CSCI/ECEN 3302
Introduction to Robotics

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Administrivia

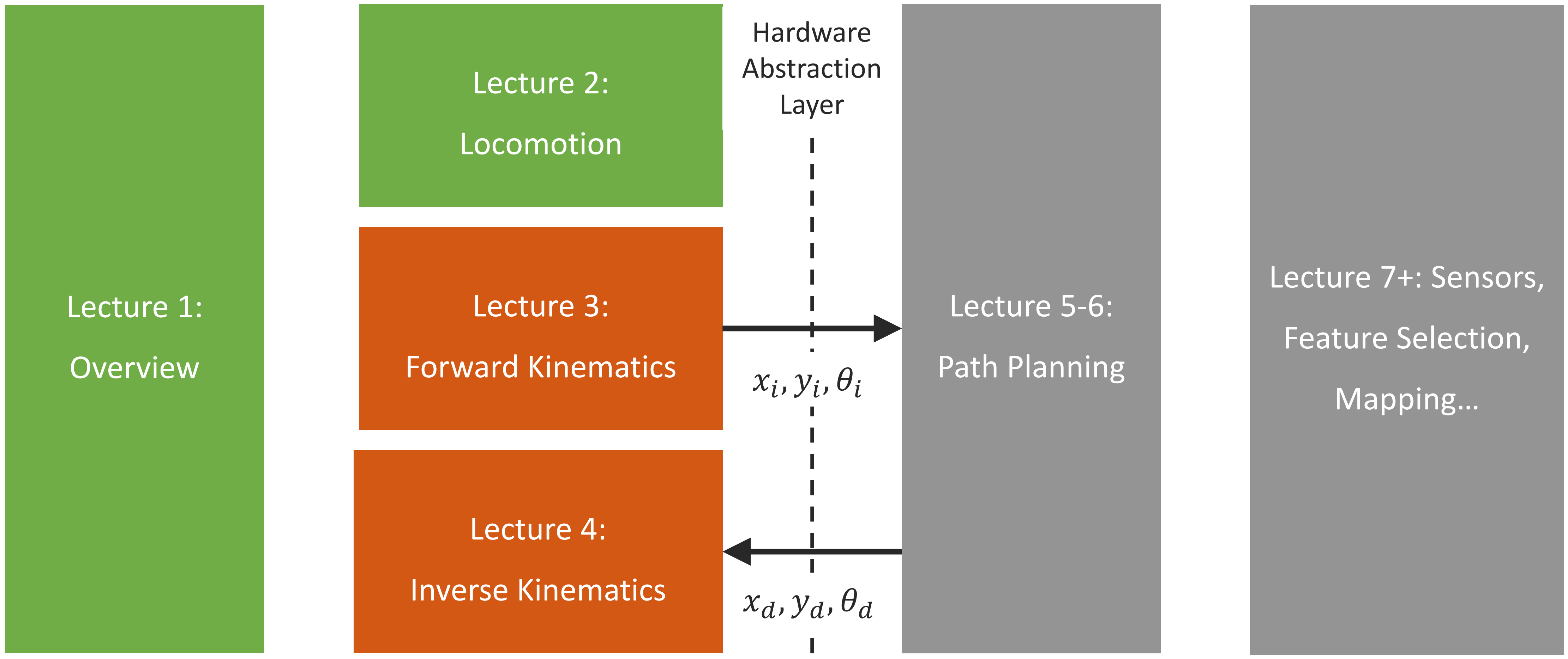
- Lab 2 is due next week (Tuesday at midnight)
  - Please see Piazza articles
  - I recommend everybody to switch from R2022a to R2021b
  - See mega-post from Shiv: https://piazza.com/class/ky8w23o9yhn4yb?cid=43
- Homework 1 is out!
  - Deadline: Feb 13, 23:59 pm
  - Please consult the book [chapters 2 and 3], slides and recordings
  - Please come to Office Hours for help!

Administrivia

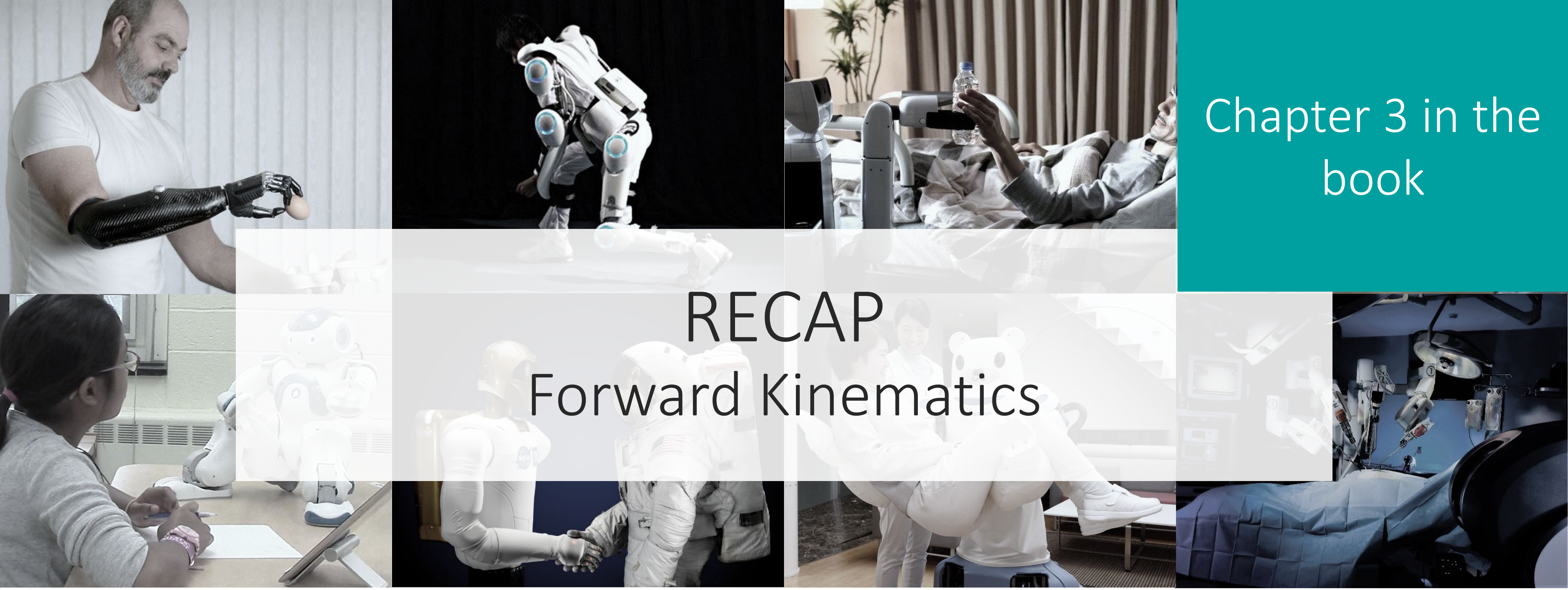
- Full hybrid mode starting from next week!
  - I expect minor adjustments to be made to the format
- Note on plagiarism
  - You should work on HWs on your own – and Labs with your team
  - You should cite your sources
  - You should not copy-paste code from the internet, but you should re-implement it in your own terms

Questions?

Roadmap

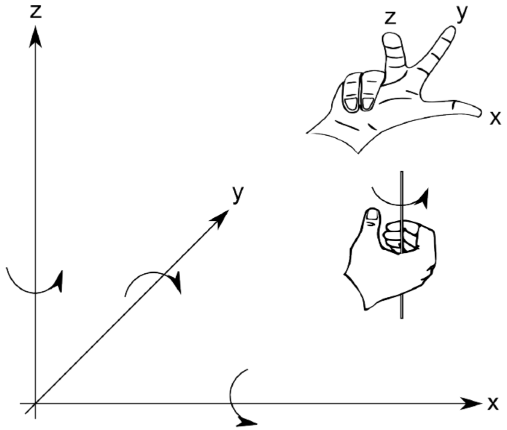


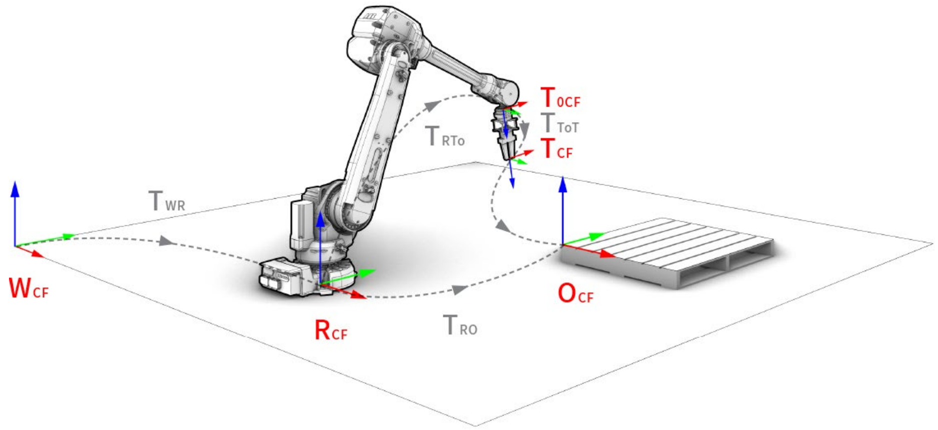
Lecture 1:
Overview
Lecture 2:
Locomotion
Lecture 3:
Forward Kinematics
Lecture 4:
Inverse Kinematics
Hardware Abstraction Layer
x_i, y_i, θ_i
x_d, y_d, θ_d
Lecture 5-6:
Path Planning
Lecture 7+: Sensors,
Feature Selection,
Mapping…



