CSCI 341 Problem Set 2

Reading Words; Language Acceptance; Finite and Infinite Automata

Due Monday, September 8

Reading Words

Problem 1 (Repeated Derivatives)**.** Let $\mathcal{A} = (Q, A, \delta, F)$ be an automaton, let $x \in Q$, and $w \in A^*$ and $a \in A$. Prove the following identity:

$$\delta(x, wa) = \{z \mid z \in \delta(y, a) \text{ and } y \in \delta(x, w)\}$$

Problem 2 (Determinstic Extension)**.** Let $\mathcal{A} = (Q, A, \delta, F)$ be a total deterministic automaton. Let $x \in Q$ and $w \in A^*$. Prove that $\delta(x, w)$ has exactly one element using Induction on Words.

Language Acceptance

Problem 3 (Let 'em Cook)**.** For each of the following languages $L_i \subseteq A^*$ below, design an automaton $\mathcal{A}_i = (Q_i, A, \delta_i, F_i)$ with a state $x \in Q_i$ such that x accepts L_i, and briefly explain why your automaton accepts L_i. Note that $A = \{a, b\}$ in all of the cases below.

(1) $L_1 = \{a, aa, aaa\}$

(2) $L_2 = \{w \in A^* \mid w \text{ ends with } b\}$

(3) $L_3 = \{w \in A^* \mid w \text{ has an even number of } a\text{'s}\}$

(4) $L_4 = \{w \in A^* \mid w \text{ has } 3k + 1 \text{ many } a\text{'s for some } k \geqslant 0\}$

(5) $L_5 = \{w \in A^* \mid w \text{ either has } 3k + 1 \text{ or } 3k + 2 \text{ many } a\text{'s for some } k \geqslant 0\}$

For each of these languages, if you drew a nondeterministic or partial automaton, also draw a total deterministic one (include both in your write-up).

Problem 4 (Pythonic Automaton I)**.** Consider the python program here: Colab Link. This Python program corresponds to a state in an automaton $\mathcal{A} = (Q, A, \delta, F)$ where $A = \{0, 1\}$.

(1) Write down a transition table for $\mathcal{A}$ and draw a state diagram of $\mathcal{A}$.

(2) Which state does the program correspond to?

(3) Using set comprehension, describe the language accepted by this state.

Make sure to thoroughly test out your state diagram of $\mathcal{A}$ against the program.

Problem 5 (Pythonic Automaton III)**.** Write a Python script in the same format as the Pythonic Automaton I that implements state s_1 in abstract state diagram (A) from the games and puzzles section. Submit your program as a .py file.

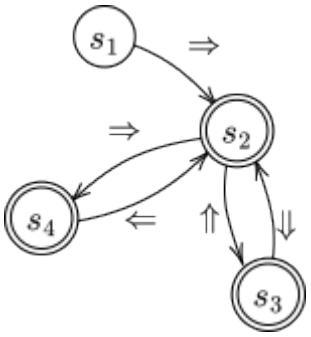

Figure 1: Abstract state diagram (A).

Finite and Infinite Automata

Problem 6 (Unravelling a Language)**.** Draw a state diagram of all of the languages that are reachable from the language $L = \{\varepsilon, aa, ba, cab, c, acab\}$ in the Brzozowski automaton (by taking derivatives). Include all of the double-circles to indicate which languages are accepting states of the Brzozowski automaton. What language is accepted by L?

Problem 7 (Language Accepts Itself)**.** Let $L \subseteq A^*$ be any language. Prove that $\mathcal{L}(\mathcal{A}_{Brz}, L) \subseteq L$.