Middle East Technical University

Electrical and Electronics Engineering Department

EE568 - Project 1 Report

Torque in a Variable Reluctance Machine

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1. Q1 – Analytical Calculations

1.1. Part A - Inductance

Minimum and maximum reluctance occurs in the electromagnetic system when a rotating part is at 0° and 90°, respectively. Reluctance can be calculated using length of mean magnetic path, area of magnetic path and relative permeability of materials. The calculatiom of minimum reluctance and is shown in Equation 1.1.

$$R_{min} = \frac{2 \times 0.5mm}{\mu_0 2\pi \times 12.5mm \times 20mm \times k} = 2 \times 1.178E06\ 1/H \qquad 1.1$$

$$k = 77.36°/360°$$

The rotating part angle at 90° results in maximum reluctance in electromagnetic system. Since the arc of rotor cannot cover the stationary part at 90°, flux area will be less than the case at 0°. Straight lines of rotor under the arc of stator are equal to %12 of the width of stator, 15mm. However, the area reduction should not be approximated as %12, but %6 because the arc of stationary part is still included in flux area. Reluctance calculation at 90° is shown in Equation 1.2.

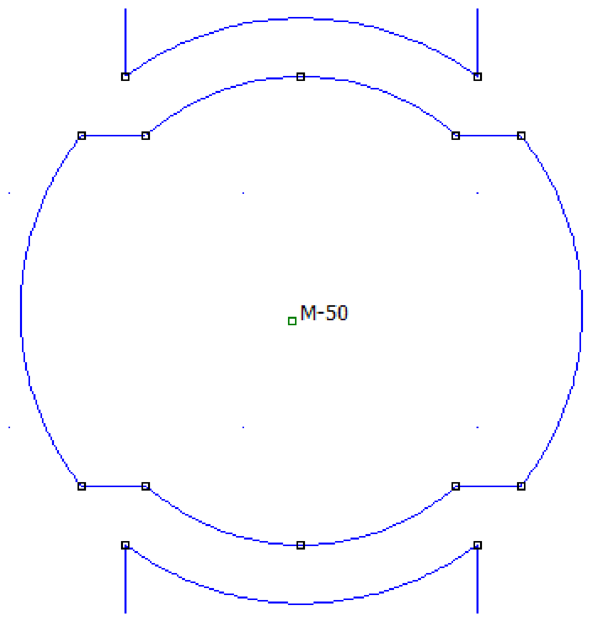

Figure 1 Representation of electromagnetic system at 90° rotor position

$$R_{max} = R_{min} \times \frac{2.5mm}{0.5mm} \times \frac{1}{0.94} = 1.253E07\ 1/H \qquad 1.2$$

Analytical calculation of minimum, maximum inductances and reluctances are given in Table 1.

Table 1 Analytical calculations of inductance and reluctance at 0° and 90°

Angle	**Reluctance (1/H)**	**Inductance (mH)**
0° degree	2.356 E06	26.528
90° degree	1.253 E07	4.988

The sinusoidal approximation for inductance variation is plotted in Figure 2 and sinusoidal approximation is given in Equation 1.3.

$$L(\theta) = \frac{L_{max} + L_{min}}{2} + \left(\frac{L_{max} - L_{min}}{2}\right)\cos(2\theta) \quad 1.3$$

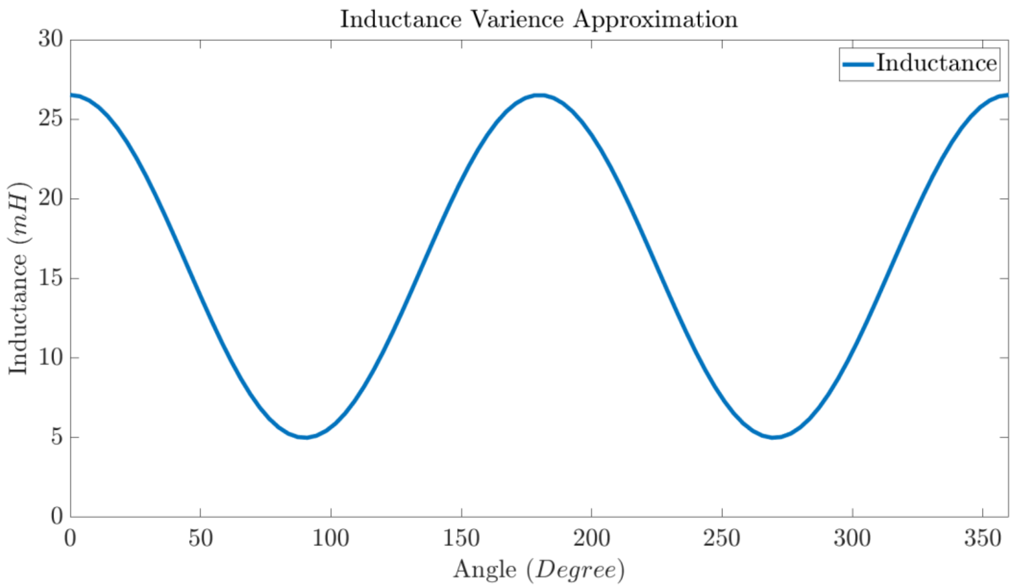

Figure 2 Inductance variation – analytical

1.2. Part B – Torque

Torque for DC excited variable inductance electromagnetic systems is derived based on Equation 1.4.

$$\tau = \frac{dL(\theta)}{d\theta}\frac{I^2}{2} = -(L_{max} - L_{min})\sin(2\theta) \times \frac{I^2}{2} \qquad 1.4$$

The plot of derived torque expression with respect to angle can be seen in Figure 3.

