ADVANCED MATHEMATICS: Test 4

December 3, 2018 (homework)

Surname Name

1.- Calculate Compute the remainder of the division of $(2^{37})^{73}$ by 37.

2.- Given the group $(G, +)$, where $G = \mathbb{Z}_6 \times \mathbb{Z}_{33}$, find the order of $g = ([5]_6, [12]_{33})$. Is there an element in G with order 198 (justify the answer)?

3.- Given $m \in \mathbb{N}\backslash\{0\}$ and

$$G_m = \left\{ \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} : \ a \in \mathbb{Z}_m \right\}.$$

Show that G_m with the operation product of matrices, $(G_m, \times)$, is a commutative cyclic group and find explicitly an isomorphism $f : (G_m, \times) \to (\mathbb{Z}_m, +)$.