1	2	
3	4	freedom

Written Homework 11 · MATH331

Markov Chain (chapter 11 in Grinstead)

Due on Thursday Apr 12 11:30AM

1. Markov Chain (Continue from Homework 10) Consider a rat in a maze with 4 cells, indexed as 1,2,3,4, and the outside (freedom), indexed by 5 (that can only be reached via cell 4). The rat starts initially in a given cell and then takes a move to another cell, continuing to do so until finally reaching freedom. We assume that at each move (transition) the rat, independent of the past, is equally likely to choose from among the neighboring cells (sharing an edge). eg. if it is in cell 4, then it is equal likely that the rat moves to cell 3, 2, or freedom.

   Please pay attention to my state labeling here $\{s_1, s_2, s_3, s_4, s_5\}$ where s_i corresponds to cell $i, i = 1, 2, 3, 4$ and s_5 corresponds to freedom

   Then corresponding transition matrix P is a 5 by 5 matrix:

   $$P = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

   (a) Find $P^{(2)} = P^2$

   (b) Find the following probability values:

   i. The probability that the rat starts in cell 2, and end up with freedom in 2 steps

   ii. The probability that the rat starts in cell 3, and end up with freedom in 2 steps

   iii. The probability that the rat starts in cell 4, and end up with freedom in 2 steps

Exercises on Linear Algebra. This is for you to review some basic linear algebra

Given matrix $P = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1 & 0 \end{pmatrix}$

1. Find $P^T :=$ the transpose of P

2. Find $P^{-1} :=$ the inverse of P