# Dynamic non-classicality

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N.B.: This is a preprint of an article whose final and definitive form will be published in the *Australasian Journal of Philosophy*, which is available online at http://www.tandf.co.uk/journals/.

**Abstract**

I show that standard dynamic approaches to the semantics of epistemic modals invalidate the classical laws of excluded middle and non-contradiction, as well as the law of 'epistemic non-contradiction'. I argue that these heretofore unnoticed facts pose a serious challenge for these approaches.

**Keywords**: dynamic semantics, epistemic modals, classical logic

## 1 Introduction

In the 1970s and 1980s, formal semantics took a dynamic turn which aimed to capture the behaviour of 'dynamic' features of natural language like anaphora resolution [Karttunen 1976; Kamp 1981; Heim 1982], presupposition projection [Karttunen 1974; Stalnaker 1974; Heim 1983], and epistemic modality [Veltman 1996; Groenendijk et al. 1996]. Our focus will be on the last of these: the use of words like 'must', and especially 'might', on a broadly epistemic interpretation.

The apparently dynamic features of epistemic modality are nicely illustrated by a contrast observed by Yalcin [2007].[1] Compare (1a) and (1b):

[1] Though Yalcin himself interpreted the data in a different, non-dynamic way.

(1) a. # Suppose it's raining and it might not be.

b. Suppose it's raining and for all we know it isn't.

Intuitively, 'It might not be raining' means roughly the same thing as 'For all we know, it's not raining'. But the two sentences in (1) sound very different: (1a) sounds incoherent, (1b) does not. One way to explain this is to hold that the information in the left conjunct in (1a)—that it's raining—is dynamically incorporated into the interpretation of the 'might' in the right conjunct. Then (1a) would end up meaning something like: 'Suppose it's raining and it might (be raining and not be raining)'. Assuming that 'For all we know' does not get interpreted along similarly dynamic lines, this would straightforwardly account for contrasts like those in (1), and similar contrasts in a wide variety of other environments.[2] Dynamic systems[3] aim to account for effects like this (along with related phenomena involving anaphora and presupposition) by treating a sentence meaning as a function which takes a context to a new context, rather than as a proposition. Conjunction is then treated as the successive application of the functions denoted by the left conjunct and the right conjunct, respectively. Given a suitable dynamic semantics for 'might', this neatly captures dynamic effects like those illustrated in (1a).

The aim of this paper is to point out that dynamic approaches to epistemic modality invalidate a wide variety of classical laws which are apparently valid. I argue that these facts,

[2] See related examples in, for instance, Groenendijk et al. [1996]; Aloni [2001]; Dorr and Hawthorne [2013]; Yalcin [2015]; Ninan [2018]; Mandelkern [2019].

[3] At least of the kind I focus on here, in the tradition following Heim [1982]; dynamic systems in the tradition of discourse representation theory [Kamp 1981], work very differently, and as far as I know do not face similar challenges to those I raise here.

which have not to my knowledge been previously observed, pose a serious challenge for dynamic systems.[4]

**2 Classical laws**

The classical laws I will focus on are the following: (where '$\phi$' ranges over sentences and '$\vDash \phi$' means that $\phi$ is a logical truth):[5]

*Law of Excluded Middle (EM):* $\vDash (\phi \vee \neg\phi)$

*Meta-Language Non-Contradiction ($NC_m$):* $(\phi \wedge \neg\phi)$ is not consistent.

*Object-Language Non-Contradiction ($NC_o$):* $\vDash \neg(\phi \wedge \neg\phi)$

Thus EM says, for instance, that a sentence like (2) is always true:

(2) Either John came to the party or John didn't come to the party.

And $NC_m$ says that (3) is never true, while $NC_o$ says that the negation of (3) is always true.

(3) John came to the party and John didn't come to the party.

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[4] The challenge I raise here also extends to systems which replicate certain features of dynamic semantics in a static setting, for instance the system in Klinedinst and Rothschild [2012].

[5] I generally omit (corner) quotes for readability.

These laws are widely taken to play a central role in characterising the behaviour of disjunction, conjunction, and negation.

