Disjunction, alternative, choice, and free choice

PHLIP 9

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1 Three mysteries of disjunction

1. The interaction of alternative questions with negative polarity licensing:

   (1) Did Amy speak to Carl or Bob? — yes/no question

   (2) Did Amy speak to Carl, or Bob? — alternative question

   (3) Did anyone speak to Carl or Bob? — y/n + NPI

   (4) *Did anyone speak to Carl, or Bob? — alt + NPI

   NPIs aren't licensed in contexts that entail a witness (Schwarz; Hoeks, Rudin)

2. The interaction of disjunction with presupposition projection:

   (5) Either this building doesn't have a bathroom, or it's in a funny place.

   (6) Does this building not have a bathroom, or is it in a funny place? — altq, *y/n

   (7) Did John not flip a coin, or did it come up heads?

   (8) Either Amy is proud of her partner, or she doesn't have a spouse.

   Sharvit: "Imagine we're in a situation in which we can assume that someone is unmarried, or they have exactly one spouse."

   Which presuppositions project depends on exclusivity assumptions.

3. Free choice permission:

   (9) You can eat an apple or a banana.

   (10) $\models$ You can eat an apple. — (Fusco: "felt entailment")

   (11) *However*: You can eat an apple or a banana ... but I don't remember which.

   (12) You can eat an apple or a banana.

   (13) [The addressee eats a banana.]

   (14) $\not\models$ You can eat an apple.

   It is part of the conventional meaning of (9) to track the amount of permission.
   (Simons 2005, Barker 2010, Fusco 2015, Willer 2018)

$\boxed{\text{Today's hypothesis: there are two kinds of disjunction: multiplicative and additive.}}$

[Willer quote]

2 Linear Logic

[delayed choice]

[refinement of classical logic, other possibilities]

[resource sensitive]

[symmetric disjunction]

Logical rules:

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \overset{*}{\wedge} B \vdash \Delta}\ \overset{*}{\wedge} L \qquad \frac{\Gamma \vdash A, \Delta \qquad \Gamma' \vdash B, \Delta'}{\Gamma, \Gamma' \vdash A \overset{*}{\wedge} B, \Delta, \Delta'}\ \overset{*}{\wedge} R$$

Multiplicative conjunction ($\overset{*}{\wedge}$)

$$\frac{\Gamma, A_i \vdash \Delta}{\Gamma, A_1 \overset{+}{\wedge} A_2 \vdash \Delta}\ \overset{+}{\wedge} L\ (i = 1, 2) \qquad \frac{\Gamma \vdash A, \Delta \qquad \Gamma \vdash B, \Delta}{\Gamma \vdash A \overset{+}{\wedge} B, \Delta}\ \overset{+}{\wedge} R$$

Additive conjunction ($\overset{+}{\wedge}$)

$$\boxed{\frac{\Gamma, A \vdash \Delta \qquad \Gamma', B \vdash \Delta'}{\Gamma, \Gamma', A \overset{*}{\vee} B \vdash \Delta, \Delta'}\ \overset{*}{\vee} L} \qquad \boxed{\frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \overset{*}{\vee} B, \Delta}\ \overset{*}{\vee} R}$$

Multiplicative disjunction ($\overset{*}{\vee}$)

$$\frac{\Gamma, A \vdash \Delta \qquad \Gamma, B \vdash \Delta}{\Gamma, A \overset{+}{\vee} B \vdash \Delta}\ \overset{+}{\vee} L \qquad \frac{\Gamma \vdash A_i, \Delta}{\Gamma \vdash A_1 \overset{+}{\vee} A_2, \Delta}\ \overset{+}{\vee} R\ (i = 1, 2)$$

Additive disjunction ($\overset{+}{\vee}$)

Structural rules:

$$\frac{\Gamma, A, B, \Gamma' \vdash \Delta}{\Gamma, B, A, \Gamma' \vdash \Delta}\ \text{Exch } L \qquad \frac{\Gamma \vdash \Delta, A, B, \Delta'}{\Gamma \vdash \Delta, B, A, \Delta'}\ \text{Exch } R$$

Exchange

Add these to get classical logic (in which case $\overset{*}{\wedge} = \overset{+}{\wedge} = \wedge$, $\overset{*}{\vee} = \overset{+}{\vee} = \vee$):

$$\frac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta}\,\text{Weak } L \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash A, \Delta}\,\text{Weak } R \qquad \text{Weakening}$$

$$\frac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta}\,\text{Con } L \qquad \frac{\Gamma \vdash A, A, \Delta}{\Gamma \vdash A, \Delta}\,\text{Con } R \qquad \text{Contraction}$$

