# Corruption and Collaborations: A Dynamic Programming Approach *

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**Abstract**

Sanction reduction policies for collaborators may be effective to deter collusive corruption. In order to test this hypothesis, this paper builds a theoretical model that solves the game of corruption with and without sanction reduction policies. It uses a dynamic programming setting to find the equilibrium strategies. Preliminary results show that the policies are effective to deter bribery. Furthermore, it shows that corruption decreases as sanction reductions are more lenient. Additionally, the model predicts that a successful anti-corruption policy shock will be followed by an increase in corruption detections, even if the probability of detection does not change.

## 1 Introduction

Collusive corruption happens when two players, a bribe payer and a bribe receiver, agree in exploiting some rent or contract. Naturally, when the bribe receiver is a public officer, the society loses from this agreement. Notably, previous works on corruption represent this relation as a three tier principal agent problem (Burguety et al., 2016). Where the principal (society) loses from an agreement between the agent (bribe payer) and the supervisor (bribe recipient).

Anti-bribery policies rely on many distinct features. One particularly well defended strategy against corruption is to reduce the sanctions for criminals who collaborate with authorities to unveil bribery schemes (Unite Nations, 2004)[1]. The idea is to break the agreement between payer and receiver. Previous literature points to the effectiveness of the policy (Buccirossi and Spagnolo, 2006; Dufwenberg and Spagnolo, 2014). However, most studies fail to measure its

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[1]This work focuses on the case where the players are the collaborators. Some whistleblowing policies rely on third parties to come up and denounce bribery schemes. This relationship implies in a distinct game

impact on the corruption level. So, this work aims to answer the following open questions. Does a sanction reduction policy to collaborators deter corruption? Is it possible to measure the effect of the policy in the overall corruption level?

In order to answer these questions, there are two main issues. First, it is necessary to comprehend the decision making from the agents that leads them to enter into corruption in an environment with collaborations and without them. This can be done by modelling a game of corruption. Secondly, it is required to find a way to measure the corruption level, and check in what degree it changes when the anti-bribery policies are introduced. If the model is successful in predicting the corruption level, then it is possible to find a real world case to test it against real data.

The model which predicts the decision from players about entering in corruption is a game. The players must have the choices to enter or not in bribery. Moreover, they also must decide whether to defect on the agreement and be rewarded for that. The profitability of the bribery is given by its terms. Or else, how much is the advantage from entering in corruption versus the costs of it. If a bribe is profitable, then bribery can happen. However, if the anti-corruption policies are well designed, it is expected that the game will work as a prisoner's dilemma, where defection is a dominant strategy. Consequently, agents would not enter into corruption in the first place. Therefore, successfully deterring corruption even if it is profitable for both players. In other words, if the sanction reduction is good enough, players would prefer to report their misconduct to get their judicial benefits. Knowing this behaviour beforehand, the other party is not paying or accepting any bribes. Also, in order to be consistent with anti-bribery legislations, even when agents are detected they can admit guilt and gain some other benefits from it. This type of analysis shows the areas where corruption can happen given the game parameters.

Notably, any realistic model that tries to predict decisions in society must account for a utility maximization strategy. Sometimes, it is not only necessary to know if the game is profitable or not. It is required to know the rewards of choosing between corruption and a substitute investment. From this tradeoff it is possible to infer the economic decision of entering into corruption. Notably, this kind of answer comes from solving a utility maximization problem. Furthermore, this type of game provides the analyst with more variables to test within the model. Such as interest rates and risk aversion. These are necessary features to solve this kind of game, and they are also handy for empirically testing the predictions.

The specific objective of predicting the change in corruption given a policy shock requires a dynamic model. More specifically, if the environment changes midstream, the decisions made in the past must be re-evaluated. Therefore, the model must carry on information through time which shows the consequences from past decisions. Consequently, the wealth or assets and the liabilities accumulated through time are crucial in the players' decision making. Therefore, the game is solved in a dynamic setting for a Markov Perfect equilibrium[2].

[2]Static games have a hard assumption of not allowing things to change over time. It is

The last necessary characteristic from the model regards the information set. Under the reasonable assumption that agent cannot perfectly observe the other player's wealth or criminal record. Players infer the other party's states based on their own state. This strategy is crucial to economize state dimensions. Given that the game requires numerical solutions, this is very convenient to overcome the problem of the curse of dimensionality. Furthermore, it provides enough volatility and uncertainty to agents, so it is possible to observe unilateral defections even under optimal strategies.

Solving the game with all the necessary features is tricky. There are multiple equilibria because the model has too many degrees of freedom. So, the model is solved for a benchmark with a reasonable configuration. Such as a balanced power relation between the players, so the profitability of the bribery is similar to both parties. Furthermore, both agents are equally risk averse. Lastly, at their initial condition they come from the same distribution of wealth and start with zero past judicial liability. Under these conditions, results show that if the bribery is profitable, it might happen, but also reporting and plea agreements too. Most importantly, the presence of sanction reduction policies decreases the expected level of corruption. Also, if sanction reductions get more lenient, corruption is less predominant in the agents' decisions, and consequently, less observed in society.

Once the model is precisely specified, it is possible to predict testable data about corruption. However, it is first necessary to come up with a good way for measuring corruption. The amount of corruption in a society is hard to measure. The quantity of money spent on bribes, or the losses in public revenue or even the number of bribes paid in a specific period is not observable. Admittedly, only a proportion of the crime is detected. Therefore, detection of corruption is an important parameter for measuring corruption in the economy[3]. Fortunately, the proportion of detected crimes of corruption is an output from this model.

The analysis of the detection of corruption over time can be misleading. Notably, an increase in corruption detection may happen because there was an increase in the number of crimes, or because there was an enhancement on the prosecution productivity. The contrary is also true. So, how is it possible to identify if changes in detection of corruption come from higher criminal activities or from higher anti-bribery enforcement? Once again, the answer relies on the chosen model. It separates enforcement variables from the decision of entering into corruption. This happens because the probability of detection is an independent input of the model, and the quantity of detection a dependent output of the model. This relation provides the necessary ingredients to predict the path of the unobservable corruption given the observable detection of

obvious that agents update their states as the game is repeated. So their decision is more consistent with repetition.

[3]There are other ways to measure corruption. The most famous being the perception indexes. Those measures have their own benefits and cons, which are properly addressed in Chapter 6

corruption under distinct scenarios[4][5].

Considering that a successful policy intervention is any one which lowers the corruption level. Then, the predictions show that a successful policy shock must increase the detections of corruption, at least in the first periods. The more successful the policy intervention is the higher is the detection of corruption and the shorter the period.

In summary, the objectives from this Chapter are twofold. The first objective is to understand the relation between the players' decisions and the relevant variables in the game. Consequently, it is necessary to model the game to predict the decisions. The second objective is to simulate what happens with the distribution of the crime of corruption in the society under distinct scenarios. These simulations are also crucial for the next chapter's empirical objective. In the following sections, the corruption game is presented.

In each new section from this work the game is expanded to account for different features or conditions. Each new feature provides an intuition about how the decision of entering in bribery changes depending on other parameters from the real world. The next section presents the simple corruption game. In this setting the main ingredients from the complete model are introduced. Such as the price of the bribe, the advantage from the bribery and the costs of corruption for each party. Notably, the players cannot self-report and collaborate with authorities to gain any judicial benefit[6]. In Section 3, the game is expanded to allow the players to self-report their misconducts [7]. Section 3 solves the game for the best possible pay-offs. In Section 4 the game is presented as a utility maximization problem. In Section 5 the game is presented as a dynamic programming problem and solved. Lastly, the more interesting results are resumed in Section 6.

[4]Such as, low enforcement low corruption; high enforcement high corruption; high corruption low enforcement; and low enforcement high corruption.

[5]Once the data from each possible scenario is generated by the model. It is possible to infer the most probable scenario using statistical and econometric routines which match real world data against the predicted ones. This strategy is tested in the next Chapter from this work.

[6]It is clear from this exercise that if the price of the bribe is constant, the players will always choose to enter in corruption if it is profitable for them. This implies that there is no strategic behaviour from players in this simple setting.

[7]In other words, if players enter in a corruption agreement they can go to the authorities and report the crime in exchange for a judicial benefit. Furthermore, if they are detected by the authorities, they have the option to admit guilt and gain some benefits from it too. Consequently, the players can now defect on the agreement and profit over it. Therefore, this feature creates a strategic interdependence in the game. Now, players need to take the best action of the other player in their own decision making. It is possible to solve this game by finding the Nash equilibria from their incentive compatibility constraints. In this equilibrium, it is possible to conclude that mitigating sanctions (or even giving bonuses) disincentivizes the corruption agreements

# 2 Simple Corruption Game

The goal from this chapter is to solve the corruption game with sanction reduction policies for collaborators. However, in order to smoothly introduce the the analytical framework. First, it is convenient to introduce a simpler game where agents are convicted when detected and they cannot self-report. From this simple example is possible to set most of the game requirements. Additionally, it is also possible to draw the first conclusions regarding the role of the bribe and the decision to enter in corruption.

## 2.1 Setting the Game

The necessary ingredients to set the game are presented here. Namely, a set of players and a set of states or stages where the system can be. Related to each estate, there are a set of possible actions or decisions. Furthermore, each pair of state and action has a pay-off that depends on the constant parameters from the game. Lastly, the game needs a timing protocol which describes how and when the players are allowed to play. These necessary features are better presented in the following subsections.

### 2.1.1 Players

The game here is played by two players[8]. A bribe payer and a bribe receiver $i \in [payer, receiver]$.

### 2.1.2 Timing and Information

The game has discrete time $t$. Here, it is assumed that in each period $t$ there is one corruption opportunity. Or, conversely, $t$ can be understood as a constant time between each corruption opportunity.

As defined in the second chapter, the players here play simultaneously. This design seems more realistic than a sequential game [9].

The design of the game imposes how information is available for players. The first simpler example has naturally more information. As the game gains complexity, agents are not able to fully observe the relevant information in the game.

For now, agents can perfectly observe each other's actions, states and, consequentially, the pay-offs. Therefore, the information here is said to be complete. Moreover, the present states contain all past information (Markovian Property)[10].

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[8] As stated in Chapter 2, there might be differences between individuals and corporations. Whenever relevant, the differences are going to be explored.

[9] This would logically imply in a take-it-or-leave-it proposition of the bribe. Which seems also not realistic.

[10] For any dependent $X_t$ in $x_t$ then $p\left(X_n = x_n | X_{n-1} = x_{n-1}, ..., X_0 = x_0\right) = p\left(X_n = x_n | X_{n-1} = x_{n-1}\right)$. In words, future events depend only on the present set of states. Or else, all the past information is embedded in the current states.

### 2.1.3 Parameters

The relevant parameters for this first examples are the price of the bribe $b$[11],the advantage from corruption $a$, the cost for the *payer* to perform the bribe $c_b$, the sanctions $s$ that players pay if detected by the authority and the probability of being detected by the authorities $\alpha$. Note that, for this first example, the sanction $s$ is given by a fine $f$[12] and everything that the agents have gained from corruption ($a$ for the *payer* and $b$ for the *receiver*).

It is possible to state that this a common criminal structure. Therefore, the conclusions made here can be applied to other types of crime. Notably, this work resembles a lot the structure used in leniency studies for anti-trust offences.

### 2.1.4 States

Any position in a game (or system) can be summarized as a state variable. Or else, in an extensive game the states are random variables that indicate each possible node in the network.

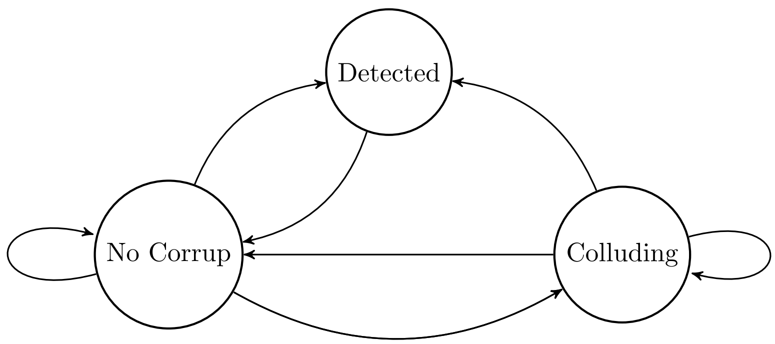
In this example, the set of states $S$ are the 'states of the world' where agents can be[13]. For instance, if agents decide not to bribe, they are in a state of '*no corruption*'. As well as if they decide to enter in bribery, then they can be in two other states. If corruption is well succeeded, agents gain their pay-offs from corruption in the state '*colluding*'. Or, they can be detected by the authorities and fined in the state '*detected*'. The Figure 1, shows the states and how agents can go from one stage to the other. Lastly, the laws that describe how agents go from one state to the other depend on the actions that they take and are better described bellow.

[11] It is possible to make the bribe $b$ as a decision for the *receiver* and not a constant. The next section solves the corruption game for a random continuous bribe and for a binary decision over a constant value for $b$.

[12] In many jurisdictions there are non-monetary sanctions both for individuals (imprisonment) and corporations (licence and activity impediments). As Becker (1968) points out, they are complementary and necessary to optimal deterrence. Here, for the moment, one can assume that agents are able to translate the non-monetary fines in the value of $f$. Nonetheless, the argument on non-monetary fines is further developed in the following sections.

[13] States can be very subjective, and they depend on the analyst design of the game. The same game can be designed using different set of states. Moreover, some solutions (such as Linear Quadratic) require the use of states that are very unintuitive. As Ljungqvist and Sargent (2012) state, finding the right state is an art.

Figure 1: Game States and Transition Rules




The arrows represent the transition from the states[14]. It is given by a rule or ‘law of motion’ which depends on the state of the players and their actions. These concepts are better explained below.

