MAE 290C, Spring 2019

CODE PROJECT

Due Sun June 16 11:59PM (google drive, email, Dropbox, etc)

Provide source code

Building on homework modules 1 and 2, write a spectral code to simulate 2D turbulence in a 1536×1536 square periodic box. Start with the 2D Navier-Stokes equations in stream-funcion / vorticity formulation and discretize them spatially using a Fourier spectral method. Use your favorite dealiasing scheme (e.g. 2/3 rule would use 1024×1024 Fourier modes) and a third-order, low storage, semi-implicit Runge-Kutta method to integrate in time. Solve these equations for a Reynolds number

$$Re = \frac{u' L}{\nu} = 5 \times 10^4,$$

where L is the size of the box, ν is the kinematic viscosity and u' is the root mean square of the velocity fluctuations at $t = 0$. Set a random initial condition with energy spectrum

$$E(k) = \widehat{u}_k \widehat{u}_k^* = \frac{4\pi}{3} u'^2 \frac{(kL)^2}{[1 + (kL)^2]^{5/3}}.$$

Run this simulation and

1. Estimate the transient time that is required for the random initial conditions to transform into a sea of compact vortices.

2. Plot the time evolution of the total enstrophy and energy in the box. Which one decays faster? Why?

3. Plot 2D vorticity fields for the instants of time when the energy is 90%, 80%, 70%, 60% and 50% of its initial value. Describe qualitatively the evolution of vortex number and size.