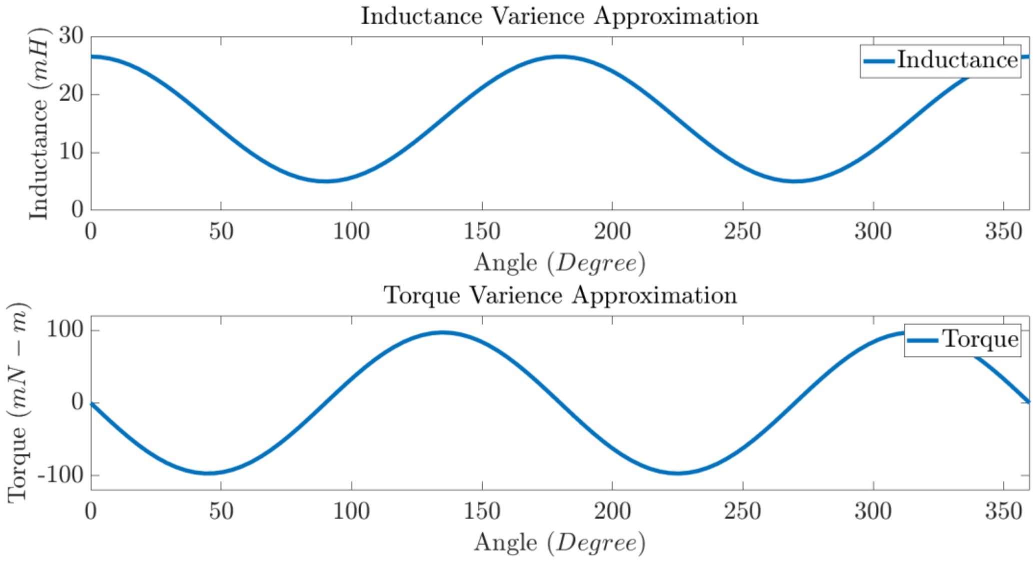

Figure 3 Inductance and torque variation - analytical

1.3. Improvement in Analytical Approximation

Flux density inner side of core is higher than the outer side since inner side of the core has smaller reluctance, which may cause saturation, so the reluctance of the electromagnetic system could be higher. Fringing flux in air-gap region crosses the air-gap through the non-direct ways and flux area increases a bit. Hence fringing flux could decrease the reluctance ance Carter's method or drawing flux tubes for approximation could be alternatives to better approximation for system reluctance calculation. Besides, the reluctance calculation at 90° is not good approximation. This can be enhanced with reluctance functions depends on angle. Finally, the alternative way, of course, is finite element analysis of the electromagnetic system.

2. 2D Linear Material Finite Element Analysis

2.1. Flux Density Vectors

The flux density vectors for positions 0°, 45°, and 90° are given in Figure 4, Figure 5, and Figure 6, respectively. Relative permeability of linear material is taken as 2250.

