Simplifying with Free Choice*

Malte Willer

Abstract

This paper offers a unified semantic explanation of two observations that prove to be problematic for classical analyses of modals, conditionals, and disjunctions: (i) the fact that disjunctions scoping under possibility modals give rise to the free choice effect and (ii) the fact that counterfactuals license simplification of disjunctive antecedents. It shows that the data are well explained by a dynamic semantic analysis of modals and conditionals that uses ideas from the inquisitive semantic tradition in its treatment of disjunction. The analysis explains why embedding a disjunctive possibility under negation reverts disjunction to its classical behavior, is general enough to predict less studied simplification patterns, and also makes progress toward a unified perspective on the distinction between informative, inquisitive, and attentive content.

1 The Plot

It is a familiar observation that disjunctions scoping under possibility modals give rise to the *free choice* effect:

(1) You may take an apple or a pear.
  a. $\rightsquigarrow$ You may take an apple.
  b. $\rightsquigarrow$ You may take a pear.

(2) Mary might be in Chicago or in New York.
  a. $\rightsquigarrow$ Mary might be in Chicago.
  b. $\rightsquigarrow$ Mary might be in New York.

In both (1) and (2), the possibility of a disjunction seems to entail the possibility of each disjunct. That is, to say the least, a bit of a puzzler for the standard analysis of modality and disjunction since the possibility of a disjunction is classically consistent with the impossibility of one of its disjuncts.[1]

The puzzler is, on first sight anyway, related to the trouble that disjunctions in conditional antecedents cause for the standard analysis of counterfactuals. Fine (1975) and Nute (1975) observe that counterfactuals in general license SIMPLIFICATION OF DISJUNCTIVE ANTECEDENTS (SDA):

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[1]Kamp's (1973, 1978) discussion of the free choice effect is seminal, though the label goes back to von Wright (1968).

SDA $(\phi \vee \psi) \mathrel{\Box\!\!\rightarrow} \chi \vDash \phi \mathrel{\Box\!\!\rightarrow} \chi, \psi \mathrel{\Box\!\!\rightarrow} \chi$

But on a textbook variably strict analysis, a *would*-counterfactual $\ulcorner\phi \mathrel{\Box\!\!\rightarrow} \psi\urcorner$ is true at some possible world w iff $f_c(w, [\![\phi]\!]) \subseteq [\![\psi]\!]$, where $f_c(w, [\![\phi]\!])$ denotes the ϕ-worlds closest to w given some contextually provided similarity relation between worlds (Lewis 1973, Stalnaker 1968). So given classical disjunction, SDA inferences are unexpected in a variably strict setting since $f_c(w, [\![\phi \vee \psi]\!]) \neq f_c(w, [\![\phi]\!]) \cup f_c(w, [\![\psi]\!])$ unless the closest ϕ-worlds are just as close to w as are the closest ψ-worlds, leaving it unexplained why (3) seems to entail that the party would have been fun if Alice had come *and* that the party would have been fun if Bert had come:

(3) If Alice or Bert had come to the party, it would have been fun.
 a. $\rightsquigarrow$ If Alice had come to the party, it would have been fun.
 b. $\rightsquigarrow$ If Bert had come to the party, it would have been fun.

The goal of this paper is to demonstrate that these and related observations receive a unified semantic explanation in a dynamic analysis of modals and conditionals that uses ideas from the inquisitive semantic tradition.

It strikes me as uncontroversial that the problem of free choice and the problem of simplification should receive a unified solution (see also Franke 2011, Klinedinst 2007, and van Rooij 2006). That such a solution could be semantic in nature might be less obvious since it is natural to treat free choice effects—and, mutatis mutandis, simplification inferences—as conversational implicatures.[2] This intuition connects with the observation that embedding a disjunctive possibility under negation reverts disjunction to its classical behavior:

(4) You may not take an apple or a pear.
 a. $\rightsquigarrow$ You may not take an apple.
 b. $\rightsquigarrow$ You may not take a pear.

(5) Mary cannot be in Chicago or in New York.
 a. $\rightsquigarrow$ Mary cannot be in Chicago.
 b. $\rightsquigarrow$ Mary cannot be in New York.

In general, free choice effects reliably disappear in downward entailing contexts. In this respect they behave like scalar implicatures, which supports the thesis that we are dealing here with a pragmatic phenomenon.

My goal here is not to argue against pragmatic approaches to free choice and simplification effects—this is a task too complex to be efficiently executed here.[3] Rather, I will focus on the more constructive project of bringing a viable semantic treatment of these problems into view. This involves explaining how the free choice effect could

[2] Pragmatic treatments of the free choice effect include Alonso-Ovalle 2006, Fox 2007, Franke 2011, Klinedinst 2007, Kratzer and Shimoyama 2002, and Schulz 2005. Semantic approaches include Aher 2012, Aloni 2007, Barker 2010, Fusco 2015a, 2015b, Geurts 2005, Simons 2005, Starr 2016, and Zimmermann 2000. Simplification has also received some attention in the literature: Franke (2011), Klinedinst (2009), and van Rooij (2010) offer pragmatic explanations for why counterfactuals should simplify in a variably strict analysis, while Alonso-Ovalle (2009) and van Rooij (2006) propose semantic approaches.

[3] Recent processing and acquisition studies suggest significant differences between free choice effects and scalar implicatures (see Chemla and Bott 2014 and Tieu et al. 2016, respectively), but of course this does not prove that free choice cannot be a pragmatic affair.

be a semantic entailment and still disappear in downward entailing environments, and I demonstrate that we can do so with a suitably sophisticated analysis of negation. It also involves offering a treatment of simplification that is more comprehensive than what existing semantic analyses have to offer. Let me explain.

One prominent semantic approach to the simplification problem goes with a Hamblin-style analysis of disjunction and then lets *if*-clauses be universal quantifiers so that $\ulcorner\phi \mathrel{\Box\!\!\rightarrow} \psi\urcorner$ is now true at some possible world w iff $f_c(w, p) \subseteq [\![\psi]\!]$ for all propositions p in the set of alternatives denoted by the antecedent ϕ—a singleton in case of non-disjunctive antecedents; a set containing all the atomic propositional disjuncts if the antecedent is a disjunct (see Alonso-Ovalle 2009). Another prominent idea is to adopt an existential analysis of disjunction—the antecedent of (3), for instance, would be of the form '$\exists x.\, \mathrm{C}x \wedge (x = \mathrm{Alice} \vee x = \mathrm{Bert})$' with 'C' denoting the property of coming to the party—and a treatment of indices of evaluation as world-assignment pairs. If we now say that two such pairs are unconnected (and hence none of them more similar to the index of evaluation than the other) if their assignments differ, we predict that the counterfactual selection function includes indices at which Alice comes to the party as well as indices at which Bert comes to the party (see van Rooij 2006).

Both approaches explain why (3) simplifies but are hand-tailored to deal with simplifications of *disjunctive antecedents*: what does the explanatory lifting in each approach is the special interaction between a non-classical analysis of disjunction with whatever is involved in interpreting *if*-clauses. And this cannot be the whole story since counterfactuals with a *negated conjunction* as antecedent also simplify (see Nute 1980) and since *might*-counterfactuals allow for simplification of their disjunctive *consequent*:

(6) If Alice and Bert had not both come to the party, it would have been fun.
   a. $\rightsquigarrow$ If Alice had not come to the party, it would have been fun.
   b. $\rightsquigarrow$ If Bert had not come to the party, it would have been fun.

(7) If Mary had not gone to Pisa, she might have gone to Lisbon or Rome.
   a. $\rightsquigarrow$ If Mary had not gone to Pisa, she might have gone to Lisbon.
   b. $\rightsquigarrow$ If Mary had not gone to Pisa, she might have gone to Rome.

It will not do to just stipulate that (6) is evaluated by checking its consequent against the union of the closest worlds in which Alice and Bert do not come the party, respectively: this fact calls for an explanation in terms of negation and conjunction just as much as SDA called for an explanation in terms of disjunction.[4] Explaining the simplification pattern exhibited by *might*-counterfactuals such as (7) is also challenging in a variably strict setting under the reasonable assumption that '$\mathrel{\Diamond\!\!\rightarrow}$' and '$\mathrel{\Box\!\!\rightarrow}$' are duals ($\phi \mathrel{\Diamond\!\!\rightarrow} \psi =_{\mathrm{def}} \neg(\phi \mathrel{\Box\!\!\rightarrow} \neg\psi)$). For observe that given some context c, the truth of $\ulcorner\neg(\phi \mathrel{\Box\!\!\rightarrow} \neg(\psi \vee \chi))\urcorner$ at some possible world w only requires that $f_c(w, [\![\phi]\!]) \cap [\![\psi]\!] \neq \varnothing$ or $f_c(w, [\![\phi]\!]) \cap [\![\chi]\!] \neq \varnothing$, not both. In any case, a comprehensive semantic story about how and why counterfactuals simplify cannot be entirely dependent on the interpretation of *if*-clauses

[4] As I will discuss more explicitly in Section 4.2, not all counterfactuals with negated conjunctions as antecedents seem to simplify, but I take this to be part of the explanandum rather than showing that there is nothing to be explained here in the first place. Nute's (1980) original example moves from 'If Nixon and Agnew had not both resigned, Ford would never have become president' to 'If Nixon had not resigned, Ford would never have become president' and 'If Agnew had not resigned, Ford would never have become president.' One may worry that the appeal of Nute's example stems from our knowledge of the historical facts, however, and so it is preferable to choose are more arbitrary case such as (6).

