Group task
(Topics 3 & 6)

Mass distribution of a distant elliptical galaxy from gravitational lensing

LENSMODEL software for modelling gravitational lens systems

http://physics.rutgers.edu/~keeton/gravlens/2012WS/

IAC Winter School

Chuck Keeton (November 2012)

Latest executables:

Mac (2017/12/22)
Fedora: 32-bit, 64-bit (old)
Ubuntu (2016/7/14)
Scientific Linux (2017/12/22)

Note -- you may need to issue the following command to make the file executable:
chmod u+x lensmodel

Basic principles and methods

see
Keeton_Slides.pdf
Keeton_TheoMods.pdf

Strong lens modeling

goal: use strong lensing data to learn about...

- mass model of a lens galaxy
- other parameters $\rightarrow$ its environment

focus:

- galaxy-scale lensing

"inverse" problem:

- fix lens data,

observational constraints: positions of two quasar images and lens galaxy, fluxes of images, time delay between images + "reasonable" *priors on model parameters*

- solve for model parameters

Least-squares fitting

general goal: minimize the difference between the model and data

quantify **goodness of fit**:

$$\chi^2 = \sum \frac{(\text{model} - \text{data})^2}{(\text{uncertainties})^2}$$

idea:

- find best fit (minimum χ^2)
- explore different mass models

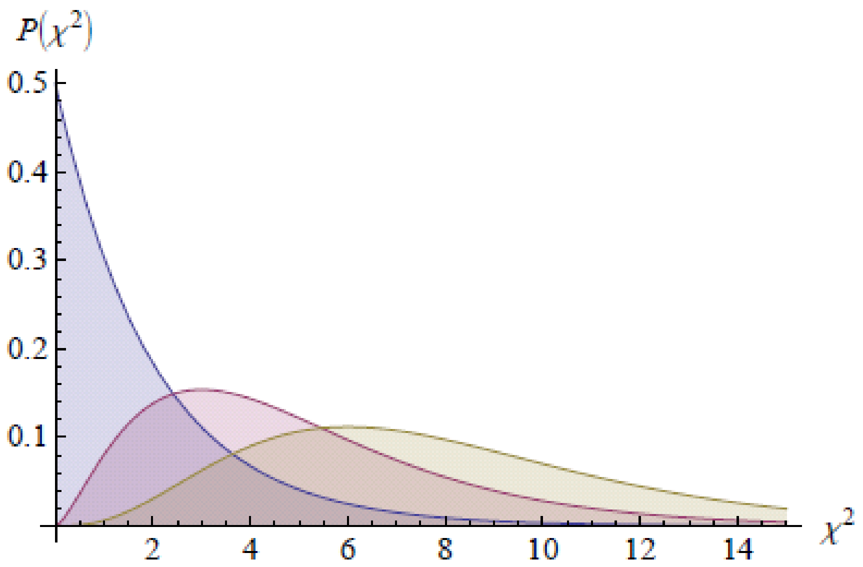
What is "good enough"?