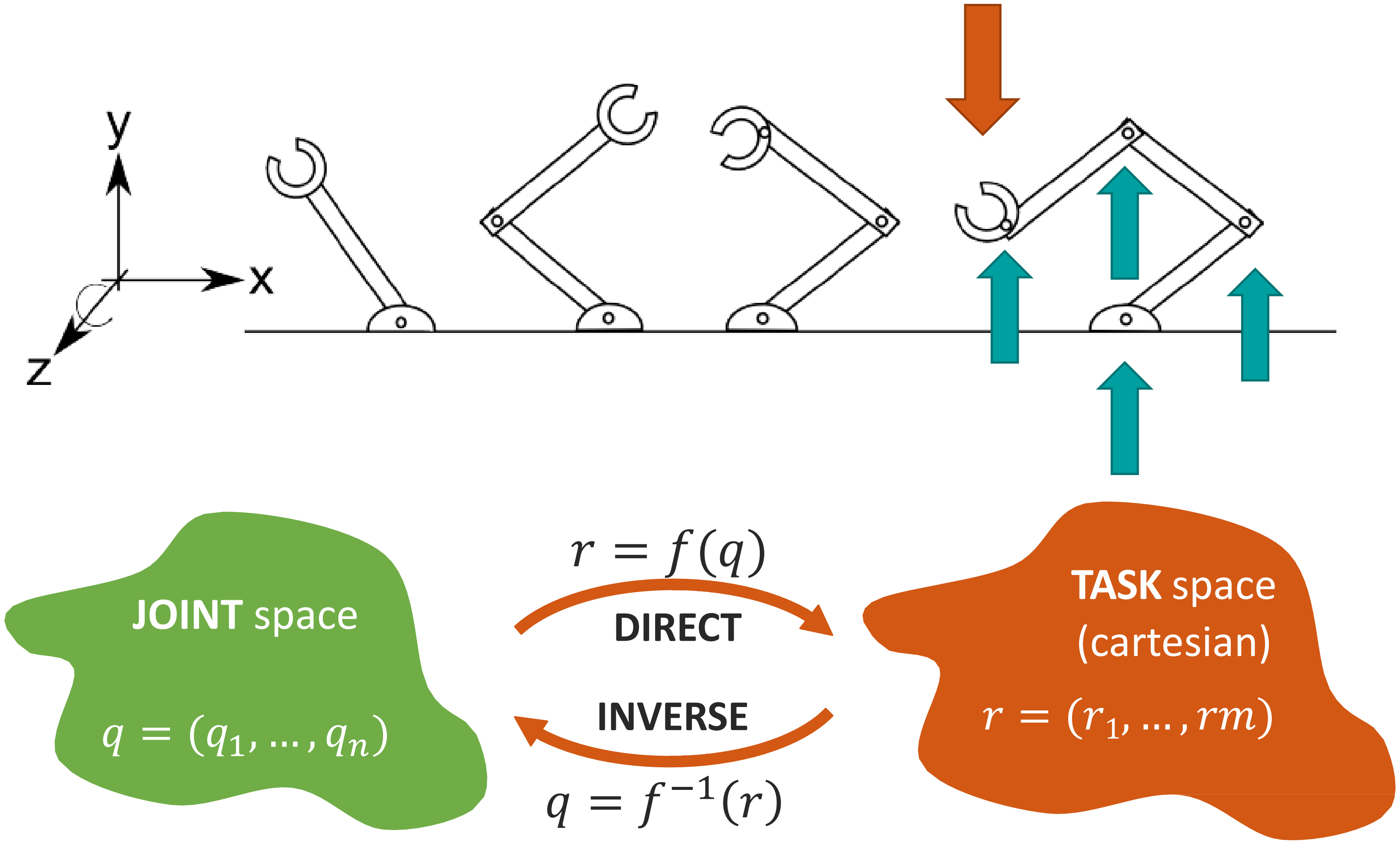
Chapter 3 in the book

RECAP
Forward Kinematics



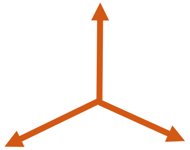


Forward (Direct) Kinematics



Rigid Body Transformations

- Rigid Body:
  - $O \in \mathbb{R}^3$
  - $\forall p, q \in O$:
  - $\|p(t) - q(t)\| = \|p(0) - q(0)\|$



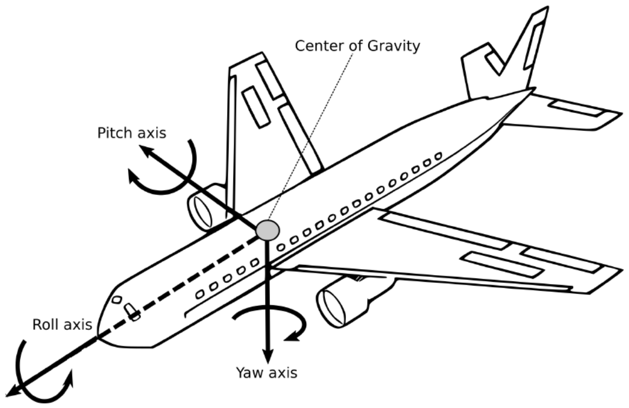
- Rigid Body Transformation:
  - A mapping **g**: $\mathbb{R}^3 \rightarrow \mathbb{R}^3$ is a rigid body transformation if:
  - Length is preserved:

$$\|g(p) - g(q)\| = \|p - q\| \;\forall p, q \in \mathbb{R}^3$$

  - Cross product is preserved:

$$g_*(v \times w) = g_*(v) \times g_*(w) \forall v, w \in \mathbb{R}^3$$

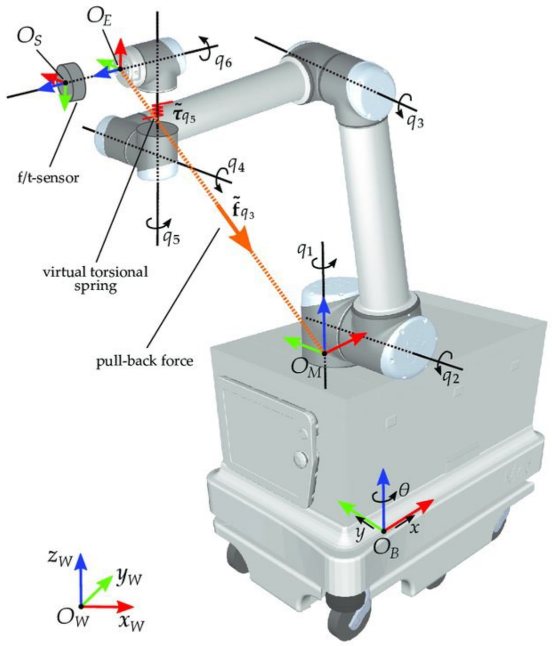
Nested Coordinate Systems

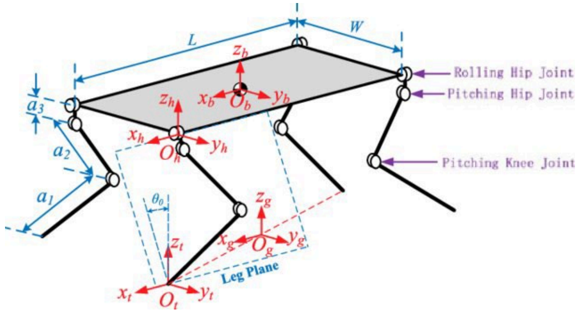


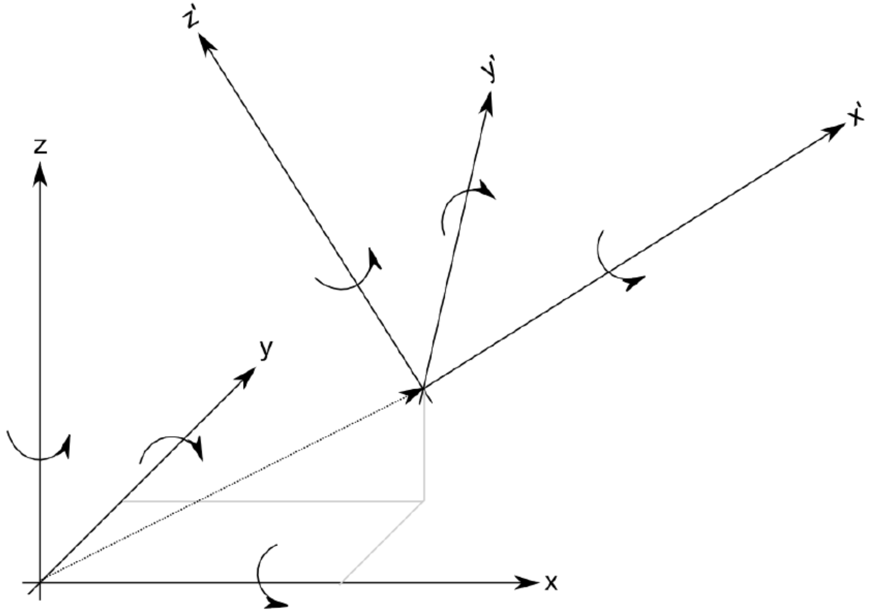
- Recall: an airplane can be modeled as a single rigid body
  - A single coordinate system (i.e. a single pose) fully characterizes its kinematic state
- How can a robot manipulator be modeled?
  - Multiple rigid bodies connected together by joints
  - Each DoF / point of actuation presents a new Coordinate System w.r.t. the previous linkage

Nested Coordinate Systems

- Each DoF / point of actuation presents a new Coordinate System w.r.t. the previous linkage
- Applies to both manipulators and mobile platforms!