**3 Dynamic semantics**

EM, $NC_o$, and $NC_m$ are all invalid in standard dynamic approaches to epistemic modals. I will illustrate this with the propositional fragment of the influential system from Groenendijk et al. [1996].[6] The points I make here extend, however, to every dynamic system I know of which includes epistemic modality,[7] except those which adopt the dynamic correlate of Boolean connectives, as I discuss in section 6.4. In this framework, a sentence $\phi$ denotes not a proposition (as in static frameworks), but rather a context change potential, written '$[\phi]$', which is a function which takes any context (a set of possible worlds) to a context, standardly written in post-fix notation ($c[\phi]$ is the result of applying $[\phi]$ to $c$). The semantics is given recursively from a set $At$ of atomic sentences $p, q, r \ldots$, a set of possible worlds $W$, and an interpretation function $I$ which assigns subsets of $W$ to elements of $At$. Our language is the set which results from closing $At$ under the one-place operators '$\neg$' and '$\Diamond$' (epistemic 'might') and two-place connectives '$\wedge$' and '$\vee$'. These have the following semantics, for any context $c$:

- $c[p] = \{w \in c : w \in I(p)\}$
- $c[\phi \wedge \psi] = c[\phi][\psi]$

[6] This comprises the semantics for atomic updates, negation, and conjunction of Heim [1983], together with the entries for disjunction from Groenendijk et al. [1996] and epistemic 'might' from Veltman [1996].
[7] Thus including developments of Groenendijk et al. [1996]'s system in for instance Dekker [1993]; Aloni [2001]; Beaver [2001]; Gillies [2001]; Yalcin [2015]; Gillies [2018]; Goldstein [2018].

- $c[\neg\phi] = c \setminus c[\phi]$
- $c[\phi \vee \psi] = c[\phi] \cup c[\neg\phi][\psi]$
- $c[\Diamond\phi] = \{w \in c : c[\phi] \neq \emptyset\}$

This system aims to capture the dynamic behaviour of epistemic modals illustrated by sentences like (1a). Conjunction is treated as successive updating, first with the left conjunct, then with the right. And $\Diamond\phi$ is treated as a check of whether a context is consistent with $\phi$. This means that, in a conjunction with the form $p \wedge \Diamond(\neg p)$, the 'might' will check a context which has already been updated with $p$. This check will always fail: a context updated with $p$ will not have any $\neg p$-worlds in it. Sentences with this form are thus contradictions in this system, accounting for the infelicity of sentences like (1a)—and, the hope is, more generally accounting for the dynamic nature of epistemic modality.

In addition to these semantic entries, it will be helpful to define dynamic notions of acceptance, entailment and consistency, following Veltman [1996]:

*Acceptance*: $c$ accepts $\phi$ iff $c = c[\phi]$

*Entailment*: A sequence $\langle \phi_1, \phi_2, \ldots \phi_n \rangle$ entails a sentence $\psi$, written $\phi_1, \phi_2, \ldots \phi_n \vDash \psi$, iff $\forall c : c[\phi_1][\phi_2]\ldots[\phi_n] = c[\phi_1][\phi_2]\ldots[\phi_n][\psi]$.

*Consistency*: A sequence of sentences $\langle \phi_1, \phi_2, \ldots \phi_n \rangle$ is consistent iff $\exists c : c[\phi_1][\phi_2]\ldots[\phi_n] \neq \emptyset$.

Thus for $\psi$ to be a logical truth (written '$\vDash \psi$') in this framework is for $\psi$ to be accepted by every context. It is important to note that this definition of entailment takes us far from the classical definitions in terms of preservation of truth, since in dynamic systems in general, truth does not play a central role. This means, for instance, that, rather than saying that $\phi\vee\neg\phi$ is true in every model, EM in a dynamic framework says that it is accepted in every context; likewise, instead of saying that $\phi\wedge\neg\phi$ is false in every model, $NC_m$ in a dynamic framework says that it takes every context to the empty set. So, in the context of dynamic semantics, these classical laws have a different interpretation than in static settings.[8] However, the dynamic notion of entailment is meant to answer to roughly the same observations that motivate static formulations: namely, the observation that a given argument is valid. Hence the considerations that motivate these classical laws on their static interpretations (for instance, the intuition that sentences with the form $\phi\vee\neg\phi$ are invariable true, and that $\phi\wedge\neg\phi$ never is) are equally motivations for these laws on their dynamic interpretation.

## 4 Failures

EM, $NC_o$, and $NC_m$ are all invalid in this dynamic system. In showing this, it will be helpful to have in hand two technical terms:

*Idempotence:* $\phi$ is idempotent iff $\forall c : c[\phi] = c[\phi][\phi]$

*Eliminativity*: $\phi$ is eliminative iff $\forall c : c[\phi] \subseteq c$

[8] See Stokke [2014] for discussion of how we might get truth back into the picture.

All sentences in our language are eliminative: updating always results in discarding worlds from a context or leaving it unchanged. But, importantly, not all of them are idempotent, as we will see.

