

# CSCI 341 Problem Set 1

Games and State Machines; Automata

Due Monday, September 1

## Games and State Machines

**Problem 1** (Always an Upper Bound). Let  $M = (G, L, S, E, A, C)$  be a directional maze with the set of legal moves  $A = \{\uparrow, \Rightarrow\}$ . Prove that  $S(M)$  is finite (there are only finitely many elements) by calculating an upper bound on the number of all possible legal paths through an  $n \times m$  directional maze.

**Problem 2** (Reverse Engineering). Find a Sokoban game that represents state  $s_1$  in abstract state diagram (A). Replace the states in the state diagram with drawings of each state of the Sokoban game.

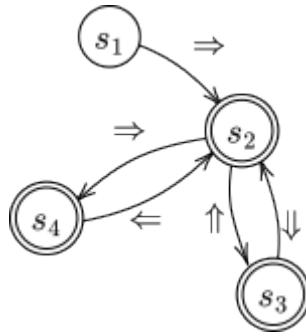


Figure 1: Abstract state diagram (A).

**Problem 3** (Impossibility). Prove that there *does not* exist a directional maze that represents state  $s_1$  in abstract state diagram (A).

## Reading Words

**Problem 4** (Repeated Derivatives). Let  $\mathcal{A} = (Q, A, \delta, F)$  be an automaton, let  $x \in Q$ , and  $w \in A^*$  and  $a \in A$ . Prove the following identity:

$$\delta(x, wa) = \{z \mid z \in \delta(y, a) \text{ for some } y \in \delta(x, w)\}$$

**Problem 5** (Deterministic Extension). Let  $\mathcal{A} = (Q, A, \delta, F)$  be a total deterministic automaton. Let  $x \in Q$  and  $w \in A^*$ . Prove that  $\delta(x, w)$  has exactly one element using Induction on Words.

## Language Acceptance

**Problem 6** (Let 'em Cook). For each of the following languages  $L_i \subseteq A^*$  below, design an automaton  $\mathcal{A}_i = (Q_i, A, \delta_i, F_i)$  with a state  $x \in Q_i$  such that  $x$  accepts  $L_i$ , and briefly explain why your automaton accepts  $L_i$ . Note that  $A = \{a, b\}$  in all of the cases below.

- (1)  $L_1 = \{a, aa, aaa\}$

- (2)  $L_2 = \{w \in A^* \mid w \text{ ends with } b\}$
- (3)  $L_3 = \{w \in A^* \mid w \text{ has an even number of } a\text{'s}\}$
- (4)  $L_4 = \{w \in A^* \mid w \text{ has } 3k + 1 \text{ many } a\text{'s for some } k \geq 0\}$
- (5)  $L_5 = \{w \in A^* \mid w \text{ either has } 3k + 1 \text{ or } 3k + 2 \text{ many } a\text{'s for some } k \geq 0\}$
- (6)  $L_6 = A^* \setminus L_2$

**Problem 7** (Pythonic Automaton). Write a Python script in the same format as the Pythonic Automaton I that implements state  $s_1$  in abstract state diagram (A) from the games and puzzles section. Submit your program as a .py file in the programming submission box (see the other Gradescope assignment).

### Finite and Infinite Automata

**Problem 8** (Unravelling an Infinite Language). Draw a state diagram of all of the languages that are reachable from the language  $L = \{(ab)^n \mid n \in \mathbb{N}\}$  in the Brzozowski automaton by taking  $a$ - and  $b$ -derivatives (the words in this language are  $\varepsilon, ab, abab, \dots$ ). Include all of the double-circled states to indicate which languages are accepting states of the Brzozowski automaton. What language is accepted by  $L$ ?

**Problem 9** (Language Accepts Itself). Let  $L \subseteq A^*$  be any language. Prove that  $\mathcal{L}(\mathcal{A}_{Brz}, L) \subseteq L$ .