(or on considerations about the similarity relation) since simplification is also a feature of disjunctive counterfactual consequents.

The problems that simplification poses thus go beyond the familiar observations about SDA in non-trivial ways. On the view developed here, a semantic explanation of why counterfactuals with disjunctive antecedents simplify flows from a semantic account of the free choice reading of disjunctions under existential modals. The proposal, in brief, is that a *would*-counterfactual $\ulcorner\phi \mathrel{\Box\!\!\rightarrow} \psi\urcorner$ is a strict material conditional $\ulcorner\Box(\phi \supset \psi)\urcorner$ over a contextually determined but dynamically evolving domain of quantification presupposing that its antecedent ϕ is a possibility in that domain (von Fintel 2001, Gillies 2007, Willer 2013b, forthcoming). Assuming duality and that presuppositions project out of negation, a *might*-counterfactual $\ulcorner\phi \mathrel{\Diamond\!\!\rightarrow} \psi\urcorner$ amounts to an assertion of $\ulcorner\Diamond(\phi \wedge \psi)\urcorner$ under the presupposition that ϕ is possible. Once we predict that $\ulcorner\Diamond(\phi \vee \psi)\urcorner$ and $\ulcorner\Diamond\neg(\neg\phi \wedge \neg\psi)\urcorner$ entail $\ulcorner\Diamond\phi\urcorner$ *as well as* $\ulcorner\Diamond\psi\urcorner$ via a semantic free choice effect, we predict that (3) as well as (6) and (7) simplify the way they do.

My plan is as follows. Section 2 offers a semantic analysis of possibility modals that delivers free choice without negative side effects, combining insights from the dynamic and the inquisitive tradition in a nonclassical semantic outlook on modals and propositional connectives. Section 3 builds an analysis of counterfactuals on top of the framework developed in the previous section. Section 4 addresses some remaining issues, including the availability of ignorance readings of free choice items and the observation that counterfactuals with disjunctive antecedents do not always seem to simplify. Section 5 summarizes the key findings and briefly outlines avenues for further research.

2 Basic Framework

The target language $\mathcal{L}$ contains a set of sentential atoms $\mathcal{A} = \{p, q, ...\}$ and is closed under negation ($\neg$), conjunction ($\wedge$), disjunction ($\vee$), the modal possibility operator ($\Diamond$), and the *would*-counterfactual ($\mathrel{\Box\!\!\rightarrow}$). Other connectives are defined in the usual manner. In this section I state the semantics for modals and propositional connectives. The subsequent section addresses counterfactuals.

2.1 Semantics: First Steps

The analysis of modals is in the spirit of Veltman's (1996) approach but here I treat input contexts as sets of consistent propositions (labeled *alternatives*).

Definition: Possible Worlds, Propositions. w is a *possible world* iff $w\colon \mathcal{A} \to \{0,1\}$. W is the set of such w's, $\mathcal{P}(W)$ is the powerset of W. The function $[\![\cdot]\!]$ assigns to nonmodal formulas of $\mathcal{L}$ a *proposition* in the familiar fashion. $\perp$ is the contradictory proposition (the empty set of possible worlds) while $\not\perp$ is any consistent proposition.

Definition: States, Alternatives. A *state* $s \subseteq \mathcal{P}(W) \setminus \perp$ is any set of consistent propositions (*alternatives*). S is just the set of all such states. The *information* carried by a state s is the set of possible worlds compatible with it so that $\texttt{info}(s) = \{\bigcup \sigma \colon \sigma \in s\}$. We refer to $\varnothing$ as the *absurd* state and speak of $s_0 = \mathcal{P}(W) \setminus \perp$ as the *initial* state.

States thus have informational content in the sense that they rule out certain ways the world could be. In addition, they encode this information as a set of alternatives (which do not have to be mutually exclusive).

States are updated by updating each of their alternatives. Updates on an alternative σ are sensitive to the state s containing it since modals perform tests on the state's informational content. Furthermore, I will think of update rules as *relations* between alternatives to capture the inquisitive effect of disjunction and distinguish between a *positive* acceptance inducing update relation $[\cdot]^+_s$ and a *negative* rejection inducing update relation $[\cdot]^-_s$ to allow for inquisitive negation.[5] So for instance we shall say:

$$(\mathcal{A}) \quad \begin{array}{l} \sigma[p]^+_s\tau \text{ iff } \tau = \sigma \cap [\![p]\!] \\ \sigma[p]^-_s\tau \text{ iff } \tau = \sigma \backslash [\![p]\!] \end{array} \qquad\qquad (\neg) \quad \begin{array}{l} \sigma[\neg\phi]^+_s\tau \text{ iff } \sigma[\phi]^-_s\tau \\ \sigma[\neg\phi]^-_s\tau \text{ iff } \sigma[\phi]^+_s\tau \end{array}$$

A positive update with a sentential atom p eliminates from an alternative all possible worlds at which p is false; a negative update with p eliminates all possible worlds at which p is true. A positive update with $\ulcorner\neg\phi\urcorner$ is a negative update with ϕ; a negative update with $\ulcorner\neg\phi\urcorner$ is a positive update with ϕ.

The basic idea about possibility modals is that they test whether their prejacent relates the information carried by a state to a contradiction $\perp$ or to a consistent proposition $\not\perp$:

$$(\Diamond) \quad \begin{array}{l} \sigma[\Diamond\phi]^+_s\tau \text{ iff } \tau = \{w \in \sigma \colon \langle \texttt{info}(s), \perp\rangle \notin [\phi]^+_s\} \\ \sigma[\Diamond\phi]^-_s\tau \text{ iff } \tau = \{w \in \sigma \colon \langle \texttt{info}(s), \not\perp\rangle \notin [\phi]^+_s\} \end{array}$$

For an alternative in a state s to pass the test imposed by a *positive* update with $\ulcorner\Diamond\phi\urcorner$, the information carried by s must not be related to the *inconsistent* proposition via a positive update with ϕ. For an alternative in a state s to pass the test imposed by a *negative* update with $\ulcorner\Diamond\phi\urcorner$—that is, a positive update with $\ulcorner\Box\neg\phi\urcorner$—the information carried by s must not be related to a *consistent* proposition via a positive update with ϕ. Differences in modal flavor (epistemic, deontic, and so on) correspond to differences in the state that context deems relevant for evaluating the modal in question. I will later say more about how context fixes modal domains (see Section 4.3) but set the ensuing complications aside for now to focus the key structural features of the proposal.

So far we only have a rewrite of classical Update Semantics but the present setup allows us to combine a test semantics for modals with an inquisitive analysis of disjunction and negated conjunction. Start by coupling each update rule for alternatives with a corresponding update procedure for states:

Definition: Updates on States. Define two update operations $\uparrow, \downarrow \colon (\mathcal{L} \times S) \mapsto S$:

1. $s \uparrow \phi = \{\tau \neq \perp \colon \exists\sigma \in s.\, \sigma[\phi]^+_s\tau\}$
2. $s \downarrow \phi = \{\tau \neq \perp \colon \exists\sigma \in s.\, \sigma[\phi]^-_s\tau\}$

[5] The appeal to acceptance and rejection inducing update procedures owes inspiration to Aher (2012) and to Groenendijk and Roelofsen (2015), who think of acceptance and rejection as static relations between inquisitive states and sentences. The idea that free choice effects are to be explained in terms of the inquisitiveness of disjunction is also prominent in Aloni 2007, Ciardelli et al. 2009, and Roelofsen 2013. Its implementation here has the important consequence of offering a straightforward explanation of why disjunctive possibilities embedded under negation behave in a classical fashion.

A positive/negative update of some state s with ϕ delivers all the alternatives that are positively/negatively related to some element of s via ϕ.