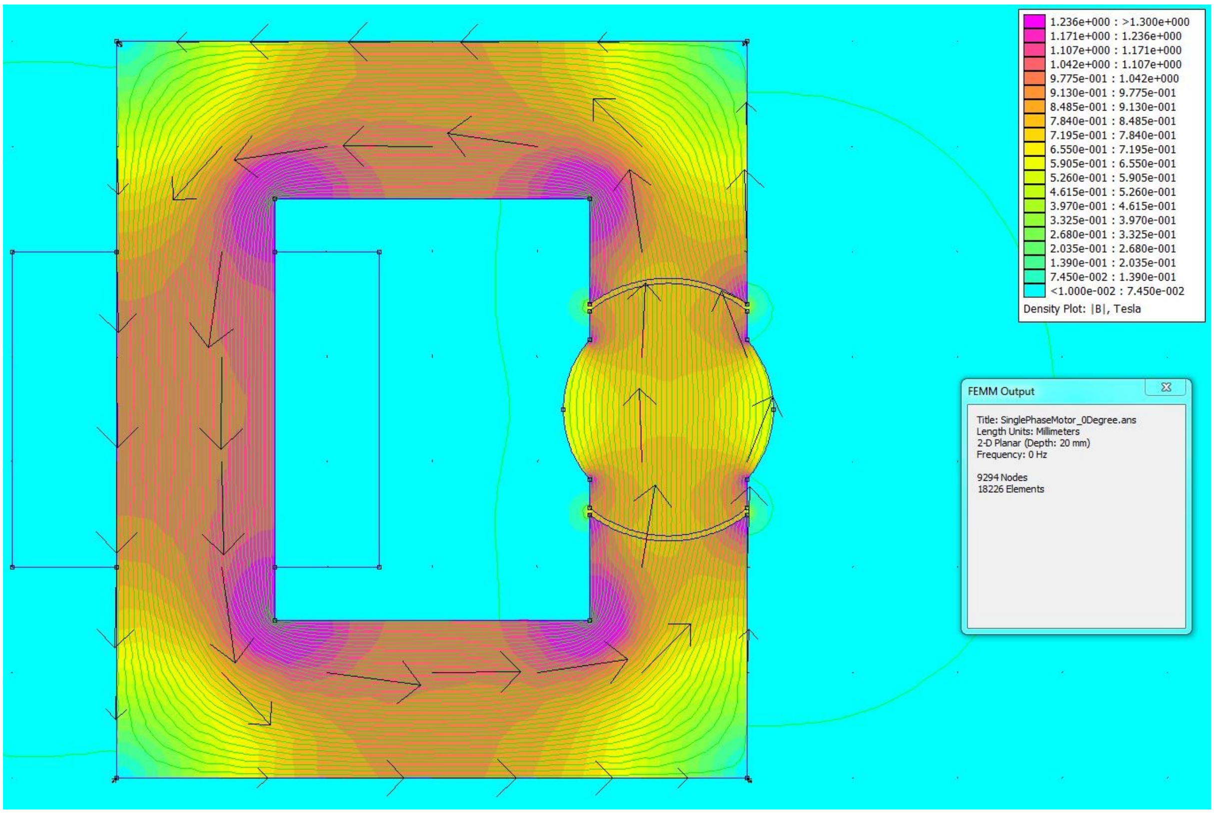

Figure 4 FEA flux density vectors - Linear material - Rotor position 0°

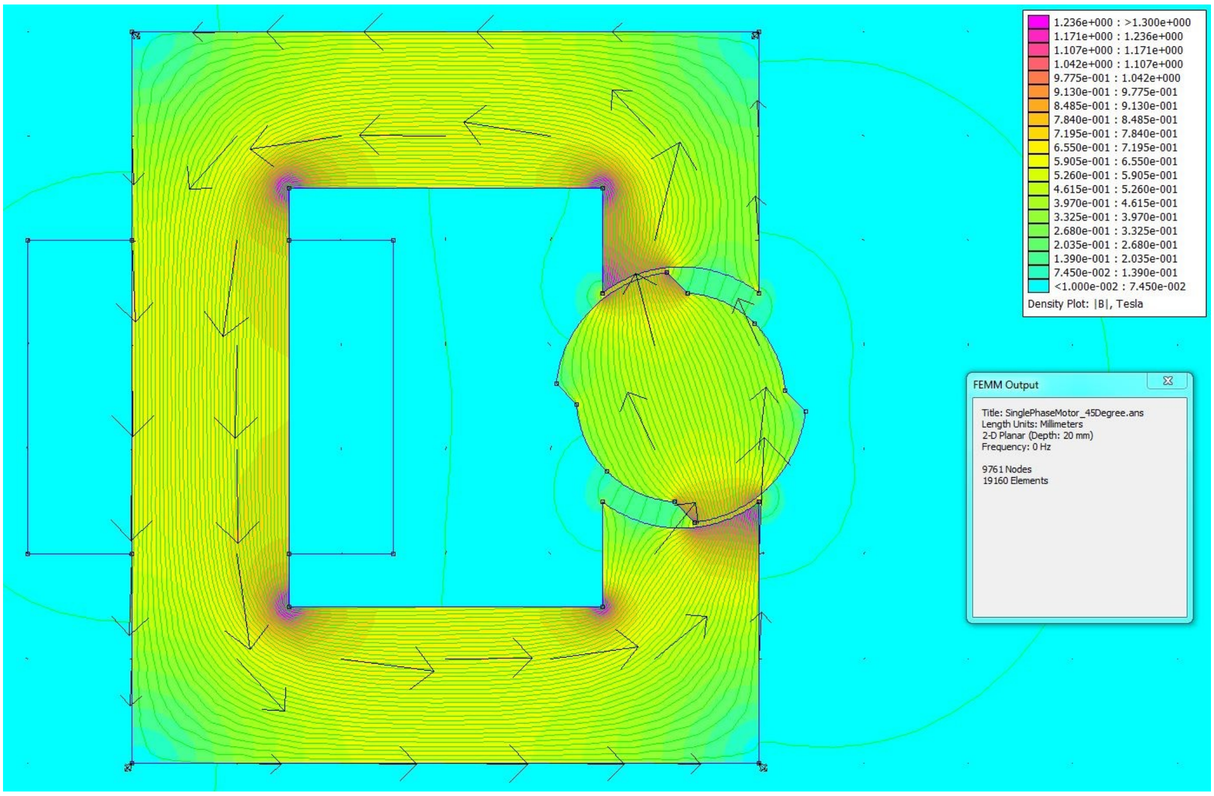

Figure 5 FEA flux density vectors - Linear material - Rotor position 45°

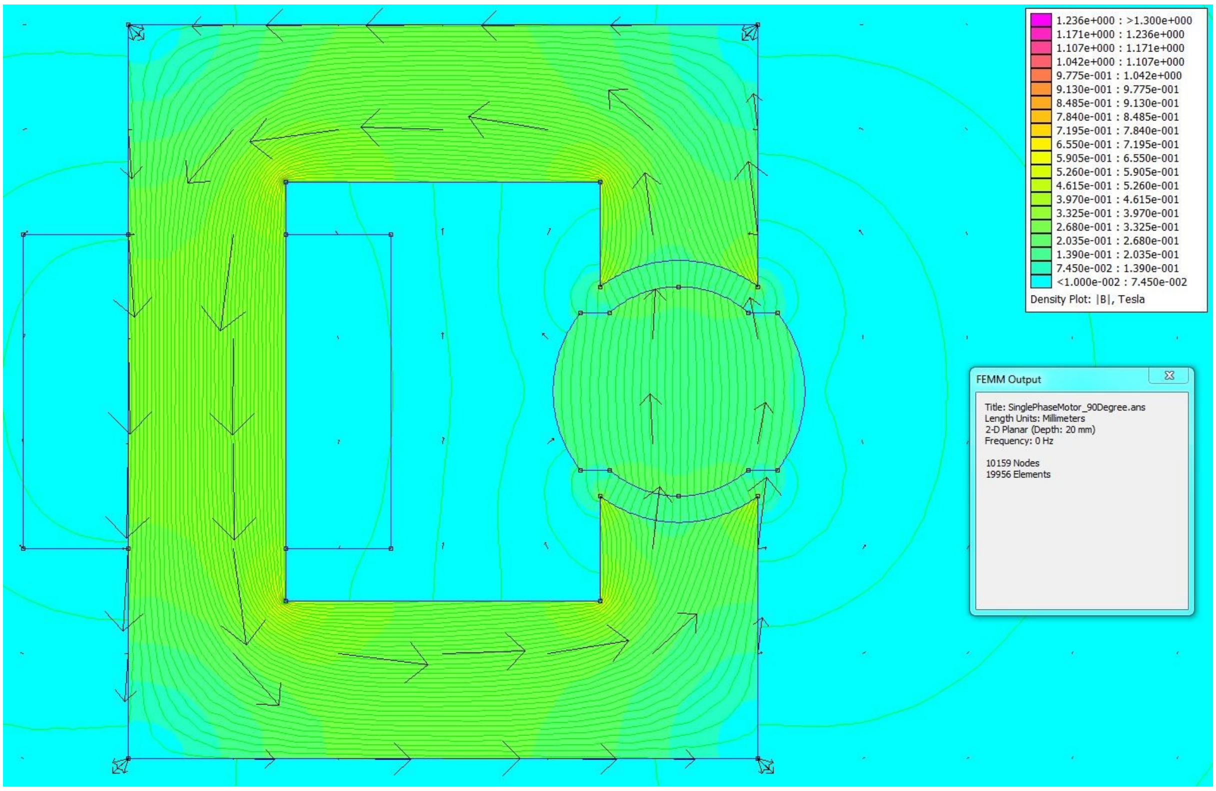

Figure 6 FEA flux density vectors - Linear material - Rotor position 90°

2.2. Part B – Inductance and Energy Calculation

Inductance values of electromangtic system for rotor positions at 0°, 45°, and 90° are calculated with finite element analysis and founded values have some discrepancies with analytical results in terms of inductance values and sinusoidal variance approximation.

Stored energy of the magnetic system is calculated in Equation 2.1.

$$E(\theta) = \frac{1}{2} L(\theta) I^2 \tag{2.1}$$

Inductance values calculated with FEA and stored energy are given in Table 2 for rotor positions 0°, 45°, and 90°

