Dynamic Programming Approach to Leniency Policies in Collusive Corruption

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Abstract

Leniency policies may be effective to deter collusive corruption. In order to test this hypothesis, this theoretical model aims to solve a corruption minimization problem using leniency policies (sanction reductions) as control variables. This problem is constrained to the agents' (criminal and potential criminals) best response to their own utility maximization dynamic programming problem. Preliminary results show that leniency can deter corruption, the main deterrent effect in the steady state comes from the possibility of exploring the game. It is observed as agents play a strategy of paying a bribe and then self-reporting, or they do not paying bribes at all, if there is a sufficiently large bonus. Results also point out that raising fines may not deter corruption, but combining leniency with high fines may determine the rate of self-reporting in a game.

1 Introduction

Using one party against the other to disrupt criminal agreements is an old prosecutorial practice. However, the design of the strategies varied throughout time. Notably, corruption or bribery requires at least one agent to pay a bribe and other receive it. Therefore, stimulating self-reporting can be particularly effective against this type of crime.

Since the seminal work from Becker (1968), economists formalized a way to analyse the decision to engage in crimes[1]. Notably Kaplow and Shavell (1994) first described how giving fine reductions to self-reporters could deter crimes[2] for sufficiently risk-averse individuals. However, their analysis was only one sided or static.

The dynamic features from the interaction between agents were initially explored in

[1] More recently Polinsky and Shavell (2007) devised a general static model based on contributions of a vast literature. The model accommodates a series of effects such as different liability rules, different risk preferences, non-monetary sanctions, expected fines, judicial errors, deterrence effect, incapacitation effect, Principal-Agent relation, settlements, repeated offenders, corruption on judgements, social norms and self-reporting. Also Garoupa (1997) resumes the relevant literature.

[2] Specific literature in corruption starts earlier. Rose-Ackerman (1975) shows that incentives to corruption rise on different market structures. Another strand of literature derived from Tirole (1986), which explored corruption as an output from agency problems in a three-tier hierarchy. Later on, Polinsky and Shavell (2001) deal with the problem of corruption for framing and extorting individuals. These works and many others were written on the same line (Burguety et al., 2016). However, here the focus is on the self-reporting dynamics and how it can deter corruption.

the seminal work of Motta and Polo (2003)[3], which was focused on anti-trust offences[4], and later Spagnolo (2005) acknowledged that the methodology could be extended to other types of criminal organizations. Later on, Buccirossi and Spagnolo (2006) framed a similar methodology as a corruption game[5]. All of them found situations in which leniency for self-reporters may deter or increase criminal activities, depending on the policy design.

Note that there are roughly two types of corruption. There are the harassment bribes[6] in which there is an asymmetry of power between parties. Consequently, one can extorts the other for bribes to access goods or services that are entitled to the extorted party. Additionally, there is also the collusive corruption, in which agents collude by the benefit of both parties to extract some rent from a third party. This work explores the last type, not only because it is related to more complex crimes, but also because leniencies may be more effective in these cases, given the cooperative nature of the offence. Moreover, the structure of the crime is very similar to the ones observed in anti-trust literature, which was heavily used in this work.

Previous works on leniency policies achieved fine results in finding optimal policies for leniency programs. However, the studies often lack prediction power to understand what happens after the policy is implemented. Most works draw a picture of the decisions and multiple equilibria in fixed environments[7]. So it is possible to predict the decisions from agents under a variety of different model configuration. However, these models fail to explore the the behaviour of agents through time given stochastic events. In this sense, this work is similar to Miller (2009). In his work, in order to predict the expected unobservable corruption, the author makes use of a simple first order Markov Chain to extract the path from the detections of cartels over time. However, the author sets all variables free to change. This paper goes deeper into the Markov process to better set the model to obey the expected laws of motion from the chosen state variables. Moreover, it may give a richer set of variables to further test empirically.

More specifically, this work uses the dynamic programming approach to solve the recursive nature from this particular problem. This methodology is widely used in applied economics since the 2000's[8]. However, there are a few works on leniency programs which use similar methodology. Notable exception is the work fromChen and Harrington (2007), in which the authors predict the price path expected from cartels in an environment with leniency. Conversely, here the focus and the structure is considerably different[9].

In this sense, this paper tries to answer how deterrent can leniency policies against collusive corruption be. Or simply, what is the optimal leniency policy against corruption?

In the next section the basic model is devised. In sections 3 the game is played with

[3]One important difference between the static and dynamic approaches so far is that the first were set on individual level, or else, the decisions were taken by natural persons. The later dynamic models were set on the corporate level, with the exception of Spagnolo (2005) extents on criminal organizations.

[4]Notable theoretical contributions in the leniency for cartels are Harrington (2008), Motchenkova (2004), Aubert et al. (2006), Motchenkova and van der Laan (2011), Buccirossi and Spagnolo (2000), Chen and Rey (2013) and Chang and Harrington (2015). Interesting experimental findings are Bigoni et al. (2012) and Bigoni et al. (2015). Also, empirical approaches are explored by Brenner (2009) and Miller (2009).

[5]Other remarkable literature on theoretical games of corruption are Dufwenberg and Spagnolo (2014) and Basu and Cordella (2016), and experimentally Engel et al. (2016) and Abbink and Wu (2017).

