Cumulative readings motivate update semantics

*Presented by Chris Barker, slides version of April*7, 2021

New argument in favor of update semantics:

- Veltman/GSV: epistemic modals need access to the full set of contextually relevant worlds
- Charlow: cumulative readings need access to the full set of contextually relevant assignment functions

Incidentally, first discussion this semester in which scope matters

Cumulative readings

Scha, Remko. 1984. Distributive, collective, and cumulative quantification. In Jeroen Groenendijk et al. (eds). *Truth, Interpretation and Information: Selected Papers from the Third Amsterdam Colloquium* De Gruyter. 131–158. Often cited as 1981.

1. 600 Dutch firms use 5000 American computers.
2. The total number of Dutch firms that use an American computer is 600 and the total number of American computers used by a Dutch firm is 5000.

Modified numbers (*at most n*, *at least n*, *exactly n*, ...)

Krifka, Manfred. 1999. At least some determiners aren't determiners. In K. Turner (ed). *The semantics/pragmatics interface from different points of view*. Elsevier Science B.V. 257-291.

1. a. Three boys ate seven apples.
   b. Meaning: $\exists x \exists y[3(x) \wedge \mathbf{boys}(x) \wedge 7(y) \wedge \mathbf{apples}(y) \wedge \mathbf{ate}(x, y)]$
   c. Alternatives:
   $\{\exists x \exists y[3(x) \wedge \mathbf{boys}(x) \wedge 7(y) \wedge \mathbf{apples}(y) \wedge \mathbf{ate}(x, y)] | n, m \in \mathbb{N}\}$

2. ASSERT a sentence with meaning M and alternatives A in c:

   a. the speaker claims M in c;
   b. for every alternative $\mathrm{M}' \in \mathrm{A}$, $\mathrm{M}' \neq \mathrm{M}$, the speaker explicitly does not claim M' in c.

3. *exactly 3 boys*

   a. Meaning: undefined
   b. Alternatives: *at least 3 boys*$_A$ $\cup$ *at most 3 boys*$_A$

Pseudo-cumulative readings

1. Exactly three boys saw exactly five movies.
2. It isn't true that exactly three boys saw exactly five movies.

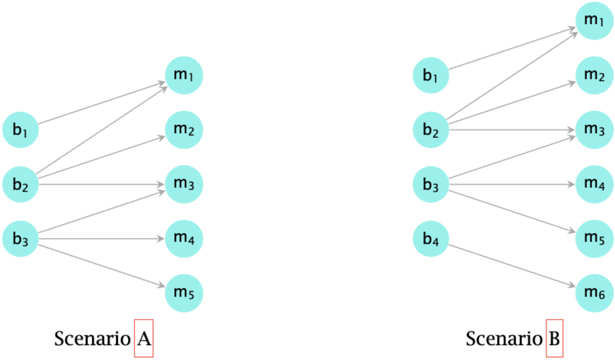

Figure 1: Diagram from Brasoveanu 2012

- "the biggest group of boys I can find who (together) saw five movies has three boys".

An asymmetry in the availability of cumulativity?

Data from Anna Alsop:

1. Exactly three boys saw exactly five movies (between them).
2. Exactly three boys saw at least five movies (between them).
3. Exactly three boys saw at most five movies (between them).
4. At least three boys saw exactly five movies (??between them).
5. At least three boys saw at least five movies (??between them).
6. At least three boys saw at most five movies (??between them).
7. At most three boys saw exactly five movies (??between them).
8. At most three boys saw at least five movies (??between them).
9. At most three boys saw at most five movies (??between them).

- Adding "between them" can try to force a cumulative reading
- If there is a systematic asymmetry, what is the explanation?

Why doesn't ordinary scope get the job done?

