## Modelling Domestic Corruption Deterrence Through Self-Reporting and Collaborations

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## 1 Profitability Constraint

The game arises when two agents, the government bureaucrat and private entrepreneur, decide whether to play or not a strategy of corruption (cooperation), constrained by probabilities of being detected and later convicted.

Given the above conditions, let:

B = Bureaucrat;

E = Entrepreneur;

 $\pi = \text{Advantage from corruption};$ 

b = Bribe; and

 $c = \text{Cost of the bureaucrat for generating } \pi$ ,

where  $\pi > 0$  is the gain of the entrepreneur from corruption and b is the gain of the bureaucrat, such that  $0 < b < \pi$ . Let the enforcement variables be:

 $\alpha = \text{probability of detection}; \text{ and }$ 

 $\beta$  = probability of conviction.

Finally, for i = B, E, let:

 $S_i = \text{Sanction}$ ; and

F = Fine.

For the moment, it is considered that  $S_i$  is given by the gains of each player with addition of F which is the same for the payer and the receiver. Consequently,  $S_E = \pi + F$  and  $S_B = b + F$ 

In order to derive the agent's decision rule, and for the sake of simplicity let,

 $p_k$  = probability that k happens for  $k = \pi, F, b$ ,  $p_F \equiv \alpha \beta$  and,

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 $p_b = p_\pi \equiv (1 - \alpha \beta).$ 

Under risk neutrality, the agents' utilities  $U_i$  are given by the the differences between the costs and the expected benefits for each agent:

$$U_E \equiv -b + (1 - \alpha\beta)\pi - \alpha\beta F = -b + p_{\pi} - \pi p_F F,$$

and

$$U_B \equiv -c + (1 - \alpha\beta)b - \alpha\beta F = -c + p_b b - p_F F.$$

Let us define some profitable bribery as being the fixed cost paid by agents which make their utilities at least greater than zero:

$$b > p_{\pi}\pi - p_F F, \tag{1}$$

and

$$c > p_b b - p_F F, (2)$$

If fine reductions are feasible and not exploitable, then corruption is performed if profitability conditions are met. In order to calculate them in a staged game with incomplete information ( $\gamma_i > 0$  and  $\omega_i > 0$ ) and distinct rules for fine reductions R and P, let:

$$p_{F_i} = f(\alpha, \beta, \gamma_E, \gamma_B, \omega_B, \omega_E, P, R, p, r), \ p_{F_E} \equiv \alpha \Gamma(\Omega \beta + (1 - \omega_B) \omega_E P + (1 - \omega_E) \omega_B + \omega_E \omega_B p) + (1 - \gamma_B) \gamma_E R + (1 - \gamma_E) \gamma_B + \gamma_E \gamma_B r, \ p_{F_B} \equiv \alpha \Gamma(\Omega \beta + (1 - \omega_B) \omega_E + (1 - \omega_E) \omega_B P + \omega_E \omega_B p) + (1 - \gamma_B) \gamma_E + (1 - \gamma_E) \gamma_B R + \gamma_E \gamma_B r,$$
 in the same way,

$$p_{\pi} = p_b \equiv \Gamma[(1 - \alpha) + \alpha\Omega(1 - \beta)].$$

Given that agents face the same problem of having a positive expected pay-off from collusion. Then, the constraints given by the positive probability of being reported may decrease expected pay-offs from corruption through bigger expected fines  $p_{F_i}F$  and lower probability of going unpunished  $p_b$  and  $p_{\pi}$  and receiving the advantage of corruption. Therefore, the decision rule for the entrepreneur can be expressed as:

$$-b + \pi \Gamma[(1-\alpha) + \alpha \Omega(1-\beta)] - F(\alpha \Gamma(\Omega\beta + (1-\omega_B)\omega_E P + (1-\omega_E)\omega_B + \omega_E \omega_B p) + (1-\gamma_B)\gamma_E R + (1-\gamma_E)\gamma_B + \gamma_E \gamma_B r) > 0,$$

and for the bureaucrat,

$$-c + b\Gamma[(1-\alpha) + \alpha\Omega(1-\beta) - F(\alpha\Gamma(\Omega\beta + (1-\omega_B)\omega_E + (1-\omega_E)\omega_BP + \omega_E\omega_Bp) + (1-\gamma_B)\gamma_E + (1-\gamma_E)\gamma_BR + \gamma_E\gamma_Br) > 0.$$

Computing the indifference curves from the above conditions and comparing with the previous calculated ones, it is clear to see the deterrent effect of the combined policies when  $\gamma_i > 0$  and  $\omega_i > 0$ . The deterrence effect is given by the reduced set of combinations of the public enforcements in which corruption would be feasible.

In order to address the effect of leniencies over corruption deterrence, let  $\theta_k$  be the linear coefficient that relates the k rules of fine reduction R and

P to the probability of self-reporting  $\gamma_i$  and  $\omega_i$ , in which  $f'(\theta_k) < 0$ , and  $f''(\theta_k) = 0$  such that,

$$\gamma_i = \gamma_{i0} - \theta_R R$$

and,

$$\omega_i = \omega_{i0} - \theta_P P$$
.

The same can be applied to the effect of the leniencies post detection over the probability to self-report after detection<sup>1</sup>.

## 2 Hold-Up Constraint

If fine reductions R or P (or their discounted values) are lower than the probabilities of being fined  $p_{F_i}$ , then the first player can credibly threat the second player to perform the favour. In other words, if fine reductions are bigger than the probability of being fined, than the 'hold up' problem constrains the corruption act. Therefore, the hold-up constraint can be expressed as:

$$min[R(1-\delta), P(1-\delta)] > p_{F_i}.$$

## 3 Exploitation Constraint

If bonuses are sufficiently big  $(R < -\frac{-b + (1-\alpha\beta)\pi - \alpha\beta F}{F}$  for the entrepreneur or  $R < -\frac{-c + (1-\alpha\beta)b - \alpha\beta F}{F}$  for the bureaucrat), agents would enter the game only to exploit it. In summary, it is possible to conclude that self-reporting is non-exploitable if:

$$R < \max \left[ -\frac{-b + (1 - \alpha\beta)\pi - \alpha\beta F}{F}; -\frac{-c + (1 - \alpha\beta)b - \alpha\beta F}{F} \right],$$

in other words, with repetition, the exploitation constraint can be written in terms of P or P in the same way by:

$$min[RF(1-\delta), PF(1-\delta)] > min[-b + p_{\pi}\pi - p_{F_E}F, -c + p_bb - p_{F_B}F]$$

<sup>&</sup>lt;sup>1</sup>This coding uses a logistical function instead of the linear bounded function for  $\gamma_i(\gamma_{i_0}, R, \theta_R)$  and  $\omega_i(\omega_{i_0}, P, \theta_P)$