[6]As defined by Basu (2011)

[7]Often, works focuses on a fixed model setting one variable free. However models can have as much degrees of freedom as reality can have.

[8]Stokey and Lucas (1989) provided first the systematic literature for applied economics using recursive problems. Nonetheless, important literature in the field are Ljungqvist and Sargent (2018) and Stachurski (2009).

[9]Heuristics used here are completely different from Chen and Harrington (2007).

more agents and there is the possibility of self-reporting. At the end, there are some final remarks regarding the potential findings from this exercise.

2 The Basic Model - Cumulated Liability, Absence of Leniency and Fixed Bribes

The basic model consists of two types of agents, an entrepreneur E and a Bureaucrat B. The agents are endowed with some certain amount of assets A, which they can choose to invest in a risk-free asset with return of r or decide to entering in a corrupt deal searching for better returns.

At each different period t, there is a contract available in which the entrepreneur can illegally extract a rent of π. In order to exploit this, the entrepreneur has to pay the bureaucrat a bribe of b. The bureaucrat faces a cost to perform the favour of ρ. In summary, there is a cost for entering corruption θ_i, where $i = [E, B]$, that assumes the value of $\theta_E = b$ for the entrepreneur and $\theta_B = \rho$ for the bureaucrat. Additionally, there is a reward ϕ_i, where $\phi_E = \pi$ and $\phi_B = b$. For now, lets assume that the rewards from corruption ϕ, and its costs θ are constant for every period[10].

Furthermore, if agents decide to enter in collusion ($\theta > 0$), there is a probability α of being detected, and if they are detected, there's a probability β of being convicted. Subsequently, if the agents are convicted, they pay a sanction of $F(L)$, where $L(C)$[11] is a liability from a is the criminal history C of the agent.

Lastly, in this first simple setting, the entrepreneur has a monopoly over the bribe payment. In other words, the entrepreneurs is the only one making decisions in this firs case. Therefore, the timing protocol is not important for the moment, since the bureaucrat will always follow the entrepreneur's decision.[12]. If the agents have to decide upon self-reporting (next section) or upon the values of the bribes, the timing structure of the game played determines the optimum solution of the problem.

The State Variables and Control Variables

The vector of state variables $\mathbf{x}$ in this model contains the assets A that an agent can hold, the liability L that an agent can accumulate and the current state of the world regarding the corruption crime S. The states of the world can be: 'not colluding', 'colluding', 'desisted', 'detected'.

Likewise, the control variable or action vector $\mathbf{u}$ contains the consumption c[13] and θ. The last one, represents the decision of engaging or not into corruption. So, in summary,

$$\mathbf{u_t} = \begin{bmatrix} c_t \\ \theta_t \end{bmatrix}, \text{ and } \mathbf{x_t} = \begin{bmatrix} A_t \\ L_t \\ S_t \end{bmatrix}$$

[10]Note that there is a perfect market for the bribes and homogeneous bureaucrats. In other words, the price of the bribes is given by the market, agents always accept the bribe at the price b because every agent has the same cost to perform the bribe ρ.

[11]Let the fines be linearly determined by the criminal history, ergo $F'(L) > 0$, and $F''(L) = 0$, also $L'(C) > 0$ and $L''(C) = 0$.

[12]A sequential with variable bribes game would imply in hold-ups due to the internal dynamics of the game. The game assumes a trust-game-like format, which can prevent rational players from entering in corruption Buccirossi and Spagnolo (2006).

[13]An equivalent decision to consume c_t is how much the agent is going to keep/save from the assets A_t.

The Laws of Motion

Agents can move from one state to the other. In order for them to transit they have to respect the laws of motions from the states.

The assets A grow at a fixed rate of return $r \in [0, 1]$ each period t. Furthermore, the agents receive an income from their activities of y and save everything else that is not consumed c or spent in corruption θ. Therefore, the assets A_t must follow the rule:

$$A_{t+1} = (1 + r)(A_t - c_t - \theta_t + y_t) \tag{1}$$

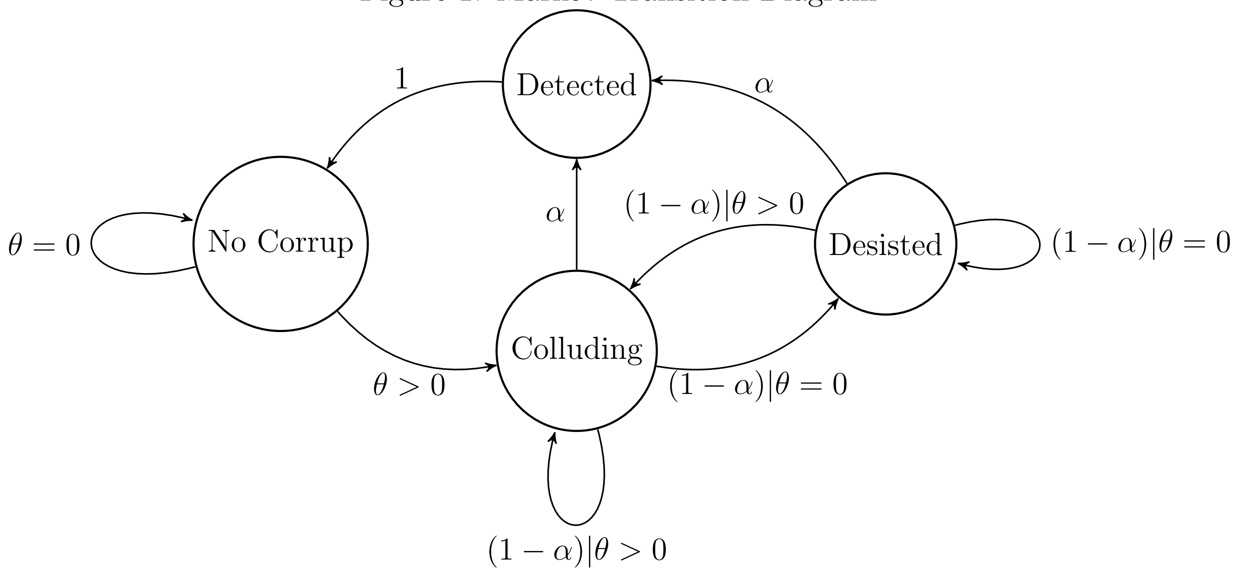
In this model, liability is a state that increases or decreases over time. Here, it is assumed that it will increase whenever the agents engage in corruption through the accumulation of crimes that they are liable (C). However, if agents stop paying bribes, and they avoid detection, they will loose liability over time. Therefore, there is a depreciation or decay $d \in [0, 1]$ over the liability[14], so its law of motion can be characterized as:

$$L_{t+1} = d(L_t + C_t) \tag{2}$$

There are a number of motives to assume that liability decays over time, the most important is that information gets lost over time. Nonetheless, anyone who works with criminal cases would sate that a crime committed recently is more easy to investigate than a crime committed a long time ago given the same available evidence[15].

Lastly, the Figure 1 shows the Markov diagram and the law of motion between states in S.

Figure 1: Markov Transition Diagram



The transition to the next state S_{t+1} is determined by a probability of detection $\alpha(S, \theta)$ which is different for every state S and conditioned to the decision on entering in collusion,

[14] In most previous similar models, the liability is constant, meaning that, convicted defendants pay a fixed fine and give back all earnings from corruption. There are two notable exceptions, in Miller (2009) agents are not liable for crimes committed in previous periods. Additionally, Motchenkova (2004) compares the difference between regimes of fixed fines and proportional fines. Results show that fixed fines can increase the duration of a cartel. Lastly, the setting proposed here allow the analyst to test all the different assumptions about the fine and liability regime by changing the coefficient d.

[15] Technically, a decay in liability would incentive agents to desist from corruption, otherwise, if agents cannot perceive that they are 'getting away from justice' they would likely never give up from corruption.

i.e. paying the cost θ_t in the period[16]. Let s be the a function that represents the rule given by Figure 1, consequently:

$$S_{t+1} = s(\alpha, \beta, S_t, \theta_t) \tag{3}$$

The set containing the combination of the laws of motion of all states $\mathbf{x}$ together is represented by the transition matrix Ω.

The Outcomes

The actions $\mathbf{u}$ determine each state in $\mathbf{x}$. Consequently, each combination of states $\mathbf{x_t}$ has an outcome[17]. The two outcomes here are y and C.

Let $y_t(S, L)$ be characterized as,

$$y_t(S, L) = \begin{cases} 0 \text{ if not colluding} \\ \phi_t \text{ if colluding} \\ 0 \text{ if desisted} \\ \text{ if detected} \begin{cases} F(L_t) \text{ with probability } \beta \\ 0 \text{ with probability } (1-\beta) \end{cases} \end{cases}$$

and,

$$C_t(S) = \begin{cases} C_t > 0 \text{ if colluding} \\ 0 \text{ else.} \end{cases}$$

2.1 The Agents' Problem

The agents want to maximize their utilities in each period given a certain time discount $\delta \in [0, 1]$. Let, the utilities be a function of consumption $u(c_t)$[18], where $u'(c_t) > 0$, and $u''(c_t) < 0$. The utility maximization is constrained by the laws of motion from the states (1), (2) and (3), such it can be characterized by:

$$\max E_0 \sum_{t=0}^{\infty} \delta^t u(c_t) \tag{4}$$

$$\begin{aligned} \text{s.t. } A_{t+1} &= (1+r)(A_t - c_t - \theta_t + y_t) \\ L_{t+1} &= d(L_t + C_t) \\ S_{t+1} &= s(\alpha, \beta, S_t) \end{aligned}$$

Where the notation E_0 means the expected value at $t = 0$.

[16]In other words, the current state S_t determines if agents can enter in corruption (not possible to pay θ if S_t is 'detected'). Then if the cost of corruption θ is paid, it is going to determine the next state. For example, If the entrepreneur pays a bribe, than she will have a positive probability of being either 'colluding' or 'detected'. Conversely, if she does not pay, depending on the current state, she can only be 'not colluding', 'desisted' or 'detected' in the next period.

[17]Note that outcomes should not be confused as the rewards. In a discrete dynamic programming problem, rewards are only the things that give the agents some utility. Outcomes here can be understood as hidden states. In fact, they could be written as states in $\mathbf{x}$, however these would increase the dimension of the problem, which is not desirable.