#### 2.1.5 Actions

Each possible state has a set of allowed actions $d$ for each player $i$. In this first simple example the players are not allowed to report their misconducts. Nonetheless, this simpler version shows some expected relations between variables and the decisions towards bribing.

The *payer* can decide whether or not to pay $d_{payer} \in [pay, not\ pay]$ a bribe of value $b$ to earn an advantage $a$ if they are not detected[15]. The *receiver* decides to accept it or not $d_{receiver} \in [accept, not\ accept]$ at a cost of $c_b$. So that $d = [d_{payer}, d_{receiver}]$.

#### 2.1.6 Laws of Motion

As shown in the Figure 1, agents go together from one state to the other. Since the game is stochastic, the next state $S_{t+1}$ is function $S(.)$ of the actions taken $d_t$ and the current state $S_t$ given some probability of detection $\alpha$.

$$S_{t+1} = S((S_t, \epsilon)|d_t) \tag{1}$$

Where $\epsilon \sim$ *Bernoulli* $(\alpha)$.

In other words, each arrow in Figure 1 happens depending on the actions taken in that state and the probability associated to that combination[16].

[14] For now, agents can be in only one state at a time. In this example, they would always go together to each state at each time $t$.

[15] For now, if agents are detected, they cannot pay or receive a bribe in that state.

[16] For instance, the arrows from Figure 1 that represents staying in the '*no corruption*' state, or going to the '*Colluding*' state can be written respectively as

$p(S_{t+1} = No\,Corruption \mid S = No\,Corruption,\ d_{t,payer} = pay,\ d_{t,receiver} = not\ accept) = 1,$ and

$p(S_{t+1} = colluding \mid S = No\ Corruption\ ,\ d_{t,payer} = pay,\ d_{t,receiver} = accept) = (1 - \alpha).$

### 2.1.7 Pay-offs, Costs and Rewards

In order to draw the player's decision rules in a similar fashion. It is possible to write the pay-offs in terms of costs $\phi$ and benefits $\pi$ from corruption to each player $i$. Let then $\pi_{payer} = a$ and, $\pi_{receiver} = b$. Also, $\phi_{payer} = b$ and, $\phi_{receiver} = c_b$. The notion of costs and benefits extends also to the decision of not entering in bribes, where $\phi_i$ and $\pi_i$ assume the value of zero.

Each state $S_t$ has an outcome or pay-off associated to it. Lets call the rewards $y$ so that $y_{i,t} = y(S_t)$, or more specifically:

$$y_{i,t}(S_t) = \begin{cases} 0 \text{ if not colluding} \\ \pi_i \text{ if colluding} \\ -f \text{ if detected} \end{cases} \tag{2}$$

As discussed in Chapter 2, rational individuals are trying to maximize some utility function. While firms may pursue the maximization of profits. For now, let the pay-off be the relevant objective from players. In the next section, the budget set and the notion of consumption is added to the agent's problem.

### 2.1.8 Expected Returns

Expected return from corruption can be written the costs of corruption $\phi_t$ plus the expected value of all possible rewards in the next period $y_{t+1}$ weighted by a time discount $\gamma$, or else:

$$E[y] = -\phi + \gamma \sum_{S} E\left[(\pi - s)\right],$$

Since $s$ is equal to zero if not detected but equal to $f$ if detected. Therefore, in this example, the cost of corruption $-\phi$ and the pay-offs from corruption $y$ to each player $i$ weighted by its expected probabilities can be written as

$$E[y_i] = -\phi_i + \gamma\left[(1-\alpha)\pi_i - \alpha f\right]. \tag{3}$$

## 2.2 Solution and Equilibria

Firstly, the algebraic analytical solutions are enough to understand the equilibria. However, as the game acquires more complexity, the equilibrium concept is narrowed to the Markovian perfect equilibria (MPE), which also requires numerical solutions.

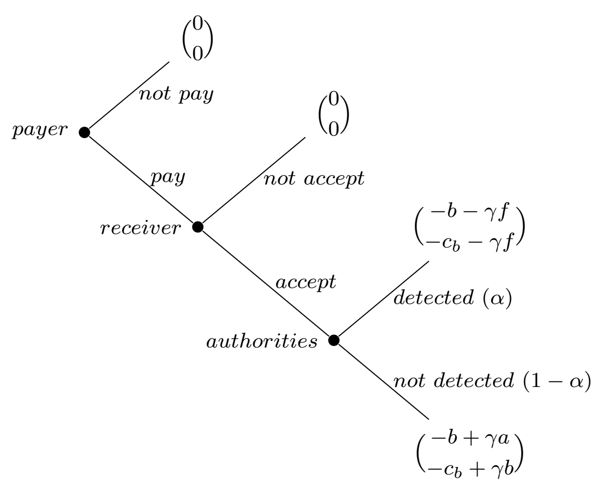
There are multiple ways to solve this game. It is possible to solve the game by taking $b$ as constant and solving for the best $d_t$ in a one shot or repeated iterations. Note that in this setting, the timing of the decision is unimportant, since corruption only happens when both parties agree. On the other hand, it is possible to solve the game for the best bribe $b$. Where, $b$ is the relevant action to be chosen b the bribe payer.

### 2.2.1 Solving for a Given Bribe

Assuming that the value of the bribe $b$ is given. Then agents must decide if they pay $d_{payer} = pay$ or accept $d_{receiver} = accept$ the bribe. In this sense[17], the solution is straightforward. If $b$ lies in any profitable interval[18], then players choose to enter in bribe, otherwise they do not.

It is possible to solve this game using backward induction. The Figure 2 shows the decision tree and the pay-offs (above for the *payer* and below for the *receiver*) associated to each decision:

Figure 2: Simple Corruption Game Tree




Importantly, note that the *payer* pays or commits to pay a bribe $b$ in the current period and the *receiver* gets it in the next period, with a value of $\gamma b$. Therefore, one must assume that this bribe is held in the present and paid in the future.

The Figure 2 shows that agents are going to enter in bribery if.

$$E[y_i] = -\phi_i + \gamma\left[(1-\alpha)\pi_i - \alpha f\right] > 0.$$

More specifically, for the *payer*,

$$E[y_{payer}] = -b + \gamma\left[(1-\alpha)a - \alpha f\right] > 0,$$

rearranging,

[17]First order conditions for the equation to maximize (3) in $b$ would lead to $\frac{\delta y_i}{\delta d_i} = y_i = 0$, since $y_i$ is a function of $d$ which is true whenever $d_i$ is true.

[18]It is assumed that agents are neutral towards risk.

$$b < \gamma\left[(1-\alpha)a - \alpha f\right] \tag{4}$$

And for the *receiver*,

$$E[y_{receiver}] = -c_b + \gamma\left[(1-\alpha)b - \alpha f\right] > 0,$$

rearranging,

$$b > \frac{(\gamma\alpha f + c_b)}{\gamma(1-\alpha)} \tag{5}$$

The equations (4) and (5) show that there will always be corruption as long as the bribe lies in the interval $\left(\gamma\left[(1-\alpha)a - \alpha f\right], \frac{(\gamma\alpha f + c_b)}{\gamma(1-\alpha)}\right)$. Importantly, it does not matter who proposes to enter in bribery first.

#### 2.2.2 Choosing the Bribe

If agents can choose the bribe, then the game design is naturally sequential. Because, one party must come up first and propose to enter in corruption. In this case, agents will choose the bribe that maximises their pay-offs. Or else, the bribe *payer* pays a bribe $b$ which is slightly bigger than $\gamma\left[(1-\alpha)a - \alpha f\right]$ the *receiver* asks for a bribe $b$ bigger enough then $\frac{(\gamma\alpha f + c_b)}{\gamma(1-\alpha)}$ [19]. Importantly, depending in the standard deviation $\sigma$[20], the utility from the uncertain returns from corruption vary according to the other player's relative risk aversion.

### 2.3 Final Remarks

Note that repetition is unimportant to this game equilibria. Here, even if the allowed actions depend on the player's state, they do not change as the stages of the game repeat. For instance, if an attempt to bribe fails and players are detected, in the next stage the decision rule is the same as if they had succeeded. Therefore, this specific design leads to the one-shot game equilibrium being equal to the same subgame perfect equilibrium (SPE) in each stage.

## 3 Corruption Game with Non-Trial Resolutions

In this section the game is expanded to encompass the non-trial resolutions (NTR). For this work, NTRs are summarized as the possibility of agreement between offenders and judicial/prosecutorial authorities to avoid a trial[21]. The agreements offer judicial benefits to the agents and can only happen if the they agree to self-report and disclose their misconduct.

[19] Notably, since $b$ is constant, its value is bigger than $E[y_i] = b$ for a risk averse agent.

[20] Which can be extracted from the binomial distribution $\sigma = nalpha(1 - alpha)$, where $n$ is the number times that one repeats the event.

[21] This is an oversimplification of the institution. NTRs can have a variety of distinct features depending on the jurisdiction.

It is important to remember that this work focuses in the ex-ante decisions. The ex-post benefits such as facilitation of prosecutions, costless judicial decisions or screening effects are not discussed here[22].

## 3.1 New Setting

As a consequence of the new setting the current model must account for three new features. First, there must be a decision to self-report or admit guilty in the action-space $d$. Second, the detection and trial phases must be separated in the state-space $S$. Lastly, the new states must have a new set of pay-offs in $y$ that account for the sanction reductions. Furthermore, the model must include a probability of being convicted $\beta$ in a trial after being detected.

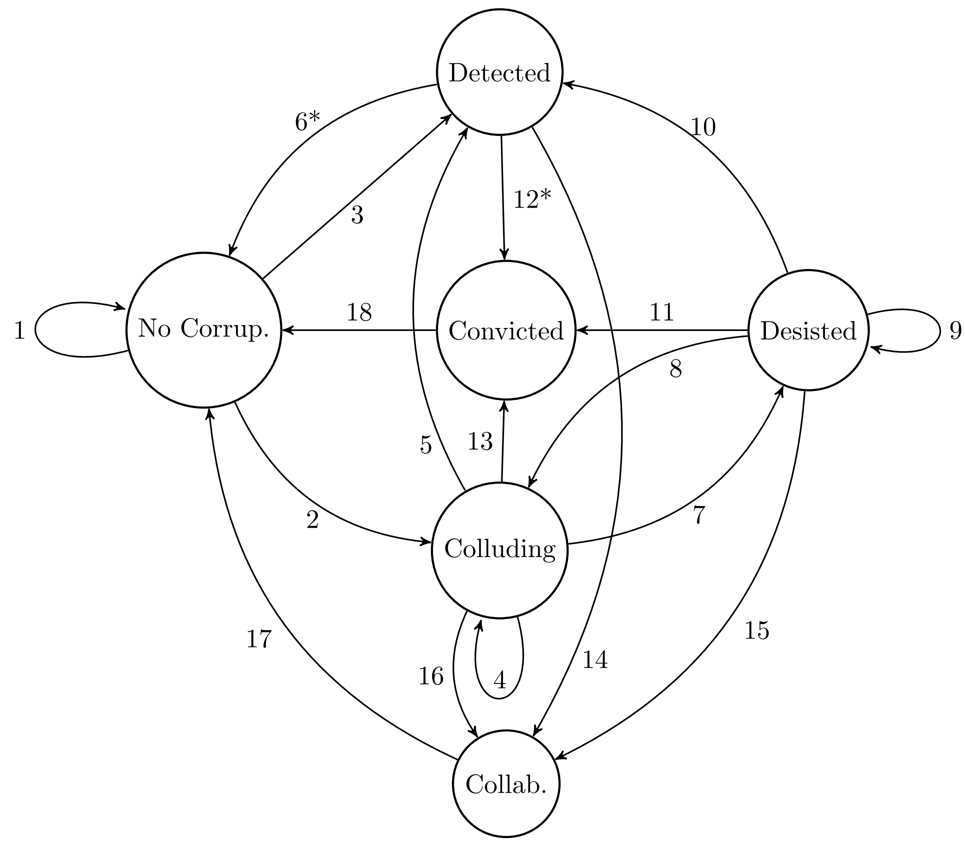
Most importantly, the model must now incorporate a set of distinct sanctions $s$ for each type of conviction/agreement. Each agreement has a rule for reducing the fine $f$, such that. If agents unilaterally self-report before detections they receive a reduced fine of $Rf$, where $R < 1$. Moreover, if both *payer* and *receiver* self-report before being detected the fine reduction is $rf$, where $1 > r > R$. However, if the agents are detected, they can plea-guilty, in this case, the fine reduction is lower, or $Pf$ if for unilateral pleading guilty and $pf$ when both players plea guilty simultaneously, where $1 > p > P$. In summary, the fine reductions are bigger (more lenient) for unilateral self-reporting than they are for simultaneous reporting. In the same way, fine reductions are more lenient to people that self-report before than after being detected. Since, admitting guilty is considered as an act of reporting after being detected. So, a collaborator is a player who either report before being detected or admit guilty after being detected.

### 3.1.1 States and Actions

In order to fully describe the new set of possible states of the world, a series of new states in $S$ must be introduced. The Figure 3 shows the new flow of states in the game.

[22] This branch of literature is better explored in Chapter 3.

Figure 3: States of the World with Non-Trial Resolutions




** The flow is represented by two different sets of states and actions.*

In a few words, if agents initially decide to pay a bribe, they can be detected (3) and go to the *detected* state or succeed (2) and go to the *colluding* state. From the latter, if they bribe again they can be detected (5) or go back to the same state (4). However, if they decide not to pay a bribe, they might be detected by their past crimes (5) or, if not detected (7), go to the state *desisted*[23]. The state *collab* stands for the states where the agents self-report (14, 15 and 16) and are collaborating with authorities. The *convicted* state is where agents pay the full fine. Admittedly, agents are convicted if they are reported (11 and 13) or when the other party admits guilty (12*). Lastly, agents that are the detected but do not report are either convicted (12) or acquitted (6*) in trials. After, this, they go back to the beginning of the game (6*, 17 and 18).

