Corruption and Collaborations: The Role of Risk Aversion *

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Abstract

Previous works show that sanction reduction policies for collaborators may be effective to deter collusive corruption. In this sense, are risk averse agents more susceptible to this type of policy? To answer this question the present paper theoretically analyses the *ex-ante* decision making of players facing a corruption game. The game features sanction reduction policies and risk averse agents. Results show that if sanctions are reduced up to zero, risk averse agents are not deterred by this policy. Notably, the increase in risk aversion decreases corruption, but not because of sanction reductions. Conversely, if there are bonuses (not popular policy), risk averse agents are deterred from corruption because of the fear of being reported. Lastly, it is expected that more risk averse agents would admit guilt or self-report more often after the bribery is committed (*ex-post*). In summary, under reasonable sanction reduction policies, the increase of risk aversion of players do not change the *ex-ante* decision of entering in bribery, but they do change the agents' *ex-post* behaviour.

1 Introduction

Collusive corruption happens when two players, a bribe payer and a bribe receiver, agree in exploiting some rent or contract. Naturally, when the bribe receiver is a public officer, the society loses from this agreement. Notably, previous works on corruption represent this relation as a three tier principal agent problem (**?**). Where the principal (society) loses from an agreement between the agent (bribe payer) and the supervisor (bribe recipient).

Anti-bribery policies rely on many distinct features. One particularly well defended strategy against corruption is to reduce the sanctions for criminals

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who collaborate with authorities to unveil bribery schemes (**?**)[1]. The idea is to break the agreement between payer and receiver. Previous literature points to the effectiveness of the policy (**??**). However, models do not address the role of risk aversion.

Since the seminal work of **?**, risk aversion is known to play a role in criminal deterrence. Notably, risk averse agents would prefer to stay out of crime to avoid the uncertainty of being caught. Therefore, a more risk averse individual demands a bigger expected premium from the crime, deterring this potential criminal from committing less profitable or more riskier crimes. **?** introduce benefits of the sanction reduction policy in this context. In their argument, since risk averse agents would prefer the safety of reporting than facing detections. Then, the policy reduces the necessity from authorities for inspecting the whole society in search of crimes. Consequently, this would make criminal detection less costly. Hence, making the policy efficient against crimes.

In another similar strand of works, authors who study antitrust crimes agree on the positive effects of sanction reductions or leniencies to deter cartels (**???????**). Notably, some point out to the role of risk aversion on the agent's decision. The main idea is that it is expected that more risk averse agents would be more affected by the risk of being cheated upon (**?**). **?** shows in his riskiness analysis that such an effect might exist in cartels. The author argues that the introduction of the sanction reduction could create a 'rush to report' that could break the collusion. Furthermore, the author affirms that this effect could also be observed in other types of collusive crimes.

In all the cited literature it is not clear the link between crimes of corruption, sanction reduction and risk aversion. Since corruption might be different from other crimes. Mainly because it is a collusive crime. Therefore, there is a strategic behaviour between players. And, differently from antitrust offences, the cost for one player (the bribe) is the benefit for the other. Making the game even more dependent on the behaviour of players. Consequently, making the players' risk aversion especially important. So, are sanction reduction policies in crimes of corruption more effective against risk averse agents? The answer is not trivial, since risk averse agents would be less inclined to enter bribery, but also more likely to accept a plea deal or a leniency agreement. This could make corruption both less or more preferred for risk averse agents.

Considering first the *ex-ante* decision of entering in bribery for risk neutral agents. In this case, players must first decide upon the value of the bribe. They do it by perceiving the probabilities of being detected and convicted, by perfectly knowing the cost and the benefit from the corruption. Additionally, players know that authorities propose a reduction on the sanction to the player who comes forward and denounces the crime before or after being detected. If one player reports then the other party is convicted. After deciding the value of the bribe, then agents decide if they will enter in corruption, if they do, then decide if they will stick to it or defect. If everything is decided beforehand, then the

[1]This work focuses on the case where the players are the collaborators. Some whistle-blowing policies rely on third parties to come up and denounce bribery schemes. This relationship leads to a distinct game

only way that agents would be deterred from entering into a profitable bribery, is if they know that the other party will defect. Notably, this can only happen if the authorities pay something better than the corruption deal, or else, a bonus. Since these policies are practically unfeasible[2], sanction reduction policies have little impact on *ex-ante* corruption deterrence. But, they might be important on *ex-post* prosecution.

The conjecture posed above was already discussed in past works, such as in **?** and **?**. However, the role of risk aversion is not at first clear. Since risk averse agents may be deterred by the uncertainty regarding the behaviour of the other party. The model here shows that, if one player knows the other's risk aversion, or have similar one, then the result does not change. It could only change if there were any source of uncertainty on the player's type of risk aversion. Therefore, if agents know the other player's propensity to risk, sanction reduction policies do not affect the *ex-ante* decision to enter in corruption. Once again, *ex-post* behaviour changes, and risk averse agents would use more sanction reduction policies if the perceived probabilities change over time. However, the best way to model the *ex-post* behaviour is by making a dynamic model in which states change over time. Admittedly, this is a matter for further studies.

The paper is organized as follows. In Section 2, the basic framework and variables are presented in a simple game. In Section 3, the game is expanded to address the sanction reductions on the players decision. Lastly, in Section 4 the game is numerically solved for distinct levels of risk aversion. At the end, in Section 5 the main conclusions and caveats are presented.

2 Simple Corruption Game

The goal from this paper is to solve the corruption game with sanction reduction policies for risk averse collaborators. However, in order to smoothly introduce the the analytical framework, it is convenient to present a simpler game first. Here, risk neutral agents are convicted when detected and they cannot self-report. From this simple example is possible to set most of the game requirements. Additionally, it is also possible to draw the first conclusions regarding the role of the bribe and the decision to enter in corruption.

2.1 The Simple Game Setting

2.1.1 Players

The game here is played by two players[3]. A bribe payer and a bribe receiver $i \in [payer, receiver]$.

[2] It is very difficult to convince society that authorities should pay the criminals for information.

[3] As stated in Chapter 2, there might be differences between individuals and corporations. Whenever relevant, the differences are going to be explored.

2.1.2 Parameters

The relevant parameters for this first examples are the price of the bribe b, the advantage from corruption a, the cost for the *payer* to perform the bribe c, the sanctions s that players pay if detected by the authority and the perceived probability of being detected by the authorities α[4]. Note that, for this first example, the sanction s is given by a fine f[5] and everything that the agents have gained from corruption (a for the *payer* and b for the *receiver*).