[18]One could change the agent from being an individual to a firm. Consequently, the agent's problem would be a profit or wealth maximization problem. This may be an alternative and less complicated way to formulate the problem. The, eventual differences from the solutions are potential interesting findings.

The Value Function

The agent's problem (4) has a recursive nature due its first-order difference equations constraints. Notably, dynamic programming is the best methodological framework to solve this kind of problem. In order to turn the problem into a dynamic programming problem, it is necessary to build the value function, which consists in the value of state vector to the agent $V(\mathbf{x})$.

Lets define a value function from the initial states $V(\mathbf{x_0})$ as being the the optimum value for the initial states A_0, L_0, and S_0. So that the value function $V(\mathbf{x})$ of the states can be defined by the following *Bellman Equation*,

$$V(\mathbf{x}) = \max_{\mathbf{x},\mathbf{x}',\mathbf{u}} \{u(\mathbf{x}) + \delta E[V(\mathbf{x}')|\mathbf{x},\mathbf{u}]\} \tag{5}$$

or,

$$V(A, L, S) = \max\{u(c) + \delta E[V(A', L', S')|A, L, S, \theta, c]\} \tag{6}$$

$$\begin{aligned} \text{s.t. } A' &= D(A - \theta - c + y) \\ L' &= d(L + C) \\ y &= y(s, L) \\ s' &= s(s, \theta). \end{aligned}$$

Where the *prime* notation represents the value of the variable in the next period[19].

2.2 Simple Solution

In order to solve dynamic programming problems the agents have to choose the best actions $\mathbf{u}$ that lead to a maximum value of current state $\mathbf{x}$ and future ones $\mathbf{x}'$ given a transition law Ω. Notably, the transition matrix $\Omega(\mathbf{x}', \mathbf{u}, \mathbf{x})$ can be understood as the probability density of the next state $\mathbf{x}'$, given the current state action pair $(\mathbf{x}, \mathbf{u})$. Hence, the dynamic programming solution to the agent's problem or the solution of the *Bellman Equation* in this case can be achieved by simple iteration of the value function given the transition matrix Ω. If the Ω matrix is stochastic, then the solution is given by maximizing each value function $V(\mathbf{x})$ given all states for each period, or else:

$$V(\mathbf{x}) = \max_{\mathbf{x},\mathbf{x}',\mathbf{u}} \{u(\mathbf{x}) + \delta \sum_{\mathbf{x}} V(\mathbf{x}')\Omega(\mathbf{x}', \mathbf{u}, \mathbf{x})\} \tag{8}$$

If V^* is a unique solution to the Bellman Equation (8), then it is possible to define an optimal policy $\sigma(\mathbf{x})$ such that:

$$\sigma(\mathbf{x}) \in \max_{\mathbf{x},\mathbf{x}',\mathbf{u}} \{u(\mathbf{x}) + \delta \sum_{\mathbf{x}} V^*(\mathbf{x}')\Omega(\mathbf{x}', \mathbf{u}, \mathbf{x})\} \tag{9}$$

In this specific case, associated with the solution of (6), there are the policy functions governing the control variables c and θ, also the states A, S and L, such that

[19] The *ex ante* value function v will depend on the current levels A, S and L drawn at period $t = 0$. It can be defined as,

$$v(A, L, S) = \sum_{s} V(A, L, S)\Omega_0(S). \tag{7}$$

$$c = f(A, S, \psi) \tag{10}$$
$$\theta = g(A, L, S, \psi) \tag{11}$$
$$A' = h(A, L, S, \psi) \tag{12}$$
$$L' = l(L, S, \psi). \tag{13}$$
$$S' = s(S, g(A, L, S, \psi)) \tag{14}$$

where ψ are the time invariant parameters.

Note that, it is possible to solve the agent's problem either by iterating the policy function or the value function. Further proofs and details about the solution can be found in Stokey and Lucas (1989), Stachurski (2009) or Ljungqvist and Sargent (2018). Nonetheless, the routines used in this article can be found in Quantecon.org[20].

Discretization

Solutions to the continuous-states dynamic programming problems are known to be tricky[21]. One way to overcome this problem is to discretize the problem, or else, turn the variables into finite grids that the agent can access. This leads to imposing boundaries to the constraints. Sometimes, it sacrifices the reality of the model.

A preliminary, single agent, dynamic problem solution for this problem is available at: `https://nbviewer.jupyter.org/github/caxaxa/Chacha_PhD_Projects/blob/master/Corruption_DP.ipynb`.

Results show that corruption is deterred under different settings. Corruption will be lower if interest rates are higher, bribes are bigger or probabilities of detection and conviction are higher. Interestingly, time discount does not seem to deter or enhance corruption. While the decay from liabilities shows an interesting interior solution. Nonetheless, the results show that corruption is deterred if agents are wealthier[22].

In the following section, a slightly different environment is defined. Agents can self-report in exchange for sanction reductions. Therefore, the analyst can understand the differences between the two regimes.

3 A Model with Leniency

Lets allow the agents to access some leniency policies. Therefore, for some specific new states in S there is a set of new legitimate decisions or control variables available in $\mathbf{u}$. Agents can now choose to self-report and earn some benefits in their outcomes y.

