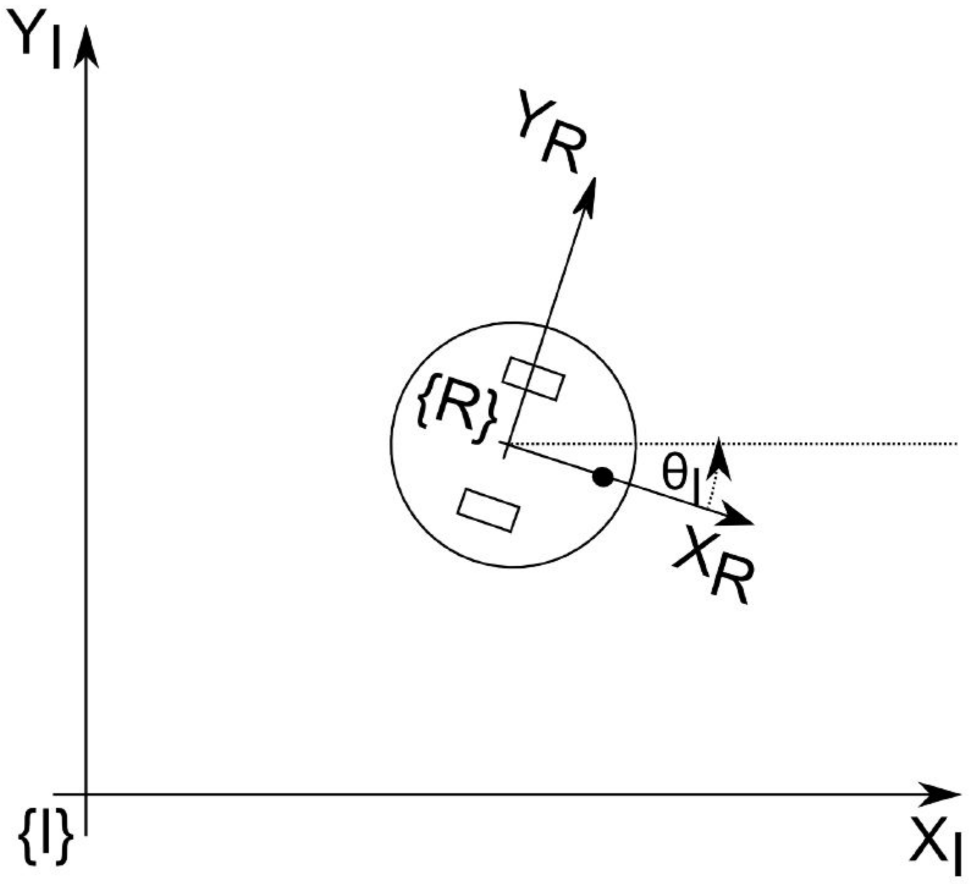
CSCI 3302
Introduction to Robotics

Inverse Kinematics

3. Forward Kinematics + Odometry



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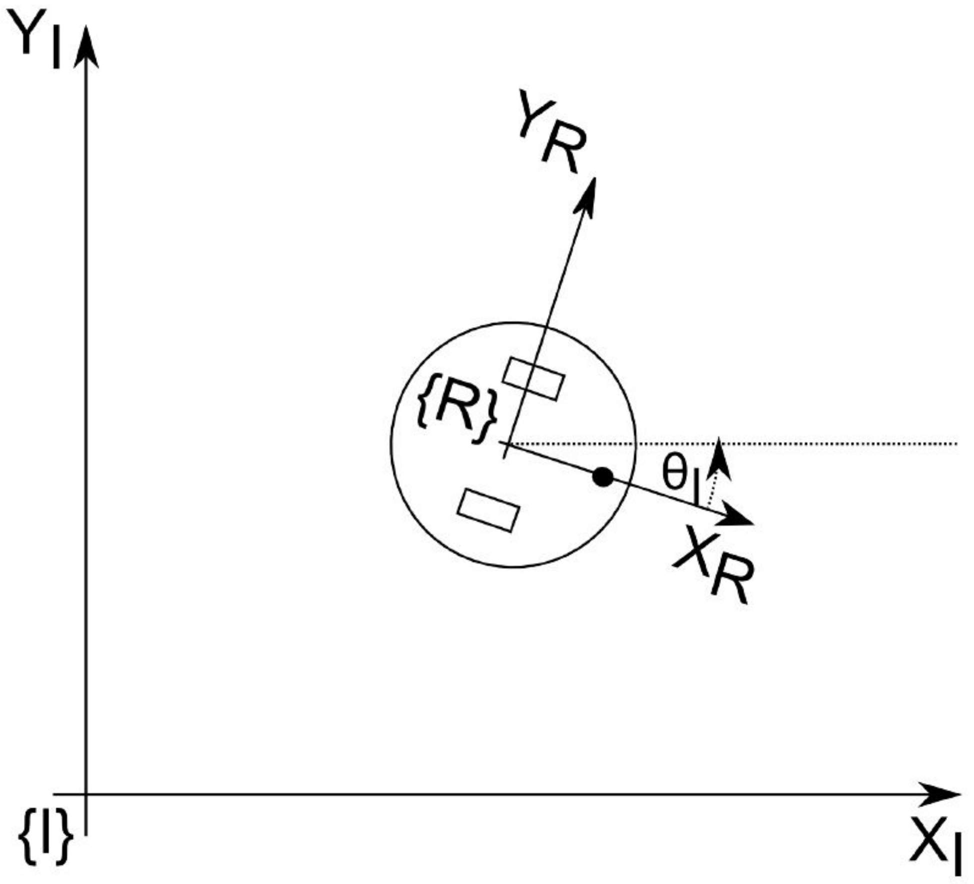
$$\begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2} \\ 0 \\ \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d} \end{bmatrix}$$

3. Forward Kinematics + Odometry