It is possible to state that this a common criminal structure. Therefore, the conclusions made here can be applied to other types of crime. Notably, this work resembles a lot the structure used in leniency studies for anti-trust offences.

2.1.3 Timing and Information

The game has discrete time t. In t_0 agents observe a bribery opportunity and simultaneously decide if they want to enter in corruption or not. In t_1, the authorities play and randomly succeed or not in detecting the crime. If bribery is detected, both agents are convicted in that stage.

The game has perfect and complete information. Information is perfect because they know the values of the advantage from corruption a, the cost of the corruption for the receiver c and the value of the bribe b. This is not such a hard assumption, since they know the bribe b, because it is exactly paid from one to the other. The cost c and the advantage a appear to be less clear for the payer and receiver respectively. However, if bribes are negotiated, then one might infer the costs and benefits before deciding the price of the bribe. Furthermore, since players know exactly all the outcomes and possible actions, game is also said to be complete.

2.1.4 Pay-offs, Costs and Rewards

In order to draw the player's decision rules. It is possible to write the pay-offs in terms of costs ϕ and benefits π from corruption to each player i. Let then $\pi_{payer} = a$ and, $\pi_{receiver} = b$. Also, $\phi_{payer} = b$ and, $\phi_{receiver} = c$.

Each state S_t has an outcome or pay-off associated to it. Lets call the rewards y so that $y_{i,t} = y(S_t)$, or more specifically:

$$y_{i,t}(S_t) = \begin{cases} 0 \text{ if not colluding} \\ \pi_i \text{ if colluding} \\ -f \text{ if detected} \end{cases} \tag{1}$$

[4]Note that, the perceived probability of detection does not need to be equal to the actual probability of detected. Since players are all making the decision *ex-ante*. However, the perceived probability must be the same for both players.

[5]In many jurisdictions there are non-monetary sanctions both for individuals (imprisonment) and corporations (licence and activity impediments). As **?** points out, they are complementary and necessary to optimal deterrence. Here, for the moment, one can assume that agents are able to translate the non-monetary fines in the value of f. Nonetheless, the argument on non-monetary fines is further developed in the following sections.

2.1.5 Expected Returns

Expected return from corruption can be written the costs of corruption ϕ_t plus the expected value of all possible rewards in the next period y_{t+1} weighted by a time discount γ, or else:

$$E[y] = -\phi + \gamma \sum_{S} E\left[(\pi - s)\right],$$

Since s is equal to zero if not detected but equal to f if detected. Therefore, in this example, the cost of corruption $-\phi$ and the pay-offs from corruption y to each player i weighted by its expected probabilities can be written as

$$E[y_i] = -\phi_i + \gamma\left[(1-\alpha)\pi_i - \alpha f\right]. \tag{2}$$

2.2 Solution and Equilibria

There are multiple ways to solve this game. It is possible to solve the game by taking b as constant and solving for the best decision in a one-shot or repeated iterations. Note that in this setting, the timing of the decision is unimportant, since corruption only happens when both parties agree. On the other hand, it is possible to solve the game for the best bribe b. Where, b is the relevant action to be chosen b the bribe payer.

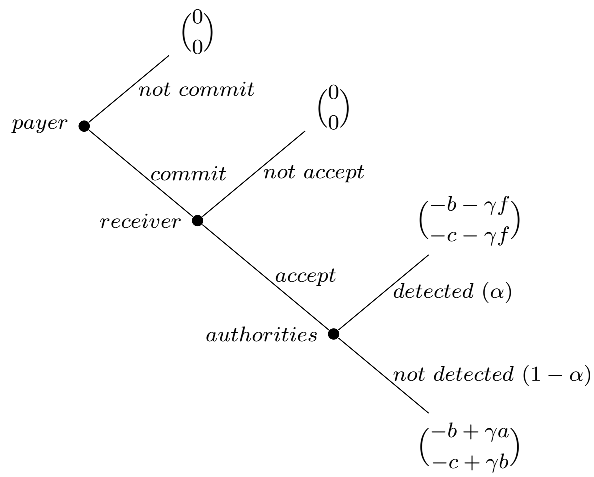
2.2.1 Solving for a Given Bribe

Assuming that the value of the bribe b is given. Or else, there is some expected bribe price which all players are price takers. Then agents must decide if they enter in corruption. In this sense, the solution is straightforward. If b lies in any profitable interval[6], then players choose to enter in bribe, otherwise they do not.

It is possible to solve this game using backward induction. The Figure 1 shows the decision tree and the pay-offs (above for the *payer* and below for the *receiver*) associated to each decision:

[6] For now, it is assumed that agents are neutral towards risk.

Figure 1: Simple Corruption Game Tree



Note that the *payer* only pays and the *receiver* only performs the corruption act after they simultaneously agree. Importantly, note that the *payer* pays or commits to pay a bribe b in the current period and the *receiver* gets it in the next period, with a value of γb. Therefore, one must assume that this bribe is held in the present and paid in the future.

The Figure 1 shows that agents are going to enter in bribery if.

$$E[y_i] = -\phi_i + \gamma\left[(1-\alpha)\pi_i - \alpha f\right] > 0.$$

More specifically, for the *payer*,

$$E[y_{payer}] = -b + \gamma\left[(1-\alpha)a - \alpha f\right] > 0,$$

rearranging,

$$b < \gamma\left[(1-\alpha)a - \alpha f\right] \tag{3}$$

And for the *receiver*,

$$E[y_{receiver}] = -c + \gamma\left[(1-\alpha)b - \alpha f\right] > 0,$$

rearranging,

$$b > \frac{(\gamma\alpha f + c)}{\gamma(1-\alpha)} \tag{4}$$

The equations (3) and (4) show that there will always be corruption as long as the 'price of the bribe' lies in the interval $\left(\gamma\left[(1-\alpha)a - \alpha f\right], \frac{(\gamma\alpha f+c)}{\gamma(1-\alpha)}\right)$. Importantly, it does not matter who proposes to enter in bribery first.