Table 2 FEA linear material - inductance and reluctane measurements

Angle	Inductance	Energy
0°	27.194 mH	122.373 mJ
45°	17.537 mH	78.917 mJ
90°	8.854 mH	39.843 mJ

Calculated inductance values from 2D linear material FEA are drawn with apply of interpolation in Figure 7.

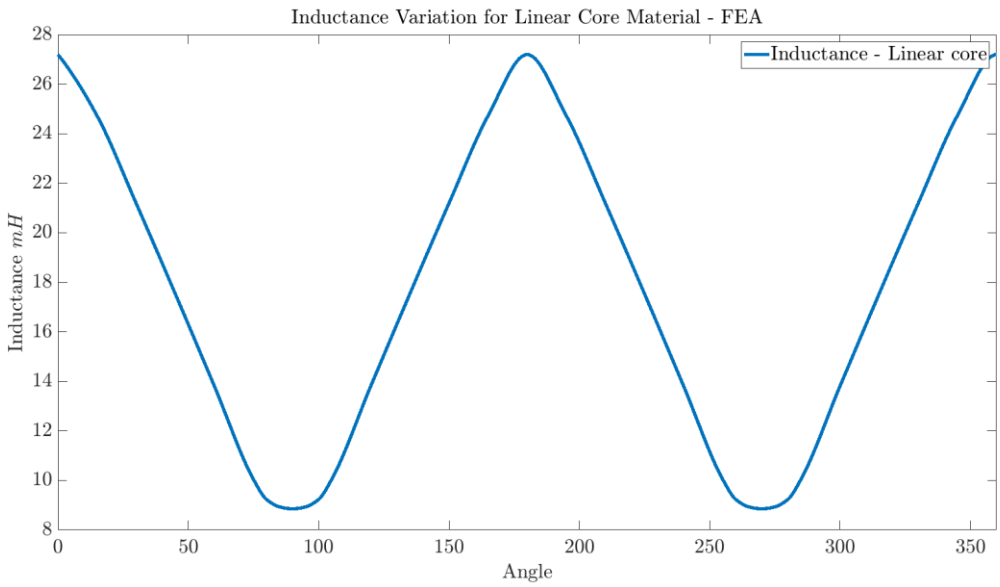

Figure 7 Inductance variation - FEA linear material

2.3. Part C – Torque

Ten sample points for inductance measurement is taken from the FEA. Interpolation and curvefit is applied to get more sample points to derive torque. Used interpolation technique is piecewise cubic hermite interpolating polynomial (PCHIP). On the other hand, sinusoidal sum is used to fit data points. Both interpolation and curvefit results can be seen in Figure 8 and Figure 9, respectively.

