Hyperbolic Embeddings for Learning Options in Hierarchical Reinforcement Learning

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Abstract

Hierarchical reinforcement learning deals with the problem of breaking down large tasks into meaningful sub-tasks. Autonomous discovery of these sub-tasks has remained a challenging problem. We propose a novel method of learning sub-tasks by combining paradigms of routing in computer networks and graph based skill discovery within the options framework [Sutton et al., 1999] to define meaningful sub-goals. We apply the recent advancements of learning embeddings using Riemannian optimisation in the hyperbolic space to embed the state set into the hyperbolic space and create a model of the environment. In doing so we enforce a global topology on the states and are able to exploit this topology to learn meaningful sub-tasks. We demonstrate empirically, both in discrete and continuous domains, how these embeddings can improve the learning of meaningful sub-tasks.

1 INTRODUCTION

Hierarchical reinforcement learning methods enable agents to tackle challenging problems by breaking them down into smaller ones. There are numerous criteria to break a problem down into smaller parts. One criteria could be re-usability of skills.

Another approach can be defining short term sub-tasks, which helps the agent by breaking a task into a sequence of meaningful chunks. For example, a human being faced with the mundane task of boarding an airplane will break the task down into multiple sub-tasks. The person has to pack their bags. Then they need to

get to the airport, which involves splitting up this sub-task into sub-sub-tasks like getting out of home, going to the bus stop and boarding the right bus. After that they will have to go through security and finally board the flight by getting to the gate. This gives them a sequence of sub-tasks, namely, pack bags, get out of the home, go to the bus stop, get on bus, get down at the airport, security clearance and finally board the flight at the gate. Even though this kind of a break down comes naturally to humans, forming meaningful sub-tasks in the context of a reinforcement learning problem is a challenging problem and falls under the purview of hierarchical reinforcement learning methods. Several mathematical frameworks for hierarchical reinforcement learning have been proposed, including *hierarchies of machines* [Parr and Russell, 1998], MAXQ [Dietterich, 2000], and the options framework [Sutton et al., 1999].

Numerous methods for learning meaningful sub-tasks have been proposed which are *heuristic* in nature [Machado et al., 2017, Şimşek and Barto, 2008, Thrun and Schwartz, 1995, Konidaris and Barto, 2009, McGovern and Barto, 2001, Konidaris and Barto, 2009]. The recently proposed option-critic framework [Bacon et al., 2017] splits tasks under the options framework by directly optimizing for the return. The Feudal network based approach [Vezhnevets et al., 2017] defines *managers* that assign a goal and *workers* that take actions moving the agent in the direction those goals. The hierarchical deep reinforcement learning framework [Kulkarni et al., 2016] works within the options framework by defining a goal set and assigning *intrinsic rewards* upon reaching these goals.

We propose a novel method for finding meaningful skills. We do so by exploiting the geometry and topology of hyperbolic spaces. The hyperbolic space, due to its underlying geometry, has been shown to be effective in capturing hierarchies [Nickel and Kiela, 2017, Ganea et al., 2018, Nickel and Kiela, 2018] and also for routing data packets in real world computer networks [Krioukov et al., 2010]. The essential idea, in the context of routing data packets using hyperbolic

space, is to ensure that every node is *aware* of it neighbours and that the underlying geometry is *congruent* with the structure of the graph. This leads to a routing mechanism with every node being unaware of the global network structure. We apply these very principles to reinforcement learning.

Our approach is to learn sub-tasks that guide the agent towards states which provide access to larger chunks of the state set. We achieve this by embedding the states in the hyperbolic space. We apply Riemannian optimisation to learn embeddings of the states. We work within the options framework in order to model these sub-tasks with the objective of learning. Our approach is closest to the graph based heuristic methods [Şimşek and Barto, 2008, Stolle and Precup, 2002, Konidaris and Barto, 2009], in the sense of finding bottlenecks in graphs.

2   NOTATION AND BACKGROUND

In RL problems, an entity that takes actions and learns from its experiences, within an environment, is called an agent. Everything outside of the agent, which the agent interacts with, is called the environment, which is usually formulated as a Markov Decision Process. A Markov Decision Process (MDP) is represented as a tuple $(\mathcal{S}, \mathcal{A}, P, R, d_0, \gamma)$, where $\mathcal{S}$ denotes the set of states that an agent can be in, $\mathcal{A}$ denotes the set of actions that an agent can take, $P : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to [0, 1]$ is the state transition function which specifies the probability of going from one state to another when an action is taken, $R : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ is the reward function which specifies the expected value of the reward for taking a action a in a state s, $d_0 : \mathcal{S} \to [0, 1]$ is the initial state distribution and is defined as $d_0(s) = P(S_0 = s)$, and $\gamma \in [0, 1]$ is the discount factor, which is used to discount the rewards based on how far into the future they occur.