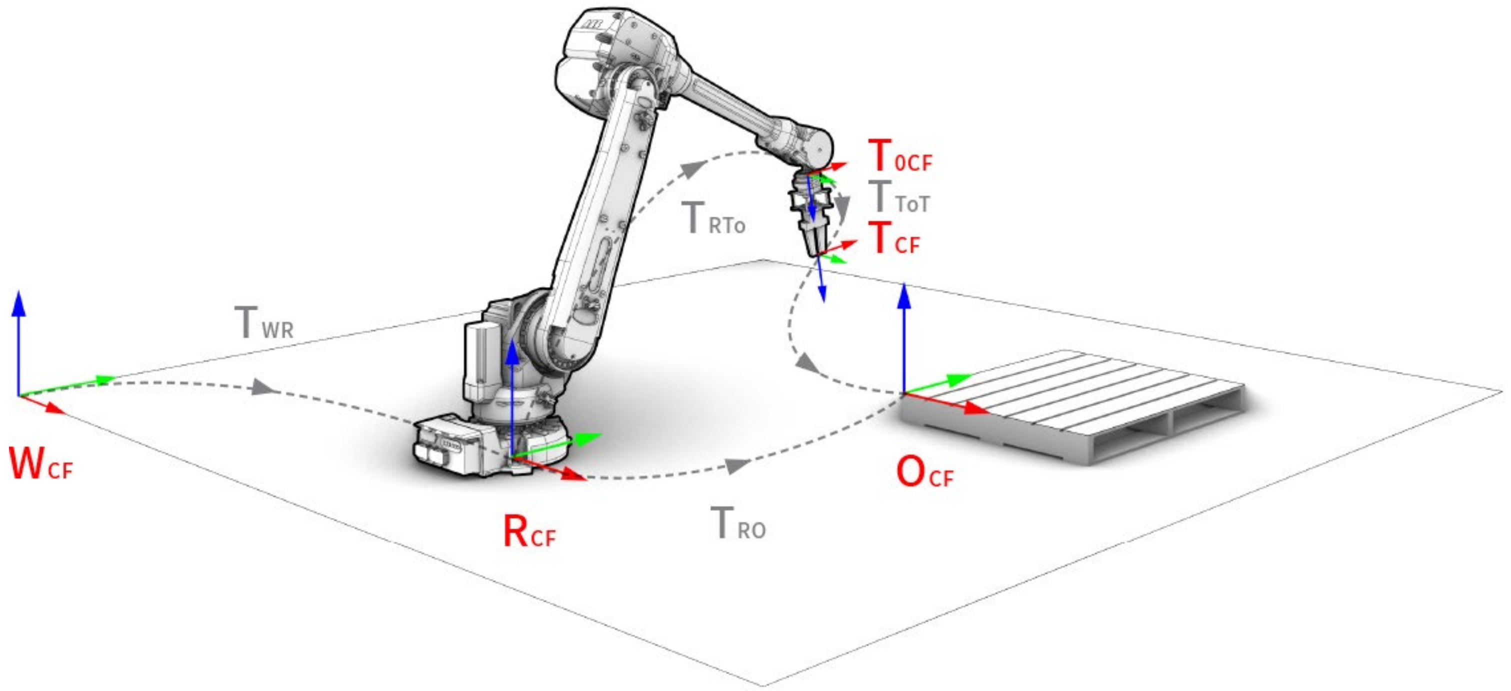


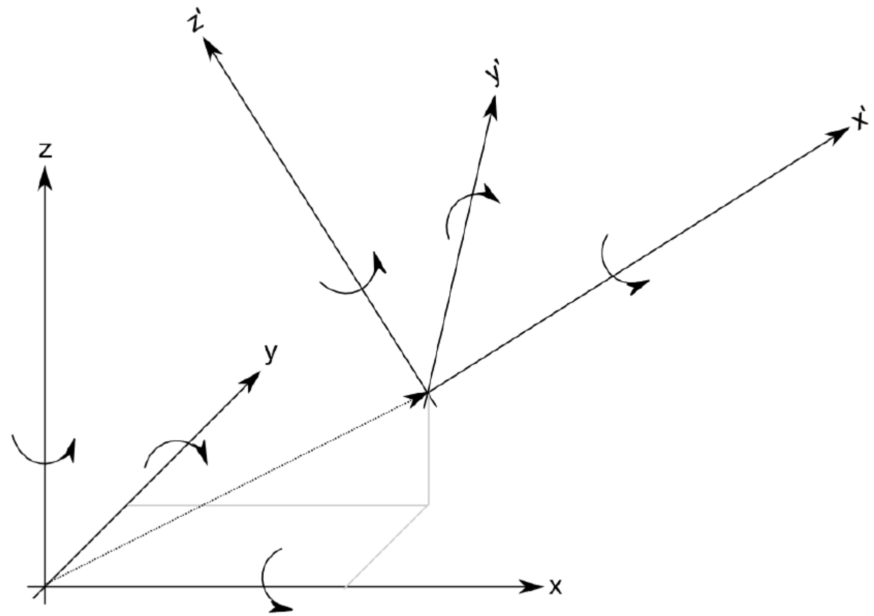




Nested Coordinate Systems

- This idea can be used beyond the robot as well!!



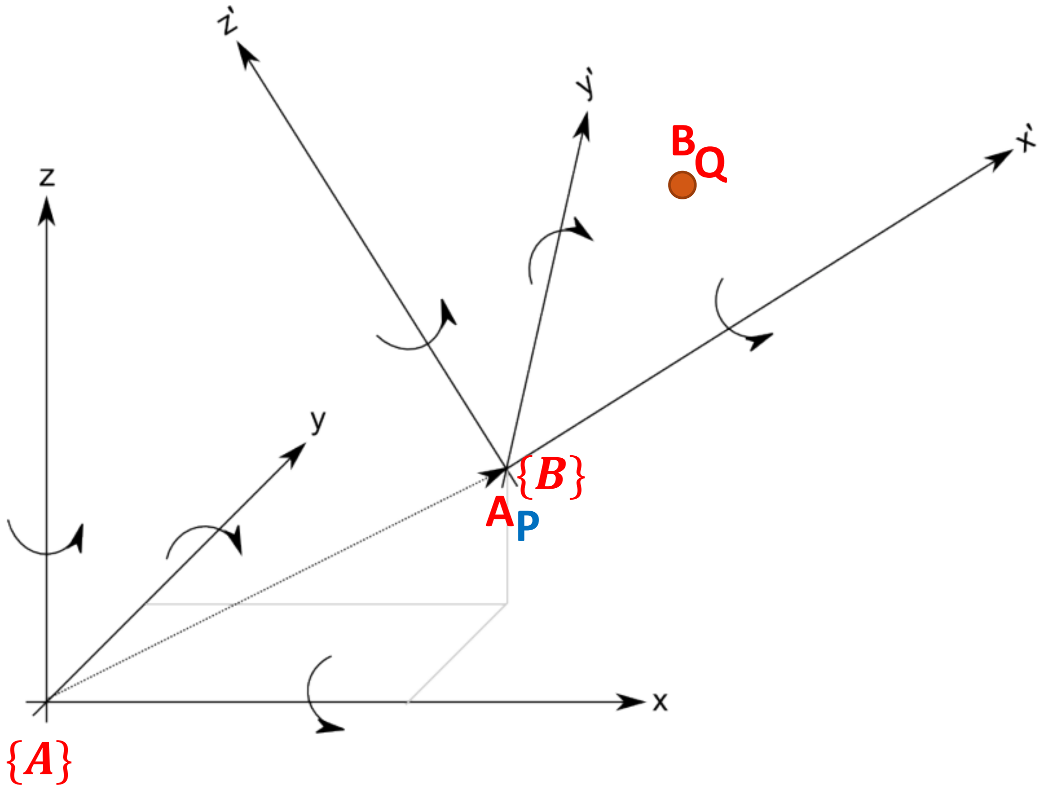


Nested Coordinate Systems

How can we express point ${}^{B}Q$, with position known in coordinate system $\{B\}$ centered at P, in coordinate system $\{A\}$ centered at the origin?

${}^{B}Q$ = Known

${}^{A}P$ = Known



Homogenous Transform

- Instead of ${}^{A}Q = {}^{A}_{B}R * {}^{B}Q + {}^{A}P$, we can express the transformation as a single matrix multiplication:

$$\begin{bmatrix} {}^{A}Q \\ 1 \end{bmatrix} = \begin{bmatrix} & {}^{A}_{B}R & & {}^{A}P \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}Q \\ 1 \end{bmatrix}$$

- Inverse Transform:

$$T^{-1} = \begin{bmatrix} R^{T} & -R^{T}\,{}^{A}P \\ 0 & 1 \end{bmatrix} \in SE(3)$$

Homogenous Transform: 2D case

- Instead of ${}^{A}Q = {}^{A}_{B}R * {}^{B}Q + {}^{A}P$, we can express the transformation as a single matrix multiplication:

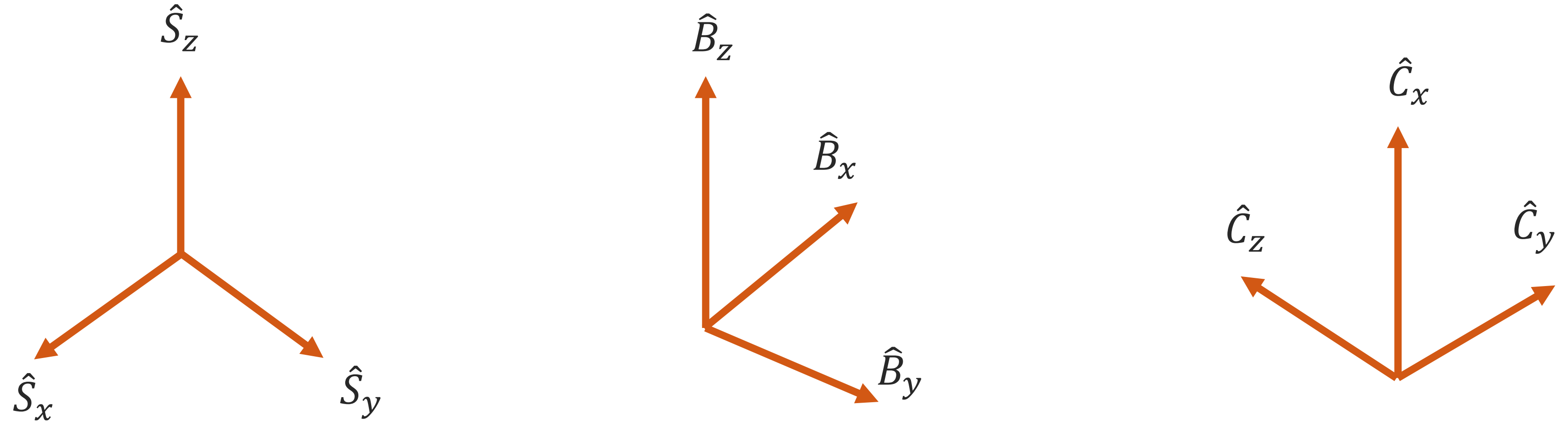
$$\begin{bmatrix} {}^{A}Q \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & t_x \\ \sin(\theta) & \cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}Q \\ 1 \end{bmatrix}$$