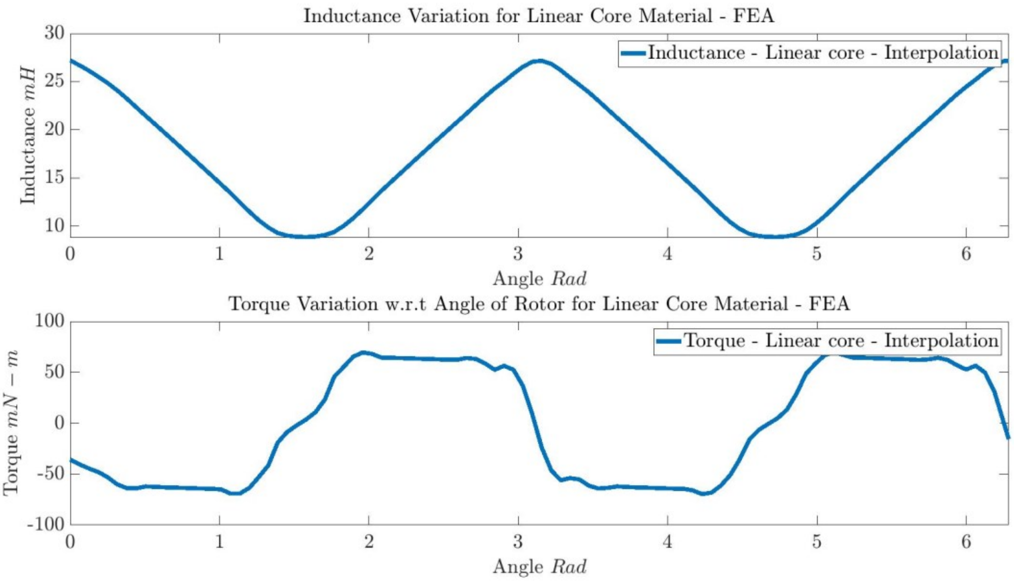

Figure 8 Inductance and torque variance - FEA linear material - Interpolation

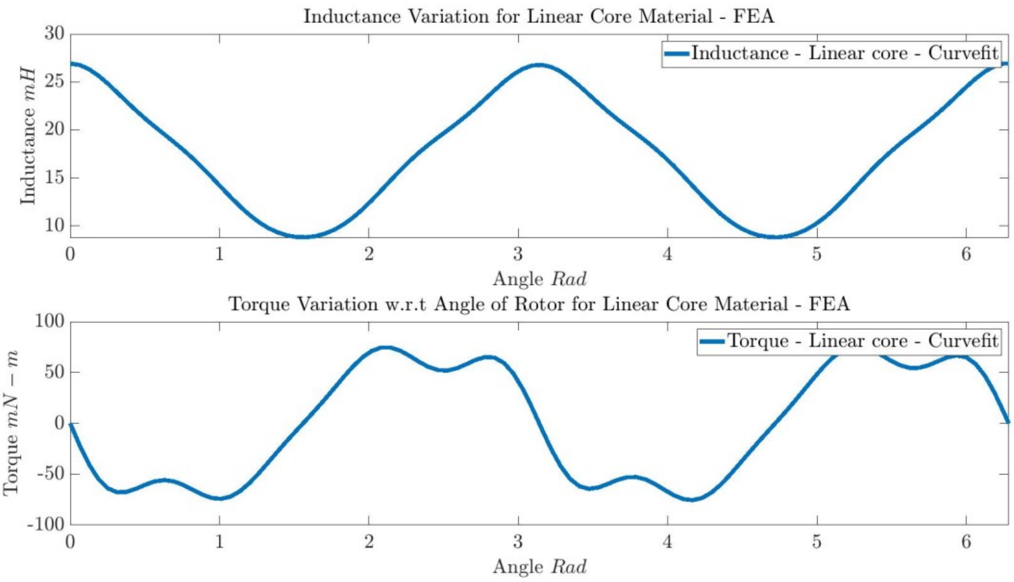

Figure 9 Inductance and torque variance - FEA linear material – Curvefit

3. 2D Non-linear Material Finite Element Analysis

3.1. Part A – Flux Density Vectors

The flux density vectors for positions 0°, 45°, and 90° are given in Figure 12, Figure 13, and Figure 14, respectively. Core material is selected as M50. B-H curve of M50 steel can be seen in Figure 10 and Figure 11.