$$\dot{x}_{I,x} = cos(\theta)\dot{x}_R.$$

$$\dot{x}_{I,y} = -sin(\theta)\dot{y}_R.$$

$$\dot{x}_I = cos(\theta)\dot{x}_R - sin(\theta)\dot{y}_R$$

$$\dot{y}_I = sin(\theta)\dot{x}_R + cos(\theta)\dot{y}_R$$

$$\dot{\theta}_I = \dot{\theta}_R$$

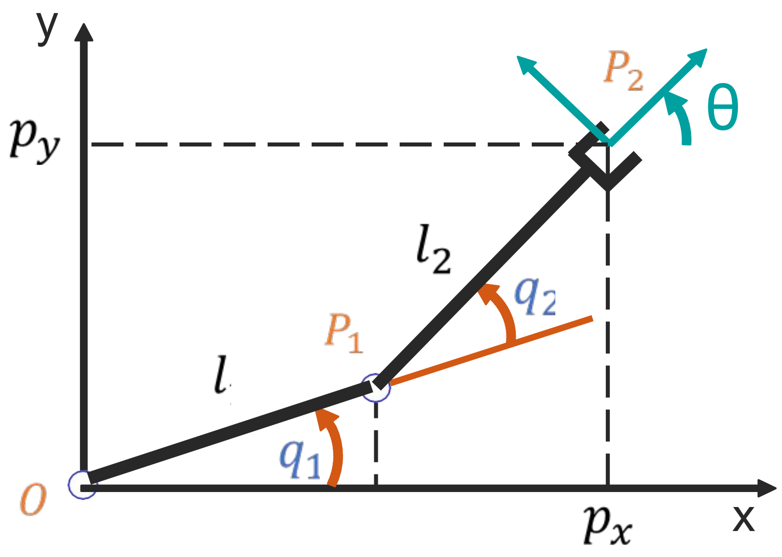
$$\begin{pmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2} \\ 0 \\ \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d} \end{pmatrix}$$