Consider first $NC_m$. $NC_m$ is invalid in this system provided there is a context $c$ and sentence $\phi$ such that $c[\phi \wedge \neg\phi] \neq \emptyset$; and there do exist such sentences and contexts. For any $c$ and $\phi$, $c[\phi \wedge \neg\phi] = c[\phi] \setminus c[\phi][\phi]$. If there is a sentence $\phi$ in our system which is eliminative but not idempotent, then there must be a $c$ such that $c[\phi][\phi]$ is a proper subset of $c[\phi]$. Then it would follow that $c[\phi] \setminus c[\phi][\phi] \neq \emptyset$, and thus that $c[\phi \wedge \neg\phi] \neq \emptyset$. And indeed there are many sentences in our system which are eliminative but not idempotent. Here are two examples:

(4) $\Diamond p \wedge \neg p$

(5) $(\Diamond p \wedge \Diamond q) \wedge (\neg p \vee \neg q)$

These are eliminative, since, again, all sentences in our system are eliminative. And these sentences are not idempotent. Consider a context $s = \{w, w'\}$, with $w \in I(p)$, $w \in I(q)$, $w' \notin I(p)$, and $w' \notin I(q)$. Then $s[(4)] = \{w'\}$, but $s[(4)][(4)] = \emptyset$. Likewise $s[(5)] = \{w'\}$ but $s[(5)][(5)] = \emptyset$.

This reasoning thus shows that $(\phi \wedge \neg\phi)$ must be consistent for some substitution instances of $\phi$; in particular, it will be consistent when we substitute (4) or (5) for $\phi$. More concretely, consider $s[(\Diamond p \wedge \neg p) \wedge \neg(\Diamond p \wedge \neg p)]$. We have:

$$s[(\Diamond p \wedge \neg p) \wedge \neg(\Diamond p \wedge \neg p)] =$$

$$s[(\Diamond p \wedge \neg p)][\neg(\Diamond p \wedge \neg p)] =$$

$$s[\Diamond p][\neg p][\neg(\Diamond p\wedge\neg p)] =$$

$$\{w'\}[\neg(\Diamond p\wedge\neg p)] =$$

$$\{w'\} \setminus (\{w'\}[\Diamond p][\neg p]) = \{w'\} \setminus \emptyset = \{w'\}$$

Thus $(\Diamond p\wedge\neg p)\wedge\neg(\Diamond p\wedge\neg p)$ fails to take every context to the empty set, and thus is consistent in the present system. Similar reasoning will show that $s[((\Diamond p\wedge\Diamond q)\wedge(\neg p\vee\neg q))\wedge\neg((\Diamond p\wedge\Diamond q)\wedge(\neg p\vee\neg q))] = \{w'\}$, and thus that $((\Diamond p\wedge\Diamond q)\wedge(\neg p\vee\neg q))\wedge\neg((\Diamond p\wedge\Diamond q)\wedge(\neg p\vee\neg q))$ is likewise consistent. Thus $NC_m$ is invalid in our system: not every sentence of the form $(\phi\wedge\neg\phi)$ is inconsistent.

Given the failure of $NC_m$, it is straightforward to show the failure of $NC_o$. Our semantics of negation says that $c[\neg\phi] = c \setminus c[\phi]$; it follows that if $\phi$ is consistent, then there is a context $c$ such that $c[\phi] \neq \emptyset$, and thus that, given eliminativity, $c \setminus c[\phi] \neq c$, and thus that there is a context $c$ which fails to accept $\neg\phi$. From the fact that $(\Diamond p\wedge\neg p)\wedge\neg(\Diamond p\wedge\neg p)$ and $[((\Diamond p\wedge\Diamond q)\wedge(\neg p\vee\neg q))\wedge\neg((\Diamond p\wedge\Diamond q)\wedge(\neg p\vee\neg q))$ are consistent it thus follows that their negations are not accepted by every context, and thus are not logical truths. Thus $NC_o$ is invalid in our system: not every sentence of the form $\neg(\phi\wedge\neg\phi)$ is a logical truth.

As a concrete example, consider the result when we update $s$ with $\neg((\Diamond p\wedge\neg p)\wedge\neg(\Diamond p\wedge\neg p))$. $s[\neg((\Diamond p\wedge\neg p)\wedge\neg(\Diamond p\wedge\neg p))] = s \setminus s[(\Diamond p\wedge\neg p)\wedge\neg(\Diamond p\wedge\neg p)]$. Since we know the latter term is $\{w'\}$, the whole expression is equal to $\{w\}$, which is not equal to $s$. Thus $s$ fails to accept $\neg((\Diamond p\wedge\neg p)\wedge\neg(\Diamond p\wedge\neg p))$. Likewise, $s$ will not accept $\neg(((\Diamond p\wedge\Diamond q)\wedge(\neg p\vee\neg q))\wedge\neg((\Diamond p\wedge\Diamond q)\wedge(\neg p\vee\neg q)))$.[9]

[9] A close corollary of these facts is that conditionals with the form $\varphi\rightarrow\varphi$ are not always accepted. The standard dynamic semantics for the conditional from Dekker [1993] says $c[\varphi\rightarrow\psi] = \{w\in c : c[\varphi] = c[\varphi][\psi]\}$. Since $c[\varphi] \neq c[\varphi][\varphi]$ for non-idempotent sentences like (4) and (5), $\varphi\rightarrow\varphi$ will not be accepted e.g. when we substitute (4) or (5) for $\varphi$.