An agent takes actions in an environment according to a *policy* $\pi : \mathcal{S} \times \mathcal{A} \to [0, 1]$, which specifies the probability of taking an action in a state. A policy π is evaluated using value function $v_\pi : \mathcal{S} \to \mathbb{R}$ which is the expected return on following policy π from a state, and action-value function $q_\pi : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ - $q_\pi(s, a)$ denotes the expected return on taking action a in state s and then following policy π thereafter. Formally, they are defined as $v_\pi(s) = \mathbb{E}_\pi[\sum_{k=0}^{T} \gamma^k R_{t+k+1}|, S_t{=}s]$ and $q_\pi(s, a) = \mathbb{E}_\pi[\sum_{k=0}^{T} \gamma^k R_{t+k+1}|S_t{=}s, A_t{=}a]$. When an agent interacts with an environment, a state-action-reward triplet is obtained for every action taken at each timestep. A sequence of such triplets over time is called a trajectory. In some environments, there may exist a state from which the agent can't transition any other state and would have to be reset to an

initial state based on d_0. Such a state is called a terminal state. The history of interactions of an agent from the initial state to the terminal state is called an episode. A MDP is said to posses a goal state if there exists a state s_g such that the MDP only terminates upon entering this state with an end-of-task reward. For MDPs without any such state, we can define a salient event as the goal state which can be a sophisticated heuristic [Singh et al., 2004, Konidaris and Barto, 2009].

2.1   Options Frameworks

Sutton et al. [1999] introduced options as a temporal abstraction over actions, which result in extended state transitions much like actions. A Markov option $o \in \mathcal{O}$ is represented as a triplet $(\mathcal{I}_o, \pi_o, \beta)$, where $\mathcal{I}_o \subseteq \mathcal{S}$ is the initiation set - the set of states in which the option can be started, $\pi_o : \mathcal{S} \times \mathcal{A} \to [0, 1]$ is the intra-option policy - the policy to be followed when the agent is in option o and $\beta_o : \mathcal{S} \to [0, 1]$ is the termination function which specifies the probability of ending the option o in each state $s \in \mathcal{S}$. Hence, if an agent were to execute option o in state s_k and end in s_T, the transitions would look like $s_k, a_k, s_{k+1}, a_{k+1}, ..., s_T$, where $a_t \sim \pi_o(s_t)$. Note that a primitive action $a \in \mathcal{A}$ is also an option with $\mathcal{I}_o$ being all the states in which the action can be taken, $\pi_a(s, a) = 1 \forall s \in \mathcal{I}_o$ and $\beta_a(s) = 1 \forall s \in \mathcal{S}$.

A policy over options is defined as $\pi_\mathcal{O} : \mathcal{S} \times \mathcal{O} \to [0, 1]$, which is used by the agent to choose the option to execute in a particular state, at which point the policy of the chosen option is executed until its termination. Sutton et al. [1999] show that any MDP with a fixed set of options is a semi-MDP. The policy $\pi_\mathcal{O}$ is learned over value function $v_\mathcal{O}$ and option-value function $q_\mathcal{O}(s, o)$ using the learning algorithms for semi-MDPs. The option-value function is learned by applying the following update after each action for the option o in which the action was performed in state s_t [Sutton et al., 1999]

$$q_\mathcal{O}(s_t, o) \leftarrow q_\mathcal{O}(s_t, o) + \alpha(r_t + \gamma((1 - \beta_o)q_\mathcal{O}(s_{t+1}, o) + \beta_o \max_{o'} q_\mathcal{O}(s_{t+1}, o')) - q_\mathcal{O}(s_t, o)). \quad (1)$$

Bacon et al. [2017] introduce the intra option policy gradient and termination gradient theorems which, in conjunction with the above step, can be used to learn the policies within the options and their termination conditions while simultaneously learning the policy over options. The intra-option policy gradient states: *Given a set of Markov options with stochastic intra-option policies differentiable in their parameters θ, the gradient of the expected discounted return with respect*

to θ and initial condition (s_0, o_0) is

$$\sum_{s,o} \mu_{\mathcal{O}}(s, o | s_0, o_0) \sum_a \frac{\partial \pi_{o,\theta}(a|s)}{\partial \theta} q_U(s, o, a),$$

where $\mu_{\mathcal{O}}(s, o | s_0, o_0) = \sum_{t=0}^{\infty} \gamma^t P(s_t = s, o_t = o | s_0, o_0)$ is the discounted weighting of the state option pairs occurring on the trajectory from (s_0, o_0); $q_U(s, o, a) = r(s, a) + \gamma \sum_{s'} P(s'|s, a) U(o, s')$ is the value of executing action a in state s while the agent is executing option o and $U(o, s') = (1 - \beta_o(s')) q_{\mathcal{O}}(s', o) + \beta_o(s') v_{\mathcal{O}}(s')$ is the value of executing o when the agent enters the state s'.