Agents can self-report in two distinct opportunities. If an agent decide to self-report after committing an act of corruption but before being detected, the agent gains a reduced fine $RF(L) < F(L)$ for $R \in (-\infty, 1]$, while the other party receives the full fine $F(L_t)$[23]. If both agents self-report at the same time, then they received a reduced fine $rF(L)$ where, $RF(L) < rF(L) < F(L)$ or $R \leq r$. Moreover, agents can plea-bargain after detected and

[20]Available at: `https://python-advanced.quantecon.org/discrete_dp.html`.

[21]There are a few known solutions, most of them use a linear quadratic constraints. They are known to lead to Euler Equations and have nice analytical properties.

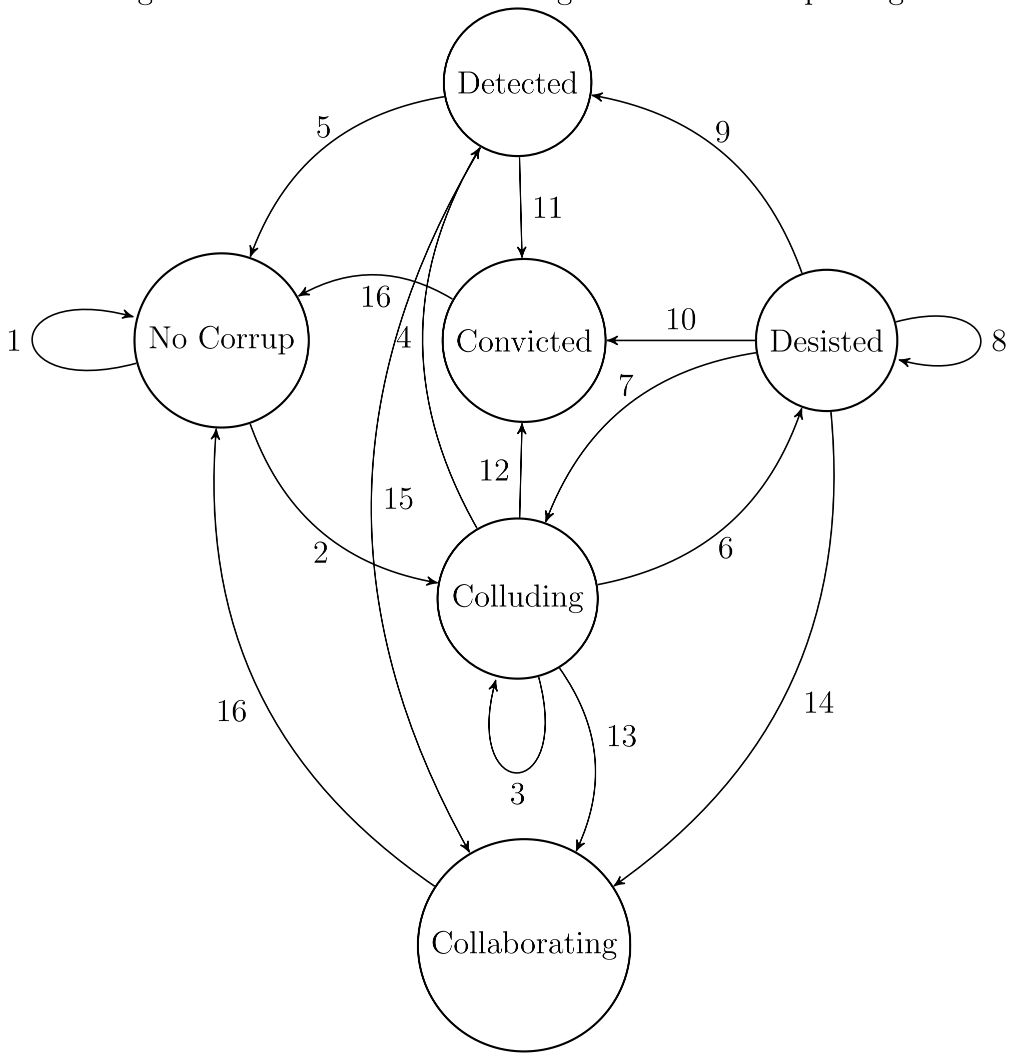
[22]The conclusion arises since corruption is less prevalent when the boundaries of the asset space are bigger. This is theoretically in line with results in the seminal paper from Mauro (1995). Interestingly, there is the hypothesis that wealth is more efficient to lower extortive corruption than it is against collusive corruption.

[23]It is assumed that when the agent report a crime, the other party is convicted for sure.

receive a reduced fine $PF(L) > RF(L)$ for $P \in (R, 1]$[24] or plea simultaneously and receive $pF(L)$ where, $PF(L) < pF(L)$ or $P \leq p$[25][26].

It is possible to introduce leniency by just increasing one dimension in S, namely, a 'collaborating with authorities' state. Which means that the offender is now in touch with the authorities to explain the crime. Both agents can be in this state if they have self-reported in the previous period. However, if only one party has self-reported, then the other party is certainly convicted. The Figure 2 shows the new states' transitions diagram.