4. Speeds → How can we compute positions?

$$\begin{pmatrix} x_I(T) \\ y_I(T) \\ \theta(T) \end{pmatrix} = \int_0^T \begin{pmatrix} \dot{x}_I(t) \\ \dot{y}_I(t) \\ \dot{\theta}(t) \end{pmatrix} dt \approx \sum_{k=0}^{k=T} \begin{pmatrix} \Delta x_I(k) \\ \Delta y_I(k) \\ \Delta\theta(k) \end{pmatrix} \Delta t$$

Inverse Kinematics

IK for manipulator

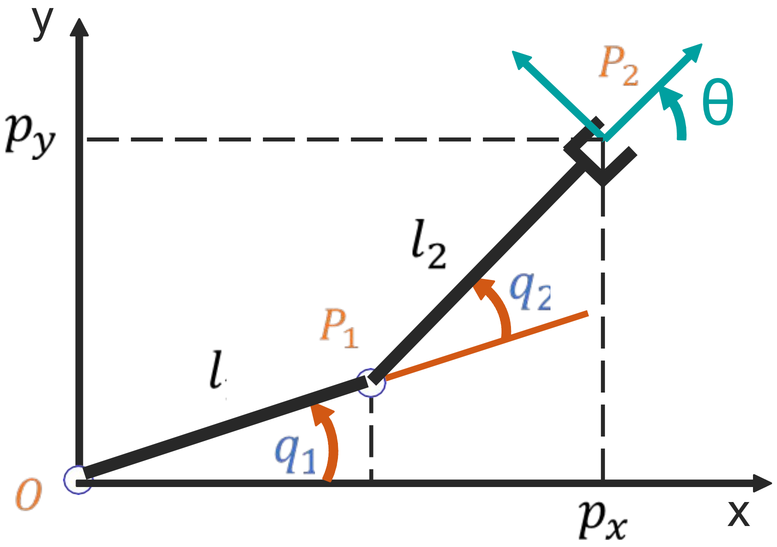


Given point (x, y, θ) Find angles (q_1, q_2)

IK for e-puck

Inverse Kinematics

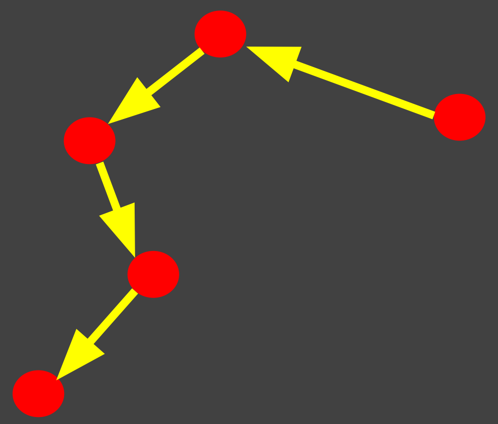
IK for manipulator



- Given point (x, y, θ) Find angles (q_1, q_2)

IK for e-puck

- Given: (x, y, θ) Find: (ϕ_l, ϕ_r)

