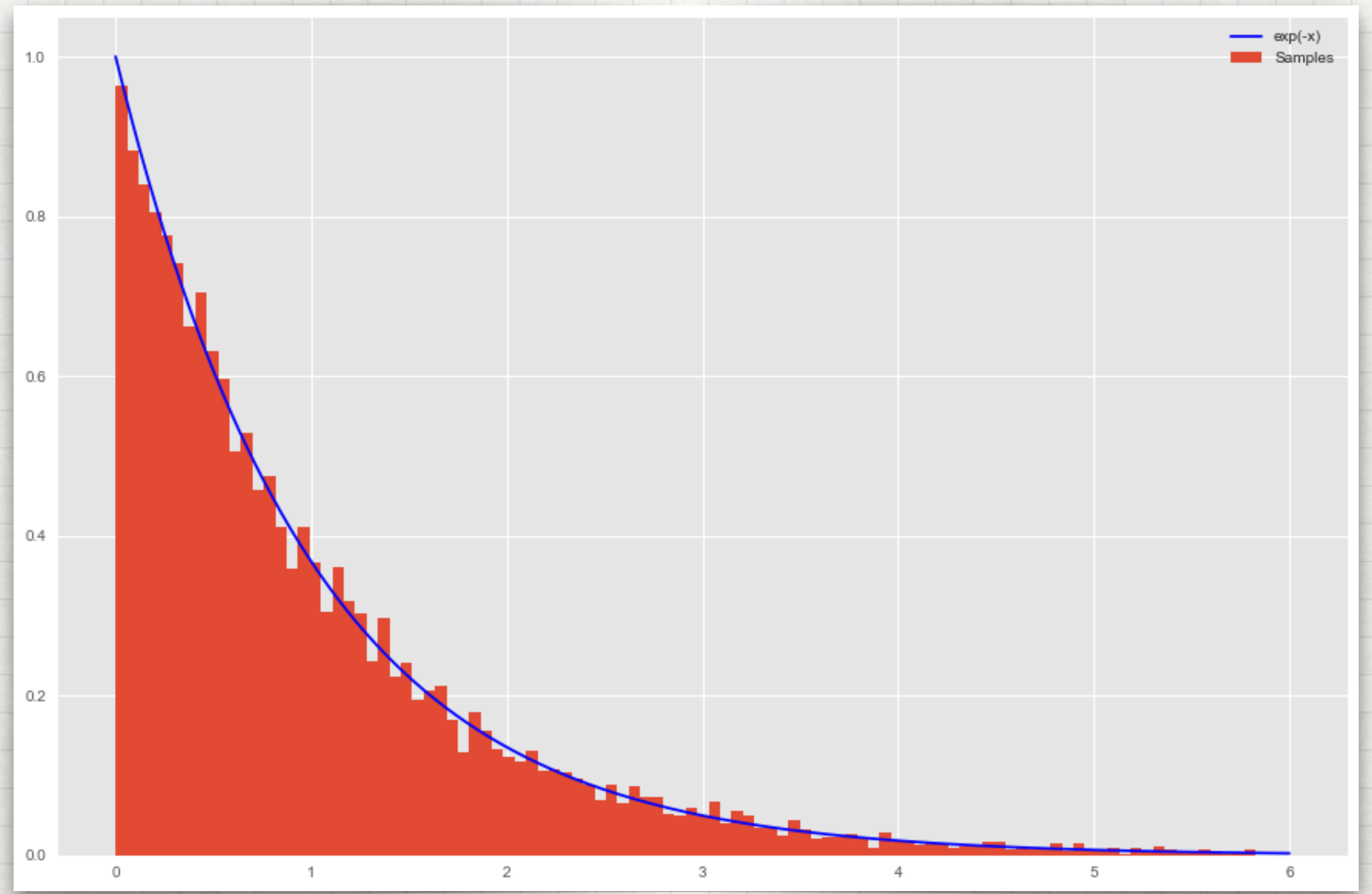


FORMALIZE

SAMPLING PROCESS

This is the process:

1. get a uniform sample u from $Unif(0, 1)$
2. solve for x yielding a new equation $x = F^{-1}(u)$ where F is the CDF of the distribution we desire.
3. repeat.