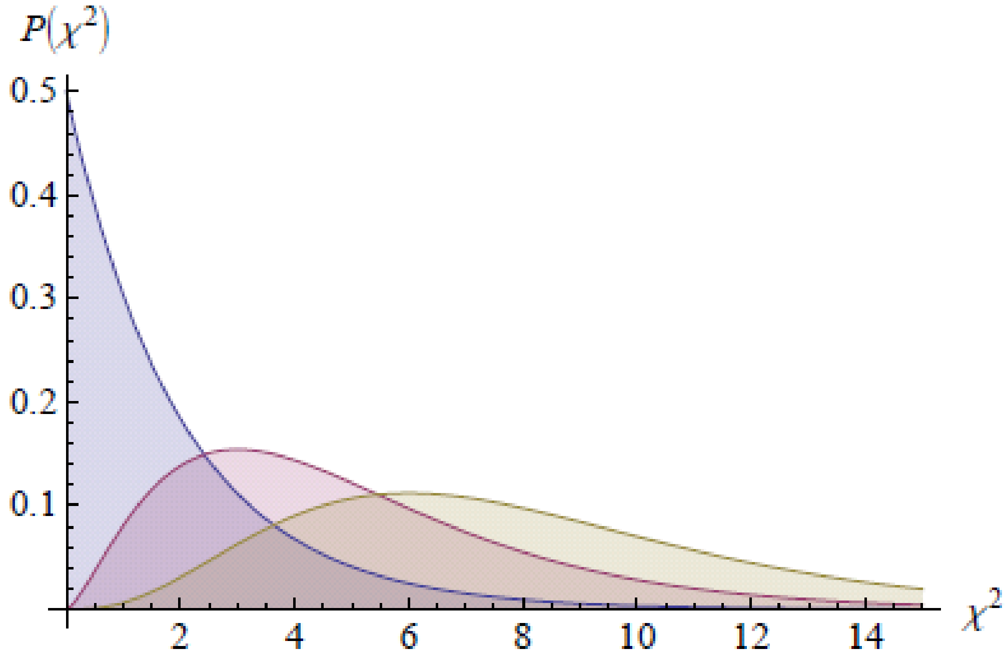
quantify **degrees of freedom**:

$$\nu = (\#\ \text{constraints}) - (\#\ \text{free parameters})$$

if errors are random, have probability distribution for χ^2:

$$p(\chi^2|\nu) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)}\ (\chi^2)^{\nu/2-1}e^{-\chi^2/2}$$





average:

$$\langle \chi^2 \rangle = \nu$$

peak:

$$\chi^2_{\text{peak}} = \max(\nu - 2, 0)$$

as a rule of thumb, we expect $\chi^2 \approx \nu$ for a "good" fit;

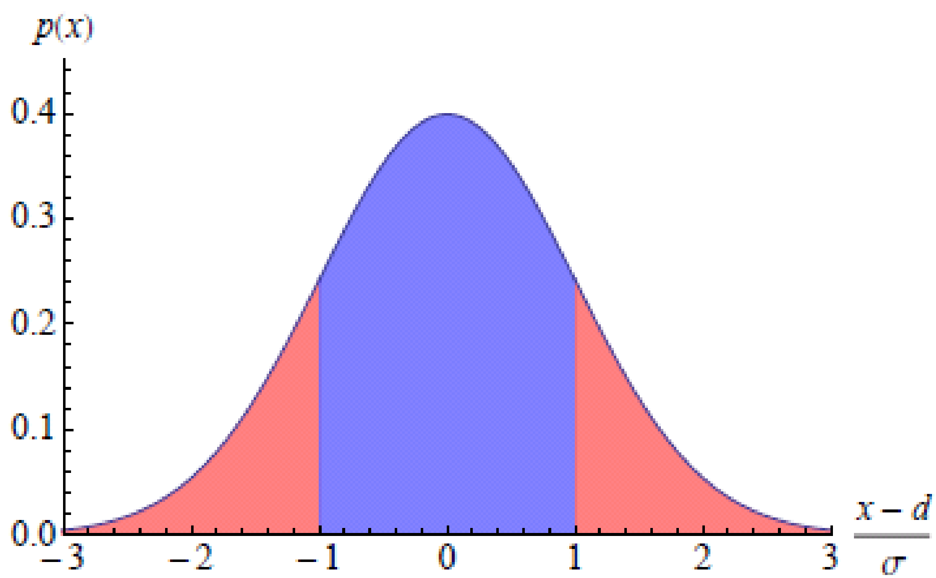
Errorbars

"likelihood"

$$\mathcal{L} \propto e^{-\chi^2/2}$$

1-d Gaussian

$$\chi^2 = \frac{(x-d)^2}{\sigma^2} \quad \Rightarrow \quad \begin{cases} \pm 1\sigma : & \Delta\chi^2 = 1 \ (68\%) \\ \pm 2\sigma : & \Delta\chi^2 = 4 \ (95\%) \end{cases}$$