(3) $[\![\text{exactly}^v \text{ three boys}]\!] = \lambda k.\,\mathbf{M}_v\,(\mathbf{E}^v\,\mathbf{boys}\,;\,k\,v)\,;\,\mathbf{3}_v$

(4) $[\![\text{saw exactly}^u \text{ five movies}]\!] = \lambda v.\,\mathbf{M}_u\,(\mathbf{E}^u\,\mathbf{movs}\,;\,\mathbf{saw}\,u\,v)\,;\,\mathbf{5}_u$

(5) $\mathbf{M}_v\,(\mathbf{E}^v\,\mathbf{boys}\,;\,\mathbf{M}_u\,(\mathbf{E}^u\,\mathbf{movs}\,;\,\mathbf{saw}\,u\,v)\,;\,\mathbf{5}_u)\,;\,\mathbf{3}_v$

Figure 2: Standard scope delivers nested scope

- We want to move that last right paren leftwards...
- Kinda like how dynamic existential scope moves parentheses rightwards...

Three solutions

1. Higher-order scope-taking
   - Easily generates cumulative readings
   - Comes for free along with the standard theory of scope-taking
   - Requires subtyping in order to block pseudo-cumulative readings
2. Postsuppositions in a bi-dimensional semantics
   - one dimension computes scope and quantification
   - second dimension measures cardinality
   - worries about island effects
3. Update semantics
   - expressions denote updates on the entire set of evaluation points
   - cardinality expressions see the full set of relevant objects
   - No need for subtyping
   - Islands work as per normal

- Desired conclusion: update semantics looks good!

Solution 1: Split scope

$$[\![\text{exactly}^v \text{ three boys}]\!] := \lambda c.\, c\,(\underbrace{\lambda k.\, \mathbf{M}_v\,(\mathbf{E}^v\, \mathbf{boys}\,;\, k\, v)}_{\mathtt{Q} ::= (\mathtt{e} \to \mathtt{t}) \to \mathtt{t}})\,;\, \mathbf{3}_v \qquad \text{type: } (\mathtt{Q} \to \mathtt{t}) \to \mathtt{t}$$

Figure 3: Higher-order denotation for *exactly^v three boys*

- Scope-taker whose trace is a generalized quantifier

Split scope example

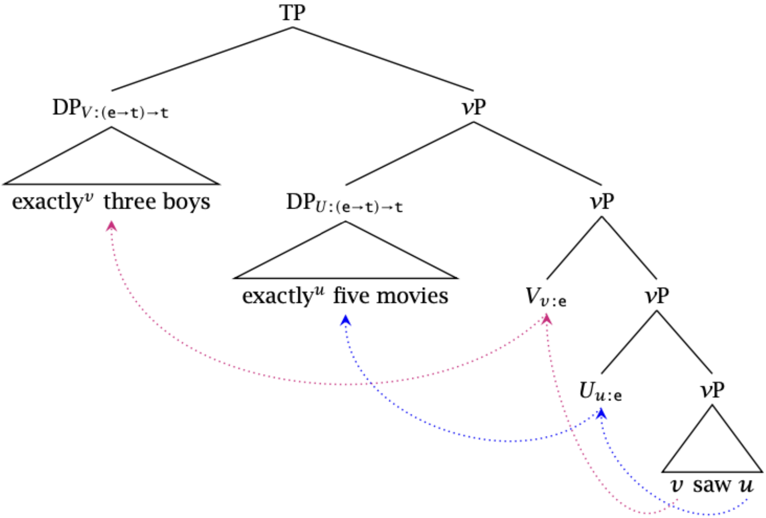

Figure 4: split scope analysis of *!3 boys saw !5 movies*

Independent motivation for split scope quantifiers: rabbits

Bumford, Dylan. 2017. Split-scope definites: Relative superlatives and Haddock descriptions *Linguistics and Philosophy* 40.6: 549–93.