Figure 2: Markov Transition Diagram with Self-Reporting



To be clear, the arrows in Figure 2 are going to be the law of motion for the entrepreneur in S if the conditions bellow are observed:

1. $\theta = 0$;

2. $\theta = b$;

[24]If the fine reduction for self-reporting before detection is less lenient than the one after $R > P$, then nobody would self-report before detected.

[25]There can be situations in which the prosecutor want to give a better plea deal to the defendant, however, this case is not addressed for now.

[26]Lets assume that $r = r(R)$ and $p = p(P)$. In other words, the reduction when both players self-report is some proportion of the total reduction rule. The usual rule of '*first come first serve*' seems to be tricky to apply in with this timing protocol.

3. $\theta = b$ with probability (1-α);

4. with probability α;

5. with probability $(1 - \beta)$ or self-reported;

6. $\theta = 0$ with probability (1-α);

7. $\theta = b$ with probability (1-α);

8. $\theta = 0$ with probability (1-α);

9. with probability α;

10. Reported by the other party;

11. with probability β or with probability $(1 - \beta)$ and reported by the other party;

12. Reported by the other party;

13. Self-reported;

14. Self-reported;

15. Self-reported (Plea Bargain); and

16. Certainty.

Note that individual decision (4) and the value function (5) have larger dimensions in $\mathbf{x}$ and $\mathbf{u}$. However, the structure is not affected by the addition of leniencies. In fact, we should expect, for $R = P = r = p = 1$, that the solution for the problem with leniency policies to be the same as the one without. This happens because the additional controls in θ would never be chosen and, consequently, the additional states in s would never be populated.

Timing and Strategic Interdependence

Note that the individual decision function g, depends on S that depends on g again. This shows the strategic interdependence of the game, where some strategy of the player i depends on the player j, or else substituting (11) in (14):

$$\theta_i = g_i(A, L, s(S', g_j(A, L, S, \psi)), \psi), \tag{15}$$

where $g = [g_E, g_B]$.

Note that the timing protocol will determine the outcome from 15. The agents play sequentially or simultaneously, and they choose the θ that maximizes its expected pay-offs given the cost of corruption. Notably, if the game is simultaneous it has a Markov Perfect Equilibrium structure, while if they play it sequentially, it is a Stackelberg structure.

Note that in the first monopolist example, there is no strategy since player E is deciding for both agents. However, the feature is important for the this setting with leniency. However, in this setting, for the sake of simplicity, the entrepreneur is still a 'bribe monopolist'[27].

[27] I expect to develop a more robust model in further studies.

3.1 The Government's Problem

It is possible to look to the problem from another perspective. The government wants to maximize society's welfare. Assuming that there is no 'greasing the wheels' hypothesis from corruption[28] and that the welfare gains from the criminals are neglectable. It is possible to infer that welfare is monotonically decreasing with the corruption level. Or else, governments want to minimize the level of corruption, or the number of agents in the state 'colluding' (lets call it S_c) over time, constrained to the agents maximization problem (6). For this, the government has the power to change the fines F and the leniency poolicies R and P. Therefore it is possible to write the governments problem as:

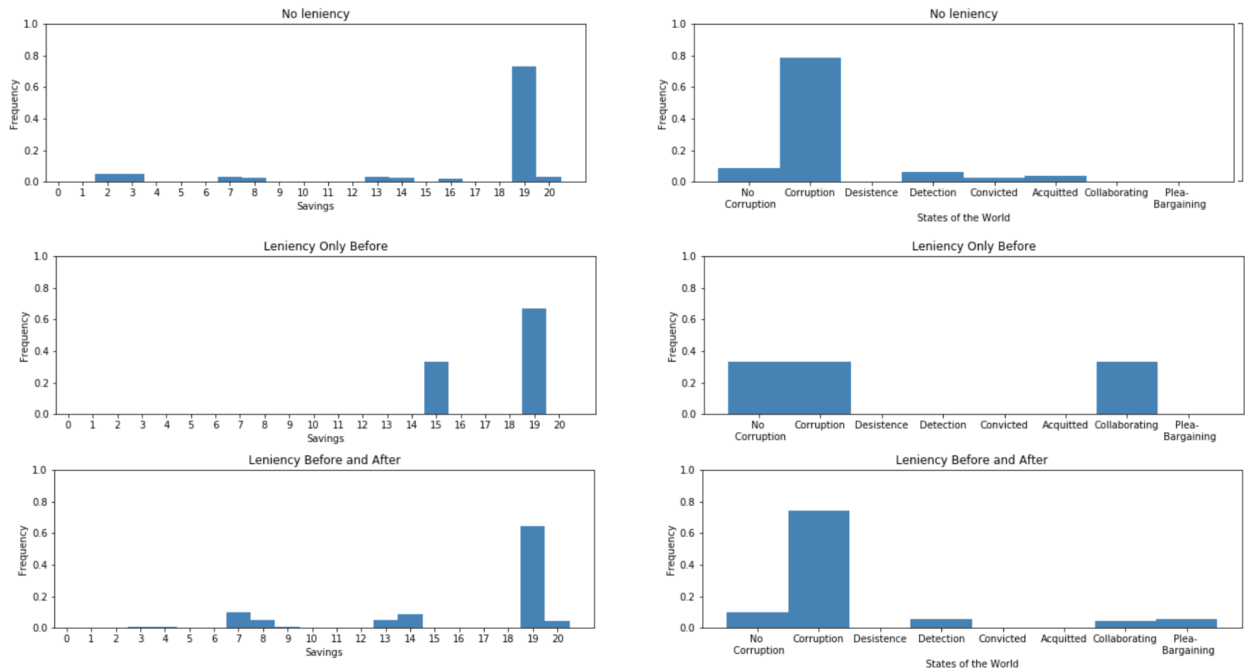
$$\min_{R,P,F} E_0 \sum_{t=0}^{\infty} \delta^t S_{c_t}(R, P, F) \tag{16}$$

$$\text{s.t. } V(A, L, S) = \max\{u(c) + \delta E[V(A', L', S')|A, L, S, \theta, c]\}.$$