Finally, EM will not be valid in our system: $(\phi \vee \neg\phi)$ is not a logical truth in this system. To see this abstractly, note that $c[\phi \vee \neg\phi\ ] = c[\phi] \cup c[\neg\phi][\neg\phi]$. Given that our system is eliminative, $c[\phi] \cup c[\neg\phi][\neg\phi]$ will always be equal to $c$ iff $\neg\phi$ is idempotent, since $c[\phi] \cup c[\neg\phi][\neg\phi] = c[\phi] \cup c[\neg\phi] = c[\phi] \cup (c \setminus c[\phi]) = c$, iff $\neg\phi$ is idempotent. But there are many sentences $\phi$ in our system such that $\neg\phi$ fails to be idempotent, and thus EM is not valid.

To find such a $\phi$, note that double negation elimination is valid in our system: $\forall\phi : [\neg\neg\phi] = [\phi]$, since $\forall c\ \forall\phi : c[\neg\neg\phi] = c \setminus c[\neg\phi] = c \setminus (c \setminus c[\phi]) = c[\phi]$ (the last step follows because of eliminativity). And thus the negation of (4), namely $\neg(\Diamond p \wedge \neg p)$, will have a negation equivalent to (4). The negation of $\neg(\Diamond p \wedge \neg p)$ will thus fail to idempotent. So consider the context $s$ from our previous example, and let us update it with disjunction that results from putting $\neg(\Diamond p \wedge \neg p)$ in $\phi$, namely $\neg(\Diamond p \wedge \neg p) \vee \neg\neg(\Diamond p \wedge \neg p)$:

$$s[\neg(\Diamond p \wedge \neg p) \vee \neg\neg(\Diamond p \wedge \neg p)] =$$

$$s[\neg(\Diamond p \wedge \neg p)] \cup s[\neg\neg(\Diamond p \wedge \neg p)][\neg\neg(\Diamond p \wedge \neg p)] =$$

$$(s \setminus s[\Diamond p \wedge \neg p]) \cup s[\Diamond p \wedge \neg p][\Diamond p \wedge \neg p] =$$

$$(s \setminus \{w'\}) \cup \{w'\}[\Diamond p \wedge \neg p] =$$

$$\{w\} \cup \emptyset = \{w\} \neq s$$

Thus $s$ does not accept $(\phi \vee \neg\phi)$ when we let $\phi = \neg(\Diamond p \wedge \neg p)$. And so $(\phi \vee \neg\phi)$ is not a logical truth in our system for all substitution instances of $\phi$, and thus EM is invalid in our system. Further counterexamples to EM are again straightforward to produce, for instance by substituting the negation of (5) for $\phi$.

## 5 Epistemic non-contradiction

I do not know of discussion of these facts in the literature. One piece of evidence that they have not been observed comes from the fact that it has been frequently claimed that the *Epistemic Non-Contradiction* principle holds in our system:[10]

*Epistemic Non-Contradiction:* For all $\phi$: $(\phi \wedge \Diamond\neg\phi)$ is not consistent.

In the present system, Epistemic Non-Contradiction is equivalent to $NC_m$. The key point is that for any $c$ and $\phi$ in our system, $c[\Diamond\phi] = \emptyset$ just in case $c[\phi] = \emptyset$. Thus it follows that $(\phi\wedge\neg\phi)$ is inconsistent for all $\phi$ if and only if $(\phi\wedge\Diamond\neg\phi)$ is as well. Invalidity of $NC_m$ thus immediately entails the invalidity of Epistemic Non-Contradiction. This fact is of independent interest (though I will not explore it here); and dialectically, it suggests that the failure of $NC_m$ (and thus of its relatives $NC_o$ and EM) in our dynamic system has not been appreciated.[11]

For a concrete counterexample to Epistemic Non-Contradiction, we need only substitute (4) or (5) for $\phi$. For instance, consider $(\Diamond p\wedge\neg p)\wedge\Diamond(\neg(\Diamond p\wedge\neg p))$, and consider again the context $s$ from above. We have:

$$s[(\Diamond p\wedge\neg p) \wedge \Diamond\neg(\Diamond p\wedge\neg p)] =$$

$$s[\Diamond p\wedge\neg p][\Diamond\neg(\Diamond p\wedge\neg p)] =$$

[10] For instance in Gillies [2001]; von Fintel and Gillies [2007]; Yalcin [2012, 2015].
[11] Though the failure of ENC is implicit in a handout of Klinedinst and Rothschild [2014] which notes that negated epistemic contradictions are not tautologies.