2.2.2 Finding the Endogenous Bribe

Notably, if agents are able to bargain, the chosen bribe is an endogenous function of the bribery parameters $b^*(a, c, \gamma, \alpha)$. If agents have different bargaining power, then the game design is naturally sequential. Because, one party must come up first and propose to enter in corruption. In this case, agents will choose the bribe that maximises their pay-offs. Or else, the bribe *payer* pays a bribe b^* which is slightly bigger than $\gamma\left[(1-\alpha)a - \alpha f\right]$ the *receiver* asks for a bribe b^* bigger enough then $\frac{(\gamma\alpha f + c)}{\gamma(1-\alpha)}$ [7]. However, if agents have similar bargaining power, then the chosen bribe b^* is given by the median point between the two intervals ((3) and (4)), or

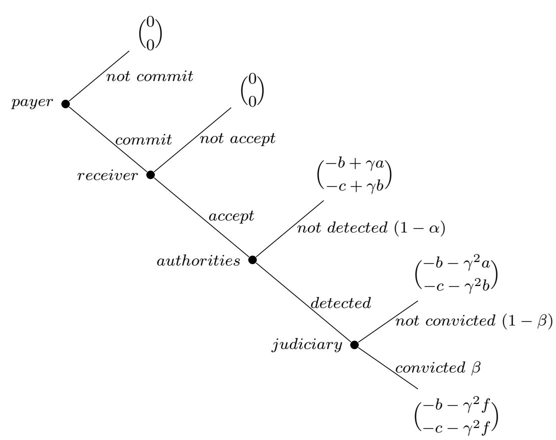
$$b^*(a, c, \gamma, \alpha) = \frac{\gamma\left[(1-\alpha)a - \alpha f\right] + \frac{(\gamma\alpha f + c)}{\gamma(1-\alpha)}}{2}. \tag{5}$$

Off course, one can change the bargaining power of agents by changing the denominator in the equation above. However, this analysis exceeds the scope of this analysis.

2.3 Setting with Trials

If agents face a trial after detection, then it is possible to draw the same game with one additional stage. The Figure 2 shows the relevant decision tree.

Figure 2: Simple Corruption Game Tree with Trials



It is possible to decompose the pay-offs by summing up all distinct possible pay-offs. First the agent pays the cost ϕ_i of entering in a bribery scheme. In the next period, there is a chance of α of being detected by the authorities, if

[7] Notably, if b is constant, its value is bigger than $E[y_i] = b$ for a risk averse agent.

detected a chance of β of being convicted. Therefore, the expected return in case of being fined is

$$\gamma(\alpha\left[\gamma(\beta f)\right],$$

or simply

$$\gamma^2\alpha\beta f.$$

There are now two possibilities of earning the corruption gains π_i. Either by not being detected in the next period

$$\gamma(1-\alpha)\pi_i.$$

Or by being detected and not convicted

$$\gamma\alpha\left[\gamma(1-\beta)\pi_i\right],$$

or rearranging,

$$\gamma^2(\alpha(1-\beta)\pi).$$

It is possible to sum the possibilities in which the agents succeed in corruption and earn π_i at the end of two periods as:

$$\gamma^2(1-\alpha)\pi_i+\gamma^2(\alpha(1-\beta)\pi),$$

or simply,

$$\gamma^2(1-\alpha\beta)\pi_i.$$

Therefore, it is possible to write the expected value as:

$$E[y_i]=-\phi_i+\gamma^2\left[(1-\alpha\beta)\pi_i-\alpha\beta f\right]. \tag{6}$$

In the absence of non-trial resolutions, it is possible to calculate the domain in which bribes are profitable, or else

$$E[y_i]=-\phi_i+\gamma^2\left[(1-\alpha\beta)\pi_i-\alpha\beta f\right]>0. \tag{7}$$

Note that corruption will occur only if corruption is profitable for both players. Given the above conditions, bribes are profitable if $\alpha<\frac{\pi_i-\phi_i}{\beta(\pi_i+f)}$ or $\beta<\frac{\pi_i-\phi_i}{\alpha(\pi_i+f)}$. Therefore, an increase in the fines f enhances the deterrent effect. On the opposite direction, an increase in $(\pi_i-\phi_i)$ makes agents more prone to bribery.

2.4 Finding the Endogenous Bribe

Letting the bribe be endogenously determined by the players bargaining process. Like in the example without sanction reduction policies, and given the equations (6) and (7), agents will inter in corruption if the proposed bribe is bigger then the expected return from corruption $E[y_i]$. Therefore, the *payer* commits to pay a bribe if

$$b < \gamma^2 \left[(1-\alpha\beta)a - \alpha\beta f\right] \tag{8}$$

And the *receiver* accepts it if,

$$b > \frac{(\gamma^2 \alpha\beta f + c)}{\gamma^2(1-\alpha\beta)} \tag{9}$$

Once again, if agents have equal bargaining power, the players surplus is equally divided and the chosen bribe b^* is the median of the interval between (8) and (9) or

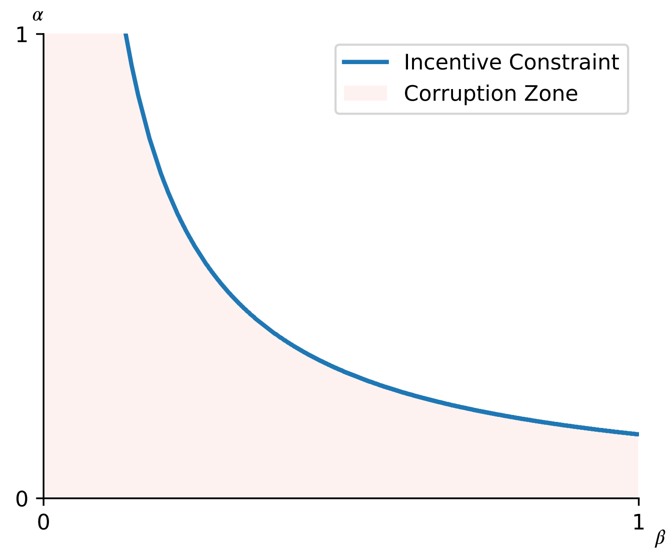
$$b^*(a, c, \beta, \alpha, \gamma) = \left[\frac{\gamma^2 \left[(1-\alpha\beta)a - \alpha\beta f\right] + \frac{(\gamma^2 \alpha\beta f + c)}{\gamma^2(1-\alpha\beta)}}{2} \right] \tag{10}$$

Note that if the denominator is bigger than 2, then the *receiver* has more bargaining power[8]. The opposite is true for the *payer*.

