CSCI 341 Problem Set 3

Determinization, the Structure of Fin, Regular Expressions, and Antimirov Derivatives

Due Friday, September 5

Determinization

Problem 1 (Determinizing is Deterministic)**.** Prove that for any automaton $\mathcal{A} = (Q, A, \delta, F)$, $\mathrm{Det}(\mathcal{A})$ is total deterministic.

Solution. □

Problem 2 (Determinized State, Completing the Proof)**.** Let $\mathcal{A} = (Q, A, \delta, F)$ be an automaton, and let $\mathrm{Det}(\mathcal{A})$ be its determinization. Prove that for any state $x \in Q$,

$$\mathcal{L}(\mathcal{A}, x) \supseteq \mathcal{L}(\mathrm{Det}(\mathcal{A}), \{x\})$$

Solution. □

Problem 3 (You Got Options)**.** Find the smallest automaton (not necessarily total or deterministic) with a state that accepts the language

$$L = \{ab^n \mid n \in \mathbb{N}\} \cup \{ac^n \mid n \in \mathbb{N}\} \cup \{a(bc)^n \mid n \in \mathbb{N}\}$$

over the alphabet $A = \{a, b, c\}$. Use determinization to find a deterministic automaton with a state that accepts the same language.

Solution. □

the Structure of Fin

Problem 4 (Finish Closed under Complement)**.** Let $\mathcal{A}' = (Q, A, \delta, Q \backslash F)$. Prove that $\mathcal{L}(\mathcal{A}', x) = A^* \backslash L$.

Solution. □

Problem 5 (Intersection-product Construction)**.** Let $\mathcal{A}_1 = (Q_1, A, \delta_1, F_1)$ and $\mathcal{A}_2 = (Q_2, A, \delta_2, F_2)$ be total deterministic automata, and let $x \in Q_1$ and $y \in Q_2$. Let $L_1 = \mathcal{L}(\mathcal{A}_1, x)$ and $L_2 = \mathcal{L}(\mathcal{A}_2, x)$. Change the accepting states in the union-product construction to obtain an automaton $\mathcal{A}_1 \otimes \mathcal{A}_2 = (Q_1 \times Q_2, A, \delta^\times, F^\otimes)$ such that $\mathcal{L}(\mathcal{A}_1 \otimes \mathcal{A}_2, (x, y)) = L_1 \cap L_2$. Explain how to obtain a proof of the Closure under Intersection Theorem from the proof of the Closure under Union Theorem (i.e., what would you change?).

Solution. □

Regular Expressions

Problem 6 (Intersections and Complements)**.** Show that the following two languages are regular over $A = \{a, b\}$.

(1) $L_6 = \mathcal{L}(b^*a(a+b)^*) \cap \mathcal{L}(a^*b(a+b)^*)$

(2) $L_7 = A^* \backslash L_6$

Solution. □

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