Written Homework 10 · MATH331

Markov Chain (chapter 11 in Grinstead), and Maximal Likelihood here
Due on Thursday March 05 11:30AM

1. Maximum Likelihood Estimates. Suppose you observe the following data set $[-1, 1, 0, 2]$, where each element in your data set follows a Gaussian Distribution $N(\mu, \sigma^2)$.

   What are the values for μ and σ that maximizes the likelihood of the observed data? Support your answer with full analytic derivations.

2. Markov Chain (see figure above) Consider a rat in a maze with 4 cells, indexed as 1,2,3,4, and the outside (freedom), indexed by 0 (that can only be reached via cell 4). The rat starts initially in a given cell and then takes a move to another cell, continuing to do so until finally reaching freedom. We assume that at each move (transition) the rat, independent of the past, is equally likely to choose from among the neighboring cells (sharing an edge). eg. if it is in cell 4, then it is equal likely that the rat moves to cell 3, 2, or freedom. Please answer the following quesitons:

   (a) write out the state space for the rat's move

   (b) write out the transition matrix for the rat's moves

   (c) (optional) what's the probability that the rat eventually gets freedom?

Exercises on Linear Algebra. This is for you to review some basic linear algebra

Given matrix $P = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1 & 0 \end{pmatrix}$ and column vector $v = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

1. find the determinant of P

2. find P^2

3. find P^3

4. find Pv

5. find $v^T P v$