$$\{w'\}[\Diamond\neg(\Diamond p\wedge\neg p)]$$

The result of updating $\{w'\}$ with $\Diamond\neg(\Diamond p\wedge\neg p)$ depends on whether the result of updating $\{w'\}$ with $\neg(\Diamond p\wedge\neg p)$ is empty.[12] It is not:

$$\{w'\}[\neg(\Diamond p\wedge\neg p)] =$$
$$\{w'\} \setminus (\{w'\}[\Diamond p\wedge\neg p]) =$$
$$\{w'\} \setminus \emptyset = \{w'\}$$

Thus $\{w'\}[\neg(\Diamond p\wedge\neg p)]$ is non-empty, and thus $\{w'\}[\Diamond\neg(\Diamond p\wedge\neg p)] = \{w'\}$. And so $s[(\Diamond p\wedge\neg p)\wedge\Diamond\neg(\Diamond p\wedge\neg p)] = \{w'\}$, not $\emptyset$. Thus Epistemic Non-Contradiction is invalid in our system.

## 6 Possible responses

What should we make of these facts?

### 6.1 Embrace the result

The most daring response on behalf of dynamic treatments of epistemic modality would be to embrace these results: to argue that EM, $NC_o$, and $NC_m$ are only valid over a non-modal fragment, and that these principles fail precisely where dynamic semantics predicts them to fail, namely, once we introduce epistemic modals.

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12 In the variant in Gillies [2018], this depends instead on whether some subset of $\{w'\}$ accepts $\neg(\Box p\wedge\neg p)$; it can readily be seen that one does, namely $\{w'\}$.

Certain failures of these laws in dynamic systems, in particular systems for anaphora, might indeed warrant this response. In such systems (for instance the system of Groenendijk and Stokhof [1991]), these laws can fail in non-modal sentences, provided that pronouns and quantifiers are co-indexed in a certain way.[13] Since there are no transparent ways to determine when quantifiers and pronouns are co-indexed in natural language (we do not pronounce numeric subscripts), it is not obvious what empirical purchases these failures have; and so, while there is perhaps no evidence *for* these failures, it is hard to find positive evidence that these predictions are wrong.

But I do not see any prospects for a defense along these lines when we turn our attention to the failures of these laws that I have pointed to above. For when it comes to these failures, it seems clear what the intended, unambiguous translation of these sentences into natural language amounts to (once we arbitrarily set meanings for our atomic sentences); and it is clear that the predicted failures of EM, $NC_o$, and $NC_m$ do not match intuitions about the corresponding sentences in natural language. To make this point, let $p$ = 'Paul is sick', and let $q$ = 'Mark is sick'. Then (4) will be translated into natural language as (6):

(6) Paul might be sick and he isn't.

And (5) will be translated as (7a) (or more colloquially as (7b)):

(7) a. Paul might be sick and Mark might be sick, and either Paul isn't sick or Mark isn't sick.

[13] Thanks to Paul Dekker for pointing this out to me and for helpful discussion.

b. Paul might be sick and Mark might be sick, but one of them isn't sick.

Now let us ask whether the predicted failures of EM, $NC_o$, and $NC_m$ match intuition. Focus on (7), since (6) is itself quite strange to begin with. The predicted failure of $NC_m$ that we get from (7) is in (8):

(8) Paul might be sick and Mark might be sick, but one of them isn't sick; and it's not the case that (Paul might be sick and Mark might be sick, but one of them isn't sick).

Its complexity notwithstanding, (8) seems to be perfectly inconsistent, like any sentence of the form $(\phi\wedge\neg\phi)$. Likewise, the negation of (8) in (9) seems to be logically true:

(9) It's not the case that: (Paul might be sick and Mark might be sick, but one of them isn't sick; and it's not the case that (Paul might be sick and Mark might be sick, but one of them isn't sick)).

But, of course, our dynamic system predicts that (8) is consistent, and thus that (9) is not a logical truth. These predicted failures of $NC_m$ and $NC_o$ do not seem to match intuition.

Nor are matters better when we turn to EM. Consider (10):

(10) Either Paul might be sick and Mark might be sick, but one of them isn't sick; or it's not the case that (Paul might be sick and Mark might be sick, but one of them isn't sick).

Again, complexity notwithstanding, (10) seems to be a logical truth as much as any disjunction of the form ($\phi \vee \neg\phi$). This is again contrary to the predictions of our theory, which says that (10) is not a logical truth.

EM, $NC_o$, and $NC_m$ are by no means sacrosanct. We may find reason to question them, for instance from semantic paradoxes or vagueness. Thus constrained failures of these laws in a formal system may be harmless or even desirable, if those failures conform with intuition about natural language. But the counterexamples to these laws predicted by the present system are in clear conflict with intuition about the corresponding natural language sentences, and thus I do not see any prospect for a defender of dynamic semantics to simply embrace these results.