Importantly, differently from the past example, agents do not go together to

[23]The *desisted* state is more meaningful in the next examples. For now, it is just a state where agents did not bribe in the past state, but have entered in bribery in any state before.

all states, they can be in distinct states at the same time. Therefore, there is a state $S_i \in S$ for each player $i$.

Note that the transition law is still a function of the present state $S_t$, the decision $d_t$ and a stochastic probability. However, now, there are two distinct stochastic events which can happen with different probabilities[24] depending on the state $S_t$ that agents are in.

$$S_{t+1} = S((S_t, \epsilon(S_{t+1}))|d_t) \tag{6}$$

Where $\epsilon(S_t)$ follows $\sim$ *Bernoulli* $(\alpha)$ or $\sim$ *Bernoulli* $(\beta)$ depending on the states and actions.

### 3.1.2 Sanctions and Pay-offs

Given the new set of states[25], there are a new set of rewards $y(S_t)$ from each state in $S$, such that

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[24] The transition rules are exhausted bellow, where the superscript $'$ represents the next period $(t+1)$:

1. $p(x' = s_{nc}|x = s_{nc}, d_i = 0 \ or \ d_j = 0) = 1$
2. $p(x' = s_{cor}|x = s_{nc}, d_i = 1 \ and \ dj_1) = 1 - \alpha$
3. $p(x' = s_{det}|x = s_{nc}, d_i = 1 \ and \ d_j = 1) = \alpha$
4. $p(x' = s_{cor}|x = s_{corr}, d_i = 1 \ and \ d_j = 1) = 1 - \alpha$
5. $p(x' = s_{det}|x = s_{nc}, d_i = 1 \ and \ d_j = 1) = \alpha$
6. $p(x' = s_{acq}|x = s_{cor}, d_i = 0 \ and \ d_j = 0) = 1 - \beta$ and $p(x' = s_{nc}|x = s_{acq}) = 1$
7. $p(x' = s_{des}|x = s_{cor}, d_i = 0 \ and \ d_j = 0) = 1 - \alpha$
8. $p(x' = s_{des}|x = s_{cor}, d_i = 1 \ and \ d_j = 1) = 1 - \alpha$
9. $p(x' = s_{des}|x = s_{des}, d_i = 0 \ and \ d_j = 0) = 1 - \alpha$
10. $p(x' = s_{det}|x = s_{cor}, d_i = 1 \ and \ dj_{=}1) = \alpha$
11. $p(x' = s_{des}|x = s_{con}, d_i \neq 2 \ and \ d_j = 2) = \alpha$
12. $p(x' = s_{con}|x = s_{det}, d_i \neq 2 \ and \ d_j \neq 2) = \beta$ and $p(x' = s_{con}|x = s_{det}, d_i \neq 2 \ and \ d_j = 2) = 1$
13. $p(x' = s_{con}|x = s_{cor}, d_i \neq 2 \ and \ d_j = 2) = 1$
14. $p(x' = s_{col}|x = s_{det}, d_i = 2) = 1$
15. $p(x' = s_{col}|x = s_{des}, d_i = 2) = 1$
16. $p(x' = s_{col}|x = s_{cor}, d_i = 2) = 1$
17. $p(x' = s_{nc}|x = s_{col}) = 1$
18. $p(x' = s_{nc}|x = s_{con}) = 1$

[25] There are hidden states within the state *collab*. They will tell if the agents collaborated unilaterally or simultaneously.

$$y_{i,t}(S_t) = \begin{cases} 0 \text{ if not colluding} \\ \pi_i \text{ if colluding} \\ 0 \text{ if desisted} \\ f \text{ if convicted} \\ 0 \text{ if acquitted} \\ Rf \text{ if reported alone before detection} \\ rf \text{ if reported simultaneously before detection} \\ Pf \text{ if the other party admits guilty} \\ pf \text{ if both parties admit guilty} \end{cases} . \tag{7}$$

For instance, if a *payer* reports a bribery to the authorities and the *receiver* does nothing, then the *payer* is convicted and pay a sanction $s = RF$ while the *receiver* is convicted with the full fine $s = f$. However, if the *receiver* simultaneously report, both agents receive a sanctions of $s = rf$. Note that the sanction reduction in the first case is smaller than in the latter. Likewise, agents can report after detection. In this case, the relation between $P$ and $p$ follows the same logic.

#### 3.1.3 Expected Returns

It is possible to decompose the pay-offs by summing up all distinct possible pay-offs. First the agent pays the cost $\phi_i$ of entering in a bribery scheme. In the next period, there is a chance of $\alpha$ of being detected by the authorities, if detected a chance of $\beta$ of being convicted. Therefore, the expected return in case of being fined is

$$\gamma(\alpha\left[\gamma(\beta f)\right],$$

or simply

$$\gamma^2\alpha\beta f.$$

There are now two possibilities of earning the corruption gains $\pi_i$. Either by not being detected in the next period

$$\gamma(1-\alpha)\pi_i.$$

Or by being detected and not convicted

$$\gamma\alpha\left[\gamma(1-\beta)\pi_i\right],$$

or rearranging,

$$\gamma^2(\alpha(1-\beta)\pi).$$

It is possible to sum the possibilities in which the agents succeed in corruption and earn $\pi_i$ at the end of two periods as:

$$\gamma^2(1-\alpha)\pi_i + \gamma^2(\alpha(1-\beta)\pi),$$

or simply,

$$\gamma^2(1-\alpha\beta)\pi_i.$$

Therefore, it is possible to write the expected value as:

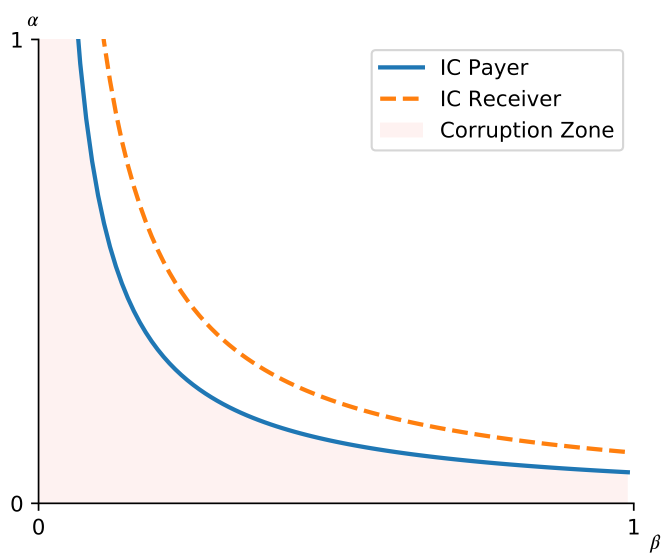
$$E[y_i] = -\phi_i + \gamma^2 \left[(1-\alpha\beta)\pi_i - \alpha\beta f\right]. \tag{8}$$

In the absence of non-trial resolutions, it is possible to calculate the domain in which bribes are profitable, ore else

$$E[y_i] = -\phi_i + \gamma^2 \left[(1-\alpha\beta)\pi_i - \alpha\beta f\right] > 0. \tag{9}$$

The Figure 4 show all the combinations in which $E[y_i]$ is positive for both players.

Figure 4: Indifference Curves for Profitable Bribes

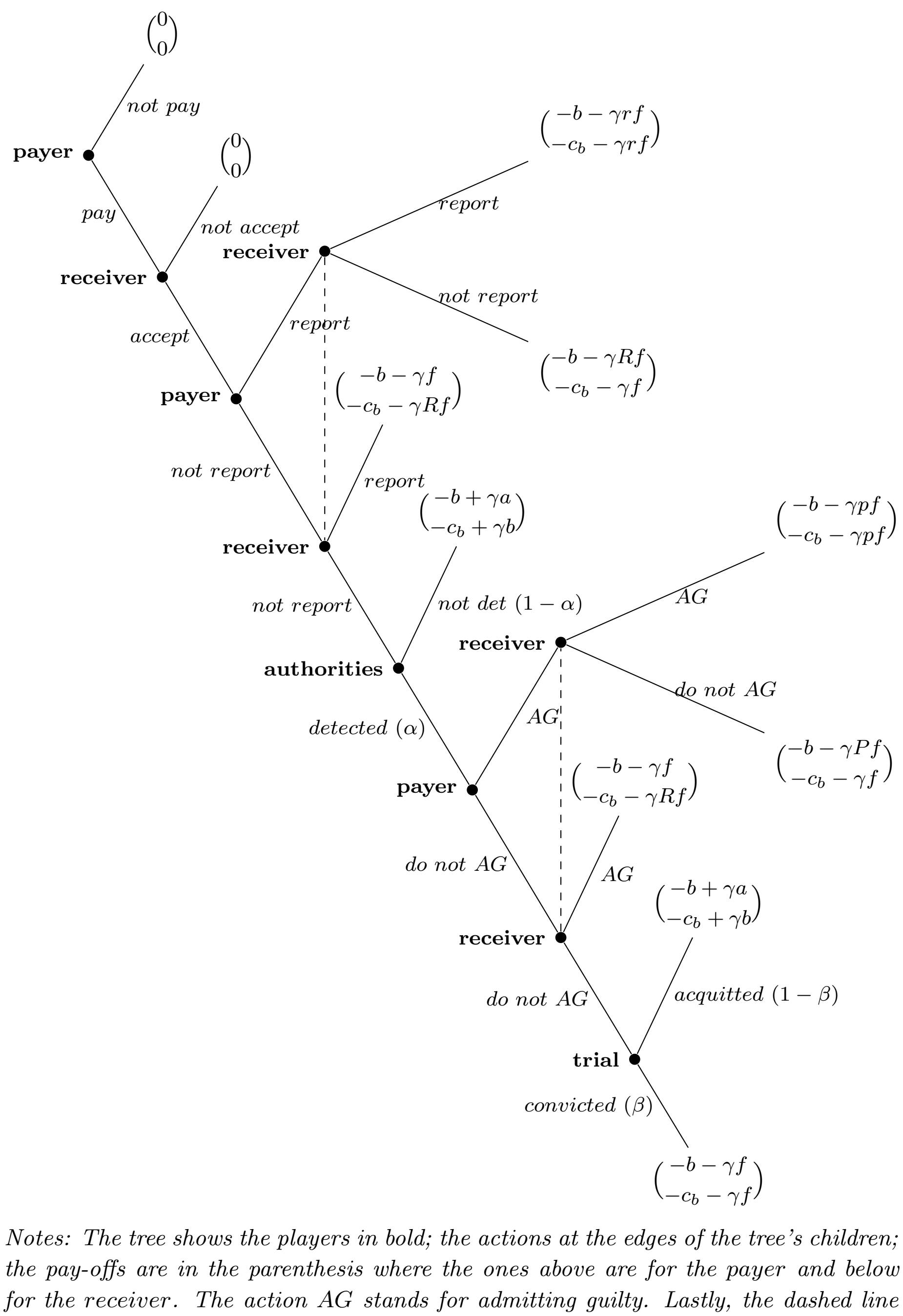



Note that corruption will occur only if corruption is profitable for both players. Given the above conditions, bribes are profitable if $\alpha < \frac{\pi_i - \phi_i}{\beta(\pi_i + f)}$ or $\beta < \frac{\pi_i - \phi_i}{\alpha(\pi_i + f)}$. Therefore, an increase in $f$ moves both curves to the left, increasing the deterrent effect. Meanwhile, a decrease in $(\pi_i - \phi_i)$ moves the indifference curves towards a more deterrent set.

## 3.2 Solution and Equilibria

In this example, the solution to the problem is more complex. However, it can be solved analytically through backward induction. The Figure 5 below shows the extensive game-tree:

Figure 5: Corruption Game with Non-Trial Resolutions




*Notes: The tree shows the players in bold; the actions at the edges of the tree's children; the pay-offs are in the parenthesis where the ones above are for the payer and below for the receiver. The action AG stands for admitting guilty. Lastly, the dashed line indicates that the players in those nodes and their parent are playing simultaneously.*

In this setting there are two stages of simultaneous decisions. Therefore, it is necessary to calculate the distinct subgame perfect Nash equilibria (SPNE)[26] that orients the players' strategy.

Lastly, note that if the bribery conditions lie in the corruption zone from Figure 4, it does not mean that there will be bribery, because players still have take the possibility of being reported into consideration. In this sense, differently from the past example. This game has different outputs if played in one-shot (one time) or repeatedly. This happens because, if players stop collaborating after a defection, then, agents have to account the benefit of a one time defection against repeated gains from cooperation.

### 3.2.1 One Shot

Figure 5 shows that there are two nodes in which the players play simultaneously. In the first one, before detection, agents have to decide if they are going to report after paying the bribe. The matrix below shows the simultaneous decision payoffs for both players. xxxx

| | Report | Not Report |
|---|---|---|
| Report | $-rf; -rf$ | $-Rf; -f$ |
| Not Report | $-f; -Rf$ | $\gamma^2 \left[(1-\alpha\beta)a - \alpha\beta f\right]; \gamma^2 \left[(1-\alpha\beta)b - \alpha\beta f\right]$ |

Assuming that the agents do not want to be criminals, or else, not be in corruption. Then they would prefer a reduced fine $Rf$ at least as good as the expected return from corruption at that node. Therefore, agents would only report if

$$-R_i f \geq \gamma^2 \left[(1-\alpha\beta)\pi_j - \alpha\beta f\right],$$

where the subscrpit $j$ represents the other player. Or else, rearranging

$$-R_i^* \geq \frac{\gamma^2 \left[(1-\alpha\beta)\pi_j - \alpha\beta f\right]}{f}. \tag{10}$$

where the $*$ means the $R$ in which agents prefer to report.