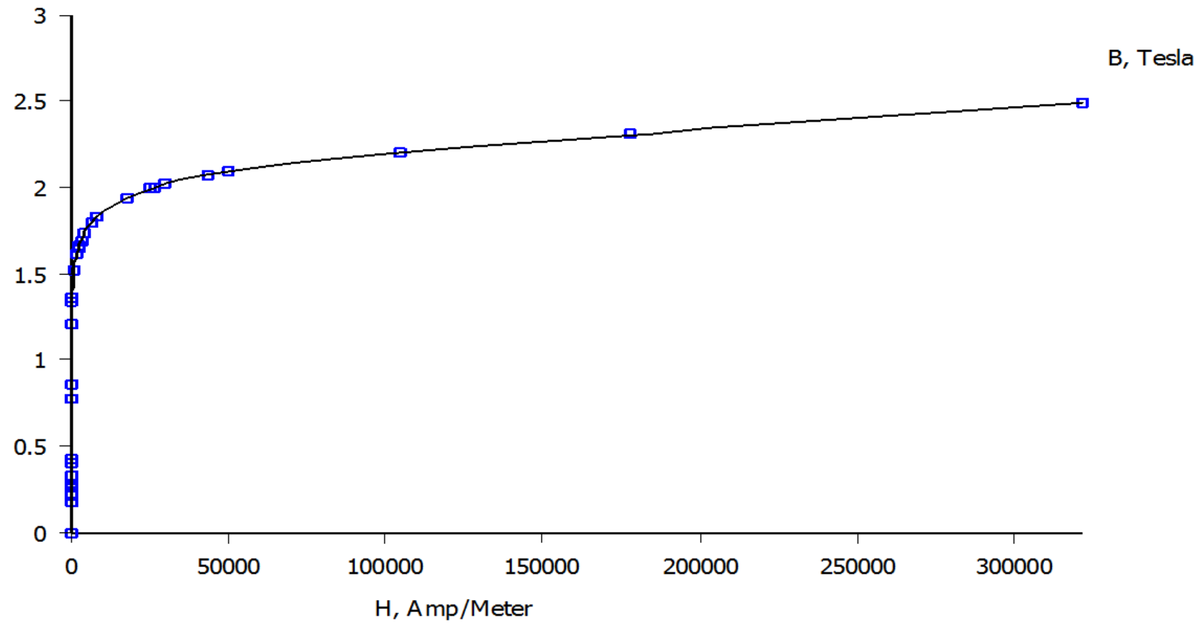

Figure 10 B-H curve of M50 material in FEMM library

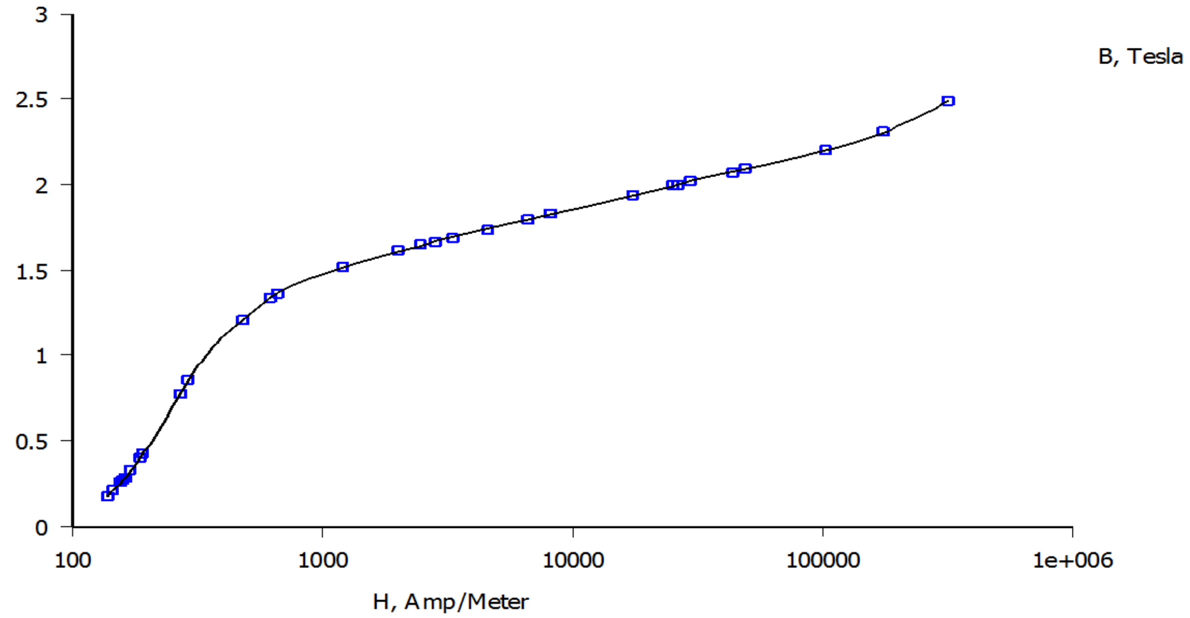

Figure 11 B-H curve of M50 material in FEMM library - log scale

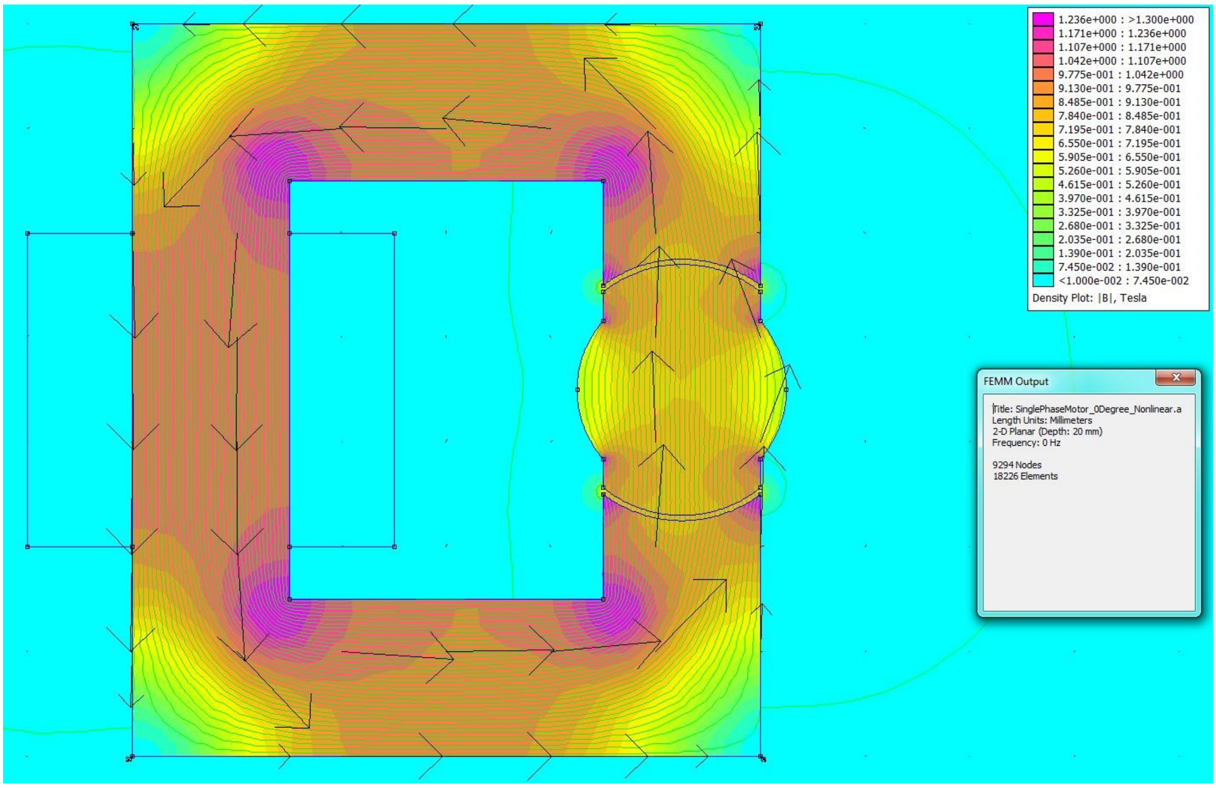

Figure 12 FEA flux density vectors - Non-linear material - Rotor position 0°

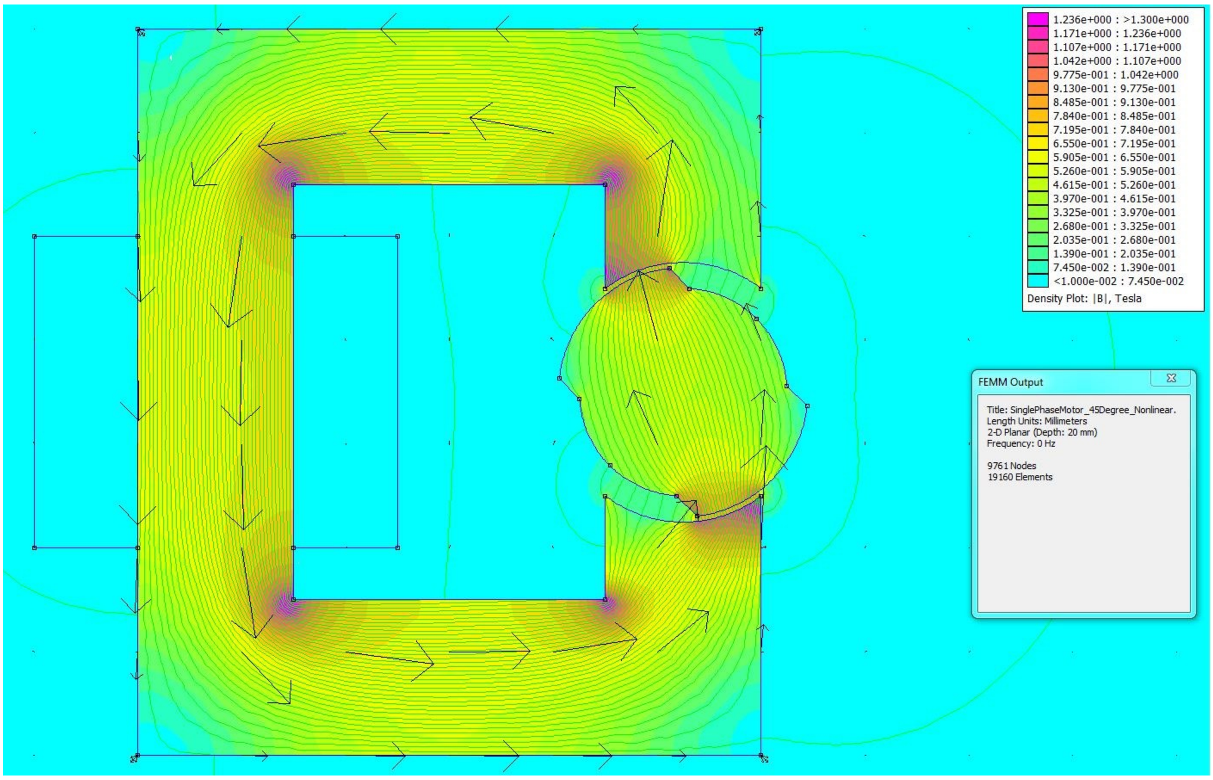

Figure 13 FEA flux density vectors - Non-linear material - Rotor position 45°

