

## Practice Midterm

**Please show your work.**

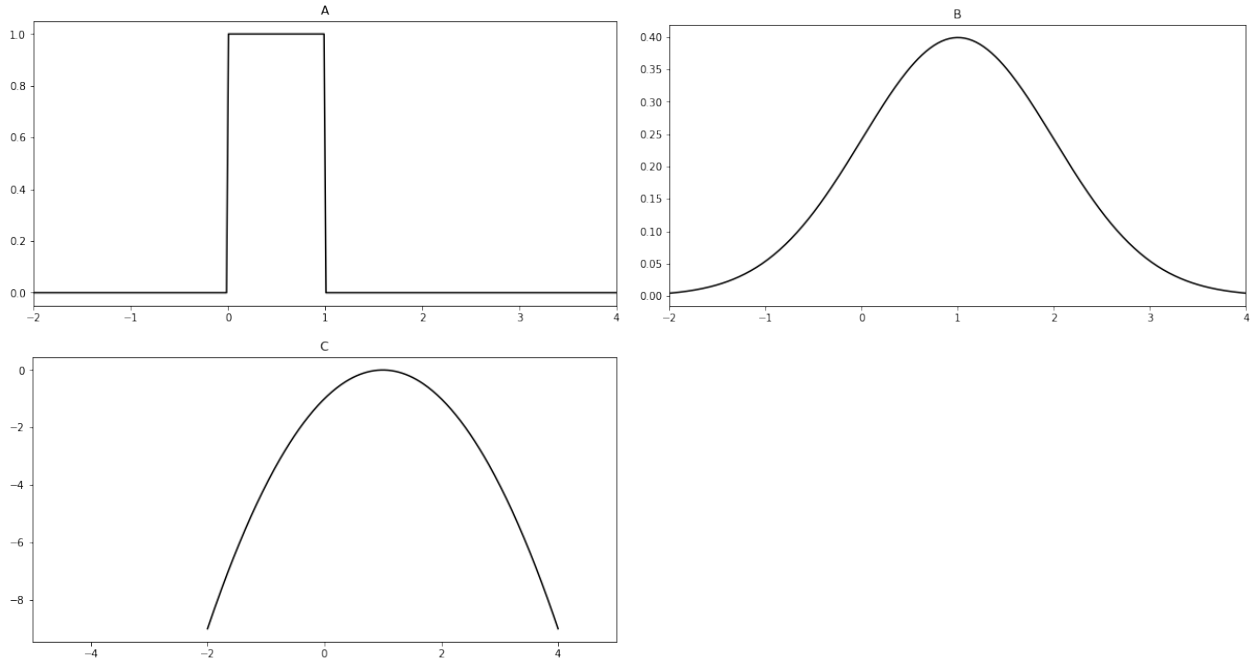
1.  $X$  is a random variable with mean 2 and variance  $1/4$ . Find expectation  $E(X/2)$  and standard deviation of  $X - 1$ .
2.  $\{X_1, X_2, \dots, X_{10}\}$  is a sequence of IID follows a Bernoulli distribution with parameter 0.1. That is

$$p(X_i = 1) = 0.1, p(X_i = 0) = 0.9, E(X_i) = 0.1, Var(X_i) = 0.09$$

- . Let  $A_{10} = \frac{\sum_{i=1}^{10} X_i}{10}$  Find the following values
- (a)  $E(A_{10})$
  - (b)  $Var(A_{10})$
  - (c)  $std(A_{10})$
  - (d) Use the Central Limit Theorem to find a normal distribution  $N(\mu, \sigma^2)$  (i.e. to find  $\mu$  and  $\sigma$ ) to approximate the distribution of  $A_{10}$ .
3. Suppose you choose a real number  $X$  from the interval  $[1, e]$  with a density function of the form:
$$f(x) = \frac{C}{x}$$
where  $C$  is a constant. Note that  $e$  is the Euler's constant 2.718281....
    - (a) Find  $C$
    - (b) Find  $P(E)$ , where  $E = [1, \frac{e}{2}]$  is a subinterval of  $[1, e]$ .
    - (c) Find the cumulative density function of  $X$
  4. Let  $U, V$  be random numbers chosen independently from the interval  $[0, 1]$  uniformly. Find the cumulative distribution and density for the random variable  $Y = \max(U, V)$
  5.  $X$  follows a Normal Distribution with mean 5 and variance 4. Calculate the following values
    - (a)  $P(|X| > 3)$
    - (b)  $P(X < 9)$
    - (c)  $P(X > 5)$
    - (d)  $P(e^X < 1)$

(e)  $E(X^2)$

6.  $X$  follows a Normal Distribution with mean  $\mu$  and standard deviation  $\sigma$ . Let  $Z$  denote the standard normal distribution. Is it true that  $P(|X - \mu| > \sigma)$  is equal to  $P(|Z| > 1)$ ? Justify your answer.
7. Let  $X$  be a random variable with  $E(X) = 0$  and  $Var(X) = 0.01$ . What integer value  $k$  will assure us that  $P(|X| \geq k) \leq 0.01$ ?
8. Which plot corresponds to a normal distribution?