Example

$$^{A}\hat{X}_B = (\hat{X}_B \cdot \hat{X}_A, \hat{X}_B \cdot \hat{Y}_A, \hat{X}_B \cdot \hat{Z}_A)^T$$

$$^{A}\hat{Y}_B = (\hat{Y}_B \cdot \hat{X}_A, \hat{Y}_B \cdot \hat{Y}_A, \hat{Y}_B \cdot \hat{Z}_A)^T$$

$$^{A}\hat{Z}_B = (\hat{Z}_B \cdot \hat{X}_A, \hat{Z}_B \cdot \hat{Y}_A, \hat{Z}_B \cdot \hat{Z}_A)^T$$



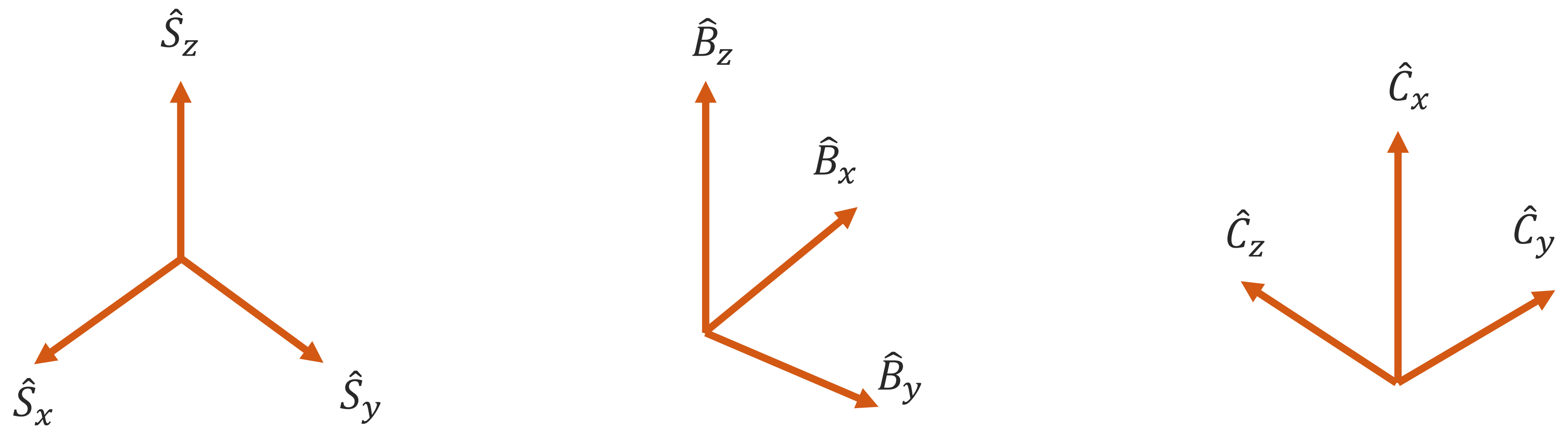
$$^{\hat{S}}_{\hat{C}}R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

X-axis of $\hat{S}$ is the negative-Y-axis of $\hat{C}$

Y-axis of $\hat{S}$ is the negative-Z-axis of $\hat{C}$

Z-axis of $\hat{S}$ is the X-axis of $\hat{C}$

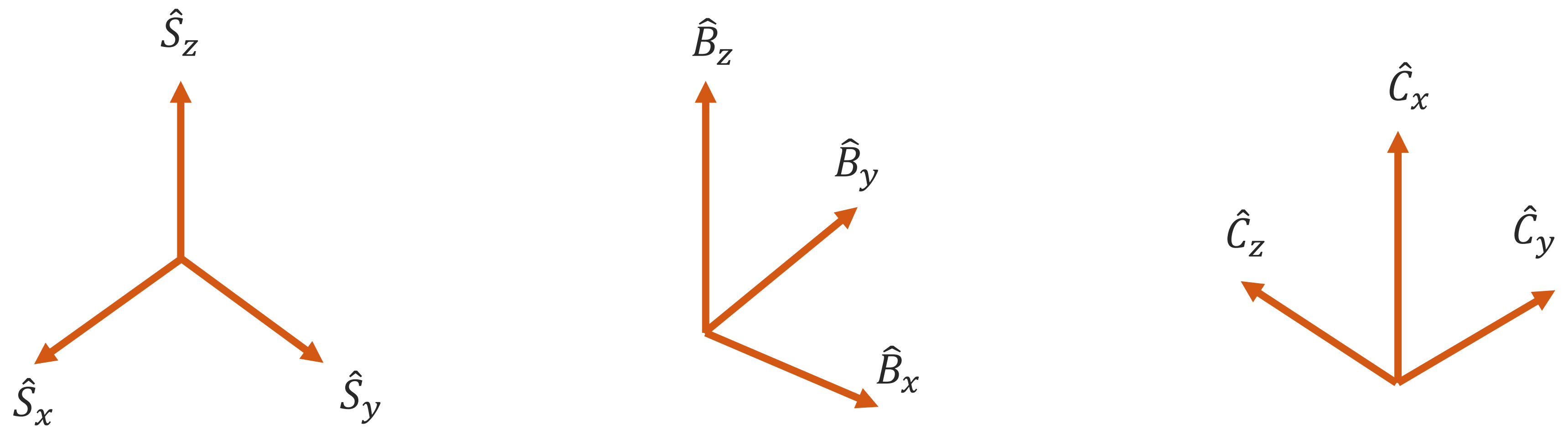
Example:
Expressing a Point in New Coordinate Frame



$$ {}^{\hat{S}}_{\hat{C}}R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \qquad {}^{\hat{C}}p = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} $$

$$ {}^{\hat{S}}p = {}^{\hat{S}}_{\hat{C}}R \; {}^{\hat{C}}p = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} $$

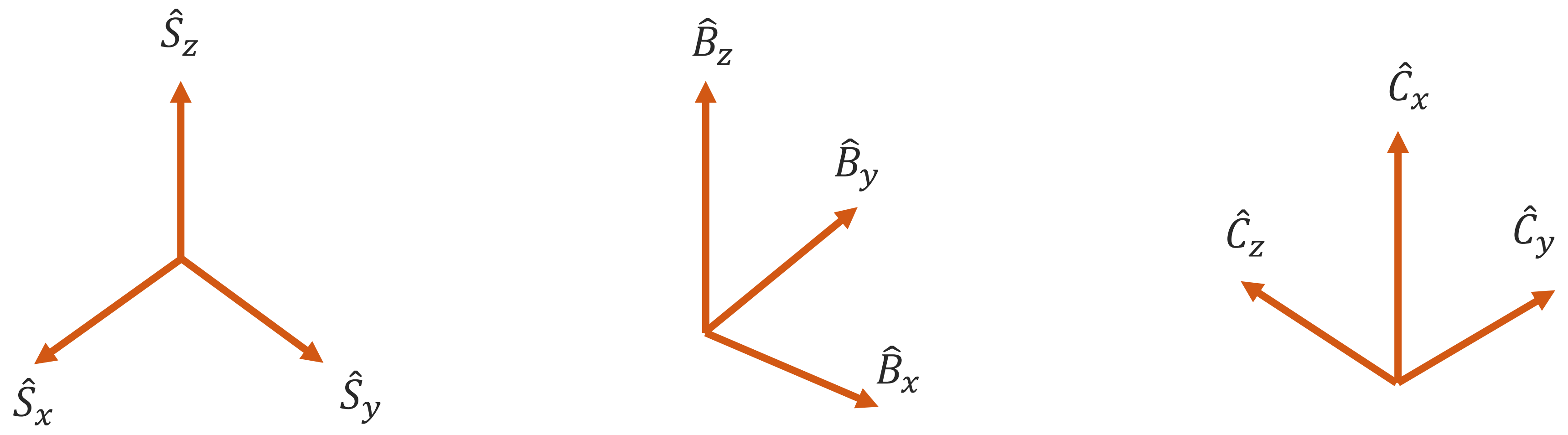
Example: Inverting a Rotation



$$
{}^{\hat{S}}_{\hat{C}}R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \qquad {}^{\hat{C}}_{\hat{S}}R = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}
$$

Transpose!

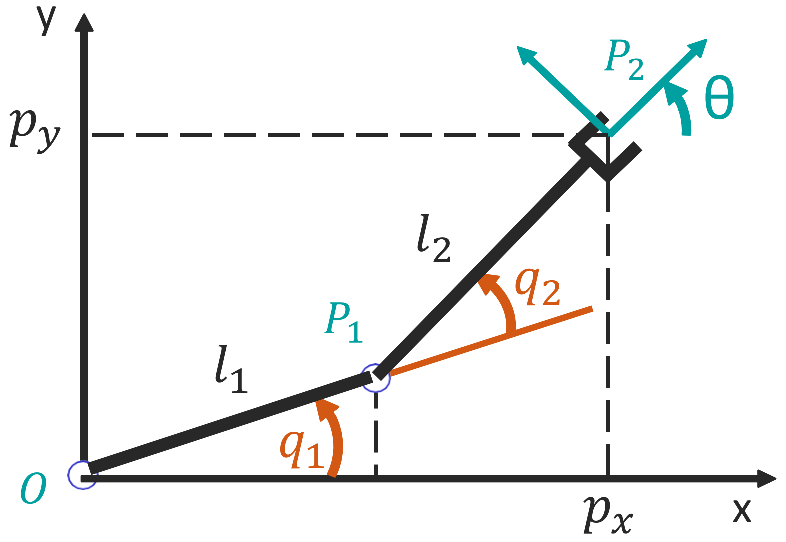
Example: Composing Rotations



$${}^{\hat{S}}_{\hat{B}}R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad {}^{\hat{B}}_{\hat{C}}R = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$${}^{\hat{S}}_{\hat{C}}R = {}^{\hat{S}}_{\hat{B}}R\,{}^{\hat{B}}_{\hat{C}}R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