exp(-x)
Samples
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6

HISTOGRAM

HISTOGRAM AND PROBABILITY DENSITY

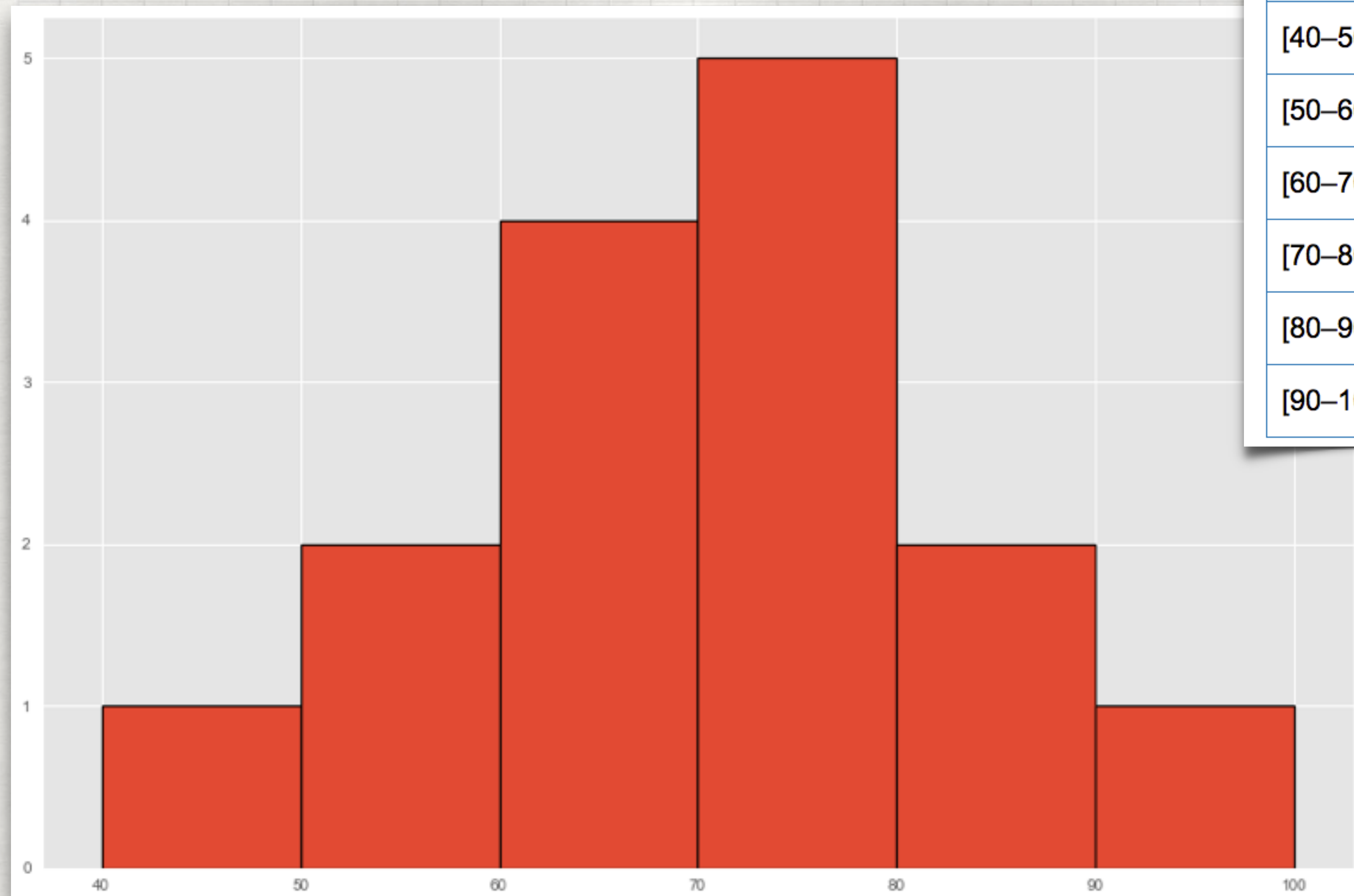
- Break the range of values into intervals and count how many observations fall into each interval.
- Here are the exam grades of 15 students:
  88, 48, 60, 51, 57, 85, 69, 75, 97, 72, 71, 79, 65, 63, 73
- We first need to break the range of values into intervals (also called "bins" or "classes").

We have 6 equal spaced bins (each of length 10)

Exam Grades

Score	Count
[40–50)	1
[50–60)	2
[60–70)	4
[70–80)	5
[80–90)	2
[90–100]	1

Histogram of Exam Grades



Score	Count
[40–50)	1
[50–60)	2
[60–70)	4
[70–80)	5
[80–90)	2
[90–100]	1

HISTOGRAM AND PROBABILITY DENSITY

- The table above can also be turned into a relative frequency table using the following steps:
- Add a row on the bottom and include the total number of observations in the dataset that are represented in the table.
- Add a column, at the end of the table, and calculate the relative frequency for each int

sum of relative sequence is 1 makes this a probability density function — in statistical sense

Score	Count (also called Frequency)	Relative Frequency
[40–50)	1	0.07
[50–60)	2	0.13
[60–70)	4	0.27
[70–80)	5	0.33
[80–90)	2	0.13
[90–100]	1	0.07
Total	15	

Step 1: Add a row at bottom of table. Put in total number of observations in the data set.

In this example, there are 15 (1+2+4+5+2+1= 15) total observations.

eg: In score [40 - 50), there are 1/15 = 0.07 much of data

density estimate histogram