3.2 Solution and Results

The results show that the effects of leniency over corruption and asset allocation are not unambiguous. The complete code and results from this simulation are available in: `https://nbviewer.jupyter.org/github/caxaxa/Chacha_PhD_Projects/blob/master/Leniency_DP.ipynb`. Moreover, Figure 3 shows the results of leniency over corruption under different leniency regimes.

Figure 3: Distinct Leniency Regimes



The charts show a setting in which corruption is possible given the simulation parameters. It can be noted that the introduction of some leniency policy can lead to corruption

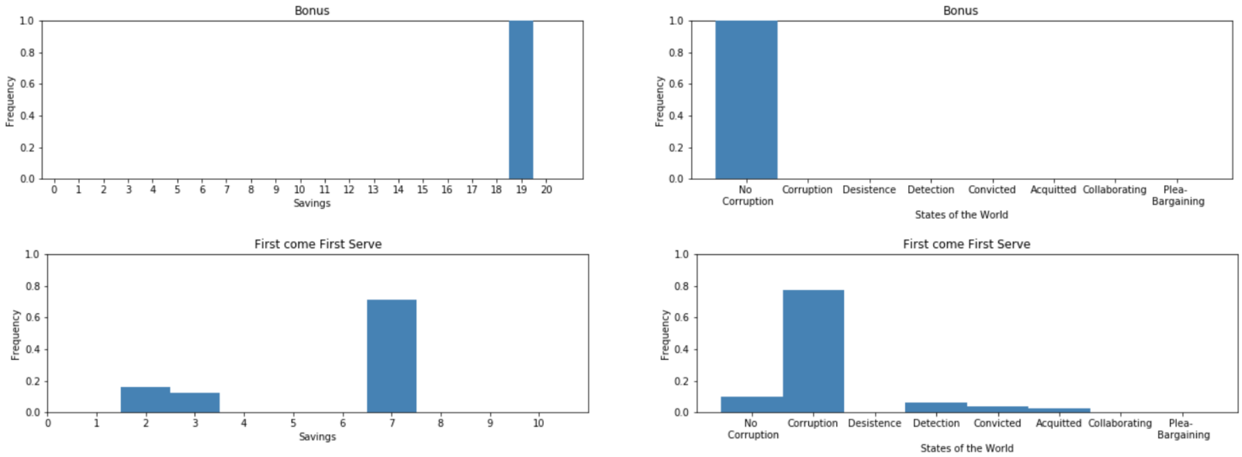
[28]The hypothesis states that bribes work as shadow prices that increase efficiency of agents' transactions. Notably, it is more realistic when dealing with extortive bribes, when lower wages from civil servants are compensated by a bribe. Since, this work deals with collusive bribery, the hypothesis is less applicable.

deterrence. In the example above, the policy of giving leniency only before detection would induce agents to exploit the game. This would naturally lead to lesser corruption. However, if agents can self-report both before and after being detected, agents would maintain their preference for entering in corruption but self-report when convenient.

It must be noted that there are multiple equilibria depending on the parameters set. Nonetheless, results seem to be in line with literature. Therefore it can be a good candidate to generate data for further testing.

Figure 4 show two distinct leniency policy regimes. In the first, there is a payment of a bonus for self-reporters (before detection) and in the last, only the first one to report is going to receive the fine reduction.