### 6.2 Adopt a different notion of entailment

A second avenue that defenders of dynamic semantics might pursue would be to adopt a different notion of entailment. For instance, we might spell out a variant of $NC_m$ in terms of Groenendijk et al. [1996]'s notion of *coherence*, rather than consistency:

*Non-Contradiction*, *Coherence Version ($NC_c$)*: ($\phi \wedge \neg\phi$) is not coherent.

Coherence is defined as follows:

*Coherence*: $\phi$ is coherent iff there is a $c : c \neq \emptyset$ and $c[\phi] = c$.

$NC_c$, by contrast to $NC_m$, is valid in our system. To see this, assume that $c$ accepts $\phi \wedge \neg\phi$, that is, $c = c[\phi \wedge \neg\phi] = c[\phi] \setminus c[\phi][\phi]$. Given eliminativity, we have two options: $c[\phi] = c$ or $c[\phi] \subsetneq c$. If the latter, then, given eliminativity, it also follows that $c[\phi] \backslash c[\phi][\phi] \subsetneq c$. But we already know that $c[\phi] \setminus c[\phi][\phi] = c$; contradiction. So $c[\phi] = c$. But then of course $c[\phi][\phi]$ is also $c$, and so $c[\phi] \setminus c[\phi][\phi] = c \setminus c = \emptyset = c$. And so $(\phi \wedge \neg\phi)$ is only accepted by the empty context; and so it is not coherent. Defenders of the dynamic approach might hold that intuitions which seem to support $NC_m$ could equally well be taken to support $NC_c$; and they validate the latter.

But this is little help. A first, milder, issue is that consistency can be bootstrapped to coherence with just the help of an epistemic modal: that is, even though sentences with the form $(\phi \wedge \neg\phi)$ are not coherent, sentences of the form $\Diamond(\phi \wedge \neg\phi)$ *are* coherent in our system (the same substitution instances as above witness this; since there are non-empty contexts $s$ and sentences $(\phi \wedge \neg\phi)$ such that $s[(\phi \wedge \neg\phi)] \neq \emptyset$, it follows immediately that $s[\Diamond(\phi \wedge \neg\phi)] = s$). But this result seems nearly as bad as the prediction that $(\phi \wedge \neg\phi)$ is coherent; the fact that $(\phi \wedge \neg\phi)$ is incoherent is little comfort if close variations like this remain coherent (variations on this point are easy to develop; for instance $\exists x(\phi(x) \wedge \neg\phi(x))$ will be coherent if we adopt Groenendijk et al. [1996]'s semantics for $\exists$).

The second, more important, issue is that this response does nothing to change our predictions about EM or $NC_o$. Adopting a different notion of entailment won't save EM or $NC_o$. Those principles are claims about logical truth; all the main dynamic theories of entailment coincide when it comes to their definitions of logical truth, and I cannot see any similar principles in the neighborhood which we might be convinced to accept in their places.[14] And so even if we could (imprudently) convince ourselves to be satisfied with $NC_c$ in place of $NC_m$, we

[14] This is so, for instance, of all the definitions of entailment given in Veltman 1996.

would remain in serious trouble from the failures of EM and $NC_o$.

### 6.3 Insist on idempotence

The final two responses worth considering acknowledge the seriousness of the failures of $NC_m$, $NC_o$, and EM in our system, and propose close modifications on the underlying system to avoid these failures. The first of these responses goes as follows. As we saw above, all the failures of our three principles stem from failures of idempotence; in a fully idempotent fragment (for instance, in the non-modal fragment of the present system), our entries for conjunction and negation suffice to validate $NC_o$ and $NC_m$; and, assuming the fragment is also eliminative, our entries for disjunction and negation suffice to validate EM. One option to take is thus to try to eliminate failures of idempotence from our system.

There are two different ways we could try to do this. One is global: ensure that no operators are included in the language which can create non-idempotent sentences. Provided that we hold onto the standard dynamic entries for the connectives given above, this would simply rule out Veltman [1996]'s semantics for '$\Diamond$'. A different approach would be local: rule out by fiat non-idempotent sentences (but still allow sentences with '$\Diamond$', provided they are idempotent). This local approach is suggested for different reasons in Yalcin [2015], building on similar proposals in Klinedinst and Rothschild [2014].

The local approach strikes me as a non-starter, because it rules out sentences with the form of (5), which, as we saw above, are non-idempotent, but which translate to perfectly felicitous sentences of natural language (like (7)). Any account of epistemic modals which predicts a sentence like (5) to be inconsistent or ill-formed is the wrong account.[15]

The global approach, by contrast, is more plausible. Indeed, my own view is that we should reject Veltman's '◊'. However, this is just to give up on the dynamic approach to epistemic modality.