The solution given by b^* imposes that $E[y_{payer}] = E[y_{receiver}]$ or the equations (6) is equal to (7). The Figure 3 show all the combinations in which $E[y_{payer}] = E[y_{receiver}] = 0$. Therefore it shows the incentive threshold. In other words,at any point (combination of probability of detection and conviction) right of the curve corruption is not going to be preferred. Likewise, at left of it, corruption is profitable for both players.

[8]The marginal gain from corruption for each player given a change in b is different for each player. Notably $\frac{\delta E[y_{payer}]}{\delta b} = -1$ while $\frac{\delta E[y_{receiver}]}{\delta b} = \gamma^2(1-\alpha\beta)$. Therefore, optimal briberies might differ from the median.

Figure 3: Incentive Constraints for Profitable Bribes



Note that, if advantage from corruption a is bigger or the cost of corruption c is smaller, then there is more rent to be extracted from the bribery. Therefore, the curve shifts to the right. Consequently, it makes the area where corruption is profitable bigger, i.e. increasing corruption. The opposite effect is also true.

Importantly, as points are near the origin, the corruption becomes more profitable.

2.5 Final Remarks

Note that repetition is unimportant to this game equilibria. Here, even if the allowed actions depend on the player's state, they do not change as the stages of the game repeat. For instance, if an attempt to bribe fails and players are detected, in the next stage the decision rule is the same as if they had succeeded. Therefore, this specific design leads to the one-shot game equilibrium being equal to the same subgame perfect equilibrium (SPE) in each stage.

3 Corruption Game with Sanction Reductions to Collaborators

In this section the game is expanded to encompass the sanction reduction policies. For this work, the policies are summarized as the possibility of agreements between offenders and the judicial or prosecutorial authorities to avoid trials[9]. The agreements offer judicial benefits to the agents and can only happen if the they agree to self-report or admit guilt and disclose their misconduct. As a result, the other party is reported and convicted because of it.

It is important to remember that this work focuses in the *ex-ante* decisions. The *ex-post* benefits such as facilitation of prosecutions, costless judicial decisions or screening effects discussed as a consequence of the model.[10].

3.1 New Setting

In this new setting the current model must account for new features. First, there must be a decision to self-report or admit guilt as a choice for the players. Second, the detection and trial phases must be two separated stages. Lastly, the new stages must have a new set of pay-offs in y that account for the sanction reductions. Furthermore, the model must include a probability of being convicted β in a trial after being detected.

Most importantly, the model must now incorporate a set of distinct sanctions s for each type of conviction/agreement. Each agreement has a rule for reducing the fine f, such that. If agents unilaterally self-report before detections they receive a reduced fine of Rf, where $R < 1$. Moreover, if both *payer* and *receiver* self-report before being detected the fine reduction is rf, where $1 > r > R$. However, if the agents are detected, they can admit guilt, in this case, the fine reduction is lower, or Pf if for unilateral pleading guilty and pf when both players plea guilty simultaneously, where $1 > p > P$. In summary, the fine reductions are bigger (more lenient) for unilateral self-reporting than they are for simultaneous reporting. In the same way, fine reductions are more lenient to people that self-report before than after being detected. Since, admitting guilty is considered as an act of reporting after being detected. So, a collaborator is a player who either report before being detected or admit guilt after being detected.

3.1.1 Sanctions and Pay-offs

Given the new set of possible outcomes, there is a new set of rewards y from each possible outcome, such that

[9] This is an oversimplification of the institution. These policies show a variety of distinct features depending on the jurisdiction.

[10] Notable works on ex-post effects of agreements come from the plea bargaining literature, such as **?**, **?**, **?** and **?**.

$$y_{i,t}(S_t) = \begin{cases} 0 \text{ if not colluding} \\ \pi_i \text{ if colluding} \\ 0 \text{ if desisted} \\ f \text{ if convicted} \\ 0 \text{ if acquitted} \\ Rf \text{ if only the player } i \text{ self-report} \\ rf \text{ if both players self-report} \\ Pf \text{ if only the player } i \text{ admits guilty} \\ pf \text{ if both players admit guilt} \end{cases} . \tag{11}$$

For instance, if a *payer* reports a bribery to the authorities and the *receiver* does nothing, then the *payer* is convicted and pay a sanction $s = RF$ while the *receiver* is convicted with the full fine $s = f$. However, if the *receiver* simultaneously report, both agents receive a sanctions of $s = rf$. Note that the sanction reduction in the first case is smaller than in the latter. Likewise, agents can report after detection. In this case, the relation between P and p follows the same logic.

