Written Homework 09 · MATH331

Hypothesis Testing here , and Maximal Likelihood here

Due on Thursday March 29 11:30AM

Please show your work.

1. This activity is based on the results of a recent study on the safety of airplane drinking water that was conducted by the U.S. Environmental Protection Agency (EPA). A study found that out of a random sample of 316 airplanes tested, 40 had coliform bacteria in the drinking water drawn from restrooms and kitchens. As a benchmark comparison, in 2003 the EPA found that about 3.5% of the U.S. population have coliform bacteria-infected drinking water. The question of interest is whether, based on the results of this study, we can conclude that drinking water on airplanes is more contaminated than drinking water in general.

   (a) If we assume $\alpha = 0.05$

   (b) If we assume $\alpha = 0.025$

   Please use hypothesis testing method to justify your answers.

2. Maximum Likelihood Estimates. Suppose you observe the following data set $\mathbf{x}^{(0)} = (1,2), \mathbf{x}^{(1)} = (3,1), \mathbf{x}^{(2)} = (2,5), \mathbf{x}^{(3)} = (10,0)$. For any vector $\mathbf{x}$, we denote the first component of $\mathbf{x}$ by x_1 and the second component by x_2. Suppose that the data is drawn from the same two-dimensional probability distribution with pdf f_X, that is, $\mathbf{x}^{(i)} \overset{iid}{\sim} f_X$, where

   $$f_X(\mathbf{x}) = 4\lambda_1^2 x_1 x_2 \exp\left\{-\lambda_0(x_1^2 + x_2^2)\right\}.$$

   You should assume that $\lambda_1, \lambda_0 > 0$ and that f_X is supported on the nonnegative quandrant of $\mathbb{R}^2$ (i.e. f_X is zero when either component is negative).

   What are the values for λ_0 and λ_1 that maximizes the likelihood of the observed data? Support your answer with full and rigorous analytic derivations.

Exercises from Grinstead

1. Section 8.2 page 323: 12 (a), (b)
   Hint: $Y_2 = X_1 + Y_1$, $Y_3 = X_2 + Y_2 = X_1 + X_2 + Y_1$,..., how to express Y_n in terms of $X_1, \ldots, X_{n-1}$ and Y_1

2. Section 9.3 page 363: 11 (a)

3. Section 9.3 page 363: 13