If the enforcement conditions $\alpha$ and $\beta$ stay constant after the bribery and knowing that agents would have paid the costs of the bribe in the past node only if it was profitable for them. It would mean that reporting only happens if $R < 0$, or else, if agents gain a bonus from reporting. This is not common in most jurisdictions and this result is in line with past results using leniency to avoid cartels (Spagnolo, 2005).

If agents are detected, they need to decide upon another set of actions. In this case,

[26] The subgames are Nash equilibrium since players build their strategies non-cooperatively.

| | Admit Guilty | Not Admit |
|---|---|---|
| Admit Guilty | $-pf; -pf$ | $-Pf; -f$ |
| Not Admit | $-f; -Pf$ | $\gamma\left[(1-\beta)a-\beta f\right]; \gamma\left[(1-\beta)b-\beta f\right]$ |

Notably, the agents admit guilty if,

$$-P_i f \geq \gamma\left[(1-\beta)\pi_j-\beta f\right]$$

or rearranging,

$$-P_i^* \geq \frac{\gamma\left[(1-\beta)\pi_j-\beta f\right]}{f} \tag{11}$$

Differently from the case of reporting before detection, the sanction reduction here does not strictly requires a bonus. Since the expected return after detection is smaller than before detection and it is probably negative. In this sense, it is possible to have plea-bargains with a $1 > P^* > 0$.

Note that, the first restriction (9) is bigger than the subsequent (10) and (11). Or else, there will be plea-bargain only if the probabilities $\alpha$ and $\beta$, or the rules of leniency change over time. Otherwise, the players would not enter in corruption at first[27].

Note that it is possible understand term on the right side of (10) and (11) as the discounted expected return from corruption[28]. So, let $\left(\frac{f+\pi}{f^2}\right)$ be constant of the return $k$, and setting the time discount $\gamma = 1$, then it is possible to rewrite (10) as,

$$(1-R_i^*) \geq k(1-\alpha\beta), \tag{12}$$

and, letting $k' \equiv \left(\frac{k-1}{k}\right)$, then

$$-P_i^* \geq k'\beta, \tag{13}$$

Now, it is possible to draw the separated equilibrium given the new boundaries (12) and (13)[29]. Therefore, for any $\alpha > \alpha(R*) = \frac{(R^*+k-1)}{\beta k}$ agents will not enter in corruption because of the fear of self-reporting [30]. Furthermore, for any

[27] However, there are situations in which agents can enter a game just to explore the payoffs. This would happen if the bonus are big enough, so agents enter in corruption just to receive it. However, this is ratter an unrealistic situation

[28] In the first case for two periods and in the latter for one period.

[29] It is possible to write

$$\alpha(R^*) \geq \frac{\left(\frac{R^* f}{\gamma^2}\right)}{\beta(a+f)},$$

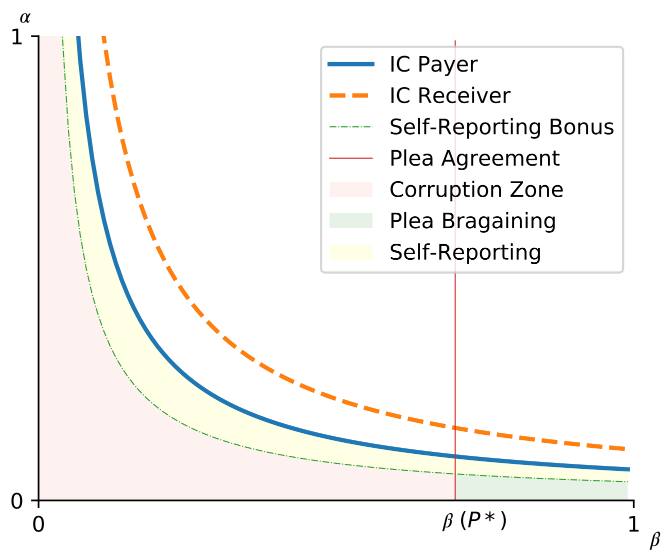
and

$$\beta(P^*) \geq \frac{\left(\frac{P^* f}{\gamma}\right)}{a+f},$$

[30] Once again, in an extreme situation, agents enter the game to self-report,Motta and Polo (2003) refer to this strategy as, CR, or collude and report. Here, this hypothesis is not considered.

$\beta > \beta(P*) = \frac{P^*}{k'}$, agents would plea guilty. The Figure 6 shows the indifference curves and the equilibrium areas.

Figure 6: Indifference Curves for Profitable Bribes with Non-Trial Resolution




The interpretation from the self-reporting area and the plea bargaining area is straightforward. In the first case, for a given leniency policy $R^*$, if the combined probability of detection and conviction is higher than $\alpha(R^*)$ and $\beta(R^*)$, it means that it is more profitable (in expected terms) to self-report and get the bonus than it is to stay in the game. Likewise, if the agents are detected, then they will always plea guilty if the observed probability of conviction $\beta$ is higher then the threshold $\beta(P^*)$. Lastly, the thresholds $\alpha\beta(R^*)$ and $\beta(P^*)$ move towards the origin as $R$ and $P$ are smaller (more lenient). Therefore, more lenient sanction reduction rules from NTR decrease the non-observable corruption.

Lastly, it must be noted that the unilateral reductions rules $R$ and $P$ are the variables that disrupt the equilibrium[31]. However, the simultaneous reduction rule $r$ and $p$ are the ones actually paid by the reporting agents[32]. Therefore a policy recommendation that focus on deterrence from lowering the pay-offs or even if it targets revenue from prosecution would aim for a smaller (even negative) fine reduction for unilateral reporters. As well as a less lenient fine reduction for simultaneous collaborators[33].

[31] This is a prisoner's dilemma.

[32] As long as $r < 1$ and $p < 1$, defection is going to be a Nash equilibrium. However, if $r > 1$ and $p > 1$, then there is no equilibrium to the game. Nonetheless, it does not mean that corruption is going to happen

[33] Spagnolo (2005) found that the first best solution would imply that the reporters receive

### 3.2.2 Repeated

In the repeated example, some assumptions are necessary to find the game's set of equilibria. Firstly, suppose that the game is infinitely repeated[34].Secondly, suppose that agents, if they are reported, they learn that the other player is not trustworthy, then they never engage in corruption again (grim trigger)[35][36].Therefore, the ex-ante decision to engage in corruption have to account for trading one-time defection against an infinitely repeated return from corruption[37]. This trade-off is expressed in the following matrix,

| | Report | Not Report |
|---|---|---|
| Report | $rf; rf$ | $Rf; f$ |
| Not Report | $f; Rf$ | $\dfrac{-b+\gamma^2\left[(1-\alpha\beta)a-\alpha\beta f\right]}{(1-\gamma)};\dfrac{-c_b+\gamma^2\left[(1-\alpha\beta)b-\alpha\beta f\right]}{(1-\gamma)}$ |

This time, the agents will enter in corruption if

$$-RF \leq \frac{-\phi_i+\gamma^2\left[(1-\alpha\beta)\pi_i-\alpha\beta f\right]}{(1-\gamma)}. \tag{14}$$

Note that, the infinitely repeated return from corruption is always bigger then the one-time expected pay-off if the time discount $\gamma > 0$, or else

$$\frac{-\phi_i+\gamma^2\left[(1-\alpha\beta)\pi_i-\alpha\beta f\right]}{(1-\gamma)} > -\phi_i+\gamma^2\left[(1-\alpha\beta)\pi_i-\alpha\beta f\right].$$

Therefore, the fine reduction $R^*$ must be more lenient[38] than it need to be in a one-shot game. Consequently, it is more difficult to combat corruption with leniencies in a repeated game. This results holds also when players are detected. The one-time plea bargain is punished by the grim-trigger, making the agents less prone to plea guilty.

### 3.2.3 Risk Aversion

XXXXXXXXXX INSERT THE TEXT AND CONCLUSIONS XXXX

## 3.3 Final Remarks

This section shows that leniency or fine reductions to corruption offenders who report their crimes may deter the crime from happening. The results found

all the gais from the other player as bonus. It would lead to complete deterrence. However, no revenues.

[34]Finite game would imply that the last game is played as a one-shot. By backward induction, it would make all sub-games played like a one-shot game.

[35]Grim trigger can also be understood as a punishment for defection.

[36]Here agents only learn to punish defectiong agents,they do not learn or update their preferences after each game. However, if players learn and update their preferences then, the the result of the game changes.This setting is explored in the next sections

[37]XXXXXXXXXXXX TO DO – Show the derivation XXXXXXXX.

[38]more positive in this case.

here replicate past findings from anti-trust literature (Motta and Polo, 2003; Spagnolo, 2005; Harrington, 2008; Chen and Harrington, 2007).

One notable outcome from this specific setting is that it shows that it is preferable to have lower fine reductions for players that unilaterally report but less lenient reductions if they do it simultaneously.

Note that the results hold under the assumption that agents are not learning or even updating any other relevant variable in their choices. In the following sections, some other variables that may evolve after repeated games are introduced. This setting make the model more realistic. However, the solutions are also more complex, and require different techniques to solve the agent's problem.

# 4 Additional States and Actions

Truly repeated games are very rare in realistic settings. As agents repeatedly interact, they gain (and accumulate) information. Therefore, it is possible to describe this additional information as states which change over time. Notably, the set of new states allows for a set of new actions.

## 4.1 Wealth and Consumption

Perhaps the most important feature missing in the previous example is the consumption. For individuals, a realist objective must account for the consumption decision $c$ and not the pay-off itself. Note that, the decision of consumption requires a notion of a finite budget or wealth $W$, otherwise there would be nothing to decide upon.

### 4.1.1 The Consumption Utility Function

Agents may present distinct propensities towards risk. Therefore it is necessary to introduce the utility operator $u(.)$ which extracts the value of the consumed goods from players given their relative risk aversion $\eta$. In the next examples, it is assumed that the utility function $u(.)$ is an isoelastic utility function. Also called constant relative risk aversion (CRRA) function, such that:

$$u(c) = \begin{cases} \frac{c^{(1-\eta)}-1}{1-\eta} \text{ if } \eta \neq 1 \\ ln(y) \text{ if } \eta = 1 \end{cases}$$

Where $\eta$ is the risk aversion parameter and $\eta > 0$ represents some degree of risk aversion.

In this setting, agents naturally search for a smoother consumption decision $c_{i,t}$.

Lastly, as discussed in chapter two, corporations and individuals might have distinct objectives. Notably, individuals are trying to maximize their utilities and corporations maximize profits. Therefore, the results from the following examples are more suitable to individuals. Admittedly, in the previous sections, the pay-off maximization strategy is more likely to portray a corporative decision. Nonetheless, corporative decisions are all made by individuals ultimately. Consequently, it is hard to attribute a set of decisions to any specific entity. Therefore, the differences between corporative and individual decision, whenever clearly spotted, are going to be pointed out.

### 4.1.2 Budget and the Asset Market

After each interaction between players, the agents either earn or lose some money from corruption. This may change the players willingness to enter again in collusion. Furthermore, even if they do not enter in a bribery, there are substitute options for player's budget[39]. Therefore, a fundamental state in the players decision making is the budget set.

Notably, the decision to consume $c_{i,t}$ is restricted to the agents budget constraint. It is possible to introduce it as the total wealth of players at each time $W_{i,t}$. Naturally, there must be a trade-off between consuming today and consuming tomorrow. This is given by the time discount $\gamma$. Consequentially, if agents choose not to consume today they need to earn some interest rates $i_r$[40] over their savings.

Note that agents might have multiple sources of income. Admittedly, there is an income from corruption $y_c$, a financial income $y_f$ and a non-financial income or wage $y_0$[41]. So that, $y_c$ is given by the expression in (7), $y_0$ is a constant autonomous income such as a wage and $y_f$ is given by the following expression:

$$y_{fi,t+1} = (W_{i,t} - c_{i,t} - \phi_{i_t})(1 + i_r). \tag{15}$$

Note that 15 implies that the player will save all amount not consumed $c_{i,t}$ or spent bribery costs $\phi_{i,t}$ at time $t$ and invest in an asset market.

### 4.1.3 Actions, States and Laws of Motion

It is possible to describe the state wealth $W_{i,t}$ as the sum of all player's incomes,

$$W_{i,t} = y_{fi,t} + y_{ci,t} + y_{0i,t} \tag{16}$$

Notably, in this setting, agents have a new allowed action to choose. They can now decide on how much to consume $c_{i,t}$[42]. This action determines how the state $W_{i,t}$ is going to evolve. Therefore

$$W_{i,t+1} = (W_{i,t} - c_{i,t} - \phi_{i_t})(1 + i_r) + y_{ci,t+1} + y_{0i,t+1} \tag{17}$$

[39] Without an asset market, there is no income and substitutions effects in play.

[40] Interest rates $ir$ and time discount $\gamma$ do not need to be equal.

[41] In this example, there is a living wage, so that agents can get enough funds to pay a bribe. This is a way to overcome the reducibility of the Markov chain. Without this artifice, agents who are convicted and lose everything would never be able to play again. Consequently, they were going to be stuck in that set of states.

[42] The decision can be on how much to save $(W_{i,t} - c_{i,t} - \phi_{i_t})$, since one decision is the complement of the other.

#### 4.1.4 The Agent's Problem

Agents want to make the best set of decision $d_i$ which maximize their utilities. Additionally, they are constrained to the other player's similar utility maximization problem. In this first simple example it is enough to present a simple maximization equation. However, in the following examples, as the complexity increases, it is convenient to introduce the value function equation.