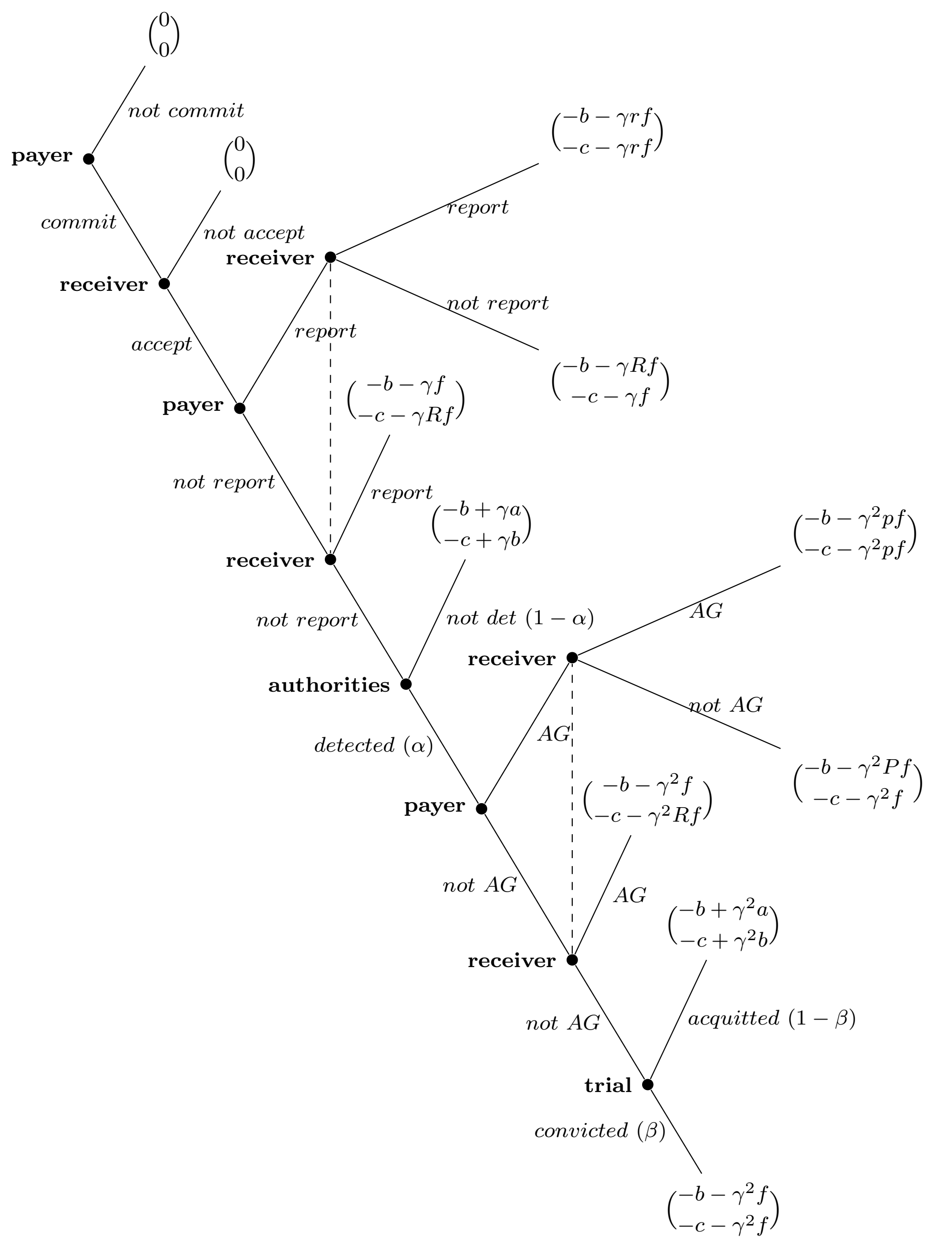
3.1.2 Choosing the Endogenous Bribe

Given the new outcomes from the game, the players decide the how much they should pay on the bribe. Notably, the options are to not entering in the bribery, entering and sticking to it or entering and reporting. Therefore, in the *ex-ante* decision, agents know for sure if they are going to use or not a sanction reduction policy, it is not stochastic. Consequently, it does not change the price of the bribe. Because the bribe is decided upon the expected return of the corruption deal. Whereas, the decision to use the sanction reductions is not stochastic.

3.2 Solution and Equilibria

In this example, the solution to the problem is more complex. However, it can be solved analytically through backward induction. The Figure 4 below shows the extensive game-tree:

Figure 4: Corruption Game with Non-Trial Resolutions



Notes: The tree shows the players in bold; the actions at the edges of the tree's children; the pay-offs are in the parenthesis where the ones above are for the payer and below for the receiver. The action AG stands for admitting guilty. Lastly, the dashed line indicates that the players in those nodes and their parent are playing simultaneously.

In this setting there are two stages of simultaneous decisions. Therefore, it is necessary to calculate the distinct subgame perfect Nash equilibria (SPNE)[11] that orients the players' strategy.

Lastly, note that if the bribery conditions lie in the corruption zone from Figure 3, it does not mean that there will be bribery, because players still have take the possibility of being reported into consideration. In this sense, differently from the past example. This game has different outputs if played in one-shot (one time) or repeatedly. This happens because, if players stop collaborating after a defection, then agents have to account for the benefit of a one time defection against repeated gains from cooperation.

3.2.1 One-Shot

Figure 4 shows that there are two nodes in which the players play simultaneously. In the first one, before detection, agents have to decide if they are going to report after paying the bribe. The matrix below shows the simultaneous decision payoffs for both players.

	Report	Not Report
Report	$-rf; -rf$	$-Rf; -f$
Not Report	$-f; -Rf$	$\gamma^2\left[(1-\alpha\beta)a - \alpha\beta f\right]; \gamma^2\left[(1-\alpha\beta)b - \alpha\beta f\right]$

Assuming that the agents do not want to be criminals, or else, not be in corruption. Then they would prefer a reduced fine Rf at least as good as the expected return from corruption at that node. Therefore, agents would only report if

$$-Rf \geq \gamma^2\left[(1-\alpha\beta)\pi_j - \alpha\beta f\right],$$

where the subscript j represents the other player. Or else, rearranging

$$-R_i^* \geq \frac{\gamma^2\left[(1-\alpha\beta)\pi_j - \alpha\beta f\right]}{f}. \tag{12}$$

where the R_i^* means the proportion of the reduction R in which agents prefer to self-report.

If the enforcement conditions α and β stay constant after the bribery and knowing that agents would have paid the costs of the bribe in the past node only if it was profitable for them. It would mean that reporting only happens if $R < 0$, or else, if agents gain a bonus from reporting. This is not common in most jurisdictions and this result is in line with past results using leniency to avoid cartels (**?**).

If agents are detected, they need to decide upon another set of actions. In this case,

[11]The subgames are Nash equilibrium since players build their strategies non-cooperatively.

	Admit Guilty	Not Admit
Admit Guilty	$-pf; -pf$	$-Pf; -f$
Not Admit	$-f; -Pf$	$\gamma\left[(1-\beta)a - \beta f\right]; \gamma\left[(1-\beta)b - \beta f\right]$

Notably, the agents admit guilt if,

$$-Pf \geq \gamma\left[(1-\beta)\pi_j - \beta f\right]$$

or rearranging,

$$-P_i^* \geq \frac{\gamma\left[(1-\beta)\pi_j - \beta f\right]}{f} \tag{13}$$

Differently from the case of reporting before detection, the sanction reduction here does not strictly requires a bonus. Since the expected return after detection is smaller than before detection and it is probably negative. In this sense, it is possible to have plea-bargains with a $1 > P^* > 0$.

Note that, the first restriction (7) is bigger than the subsequent (12) and (13). Or else, there will be plea-bargain only if the probabilities α and β, or the rules of leniency change over time (*ex-post*). Otherwise, the players would not enter in corruption at first (*ex-ante*)[12].