The proposal for disjunction is as follows:

$$(\vee)\quad \begin{array}{l} \sigma[\phi \vee \psi]^+_s\tau \text{ iff } \sigma[\phi]^+_s\tau \ \underline{\text{or}}\ \sigma[\psi]^+_{s\downarrow\phi}\tau \\ \sigma[\phi \vee \psi]^-_s\tau \text{ iff } \exists\nu\colon \sigma[\phi]^-_s\nu \ \underline{\text{and}}\ \nu[\psi]^-_{s\downarrow\phi}\tau \end{array}$$

This analysis captures two important intuitions about disjunctions: first, in addition to ruling out certain possibilities they raise each of their disjuncts as an issue in discourse. We capture this by letting a disjunction relate an input alternative to two potentially distinct alternatives: the result of updating with the first and the result of updating with the second disjunct. Moreover, in a sentence such as 'Mary is in Chicago or she must be in New York' the modal in the second disjunct naturally receives a modally subordinated interpretation: it is interpreted under the supposition that Mary is not in Chicago. We achieve this result by saying that whenever a disjunction is processed in light of some state s, its second disjunct is processed in light of a negative update of s with the first disjunct.

Given some state s, a positive update with a conjunction $\ulcorner\phi \wedge \psi\urcorner$ proceeds via a positive update with ϕ light of s and then via a positive update with ψ in light of $s \uparrow \phi$:

$$(\wedge)\quad \begin{array}{l} \sigma[\phi \wedge \psi]^+_s\tau \text{ iff } \exists\nu\colon \sigma[\phi]^+_s\nu \ \underline{\text{and}}\ \nu[\psi]^+_{s\uparrow\phi}\tau \\ \sigma[\phi \wedge \psi]^-_s\tau \text{ iff } \sigma[\phi]^-_s\tau \ \underline{\text{or}}\ \sigma[\psi]^-_{s\uparrow\phi}\tau \end{array}$$

The rules for negative updates with disjunctions and conjunctions enforce the validity of De Morgan's Laws.

It remains to define the notions of support, entailment, and consistency. I will say that s supports ϕ just in case a positive update with ϕ does not add anything to the informational content of s, and then define entailment and consistency on that basis. It will also be useful to define a notion of equivalence between two sentences ϕ and ψ, which requires that any state that has been updated with ϕ does not undergo any further modification if updated with ψ, and vice versa.

Definition: Support, Entailment, Consistency, Equivalence. Take any $s \in S$ and formulas of $\mathcal{L}$:

1. s *supports* ϕ, $s \Vdash \phi$, iff $\texttt{info}(s \uparrow \phi) = \texttt{info}(s)$
2. $\phi_1, \ldots, \phi_n$ *entails* ψ, $\phi_1, \ldots, \phi_n \vDash \psi$, iff for all $s \in S$, $s \uparrow \phi_1 \ldots \uparrow \phi_n \Vdash \psi$
3. $\phi_1, \ldots \phi_n$ is *consistent* iff for some $s \in S$: $s \uparrow \phi_1 \ldots \uparrow \phi_n \neq \varnothing$
4. ϕ and ψ are *equivalent*, $\phi \cong \psi$, iff for all $s \in S$, $s \uparrow \phi = s \uparrow \psi$

It would, of course, be possible to define all of the relevant notions on the basis of '$\downarrow$' but I set an exploration of this interesting avenue aside for now. Instead, let me a highlight a few crucial predictions that the framework developed so far makes.

2.2 Output

Disjunctions embedded under a possibility operator exhibit the free choice effect:

Fact 1. $\Diamond(p \vee q) \vDash \Diamond p \wedge \Diamond q$

The underlying observation here is that $s \uparrow \Diamond(p \vee q) \neq \varnothing$ only if $\langle \texttt{info}(s), \underline{\perp} \rangle \notin [p \vee q]^+_s$. But suppose that $\texttt{info}(s)$ fails to contain both p- and q-worlds: then $[p]^+_s$ or $[q]^+_s$ *does* relate $\texttt{info}(s)$ to $\underline{\perp}$, hence $\texttt{info}(s)[p \vee q]^+_s \underline{\perp}$ and thus $\langle \texttt{info}(s), \underline{\perp} \rangle \in [p \vee q]^+_s$ after all. So if $s \uparrow \Diamond(p \vee q) \neq \varnothing$ then $s \uparrow \Diamond p = s$ and $s \uparrow \Diamond q = s$ and so clearly $\texttt{info}(s) = \texttt{info}(s \uparrow \Diamond p) = \texttt{info}(s \uparrow \Diamond q)$.

Note that $\Diamond(p \vee q) \nvDash \Diamond(p \wedge q)$ since passing the test conditions under consideration does not require the presence of a $p \wedge q$-world in $\texttt{info}(s)$. Furthermore, it is easy to see that the free choice effect also arises if '$\Diamond$' scopes over a negated conjunction since for all choices of $s \in S$ we have $[\neg(\neg\phi \wedge \neg\psi)]^+_s = [\phi \vee \psi]^+_s$ by design.

We also account for the earlier stated observation that embedding a disjunctive possibility under negation reverts disjunction to its classical behavior:

Fact 2. $\neg\Diamond(p \vee q) \vDash \neg\Diamond p \wedge \neg\Diamond q$

Observe that $s \uparrow \neg\Diamond(p \vee q) \neq \varnothing$ only if $\langle \texttt{info}(s), \underline{\not\perp} \rangle \notin [p \vee q]^+_s$. But suppose that $\texttt{info}(s)$ contains a p- or a q-world: then $[p]^+_s$ or $[q]^+_s$ *does* relate $\texttt{info}(s)$ to $\underline{\not\perp}$, hence $\texttt{info}(s)[p \vee q]^+_s \underline{\not\perp}$ and thus $\langle \texttt{info}(s), \underline{\not\perp} \rangle \in [p \vee q]^+_s$ after all. So if $s \uparrow \neg\Diamond(p \vee q) \neq \varnothing$ then $s \uparrow \neg\Diamond p = s$ and $s \uparrow \neg\Diamond q = s$ and so clearly $\texttt{info}(s) = \texttt{info}(s \uparrow \Diamond\neg p) = \texttt{info}(s \uparrow \Diamond\neg q)$.

It follows immediately from the previous observations that dynamic entailment is nonclassical, and specifically that $\phi \vDash \psi$ fails to guarantee $\neg\psi \vDash \neg\phi$. For instance, we know now that $\Diamond(p \vee q) \vDash \Diamond p$ but also that $\neg\Diamond p \nvDash \neg\Diamond(p \vee q)$.

We may also observe that the framework developed here preserves key insights from the dynamic analysis of modals, including the internal dynamics of conjunction:

Fact 3. $\neg p \wedge \Diamond p$ is inconsistent

Here it pays off that updates are defined relative to a shifty state parameter s. Clearly $\texttt{info}(s \uparrow \neg p)$ does not contain any p-worlds and so any update with with '$\Diamond p$' in light of $s \uparrow \neg p$ is guaranteed to result in the absurd state.

Another noteworthy observation is that two sentences may be mutually entailing without being equivalent. Here is one case that will become relevant at a later stage:

Fact 4. $p \dashv\vDash (p \wedge q) \vee (p \wedge \neg q)$ but $p \ncong (p \wedge q) \vee (p \vee \neg q)$

The first part of the claim is obvious. To see the second part of the claim, suppose that $s = \{W\}$ and observe that $s \uparrow p = \{[\![p]\!]\}$ while $s \uparrow (p \wedge q) \vee (p \wedge \neg q) = \{[\![p \wedge q]\!], [\![p \wedge \neg q]\!]\}$. The two states have identical informational contents but differ in their inquisitive profile and thus fail to be identical.

Finally, let me state some observations about the material conditional and the necessity operator that are of relevance for the upcoming discussion:

Fact 5. $\Box(\phi \supset \psi) \cong \neg\Diamond(\phi \wedge \neg\psi)$ and $\neg\Box(\phi \supset \psi) \cong \Diamond(\phi \wedge \neg\psi)$

These equivalence facts follow immediately from our treatment of '$\Diamond$' and '$\Box$' as duals together with the standard analysis of the material conditional in terms of conjunction and negation. For parallel reasons, we can observe that a negated conjunction is equivalent to the disjunction of its negated conjuncts, and that a negated disjunction is equivalent to the conjunction of its negated disjuncts.

In sum, the proposal developed so far combines a test semantics for modals with an inquisitive approach to disjunction and negated conjunction in a way that captures the scope as well as the limits of the free choice effect. That is no small achievement, since it demonstrates that a semantic explanation of the free choice can handle the problem of negated disjunctive possibilites. But I also said that a good story about the free choice should be able to explain why counterfactuals simplify, so let me now explain how we can do this.

3 Counterfactuals

I will first present an analysis of counterfactuals that explains the basic simplification patterns (§3.1). The semantics does not give us everything one might hope for but—as I will show in §3.2—the most immediate shortcomings are avoided by adding just a few bells and whistles to the basic story.

