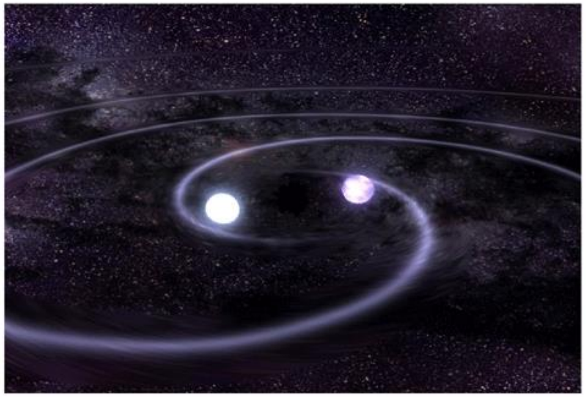
Individual task #3
(Topics 3–7)

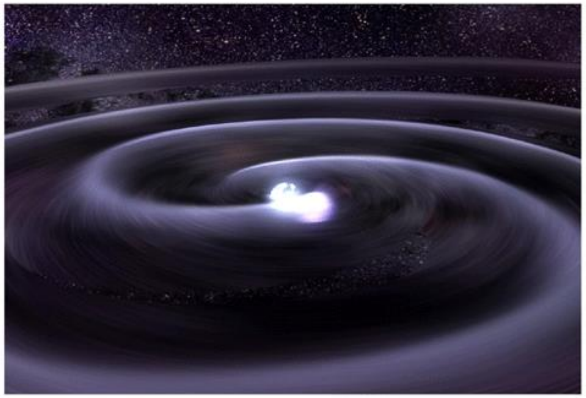
Analysis of the gravitational wave event LIGO/GW150914: is dark matter made of primordial stellar-mass black holes?

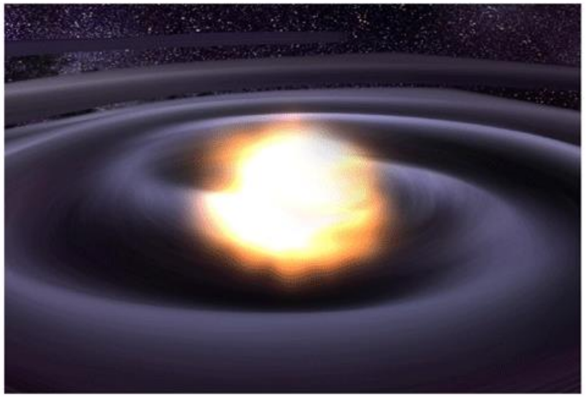
Gravitational waves are fluctuations in the curvature of the space-time (General Relativity) that propagate at the speed of light, and they come from sources including accelerated masses ($\partial^2\rho/\partial t^2 \neq 0$), e.g. in **binary systems**. While accelerated electrical charges are related to the emission of electromagnetic radiation (electromagnetic waves), accelerated masses are responsible for the emission of gravitational radiation (gravitational waves)

[ver PDFs ***Schutz*** & ***Burko***]

Two objects orbiting around their common center of mass produce gravitational waves. These waves transport energy, which leads to the coalescence of both objects after a certain time. The effect is only important when the masses and energies (potential and kinetic) are large, e.g. final phases of compact binaries including neutron stars (NS) or black holes (BH)







