MAE 290C, Spring 2019
HOMEWORK 4
Due May 27 11:59 PM (google drive, email, dropbox, etc)
Provide source codes used to solve all questions

Integrate numerically the linear wave equation

$$\partial_t u + c\partial_x u = 0,$$

in the domain $0 \leq x < 10$ with homogeneous initial conditions and $u(x = 0, t) = \sin(At)$ Solve using second-order centered finite difference schemes with $N = 200$ grid points, $\Delta t = 0.01$ and the following boundary conditions at the artificial exit:

1. Homogeneous boundary conditions, $u_N = 0$.
2. Linear extrapolating boundary conditions, $u_N = 2u_{N-1} - u_{N-2}$.
3. Quadratic extrapolating boundary conditions, $u_N = 3u_{N-1} - 3u_{N-2} + u_{N-3}$.
4. Homogeneous Neumann boundary conditions, $u_N = u_{N-1}$.
5. Antisymmetric boundary conditions, $u_N = -u_{N-1}$.
6. First-order upwinding convective boundary conditions,
$$d_t u_N = -c\frac{u_N - u_{N-1}}{\Delta x}.$$
7. Second-order upwinding convective boundary conditions,
$$d_t u_N = -c\frac{3u_N - 4u_{N-1} + u_{N-2}}{2\Delta x}.$$

Perform an analytical study of the reflection of waves generated by the each scheme at the artificial boundary and discuss the results obtained for $A = 0.1$ and $A = 3$. Compare your numerical results with the analysis of each boundary condition.