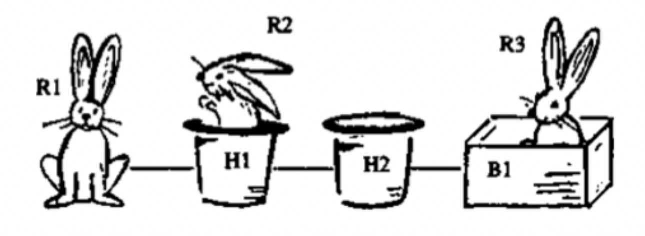

Figure 5: the rabbit in the hat

- Puzzle for the definite descriptions:
  - The uniqueness of the designated rabbit and hat appears circular
  - Uniqueness is global:
    - collect all rabbits in hats
    - collect all hats with rabbits in them

Split scope analysis of Haddock descriptions

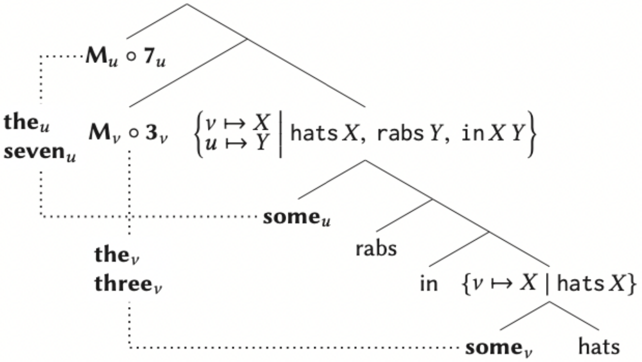

Figure 6: the seven rabbits in the three hats

- So what's the problem? Pseudo-cumulative readings...

Unfortunately, split scope enables pseudo-scope readings

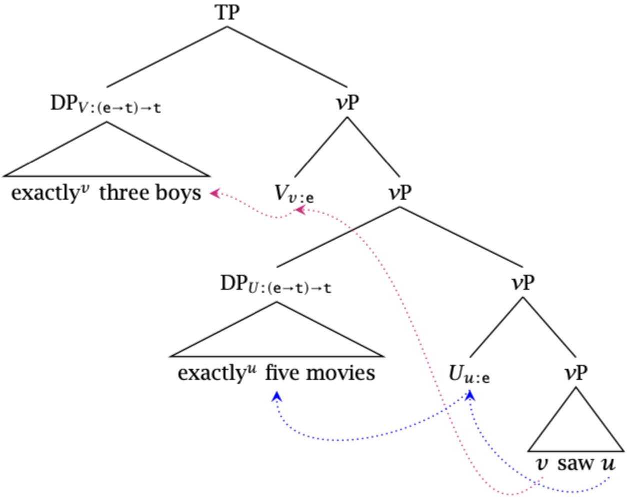

Figure 7: Exactly three boys saw exactly five movies

- diagnosis: a quantifier takes scope over a quantity measure

Subtypes

- Cool!
- Unfortunately, also derives pseudo-cumulative reading
- Refinement: subtyping can rule out pseudo-cumulative reading
- Arguments that subtyping is linguistically useful are (too) rare
  - Johnson and Bayer 1995
  - Bernardi 2002
  - Bernardi and Szabolcsi 2008

Subtyping example

- Atomic individuals and sum individuals are subtypes of `e`.
  - Some predicates require atomic individuals (*dies*)
  - Some predicates require proper sum individuals (*gather*)
  - Some predicates allow either (*lifted a piano*)
- Suppose type `t` (almost-complete sentence) is a subtype of `T` (definitely complete sentence)
  - quantifiers can only take scope over `t` domains
  - quantity measures can only take scope over `T` domains
  - Rule: each tower level must contain a uniform type

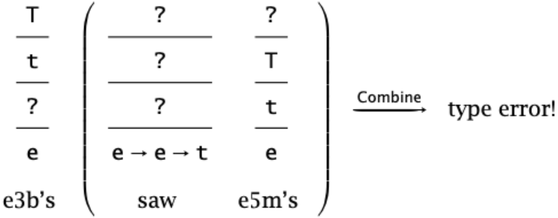

Figure 8: attempted psuedo-cumulative derivation

Solution 2: Postsuppositions in a bi-dimensional semantics

- Postsuppositions: Brasoveanu and collaborators
- Adrian Brasoveanu. 2012. Modified Numerals as Post-Suppositions *Journal of Semantics* 30.2: 155–209. https://doi.org/10.1093/jos/ffs003
  - ***pre***suppositions place requirements on the context *before* evalaution
  - ***post***suppositions place requirements on the context *after* evaluation
- First, find the maximal set of boys and the maximal set of movies in the see relation
- Later, check that the cardinality of these sets is 3 and 5

