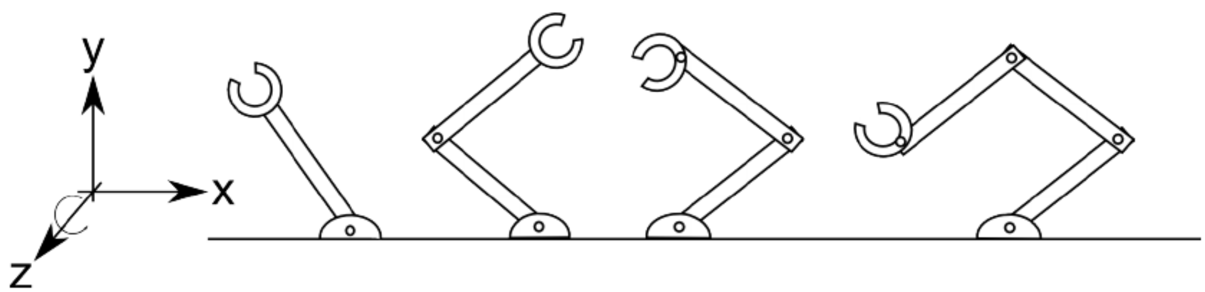
Homework 1: Kinematics and Odometry
CSCI 3302: Introduction to Robotics

Total points: 100

Degrees of Freedom (DOFs)

1. **(8 pts)** For each of the 4 robots operating in X-Y plane provide the
   a. Joint-space DOF
   b. Operational space DOF (of the end-effector)

Note that the tiny circles represent the joints that allow rotation about the Z axis.



2. **(4 pts)** What are the maximum non-redundant degrees of freedom for rigid bodies in the real world? Please provide an explanation.

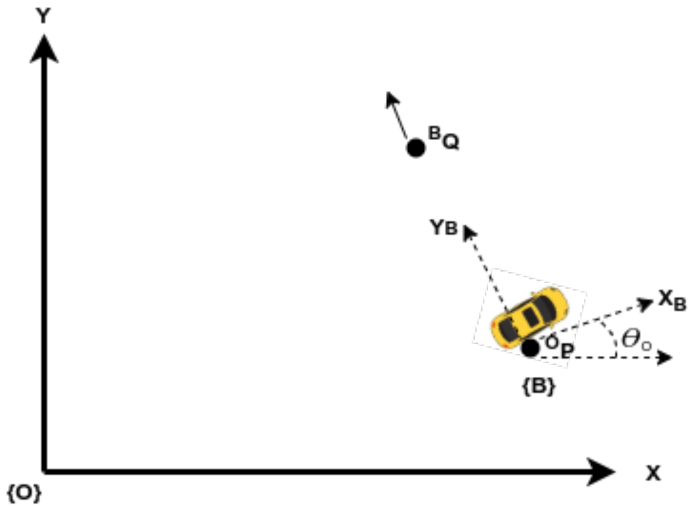
3. **(12 pts)** Answer the following briefly.
   a. How many DOF does a human arm have in joint space?
   b. What is redundancy?
   c. Does a human arm have redundancy? Explain your answer in one sentence.
   d. What is the consequence of redundancy?

Homogeneous Transformations

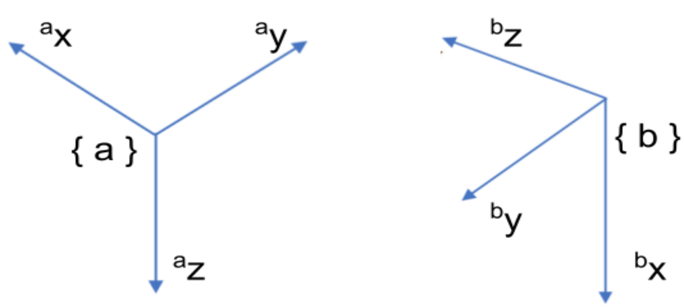
(**Solutions without calculations will not receive any credit**)

4. **(8 pts)** Calculate and provide a third vector that forms a coordinate system with $(\sin(30^{o}), -\cos(60^{o}), 0)^T$ and $(\cos(60^{o}), \sin(30^{o}), 0)^T$. Note that the given vectors are not unit length.