3 Semantics

Phase space semantics (Girard 1987): Let M be a commutative monoid with $\bot \subset M$.

$$\neg A \mapsto \{m | \forall a \in A, mn \in \bot\} \tag{15}$$

$$A \overset{*}{\wedge} B \mapsto \neg\neg\{ab | a \in A, b \in B\} \tag{16}$$

$$A \overset{+}{\wedge} B \mapsto \neg\neg(A \cap B) \tag{17}$$

The negation is involutive ($\neg\neg A = A$), and the following DeMorgan equivalences hold:

$$A \overset{*}{\vee} B \equiv \neg(\neg A \overset{*}{\wedge} \neg B) \qquad A \overset{+}{\vee} B \equiv \neg(\neg A \overset{+}{\wedge} \neg B)$$

Not so helpful with intuitions

Possible world semantics (Allwein and Dunn 1993, Gehrke 2005):

- Formulas can be true at a world, false, or indeterminate.
- There is a relational semantics using Kripke frames (see Barker 2010, 2019)
- Modal operators are one-place relations over worlds; the binary multiplicative operations give rise to two-place relations. So multiplicative disjunction is "intensional"

Game semantics (van Benthem 2014, chapter 20):

- Formulas correspond to two-person games with potentially infinite alternating turns
- Games can have winning strategies for the first player (true), for the second player (false), or no winning strategy (indeterminate)
- Proponent (P) seeks to prove truth, Opponent (O) seeks to prove false
- $\neg A$: P and O switch roles and play game A
- $A \overset{*}{\wedge} B$: In order to win the complex game, P must win both subgames.
  O chooses which subgame must be played next.
- $A \overset{+}{\wedge} B$: O chooses whether to play game A or game B

Example: **excluded middle**: if A is indeterminate, $A \overset{+}{\vee} \neg A$ is also indeterminate.
But $A \overset{*}{\vee} \neg A$ always has a winning strategy. Let A be chess (so $\neg A$ is role-reversed)

Player	Chess	¬Chess
O	m_1	
P		m_1
O		m_2
P	m_2	
...	...	...

Table 1: From van Benthem 2014:429. P forces O to make the first move in game A. P switches to game $\neg A$ and copies O's move. P then forces O to continue game $\neg A$; when O moves (m_2), P returns to game A and plays m_2. Because P is copying O's strategy perfectly, there is no way O can win both games.

Crucially, P is able to use information from one subgame to guide play in the other subgame.

4 Hypothesis: *or* is ambiguous between $\overset{*}{\vee}$ and $\overset{+}{\vee}$

1. Did Amy/anyone talk to Carl or Bob?
   *Did John not flip a coin or did it come up heads?
   The disjunction in yes/no questions: $A \overset{+}{\vee} B \nvdash \neg A \rightarrow B$
   No guarantee that if Amy didn't talk to Carl that she talked to Bob

2. Did Amy/*anyone talk to Carl, or Bob?
   Did John not flip a coin, or did it come up heads?
   The disjunction in alternative questions: $A \overset{*}{\vee} B \vdash \neg A \rightarrow B$
   If Amy didn't talk to Carl, she must have talked to Bob

5 Free Choice is futurate choice

1. Analyses of free choice rarely explicitly model the connection between free choice implications and choice.

2. A exception: Fusco 2015 draws on decision theory to characterize the outcome after the addressee has chosen.

3. Likewise, analyses of free choice rarely (never?) explicitly rely on the tenseless, futurate nature of the expression characterizing the disjunction. That is, the bracketed expressions are all tenseless and (arguably) futurate:

   (18) You can [eat an apple or a banana].

   (19) Amy might [be in Boston or New York]. (we may learn that ...)

   (20) I can [sing or dance].

4. P. T. Geach. 1982. Whatever happened to deontic logic? *Philosophia* **11**: 1–12.

   > (p. 9) I hold, therefore, that various well-known paradoxes of deontic logic have puzzled people only because of two simple mistakes: thinking of *Sein-sollen* instead of the obligations of agents, and forgetting that **obligations arise and are extinguished in time**. These errors have been made hard to detect by the use of an unsuitable formal system.

5. Ron van der Meyden. 1996. The Dynamic Logic of Permission. *Journal of Logic and Computation* **6.3**: 465–479. [A logic of actions that models free choice permission.]

   > (p. 470) Rather than deal with a new operator for permission, [Meyer] introduces a new action construct $\oplus$ such that $a \oplus b$ denotes an agent's **'internally choosing between** a **and** b**'**, and distinguishes this from $a+b$ (essentially equivalent to $a \cup b$, in our notation) which represents **an external choice, made by the environment**.