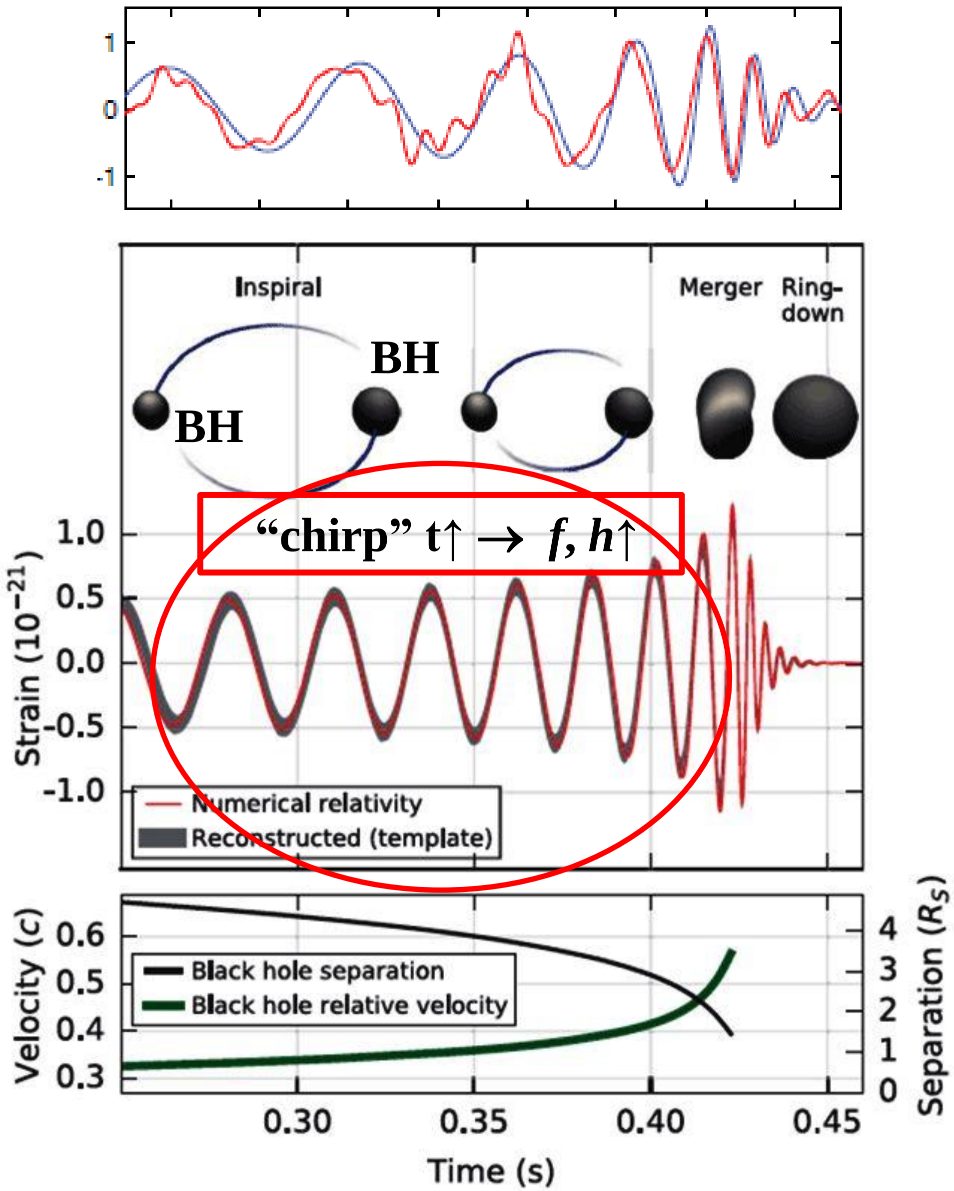
inspiral phase

merger phase

GRAVITATIONAL WAVE EVENT LIGO/GW150914

On September 14th 2015, the **LIGO collaboration detected the GW150914 event**, and later related the observed signal (strain *h*) with the **coalescence of two BHs**. When the two objects are not very close to each other (during the inspiral phase), only small relativistic corrections are required to describe the signal, while complex calculations (numerical relativity) are used to understand the merger and post-merger phases. Here, we focus on the inspiral phase and consider equations describing gravitational waves produced by objects that are well separated from each other

[ver PDFs ***LIGO1*** & ***LIGO2***]



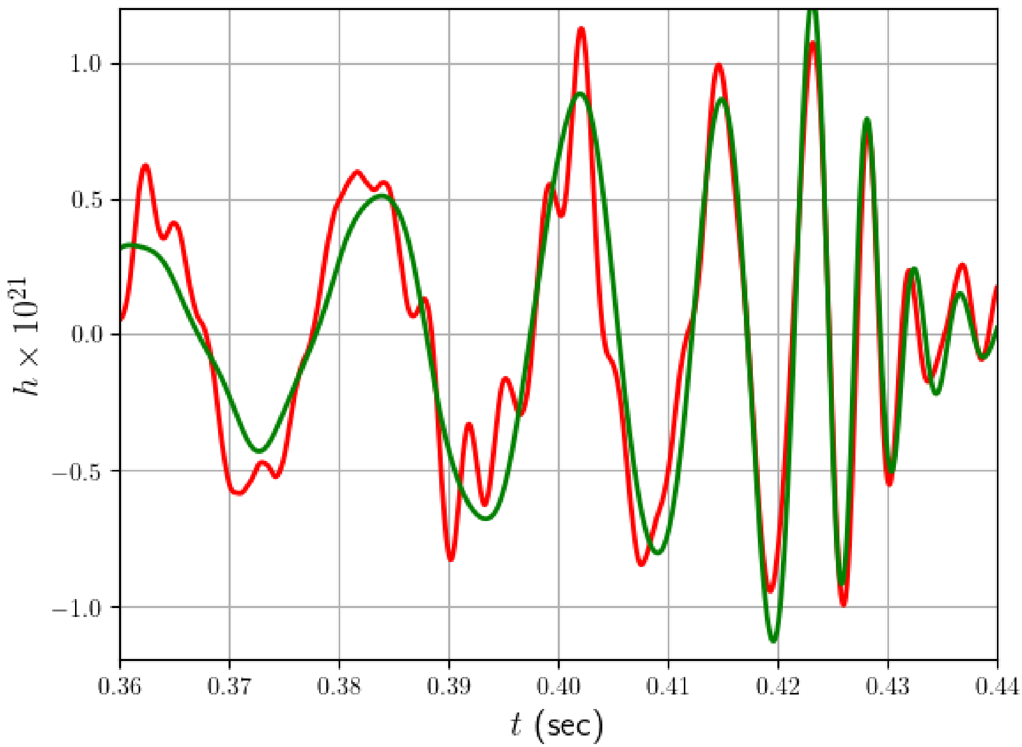
Q1: See the information on GW150914 at https://www.gw-openscience.org/events/GW150914, and then build the data files:

H1f.dat = filtered signal from the Hanford detector (H1)

H1fmod.dat = numerical relativity model for the filtered signal from H1, and

H1tmod.dat= numerical relativity model for the unfiltered signal from H1.