3.1 Simplification

A *would*-counterfactual is a strict material conditional presupposing that its antecedent is possible in its domain of quantification (again, I will set aside the question of how exactly context fixes that domain for now). Following standard protocol I treat *might*- and *would*-counterfactuals as duals and presuppositions as definedness conditions on updating (Heim 1982, Beaver 2001):

$$(\Box\!\!\rightarrow) \quad \begin{array}{l} \sigma[\phi \Box\!\!\rightarrow \psi]^+_s\tau \text{ iff } \sigma[\Diamond\phi]^+_s\sigma \text{ \underline{and} } \sigma[\Box(\phi \supset \psi)]^+_s\tau \\ \sigma[\phi \Box\!\!\rightarrow \psi]^-_s\tau \text{ iff } \sigma[\Diamond\phi]^+_s\sigma \text{ \underline{and} } \sigma[\Box(\phi \supset \psi)]^-_s\tau \end{array}$$

Given some state s, a positive or negative update with $\ulcorner\phi \Box\!\!\rightarrow \psi\urcorner$ fails to relate an input alternative σ to any output in case the information carried by s is incompatible with ϕ (the presupposition thus projects out of negation). Assuming that the presupposition is satisfied, a positive update with $\ulcorner\phi \Box\!\!\rightarrow \psi\urcorner$ then tests whether s supports $\ulcorner\phi \supset \psi\urcorner$ while a negative update effectively asks whether $\ulcorner\phi \wedge \neg\psi\urcorner$ is compatible with s.[6]

For convenience, let me state explicitly the update rules for *might*-counterfactuals:

$$(\Diamond\!\!\rightarrow) \quad \begin{array}{l} \sigma[\phi \Diamond\!\!\rightarrow \psi]^+_s\tau \text{ iff } \sigma[\Diamond\phi]^+_s\sigma \text{ \underline{and} } \sigma[\Diamond(\phi \wedge \psi)]^+_s\tau \\ \sigma[\phi \Diamond\!\!\rightarrow \psi]^-_s\tau \text{ iff } \sigma[\Diamond\phi]^+_s\sigma \text{ \underline{and} } \sigma[\Diamond(\phi \wedge \psi)]^-_s\tau \end{array}$$

[6]Bringing presuppositions into the picture also raises the question of how they project and a proper answer requires minor modifications to some of our original update rules. For instance, in order to predict that presuppositions project out of the possibility operator one would need to say that $\sigma[\Diamond\phi]^+_s\tau$ holds just in case $\tau = \{w \in \sigma : \langle \mathtt{info}(s), \underline{\perp}\rangle \notin [\phi]^+_s\}$ and, moreover, $\exists\nu.\, \sigma[\phi]^+_s\nu$. Likewise for the negative entry: $\sigma[\Diamond\phi]^-_s\tau$ holds just in case $\tau = \{w \in \sigma : \langle \mathtt{info}(s), \underline{\not\perp}\rangle \notin [\phi]^+_s\}$ and, moreover, $\exists\nu.\, \sigma[\phi]^-_s\nu$. I set these additional complexities, which would also affect the update rules to disjunction, aside to streamline the notation and since getting all the facts about presupposition projection right goes beyond the scope of this investigation.

These update rules are an immediate consequence of treating '$\Box\!\!\rightarrow$' and '$\Diamond\!\!\rightarrow$' as duals.

It is of course uncontroversial that this analysis predicts that counterfactuals simplify if their antecedents involve a disjunction or a negated conjunction:

Fact 6. $(p \vee q) \Box\!\!\rightarrow r \models p \Box\!\!\rightarrow r, q \Box\!\!\rightarrow r$ and $\neg(p \wedge q) \Box\!\!\rightarrow r \models \neg p \Box\!\!\rightarrow r, \neg q \Box\!\!\rightarrow r$

This is an immediate consequence of analyzing *would*-counterfactuals as strict material conditionals. The claim that counterfactuals presuppose the possibility of their antecedents, however, immediately predicts that $s \uparrow (p \vee q) \Box\!\!\rightarrow r = \varnothing$ unless $\texttt{info}(s)$ includes p- as well as q-worlds due to the free choice effect. I will come back to this fact momentarily, but we can already at this stage observe the following fact about *might*-counterfactuals:

Fact 7. $p \Diamond\!\!\rightarrow (q \vee r) \models p \Diamond\!\!\rightarrow q, p \Diamond\!\!\rightarrow r$

A state s supports '$p \Diamond\!\!\rightarrow (q \vee r)$' just in case it supports '$\Diamond(p \wedge (q \vee r))$.' But s supports '$\Diamond(p \wedge (q \vee r))$' just in case it supports '$\Diamond((p \wedge q) \vee (p \wedge r))$' due to distribution and so—due to the free choice effect—only if s also supports '$\Diamond(p \wedge q) \wedge \Diamond(p \wedge r)$.' And that is just what it takes for s to support '$p \Diamond\!\!\rightarrow q$' and '$p \Diamond\!\!\rightarrow r$.' So we predict that *might*-counterfactuals such as (7) simplify in the way they do. In contrast, *would*-counterfactuals with disjunctive consequents rightly fail to simplify: for instance, a nonempty state supporting '$p \wedge (q \wedge \neg r)$' supports '$p \Box\!\!\rightarrow (q \vee r)$' without supporting '$(p \Box\!\!\rightarrow r)$.'

The free choice effect is also key for the following prediction:

Fact 8. $\neg((p \vee q) \Box\!\!\rightarrow r)) \models \neg(p \Box\!\!\rightarrow r), \neg(q \Box\!\!\rightarrow r)$

Given duality, $\neg((p \vee q) \Box\!\!\rightarrow r)) \cong (p \vee q) \Diamond\!\!\rightarrow \neg r$, and it is easy to verify that the right-hand side entails '$p \Diamond\!\!\rightarrow \neg r$' and '$q \Diamond\!\!\rightarrow \neg r$' due to free choice. Appealing to duality once more then establishes the fact.

Fact 8 helps explain Santorio's (2016) observation that downward entailing operators scoping over counterfactuals with disjunctive antecedents seem to distribute over their simplifications:

(8) It's not the case that, if Alice or Bob came to the party, it would be fun.
  a. $\rightsquigarrow$ It's not the case that, if Alice came to the party, it would be fun.
  b. $\rightsquigarrow$ It's not the case that, if Bert came to the party, it would be fun.

(9) I doubt that if Alice or Bert had come to the party, it would have been fun.
  a. $\rightsquigarrow$ I doubt that if Alice had come to the party, it would have been fun.
  b. $\rightsquigarrow$ I doubt that if Bert had come to the party, it would have been fun.

The first case is just an instance of Fact 8. And under the reasonable assumption that Jones's doubting that ϕ amounts to there being some distinguished state supporting $\ulcorner \neg\phi \urcorner$, we also predict the entailment pattern exemplified by (9).

All of this is good news but readers familiar with the literature on simplification may wonder what the current proposal has to say about ANTECEDENT STRENGTHENING:

AS $\phi \Box\!\!\rightarrow \chi \models (\phi \wedge \psi) \Box\!\!\rightarrow \chi$

It is a familiar point from Lewis (1973) that AS is undesirable since so-called *Sobel sequences* appear to be perfectly consistent:

(10) If Alice had come to the party, it would have been fun. But if Alice and Bert had come to the party, it would not have been fun.

And yet both Fine (1975) and Warmbrōd (1981) worry that AS is a consequence of SDA assuming substitution of equivalents in conditional antecedents. The simple argument here is that ϕ is equivalent with $\ulcorner(\phi \wedge \psi) \vee (\phi \wedge \neg\psi)\urcorner$, hence $\ulcorner\phi \mathrel{\Box\!\!\rightarrow} \chi\urcorner$ entails $\ulcorner((\phi \wedge \psi) \vee (\phi \wedge \neg\psi)) \mathrel{\Box\!\!\rightarrow} \chi\urcorner$ and thus, given SDA, must also entail $\ulcorner(\phi \wedge \psi) \mathrel{\Box\!\!\rightarrow} \chi\urcorner$. What can we say about this argument?

In response, the framework developed here supports substitution of equivalents in conditional antecedents, but we already saw that ϕ and $\ulcorner(\phi \wedge \psi) \vee (\phi \wedge \neg\psi)\urcorner$ are not equivalent in the relevant sense. For instance, 'p' and '$(p \wedge q) \vee (p \wedge \neg q)$' differ in their inquisitive update potential since the latter, but not the former, raises the question whether or not q is the case (cf. §2.2). Since the semantics of the possibility modal and thus the semantics of counterfactuals are sensitive to this inquisitive dimension, it is not innocent to replace 'p' with '$(p \wedge q) \vee (p \wedge \neg q)$' in the antecedent of a counterfactual. For sure, ϕ and $\ulcorner(\phi \wedge \psi) \vee (\phi \wedge \neg\psi)\urcorner$ are mutually entailing, but this kind of 'equivalence' is not enough to license substitution, and so the problematic derivation of AS from SDA is blocked.

The fact remains, however, that (10) is predicted to be inconsistent since no single state can support both members of the sequence. The positive news is that it does not take much to modify the basic story so that it avoids the problem. Presuppositions in general and possibility presuppositions in particular are normally *accommodated* as discourse proceeds, allowing counterfactual domains of quantification to evolve dynamically, and this is what underlies the consistency of Sobel sequences. Let me explain.