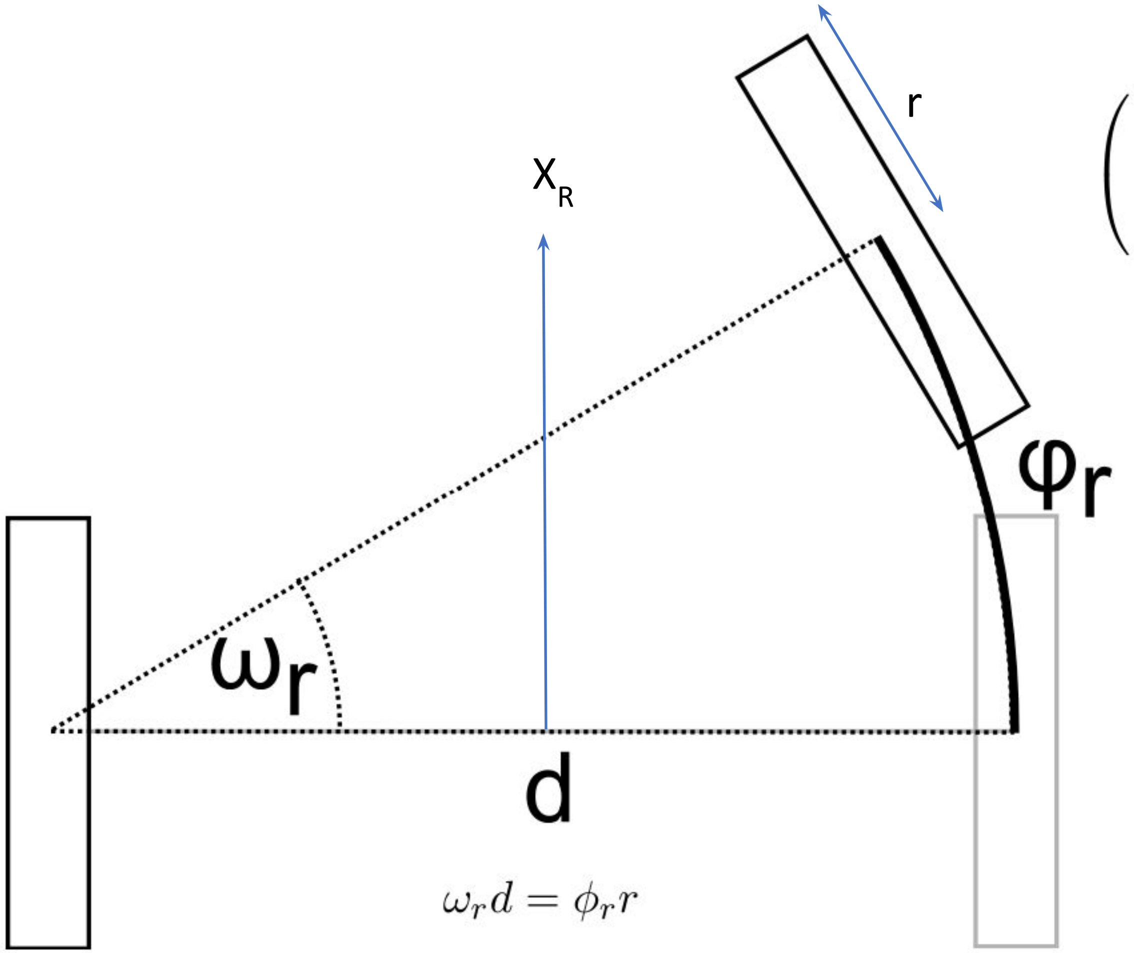


This Week's Lab: IK and Feedback Control on a Mobile Robot!

Implement a feedback controller to allow the robot to navigate to a given pose.

Recall: Kinematics of a Mobile Robot

$$\begin{pmatrix} \dot{x_R} \\ \dot{y_R} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \frac{r\dot{\phi_l}}{2} + \frac{r\dot{\phi_r}}{2} \\ 0 \\ \frac{\dot{\phi_r} r}{d} - \frac{\dot{\phi_l} r}{d} \end{pmatrix}$$



$$\omega_r d = \phi_r r$$

Inverse Kinematics of Mobile Robots

$$\begin{pmatrix} \dot{x_I} \\ \dot{y_I} \\ \dot{\theta} \end{pmatrix} = \underbrace{\begin{pmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}}\begin{pmatrix} \frac{r\dot{\phi_l}}{2} + \frac{r\dot{\phi_r}}{2} \\ 0 \\ \frac{\dot{\phi_r} r}{d} - \frac{\dot{\phi_l} r}{d} \end{pmatrix}$$

$$\dot{\xi_I} = T(\theta)\dot{\xi_R}$$

$$T^{-1}(\theta)\dot{\xi_I} = T^{-1}(\theta)T(\theta)\dot{\xi_R}$$

$$T^{-1} = \begin{pmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\dot{\xi_R} = T^{-1}(\theta)\dot{\xi_I}$$