Let $\gamma$ be a time discount for both agents and $E_0$ is the expected value at $t = 0$, then the objective function can be written as:

$$\max_{c,d} E_0 \sum_{t=0}^{\infty} \gamma^t u(c_t) \tag{18}$$

$$\begin{aligned} \text{s.t.} \qquad S_{t+1} &= S((S,\epsilon)|d_t) \\ W_{i,t+1} &= (W_{i,t} - c_{i,t} - \phi_{i_t})(1 + i_r) + y_{ci,t+1} + y_{0i,t+1} \end{aligned}$$

Where the notation $E_0$ means the expected value at $t = 0$.

Note that, in this specific example, the decision of the player $i$ depends on the decision of $j$[43] through the state transition rule. Therefore there is a strategic interdependence in the players' best decision, or else the decision of one player depends on the other $d_i = d(d_j)$.

#### 4.1.5 Solution and Equilibria

Note that in the previous example, the expected return from corruption needs only to be greater than zero to be preferred in a subgame. Admittedly, now there are a set of new conditions for acceptance. First, agents can choose upon financial gains with certainty or stochastic returns from corruption. Notably here, the risk aversion plays a role. Secondly, agents can trade some instant pay-off for a better position in the game regarding all states. Given that, the current states are sufficient for agents to make their decision[44], it is possible to find a set of equilibria known as the Markov Perfect Equilibria (MPEs).

Before solving the problem in (18), it is convenient to introduce a new set of states that account for the judicial liability.

### 4.2 Judicial Liability

In the previous example and for most of the literature in leniency it is assumed that players are not liable for crimes committed in the past (Marvão and Spagnolo, 2016). In other words, if the crime is not detected immediately after it is committed, then agents are not liable for it in future stages. This is a convenient but unrealistic assumption about the game[45]. In order to overcome this

[43] Where $j$ represents the other player.

[44] A stochastic process $x_t$ is said to have the Markov property if for all $k \geq 1$ and all t , $Prob(x_{t+1}|x_t, x_{t-1}, ..., x_{t-k}) = Prob(x_{t+1}|x_t)$ (Ljungqvist and Sargent, 2012) p. 24.

[45] Also, without liability from past crimes, once in corruption, agents would never desist from entering in corruption again in a next period.

shortcoming, this section introduces the notion of judicial liability. Notably, it evolves as the agents play the game of corruption repeatedly.

#### 4.2.1 Criminal Liability

Agents are liable for each crime they have committed. Each bribe paid generates a criminal liability $l$ in the next period. Nonetheless, the liability from crimes tend to depreciate at a rate $\delta$ over time. Such that,

$$l_{t+1}(d) = \begin{cases} 1 \text{ if players agree on the bribe in } t \\ 0 \text{ if at least one player does not enter in corruption in } t \end{cases}$$

#### 4.2.2 Liability Transition Rule

$$L_{i,t+1} = (1-\delta)L_{i,t} + l_{i,t+1} \tag{19}$$

There are a number of reasons to assume that the judicial liability decays over time. It may happen because information gets lost over time. Furthermore, prescription rules eventually cease the agents from culpability for old enough offences.

## 5 Complete Game

In this section, the corruption game with non-trial resolutions is solved in a more complex environment. Here, agents need to solve their consumption maximization problem given their choices to enter in corruption and face all implications from criminal law enforcements or make a living out of wages and financial markets.

At this point it is important to reorganize the notations so it is possible to write the problems more clearly. Following the tradition in the dynamic programming literature, the subscript $(t+1)$ is substituted by a *prime* $(')$ superscript. So, all the variables without a subscript $(t)$ are referring to the present period. Additionally, lets define the vector containing all possible states as $\mathbf{x}$ such that $\mathbf{x}_i \supset [W_i, L_i, S_i]$. Similarly, the set of all actions as $\mathbf{d}$, such that $\mathbf{d}_i \supset [d_i, c_i]$. Lastly, $\theta$ is the vector containing all constant parameters. In summary, there are two vectors that contain all the states and actions,

$$\mathbf{d} \equiv \begin{bmatrix} d_i \\ c_i \\ d_j \\ c_j \end{bmatrix}, \text{ and } \mathbf{x} \equiv \begin{bmatrix} W_i \\ L_i \\ S_i \\ W_j \\ L_j \\ S_j \end{bmatrix}$$

Where, $j$ is the other player.

## 5.1 The Agent's Problem

Both agents $i$ face the same economic problem[46]. They want to maximize their utilities. Therefore, it is possible to rewrite (18) as

$$\max_{\mathbf{d}} E_0 \sum_{t=0}^{\infty} \gamma^t u(c_t(\mathbf{x})) \tag{21}$$

$$\text{s.t. } \mathbf{x}' = f(\theta, \mathbf{x}, \mathbf{d}, \epsilon')$$

The complete description of the constraint in (21), is given by the laws of motion (6), (17) and (19). Or else,

$$\begin{aligned} \text{s.t. } W' &= (W_i - c_i - \phi_i)(1 + i_r) + y'_{ci} + y'_{0i} \\ L' &= (1 - \delta)L_i + l'_i \\ S' &= S((S, \epsilon')|d) \end{aligned}$$

## 5.2 The Value Function

The agent's problem (21) has a recursive nature due its first-order difference equations constraints. Importantly, some future states are stochastically determined. Notably, this kind of stochastic processes are known as Markov decision processes[47]. In this sense, dynamic programming is the best methodological framework to solve this kind of problem (Putterman, 2005). In order to turn the problem into a dynamic programming problem, it is necessary to build the value function, which consists in the value of state vector to the agent $V(\mathbf{x})$.

Lets define a value function from the initial states $V(\mathbf{x_0})$ as being the the optimum value for the initial states $W_0$, $L_0$, and $S_0$. So that the value function $V(\mathbf{x})$ of the states can be defined by the following *Bellman Equation*,

[46] It is possible to look to the problem from another perspective. The government wants to maximize society's welfare. Assuming that there is no 'greasing the wheels' hypothesis from corruption (The hypothesis states that bribes work as shadow prices that increase efficiency of agents' transactions. Notably, it is more realistic when dealing with extortive bribes, when lower wages from civil servants are compensated by a bribe. Since, this work deals with collusive bribery, the hypothesis is less applicable.) and that the welfare gains from the criminals are neglectful. It is possible to infer that welfare is monotonically decreasing with the corruption level. Or else, governments want to minimize the level of corruption, or the number of agents in the state 'colluding' (lets call it $S_c$) over time, constrained to the agents maximization problem (23). For this, the government has the power to change the fines $F$ and the leniency policies $R$ and $P$. Therefore it is possible to write the governments problem as:

$$\min_{R,P,F} E_0 \sum_{t=0}^{\infty} \gamma^t S_{c_t}(R, P, F) \tag{20}$$

$$\text{s.t. } V(W, L, S) = \max\{u(c) + \gamma E[V(W', L', S')|W, L, S, \mathbf{d}, c]\}.$$

[47] It is also referred in engineering as stochastic control problems.

$$V(\mathbf{x}) = \max_{\mathbf{d}}\{u(\mathbf{x}) + \gamma E[V(\mathbf{x}')|\mathbf{x}, \mathbf{d}]\} \tag{22}$$
$$\text{s.t. } \mathbf{x}' = f(\theta, \mathbf{x}, \mathbf{d}, \epsilon')$$

or,

$$V(W, L, S) = \max_{d,c}\{u(c) + \gamma E[V(W', L', S')|W, L, S, d, c]\} \tag{23}$$

$$\begin{aligned} \text{s.t. } W' &= (W - c - \phi)(1 + i_r) + y_c' + y_0' \\ L' &= (1 - \delta)L + l' \\ S' &= S((S, \epsilon)|d) \end{aligned}$$

Therefore, the value function $V(.)$ takes the local maximum of the utility from consumption $u(c)$ today given all the possible consumption from all possible next states. This is an exhaustive exercise of finding the path that leads to the best expected results given all possible states.

## 5.3 Dynamic Programming Solution

The objective here is to find the strategies that are optimal for both players in an equilibrium. In order to solve dynamic programming problems the agents have to choose the best actions $\mathbf{d}$ that lead to a maximum value of current state $\mathbf{x}$ and future ones $\mathbf{x}'$ given a stochastic transition law $\Omega$. Notably, the transition matrix $\Omega(\mathbf{x}', \mathbf{d}, \mathbf{x})$ can be understood as the probability density of the next state $\mathbf{x}'$, given the current state action pair $(\mathbf{x}, \mathbf{d})$[48]. Hence, the dynamic programming solution to the agent's problem, or the solution of the *Bellman Equation* in this case, can be achieved by simply iterating the value function given the transition matrix $\Omega$ up to a fixed point[49]. If the $\Omega$ matrix is stochastic, then the solution is given by maximizing each value function $V(\mathbf{x})$ given all states for each period, or else:

$$V(\mathbf{x}) = \max_{\mathbf{d}}\{u(\mathbf{x}) + \gamma \sum_{\mathbf{x}} V(\mathbf{x}')\Omega(\mathbf{x}', \mathbf{d}, \mathbf{x})\} \tag{24}$$

[48]It is possible to calculate the matrix $\Omega(\mathbf{x}', \mathbf{d}, \mathbf{x})$ for a given number of states $n$ and actions $m$ as:

$$\Omega(\mathbf{x}, \mathbf{u}, \mathbf{x}') = \begin{bmatrix} \begin{bmatrix} p(\mathbf{x}_0'|\mathbf{x}_0, \mathbf{d}_0) & \cdots & p(\mathbf{x}_0'|\mathbf{x}_0, \mathbf{d}_m) \\ \vdots & \ddots & \vdots \\ p(\mathbf{x}_0'|\mathbf{x}_n, \mathbf{d}_0) & \cdots & p(\mathbf{x}_0'|\mathbf{x}_n, \mathbf{d}_m) \end{bmatrix} \\ \vdots \\ \begin{bmatrix} p(\mathbf{x}_n'|\mathbf{x}_0, \mathbf{d}_0) & \cdots & p(\mathbf{x}_n'|\mathbf{x}_0, \mathbf{d}_m) \\ \vdots & \ddots & \vdots \\ p(\mathbf{x}_n'|\mathbf{x}_n, \mathbf{d}_0) & \cdots & p(\mathbf{x}_n'|\mathbf{x}_n, \mathbf{d}_m) \end{bmatrix} \end{bmatrix}.$$

[49]The existence of a fixed point is guaranteed for a $\gamma < 1$, which implies that the system is a Banach contraction (Putterman, 2005) (p. 149-151).

If $V^*$ is a unique solution to the Bellman Equation (24)., then it is possible to define an optimal policy $\sigma(\mathbf{x})$ such that:

$$\sigma(\mathbf{x}) \in \text{argmax}_{\mathbf{d}}\{u(\mathbf{x}) + \gamma \sum_{\mathbf{x}} V^*(\mathbf{x}')\Omega(\mathbf{x}', \mathbf{d}, \mathbf{x})\} \tag{25}$$

The optimal policy $\sigma_i$ is a vector with dimensions $(n \times 2)$, where $n$ is the number of all possible combination of states. Consequently, for each row in $\sigma_i$ there is an optimal pair of actions ($d_i$ and $c_i$) that maximizes the value of all possible outcomes, not only for the next period, but for all other periods. In other words, the best optimum policy $\sigma_i$ is a rule for finding not only the best outcome for the next period, but also to find the best position in the state space which leads to better outcomes.

### 5.3.1 Strategic Interdependence

Note that the individual decision function $c$ and $d$, depends on $S$ that depends on $d$ again. This shows the strategic interdependence of the game, where some strategy of the player $i$ depends on the other player $j$, or else substituting (6) in $d_i$, then

$$d_i = d(W_i, L_i, S(S_i', d_j(W_j, L_j, S_j, \theta)), \theta), \tag{26}$$

where $d_i$ and $d_j \in [d_{payer}, d_{receiver}]$. Similarly, using the same logic $c_i$ depends on $c_j$.

From this dependence it is clear to identify the strategic game from the problem. Therefore, it is possible to rewrite the policy function (24) in terms of the separate decisions of $i$ and $j$,

$$\sigma_i(\mathbf{x}) \in \text{argmax}_{\mathbf{d_i},\mathbf{d_j}}\{u(\mathbf{x}) + \gamma \sum_{\mathbf{x}} V^*(\mathbf{x}')\Omega(\mathbf{x}', \mathbf{d_i}, \mathbf{d_j}, \mathbf{x})\} \tag{27}$$

In this way, it is clear to see that each player has a $\sigma_i$ strategy that depends on the other player's strategy $\sigma_j$ and vice versa. The equilibrium (MPE) arises when both players identify the other players optimal decision, given their own ones and still hold to their decisions (do not want to change their policy $\sigma$). Therefore, the solution to the agent's problem is a pair of optimal policy vectors ($\sigma_i$ and $\sigma_j$) which maximizes the value function $V(\mathbf{x})$ for both players.

### 5.3.2 Discretization

Solutions to the continuous-states dynamic programming problems are known to be tricky[50]. One way to deal with this shortcoming is to discretize the problem, or else, turn the states and action variables into finite grids or lattices which agents can access[51].

[50] There are a few known solutions, most of them use a linear quadratic constraints. They are known to lead to Euler Equations and have nice analytical properties.

[51] The discretized variables are presented in the Appendix **??**.

It is important to point out that the discretization of the problem sometimes sacrifices the reality of the model. This happens because it is necessary to impose boundaries to the grids. Consequently, this finite lattice creates conditions to strange decisions near the boundaries. For instance, in this setting, the minimum wealth $W$ that an agent can have is zero. So, if the agents see themselves in situations where the sum of the bad outcomes are greater than their wealth, they may act recklessly. This is a situation known as limited liability, and in a small grid it would occur more than in reality. The same thing can happen on the other extreme of the grid. If agents reach the maximum wealth, they have no incentive to invest in growing their assets, which also leads to strange decisions.