central region = 68% of the probability; each tail = 16%

Non-linear parameters

must explicitly search parameter space

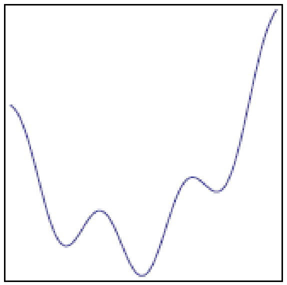
use established algorithms to search for minimum of a function in multiple dimensions

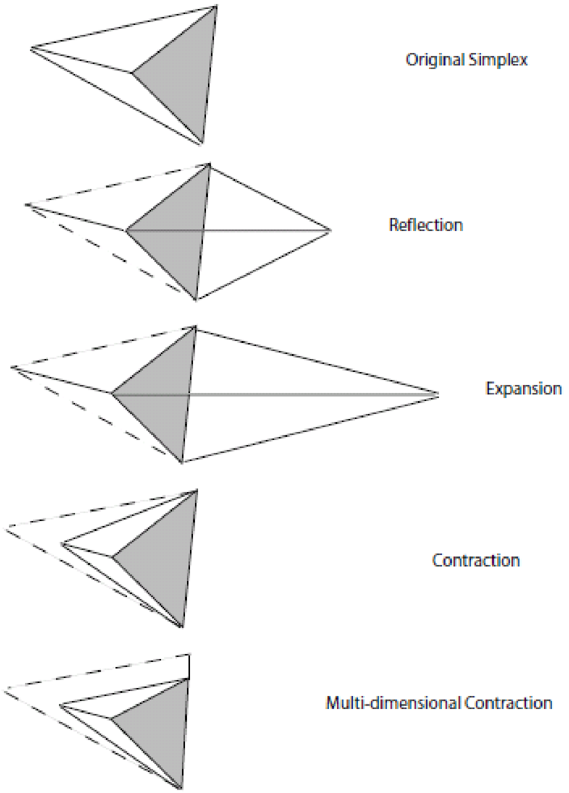
Downhill simplex method ("amoeba")

http://www.cs.usfca.edu/~brooks/papers/amoeba.pdf — also Numerical Recipes

challenges:

- computational effort
- local minima
- long, narrow valleys
- degeneracies





Linear + non-linear parameters

suppose we have parameters a and b such that

$$d^{\rm mod} = a\, f(b)$$

then

$$\chi^2(a,b) = \sum \frac{[a\, f(b) - d^{\rm obs}]^2}{\sigma^2}$$

optimal value of a:

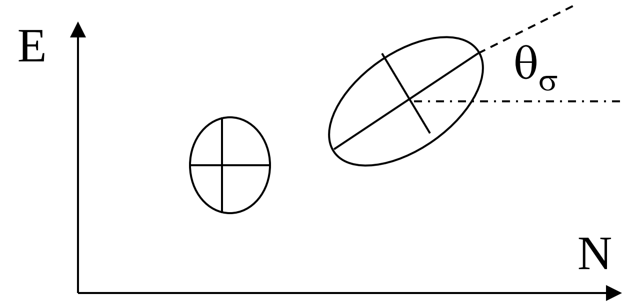
$$0 = \frac{\partial \chi^2}{\partial a} = 2 \sum \frac{f(b)[a\, f(b) - d^{\rm obs}]}{\sigma^2} \quad \Rightarrow \quad a_{\rm opt} = \frac{\sum f(b) d^{\rm obs}/\sigma^2}{\sum f(b)^2/\sigma^2}$$

then

$$\chi^2(b) = \chi^2(a_{\rm opt}(b), b)$$

we can still optimize the linear parameters analytically

Position constraints



"exact" position χ^2:

$$\chi^2_{\rm pos} = \sum_{\rm images} (\mathbf{x}_i^{\rm mod} - \mathbf{x}_i^{\rm obs})^t \, \mathbf{S}_i^{-1} \, (\mathbf{x}_i^{\rm mod} - \mathbf{x}_i^{\rm obs})$$

astrometric uncertainties: error ellipse with axes $(\sigma_{1i}, \sigma_{2i})$ and position angle $\theta_{\sigma i}$ (East of North) $\rightarrow$ covariance matrix

$$\mathbf{S}_i = \mathbf{R}_i \begin{bmatrix} \sigma_{1i}^2 & 0 \\ 0 & \sigma_{2i}^2 \end{bmatrix} \mathbf{R}_i^t \qquad \mathbf{R}_i = \begin{bmatrix} -\sin\theta_{\sigma i} & -\cos\theta_{\sigma i} \\ \cos\theta_{\sigma i} & -\sin\theta_{\sigma i} \end{bmatrix}$$

if symmetric uncertainties:

$$\mathbf{S}_i = \begin{bmatrix} \sigma_i^2 & 0 \\ 0 & \sigma_i^2 \end{bmatrix}$$

note: define source position associated with each observed image

$$\mathbf{u}_i^{\rm obs} = \mathbf{x}_i^{\rm obs} - \alpha(\mathbf{x}_i^{\rm obs})$$

also

$$\mathbf{u}^{\rm mod} = \mathbf{x}^{\rm mod} - \alpha(\mathbf{x}^{\rm mod})$$

subtract:

$$\delta\mathbf{u}_i \;=\; \delta\mathbf{x}_i - \left[\alpha(\mathbf{x}^{\rm mod}) - \alpha(\mathbf{x}_i^{\rm obs})\right] \;\approx\; \mu_i^{-1}\cdot\delta\mathbf{x}_i$$

provided that model is decent, such that $\delta\mathbf{x}_i$ and $\delta\mathbf{u}_i$ are "small"

then $\delta\mathbf{x}_i \approx \mu_i \cdot \delta\mathbf{u}_i$ yields "approximate" position χ^2:

$$\chi^2_{\rm pos} \approx \sum_i (\mathbf{u}^{\rm mod} - \mathbf{u}_i^{\rm obs})^t\, \mu_i^t\, \mathbf{S}_i^{-1} \mu_i\, (\mathbf{u}^{\rm mod} - \mathbf{u}_i^{\rm obs})$$

$$\chi^2_{\text{pos}} \approx \sum_i (\mathbf{u}^{\text{mod}} - \mathbf{u}_i^{\text{obs}})^t \, \mu_i^t \, \mathbf{S}_i^{-1} \mu_i \, (\mathbf{u}^{\text{mod}} - \mathbf{u}_i^{\text{obs}})$$

advantages:

- don't need to solve lens equation
- $\mathbf{u}^{\text{mod}}$ is a linear parameter, so optimize it analytically

$$\mathbf{u}^{\text{mod}} = \mathbf{A}^{-1} \, \mathbf{b}$$

where $$\mathbf{A} = \sum_i \mu_i^t \, \mathbf{S}_i^{-1} \, \mu_i \qquad \mathbf{b} = \sum_i \mu_i^t \, \mathbf{S}_i^{-1} \, \mu_i \, \mathbf{u}_i^{\text{obs}}$$

concerns:

- approximation is valid only when residuals are small ... but χ^2_{pos} yields a large value (i.e., bad fit) in either case
- since we do not solve the lens equation, we cannot check that the model predicts correct number of images ... only worry about models yielding *too many* images