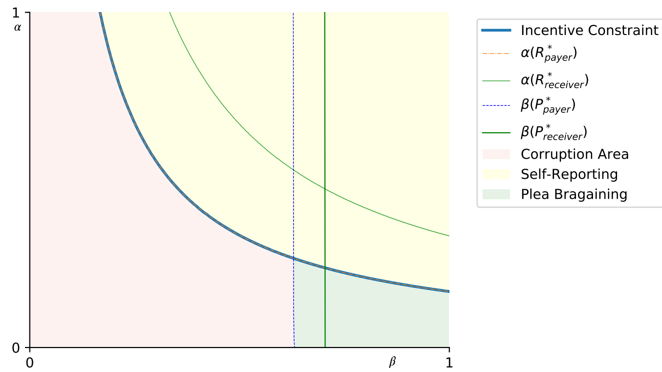
Assuming that agents choose a bribe $b = b^*$. Then the thresholds which make agents report R_i^* or admit guilt P_i^* depend on b^* for the *payer* and on a for the receiver[13]. Therefore, there is combination of probabilities $\alpha(R_i^*)$ and $\beta(R_i^*)$ which makes agents choose to enter in corruption, or avoid bribery with fear of reporting. By the same way, there is an $\alpha(P_i^*)$ and mainly a $\beta(P_i^*)$ which makes agents enter in corruption but admit guilt as soon as reported[14]The Figure 5 shows the and the equilibrium areas.

[12]However, there are situations in which agents can enter a game just to explore the payoffs. This would happen if the bonus are big enough, so agents enter in corruption just to receive it. However, this is ratter an unrealistic situation

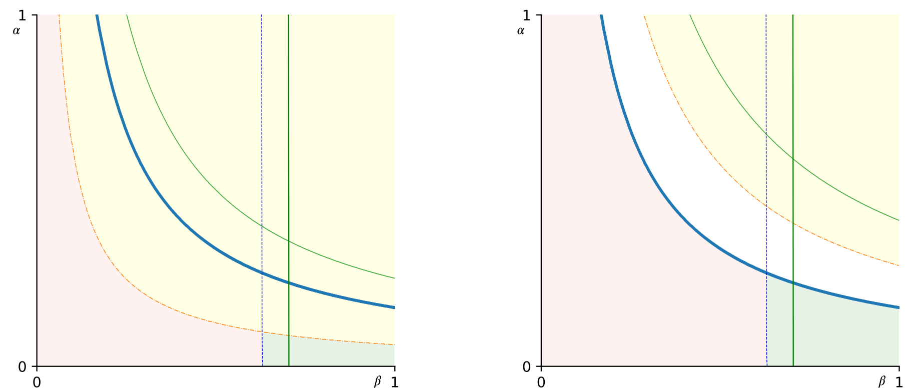
[13]Since equation (10) cannot be analytically solved for any α or β, then R^*_{payer} and P^*_{payer} have no analytical solution for α or β. Furthermore, P_i^* is going to be constant in α for the *receiver* but dependent on probabilities α and β for the *payer*

[14]Once again, in an extreme situation, agents enter the game to self-report,**?** refer to this strategy as, CR, or collude and report. Here, this hypothesis is not considered.

Figure 5: Incentive Constraints for Profitable Bribes with Sanction Reductions for Collaborators



(a) Full reduction ($R = 0.5$)



(b) Bonuses ($R = -0.1$)

(c) Moderate reduction ($R = 0.1$)

The interpretation from the self-reporting area and the plea bargaining area is straightforward. In the first case, for a given leniency policy R^*, if the combined probability of detection and conviction is higher than $\alpha(R^*)$ and $\beta(R^*)$, it means that it is more profitable (in expected terms) to self-report and get the bonus than it is to stay in the game. Likewise, if the agents are detected, then they will always admit guilt if the observed probability of conviction β is higher then the threshold $\beta(P^*)$. Lastly, the thresholds R_i^* and P_i^* move towards the origin as R and P are smaller (more lenient). Therefore, more lenient sanction reduction rules are the less corruption there is. In summary, the yellow area in Figure 5 shows all points in which corruption is deterred because of bonus for self-reporters. Whereas the green area shows all points in which agents would admit guilt if detected by the authorities.

The Figure 5 shows clearly that corruption is only deterred under bonuses.

Because in Subfigure 5a when agents receive full redemption of the fines, they are still not afraid of other players entering in the game just to self-report. This only happens when the other party receives a bonus for reporting, such as Subfigure 5b. In that case, the bribe payer will always self-report in the yellow area bellow the incentive constraint. Knowing this, the bribe receiver would not enter in corruption, and corruption would be deterred.

Lastly, it must be noted that the unilateral reductions rules R and P are the variables that disrupt the equilibrium[15]. However, the simultaneous reduction rule r and p are the ones actually paid by the reporting agents[16]. Therefore a policy recommendation that focus on deterrence from lowering the pay-offs or even if it targets revenue from prosecution would aim for a smaller (even negative) fine reduction for unilateral reporters. As well as a less lenient fine reduction for simultaneous collaborators[17].

3.2.2 Repeated

In the repeated example, some assumptions are necessary to find the game's set of equilibria. Firstly, suppose that the game is infinitely repeated[18].Secondly, suppose that agents, if they are reported, they learn that the other player is not trustworthy, then they never engage in corruption again (grim trigger)[19][20].Therefore, the ex-ante decision to engage in corruption have to account for trading one-time defection against an infinitely repeated return from corruption. This trade-off is expressed in the following matrix,

	Report	Not Report
Report	$rf; rf$	$Rf; f$
Not Report	$f; Rf$	$\dfrac{-b+\gamma^2\left[(1-\alpha\beta)a-\alpha\beta f\right]}{(1-\gamma)};\dfrac{-c+\gamma^2\left[(1-\alpha\beta)b-\alpha\beta f\right]}{(1-\gamma)}$

This time, the agents will enter in corruption if

$$-RF \leq \frac{-\phi_i+\gamma^2\left[(1-\alpha\beta)\pi_i-\alpha\beta f\right]}{(1-\gamma)}. \tag{14}$$

[15]This is a prisoner's dilemma.

[16]As long as $r<1$ and $p<1$, defection is going to be a Nash equilibrium. However, if $r>1$ and $p>1$, then there is no equilibrium to the game. Nonetheless, it does not mean that corruption is going to happen

[17]**?** found that the first best solution would imply that the reporters receive all the gains from the other player as bonus. It would lead to complete deterrence. However, no revenues.

[18]Finite game would imply that the last game is played as a one-shot. By backward induction, it would make all sub-games played like a one-shot game.

[19]Grim trigger can also be understood as a punishment for defection.

[20]Here agents only learn to punish defecting agents,they do not learn or update their preferences after each game. However, if players learn and update their preferences then, the the result of the game changes.This setting can only be examined in a dynamic in which states change over time.