Example: direct kinematics of 2R arm



$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \Leftarrow$ Joint-space DOFs

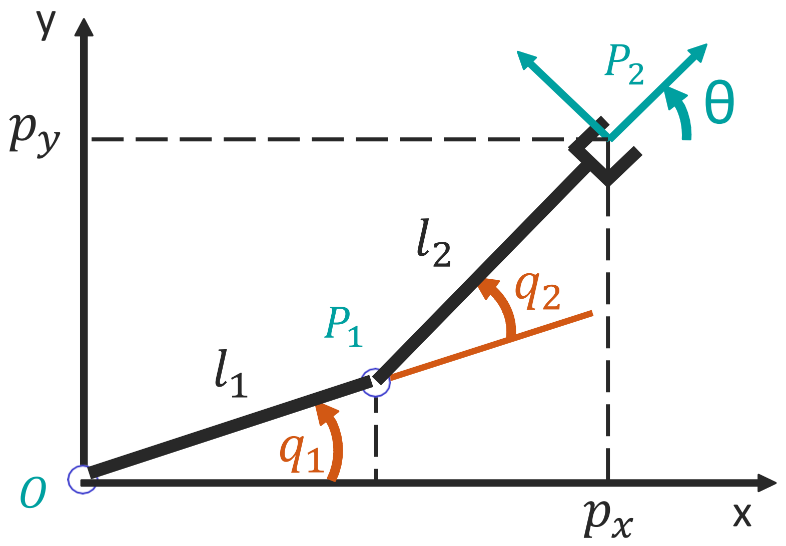
$r = \begin{bmatrix} p_{2,x} \\ p_{2,y} \\ \theta \end{bmatrix} \Leftarrow$ Operational-space DOFs

r=$f(q)$

$$
\begin{aligned}
p_{2,x} &= p_{1,x} + \overline{P_1 P_2}_x \\
p_{2,y} &= p_{1,y} + \overline{P_1 P_2}_y \\
\theta &= q_1 + q_2
\end{aligned}
$$

Please take a look at the example in the book too!!! Section 3.1.1

Example: direct kinematics of 2R arm



$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \Leftarrow$ Joint-space DOFs

$r = \begin{bmatrix} p_{2,x} \\ p_{2,y} \\ \theta \end{bmatrix} \Leftarrow$ Operational-space DOFs

r=f(q)

$$p_{2,x} = l_1 \cos(q_1) + l_2 \cos(q_1 + q_2)$$
$$p_{2,y} = l_1 \sin(q_1) + l_2 \sin(q_1 + q_2)$$
$$\theta = q_1 + q_2$$

For more general cases, we need a “method”!

Please take a look at the example in the book too!!! Section 3.2.1

Forward Kinematics of a differential wheel robot (i.e. the e-puck)

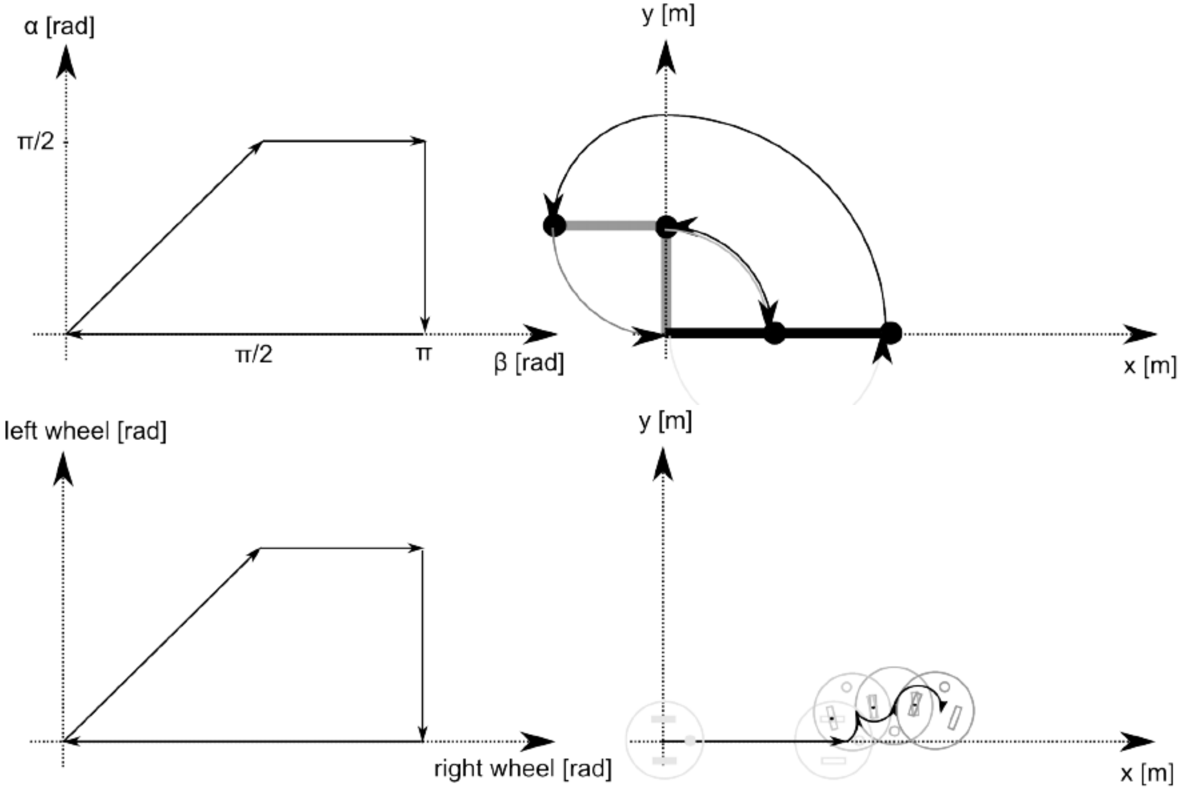
- Manipulator:
  - Forward kinematics is uniquely defined by its joint angles
  - Measure of joint angles through encoders is absolute
- Differential Wheels robot:
  - Measure of joint angles through encoders is relative
  - Encoders' values need to be integrated over time

Please take a look at Section 3.2.2 in the book

Forward Kinematics of a differential wheel robot (i.e. the e-puck)

Manipulator
[HOLONOMIC]

Differential Wheels
[NON-HOLONOMIC]



Configuration [joint] Space

Work [task] space

Please take a look at Section 3.2.2 in the book

Holonomic or Non-Holonomic on the 2D plane?

- Steering wheel is rotated 90 degrees then acceleration is applied for 1 second

vs.

- Acceleration is applied for 1 second then steering wheel rotated 90 degrees!

- Different ending configuration = Non-holonomic!

Forward Kinematics – Odometry

- How to model wheel motion
- Wheel motions to position updates
- Position updates to Forward Kinematics in Inertia frame

Chapter 3.2

Odometry

- Derived from Greek words for “measure route”
- Utilization of sensors to estimate changes in position over time
- Useful for position/pose estimation!

Lab 2:

Forward Kinematics and Odometry!

Odometry aka: Where is the e-puck?

- How do we figure out where the e-puck is in the world?

What is our state vector?

- What are we able to control?
  - [JOINT SPACE DoFs]

- What are we able to measure?

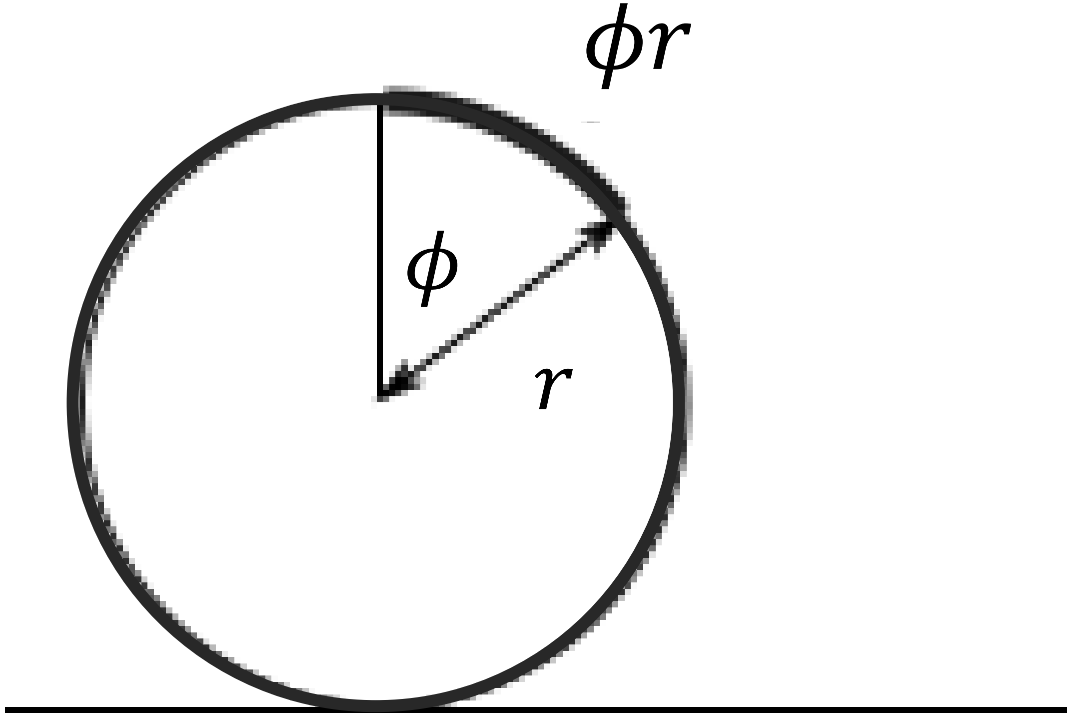
- What variables do we need to measure the robot’s motion in space?
  - [OPERATIONAL SPACE DoFs]

Odometry aka: Where is the e-puck?

- How do we figure out where the e-puck is in the world?