5. **(20 pts)** In our fixed coordinate frame {O}, the car is at point P and shows odometry readings of $(x_O, y_O, \theta_O) = (8\text{ m}, 2\text{ m}, \pi/6\text{ rad})$. Suppose the car using a local coordinate frame {B} detects an object Q at position $(x_B, y_B, \theta_B) = (0\text{ m}, 3\text{ m}, \pi/2\text{ rad})$. Using a homogeneous transform, find the pose [position + orientation] of Q in the coordinate frame {O}. Show the complete process accompanied with explanation for each step using the relevant rotation matrix to receive credit.



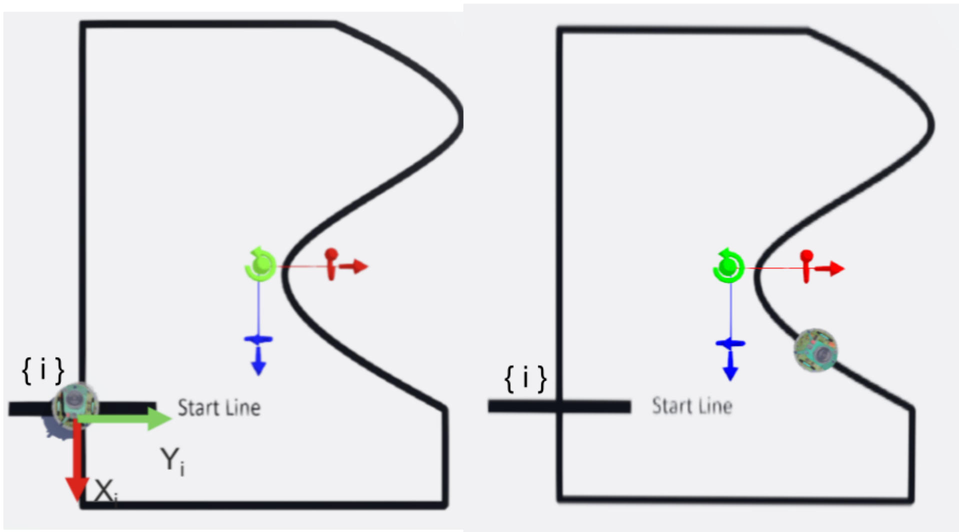
6. **(10 pts)** Consider the coordinate system {a} and {b} which have the same origin and follow the right-hand rule but are rotated differently.



Suppose we rotate {b} around $^b\mathbf{x}$ for **-90°** (note that it is a negative rotation) degrees to form a new coordinate system {c}. Draw the resulting rotated frame. Find a single rotation matrix that transforms a point in {a} to a point in {c}. Show how you arrived at this matrix for credit.

7. **(38 pts)** Suppose the ePuck starts at $x_i, y_i, \theta_i = (0, 0, 0)$ in coordinate system {i} which has its origin at the 'Start Line'. The subscript also helps you identify the coordinate system). X_i and Y_i show the positive x-axis and y-axis respectively in {i}. The ePuck starts in the positive X_i direction. In this case the **Webot's coordinate system {w}** also has the same origin 'Start

Line’ but is rotated differently than {i}. Red-Green-Blue (RGB) corresponds to XYZ respectively and the axes of {w} are shown through the RGB axes in the middle of the lap. Note that {w} has its origin at the ‘Start Line’ and not in the middle.
After some time, the ePuck is at x_i, y_i, θ_i = (-0.10 m, 0.33 m, 0.47 rad).



a. (8 pts) Provide the rotation matrix that correctly rotates a point in {i} to a point in {w}.

b. (10 pts) What is the pose (x_i, y_i, θ_i) = (-0.10 m, 0.33 m, 0.47 rad) in the coordinate system {w}? Show all the steps to receive credit.

c. (20 pts) Suppose the ePuck has a maximum wheel velocity of 0.12 m/s. You set the left wheel velocity to maximum and the right wheel velocity to maximum/2 and let the ePuck run at this setting for 2 seconds. What is the new pose (round up to 2 decimal places) of ePuck in the {i} coordinate system. Show each step utilizing the correct equations to get credit.