1

$$y' = \tan(y)\cos(x)$$
$$y(0) = \pi/2$$

$$\int \frac{dy}{\tan(y)} = \int \cos(x)\,dx$$

$$\int \frac{\cos(y)\,dy}{\sin(y)} = \sin(x) + C$$

$$\int \frac{d\sin(y)}{\sin(y)} = \ln|\sin(y)| = \sin(x) + C$$

$$\boxed{\sin(y) = A e^{\sin(x)}}$$

<u>Check:</u> $$y' = \frac{\sin(y)}{\cos(y)}\cos(x)$$

$$\cos(y)\,y' = \sin(y)\cos(x)$$

$$(\sin(y))' = \sin(y)\cos(x)$$

Using the solution $\longrightarrow$ $Ae^{\sin(x)}\cos(x) = Ae^{\sin(x)}\cos(x)$ ok!

<u>Constant:</u> $y(0) = \pi/2$

$$\sin\left(\frac{\pi}{2}\right) = A e^{\sin(0)}$$

$$1 = A$$

Final solution: $\boxed{\sin(y) = e^{\sin(x)}}$

(For $y \in [-\pi/2, \pi/2]$ $y = \arcsin(e^{\sin(x)})$)

2

$$\begin{cases} y'' + y' = -8\sin 2x - 6\cos 2x & A \\ y'' + 4y = 5e^{-x} & B \end{cases}$$

$$y' - 4y = -8\sin 2x - 6\cos 2x - 5e^{-x}$$

Deriving:

$$y'' - 4y' = -16\cos(2x) + 12\sin(2x) + 5e^{-x}$$

Subtracting form A:

$$5y' = -20\sin(2x) + 10\cos(2x) - 5e^{-x}$$

$$y' = -4\sin(2x) + 2\cos(2x) - e^{-x}$$

$$\boxed{y = y_0 + 2\cos(2x) + \sin(2x) + e^{-x}}$$

Let's test the solution:

$$y'' = (y')' = -8\cos(2x) - 4\sin(2x) + e^{-x}$$

Eq. A: $y'' + y' = -8\cos(2x) - 4\sin(2x) + e^{-x} - 4\sin(2x) +$

$+2\cos 2x - e^{-x} = -6\cos(2x) - 8\sin(2x)$ OK

Eq. B: $y'' + 4y = -8\cos(2x) - 4\sin(2x) + e^{-x} + 4y_0 + 8\cos(2x)$

$+4\sin(2x) + 4e^{-x} = 5e^{-x} + 4y_0 = 5e^{-x}$

Thus $\boxed{y_0 = 0}$

Final solution: $\boxed{y(x) = 2\cos(2x) + \sin(2x) + e^{-x}}$