3.2 Hyperstates

The twist to the basic story is the idea that modals and counterfactuals are quantifiers over a *minimal* and *dynamically evolving* domain of quantification. To make sense of this idea we let context determine a slightly more complex background for processing counterfactuals: instead of thinking of a context as providing a single state, we will think of it as providing a *set* of (nonempty) states. Intuitively, the information carried by each state can be understood as a domain of quantification, and counterfactuals then pertain to whatever states come with the strongest informational content.

Definition: Hyperstates. A *hyperstate* $\pi \subseteq \mathcal{P}(S)\backslash\varnothing$ is any set of nonempty states. We say that $s' \preceq_\pi s$, s' is *at least as strong as* s in π, iff $s, s' \in \pi$ and $\texttt{info}(s') \subseteq \texttt{info}(s)$. Π is the set of all hyperstates. We refer to $\varnothing$ as the *absurd* hyperstate and treat $\pi_0 = \mathcal{P}(S)\backslash\varnothing$ as the *initial* hyperstate.

Thinking of contexts as hyperstates requires some modifications to our update system. Fortunately, our update procedures for alternatives can stay the same. But states are now updated in light of a hyperstate and updates with modals now pertain to those elements of a hyperstate whose informational content is strongest. We achieve this by slightly modifying the update functions for states in the following manner:

Definition: Hyper-updates on States Define update functions $\uparrow_\pi, \downarrow_\pi : \mathcal{L} \mapsto (S \mapsto S)$ as follows:

1. $s \uparrow_\pi \phi = \{\tau \neq \underline{\perp} : \exists \sigma \in s\ \exists s' \preceq_\pi s.\ \sigma[\phi]^+_{s'}\tau\}$
2. $s \downarrow_\pi \phi = \{\tau \neq \underline{\perp} : \exists \sigma \in s\ \exists s' \preceq_\pi s.\ \sigma[\phi]^-_{s'}\tau\}$

To see why these modifications matter, consider how a modal now interacts with some $s \in \pi$: an update of s with '$\Diamond p$' now tests whether there is some $s' \preceq_\pi s$ whose informational content $\texttt{info}(s')$ includes a p-world. Clearly, this is so just in case $\texttt{info}(s)$ includes a p-world as well, and so possibility modals work exactly as before. But the twist does matter when it comes to an update with '$\Box p$': this now tests whether there is some $s' \preceq_\pi s$ whose informational content $\texttt{info}(s')$ exclusively consists of p-worlds, and that may be so even if $\texttt{info}(s)$ itself includes a possible world at which p is false (though of course no state stronger than s' may contain a $\neg p$-world). So in this sense '$\Box$' becomes a strict quantifier over the informational content of the strongest members of a hyperstate. Updating with nonmodal formulas of $\mathcal{L}$ stays the same.

We now define what it takes for a context understood as selecting a hyperstate to accept and admit ϕ and define updates on hyperstates on that basis:

Definition: Acceptance, Admission, Updates on Hyperstates. Consider arbitrary $\pi \in \Pi$ and $\phi \in \mathcal{L}$:

1. π *accepts* ϕ, $\pi \Vdash \phi$, iff for all $s' \in \pi$ there exists some $s \preceq_\pi s'$: $\texttt{info}(s \uparrow_\pi \phi) = \texttt{info}(s)$
2. π *admits* ϕ, $\pi \rhd \phi$, iff $\pi \nVdash \neg\phi$
3. $\pi + \phi = \{s \uparrow_\pi \phi : s \in \pi\ \&\ \pi \rhd \phi\} \backslash \varnothing$

Acceptance of ϕ amounts to support by the strongest states in a hyperstate. An update with ϕ is admitted as long as its negation is not accepted. And finally, a hyperstate is updated with ϕ by updating each of its elements with ϕ and collecting the nonempty results, provided that an update with ϕ is admissible.

Following Willer (2013a), we can distinguish between two senses in which a proposition $[\![\phi]\!]$ is possible according to a hyperstate. First, $[\![\phi]\!]$ may be possible in the sense that it is compatible with the informational content of some element of a hyperstate: $[\![\phi]\!]$ is simply not ruled out. Second, $[\![\phi]\!]$ may be possible in the sense that each minimal element of the hyperstate includes a world at which it is true: $[\![\phi]\!]$ is a *live possibility* in the sense that is settled to be among the possibilities immediately relevant for discourse and reasoning.

Definition: Possibilities, Live Possibilities. Consider any $\pi \in \Pi$ and $[\![\phi]\!] \subseteq W$:

1. $[\![\phi]\!]$ is a *possibility* in π iff $\exists s \in \pi \exists w \in s : w \in [\![\phi]\!]$
2. $[\![\phi]\!]$ is a *live possibility* in π iff $\forall s \in \pi \exists w \in s : w \in [\![\phi]\!]$

Importantly, we can now interpret an update with '$\Diamond p$' as a process that draws attention to a certain possibility: it is designed to transform $[\![p]\!]$ from a plain into a live possibility. It achieves this by eliminating from a hyperstate each element whose informational content is incompatible with p.

Entailment, consistency, and equivalence receive their by now familiar treatment:

Definition: Entailment and Consistency (Hyperstates). Take any $\pi \in \Pi$ and formulas of $\mathcal{L}$:

1. $\phi_1, \ldots, \phi_n$ *entails* ψ, $\phi_1, \ldots, \phi_n \vDash \psi$, iff for all $\pi \in \Pi$, $\pi + \phi_n \ldots + \phi_n \Vdash \psi$
2. $\phi_1, \ldots \phi_n$ is *consistent* iff for some $\pi \in \Pi$: $\pi + \phi_1 \ldots + \phi_n \neq \varnothing$
3. ϕ and ψ are *equivalent*, $\phi \cong \psi$, iff for all $s \in S$, $s \uparrow \phi = s \uparrow \psi$

This setup preserves everything said in Section 3.1 but in addition allows for a Sobel sequence like '$p \mathbin{\Box\!\!\rightarrow} r$' followed by '$(p \wedge q) \mathbin{\Box\!\!\rightarrow} \neg r$' to be consistent. To see why, assume that $w_1 \in [\![p \wedge \neg q \wedge r]\!]$ and that $w_2 \in [\![p \wedge q \wedge \neg r]\!]$, and let $\pi = \{s, s'\}$ be such that $\texttt{info}(s) = \{w_1\}$ while $\texttt{info}(s') = \{w_1, w_2\}$. Then clearly both s and s' satisfy the presupposition carried by the first counterfactual in the sequence and since $s \preceq_\pi s'$ and $s \uparrow_\pi p \mathbin{\Box\!\!\rightarrow} r = s$, we have $\pi + p \mathbin{\Box\!\!\rightarrow} r = \pi$. Notice furthermore that $\pi \not\Vdash \neg((p \wedge q) \mathbin{\Box\!\!\rightarrow} r)$ since we have $s \uparrow_\pi \neg((p \wedge q) \mathbin{\Box\!\!\rightarrow} r) = \varnothing$: here the underlying observation is that s is the strongest state in π but fails support the counterfactual's possibility presupposition '$\Diamond(p \wedge q)$.' So π admits an update with the second member of the Sobel sequence, resulting in a consistent hyperstate $\pi' = \{s'\}$, as desired. More complex Sobel sequences can be consistently processed in more complex hyperstates.

Intuitively, what is going on in a felicitous Sobel sequence such as (10) is that its second member draws attention to a possibility—that Alice *and* Bert might have come to the party—that we did not take into consideration when we just asked what would have happened if Alice had come to the party. This idea can be further elaborated in order to capture other crucial observations about Sobel sequences (see Willer forthcoming). For current purposes, it suffices to observe that the validity of SDA is compatible with the fact that counterfactuals resist AS, and that this is so because the set of live possibilites in counterfactual discourse and reasoning may expand as hitherto ignored possibilities come into view.

4 Loose Ends

In this section I address a few remaining issues by embellishing the core semantic proposal with a few additional and indepedently plausible assumptions.

4.1 Exclusivity and Ignorance

The semantic analysis developed here correctly predicts that a permission to take an apple or a pear communicates that one is permitted to take an apple and permitted to take a pear, without communicating that one is permitted to do both. In ordinary conversations, however, a permission to take an apple or a pear also tends to communicate that the options are *exclusive* in the sense that one is not permitted to do both (and likewise for epistemic possibility statements). Unlike the free choice effect, this exclusivity effect is easily cancelable:

(11) a. ✓ Mary ate most of the cookies. In fact, she ate all of them.
b. ✓ You may have an apple or a pear. In fact, you may have both.
c. # You may have an apple or a pear. But/In fact you cannot have a pear.