It is very clear that these grid limitations affect also the steady state from the game. Therefore, it is prudent to analyse the results excluding these extreme conditions. In the Section 5.4, the results for the society are sampled from the middle of the grid to avoid these problems.

#### 5.3.3 Curse of Dimensionality

Discretization implies that we abandon traditional analytical mathematics for numerical computing. Notably, each discrete point in the state-action-space represents one point in the lattice of possibilities. Consequently, the number of possible points is obtained by the multiplication of all elements of each state and action. Therefore, it grows exponentially in the number of states or actions. This problem was called by Bellman and Dreyfus (1962) as the '*curse of dimensionality*'. Consequently, if the game is played in its full complexity, it is expected that the number of different combination in the state-action lattice to be excessively large.

For instance, in the complete game with only natural numbers as allowed states and action, with maximum wealth of $\overline{W} = 40$ and maximum liability of $\overline{L} = 5$, them the total number of point is equal to $(\overline{W}^4 \times \overline{L}^2 \times 8^2 \times 4^2 = 36.864 \times 10^9)$[52]. Therefore, it would require a lot of computational power to operate with matrices and vectors of length bigger than 36 billion elements.

#### 5.3.4 Dimension Reduction

There are a couple of ways to avoid the course of dimensionality. Most of them are based on approximation of the necessary integrals (Pakes and McGuire, 2001).However, here, the solution to overcome this problem is to simply shorten the vector of states and action. The strategy is based in two main ingredients.

First, it is necessary to work only with feasible states. In other words, states that are impossible to reach do not need to be on the state-space[53]. For

[52]There are 4 spaces with length 40, the wealth space and the consumption space for both players. Then there are the liability spaces for both players. Also, the states of the world for both players, and lastly, the actions for both players.

[53]Importantly, the excluded spaces are only the ones that are impossible to be reached. The states that can be reached, but are eventually never be chosen still need to be on the space.

instance, if agents are not allowed to pay a bribe if they are detected, than the combination of state '*detected*' and the action '*pay the bribe*' is not accounted for[54].

Lastly, it is possible to work with incomplete informations. Meaning that, in a realistic setting, players are not able to fully observe each others states. For instance, it is unlikely that players can perfectly observe the other players budget set. Therefore, it is possible to use their own wealth as a parameter of the other player's budget.

There are a couple of additional good practices to eliminate some more excessive degrees of freedom in the model. For instance, find clever ways to structure the game such that the number of dimensions decrease by using additional feasibility rules[55]. Additionally, at each iteration the feasible states are recalculated. Unfortunately, it implies in distinct dimensions of the policy vector $\sigma$ for each iteration[56].

#### 5.3.5 Solution

It is possible to solve the agent's problem by iterating the policy function[57]. However, the optimum solution for one player, may not be the best for the other. In this sense, it is necessary to find the pair of strategies that solve the problem for both players[58].

The solution used here starts by finding the optimal policy $\sigma_i$ given that $j$ plays always a greed strategy. After, the problem is solved for $\sigma_j$ using the $\sigma_i$ from the previous result. By the definition of a MPE given by Maskin and Tirole (1988), the equilibrium is settled when both players observe the other player's best strategy $\sigma$ but they do not change their strategies. Most importantly, if strategy start alternating, then the solution does not converge and the game has no equilibrium. The complete code containing the solution method and the results used in this work is available at: https://github.com/caxaxa/Corruption_Game/blob/main/Corruption_game_leniency_and_non_trial_resolutions.ipynb.

### 5.4 Results and Equilibria XX PRELIMINARY XX

The results can be summarized in the best strategies or policy vectors $\sigma_i$ that both players play to maximize their own objectives. These, policies depend on

[54] I also conveniently implies that the $\Omega$ matrix is not sparse. In other words, all rows in the $\Omega$ sum up to one. Which makes the Markov chain easier to analyse. Consequently, it is easier to avoid problems such as reducibility in the Markov chain

[55] The set of all feasibility rules is available in the code

[56] If a decision rule is not in *sigma*, then the algorithm chooses a greedy decision.

[57] Putterman (2005) provides four methods to find the solution to discrete dynamic problems. Namely, value iteration, policy iteration, modified policy iteration and linear programming. Nonetheless, the routines used in this article can be found in Quantecon.org (Available at: https://python-advanced.quantecon.org/discrete_dp.html).

[58] Importantly, the agents must separately maximize their own objective functions given their own rewards. So, it is not enough to maximize a global objective function, since this solution would resemble a monopolistic solution rather then a Markov Perfect equilibrium.

every parameter in the game. Therefore, there is one equilibrium (if it converges) for each combination of parameters. So, in order to systematically analyse the numerical results. First, it is necessary to choose a benchmark and analyse it. Then, it is possible to analyse the vicinity of the benchmark in order to observe how changes in variables affect the equilibrium.

### 5.4.1 The Benchmark

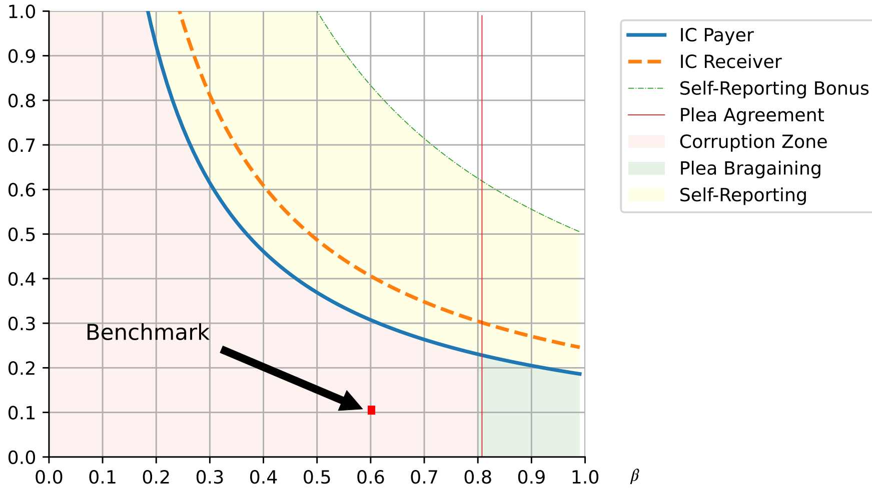
Let the benchmark be an arbitrary point in which players are expected to choose corruption, and not self-report or plea guilty. Importantly, the bargain power of the parties is set to be equal. This is done by choosing a the median point in the interval between the lowest bribe accepted for the payer (4) and the receiver (5). The Table 1 shows the chosen parameters for this case.

Table 1: Game Parameters

| Var | Value | Meaning |
|---|---|---|
| | | Bribery Parameters |
| $a$ | 5 | Benefit from corruption |
| $b$ | 3 | Bribe |
| $c_b$ | 1 | Cost of corruption |
| $f$ | 5 | Fine |
| | | Time Discounts |
| $\gamma$ | 0.975 | Time discount |
| $i_r$ | 0.05 | Interest Rate |
| | | Leniency Rules |
| $R$ | 0 | Unilateral reporting before detection |
| $r$ | 0.5 | Simultaneous reporting before detection |
| $P$ | 0.6 | Unilateral pleading guilty |
| $p$ | 0.8 | Simultaneous pleading guilty |
| | | Leniency Rules |
| $\alpha$ | 0.1 | Probability of detection |
| $\beta$ | 0.6 | Probability of conviction |

The Figure 7 shows the profitable areas for corruption in and the indifference curves.

Figure 7: The Benchmark Indifference Curves

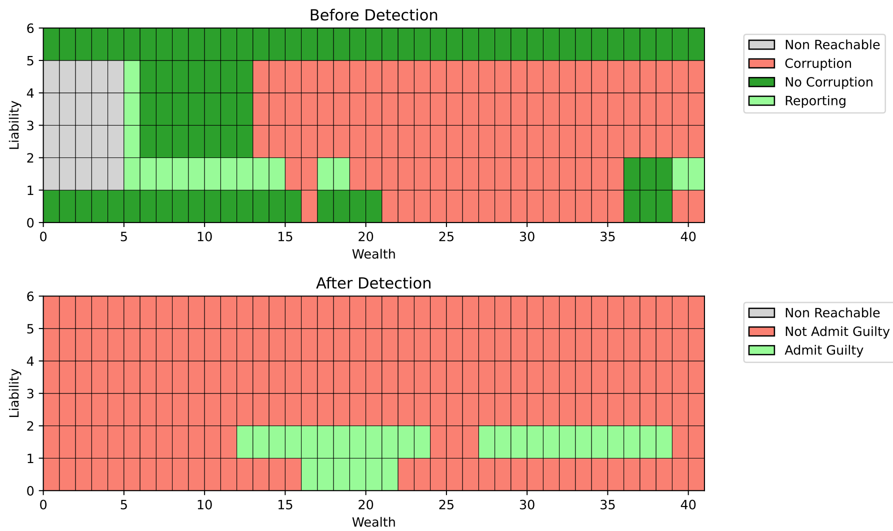



**Individual Strategies**

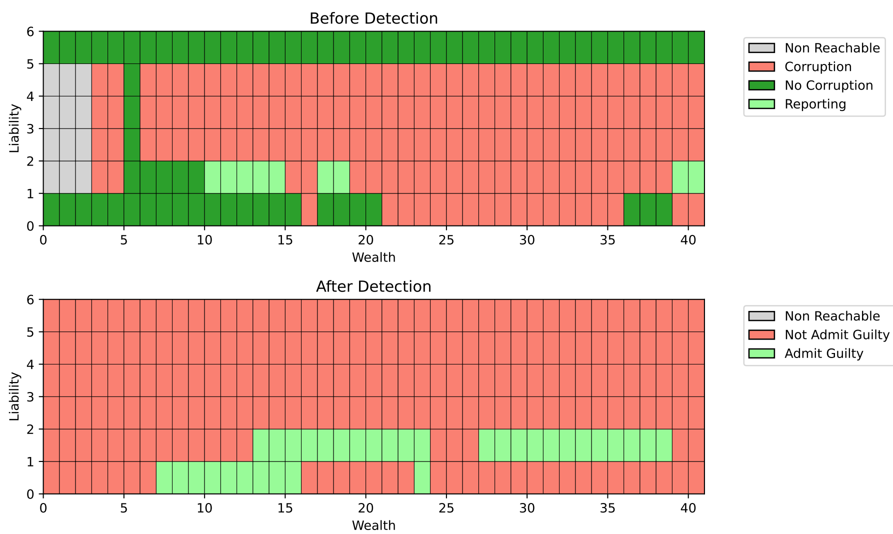
The players' optimal policy consists in a rule that tells the players how to play at each different point in the game. It is possible to visualize the best policy given the parameters in Table 1 and the agent's current wealth and judicial liability. The Figure 8 shows all the combinations of wealth and liability and the player's decisions before and after detection for the parameters in Table 1.

Figure 8: Players' Optimal Policy

(a) Payer's Policy




(b) Receiver's Policy



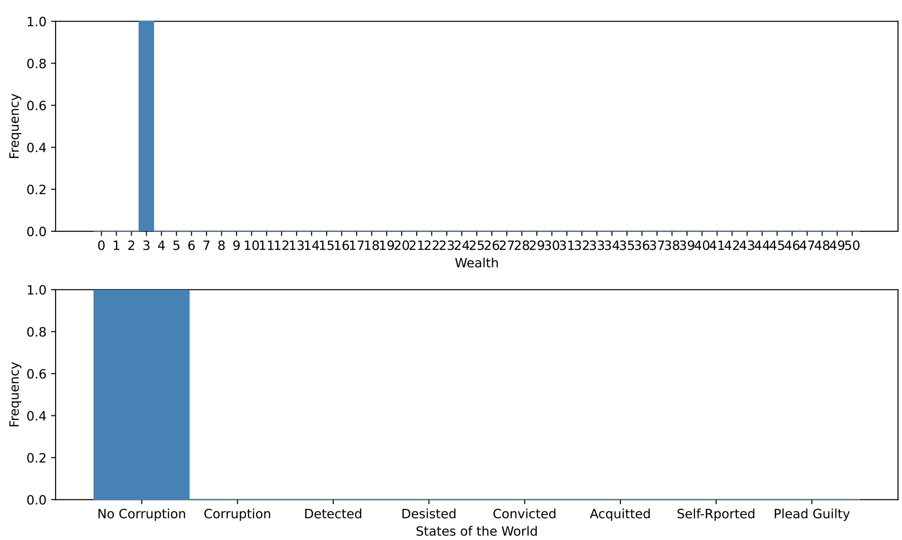

The first takeaway from the players' optimum policy is that the they are different between agents. Note that, even that Figure 7 shows that corruption would be preferred at that point, there are combinations of wealth and liability that would make the agents prefer not to enter in corruption. Moreover, Figure

7 also shows that agents would never prefer to plea guilty if detected in that point. However, since the players do not observe the other player's budget[59], they might be reported if they fail to assert the other player's wealth level. Lastly, there are some combinations that would make players self-report before being detected, even when it is not profitable.

It is necessary to carefully analyse the action choices near the beginning and the end of the grid. In these extreme points, agents may see themselves in special situations as pointed out in Section 5.3.2. Notably, player's choices at the beginning of the grid were expected to be more risky, since the fines can go only up to the player's maximum wealth. However, agents prefer not to enter in corruption in that region. On the other hand, the end of the grid is more complex to analyse. In these points agents cannot have more wealth than $\overline{W}$, so they might choose to save less, or consume more. It is hard to say if it would change their decision towards entering in corruption. In this case, for most points, if agents are not in corruption (liability = 0) they do not enter in corruption. While, if they are liable for one crime in the past, they choose to collaborate and if they are liable for more crimes, than, they choose not to desist.

One of the important characteristics from the Markov Perfect Equilibrium is that it is stationary. In other words, the set of optimal strategies drive the system to an stable equilibrium. The Figure 9 shows the stationary equilibrium.