- Measuring the e-puck’s displacement

1. Wheel motion

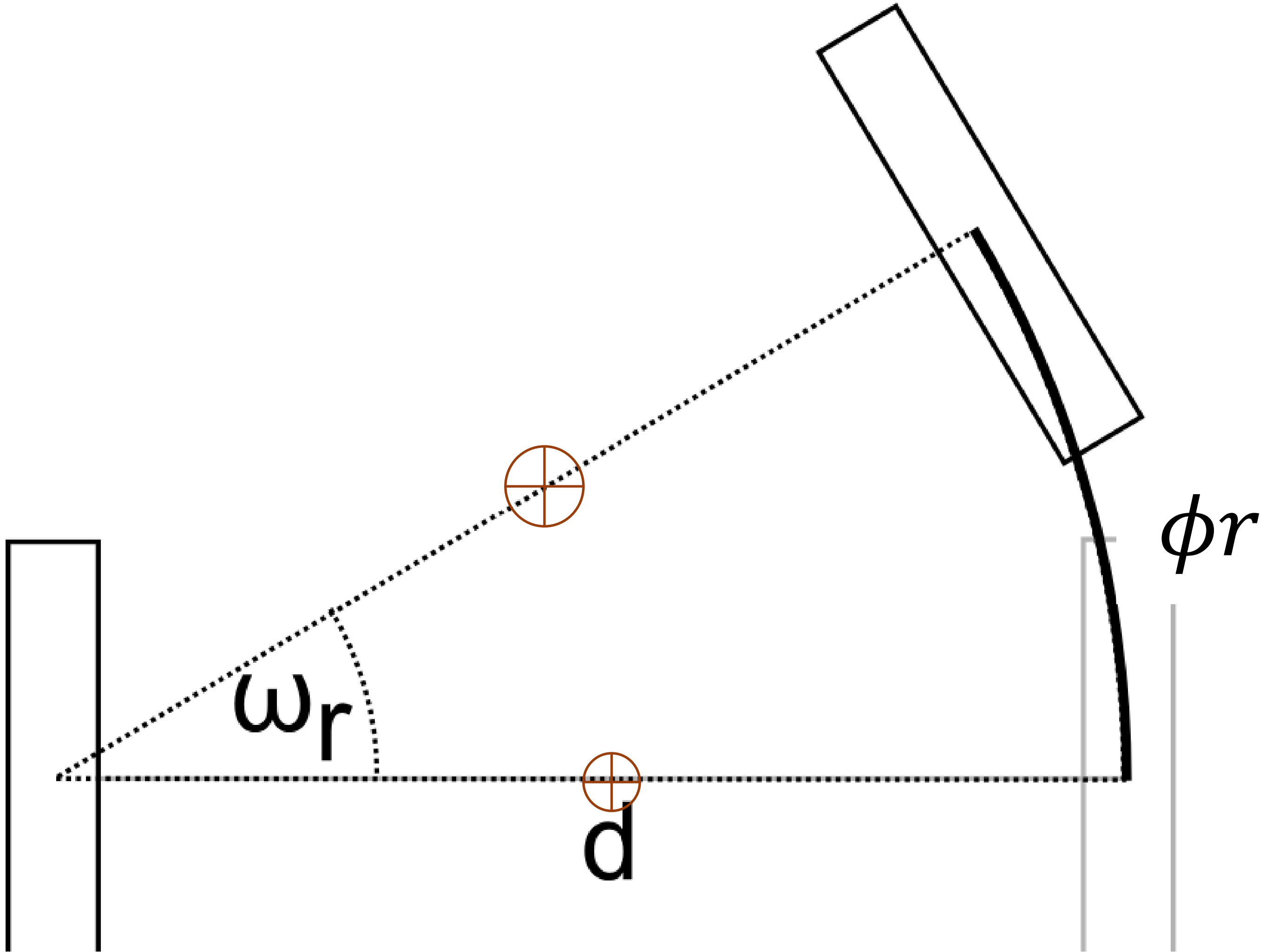


Distance traveled is angle of rotation times wheel radius:

$$x = r\phi$$

$$\dot{x} = r\dot{\phi}$$

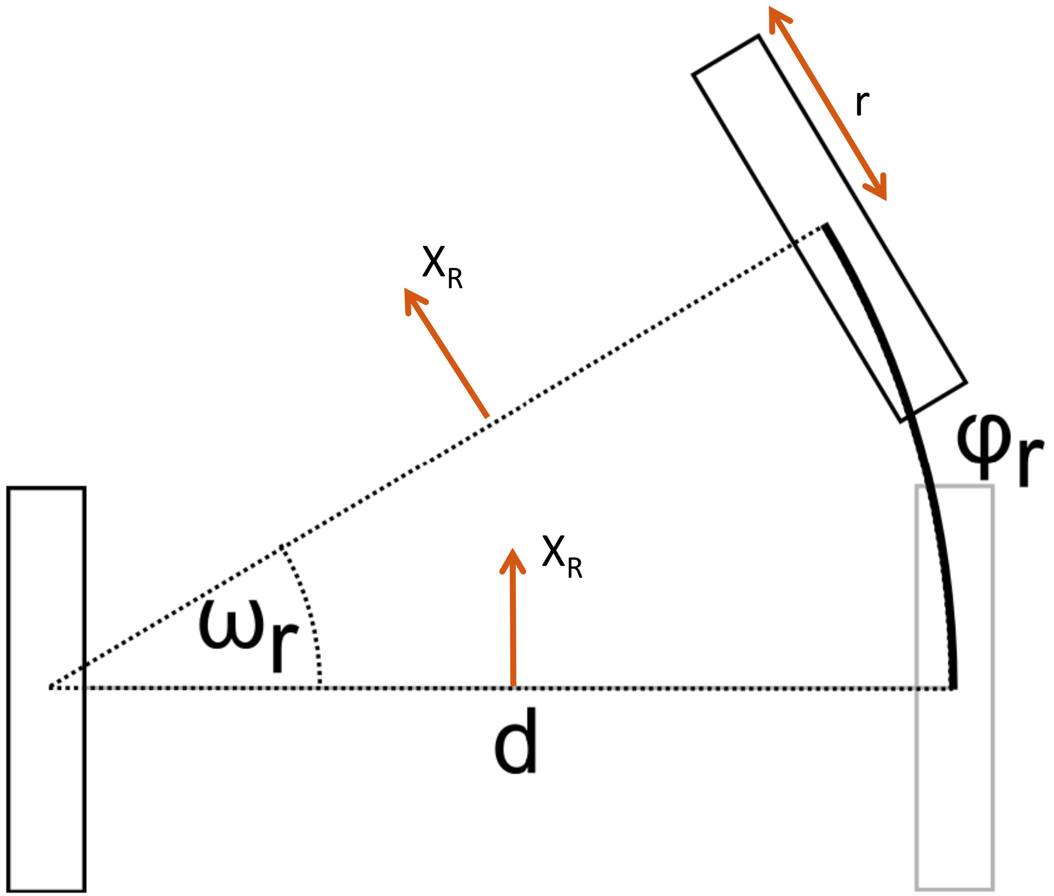
2. Wheel motion → Position Updates



What about the case where only one wheel is moving?

Its center of mass will move by $\frac{1}{2}\dot{\phi} r$!

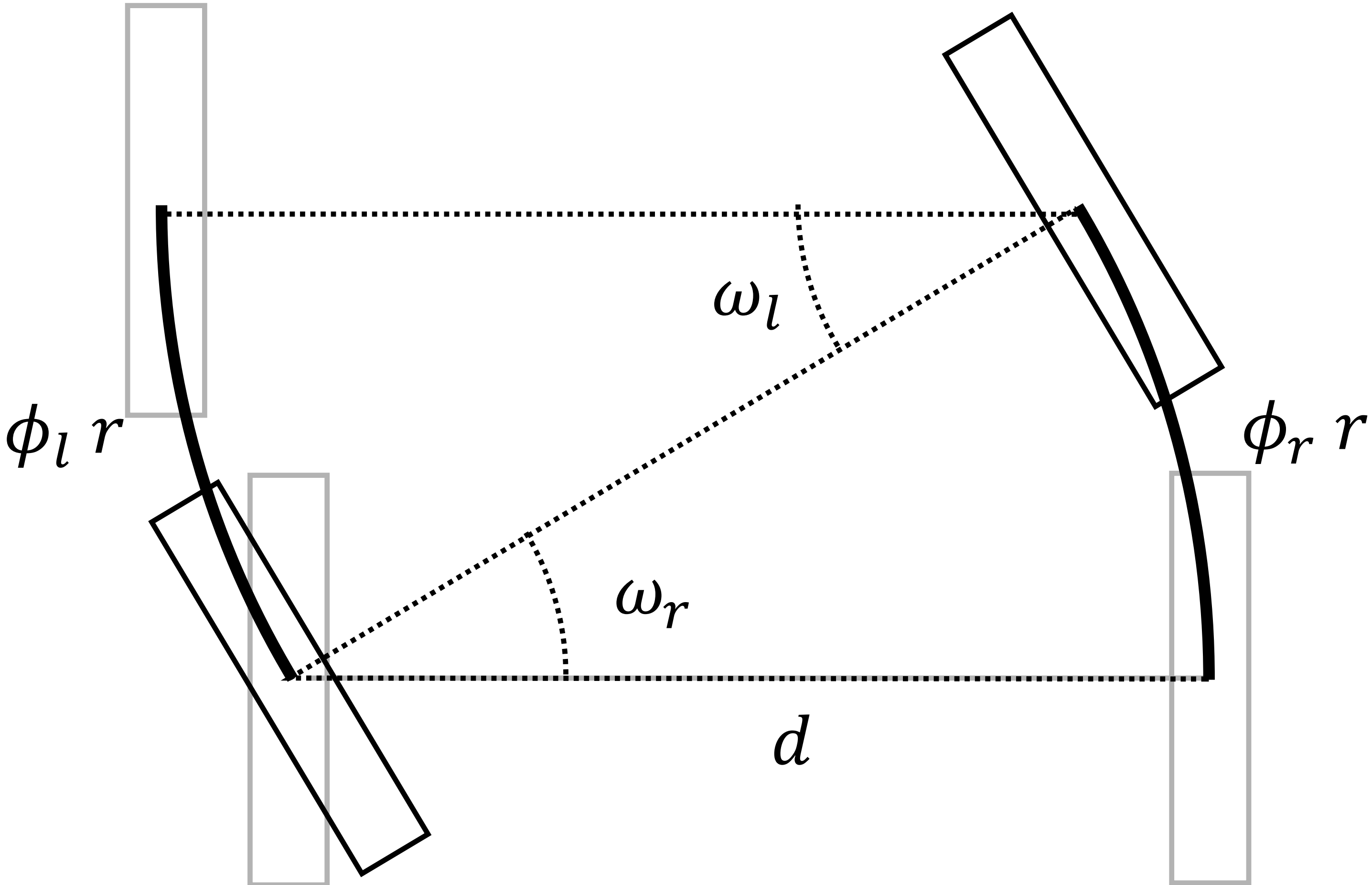
2. Wheel motion → Position Updates



$$\dot{x}_r = \frac{1}{2}\dot{\phi} r$$

$$\omega_r d = \phi_r r \;\rightarrow\; \dot{\omega}_r = \frac{\dot{\phi}_r r}{d}$$

