CSCI 341 Problem Set 3

Determinization, the Structure of Fin, Regular Expressions, and Antimirov Derivatives

Due Friday, September 16

Don't forget to check the webspace for hints and additional context for each problem!

Determinization

Problem 1 (Determinizing is Deterministic). Prove that for any automaton $\mathcal{A} = (Q, A, \delta, F)$, $\mathrm{Det}(\mathcal{A})$ is total deterministic.

Solution. □

Problem 2 (Determinized State, Completing the Proof). Let $\mathcal{A} = (Q, A, \delta, F)$ be an automaton, and let $\mathrm{Det}(\mathcal{A})$ be its determinization. Prove that for any state $x \in Q$,

$$\mathcal{L}(\mathcal{A}, x) \supseteq \mathcal{L}(\mathrm{Det}(\mathcal{A}), \{x\})$$

Solution. □

Problem 3 (You Got Options). Find the smallest automaton (not necessarily total or deterministic) with a state that accepts the language

$$L = \{ab^n \mid n \in \mathbb{N}\} \cup \{ac^n \mid n \in \mathbb{N}\} \cup \{a(bc)^n \mid n \in \mathbb{N}\}$$

over the alphabet $A = \{a, b, c\}$. Use determinization to find a deterministic automaton with a state that accepts the same language.

Solution. □

The Structure of Fin

Problem 4 (Finish Closed under Complement). Let $\mathcal{A} = (Q, A, \delta, F)$ be a total deterministic automaton with a state x and let $L = \mathcal{L}(\mathcal{A}, x)$. Let $\mathcal{A}' = (Q, A, \delta, Q \backslash F)$. Prove that $\mathcal{L}(\mathcal{A}', x) = A^* \backslash L$.

Solution. □

Problem 5 (Intersection-product Construction). Let $\mathcal{A}_1 = (Q_1, A, \delta_1, F_1)$ and $\mathcal{A}_2 = (Q_2, A, \delta_2, F_2)$ be total deterministic automata, and let $x \in Q_1$ and $y \in Q_2$. Let $L_1 = \mathcal{L}(\mathcal{A}_1, x)$ and $L_2 = \mathcal{L}(\mathcal{A}_2, x)$. Change the accepting states in the union-product construction to obtain an automaton $\mathcal{A}_1 \otimes \mathcal{A}_2 = (Q_1 \times Q_2, A, \delta^{\times}, F^{\otimes})$ such that $\mathcal{L}(\mathcal{A}_1 \otimes \mathcal{A}_2, (x, y)) = L_1 \cap L_2$. Explain how to obtain a proof of the Closure under Intersection Theorem from the proof of the Closure under Union Theorem (i.e., what would you change?).

Solution. □

Regular Expressions

Problem 6 (Intersections and Complements)**.** Show that the following two languages are regular over $A = \{a, b\}$.

(1) $L_6 = \mathcal{L}(b^*a(a + b)^*) \cap \mathcal{L}(a^*b(a + b)^*)$

(2) $L_7 = A^*\backslash L_6$

Solution. □

Antimirov Derivatives

Problem 7 (Some Nested Derivatives)**.** Consider the regular expression $r = (a(b + c^*) + b)^*$ over the alphabet $A = \{a, b, c\}$.

(1) Name three different words $w_0, w_1, w_1 \in A^*$ that are not in $\mathcal{L}(r)$, i.e., $w_0, w_1, w_2 \notin \mathcal{L}(r)$.

(2) Use the inequalities in the proof of the Linear Bound on Antimirov Derivatives Lemma to determine an upper bound on the number of states in the automaton $\langle r \rangle_{\mathcal{A}_{Ant}}$ generated by r in $\mathcal{A}_{Ant}$, i.e., $\#(r)$.

(3) Now draw a state diagram of $\langle r \rangle_{\mathcal{A}_{Ant}}$.

(4) How many formation rules were used to form the regular expression r? How does this number of formation rules compare to the number of states in $\langle r \rangle_{\mathcal{A}_{Ant}}$?

Solution. □