Note that, the infinitely repeated return from corruption is always bigger then the one-time expected pay-off if the time discount $\gamma > 0$, or else

$$\frac{-\phi_i + \gamma^2 \left[(1-\alpha\beta)\pi_i - \alpha\beta f\right]}{(1-\gamma)} > -\phi_i + \gamma^2 \left[(1-\alpha\beta)\pi_i - \alpha\beta f\right].$$

Therefore, the fine reduction R^* must be more lenient[21] than it need to be in a one-shot game. Consequently, it is more difficult to combat corruption with leniencies in a repeated game. This results holds also when players are detected. The one-time plea bargain is punished by the grim-trigger, making the agents less prone to plea guilty.

4 Corruption game with Risk Aversion

In this section, the monetary pay-offs are substituted by a utility function to check if risk aversion plays a role on the model equilibrium.

Past studies on corruption or sanction reduction policies for crimes assume that agents are risk-neutral. **?** show that the sanction reduction policies are important to lower the cost of detection of crimes. In their analysis, if players are risk-averse, they would prefer the certainty of the reduced sanction rather than gambling on the probability of detections (and trials). This would lower the number of necessary investigations in the society, reducing the cost for authorities. They also argue that risk aversion could lower the optimal sanctions, and consequently the optimal monetary fines f.

It should be pointed out that one of the biggest drivers of self-reporting is the fear of being reported, as stated **?** about leniency policies for cartels. Notably, this effect must increase with uncertainty. In order to address the risk of being cheated on, **?** makes a riskiness analysis to answer this question. The author finds support that providing sanction reductions only to the first reporter creates a 'rush to report', which might increase the deterrent effect of the policies. Here, this hypothesis is not considered by this methodology. Because, given the relation in (12), (13) and (14), it is clear that the players' decision depend only on R and P and not on r and p. Which implies that it doesn't matter for the decision maker the size of the sanction reduction when they self-report or admit guilt simultaneously. Only unilateral defection disrupts the equilibrium.

Lastly, it is important to remember that this methodology only captures the ex-ante decisions of entering into corruption. *Ex-post* effects might consider a change in the variables for the next periods, which might change the conditions of the game. The correct way to address this type of decision is by making a dynamic game.

[21] more positive in this case.

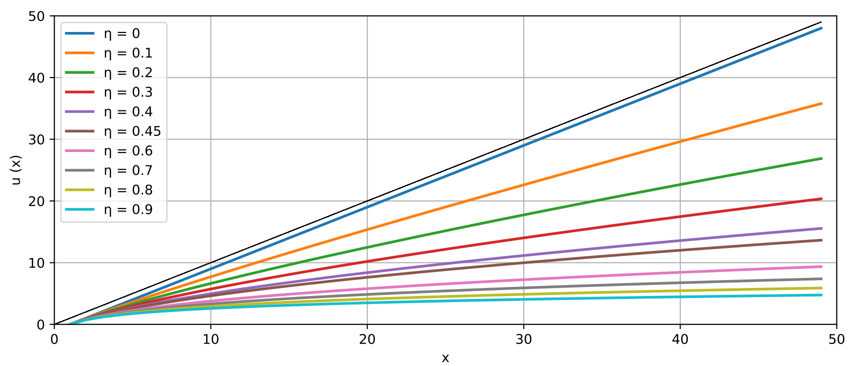
4.1 The Utility Function

In this section, it is assumed that the utility function $u(.)$ is an isoelastic utility function, or Constant Relative Risk Aversion (CRRA) function, such that

$$u(x) = \begin{cases} \frac{x^{(1-\eta)} - 1}{1-\eta} \text{ if } \eta \neq 1 \\ ln(y) \text{ if } \eta = 1 \end{cases}$$

Where η is the relative risk aversion parameter, such that $\eta > 0$ represents some degree of risk aversion. The Figure 6 shows the behaviour of the utility function as the risk aversion η increases.

Figure 6: The CRRA Utility Function



4.2 New Constraints

In order to measure the effect of risk aversion over *ex-ante* corruption deterrence. The previous constraints are recalculated applying the utility function $u(.)$ to all pay-offs. First, since the CRRA utility function misbehaves in the negative domain. It is necessary to transform the relevant inequalities into strictly positive pay-offs. Consequently, agents are endowed with the worst pay-off in the present period, so that their worst outcome is losing everything, or else, $u(0)$. Notably, the expected return from corruption $E[y_i]$, in condition (7) can now be represented as

$$- \underbrace{\left(\frac{f}{\gamma^2} + \phi\right)}_{\text{Initial endowment}} + \gamma^2 \left[\underbrace{(1-\alpha\beta)}_{\text{Prob. of success}} \overbrace{\left(\pi_i + f - \frac{\phi}{\gamma^2}\right)}^{\text{Return if succeed}} + \underbrace{\alpha\beta}_{\text{Prob. of failure}} \overbrace{(0)}^{\text{Return if failure}} \right] > 0. \tag{15}$$

Note that the expected value of $E[y_i]$ in (15) is still equal from the one in (7). Now, it is possible to apply the utility function $u(.)$ over the returns and compare it to the utility of the initial endowment, such that

$$E[u(y_i)] = \gamma^2 \left[(1-\alpha\beta)u\left(\pi_i + f - \frac{\phi}{\gamma^2}\right) - \alpha\beta u(0)\right] > u\left(\frac{f}{\gamma^2} + \phi\right). \quad (16)$$

Therefore, agents have to choose between the utility of the initial endowment today $u(\frac{f}{\gamma^2} + \phi)$ against the expected utility of entering in corruption.

By the same way, the conditions (12) and (13) can be written as

$$u\left(-R_i^* f + \frac{f}{\gamma^2}\right) \geq \gamma^2 \left[(1-\alpha\beta)u(\pi_j + f) - \alpha\beta u(0)\right] \quad (17)$$

and,

$$u\left(-P_i^* f + \frac{f}{\gamma}\right) \geq \gamma \left[(1-\beta)u(\pi_j + f) - \beta u(0)\right] \quad (18)$$

There are three important issues from this rearrangement. First note that, ϕ is not present at this stage, since it has been already spent. Therefore, as it is as 'sunkcost' it does not count as an initial endowment. Second, P and R are reductions of the sanction, not probabilities. Therefore, they are inside the utility function. Furthermore, since the CRRA utility function is not invariant to scaling[22], then it is not possible to isolate the R_i^* and P_i^*. Lastly, note also that b^* is a function of the pay-offs. So it is necessary to apply the utility function on these parameters also. Consequently, b^* is also a function of η.