Show in the same figure the data in *H1f.dat* (sometimes signals are filtered to remove observational noise) and *H1fmod.dat* around the coalescence epoch, i.e. from 0.36 to 0.44 s. Is there agreement between observed and model signal?



Q2: Focus on *H1tmod.dat* (model for the unfiltered signal) at times $t \leq 0.4$ s, i.e. before the coalescence. The strain has the form

$$h(t) = A(t) \sin [\omega (t) \times t],$$

where $\omega(t) = 2\pi f(t)$ and $f(t)$ is the frequency. Thus, both amplitude and frequency depend on time. However, reasonably assuming that variability timescales of A and ω are longer than the time interval between consecutive data ($\Delta t < 100$ μs), consider the approach

$$h(t) \approx \alpha \sin (\omega t),$$

where α and ω are constants in a time interval $[t - \Delta t, t + \Delta t]$. Find the relationship between f, h and d^2h/dt^2 in such interval, calculate the values of f at maxima and minima of h, and show the corresponding behaviour of $f(t)$, i.e. time evolution of the frequency

NOTE: Once a maximum/minimum of h is found, it is possible to use the triplet of adjacent data centred at such extreme value to obtain

$$dh/dt \approx [h(t + \Delta t) - h(t - \Delta t)] / (2\Delta t) \text{ y}$$
$$d^2h/dt^2 \approx [h(t + \Delta t) - 2h(t) + h(t - \Delta t)] / \Delta t^2$$

Data of the inspiral phase allow us to **determine a characteristic mass of the binary system**. Instead of the total mass or the reduced mass, we can estimate **the chirp mass** (***M_c***). For two objects with masses m_1 and m_2, the basic equation is

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \simeq \frac{c^3}{G} \left[\frac{5}{96} \pi^{-8/3} f^{-11/3} \dot{f} \right]^{3/5}$$

Q3: Estimate M_c of the binary system using the behaviour of $f(t)$ in **Q2** and values of df/dt from triplets of adjacent data. Show the M_c estimates in solar masses, and assume that our basic equation works better when time is smaller (well separated objects). At epochs close to the coalescence (merger), important relativistic corrections are required. Therefore, perform a linear extrapolation at t = 0.2 s (using the nine measurements), and compare the extrapolated chirp mass with the chirp mass detected by the LIGO collaboration. Are the two masses consistent to each other?

NOTE: Detected masses (m) are different to masses in the rest frame of the source (m_{RFS}), since $m_{RFS} = m/(1 + z)$ and $z \approx 0.09$

Q4: Estimate the masses of the two objects of the binary system, assuming:
(a) both members have similar mass ($m_1 \approx m_2$), and
(b) one is much more massive than other ($m_1 >> m_2$).
For the case (b), if we deal with stellar-type objects (m_1 and m_2 within the range 0.1–150 solar masses), discuss possible values of the pair (m_1, m_2). Taking into account results for cases (a) and (b), in what binary scenario have the gravitational waves been produced? NS+NS, NS+BH or BH+BH? Compare the results for both cases and detailed measurements of the LIGO collaboration (detected masses), and decide what case is more realistic

In addition to the chirp mass, it is also possible to **estimate the distance to the source of gravitational radiation**. There is a relationship between **the luminosity distance** (***D*$_{\mathbf{L}}$**), the absolute value of the signal at its extremes (maxima/minima; $|h|_0$), f and df/dt:

$$D_L \simeq \frac{5}{96\pi^2}\frac{2c}{|h|_0}\frac{\dot{f}}{f^3}$$

Q5: Show the D_L estimates at different (nine) epochs of the inspiral phase. To obtain a reliable value of D_L, do a linear extrapolation at t = 0.2 s (using the nine measurements). Compare the extrapolated value and that obtained by the LIGO collaboration. Does the signal come from a region in the MIlky Way?

Q6: As a final test, go to the web simulator at http://data.cardiffgravity.org/waveform-fitter/ and read the information about it ("About"). The observed signal (blue) corresponds to GW150914, and this signal can be compared with general relativitic simulations for a binary system formed by two objects of equal mass (yellow). Use your results (total mass in the RFS for the case (a) and D_L) to check if the simulated waves reproduce or not the observations. Show a screenshot