And indeed it is easy to derive exclusivity as a pragmatic effect since $\ulcorner\Diamond(\phi \wedge \psi)\urcorner$ is strictly stronger than $\ulcorner\Diamond(\phi \vee \psi)\urcorner$. Together with a competence assumption we may then derive $\ulcorner\neg\Diamond(\phi \wedge \psi)\urcorner$ and thus exclusivity as a conversational implicature in the familiar fashion.[7]

While (11c) demonstrates that the free choice effect resists ordinary cancelation, this does not prove that it is a semantic entailment: after all, not all pragmatic implicatures need to be created equal. Moreover, it is uncontroversial that disjunctive possibilities allow for *ignorance* readings and that such readings can be explicitly enforced, as in the following example involving deontic permission:

(12) (a) You may have an apple or a pear, (b) but I do not know which.

So is free choice a pragmatic affair after all? A plausible response on behalf of a semantic approach is that ignorance readings arise whenever the disjunction takes scope over the possibility modal so that (12a) is read as the disjunction of two permissions, which in the framework developed here does not entail its corresponding conjunction.[8] On this view, the availability of an ignorance reading does not flow from the cancelability of the free choice effect but from the flexible scope interaction between modals and connectives.

My response to the availability of ignorance readings presupposes that flexible scope interactions between modals and connectives make sense from a syntactic point of view. Defending this presupposition in detail goes beyond the scope of this paper, but it strikes me as uncontroversial that disjunctions can scope over other operators in constructions that are structually alike to (12a). Consider:

(13) (a) Mary is looking for a maid or a cook, (b) but I do now know which.

(14) (a) Everyone ordered the tuna or the salmon, (b) but I do not know which.

The example in (13a) as well as the fact that it allows for an ignorance reading are familiar from Rooth and Partee 1983, and (14a) clearly allows for a reading on which everyone ordered the same kind of dish but the speaker is uncertain as to whether it was the tuna or the salmon that everyone ordered.

The claim that disjunction is flexible in its scope is empirically well attested, and so every syntactic story worth its salt must explain why this is so. Such a story may postulate a syntactic level of Logical Form that differs in configuration in certain well-defined ways from the syntactic level that feeds pronunciation (see Simons 2005 for a discussion of this strategy and its ramifications, including the interplay with Larson's (1985) claims about the role of 'either' in restricting the scope of 'or'). But, importantly, this is not the only option: most notably, adopting the continuation based approach by Barker and Shan (2014) would allow us to treat connectives as generalized coordinators so that we can account for free choice and ignorance readings of disjunctive possibilities while interpreting the disjunction in situ. Doing so would ultimately require rewriting (i.e. *continuizing*) the framework developed here, but the needed modifications are fairly straightforward, not least because continuation based grammars are naturally understood as generalizations of dynamic semantics. The fact that disjunctive possibilities allow for an ignorance reading receives a plausible explanation in the semantic framework developed here.

[7] I appeal here to the 'standard recipe' for deriving quantity implicatures. See, for instance, Geurts 2014 and references therein.

[8] Fusco (2015a) suggests that it is the very availability of the sluice in (12b) which shows that the disjunction takes scope over the modal in (12a).

4.2 Simplification Failures?

I suggest that scope issues are also at play when it comes to the observation that not all counterfactuals with negated conjunctions as antecedents seem to simplify:

(15) If John had not had that terrible accident last week and died, he would have been here today.

Here the intuition is that (15) does not license the inference of 'If John had not died last week, he would have been here today' since he still might have had that accident.[9] What underlies this observation, I suggest, is the familiar fact that negated conjunctions sometimes give rise to a 'neither' rather than a 'not both' reading (Szabolsci and Haddican 2004). Not surprisingly, counterfactuals whose antecedents receive the former interpretation are not predicted to simplify.

Certain apparent counterexamples to simplification can thus be dispelled by indepedently motivated scope considerations. Such considerations do not apply to McKay and van Inwagen's (1977) well-known case against simplification of disjunctive antecedents:

(16) If Spain had fought for the Axis or the Allies, she would have fought for the Axis.
a. $\rightsquigarrow$ If Spain had fought for the Axis, she would have fought for the Axis.
b. ??? If Spain had fought for the Allies, she would have fought for the Axis.

(16a) is of course trivial but (16b) is objectionable, contrary to what SDA seems to predict. Let me explain why this observation does not upset what I have said about simplification so far.

McKay and van Inwagen's case reminds us that we need to distinguish between the formal validity of a rule of inference and its applicability in a specific discourse situation (or line of reasoning). The latter requires full uptake of the premises in discourse, which is not guaranteed since other factors may sometimes intervene. Specifically, it is obvious that an uptake of McKay and van Inwagen's conditional does not involve accommodating the presupposition that Spain might have fought with the Allies. In fact, acknowledging that possibility would render (16) plain weird (cf. Starr 2014):

(17) Spain might have fought for the Allies. ???But if Spain had fought for the Axis or the Allies, she would have fought for the Axis.

Accordingly, the effect of (16) in discourse and reasoning does not amount to a complete update with a conditional of the form '$(\mathtt{Ax} \vee \mathtt{Al}) \mathrel{\Box\!\!\rightarrow} \mathtt{Ax}$' since that would inter alia require an update with '$\Diamond\mathtt{Al}$.' The formal validity of SDA is thus compatible with the observation that McKay and van Inwagen's conditional does not seem to simplify.

One may think that the previous observation immediately undermines the proposal that counterfactuals with disjunctive antecedents presuppose the possibility of each disjunct. But this is not so: in ordinary circumstances anyway, the indicative conditional 'If John wins the competition, then I am the Flying Dutchman' does not carry the presupposition that John possibly wins the competition, but there is no serious doubt that indicative conditionals *in general* presuppose that their antecedent is a possibility in the

[9] Thanks to an anonymous reviewer for the 20th Amsterdam Colloquium for drawing my attention to this case.

common ground (Stalnaker 1975). So what is really needed is a story about how the possibility presupposition carried by a conditional may at times be cancelled.

The basic idea is that presuppositions impose a preference on the input context that is *defeasible* in the sense that it has to be balanced with other constraints such as the ones flowing from asserted content and conversational implicatures. In the Flying Dutchman conditional, for instance, the presupposition carried by the antecedent imposes a possibility preference on the input context that conflicts with the implicature-based constraint on the context not to treat John's winning the competition as a serious possibility (since, after all, the speaker could not seriously intend to communicate that he or she might be the Flying Dutchman). Insofar as the implicature based constraint is given priority, we predict that the possibility presupposition is canceled.[10]

Going back to McKay and van Inwagen's example, the general idea is that (16) carries two preferences in virtue of its presupposed content: that the context supports the possibility of Spain's fighting with the Axis ($\Diamond\mathtt{Ax}$) and that it supports the possibility of Spain's fighting with the Allies ($\Diamond\mathtt{Al}$). The latter presupposition—but not the former—is cancelled since it conflicts with the implicature-based constraint not to treat Spain's fighting with the Allies as a live possibility. It is straightforward to see how the implicature may be derived. Take any hyperstate π such that $\pi \Vdash \Diamond\mathtt{Al}$ and consider the result of updating π with the asserted content of (16), that is '$\Box((\mathtt{Ax} \vee \mathtt{Al}) \supset \mathtt{Ax})$:' clearly, $\pi + \Box((\mathtt{Ax} \vee \mathtt{Al}) \supset \mathtt{Ax}) \Vdash \Diamond(\mathtt{Al} \wedge \mathtt{Ax})$, that is, an update of π with the asserted content of (16) results in a state according to which Spain might have fought with the Allies *and* the Axis. But ordinary speakers believe—and are commonly believed to believe—that Spain would not have fought on both sides of the war. Hence an utterance of (16) communicates, in virtue of its asserted content, that the speaker does not treat the possibility of Spain's fighting with the Allies as a live possibility.

Note here that a parallel line of reasoning does not go through if we start with a hyperstate π such that $\pi \Vdash \Diamond\mathtt{Ax}$—updating π with the asserted content of (16) does not lead to implausible possibility commitments—and so we correctly predict that the possibility presupposition of Spain's fighting with the Axis remains unconflicted.

I thus conclude that there is a principled pragmatic explanation for why certain counterfactuals resist simplification. The reason, in brief, is that presuppositions in general, and possibility presuppositions in particular, are cancelable in case of a conflict with other discourse constraints like conversational implicatures. Such cases do not undermine the validity of simplification but create contexts in which this rule of inference has no purchase. While this account taps into pragmatic resources to account for the problematic data, the needed assumptions are modest and well-motivated.

4.3 Modals and Conditionals in Context

Section 2.1 talked about the possibility modal operator as performing a test on states without doing justice to the observation that natural language modal expressions come in a variety of flavors: epistemic, deontic, telic, and so on. Relatedly, Section 3.1 talked about counterfactuals as tests on states without mentioning epistemic conditionals, deontic conditionals, and indicative conditionals more generally. This was intentional since

[10]The claim that implicatures may cancel presuppositions is empirically well-attested, though the former are not always given priority over the latter in case of a conflict. See Beaver 2010 and references therein for detailed discussion.

it allowed us to highlight the framework's potential to account for free choice and simplification effects. Let me briefly indicate how the proposal made here figures in a more general theory about the interpretation of modals and conditionals in discourse.