Flux constraints

$$\chi^2_{\text{flux}} = \sum_i \frac{(F_i^{\text{obs}} - \mu_i F^{\text{src}})^2}{\sigma^2_{f,i}}$$

if desired, include parity by letting F_i^{obs} and μ_i be signed

optimal source flux can be found analytically

$$F^{\text{src}} = \frac{\sum_i F_i^{\text{obs}} \mu_i / \sigma^2_{f,i}}{\sum_i \mu_i^2 / \sigma^2_{f,i}}$$

if desired, straightforward to switch to magnitudes

$$m_i^{\text{mod}} = m^{\text{src}} - 2.5 \log |\mu_i|$$

note: photometric units are arbitrary — absolute fluxes or magnitudes, or relative values

Time delay constraints

predicted time delay

$$t_i^{\rm mod} = t_0 \tau_i^{\rm mod} + T_0$$

$$\begin{aligned}
\text{model:} \quad \tau_i^{\rm mod} &= \frac{1}{2}\left|\mathbf{x}_i^{\rm mod} - \mathbf{u}^{\rm mod}\right|^2 - \phi\left(\mathbf{x}_i^{\rm mod}\right) \\
\text{cosmol:} \quad t_0 &= \frac{1+z_l}{c}\,\frac{D_l D_s}{D_{ls}} = H_0^{-1} \times f(\Omega_M, \Omega_\Lambda; z_l, z_s)
\end{aligned}$$

note: time zeropoint T_0 does not affect differential time delays; but let's make framework general

then

$$\chi^2_{\rm tdel} = \sum_i \frac{(t_i^{\rm obs} - t_0 \tau_i^{\rm mod} - T_0)^2}{\sigma_{t,i}^2}$$

Main galaxy

Keeton_SoftManual.pdf:
warning at page 23!

softened power law ellipsoid

$$\kappa = \frac{1}{2}(b)^{2-\alpha}\left[(s)^2 + \zeta^2\right]^{\alpha/2-1}$$

$$\zeta^2 = (1-\varepsilon)X^2 + (1+\varepsilon)Y^2$$

$$q^2 = (1-\varepsilon)/(1+\varepsilon)$$

where

SE $\boxed{s = 0}$ **SIE**

$$M(r) \sim r^\alpha \quad \Rightarrow \quad \alpha \begin{cases} < 1 & \text{steeper than isothermal} \\ \boxed{= 1 \quad \text{isothermal}} & \\ > 1 & \text{shallower than isothermal} \end{cases}$$

lensmodel has many other model classes: point mass, pseudo-Jaffe, de Vaucouleurs, Hernquist, Sersic, NFW, Nuker, exponential disk, . . .

elliptical with M/L = constant

Environmental effects

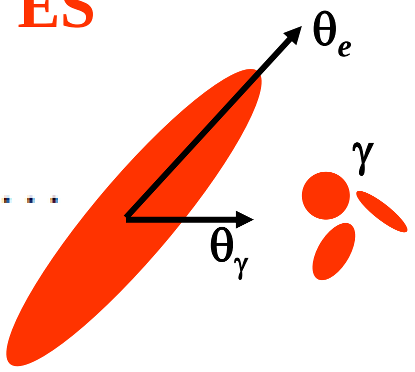
few lens galaxies are isolated — they have neighbors, and may be embedded in groups or clusters of galaxies

environments can affect the light bending by an amount larger than the measurement uncertainties

if neighboring galaxies are "far" from the lens (compared with Einstein radius), make Taylor series expansion

ES

$$\begin{aligned}\phi_{\rm env} \quad &= \quad \phi_0 + \mathbf{a}\cdot\mathbf{x} + \frac{\kappa_c}{2} r^2 + \boxed{\frac{\gamma}{2} r^2 \cos 2(\theta - \theta_\gamma)} \\ &\quad + \frac{\sigma}{4} r^3 \cos(\theta - \theta_\sigma) + \frac{\delta}{6} r^3 \cos 3(\theta - \theta_\delta) + \ldots\end{aligned}$$



structures along the line of sight can also affect the light bending
. . . more complicated

How does it work with a double QSO?