Inverse Kinematics of (Non-Holonomic) Mobile Robots

Forward Kinematics Equations

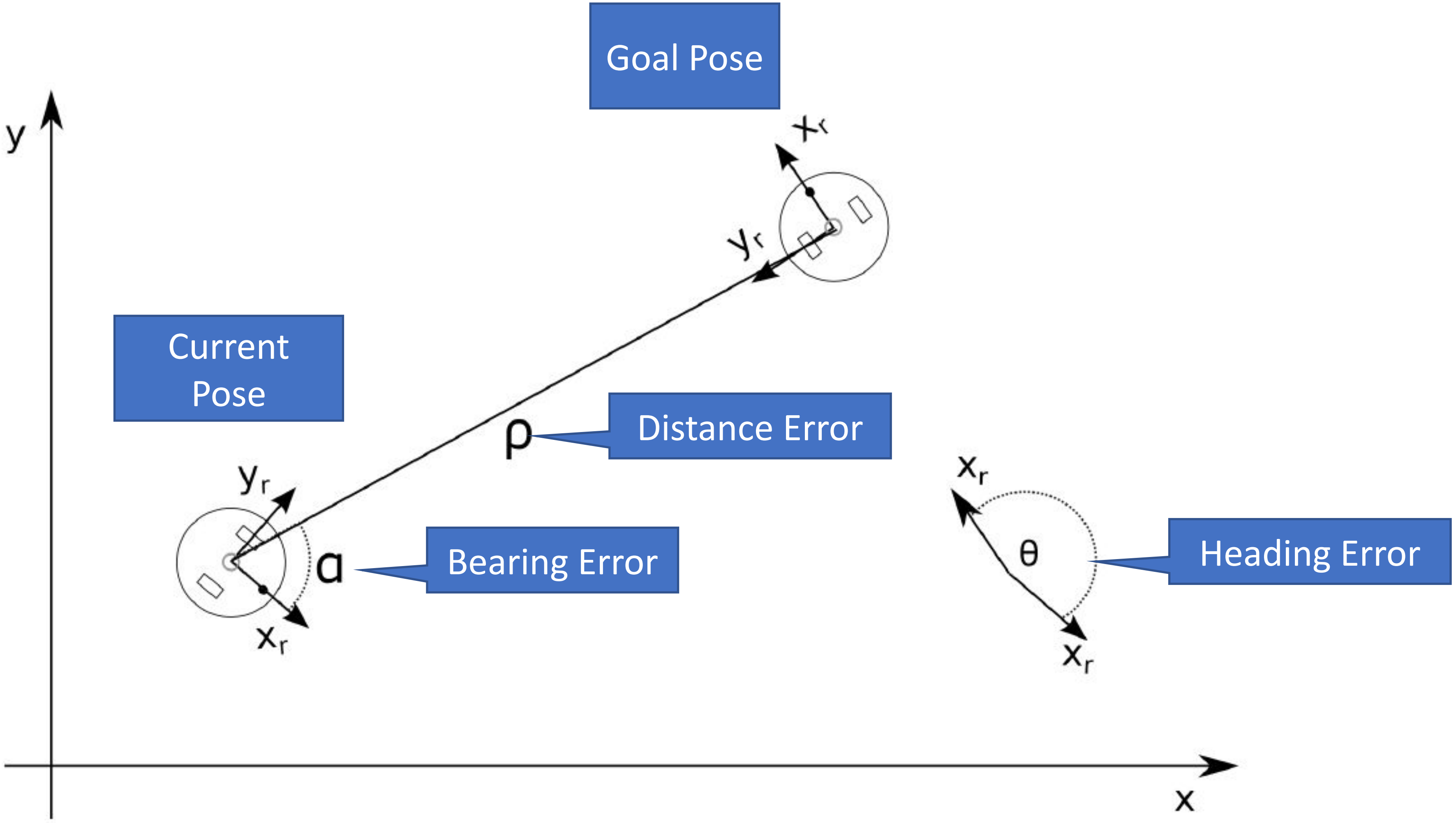
$$\begin{pmatrix} \dot{x_R} \\ \dot{y_R} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2} \\ 0 \\ \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d} \end{pmatrix}$$