4.3 Numerical Solution

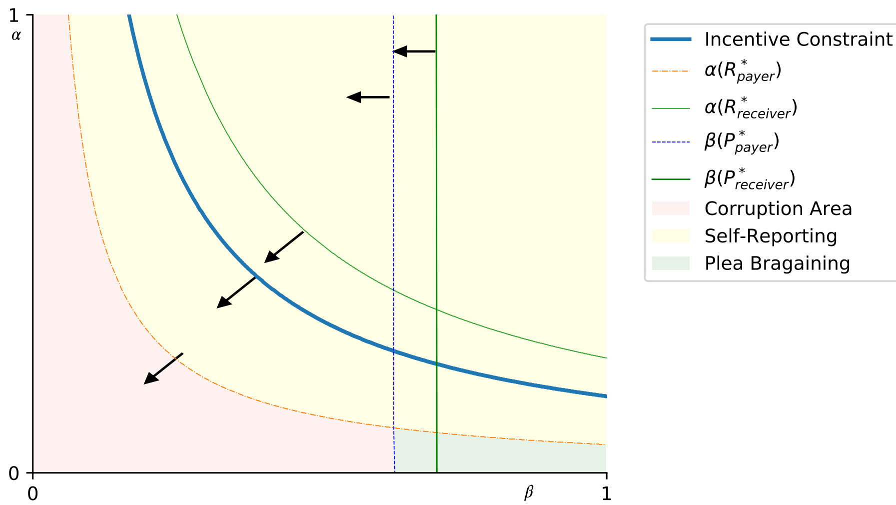
Given the complex structure of the utility function. It is not possible to analytically solve the conditions (16), (17) and (18) for the risk aversion η. Nonetheless, it is possible to find the numerical solutions for all relevant points. The reader can try the multiple arrangements of variables in an interactive application available at https://caxaxa.github.io/Interactive_Corruption_Game/.

From the numerical solutions it is possible to observe that an increase in the risk aversion η reduces the observed corruption. It happens both because the risky agreement from corruption is less preferred (change in the incentive constraint), or because more self-reporting is more expected (change in $\alpha(R_i^*)$). Moreover, more plea-agreements are also expected (changes in $\beta(P_i^*)$).

The arrows on the Figure 7 show the direction in which curves move when the risk aversion η increases. It is possible to see that the 'corruption zone' shrinks, while the 'self-reporting area' and 'plea bargain area' increase. Consequently, leading to a more deterrent setting.

[22] Preferences are not homothetic. Or else, the utility function is not homogeneous of degree 1, such that $u(zx) \neq zu(x)$, for z being a constant.

Figure 7: Incentive Constraints for Profitable Bribes with Sanction Reductions for Collaborators



There is one notable outcome from this setting. Note that, even though the self-reporting area increases as agents are more risk avers. It can only be bigger than the deterrent effect from the reduction in the incentive constraint curve if there are bonuses ($R < 0$). In other words, the most favourable setting for the bribe payer without bonuses is when sanctions reductions are maximum $R = 0$, costs for the other party are zero $c = 0$ and time discount is absent. Even in this situation, the constraint from self-reporting (17) is at most equal to the incentive constraint (16). This can be shown by substituting $\gamma = 0$, $c = 0$ and $R = 0$ in (17) or in (16) leads to the same expression

$$\gamma^2 \left[(1-\alpha\beta)u\left(\pi_i + f\right) - \alpha\beta u(0)\right] > u\left(\frac{f}{\gamma^2}\right).$$

In other notation, if

$$\lim \alpha(R_i^*)_{\gamma\to 0, R\to 0, c\to 0} = \alpha(E[y_i] = 0)$$

So, if there are no bonuses, as the agents are more averse to risk, then the uncertainty from entering in corruption itself is at most equal to the risk of being cheated upon. Therefore, the only way to increase the deterrent effects from the policies of sanctions reductions for risk averse players is to give them bonuses.

The result here is in line with past studies in which moderate 'leniency policies' or sanction reductions might not be effective to deter *ex-ante* criminal behaviour. Here, the only deterrent effect comes from giving bonuses to reporters. Whereas, allowing players to admit guilt only makes them report more after detection, but it does not deter crimes. Not even to enough risk-averse individuals.

5 Final Remarks

This paper relates risk aversion to a corruption game with sanction reduction policies. It builds a model of the game to analyse the relation between the relevant variables. The model has multiple equilibria, given its various degrees of freedom. The reader can check all possible configuration from the model by accessing this paper's application available at https://caxaxa.github.io/Interactive_Corruption_Game/.

5.1 Conclusions

Previous literature has already shown that moderate sanction reduction policies might not be efficient against corruption. This work does not care about the costs of the policies. Therefore, it considers only their efficacy.

In this regard, it is possible to conclude that sanction reduction policies would only deter corruption in a very unrealistic scenario. Admittedly, players are only deterred from corruption if they receive some extra bonus for reporting the other party before being detected by the authorities. In this scenario, the *ex-ante* best decision is not to enter into corruption, because they know that the other party would exploit the game.

Agents are also expected to admit guilt more if they are detected. However, this does not change their decision to enter into corruption. Agents would just enter in the game knowing that they would admit guilt if detected.

The propensity to risk does not change this conclusion. Although more risk averse agents are expected to practice less corruption. They do it because the crime is less preferable, not because they are more afraid of being reported.

If the environment changes, then players have to change their decision. In this *ex-post* scenario, agents respond to the sanction reduction policies. Notably, less lenient sanction reduction or more risk averse agents would self-report and plea bargain more. Importantly, it is preferable to have lower fine reductions for players that unilaterally report but less lenient reductions if they do it simultaneously.

5.2 Caveats

This work assumes rational behaviour from agents. Naturally, risk aversion might play a role under bounded rationality in the form of other types of aversion. Such as betrayal aversion, or loss aversion. Moreover, there might be all sources of uncertainty on a realistic game. If the game's information is imperfect then risk aversion might play a bigger role.

This model predicts *ex-ante* deterrent effects from sanction reductions. If the environment changes over time, then other strategies are going to be observed. For instance, if probabilities of detection or conviction change over time, there are areas in which agents would self-report, as **?** predicted. Note that the results hold under the assumption that agents are not learning or even updating any other relevant variable in their choices. A dynamic model, where states change over time is developed as a follow up from this one. This dynamic model may answer questions about *ex-post* criminal behaviour and it may also predict criminal activity through time.

References