http://www.physics.rutgers.edu/~keeton/gravlens/manual.pdf

Keeton_SoftManual.pdf

First, you need a data set to be fitted (observational constraints), e.g.,
QSOname.dat [#9] + **prior.dat** [#2]

optical position of G [#2]
optical positions of A&B [#4]
fluxes of A&B [#2]
time delay between A&B [#1]

optical ellipticity and position angle of G

```
2                # Npri
1 4 e_G error    # G: ellip and its error
1 5 θ_eG error   # G: PA and its error
```

```
1                # Ngal
x_G y_G errxy    # G: position and error (in ")
0.0 1000
0.0 1000
0.0 1000

1                # Nsrc
2                # Nimg
x_A y_A F_A errxy errF delayA errdelay    # image A
x_B y_B F_B errxy errF delayB errdelay    # image B
```

p[1]	=	b	=	mass scale
(p[2], p[3])	=	(x_0, y_0)	=	galaxy position
(p[4], p[5])	=	(e, θ_e)	=	ellipticity parameters
(p[6], p[7])	=	(γ, θ_γ)	=	external shear parameters
(p[8], p[9])	=	(s, a)	=	misc., often scale radii
p[10]	=	α	=	misc., often a power law index

Second, choose a mass model (and more things), e.g., **QSOnameSES.ini** [#11]

SE + ES mass model for L [#8]: $(b, x_0, y_0, e, \theta_e, \alpha) + (\gamma, \theta_\gamma)$
source [#3]: x_s, y_s, F_s

$\chi^2 \sim 0$!

Standard cosmology

PROCEDURE

```
set omega = 0.3                    # cosmology & redshifts
set lambda = 0.7
set hval = 0.7
set hvale = 0.0
set zlens = zlens
set zsrc = zsrc
#         observations, priors and optimization parameters
set omitcore = 0.01
data QSOname.dat
plimits prior.dat
set checkparity = 0    # 0: don't check parities of images, 1: check parities (default)
set chimode = 1        # 0: chi2 in the source plane, 1: chi2 in the image plane (default)
set optmode = 1        # optimization algorithm: 1 -> amoeba (default), 2 -> Powell
```

h = H_0/100 (reduced Hubble constant)

continue in the next page …

```
set ftol = 1.0e-5          # tolerance in the optimization routine (default = 1.0e-4)
set restart = 5            # number of times to run the optimization routine
set alphanorm = 1          # alpha models: 0 -> catalog, 1 -> code (default)
#          mass model
startup 1 1
           alpha 1.0 x_G y_G e_G θ_eG 0.0 0.0 0 0 1
           1 1 1 1 1 1 1 0 0 0
varyone 1 10 0.8 1.2 41  QSOnameSES
quit
```

$\alpha = 1.0$

By varying α around $\alpha = 1.0$ (SIE): BEST SOLUTION FOR THE SES ≡ SE + ES MASS MODEL ($\chi^2 \sim 0$?)

&

1σ CONFIDENCE INTERVAL FOR α (χ^2 vs. α)

p[1]	=	b	=	mass scale
(p[2], p[3])		(x_0, y_0)	=	galaxy position
(p[4], p[5])	=	(e, θ_e)	=	ellipticity parameters
(p[6], p[7])	=	(γ, θ_γ)	=	external shear parameters
(p[8], p[9])	=	(s, a)	=	misc., often scale radii
p[10]	=	α	=	misc., often a power law index

Third, run the model:

./lensmodel QSOnameSES.ini

Computes χ^2 (total) and χ^2_{pos} χ^2_{flux} χ^2_{tdel} χ^2_{gal} χ^2_{plim} for each value of α