9. The probability density function of  $X$  is  $f(x) = \begin{cases} 3(x-1)^2, & \text{if } 2 \geq x \geq 1 \\ 0, & \text{otherwise} \end{cases}$

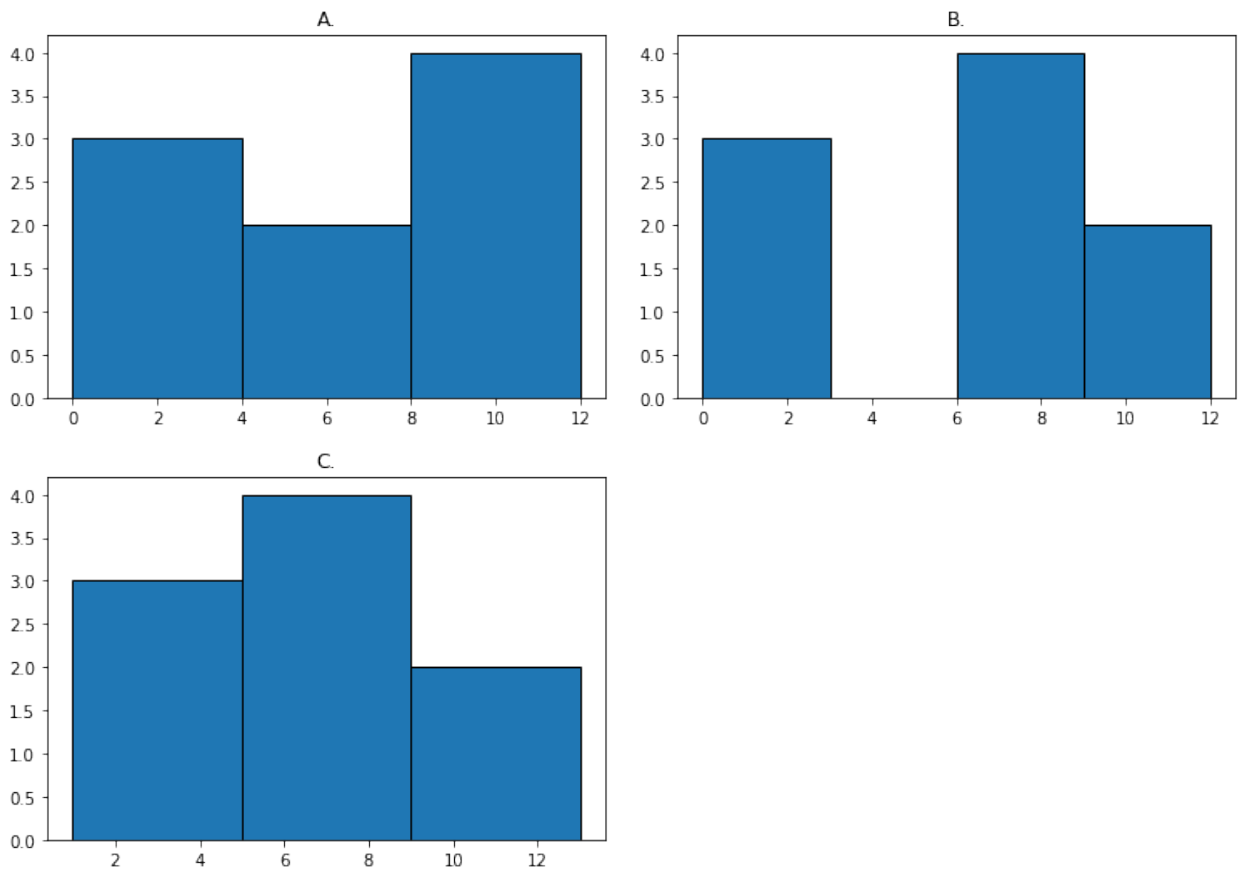
Find the inverse Cumulative Density Function of  $X$

10. Use Hypothesis Testing to answer the question. There are rumors that students at a certain liberal arts college are more inclined to use Gahoo email than U.S. college students in general. Suppose that in a simple random sample of 100 students from the college, 50 admitted to Gahoo email use. Do the data provide enough evidence to conclude that the proportion of Gahoo email users among the students in the college ( $p$ ) is higher than the national average, which is 0.3?
- (a) Determine Null Hypothesis  $H_0$
- (b) Determine Alternative Hypothesis  $H_a$
- (c) Determine z-score.

- (d) Assume the confidence level is  $\alpha = 5\%$ . Do the data provide enough evidence to conclude that the proportion of Gahoo email users among the students in the college ( $p$ ) is higher than the national proportion, which is 0.3?

11. In python, if I run the following code, select the right output graph.

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3
4 data = [1,2,2,7,7.5,8,8.5,9,12]
5 plt.hist(data,bins = 3, range = (0,12),edgecolor='black', linewidth=1)
6 plt.show()
7
```



12. In python, if I run the following code, select the right output.

```
1 arr = np.array([8,9,10,-1])
2 print(np.where(arr<0)[0])
3
```

- A. [4]  
B. [3]

C.  $[-1]$

D.  $[10]$

E. None of the above