Implementing postsupposition and bi-dimensionality

- For any type τ (including the types of dynamic expressions),

$$\tau^+ = \tau \mid \tau \times \tau \mid \tau^+ \to \tau^+$$

(53) $[\![\text{exactly}^{v} \text{ three boys}]\!]^{+} := \left(\dfrac{\mathbf{M}_{v}\ (\mathbf{E}^{v}\ \mathbf{boys}\ ;\ [\ ])}{v}, \mathbf{3}_{v} \right)$ type: $\dfrac{\mathtt{t}}{\mathtt{e}} \times \mathtt{t}$

Figure 9: Example of a bidimensional meaning

- Implemented in the appendix via a Writer monad
- See Haskell code here

Combining bidimensional meanings

$$
\begin{array}{cccc}
\dfrac{C}{A \to B} \times \texttt{t} & \dfrac{C}{A} \times \texttt{t} & & \dfrac{C}{B} \times \texttt{t} \\
\\
\text{left} & \text{right} & \xrightarrow{\text{Combine}^{+}} & \text{left right} \\
\\
\left(\dfrac{g[\ ]}{f}, p \right) & \left(\dfrac{h[\ ]}{x}, q \right) & & \left(\dfrac{g[h[\ ]]}{f\,x}, p\,; q \right)
\end{array}
$$

Figure 10: Combining bidimentional meanings

- "Combine+ is a post-suppositional extension of tower combination — it does Combine in the first dimension, and dynamic conjunction of post-supposed content in the second dimension."

Viola!

$$\left(\frac{\mathbf{M}_v\,(\mathbf{E}^v\,\mathbf{boys}\,;[\ ])}{v}, \mathbf{3}_v\right)\left(\left(\left(\frac{[\ ]}{\mathbf{saw}}, \mathbf{T}\right)\left(\frac{\mathbf{M}_u\,(\mathbf{E}^u\,\mathbf{movs}\,;[\ ])}{u}, \mathbf{5}_u\right)\right)\right)$$

$$\xrightarrow{\text{Combine}^+,\ \text{Lower}^+} (\mathbf{M}_v\,(\mathbf{E}^v\,\mathbf{boys}\,;\mathbf{M}_u\,(\mathbf{E}^u\,\mathbf{movs}\,;\mathbf{saw}\,u\,v)), \mathbf{3}_v\,;\mathbf{5}_u)$$

Figure 11: Exactly 3 boys saw exactly 5 movies

Worries about islands [skip]

65. Exactly^v three boys waved to a^z girl [who owns exactly^u five donkeys].

- A reading cumulating boys and donkeys appears to be unattested
- diagnosis: the relative clause traps the postsupposition
- "Could the absence of cumulative readings in such cases be due to a silent distributivity operator that guards the edge of the island, and which prevents any post-supposed content from projecting further?"
- Or "a 'reifying' operator like (▪)"?
- See work of Bumford, Kuhn, Law

Interim assessment

- Second dimension corresponds to the `T` domain from the subtyping solution
- "[I]n both theories, the flow of information out of the separate realm where cardinality tests abide must sometimes be explicitly managed"
- Scope-based theory scores a point:
  - Scope-based theory: islands are automatically enforced
  - Bi-dimensional: post-suppositional meaning should be independent of islands, perhaps contrary to fact

Solution 3: move to an update semantics

- Back to one dimension

Update semantics:

- DPL: relation over pairs of assignment functions
- GSV: relation over contexts (because of the modals)
- This paper: relation over sets of assignment functions
- "The more global perspective afforded by an update semantics allows maximization operators to survey the entire context as it is progressively assembled in the course of a complex dynamic update."