Inverse Kinematics Equations

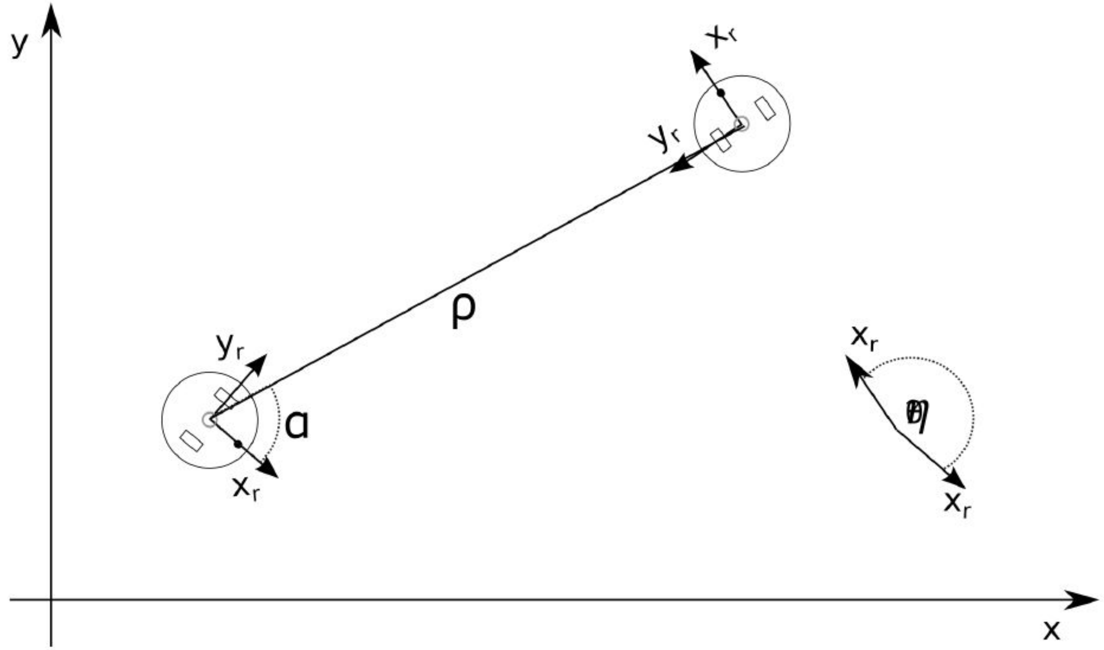
$$\dot{\phi}_L = \frac{\dot{X}_R - \frac{\dot{\theta} d}{2}}{r}$$