It should be emphasized that other parts of the dynamic system—for instance, dynamic approaches to presupposition and anaphora—could survive this. Having said that, this more limited approach would undermine one of the motivations for dynamic semantics, namely providing a unified treatment of the dynamics of anaphora, presupposition, and epistemic modality. The present considerations also bring out a more abstract consideration which tells against even a limited dynamic approach like this. Any theory which incorporates the standard entries for conjunction, disjunction, and negation above will have to adopt further stipulations to ensure that the theory validates the classical laws we are discussing: those entries for the connectives do not on their own suffice to validate those laws, as we have seen. By contrast, standard static Boolean connectives validate these principles without further stipulation. This strikes me as a theoretical benefit of static approaches over dynamic competitors, even those competitors which succeed in stipulating away actual failures of those laws by rejecting the standard dynamic treatment of epistemic modality.

### 6.4 Adopt different connectives

[15] Klinedinst and Rothschild [2014] propose an interesting modification of a semantic system like ours which rules out some but not all idempotence. On that system, context updates are, in essence, repeated until they reach a fixed point. This helps with issues concerning non-contradiction, without falling into the present pitfall. It is, however, no help in avoiding the issues concerning excluded middle.

In light of this last point, it is worth asking, finally, whether dynamic semanticists can have what static semanticists do: a semantics for connectives which immediately validates EM, $NC_o$, and $NC_m$, without stipulations about what vocabulary is admissible in the language.

Indeed they can, or at least nearly so. The following semantics for conjunction, from Veltman [1996], together with the dynamic negation from above, guarantees the validity of $NC_o$ and $NC_m$, *no matter how the language is extended*:

- $c[\phi\wedge\psi] = c[\phi] \cap c[\psi]$

This is because, in this system, $\forall c \forall \phi : c[\neg(\phi\wedge\neg\phi)] = c \setminus (c[\phi] \cap c[\neg\phi]) = c \setminus (c[\phi] \cap (c \setminus c[\phi]))$ $= c$, so every context accepts $\neg(\phi\wedge\neg\phi)$. Likewise, $(\phi\wedge\neg\phi)$ will take every context to $\emptyset$, since $\forall c \forall \phi : c[\phi\wedge\neg\phi] = c[\phi] \cap (c \setminus c[\phi]) = \emptyset$. So $NC_o$ and $NC_m$ will be valid, no matter how the system is extended.

Likewise, suppose we adopt this disjunction from Veltman [1996]:

- $c[\phi\vee\psi] = c[\phi] \cup c[\psi]$

Then EM will be guaranteed to be valid for any extension of the fragment *which remains eliminative*. In that case we will have that $\forall c \forall \phi : c[\phi\vee\neg\phi] = c[\phi] \cup (c \setminus c[\phi])$, which will be $c$, thanks to eliminativity. This is not yet quite as general as we might want—if we extend our language with non-eliminative sentences, we will still invalidate EM—but it is a major improvement on the present situation.

So could we hold onto the dynamic 'might', reject the standard dynamic connectives in favor of these entries, and avoid the challenge I have raised? There is reason to hesitate. First, although these entries validate $NC_o$ and $NC_m$, they still face a closely related problem. These entries validate $NC_o$ and $NC_m$ because, unlike the standard entry for conjunction, they treat conjunction as set intersection, rather than consecutive update. But consecutive assertions, of course, are still treated as consecutive updates. And so, even though this approach predicts that all sentences of the form $(\phi\wedge\neg\phi)$ are inconsistent, it predicts that *sequential assertions* of the form $\langle\phi,\neg\phi\rangle$ can still be perfectly consistent (the same substitution instances as above witness this, for precisely the same reasons). This seems nearly as problematic to me as the failure to validate $NC_o$ or $NC_m$, and so it is not really clear that this approach solves our basic problem.[16]

Second, these entries strip the dynamic system of much of what is attractive and interesting about it: namely, its ability to capture the characteristic dynamic *intrasentential* behaviour of epistemic modality, presupposition, and anaphora. This, after all, is the whole point of dynamic semantics. The entries under consideration validate none of the intrasentential patterns which have been used to motivate dynamic semantics, and which have been claimed as successes for dynamic semantics. I will mention just two examples here.

First, although, as I have shown, Epistemic Non-Contradiction is not valid in general in the dynamic system laid out above, it *is* valid in that system when restricted to the non-modal fragment of the system (that is, $(\phi\wedge\Diamond\neg\phi)$ will be inconsistent provided $\phi$ does not contain '$\Diamond$'). This remains a success of dynamic semantics—albeit more limited than has been claimed—

[16] Though it should be noted that these sequences *will* be incoherent, which might provide some explanation of what is wrong with them.

which illustrates some of what is attractive about the approach. This success would be lost if we adopted the entry for conjunction under consideration.

