CSCI 341 Problem Set 1

Games and State Machines; Automata

Due Monday, September 1

Games and State Machines

Problem 1 (Always an Upper Bound)**.** Let $M = (G, L, S, E, A, C)$ be a directional maze with the set of legal moves $A = \{\Uparrow, \Rightarrow\}$. Prove that $\mathcal{S}(M)$ is finite (there are only finitely many elements) by calculating an upper bound on the number of all possible legal paths through an $n \times m$ directional maze.

Problem 2 (Reverse Engineering)**.** Find a Sokoban game that represents state s_1 in abstract state diagram (A). Replace the states in the state diagram with drawings of each state of the Sokoban game.

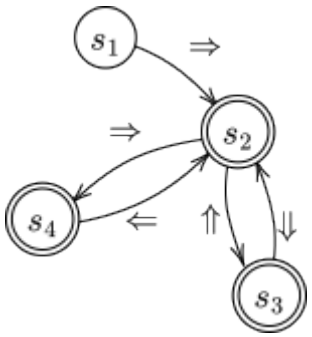

Figure 1: Abstract state diagram (A).

Problem 3 (Impossibility)**.** Prove that there *does not* exist a directional maze that represents state s_1 in abstract state diagram (A).

Reading Words

Problem 4 (Repeated Derivatives)**.** Let $\mathcal{A} = (Q, A, \delta, F)$ be an automaton, let $x \in Q$, and $w \in A^*$ and $a \in A$. Prove the following identity:

$$\delta(x, wa) = \{z \mid z \in \delta(y, a) \text{ for some } y \in \delta(x, w)\}$$

Problem 5 (Determinstic Extension)**.** Let $\mathcal{A} = (Q, A, \delta, F)$ be a total deterministic automaton. Let $x \in Q$ and $w \in A^*$. Prove that $\delta(x, w)$ has exactly one element using Induction on Words.

Language Acceptance

Problem 6 (Let 'em Cook)**.** For each of the following languages $L_i \subseteq A^*$ below, design an automaton $\mathcal{A}_i = (Q_i, A, \delta_i, F_i)$ with a state $x \in Q_i$ such that x accepts L_i, and briefly explain why your automaton accepts L_i. Note that $A = \{a, b\}$ in all of the cases below.

(1) $L_1 = \{a, aa, aaa\}$

(2) $L_2 = \{w \in A^* \mid w \text{ ends with } b\}$

(3) $L_3 = \{w \in A^* \mid w \text{ has an even number of } a\text{'s}\}$

(4) $L_4 = \{w \in A^* \mid w \text{ has } 3k+1 \text{ many } a\text{'s for some } k \geqslant 0\}$

(5) $L_5 = \{w \in A^* \mid w \text{ either has } 3k+1 \text{ or } 3k+2 \text{ many } a\text{'s for some } k \geqslant 0\}$

(6) $L_6 = A^* \backslash L_2$

Problem 7 (Pythonic Automaton)**.** Write a Python script in the same format as the Pythonic Automaton I that implements state s_1 in abstract state diagram (A) from the games and puzzles section. Submit your program as a .py file in the programming submission box (see the other Gradescope assignment).

Finite and Infinite Automata

Problem 8 (Unravelling an Infinite Language)**.** Draw a state diagram of all of the languages that are reachable from the language $L = \{(ab)^n \mid n \in \mathbb{N}\}$ in the Brzozowski automaton by taking a- and b-derivatives (the words in this language are ε, ab, $abab$, ...). Include all of the double-circled states to indicate which languages are accepting states of the Brzozowski automaton. What language is accepted by L?

Problem 9 (Language Accepts Itself)**.** Let $L \subseteq A^*$ be any language. Prove that $\mathcal{L}(\mathcal{A}_{Brz}, L) \subseteq L$.