$$\dot{\phi}_R = \frac{\dot{X}_R + \frac{\dot{\theta} d}{2}}{r}$$

Position Change Using Feedback Control



Position Change Using Feedback Control



Error Terms

distance $\rho = \sqrt{(x_r - x_g)^2 + (y_r - y_g)^2}$

bearing $\alpha = \tan^{-1}\left(\frac{y_g - y_r}{x_g - x_r}\right) - \theta_r$

heading $\eta = \theta_g - \theta_r$

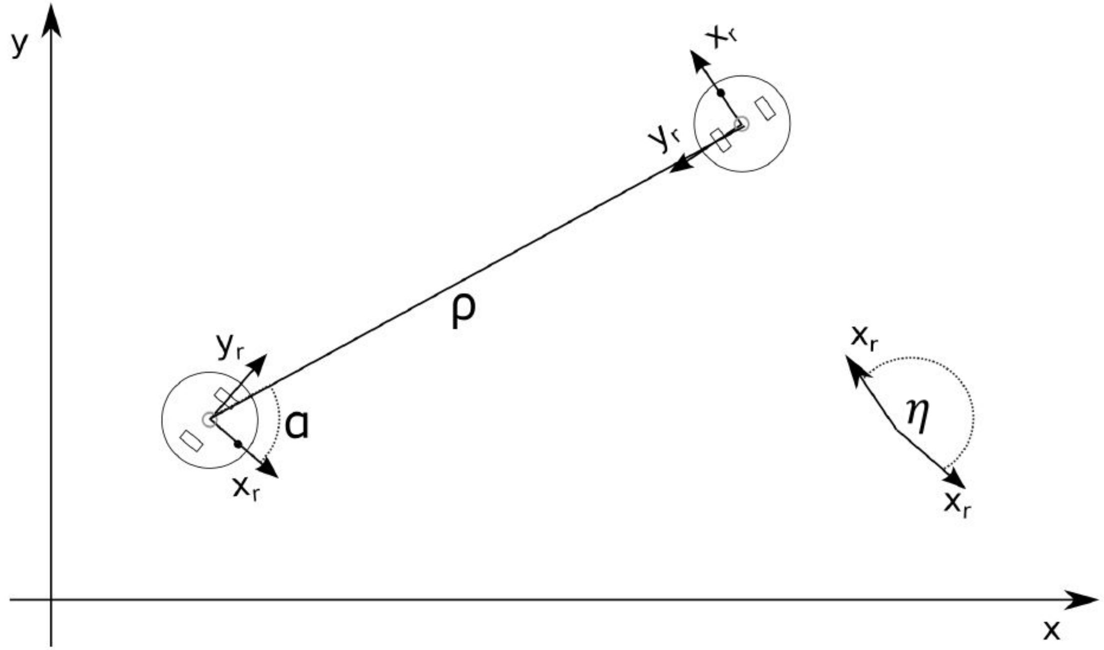
Update Rules

translation $\dot{x} = f(\rho)$

rotation $\dot{\theta} = f(\alpha, \eta)$

How can we regulate α, η?

Position Change Using Feedback Control



Update Rules

translation $\dot{x} = f(\rho)$

rotation $\dot{\theta} = f(\alpha, \eta)$

How can we regulate between α and η?