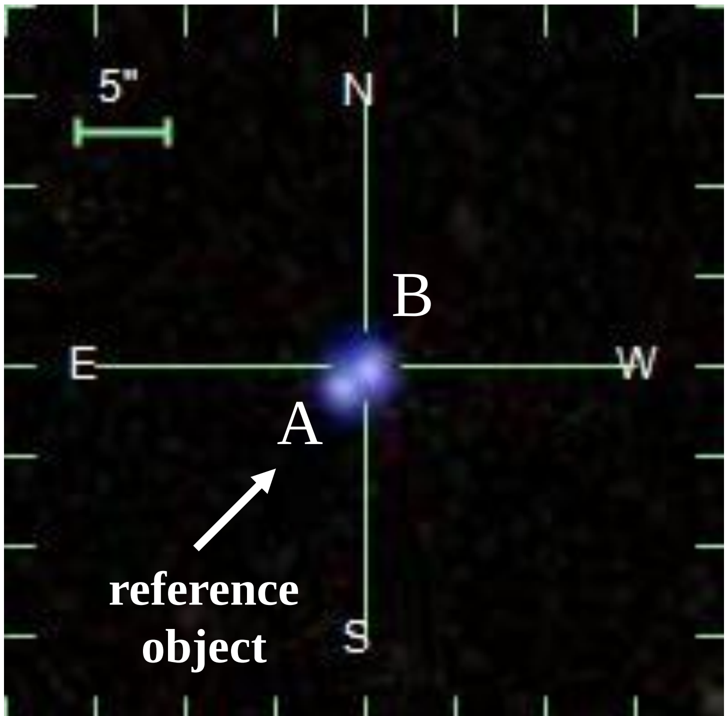
One obtain the best solution for the SE (b, x_0, y_0, e, θ_e, α) and the ES (γ, θ_γ)

One can also discuss a M/L = constant mass model, and thus compare results from both approaches: standard (SE) mass halo vs. De Vaucouleurs (DV) mass halo

We focus on the double quasar

SDSS J1339+1310

OPTICAL DATA



SDSS

<u>*QSO IMAGES*</u>:

zsrc = 2.231

$x_A = y_A = 0.0 \pm 0.0001''$

$x_B = 1.419 \pm 0.001''$, $y_B = 0.939 \pm 0.001''$

$F_A = 1.000 \pm 0.001$ a.u. , $F_B = 0.175 \pm 0.015$ a.u.

$\Delta t_{AB} = 48 \pm 2$ days

(delayA = 0.0 ± 0.1 days , delayB = 48.0 ± 2.0 days)

<u>*LENSING GALAXY*</u>: zlens (**galaxy G in Individual task #2**)

$x_G = 0.981 \pm 0.010''$, $y_G = 0.485 \pm 0.010''$

DV light model: $R_{\mathrm{eff}} = 0.96 \pm 0.07''$, $e_G = 0.18 \pm 0.05$, $\theta_{eG} = 32 \pm 10$ deg

Q1: Using optical data of SDSS J1339+1310, construct the files QSO1339.dat, prior.dat and QSO1339SES.ini, i.e., design a program to model the lensing mass as a SE + ES. Taking the degrees of freedom (ν) into account, do you obtain good results ($\chi^2 \sim 0$ for $\nu = 0$) when running the program? What is the best solution for the full mass model (including α)? Do you obtain $\alpha_{best} = 1$ (SIE mass model for the lens galaxy)? Give the 1σ confidence interval for α. Is the mass of G aligned with galaxies around it?

Q2: Using optical data of SDSS J1339+1310, construct the files QSO1339.dat, prior2.dat (incorporating a prior on R_{eff}) and QSO1339DVES.ini, i.e., design a program to model the lensing mass as a DV + ES, where we are assuming that light traces mass. Taking the degrees of freedom (ν) into account, do you obtain good results ($\chi^2 \sim 1$ for $\nu = 1$) when running the program? What conclusion do you draw?

Q3: Compare results in Q1 and Q2, and put your conclusions into perspective. For example, does the light of G trace its mass? is it reasonable to assume a SIE model for the total (dark+luminous) mass distribution of G? are your results consistent with those for local galaxies with different morphology (e.g. the Milky Way)?…