CSCI 341 Problem Set 1

Games and State Machines; Automata

Due Friday, September 5

Games and State Machines

Problem 1 (Always an Upper Bound)**.** Let $M = (G, L, S, E, A, C)$ be a directional maze with the set of legal moves $A = \{\Uparrow, \Rightarrow\}$. Prove that $\mathcal{S}(M)$ is finite (there are only finitely many elements) by calculating an upper bound on the number of all possible legal paths through an $n \times m$ directional maze.

Problem 2 (Reverse Engineering)**.** Find a Sokoban game that represents state s_1 in abstract state diagram (1). Replace the states in the state diagram with drawings of each state of the Sokoban game.

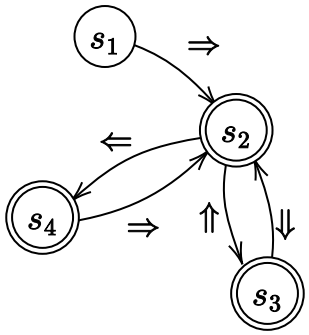

Figure 1: Abstract state diagram (1).

Problem 3 (Impossibility)**.** Prove that there *does not* exist a directional maze that represents state s_1 in abstract state diagram (1).

Reading Words

Problem 4 (Repeated Derivatives)**.** Let $\mathcal{A} = (Q, A, \delta, F)$ be an automaton, let $x \in Q$, and $w \in A^*$ and $a \in A$. Prove the following identity:

$$\delta(x, wa) = \{z \mid z \in \delta(y, a) \text{ for some } y \in \delta(x, w)\}$$

Problem 5 (Determinstic Extension)**.** Let $\mathcal{A} = (Q, A, \delta, F)$ be a total deterministic automaton. Let $x \in Q$ and $w \in A^*$. Prove that $\delta(x, w)$ has exactly one element using Induction on Words.