$$\dot{\phi}_L = \frac{\dot{X}_R - \frac{\dot{\theta} d}{2}}{r}$$

$$\dot{\phi}_R = \frac{\dot{X}_R + \frac{\dot{\theta} d}{2}}{r}$$

Inverse Kinematics Lab

First challenge

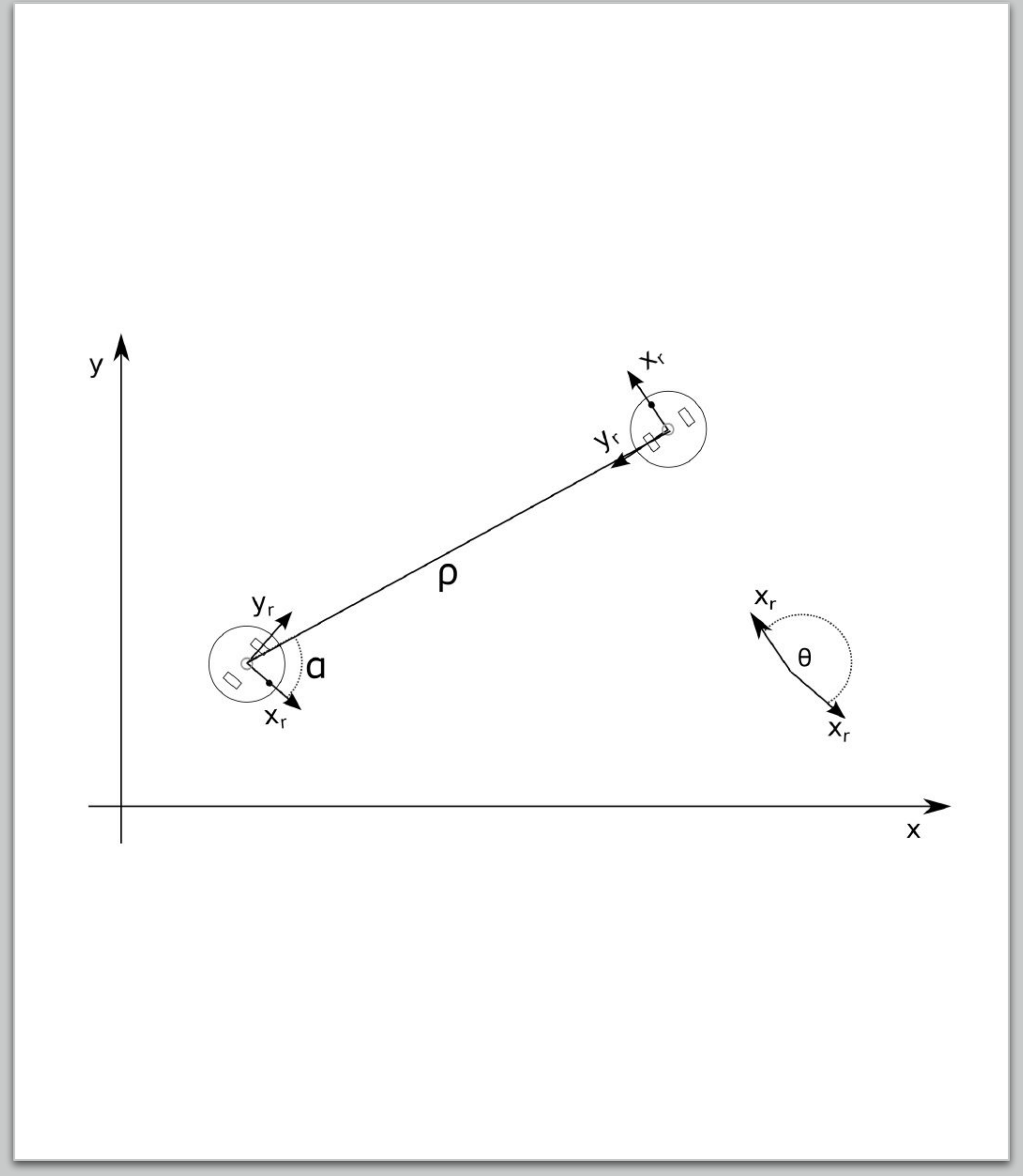
Derive the Inverse Kinematics for a differential drive robot and the Position / Bearing / Heading Error equations and test your results.

Second challenge

Implement a feedback controller to continuously move your robot to a desired pose.

Third challenge

Implement a state machine to use your feedback controller to navigate a *series of waypoints* to reach a goal! Consider what your “goal criteria” is for each waypoint that the robot seeks to reach.



Inverse Kinematics Lab Summary

Error Terms

distance $\rho = \sqrt{(x_r - x_g)^2 + (y_r - y_g)^2}$

bearing $\alpha = \tan^{-1}\left(\frac{y_g - y_r}{x_g - x_r}\right) - \theta_r$

heading $\eta = \theta_g - \theta_r$

Three Problems to overcome:

- ϕ_L' and ϕ_R' saturate over low Δ_t
- $\dot{X}_R \gg \dot{\theta}$ causes angle to be ignored
- α and η both influence $\dot{\theta}$

Forward Kinematics Equations

$$\begin{pmatrix} \dot{x_R} \\ \dot{y_R} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2} \\ 0 \\ \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d} \end{pmatrix}$$

$\longrightarrow$

Inverse Kinematics Equations

$$\phi_L = \frac{\dot{X}_R - \frac{\dot{\theta} d}{2}}{r}$$

$$\phi_R = \frac{\dot{X}_R + \frac{\dot{\theta} d}{2}}{r}$$