CSCI 341 Problem Set 4

The Algebra of Regular Expressions; Kleene's Theorem; Silent Transitions

Due Friday, September 26

Don't forget to check the webspace for hints and additional context for each problem!

Algebra of Regular Expressions

Problem 1 (Distributing on the Left). Let $r_1, r_2, r_3 \in \mathit{RExp}$. Prove the equation

$$r_1 \cdot (r_2 + r_3) =_{\mathcal{L}} (r_1 \cdot r_2) + (r_1 \cdot r_3)$$

by calculating the language on either side of the equation and arguing that these two languages are equal.

Solution. □

Problem 2 (Air Flare). Let $a, b \in A$. Use the equations in the Lemmas above to prove the following equations:

(1) $a^*a^* =_{\mathcal{L}} a^*$

(2) $(a + b)^* =_{\mathcal{L}} b^*(ab^*)^*$

Label each equation you use in your proof to indicate which lemma was used where.

Solution. □

Kleene's Theorem

Problem 3 (Some String Matching). Let $A = \{0, 1\}$. In the following automaton, the state x_0 represents a program that checks that in a given input string, every instance of 01 is eventually followed by a 0.

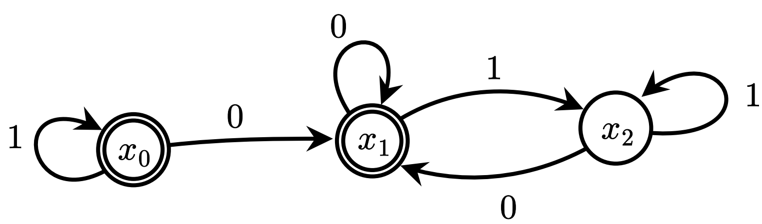

Figure 1: An automaton $\mathcal{A}$ with a state x_0 that accepts a string of 0s and 1s if and only if every instance of 01 is eventually followed by 0.

(1) Use Kleene's algorithm to derive a regular expression $s_0 \in \mathit{RExp}$ such that $\mathcal{L}(\mathcal{A}, x_0) = \mathcal{L}(s_0)$.

(2) Draw the portion of the Antimirov automaton generated by s_0, $\mathcal{A}' = \langle s_0 \rangle_{\mathcal{A}_{Ant}}$. There is a state y_0 is the Antimirov automaton such that $\mathcal{A} = \langle y_0 \rangle_{\mathcal{A}'}$. Find y_0.

Solution. □

Silent Transitions

Problem 4 (Follow Through 2)**.** Use the dagger construction to transform the automaton with silent transitions into a standard automaton and then determine the languages accepted by each state.

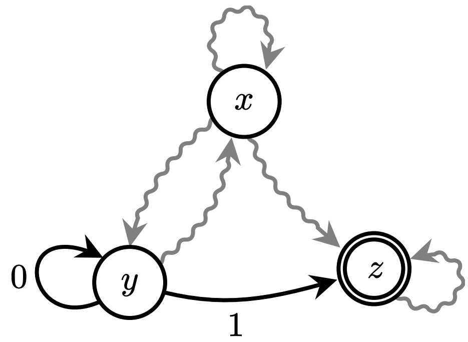

Figure 2: An automaton with silent transitions $\mathcal{B} = (Q, A, \delta, \rightsquigarrow, F)$.

Solution. □