Sentences are interpreted in context and we shall assume that context at least fixes a state s_c representing what is common ground between the discourse participants. Differences in modal flavor correspond to differences in the set of possible worlds that context deems contextually relevant for evaluating the modal in question (see Kratzer 1981, 1991), and in our setup it makes sense to say that modal quantifier domains are functions of the common ground. I shall thus assume that context fixes not only a common ground but also a set of modal selection functions mapping states to states, and that each such selection function corresponds to a distinct flavor that a modal may carry in discourse. So while epistemic *might* runs a test on a state that is determined by an epistemic selection function e_c together with s_c, deontic *may* runs a test on a state that is determined by the deontic selection function d_c together with s_c, and so on. We will use subscripts (as in '$\Diamond_e$' and '$\Box_d$') to indicate whether the modal is interpreted epistemically, deontically, and so on.

Update values are now relative to a context c rather than a simple state s. If ϕ is a sentence, then $c \circ \phi$ is just like c except that s_c is replaced with $s_c \uparrow \phi = \{\tau \neq \underline{\perp} \colon \exists \sigma \in s.\, \sigma[\phi]_c^+ \tau\}$; and if Δ is any modal selection function, then $c \circ \Delta$ is just like c except that s_c is replaced with $\Delta(s_c)$. The general entry for the possibility modal now looks as follows:

$$(\Diamond_\Delta) \quad \begin{array}{l} \sigma[\Diamond_\Delta \phi]_c^+ \tau \text{ iff } \tau = \{w \in \sigma \colon \langle \mathtt{info}(\Delta_c(s_c)), \underline{\perp} \rangle \notin [\phi]_{c \circ \Delta}^+\} \\ \sigma[\Diamond_\Delta \phi]_c^- \tau \text{ iff } \tau = \{w \in \sigma \colon \langle \mathtt{info}(\Delta_c(s_c)), \underline{\not\perp} \rangle \notin [\phi]_{c \circ \Delta}^+\} \end{array}$$

Updates with possibility modals continue to run tests on states, but now we let context decide which state is relevant for the modal in question, and it does so by fixing a selection function Δ_c for each modal flavor Δ. Rewrite the other semantic entries in the obvious way so that, for instance, processing a positive update with $\ulcorner \phi \wedge \psi \urcorner$ in c involves processing ψ in light of $c \circ \phi$. Conjunctions like '$\Diamond_e p \wedge \Diamond_d q$' are thus interpreted as first running a test on $\mathtt{info}(e_c(s_c))$ followed by a test on $\mathtt{info}(d_{c'}(s_{c'}))$, where $c' = c \circ \Diamond_e p$ (note that $s_{c'}$ is just s_c in case c passes the first test).

It is straightforward to expand the notions of support and logical consequence from Section 2.1. Most significantly, we can say that a context supports ϕ just in case $\mathtt{info}(s_{c \circ \phi}) = \mathtt{info}(s_c)$ and that $\phi_1, \ldots, \phi_n$ entails ψ just in case for all c, $c \circ \phi_1 \circ \ldots \circ \phi_n$ supports ψ. If we assume that $e(s) = s$ for all choices of s, epistemic *might* becomes a test on the common ground and '$\neg p \wedge \Diamond_e p$' is a contradiction. Other selection functions work differently: in particular, since for some d and s, $\mathtt{info}(d(s)) \nsubseteq \mathtt{info}(e(s))$—what is settled to be the case need not coincide with what is deontically ideal—sentences of the form '$\neg p \wedge \Diamond_d p$' are consistent.

What all existential modals are predicted to have in common regardless of flavor is that they exhibit the free choice effect. In particular, we have $\Diamond_\Delta(p \vee q) \vDash \Diamond_\Delta p \wedge \Diamond_\Delta q$ for all choices of Δ. Take arbitrary c and let c' be the result of updating c with '$\Diamond_\Delta(p \vee q)$:' observe that $s_{c'} \neq \varnothing$ only if $\langle \mathtt{info}(\Delta_c(s_c)), \underline{\perp} \rangle \notin [p \vee q]_{c \circ \Delta}^+$ and thus only if $\mathtt{info}(\Delta_c(s_c))$ includes both a p-world and a q-world. The fact that $s_{c'} = s_c$ if $s_{c'} \neq \varnothing$ establishes the point. Relatedly, $\neg \Diamond_\Delta(p \vee q) \vDash \neg \Diamond_\Delta p \wedge \neg \Diamond_\Delta q$: if c' is the result of updating c with '$\neg \Diamond_\Delta(p \vee q)$,' then $s_{c'} \neq \varnothing$ only if $\langle \mathtt{info}(\Delta_c(s_c)), \underline{\not\perp} \rangle \notin [p \vee q]_{c \circ \Delta}^+$ and so only if $\mathtt{info}(\Delta_c(s_c))$ contains neither a p-world nor a q-world. In brief, all the ideas that proved to be central to explaining free choice in the earlier discussion carry over to the more general story about how modals are interpreted in context.

Moving on to conditional constructions more generally, we can say that bare indicative conditionals such as ‘If Mary is not in Chicago, then she is in Rome’ presuppose that their antecedent is an epistemic possibility and assert the epistemic necessity of the corresponding material conditional. Let ‘$\rightarrow$’ represent the basic conditional connective:

$$(\rightarrow) \quad \begin{array}{l} \sigma[\phi \rightarrow \psi]^+_c\tau \text{ iff } \sigma[\Diamond_e\phi]^+_c\sigma \text{ \underline{and} } \sigma[\Box_e(\phi \supset \psi)]^+_c\tau \\ \sigma[\phi \rightarrow \psi]^-_c\tau \text{ iff } \sigma[\Diamond_e\phi]^+_c\sigma \text{ \underline{and} } \sigma[\Box_e(\phi \supset \psi)]^-_c\tau \end{array}$$

It is a familiar idea that counterfactuals differ from their indicative cousins in that they may pertain to possibilities that are incompatible with the common ground (see e.g. Stalnaker 1975) and that this is so in virtue of their carrying a past tense morphology that receives a modal instead of a temporal interpretation (see e.g. Iatridou 2000). We can model the semantic contribution of a modal past operator ‘$\lhd$’ using a counterfactual selection function κ:

$$(\lhd) \quad \begin{array}{l} \sigma[\lhd\phi]^+_c\tau \text{ iff } \sigma[\phi]^+_{c\circ\kappa} \\ \sigma[\lhd\phi]^-_c\tau \text{ iff } \sigma[\phi]^-_{c\circ\kappa} \end{array}$$

The proposal then is that a *would*-counterfactual is a plain conditional with an additional meaning component that corresponds to the presence of past tense morphology that receives a modal instead of its usual temporal interpretation. Specifically, the proposal is that $\phi \mathrel{\Box\!\!\rightarrow} \psi \cong \lhd(\phi \rightarrow \psi)$ and that $\phi \mathrel{\Diamond\!\!\rightarrow} \psi \cong \lhd\neg(\phi \rightarrow \neg\psi)$. The earlier offered analysis of counterfactuals thus flows from a basic analysis of the conditional connective together with a semantic proposal for the fake past.

It turns out that a bare indicative conditional $\ulcorner\phi \rightarrow \psi\urcorner$ is equivalent to the result of embedding its consequent under epistemic *must*, i.e. $\ulcorner\phi \rightarrow \Box_e\psi\urcorner$. This is a welcome result since plain indicatives do have a distinct epistemic flavor in the sense that the consequent is claimed to be settled under the supposition of the antecedent. It also suggests a method for deriving the semantics of modalized conditionals more generally. For instance, let formulas of the form $\ulcorner\phi \mathrel{\bigcirc\!\!\rightarrow} \psi\urcorner$ represent statements articulating conditional obligations such as ‘If Paul does not clean the dishes, he has to mow the lawn.’ An adequate semantics for ‘$\bigcirc\!\!\rightarrow$’ can be given if we treat it as an indicative conditional with a deontically necessitated consequent:

$$(\bigcirc\!\!\rightarrow) \quad \begin{array}{l} \sigma[\phi \mathrel{\bigcirc\!\!\rightarrow} \psi]^+_c\tau \text{ iff } \sigma[\Diamond_e\phi]^+_c\sigma \text{ \underline{and} } \sigma[\Box_e(\phi \supset \Box_d\psi)]^+_c\tau \\ \sigma[\phi \mathrel{\bigcirc\!\!\rightarrow} \psi]^-_c\tau \text{ iff } \sigma[\Diamond_e\phi]^+_c\sigma \text{ \underline{and} } \sigma[\Box_e(\phi \supset \Box_d\psi)]^-_c\tau \end{array}$$

Earlier we defined a *might*-counterfactual as the dual of a *would*-counterfactual, and likewise we can define conditional permissions as the dual of conditional obligations. In general, differences in flavor between iffy necessities are captured by differences in the flavor of their necessitated consequents, and iffy possibilities fall out as the dual.