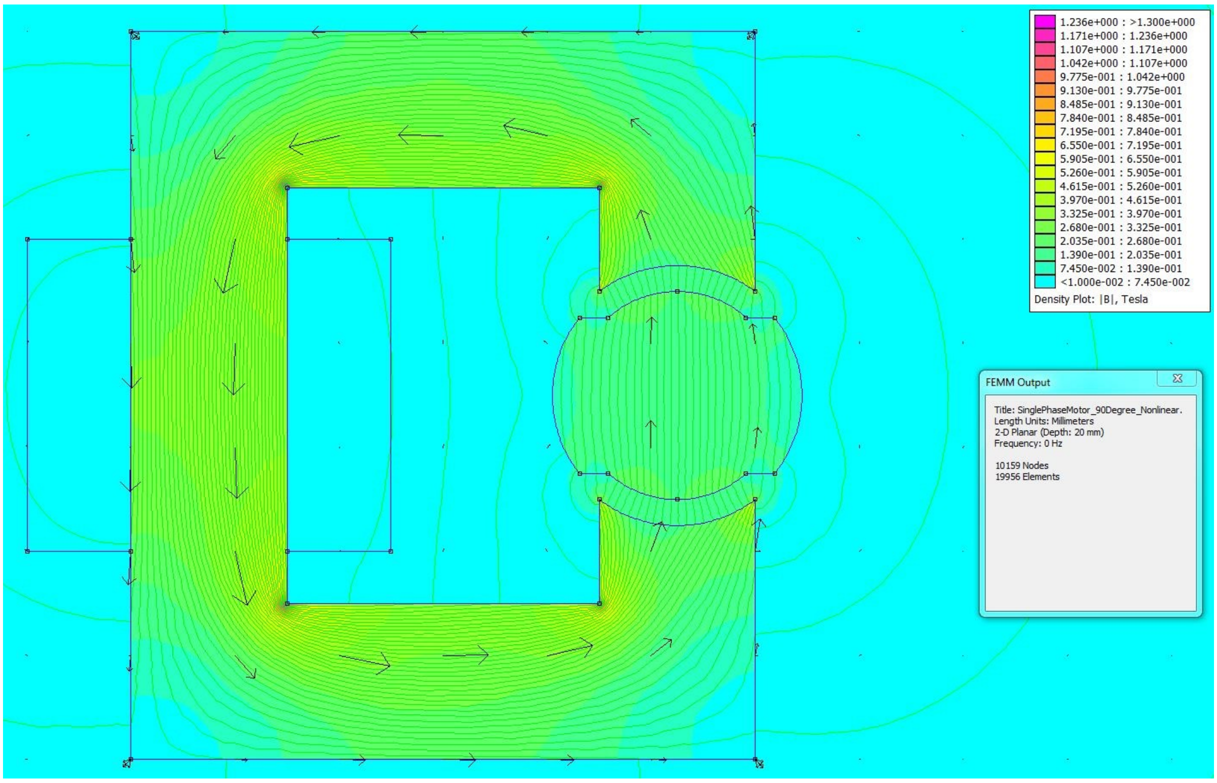

Figure 14 FEA flux density vectors - Non-linear material - Rotor position 90°

3.2. Part B – Inductance and Energy Calculation

Inductance values of electromangtic system for rotor positions at 0°, 45°, and 90° are calculated with finite element analysis. Non-linearity of material cause reduction inductance values slightly since observed flux density values doesn't make obvious non-linearity of M50 material.