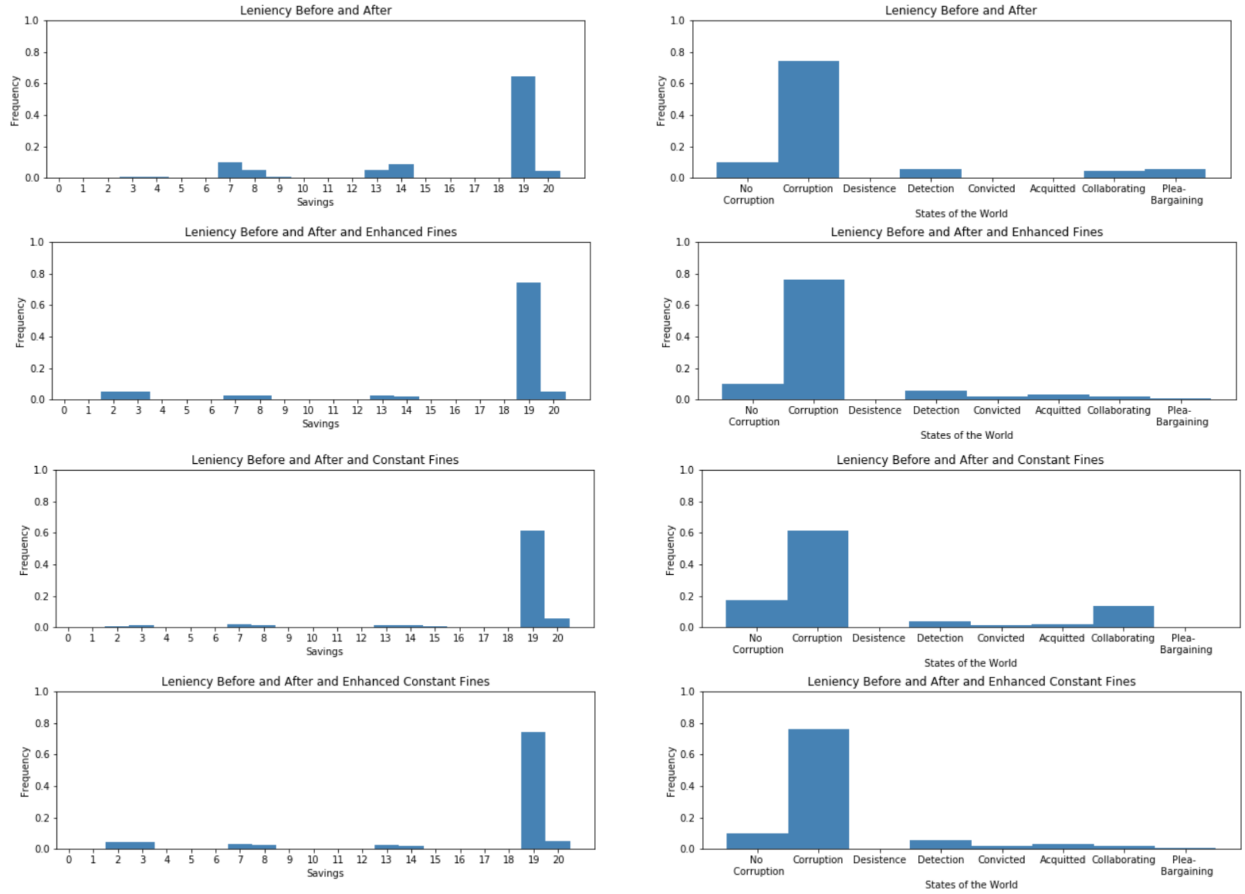
Figure 4: Distinct Leniency Regimes



The results show that bonuses can fully deter corruption. This may happen because agents would anticipate that the other agent would always self-report, as Spagnolo (2005) predicted. However, the 'first come first serve' policy, does not seem to have any impact in deterring corruption. Once again, these equilibria in steady state may change depending on the other free parameters.

Lastly, the Figure 5 show the results under distinct fining policies. Namely, under a progressive fine policy and under a constant fine policy.

Figure 5: Distinct Fining Regimes



The findings show that progressive fines lead to higher corruption, in contrast with previous literature (Motchenkova, 2004). Moreover, the increase in the severity of the fines, change the rate in which agents report, but not the rate in which they collude (first and second charts from Figure 5). If fines are constant, we can observe that the increase in the fine, lowers the prevalence of corruption in the steady state. This result may seem counterintuitive at first, because one should expect that an increase in fines should lower the criminal activity. However, the Beckerian approach would also state that the agent's wealth is a limit for the fines, everything beyond this would not improve deterrence.

4 Final Remarks

This working paper describes a corruption game with leniencies as a dynamic programming problem. This framework is widely used in economic models since the 2000's. However, there is not any other work[29] that uses the same approach to analyse the issue of corruption and leniencies. Therefore, the potential results from the solution of this problem may be of great interest to the scientific community.

Further Studies

From this exercise, it is possible to achieve further better results. I enumerate bellow some possible and interesting potential findings.

[29]Except Chen and Harrington (2007), in the anti-trust literature.

Some alternative interesting settings (based on Ljungqvist and Sargent (2018)):

1. How to consider multiple agents? Do they have similar information and beliefs? How to solve the discrete problem with two Bellman Equations?

2. Do the different types of agents have distinct utility functions?

3. Should the model use quadratic rewards or costs? Most of the stable models using DP apply these kind of structures;

4. There might be multiple ways to write the laws of motion and transition rules to avoid the curse of dimensionality; and

5. When dealing with multiple agents, is it a Stackelberg or a Markov perfect equilibrium structure?

What else to Find?

1. Set of optimal rules mapping spaces into agents decisions;

2. The data generation process based on the parameters (testable);

3. A density of the possible outcomes (testable); and

4. A density of the possible observable and unobservable states (testable).

What and How to Test?

After discovering the estimated density from the spaces, it is possible to create a Bayesian updating machine to infer the behaviour of the unobservable state of corruption based on other observable states and parameters. Examples are: crime detection, crime conviction, interest rates, income and others.

One promising experiment is to test the impact from the Brazilian anti-corruption law reform from 2013, that introduced self-reporting for cases of corruption on its provisions. The Brazilian Public Prosecution Office provides the information over 885.675 investigations of multiple offences conduced by them, from January 2009 to January 2020. It is possible to select and observe only the investigations related to corruption crimes and test for significant changes after the introduction of the policies.

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