It follows that simplification is a feature not only of counterfactuals but of conditionals more generally. To illustrate, let c' be the context of updating c with ‘$(p \vee q) \mathrel{\bigcirc\!\!\rightarrow} r$,’ assume that $s_{c'} \neq \varnothing$: then $\texttt{info}(s_c)$ includes both p-worlds and q-worlds. Moreoever, it follows that $\langle\texttt{info}(s_c), \underline{\angle}\rangle \notin [(p \vee q) \wedge \Diamond_d\neg r]^+_c$ and so we get $\langle\texttt{info}(s_c), \underline{\angle}\rangle \notin [p \wedge \Diamond_d\neg r]^+_c$ and $\langle\texttt{info}(s_c), \underline{\angle}\rangle \notin [q \wedge \Diamond_d\neg r]^+_c$, which shows that c supports both ‘$p \mathrel{\bigcirc\!\!\rightarrow} r$’ and ‘$q \mathrel{\bigcirc\!\!\rightarrow} r$.’ Relatedly, observe that for a context to pass an update with ‘$\neg(p \mathrel{\bigcirc\!\!\rightarrow} \neg(q \vee r))$,’ s_c

must include a p-world and, moreover, $\langle \mathtt{info}(s_c), \bot\rangle \notin [\Diamond_e p \wedge \Diamond_d(\neg q \wedge \neg r)]^+_c$. It follows straightaway that $\neg(p \mathrel{\Diamond\!\!\rightarrow} \neg(q \vee r)) \vDash \neg(p \mathrel{\Diamond\!\!\rightarrow} \neg q), \neg(p \mathrel{\Diamond\!\!\rightarrow} \neg r)$ and thus that *may*-conditionals with disjunctive consequents simplify. Once again we see that everything said earlier carries over to the more general framework.

Finally, just as we made a transition from a state s to a set π of states to arrive at a better picture of how modals and conditionals change the common ground, so we can make a transition from a single context c a set χ of such contexts. We say that c' is at least as strong as c in χ, $c' \preceq_\chi c$, just in case $\mathtt{info}(\Delta_{c'}(s_{c'})) \subseteq \mathtt{info}(\Delta_{c'}(s_{c'}))$ for all modal selection functions Δ (note that this requires that $s_{c'} \subseteq s_c$ since e is the identity function). Updating a context with some formula is now defined relative to some choice of χ. Specifically, $c \circ_\chi \phi$ is just like c expect that we update s_c with ϕ in light of χ: collect all the nonempty τ's such that for some σ in s_c and some $c' \preceq_c c$, $\sigma[\phi]^+_{c'}\tau$ (if this results in the empty set, the context is eliminated). It is straightforward to rewrite the definitions from Section 3.2 in these lights. Epistemic possibility modals continue to bring certain epistemic possibilities into view, while deontic possibility modals bring certain deontic possibilities into view, and so on.

I submit that the key ideas developed in this paper can be implemented in a theory that captures the rich variety of flavors that modals and conditionals exhibit in everyday discourse.

5 Conclusion

Combining insights from the dynamic and the inquisitive semantic tradition leads to a semantic explanation of the free choice effect that has no trouble with negation. Embellished with a dynamic analysis of counterfactuals, the framework also explains a range of simplification patterns that prove to be problematic for the standard variably strict analysis. Other data such as the availability of ignorance readings of disjunctive possibilities can be accommodated given modest syntactic and pragmatic assumptions. Let me conclude the discussion by pointing to a few remaining tasks left for another day.

First, free choice effects go beyond possibility modals scoping over disjunction, and so there is the general question what the framework can say about these. It would go far beyond the scope of this paper to address the full range of free choice effects in natural language, but let me highlight one important aspect of the story told here. What explains the free choice effect in formulas of the form $\ulcorner\Diamond(\phi \vee \psi)\urcorner$ is the interaction between an issue raising operation with a test that effectively asks whether each alternative generated under its scope is consistent. There is no reason to think that the operations at play exclusively manifest in natural language as disjunctions and possibility modals, respectively. In fact, the inquisitive proposal in Coppock and Brochhagen (2013) for scalar modifiers immediately allows us to expand the current proposal so that it explains why, for instance, 'You may take at most two apples' grants permission to take fewer than two apples.[11]

Second, the role of negation and conjunction in counterfactual antecedents deserves more attention than it has received (and could have received) here. In a recent study, for instance, Champollion et al. (2016) presented to the participants a scenario in which a light is on whenever two switches A and B are in the same position (both up or both

[11]On the other hand, the fact that not all existential quantifiers need to receive a test semantics may account for Klinedinst's (2007) observation that singular existentials do not behave like free choice items.

down). Right now switch A and switch B are both up, and the light is on. Participants were then asked to evaluate the following two judgments:

(18) If switch A or switch B was down, the light would be off.

(19) If switch A and switch B were not both up, the light would be off.

The interesting observation is that while a large majority of the participants judged (18) true, only a minority did so for (19).

A natural suggestion is to say that (19), but not (18), draws attention to the possibility of both switches being down in virtue of its antecedent. If counterfactuals are strict over the domain including that possibility, we can account for why (19) appears less acceptable than (18). But we would still need to explain why (18) and (19) draw attention to different possibilities. One option is to revise the negative entry for conjunction. But another option is to keep the entry as it is and to explore the role of expressions like 'both' in generating alternatives over and above the ones predicted by the clause for negated conjunction. Further inquiry, including work at the empirical level, is needed to decide which of these options is more viable.

Third, the attempt to combine a dynamic treatment of modals with an inquisitive treatment of disjunction brings to mind the question, discussed by Ciardelli et al. (2009) and Roelofsen (2013), of how to distinguish between informative, inquisitive, and attentive content. Here I want to briefly observe that the notion of a hyperstate is fine-grained enough to keep track of various kinds of discourse information (it will be good enough to focus on the simple version from Section 2). First, there is the *informational content* of a hyperstate π, understood as the possible worlds compatible with what is taken for granted; second, we may associate with π an *issue* understood as the set of its maximal alternatives. So if $\mathtt{Alt}(\pi) = \bigcup\{s \colon s \in \pi\}$, it makes sense to say the following:

1. $\mathtt{Info}(\pi) = \bigcup\{\mathtt{info}(s) \colon s \in \pi\}$
2. $\mathtt{Issue}(\pi) = \{\sigma \colon \sigma \in \mathtt{Alt}(\pi) \;\&\; \neg\exists\tau \in \mathtt{Alt}(\pi).\, \sigma \subset \tau\}$

Looking at the initial hyperstate π_0, we can then say that an atomic sentence p has $[\![p]\!]$ as its *informational* content in the sense that the informational content of $\pi_0 + p$ is just $[\![p]\!]$. For parallel reasons we can say that '$p \vee q$' has $[\![p]\!] \cup [\![q]\!]$ as its informational content and $\{[\![p]\!], [\![q]\!]\}$ as its *inquisitive* content since $[\![p]\!]$ and $[\![q]\!]$ are among the issues in $\pi_0 + p \vee q$. The distinction between the informativeness of a formula and its inquisitiveness is thus analyzed in terms of the potential to eliminate possibilities and to raise issues in discourse.

The strategy pursued in the previous paragraph allows us to identify another dimension of content. Earlier I pointed out that in addition to the possibilities compatible with a hyperstate, we can also identify the possibilities that the state treats as *live*:

3. $\mathtt{Live}(\pi) = \{\sigma \colon \forall s \in \pi \; \exists w \in \mathtt{info}(s).\, w \in \sigma\}$

We can then say that a sentence has *attentive* content in virtue of its potential to bring hitherto ignored possibilities into view: '$\Diamond p$', for instance, has $\{[\![p]\!]\}$ as its attentive content since $\pi_0 + \Diamond p$ treats $[\![p]\!]$ as a live possibility, and for parallel reasons '$\Diamond(p \vee q)$' has $\{[\![p]\!], [\![q]\!]\}$ as its attentive content.

A comprehensive discussion would explore in more detail what predictions the setup sketched here makes about the interaction between informational, inquisitive, and attentive content, and how its predictions differ from those of other frameworks. For now I conclude that the framework is of general semantic interest beyond its capacity to account for the free choice effect by combining a very attractive dynamic semantic treatment of possibility modals as highlighting the significance of certain possibilities in discourse with a—no less attractive—inquisitive treatment of disjunction as refining issues in discourse. The fact that this treatment allows us to make substantial progress toward a better understanding of why counterfactuals simplify gives us all the more reason to think that the dynamic inquisitive story told here deserves further exploration.

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