Table 3 FEA non-linear material - inductance and reluctane measurements

Angle	**Inductance**	**Energy**
0°	27.025 mH	121.625 mJ
45°	17.416 mH	78.372 mJ
90°	8.748 mH	39.366 mJ

3.3. Part C – Torque

Same interpolation and curvefit techniques apply for non-linear core analysis. Derivation of inductance is solved with numeric derivative and continous derivative (symbolic library derivative). Both interpolation and curvefit results can be seen in Figure 15 and Figure 16, respectively.

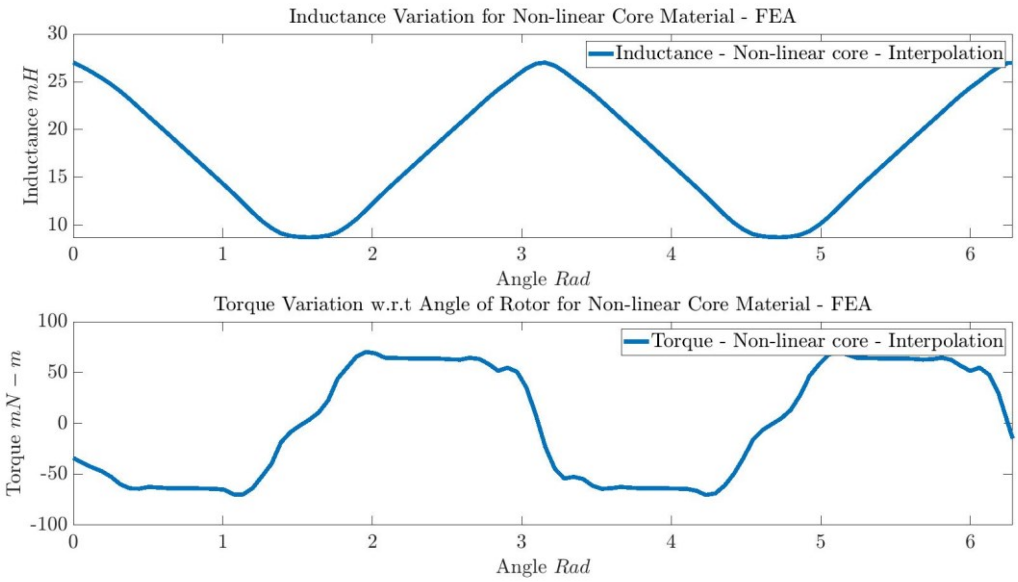

Figure 15 Inductance and torque variance - FEA non-linear material - Interpolation

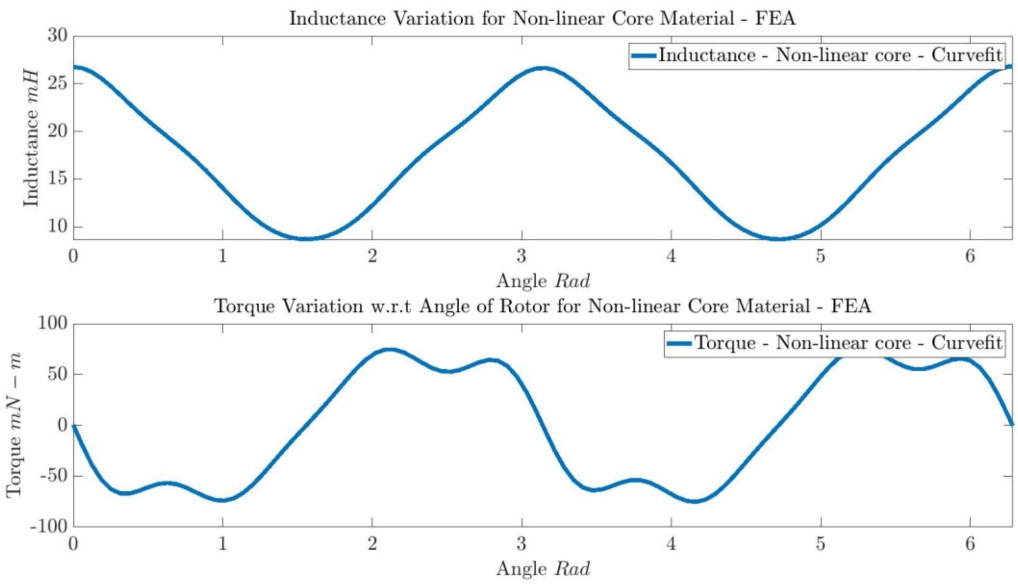

Figure 16 Inductance and torque variance - FEA non-linear material – Curvefit

3.4. Part D – Nonlinear Comparsion

Saturation in non-linear material is due to the diminish in permeability of the core. The decrease in permeability in saturated regions result in mroe fringing flux since the ratio between permeability of air and saturated core becomes comparable. On the other hand, the selected relative permeability for linear material is chosen with consideration of B-H curve of material. Since the applied H field is not so high that only inner side of the core is saturated. Hence, the big difference between linear and non-linear material could not be observed.

4. Q4 – Control Method

Net torque on rotating part of electromagnetic system is equal to zero in a half mechanical cycle. Basically, control and continous motion of machine can be succeeded with a rectification process. Supply connection should be seperated for half of electrical cycle to get net positive or net negative torque.

Since it is the first time I have used FEMM or any finite element program, I didn't want to go further with bonus parts. It would be unefficient, I chose pregnant pause to reinforce and reconsider what I have done and learn in this project.