6. The analysis here will depend on the slaves's choice among possible future courses of action.

7. Let $\leq$ be a partial order over states that models branching timelines in the usual way. Then $a \leq b$ iff b is a temporal development of a. Everything in b happens just as it did in a, up to the point at which a ends.

8. Then $\leq$ constrains the modal base f in the following way: $\forall w, w'. w' \in fw \rightarrow w \leq w'$. That is, deontically accessible worlds must all be temporal descendants of the world of evaluation.

9. When an agent makes the decision to refrain from eating an apple, the following proposition becomes true: $\neg\Diamond A$. Here's why: the only worlds that end up in the modal base are temporal descendants of the world of evaluation. If the addressee sees to it that none of those descendants are apple-eating worlds, then none of them will be worlds in which A is true. It follows that $\neg\Diamond A$ must be true.

10. Bottom line: $\neg\Diamond A$ can either be prohibition by the master, or else a decision on the slave's part to refrain from eating an apple. Even slaves can have agency!

11. You can eat an apple or a banana.

$$\Diamond(A \overset{*}{\vee} B)$$

$$\neg\Box\neg(A \overset{*}{\vee} B) \qquad \text{(duality of } \Diamond \text{ and } \Box\text{)}$$

$$\neg\Box(\neg A \overset{*}{\wedge} \neg B) \qquad \text{(DeMorgan's)}$$

$$\neg(\Box\neg A \overset{*}{\wedge} \Box\neg B) \qquad \text{(standard deontic logic axiom K)}$$

$$\neg\Box\neg A \overset{*}{\vee} \neg\Box\neg B \qquad \text{(DeMorgan's)}$$

$$\Diamond A \overset{*}{\vee} \Diamond B \qquad \text{(duality of } \Diamond \text{ and } \Box\text{)}$$

$$\neg\Diamond A \rightarrow \Diamond B \qquad (X \overset{*}{\vee} Y \equiv \neg X \rightarrow Y)$$

12. Likewise, by a symmetric argument, $\neg\Diamond B \rightarrow \Diamond A$.

13. Here's the thought: disjunction under permission does not guarantee either $\Diamond A$ or $\Diamond B$ outright. Rather, it only guarantees permission to eat a banana if the addressee doesn't eat an apple in the meantime.

14. The reason we feel there is an entailment from $\Diamond(A \overset{*}{\vee} B)$ to $\Diamond A$ and to $\Diamond B$ is because we understand that it is within the power of the slave to acquire either permission.

15. Using up permission:

    (a) The master says to the slave $\Diamond(A \overset{*}{\vee} B)$.

    (b) Then the slave eats an apple.

(c) The granted permission has been used up.

(d) There is no longer an entailment to $\Diamond B$.

(e) (More precisely, if $\Diamond B$ remains true, it is not by virtue of the permission afforded by the master's utterance.)

16. Does (1) guarantee the right to eat an apple? Yes, but only at the cost of giving up the right to eat a banana. Alas, it is the logical nature of choice that selecting one option means forsaking another.

Empirical claim: it is part of the conventional meaning of free choice that choosing either option deactivates the permission for the other option.

(21) Mail this letter. $\not\Rightarrow$ Mail this letter or burn it. Ross's paradox

(22) You may eat an apple. $\not\Rightarrow$ You may eat an apple or a banana. Free Choice Ross

Addition requires $\overset{+}{\vee}$, which does not validate $\neg\Diamond A \rightarrow \Diamond B$

(23) Amy might be in Boston. $\Rightarrow$ So Amy might be in Boston or New York. [Fusco]

6 The exhaustivity approach

Given a sentence p and a set of alternatives C:

a. $IE(p,C) = \bigcap\{C' \subseteq C : C'$ is a maximal subset of C, s.t. $\{\neg q : q \in C'\} \cup \{p\}$ is consistent$\}$ [=(15b)]

b. $II(p,C) = \bigcap\{C'' \subseteq C : C''$ is a maximal subset of C, s.t. $\{r : r \in C''\} \cup \{p\} \cup \{\neg q : q \in IE(p,C)\}$ is consistent$\}$

Innocent Exclusion+Innocent Inclusion–based exhaustivity operator:

$$[\![\mathcal{E}xh^{\text{IE+II}}]\!](C)(p)(w) \Leftrightarrow \forall q \in IE(p,C)[\neg q(w)] \wedge \forall r \in II(p,C)[r(w)]$$