Figure 9: Players' Optimal Policy




[59]By construction, they estimate the other player's wealth based on their own. In a complete game, it is possible to make the decision with players perfectly observing the other players budget set. However, it takes an entire new set of states, with length $\overline{W}$. Consequently, it requires a lot more computational power.

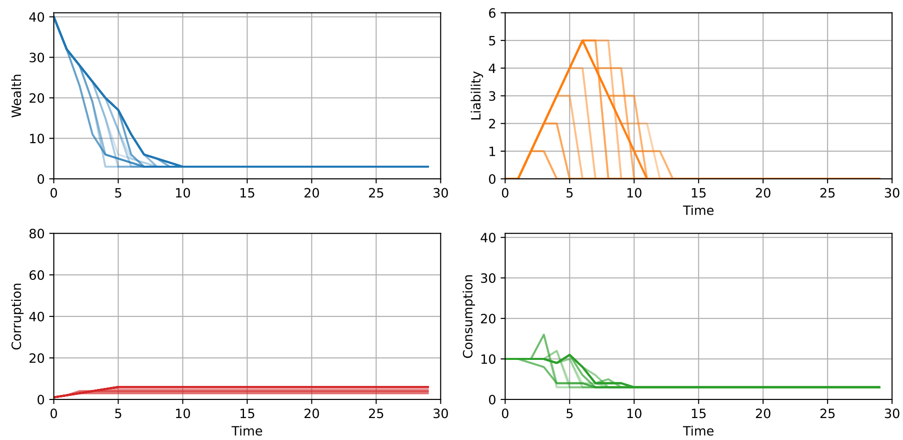
For the benchmark, the steady state equilibrium is the same for both players[60]. Note that, by construction, in order to avoid reducibility of the Markov Chain in a situation where agents would have no funds to pay a bribe or to save. The model gives the players a '*living wage*', so they will always have at least an income equal to the value of the bribe. In this case, the agents opted to live with the living wage, and consume everything at each state.

Notably, even if there is no corruption in the steady state, it does not mean that there wouldn't be corruption in the path to the steady state. It is possible to observe how the strategies lead to the steady state over time. The Figure 10 shows the path to the steady state.

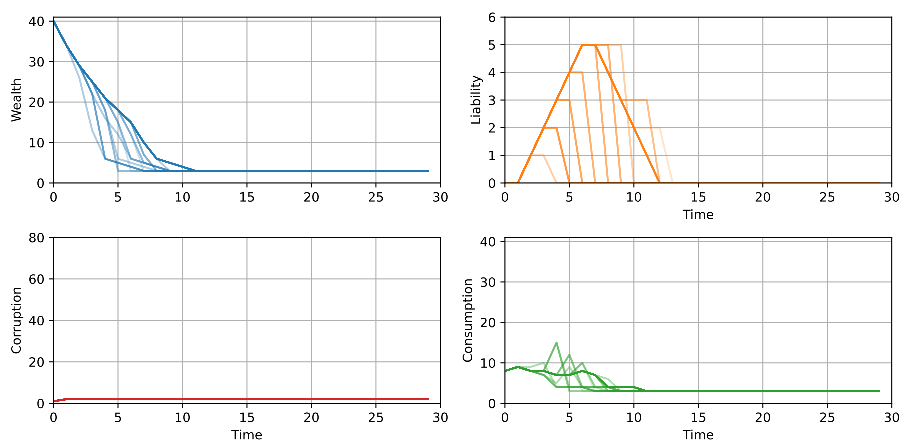
[60]It is not always the case

Figure 10: Path to Steady State

(a) Payer's Path




(b) Receiver's Path



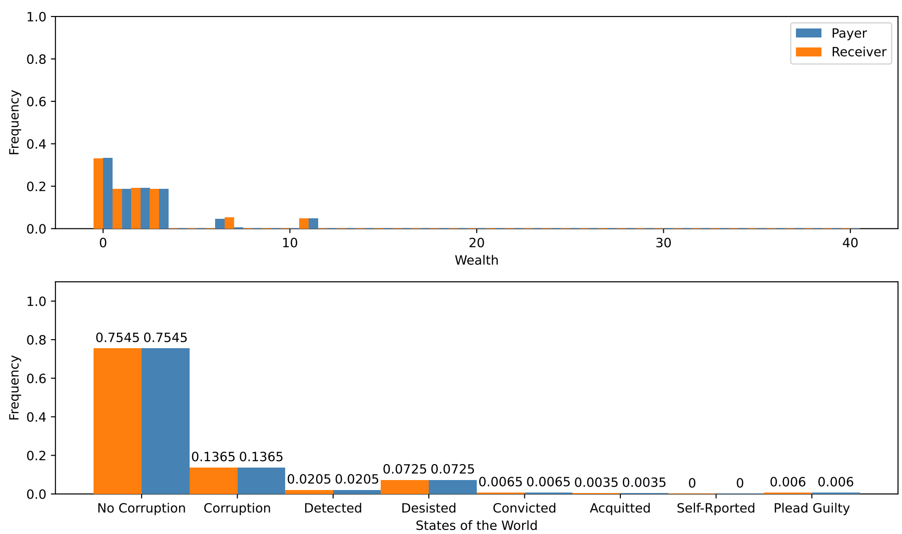

The Figure 10 shows how the agents alone would play this game for 30 periods. It also shows 50 stochastic outcomes from the decisions. Therefore, in some cases, agents are detected and in others they succeed. The multiple paths are superposed so the thicker the line, the more probable is the outcome. It clearly shows that, when agents start with the maximum wealth $\overline{W}$ and then play the game, they choose some corruption in the way, but eventually they consume all the wealth and stay in the steady state indefinitely.

Note that Figure 10 shows the strategy of the players if they play alone and they accurately predict the other player's decision. Once again, this game has incomplete information about the other players budget. Therefore, the results observed if we put players to play against each other might differ substantially. Below, the results of the interaction between players is exposed as the distribution of decisions in the society.

### Corruption Distribution in the Society

One of the goals from this chapter is to understand the distribution of the unobservable corruption in the society as the parameters from the game change. In order to obtain such distribution, it is possible to make both players play their best strategy against each other and sample from this interaction. To avoid the unwanted weird decisions on the extremes of the grids. Let the players' wealth be normally sampled from the wealth distribution, with mean 20 and standard deviation 4. So the majority of the sample is going to be centred in the middle of the grid. Also, the players are paired from a situation of '*no corruption*' and then interact for 5 rounds. After this, we keep the results and proceed with another sampling for 200 rounds. The Figure 11 shows the distribution of states on the society.

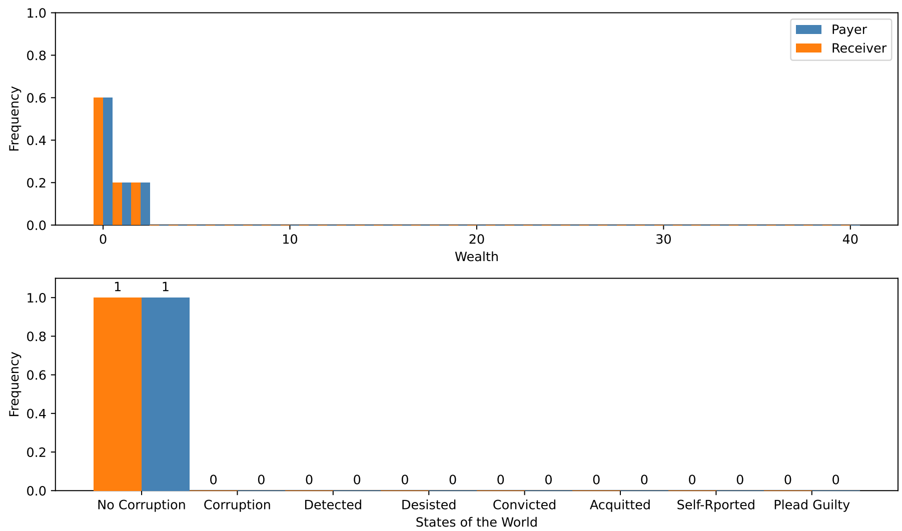
Figure 11: Distribution Sampling



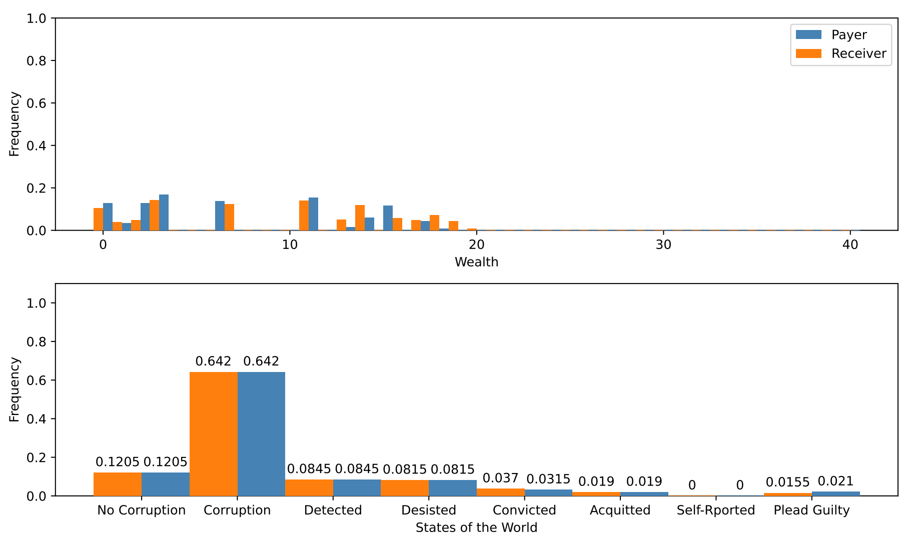

Note that, differently from the steady states, corruption is observed. One other interesting result may arise from testing a slightly different level of initial wealth. In other words, we can test the perseverance of corruption in a poorer or richer environment.

Figure 12: Distribution Sampling from Unevenly Wealthy Societies

(a) Poorer Society




(b) Richer Society



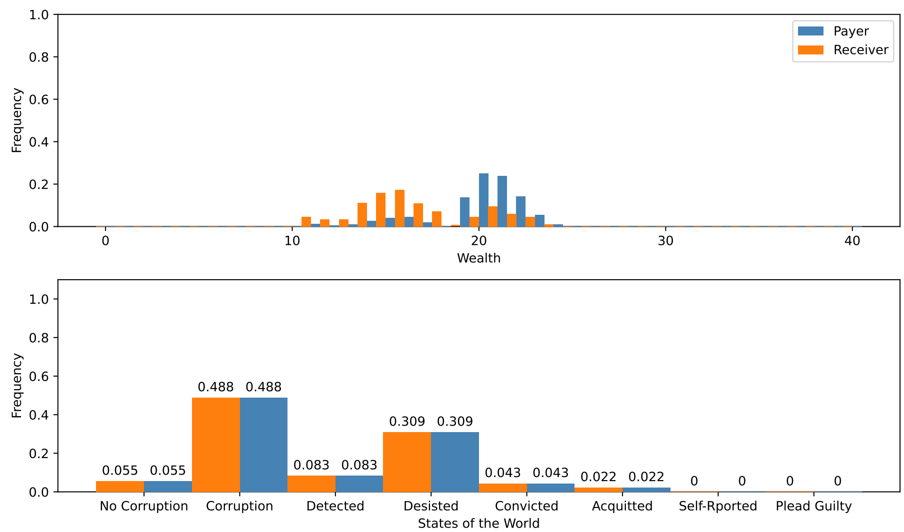

The Figure 12 shows that less wealthy society, tend not to choose corruption. While richer societies would. This results seems to be counter-intuitive. But it does shows that collusive corruption is a type of corruption that affects richer countries, while more extortive corruptions tend to be bigger in smaller countries

(Søreide, 2018). The results follow directly from the optimal policies shown in Figure 8.

### 5.5 The Effectiveness of The Sanction Reductions

In the benchmark, the players receive lenient fine reductions. Notably, they pay no fines if they unilaterally report before being detected $R = 0$, or they pay half of the fine if agents simultaneously report $r = 0.5$. Lastly, if they unilaterally admit guilty they pay half od the fine $P = 0.5$ but if they do it simultaneously they receive only 20% discount each $p = 0.8$. The Figure 13 shows the distribution of corruption in the society in the case in which there are no fine reductions ($R = 1$, $r = 1$, $P = 1$ and $r = 1$).

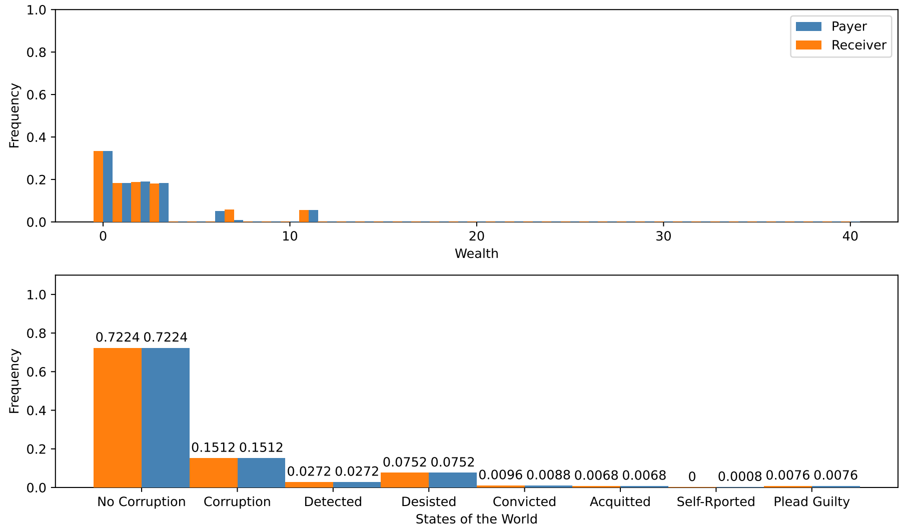
Figure 13: Distribution of States without Sanction Reductions




Now it is possible to answer if sanction reductions deter corruption. For this, the lenient sanction reduction case in Figure 11 is compared against the case without sanction reductions in Figure 13. Note that, corruption falls 72% when the lenient reductions are applied in this context. Furthermore, the case with mild (Moderate) sanction reductions ($R = 0.5$, $r = 0.75$, $P = 0.75$ and $p = 0.9$) also shows a reduction in corruption of roughly 10% from the case without it [61]

[61] This result is in line with Spagnolo (2005) where the author discuss the extensions of the results from Motta and Polo (2003) which claim that the 'First come first serve' is the only optimal.