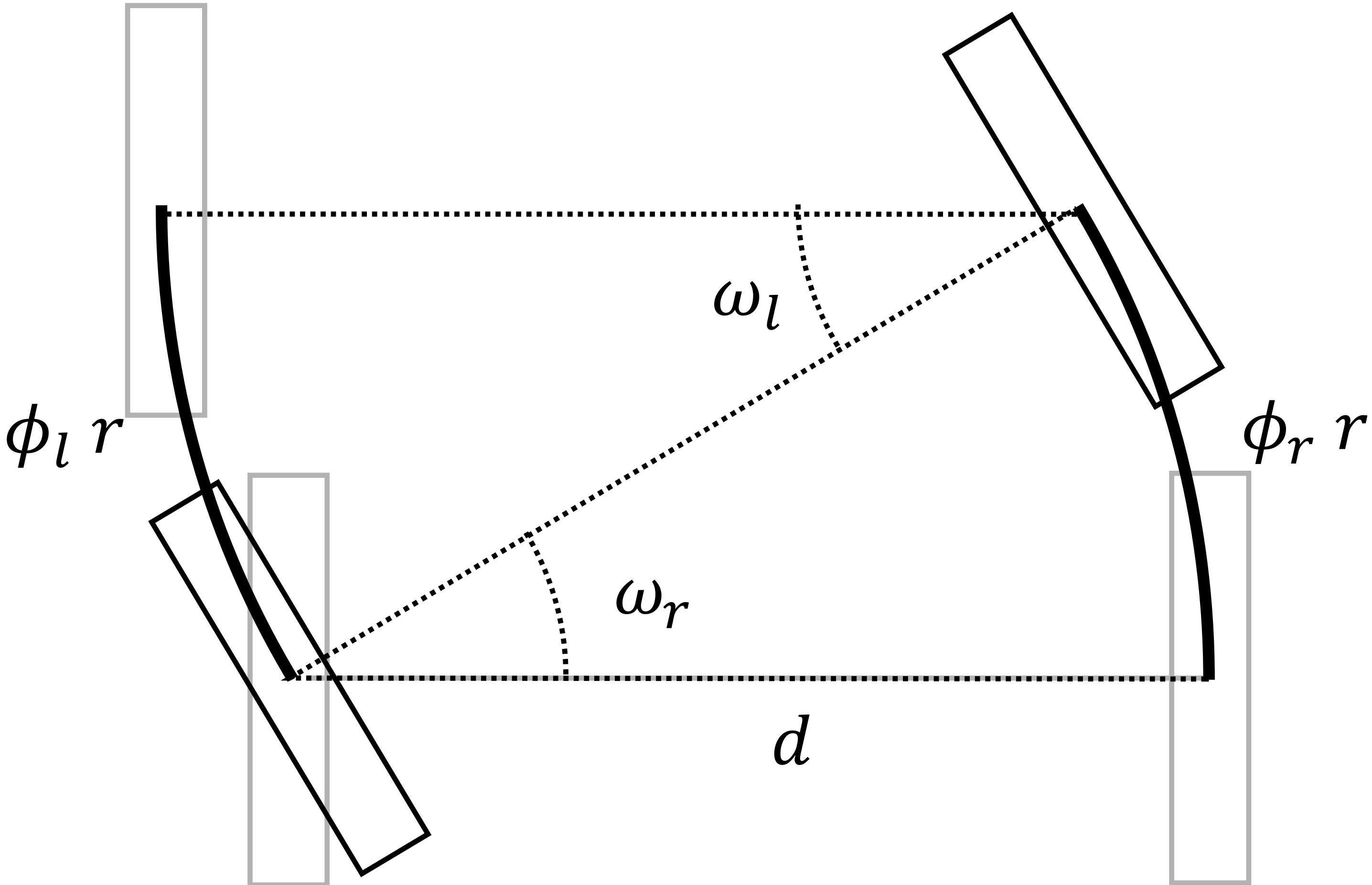
2. Wheel motions → Position Updates



What about the case where both wheels are moving at different speeds $\dot{\phi}_l$ and $\dot{\phi}_r$?

We can decouple the instantaneous contributions of $\dot{\phi}_l$ and $\dot{\phi}_r$!!

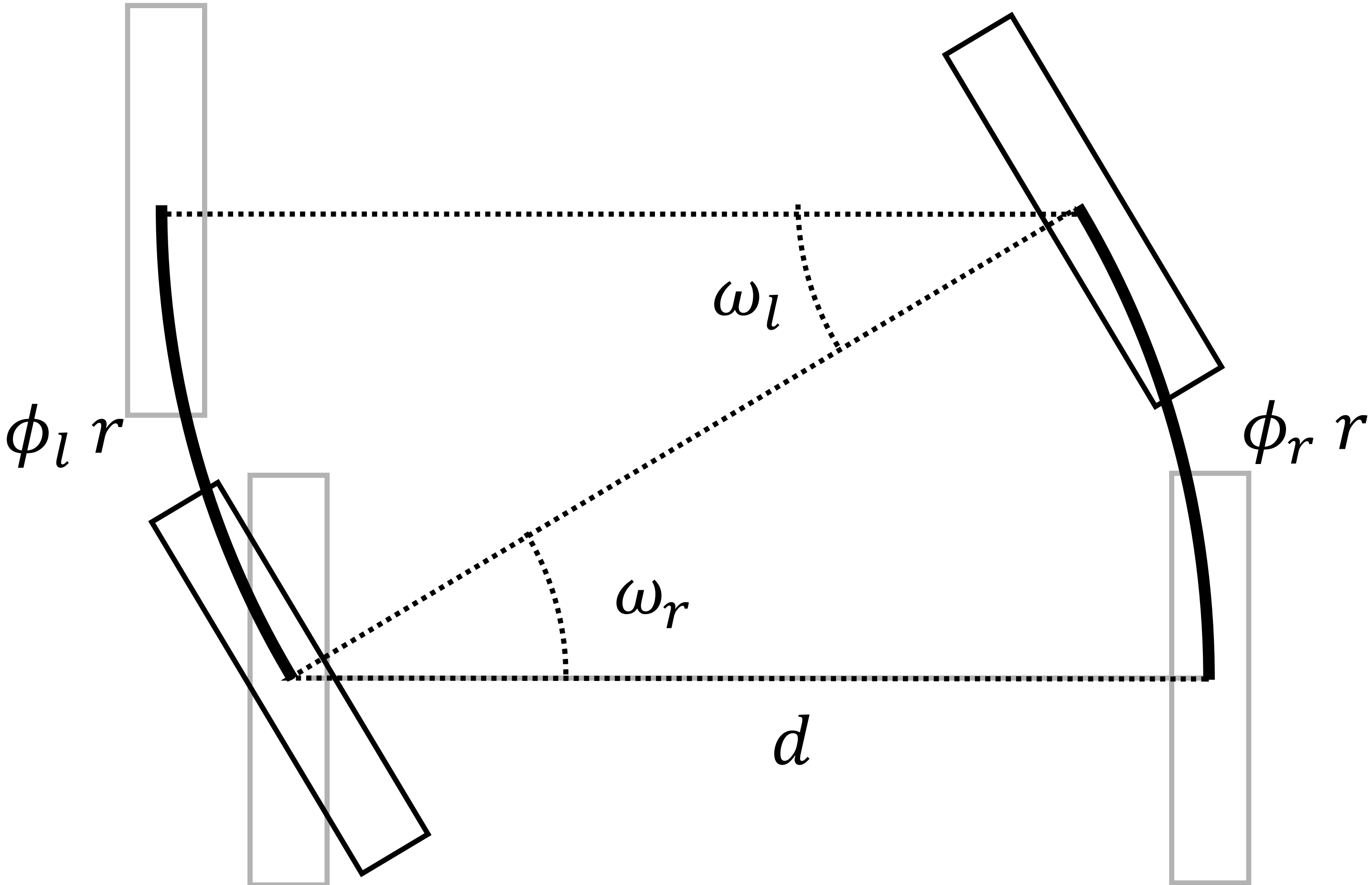
2. Wheel motions → Position Updates



What about the case where both wheels are moving at different speeds $\dot{\phi}_l$ and $\dot{\phi}_r$?

