1 Notes on Dekker's 2004 Predicate Logic with Anaphora

Here is a re-presentation of PLA that I hope is easier to grasp. This is intended to be faithful to the original in every substantive respect, although I've made a number of cosmetic changes. In particular, I've suppressed mention of the model M that gives values to predicates and constants, and I've reversed the direction of the witness sequences, since that is more iconic: witnesses corresponding to existentials earlier in the formula now appear earlier in the witness sequence.

Expressions are evaluated with respect to an assignment function g and a (possibly empty) sequence of individuals σ.

Variables get their values from the assignment function:

$$[\![x]\!]^{g,\sigma} = g(x)$$

Pronouns get their values from the sequence of individuals:

$$[\![p_i]\!]^{g,\sigma} = \sigma_i$$

where σ_i is the ith element of σ.

Predicates. Predicates are applied to terms in the usual way:

$$[\![R(t_1, ..., t_n)]\!]^{g,\sigma} = [\![R]\!]^{g,\sigma}([\![t_1]\!]^{g,\sigma}, ..., [\![t_n]\!]^{g,\sigma})$$

Existentials. Let $\sigma \cdot a$ be the sequence formed by adding the individual a as the new last element of the sequence σ. So the length of $\sigma \cdot a$ is one greater than the length of σ.

$$[\![\exists x\phi]\!]^{g,\sigma\cdot a} = [\![\phi]\!]^{g[x\mapsto a],\sigma}$$

Note that there is no quantification! (But consider the net effect of this definition in combination with the definition of truth given below.) This rule simply checks whether ϕ is true when all free occurrences of x are assigned a as a value. It may help to read this equation from right to left: if setting x to a makes ϕ true, then we add the witness a to σ in order to underwrite subsequent anaphora. This is a strange and beautiful rule, worth contemplating.

Negation. Let $|\phi|$ be the number of indefinites in ϕ.

$$[\![\neg\phi]\!]^{g,\sigma} = \neg\exists a_1, ..., a_{|\phi|} : [\![\phi]\!]^{g,\sigma\cdot a_1\cdot ...\cdot a_{|\phi|}}$$

A negated formula $\neg\phi$ is true with respect to a sequence σ just in case there is no way to extend σ that makes ϕ true. How many witnesses a_i should we try adding? Answer: one for each indefinite in ϕ.

Conjunction.

$$[\![\phi \wedge \psi]\!]^{g,\sigma\tau} = [\![\phi]\!]^{g,\sigma} \wedge [\![\psi]\!]^{g,\sigma\tau}$$

where $\sigma\tau$ is the sequence consisting of σ followed by τ, and $\tau = \tau_1 \cdot ... \cdot \tau_{|\psi|}$. That is, we have to be able to make ϕ true without the benefit of the witnesses contributed by evaluating ψ.

Truth. A sentence ϕ is true with respect to an assignment g and a sequence σ iff $\exists a_1, ..., a_{|\phi|} : [\![\phi]\!]^{g,\sigma \cdot a_1 \cdot ... \cdot a_{|\phi|}}$, that is, just in case we can find suitable witnesses for the indefinites in ϕ. It is the existential in this definition that provides the engine for the existential quantification triggered by indefinites. Note that the negation rule in effect checks for the truth of the prejacent.

2 Examples

(1) A^x woman nominated herself$_x$ and she$_1$ made a^y speech.

$$[\![(\exists x.\textbf{nominated}(x,x)) \wedge (\exists y.\textbf{made}(p_1,y))]\!]^{g,a\cdot s} \tag{2}$$
$$=[\![(\exists x.\textbf{nominated}(x,x))]\!]^{g,a} \wedge [\![(\exists y.\textbf{made}(p_1,y))]\!]^{g,a\cdot s} \tag{3}$$
$$=[\![\textbf{nominated}(x,x))]\!]^{g[x\mapsto a],[]} \wedge [\![(\textbf{made}(p_1,y))]\!]^{g[y\mapsto s],a} \tag{4}$$
$$=\textbf{nominated}([\![x]\!]^{g[x\mapsto a],[]}, [\![x]\!]^{g[x\mapsto a],[]}) \wedge \textbf{made}([\![p_1]\!]^{g[y\mapsto s],a}, [\![y]\!]^{g[y\mapsto s],a}) \tag{5}$$
$$=\textbf{nominated}(a,a) \wedge \textbf{made}(a,s) \tag{6}$$

There's a lot going on here. In step (3), the conjunction rule evaluates the left conjunct with respect to a sequence with the second conjunct's one witness lopped off. In step (4), each existential uses the last individual in its evaluation sequence as the value for its variable, then evalautes the prejacent against a sequence with that witness removed. I'm using [] to represent the empty sequence. Note that *herself* and *she* end up indicating the individual a via different mechanisms: within the scope of the first conjunct, a will be the value of the variable x; within the scope of the second conjunct, pronouns can access a from the list of witnesses established by the earlier conjunct.

The net prediction is that (1) will be true just in case we can find a woman a and a speech s such that a nominated herself and made s.

(7) If a^x farmer owns a^y donkey, he$_x$ beats it$_y$.

Dekker approximates the conditional (as do other discussions of dynamic semantics, e.g., Heim 1983) with material implication, expressed here via conjunction and negation: $A \to B \equiv \neg(A \wedge \neg B)$.

$$\begin{aligned}
&[\![\neg(\exists x \exists y.\mathbf{owns}(x, y)) \wedge \neg(\mathbf{beats}(p_2, p_1))]\!]^{g,[]} && (8)\\
=&\neg\exists f, d : [\![(\exists x \exists y.\mathbf{owns}(x, y)) \wedge \neg(\mathbf{beats}(p_2, p_1))]\!]^{g,d \cdot f} && (9)\\
=&\forall f, d : \neg[\![(\exists x \exists y.\mathbf{owns}(x, y)) \wedge \neg(\mathbf{beats}(p_2, p_1))]\!]^{g,d \cdot f} && (10)\\
=&\forall f, d : \neg([\![\exists x \exists y.\mathbf{owns}(x, y)]\!]^{g,d \cdot f} \wedge [\![\neg(\mathbf{beats}(p_2, p_1))]\!]^{g,d \cdot f}) && (11)\\
=&\forall f, d : \neg([\![\mathbf{owns}(x, y)]\!]^{g[x \mapsto f][y \mapsto d],[]} \wedge \neg[\![\mathbf{beats}(p_2, p_1)]\!]^{g,d \cdot f}) && (12)\\
=&\forall f, d : \neg(\mathbf{owns}(f, d) \wedge \neg\mathbf{beats}(f, d)) && (13)\\
=&\forall f, d : \neg\mathbf{owns}(f, d) \vee \mathbf{beats}(f, d) && (14)
\end{aligned}$$

The net truth conditions say that (7) is true just in case there is no way of choosing a farmer and a donkey such that the farmer owns the donkey but doesn't beat it. The universal force comes from the outer negation in the translation of the material implication operating on the existential introduced by the semantics of negation. The indefinites in the antecedent in effect control the value of the pronouns in the consequent by linking the variables associated with the indefinites to the same individuals that serve as the witnesses indexed by the pronouns.