Figure 14: Distribution of States with Moderate Sanction Reductions




Note that the difference between the more lenient and the moderate sanction reduction is small [62]. Therefore, any policy focused in efficiency, or in revenue from combating corruption should consider the intensity of the corruption reduction. However, this result is sufficient to show that sanction reductions are effective against corruption.

## 5.6 The Effects of a Policy Shock

From the analysis of the model, it is possible to better understand the decision making of the agents. Notably, the very same model can be used to predict the path of the unobservable level of corruption given observable inputs. More specifically, the model connects the amount of states in which there is corruption with the number of times that corruption is detected. Be it by being reported by agents or randomly detected by the authorities. Therefore, it is possible to predict how the changes on the real unobservable level of corruption given the changes observed in the detection of corruption.

As an example, let the *Regime 0* be a hypothetical scenario where there are no sanction reductions from collaborators. The policies are resumed in Table **??**. More specifically, *Regime one* represents a new scenario where sanction reductions are more lenient. Whereas *Regime 2* shows a scenario where sanction reductions do not change, and corruption becomes harder to detect and convict. Furthermore, *Regime 3* shows a scenario where not even sanction reductions

[62]Although statistically significant XXXXX Place the statistic here n = 500 var = ?? XXXX.)

are more lenient, but corruption is more detected than before. Lastly, *Regime 4* shows a case where corruption is even harder to detect than *Regime2*. The Table 2 resumes the regimes' features.

Table 2: Policy Shocks

| Regime | Sanction Reductions | Probability of Detection | Corruption Change |
|---|---|---|---|
| 0 | No Reductions | Average Detection | - |
| 1 | Lenient Reductions | Average Detection | Decreases |
| 2 | No Reductions | High Detection | Increases |
| 3 | Lenient Reductions | Low Detection | Decreases |
| 4 | No Reductions | Very High of Detection | Increases |

The Figure 15 shows path of the corruption detection after four distinct policy shocks.

Figure 15: Policy Shocks over Corruption and Detections

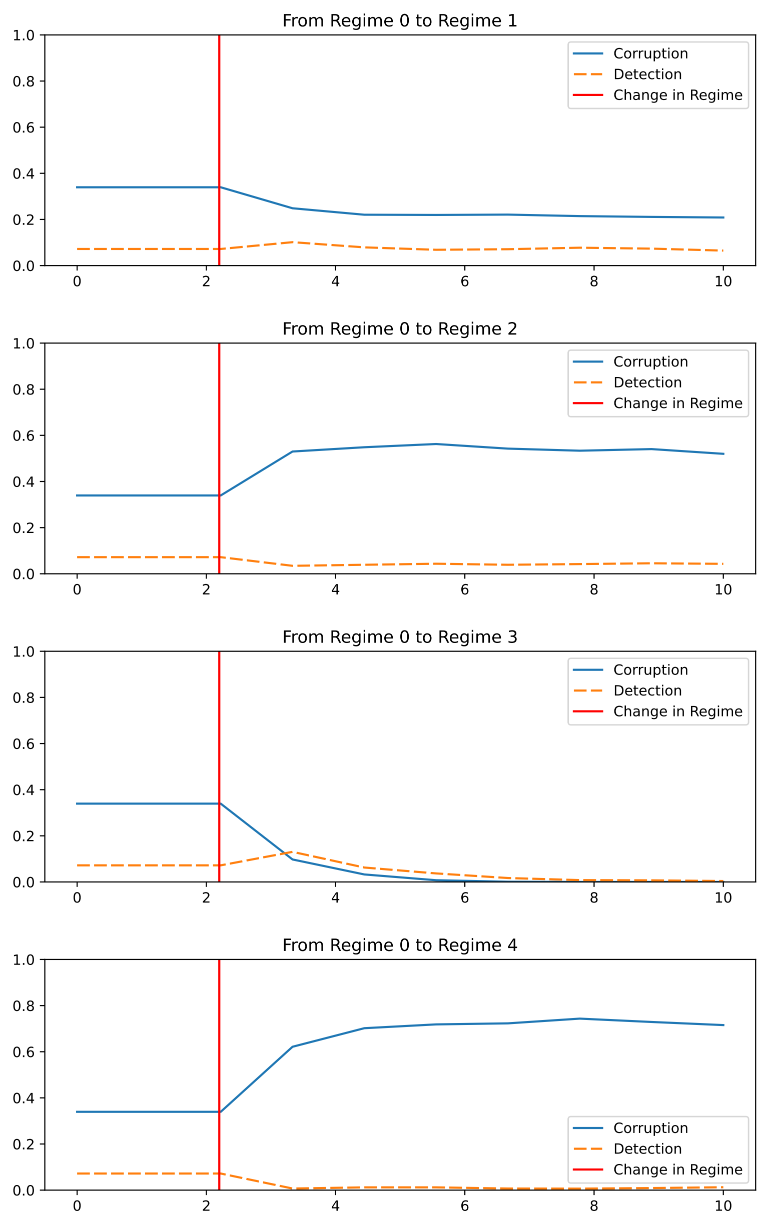



Figure 15 shows that the observable detections tends to increase in scenarios where corruption decreases. Conversely, it goes down when corruption increases. This behaviour may appear to be counter intuitive, since people would expect to find more detection of corruption in more corrupt countries. Note that, the path predicted here, states something about the observable detection of corruption over time in the same place. If two distinct places are compared against each other, the place with more corruption detection is most likely more corrupt.

The result presented here is in line with Miller (2009). However, it provides a fundamental analytical difference. Here, it is possible to decouple the effect from the sanction reductions from the effect of the probability of detection. More specifically, it is possible to show, how much corruption is deterred due to the effect of the sanction reduction policies. And how it is deterred due to the increase in the capacity of prosecution from the authorities. This distinction is important for the analysis of anti-corruption policies. For instance, in the Brazilian anti corruption reforms from 2013. Besides the mechanisms of sanction reduction and collaboration, several administrative efforts were put in place. Such as the use of task-forces to combat corruption[63].

# 6 Final Remarks

This work is in progress. The results found here are only preliminary and require further scrutiny. Therefore, the conclusions so far need to be taken with caution.

The major obstacle for delivering further results is the lack of researching infrastructure. The model is numerically intensive, so it requires a lot of computational power. Consequently, it would be necessary hours of supercomputing to run the code using higher dimensions.

It is important to note that, even if the results are inconclusive for the moment, they are part of the ingredients for the empirical part. Notably, for the empirical part it is necessary to have a model that connects observable data (such as corruption detection, interest rates and wealth) with unobservable (such as number of corruption crimes, or probability of detection).

## Conclusion

This work explores the game of corruption. It shows how agents make decisions towards entering into corruption. Additionally, it allows agents to report their misconduct and collaborate with authorities in exchange for judicial benefits. In order to predict the agent's decision, the model takes into account the tradeoff between entering into bribery investing in the asset market. Consequently, profitable bribes are necessary but not sufficient conditions for agents to enter in corruption. Furthermore, this model also analyses the dynamics of the game. Meaning that, agents make their best decisions taking into account their conditions in the present state, and how it affects their future in the game. Consequently, wealth and criminal liability are crucial for agents to make their decision.

Under the conditions of the model, it is possible to conclude that sanction reductions for collaborators reduce corruption. Furthermore, the more lenient

[63]The case of the Brazilian anti-corruption experience is explored in the empirical extension from this work

the reductions are, the less corruption there is.

Successful anti-corruption policy shocks are likely to increase corruption detection, at least at first. Detections may increase either because of new reporting or because of the increase in the prosecution productivity.

### Caveats

As stated before, the curse of dimensionality implies using lower dimensional grids. They notably oversimplify the space of possibilities in which players can be.

In order to save dimensions to avoid the curse of dimensionality. The players estimate the other player's wealth as being equal to their own. If the game is played too long with the same players, the will start to systematically mispredict each other's wealth level. Therefore, the game must not be repeated to many rounds with the same players.

### Further Studies

The model constructed here has many degrees of freedom. It is always possible to combine features for predicting distinct scenarios. In other words, the very same model can answer lots of different questions. Such as the impact of changes in interest rates, or in the bribery terms or even in agents risk aversion.

Lastly, from the estimated path of corruption and detection (Figure 15), it is possible to use real world data to calibrate the model to the observed detection to infer changes in the unobservable corruption. Notably, the Brazilian anti-corruption policies from 2013 are the perfect candidate. In that case, the new laws introduced leniency agreements and new plea bargaining conditions for corruption criminals. Therefore, it is possible to infer the behaviour of the level of corruption from the country's data.

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# A Appendix

This appendix contains a short description of all variables, states and actions that contained in the model.

## A.1 Constants $\theta$

Given the following constants $\theta$ for all periods $t$ and all agents $i \in (payer, receiver)$:
* Let $b$ be the fixed price of the bribes;
* Let $a$ be the gains from corruption to the payer;
* The cost from performing the corruption favour for the receiver is $c_b$ ;
* Let $f$ be the monetary fines for agents caught in corruption;
* $i_r$ is the interest rate;
* $\delta$ is the liability depreciation or prescription;
* $\gamma$ is the time discount;
* $\eta$ is the agents' risk aversion;
* $y_0$ is an autonomous non-financial income;
* Probability of detection is $\alpha$; and
* Probability of conviction is $\beta$.

### A.1.1 Leniency rules

Regarding the rules of leniency, they can be:
* $R$ sanction reduction for unilateral self-reporting before detection;
* $r$ sanction reduction for simultaneously self-reporting before detection;
* $P$ sanction reduction for unilateral self-reporting after detection; and
* $p$ sanction reduction for simultaneously self-reporting after detection.

### A.1.2 Leniency rules

A set of higher level notation to identify cost $\phi_i$ and benefits $\pi_i$ of corruption to each agent $i$:
* $\pi_{payer} = a$;
* $\pi_{receiver} = b$;
* $\phi_{payer} = b$; and
* $\phi_{receiver} = c_b$.

## A.2 A set of discrete actions or decisions (controls) $\mathbf{d}_{i,t}$

### A.2.1 Action of Entering in Corruption $d$

The decision regarding corruption $di, t$ can assume 3 different values:
* $not_{i,t}$ agents can choose to do nothing;
* $cor_{i,t}$ is the decision from player $i$ to pay/receive a bribe in time $t$; and
* $rep_{i,t}$ is the decision from player $i$ to self-report.

Agents decide how much they are going to consume at each time $c_t$. A complementary way to represent the consumpion decision, is the decision of how much to save $w$, or buy financial assets.

### A.2.2 Action of Consuming or Saving $c$ and $w$

* $w_{i,t}$ is the decision of how much to save, can go from 0 to $(W_t)$; and

* $c_{i,t}$ is the decision of how much to consume, can go from 0 to $(W_t)$.

Note that de decisions are complementary and equal to $W_{i_t} = c_{i,t} + w_{i,t} + \phi(d_{i,t})$.

## A.3 A set of discrete states $\mathbf{x}_{i,t}$

### A.3.1 State of the world $S$

* $S_{i,t}$ is the state of the world at time $t$;

* $s_{nc}$ is the state of the world where neither players are in corruption;

* $s_{cor}$ is the state of the world where both agents succeded in corruption;

* $s_{det,i}$ is the state of the world where the agent $i$ is detected for corruption;

* $s_{con,i}$ is the state of the world where the agent $i$ is convicted;

* $s_{acq,i}$ is the state of the world where the agent $i$ is acquitted;

* $s_{sr,i}$ is the state of the world where the agent $i$ self-reported before being detected;

* $s_{pg,i}$ is the state of the world where the agent $i$ plead guilty;

and

* The states of the world $(s_{nc}, s_{cor}, s_{det,i}, s_{col,i}, s_{con,i}, s_{acq,i}, s_{sr,i}, s_{pg,i}) \in S$;

### A.3.2 Wealth from Players $W$

* $W_{i,t}$ are the assets from player $i$ at time $t$. It goes from 0 to a maximum of $\overline{W}$;

### A.3.3 Liability from Players $L$

* $L_{i,t}$ is the wealth from player $i$ at time $t$. It goes from 0 to a maximum of $\overline{L}$;

## A.4 Discretization, State and Action Spaces

Let $\mathbf{x}$ be the entire state-space. If,

$W \in [0, ..., \bar{W}]$,

$L \in [0, ..., \bar{L}]$, and

$S \in [s_{nc}, s_{cor}, s_{des}, s_{det}, s_{con}, s_{acq}, s_{sr}, s_{pg}]$,

then,

$\mathbf{x} = [W, L, S]$ and has dimention $(3 \times (\bar{W} * \bar{L} * 8))$.

* Let $\mathbf{d}$ be the entire action or control-space. If,

$d \in [not_{i,t}, cor_{i,t}, rep_{i,t}]$ and

$w = [0, ..., \bar{W}]$, then

$\mathbf{d} = [w, d]$ and has dimension $(2 \times (\bar{W} * 3))$.