

CSCI 341 Problem Set 3

Determinization, the Structure of Fin, Regular Expressions, and Antimirov Derivatives

Due Friday, September 5

Determinization

Problem 1 (Determinizing is Deterministic). Prove that for any automaton $\mathcal{A} = (Q, A, \delta, F)$, $\text{Det}(\mathcal{A})$ is total deterministic.

Solution. □

Problem 2 (Determinized State, Completing the Proof). Let $\mathcal{A} = (Q, A, \delta, F)$ be an automaton, and let $\text{Det}(\mathcal{A})$ be its determinization. Prove that for any state $x \in Q$,

$$\mathcal{L}(\mathcal{A}, x) \supseteq \mathcal{L}(\text{Det}(\mathcal{A}), \{x\})$$

Solution. □

Problem 3 (You Got Options). Find the smallest automaton (not necessarily total or deterministic) with a state that accepts the language

$$L = \{ab^n \mid n \in \mathbb{N}\} \cup \{ac^n \mid n \in \mathbb{N}\} \cup \{a(bc)^n \mid n \in \mathbb{N}\}$$

over the alphabet $A = \{a, b, c\}$. Use determinization to find a deterministic automaton with a state that accepts the same language.

Solution. □

the Structure of Fin

Problem 4 (Finish Closed under Complement). Let $\mathcal{A}' = (Q, A, \delta, Q \setminus F)$. Prove that $\mathcal{L}(\mathcal{A}', x) = A^* \setminus L$.

Solution. □

Problem 5 (Intersection-product Construction). Let $\mathcal{A}_1 = (Q_1, A, \delta_1, F_1)$ and $\mathcal{A}_2 = (Q_2, A, \delta_2, F_2)$ be total deterministic automata, and let $x \in Q_1$ and $y \in Q_2$. Let $L_1 = \mathcal{L}(\mathcal{A}_1, x)$ and $L_2 = \mathcal{L}(\mathcal{A}_2, y)$. Change the accepting states in the union-product construction to obtain an automaton $\mathcal{A}_1 \otimes \mathcal{A}_2 = (Q_1 \times Q_2, A, \delta^\times, F^\otimes)$ such that $\mathcal{L}(\mathcal{A}_1 \otimes \mathcal{A}_2, (x, y)) = L_1 \cap L_2$. Explain how to obtain a proof of the Closure under Intersection Theorem from the proof of the Closure under Union Theorem (i.e., what would you change?).

Solution. □

Regular Expressions

Problem 6 (Intersections and Complements). Show that the following two languages are regular over $A = \{a, b\}$.

(1) $L_6 = \mathcal{L}(b^*a(a+b)^*) \cap \mathcal{L}(a^*b(a+b)^*)$

(2) $L_7 = A^* \setminus L_6$

Solution. □

Antimirov Derivatives

Problem 7 (Some Nested Derivatives). Consider the regular expression $r = (a(b + c^*) + b)^*$ over the alphabet $A = \{a, b, c\}$.

- (1) Name three different words $w_0, w_1, w_2 \in A^*$ that are not in $\mathcal{L}(r)$, i.e., $w_0, w_1, w_2 \notin \mathcal{L}(r)$.
Use the inequalities in the proof of the Linear Bound on Antimirov Derivatives Lemma to determine an upper bound on the number of states in the automaton $\langle r \rangle_{\mathcal{A}_{Ant}}$ generated by r in \mathcal{A}_{Ant} , i.e., $\#(r)$.
- (2) Now draw a state diagram of $\langle r \rangle_{\mathcal{A}_{Ant}}$.
- (3) How many formation rules were used to form the regular expression r ? How does this number of formation rules compare to the number of states in $\langle r \rangle_{\mathcal{A}_{Ant}}$?

Solution.

□