$$\dot{x}_r = \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2}$$

2. Wheel motions → Position Updates

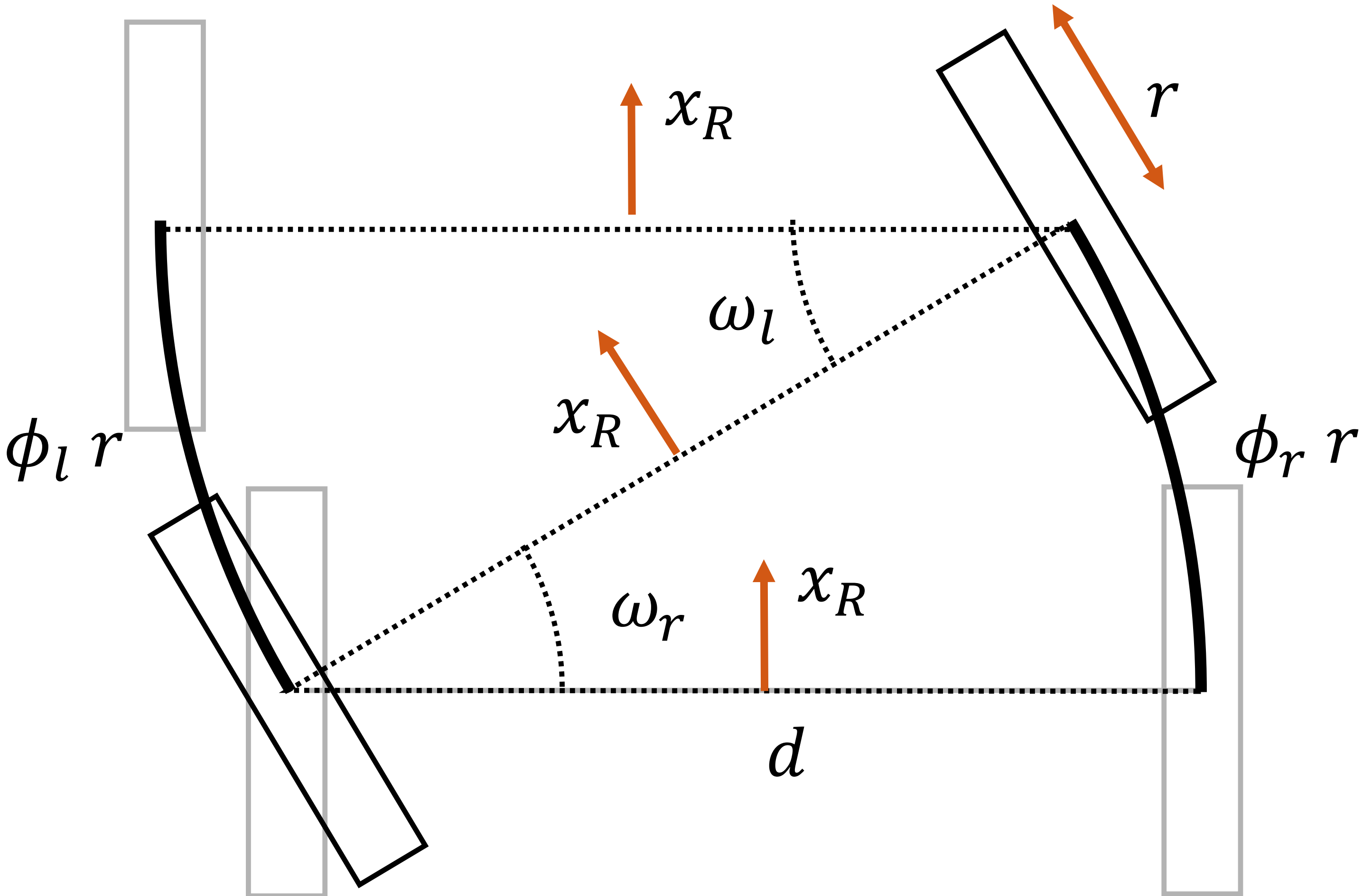


$$\dot{\omega}_r = \frac{\dot{\phi}_r r}{d}$$

$$\dot{\omega}_l = \frac{\dot{\phi}_l r}{d}$$

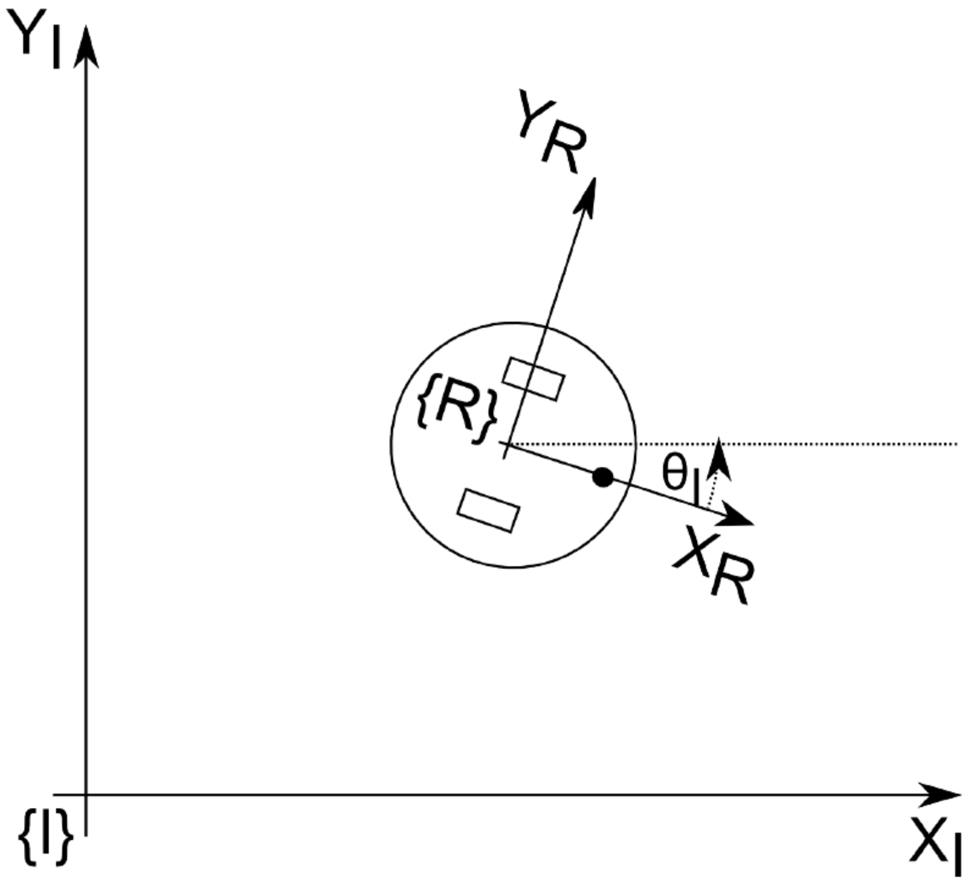
$$\dot{\theta} = \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d}$$

2. Forward Kinematics of mobile robot



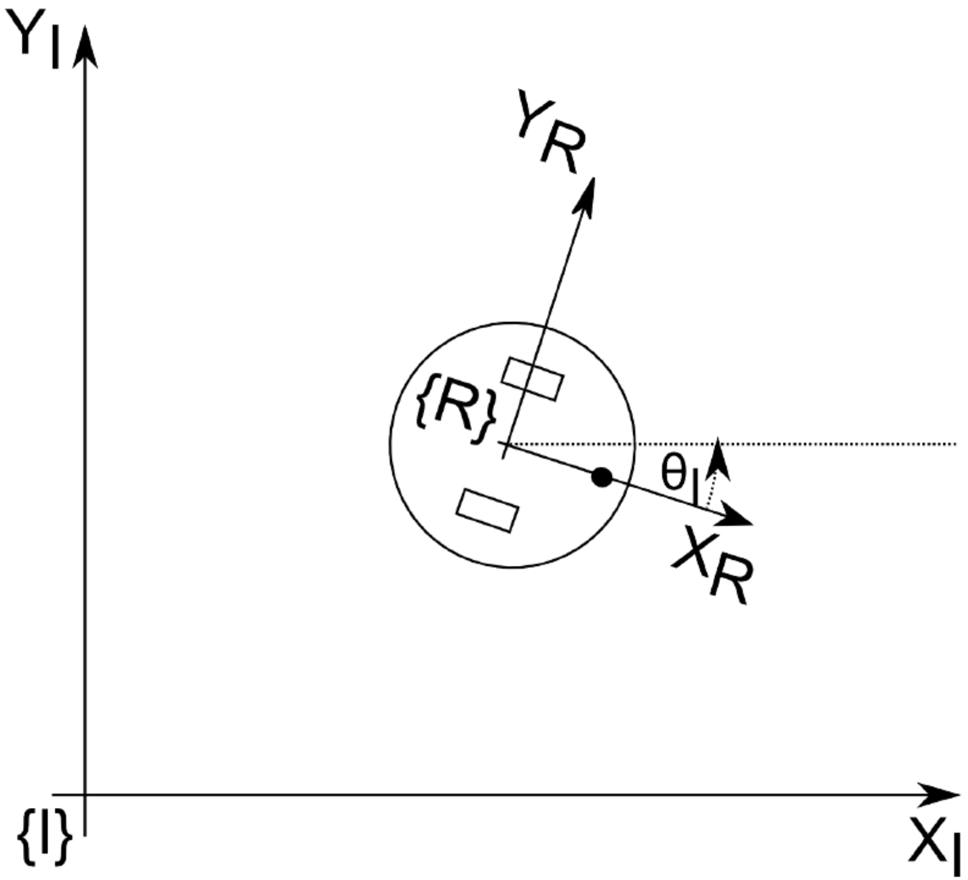
$$\begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2} \\ 0 \\ \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d} \end{bmatrix}$$

3. Forward Kinematics + Odometry



$$\begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2} \\ 0 \\ \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d} \end{bmatrix}$$

3. Forward Kinematics + Odometry



$$\dot{x}_{I,x} = cos(\theta)\dot{x}_R.$$

$$\dot{x}_{I,y} = -sin(\theta)\dot{y}_R.$$

$$\dot{x}_I = cos(\theta)\dot{x}_R - sin(\theta)\dot{y}_R$$

$$\dot{y}_I = sin(\theta)\dot{x}_R + cos(\theta)\dot{y}_R$$

$$\dot{\theta}_I = \dot{\theta}_R$$

$$\begin{pmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{r\dot{\phi}_l}{2} + \frac{r\dot{\phi}_r}{2} \\ 0 \\ \frac{\dot{\phi}_r r}{d} - \frac{\dot{\phi}_l r}{d} \end{pmatrix}$$

4. Speeds → How can we compute positions?

$$\begin{pmatrix} x_I(T) \\ y_I(T) \\ \theta(T) \end{pmatrix} =$$