Second, there are apparent asymmetries in how we process conjunctions containing anaphora and presupposition; for instance, 'A man walked in and he sat down' feels quite different from 'He sat down and a man walked in'. Likewise 'France has a king, and the king of France is bald' is quite different from 'The king of France is bald, and France has a king'. The standard dynamic connectives are designed to predict precisely these asymmetries, and their ability to do so is a central success story of dynamic semantics. By contrast, the entries under consideration here are symmetric, and so do not predict these order asymmetries. So going this way would sacrifice much of what is attractive not only in dynamic treatments of epistemic modality, but also in dynamic treatments of anaphora and presupposition.

## 7 Conclusion

The challenge which these considerations raise, and which I leave open here, is whether dynamic semantics can capture the characteristic dynamic patterns of epistemic modality without simultaneously invalidating classical patterns which we have every reason to preserve.

**Acknowledgements**

Many thanks to Paul Dekker, Anthony Gillies, Simon Goldstein, Daniel Rothschild, Ginger Schultheis, and two anonymous referees for this journal for very helpful comments and discussion.

## REFERENCES

Aloni, Maria D. 2001. *Quantification Under Conceptual Covers*. University of Amsterdam dissertation.

Beaver, David. 2001. *Presupposition and Assertion in Dynamic Semantics*. CSLI Publications: Stanford, CA.

Dekker, Paul. 1993. *Transsentential Meditations: Ups and Downs in Dynamic Semantics*. University of Amsterdam dissertation.

Dorr, Cian and John Hawthorne. 2013. Embedding Epistemic Modals. *Mind* 122/488: 867–913.

von Fintel, Kai and Anthony Gillies. 2007. An Opinionated Guide to Epistemic Modality. In *Oxford Studies in Epistemology 2*, 32–62, eds. Tamar Gendler and John Hawthorne. Oxford: Oxford University Press.

Gillies, Anthony. 2001. A New Solution to Moore's Paradox. *Philosophical Studies* 105/3: 237–250.

Gillies, Anthony S. 2018. Updating Data Semantics. *Mind*.

Goldstein, Simon. 2018. Generalized Update Semantics. *Mind*.

Groenendijk, Jeroen and Martin Stokhof. 1991. Dynamic Predicate Logic. *Linguistics and Philosophy* 14/1: 39–100.

Groenendijk, Jeroen, Martin Stokhof and Frank Veltman. 1996. Coreference and Modality. In *Handbook of Contemporary Semantic Theory*, ed. Shalom Lappin, 179–216. Oxford: Blackwell.

Heim, Irene. 1982. *The Semantics of Definite and Indefinite Noun Phrases*. University of Massachusetts, Amherst dissertation.

Heim, Irene. 1983. On the Projection Problem for Presuppositions. In *The West Coast Conference on Formal Linguistics (WCCFL)*, eds. Michael Barlow, Daniel P. Flickinger and Nancy Wiegand, vol. 2, 114–125. Stanford: Stanford University Press.

Kamp, Hans. 1981. A theory of truth and semantic representation. In *Formal Methods in the Study of Language*, ed. J. Groenendijk et al. Amsterdam: Mathematisch Centrum, University of Amsterdam.

Karttunen, Lauri. 1974. Presupposition and Linguistic Context. *Theoretical Linguistics* 1/1-3: 181–194.

Karttunen, Lauri. 1976. Discourse Referents. In *Syntax and semantics: Notes from the Linguistic Underground*, vol. 7, Academic Press.

Klinedinst, Nathan and Daniel Rothschild. 2012. Connectives Without Truth Tables. *Natural Language Semantics* 20/2: 137–175.

Klinedinst, Nathan and Daniel Rothschild. 2014. Epistemic Contradictions: Why Idempotence is Hygienic. Handout.

Mandelkern, Matthew. 2019. Bounded Modality. *The Philosophical Review* 128/1: 1–61.

Ninan, Dilip. 2018. Quantification and Epistemic Modality. *The Philosophical Review* 127/2: 433–485.

Stalnaker, Robert. 1974. Pragmatic Presuppositions. In *Semantics and Philosophy*, eds. Milton K. Munitz and Peter Unger, 197–213. New York: New York University Press.

Stokke, Andreas. 2014. Truth and Context Change. *Journal of Philosophical Logic* 43/1: 33–51.

Veltman, Frank. 1996. Defaults in Update Semantics. *Journal of Philosophical Logic* 25/3: 221–261.

Yalcin, Seth. 2007. Epistemic Modals. *Mind* 116/464: 983–1026.

Yalcin, Seth. 2012. Dynamic Semantics. In *Routledge Companion to Philosophy of Language*, eds. Gillian Russell and Delia Graff Fara, Abingdon-on-Thames: Routledge.

Yalcin, Seth. 2015. Epistemic Modality *de re*. *Ergo* 2/19: 475–527.

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