MAE 290B. Final project

Winter 2019

Submission before March 19th, 5 PM

Note: *Plots and sketches must be labeled and the axis well defined. Staple the report and number the pages. Torn-out pages are not allowed. Submit a hard copy including all the plots and codes in EBU2 238. Computer programs must be* **written on your own**, *included in the report and also submitted through TritonEd.*

1. The two-dimensional unsteady diffusion equation with a heat source/sink is to be solved in the square domain $0 \leq x \leq 1$, $0 \leq y \leq 1$,

$$T_t = \alpha(T_{xx} + T_{yy}) + Q(x, y, t), \tag{1}$$

where $Q(x, y, t)$ is the forcing term. The temperature at the four boundaries is $T = 0$ and the initial temperature is also $T = 0$. The source term is defined as

$$Q(x, y, t) = 2.5 \sin(3\pi x) \sin(4\pi y) f(t)\,, \tag{2}$$

where the time dependence is given by

$$f(t) = 1 - \mathrm{e}^{-at} \cos(\Omega t) \cos(2\Omega t)\,. \tag{3}$$

The ADI method will be used to obtain a time accurate solution for $T(x, y, t)$. Use second-order FD and *your own Thomas algorithm* for inverting the tridiagonal matrix.

a) Perform an accuracy and stability analysis of the ADI method.

10 points

b) Implement the numerical method to obtain a time-accurate solution for $T(x, y, t)$ with $N_x = N_y = 81$, i.e. a uniform grid resolution of $\Delta x = \Delta y = 0.0125$. Let $\alpha = 0.1$, $a = 3$ and $\Omega = 30$. Explain with detail your choice of the time step Δt. Plot the time evolution of $T(x = 0.55, y = 0.45, t)$ and obtain the required time to reach steady state. Plot the 2D-contour plot, $T(x, y)$ at final time (Hint: you can use Matlab's *contourf* command).

50 points

c) At a given time, t_n, the solution is known as $T_n(x, y)$. The error during the time advance of the solution to t_{n+1} has to be smaller than a tolerance ε. Ignore the spatial discretization error for this part. Analytically, estimate the maximum value of the time step as a function of ε.

15 points

d) Explore the influence of varying Ω and α by comparing the time evolution of $T(x = 0.55, y = 0.45, t)$.

10 points

e) Considering all the tools that you have learnt in this course, what is the most accurate numerical method that you could apply to this problem? Give *details* of your method. No programming required.

15 points