MAT331 Midterm
March 06 2018

Name:
Student ID:

1. (10pts) X is a random variable with mean 3 and variance 9. Find expectation $E(-X)$ and standard deviation $\text{Std}(-2X+1)$.

$E(-X) = -E(X) = -3$

$\text{Std}(-2X+1) = \text{Std}(-2X) = |-2|\,\text{Std}(X) = 2\sqrt{9} = 6$

or $\text{Std} = \sqrt{\text{Var}(-2X+1)} = \sqrt{\text{Var}(-2X)} = \sqrt{(-2)^2\text{Var}(X)} = \sqrt{4\times 9} = 6$.

2. (20pts) $\{X_1, X_2, \ldots X_9\}$ is a sequence of IID follows a exponential distribution with parameter 1. Let $A_9 = \frac{\sum_{i=1}^{9} X_i}{9}$. Use the Central Limit Theorem to find a normal density $N(\mu, \sigma^2)$ (i.e. to find μ and σ) to approximate the density of A_9.

$$\begin{cases} \mu = E(X_i) = 1 \\ \sigma^2(X_i) = 1 \end{cases}$$

$A_9 \overset{\cdot}{\sim} N(1, \frac{1}{9})$

$$\begin{cases} E(A_9) = 1 \\ \text{Var}(A_9) = \frac{1}{9} \end{cases}$$

3. (20pts) X follows a Normal Distribution with mean -2 and variance 1. Calculate the following values

   (a) $P(|X+2|>1)$

   (b) $E(X^2)$

a) $Z \sim N(0,1)$

$X = Z - 2$

$$P(|X+2|>1) = P(|Z-2+2|>1) = P(|Z|>1)$$

$$= P(Z>1) + P(Z<-1)$$

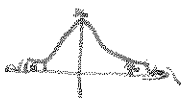
$$= 1 - P(Z<1) + P(Z<-1)$$

$$= 1 - 0.8413 + 0.1587$$

$$= 0.1587 + 0.1587 = 2 \cdot 0.1587$$

$$= 0.3174$$

or $P(|X+2|>1) = P(|Z|>1) = 2 \cdot P(Z<-1) = 2 \cdot 0.1587 = 0.3174$



b) $Var(X) = E(X^2) - E(X)^2 \Rightarrow E(X^2) = Var(X) + E(X)^2$

$= 1 + (-2)^2 = 5$

4. (20pts) Suppose you choose a real number X from the interval $(-\infty, 0]$ with a density function of the form:

$$f(x) = \begin{cases} \frac{C}{(x-1)^2}, & \text{if } x \in (-\infty, 0] \\ 0, & \text{otherwise} \end{cases}$$

where C is a constant.

(a) Find C

(b) Find the Inverse Cumulative Density Function of X

a) $\int_{-\infty}^{0} \frac{C}{(x-1)^2} dx = 1 \qquad -\frac{C}{(x-1)} \Big|_{-\infty}^{0} = 1 \Rightarrow -(-C+0) = 1$

$C = 1$ & $f(x) = \begin{cases} \frac{1}{(x-1)^2}, & x \in (-\infty, 0] \\ 0, & \text{otherwise.} \end{cases}$ ≥ 0 ✓

$f(x) \geq 0$ all x

b) $F(t) = 1$ if $t \geq 0$.

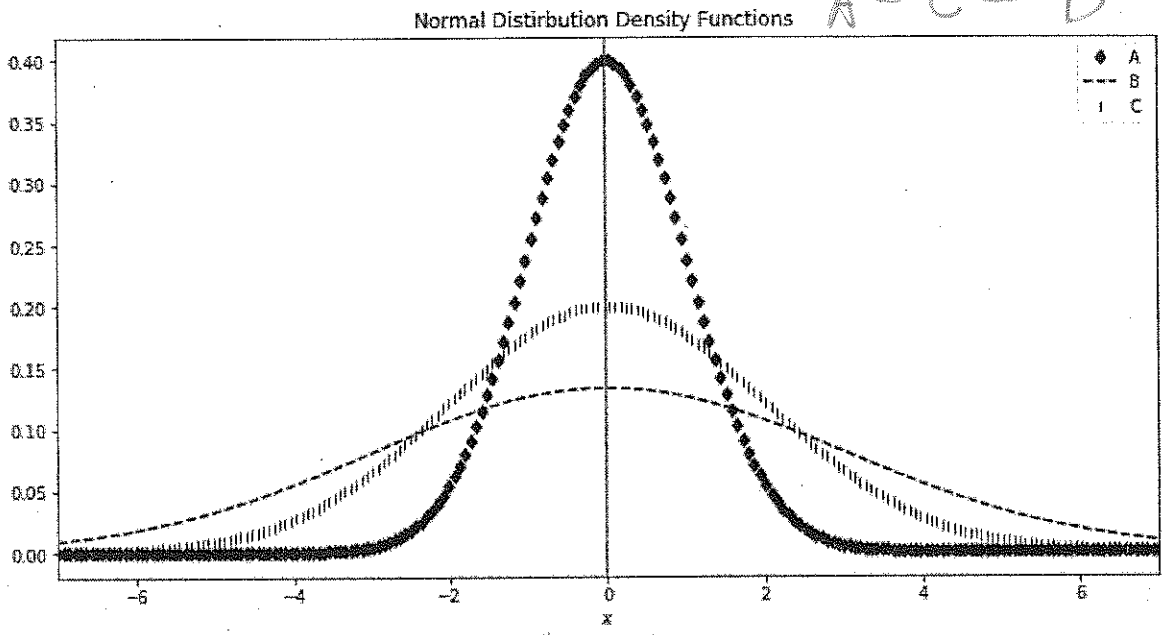
for $t \leq 0$: $F(t) = \int_{-\infty}^{t} \frac{1}{(x-1)^2} dx = -\frac{1}{(x-1)} \Big|_{-\infty}^{t}$

$u = -\frac{1}{t-1}$

$t = 1 - \frac{1}{u} \qquad u \in (0, 1]$

5. (15pts) The following graph shows plots of 3 normal distributions – labeled as A, B, and C. They have different standard deviations. Order them **from the smallest to the largest** by their standard deviations.

A - C - B



the skinnier the "belt" is the smaller the σ is

$\sigma_A < \sigma_C < \sigma_B$

6. (15pts) In python, if I run the following code, what are the outputs?

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3
4 x = np.arange(2)
5
6 print(len(x))
7 print(x)
8 print(type(x[0]))
9
```

2

[0 1]

int

x is an array of 2 elts. [0 1] each of them is an integer type