Computer Assignment #1 Due January 30, 2020

An elastic spherical ball is subjected to an internal pressure as shown below:

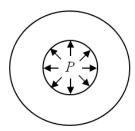


Figure 1 Cross-sectional view of the hollow sphere

The spherical ball is made of a linear elastic material with inner radius $R_i = 10.0$ in, outer radius $R_o = 20.0$ in, Young's modulus E = 1000.0 psi, Poisson's ratio v = 0.3, and is subjected to an internal pressure P = 10.0 psi.

The analytical solution of this problem is given below:

$$\begin{split} \sigma_{rr} &= \frac{PR_i^3(R_o^3 - r^3)}{r^3(R_i^3 - R_o^3)} \\ \sigma_{\theta\theta} &= \sigma_{\phi\phi} = -\frac{PR_i^3(2r^3 + R_o^3)}{2r^3(R_i^3 - R_o^3)} \\ \mathbf{\varepsilon} &= \mathbf{C}^{-1}\mathbf{\sigma} \\ u_r &= r\varepsilon_{\theta\theta} = r\varepsilon_{\phi\phi} \end{split}$$

Obtain FEM solutions using 11, 21, and 41 2-node linear elements and 3-node quadratic elements.

- 1. Compare FEM radial displacement solutions u_r with analytical solution at the inner and outer surfaces of the ball for different mesh refinements, and make discussions.
- 2. Compare FEM radial stress solutions σ_{rr} with analytical solution at the middle of the ball thickness for different mesh refinements, and make discussions.
- 3. Compare FEM circumferential stress solutions $\sigma_{\theta\theta}$ with analytical solution at the middle of the ball thickness for different mesh refinements, and make discussions.