Reminder: update semantics for modals

- GSV: *might* checks to see if any world in the local context verifies the prejacent
  - This requires having the entire local context in view
  - So pointwise evaluation isn't adequate

Mandelkern, Matthew. 2019. Bounded Modality. The Philosophical Review 128.1: 1–61. https://doi.org/10.1215/00318108-7213001
"epistemic modals are quantifiers over accessible worlds, as the standard theory has it; but, crucially, their domain of quantification is limited by their local contexts."

- Evaluation is pointwise, ***but***
  - the set of worlds that are epistemically accessible from any given point is always a subset of the local context

1. c + Ann arrived and Ann might$^{(c+\text{Ann arrived})}$ not have arrived.

- So computing truth conditions requires tracking local context

Sketch of the update semantics

Types (pay attention to italic face):

$$\mathsf{e} := \mathcal{V} \tag{1}$$

$$a := \mathcal{V} \to e \tag{2}$$

$$\mathsf{u} := (a \to t) \to a \to t \tag{3}$$

$$\tau := \mathsf{e} \mid \mathsf{u} \mid \tau \to \tau \tag{4}$$

- Dynamic conjunction ';' : $\mathsf{u} \to \mathsf{u} \to \mathsf{u}$

$$s[L; R] := s[L][R]$$

[exactly^v three boys] $:= \lambda k.\mathbf{M}_v\,(\mathbf{E}^v\,\mathbf{boys}\,;\,k\,v)\,;\,\mathbf{3}_v$ type: $(\mathsf{e} \to \mathsf{u}) \to \mathsf{u}$

Figure 12: modified numeral denotation

Key example showing the update semantics

$$\frac{\mathbf{M}_v\,(\mathbf{E}^v\,\mathbf{boys}\,;[\ ]\,;\mathbf{3}_v)}{v}\left(\frac{[\ ]}{\mathbf{saw}}\;\frac{\mathbf{M}_u\,(\mathbf{E}^u\,\mathbf{movs}\,;[\ ]\,;\mathbf{5}_u)}{u}\right)$$

$$\xrightarrow{\text{Combine, Lower}}\ \mathbf{M}_v\,(\mathbf{E}^v\,\mathbf{boys}\,;\mathbf{M}_u\,(\mathbf{E}^u\,\mathbf{movs}\,;\mathbf{saw}\,u\,v\,;\mathbf{5}_u)\,;\mathbf{3}_v)$$

Figure 13: Exactly 3 boys saw exactly 5 movies

- Note placement of parentheses—identical to (5) above!
- Standard second-order quantifiers

Explanation

Point-wise:

$$\lambda i.\,\{j \in (\mathbf{E}^u\,\mathbf{movs}\,;\mathbf{saw}\,u\,v)\,i \mid \neg\exists h \in \underbrace{(\mathbf{E}^u\,\mathbf{movs}\,;\mathbf{saw}\,u\,v)\,i}\,: j_u < h_u\}$$

Update-theoretic:

$$\lambda s.\,\{j \in (\mathbf{E}^u\,\mathbf{movs}\,;\mathbf{saw}\,u\,v)\,s \mid \neg\exists h \in \underbrace{(\mathbf{E}^u\,\mathbf{movs}\,;\mathbf{saw}\,u\,v)\,s}\,: j_u < h_u\}$$

Figure 14: the key difference

- "In a point-wise system, maximization over *u* happens many times over — once per input assignment. Thus, a plurality of movies *m* that looks maximal relative to some input assignment *i* may turn out looking not-so-maximal relative to some other input *i*′."
- "In the update-theoretic setting, by contrast, maximization over *u* happens only once. Since the incoming context *s* harbors all possible pluralities of boys, [the argument to the maximalization oprator] contains all the movies seen by any plurality of boys, whatsoever. It is with respect to this global boys-seeing-movies **context** that the movies are maximized."

The punchline

- Update semantics (e.g., GSV) are motivated by *might*
- "The ease with which the update-theoretic account of modified numerals generates cumulative readings, and fails to generate pseudo-cumulative readings, is thus a novel argument for such an approach."