3 HYPERBOLIC SPACE

Hyperbolic spaces are defined as spaces with constant negative curvature. The idea of positive to zero to negative curvature as continuum is achieved through spherical, Euclidean and hyperbolic spaces. The hyperbolic space was introduced by relaxing the fifth axiom of euclidean geometry: given a line and a point not on it, there is exactly one line going through the given point that is parallel to the given line. This yields an intriguing set of properties, all of which can be arrived at by defining the appropriate *Reimmanian metric* in the Poincare ball model of the hyperbolic space.

Hyperbolic spaces are particularly efficient in representing hierarchical structures like trees, which grow exponentially with depth. For example, in two dimensions, hyperbolic disc area $(2\pi(\cosh r - 1))$ and circumference $(2\pi \sinh r)$ grow exponentially with the radius, as compared to only quadratically (πr^2) and linearly$(2\pi r)$ in $\mathbb{R}^2$.

There are various models of the hyperbolic space [Cannon et al., 1997], which are isomorphic to each other. We focus on the Poincaré ball model for our purposes. For all purposes we deal with a negative curvature of magnitude 1. Let $\mathcal{B}^d = \{\mathbf{x} \in \mathbb{R}^d; \|\mathbf{x}\| < 1\}$ represent a d-dimensional unit ball in Euclidean space. The Poincaré ball model in hyperbolic space is the Reimannian manifold $(\mathcal{B}^d, \mathbf{G}_p)$, with $\mathbf{G}_p$ being the Reimannian metric tensor

$$\mathbf{G}_p(\mathbf{x}) = \left(\frac{2}{1 - \|\mathbf{x}\|^2} \right) \mathbf{G}_e, \qquad (2)$$

where $\mathbf{G}_e = \mathbf{I}_d$ is the Euclidean metric tensor. This results in the distance function between two points $\mathbf{x}$ and $\mathbf{y}$ on this manifold

$$d_p(\mathbf{x}, \mathbf{y}) = \cosh^{-1} \left(1 + 2 \frac{\|\mathbf{x} - \mathbf{y}\|^2}{(1 - \|\mathbf{x}\|^2)(1 - \|\mathbf{y}\|^2)} \right),$$

where $\|.\|$ represents the euclidean norm. As can be seen, this distance function is symmetric and changes

smoothly with the norms of the vectors involved. Also, the distance grows exponentially to infinity as the norm of either nodes approaches 1, for a fixed value of $\|\mathbf{x} - \mathbf{y}\|^2$.

3.1 Applications To Complex Networks

Travers and Milgram [1969] conducted an experiment in which random individuals, called sources, were asked to send a letter to a person, given the persons name, age, occupation and city, by passing the letter onto their friends who have the maximum probability of knowing the person. The source, in this case, is unaware of the exact address of the destination. Surprisingly, 30% of the letters reached the destination. The sources and their friends passed on the letters without a global knowledge of the world. One way the phenomenon is explained is that in addition to being a part of a global graph each node (sources, friends and the destination) also resides in a coordinate space [Kleinberg, 2000]. The forwarding takes place in a greedy manner, with every individual passing the letter on to the node closest to the destination in their immediate neighbourhood.

This greedy routing can be efficient only if the coordinate space, in which the nodes reside, is congruent with the underlying coordinate space [Krioukov et al., 2008]. Krioukov et al. [2010] show why nodes of a *complex network* possess an underlying hyperbolic geometry. They demonstrate and prove theoretical guarantees that lead to a tree like structure of nodes embedded in the hyperbolic space. Krioukov et al. [2010] introduce two conditions, under which the tree like structure holds: **1)** that nodes are exponentially and randomly distributed with r: $\Pr(\mathbf{x}|r) \sim e^r$, where r is the distance from origin in the Poincaré ball and $\Pr(\mathbf{x}|r)$ is the probability of node $\mathbf{x}$ given distance r. **2)** The probability of an edge between two nodes, represented in $\mathcal{B}^d$ by $\boldsymbol{x}$ and $\boldsymbol{y}$, is defined by

$$\Pr((\mathbf{x}, \mathbf{y}) \in \mathcal{D}) = 1 / \left(e^{(d_p(\mathbf{x}, \mathbf{y}) - r)/t} + 1 \right), \quad (3)$$

where $(\mathbf{x}, \mathbf{y})$ represents an edge between the corresponding two nodes, d_p is the Poincaré distance and $\mathcal{D}$ represents the set of edges. Also, r and t are the distance and temperature hyper-parameters. The above distribution is called the Fermi-Dirac distribution.

Building on the work of Krioukov et al. [2010], Candellero and Fountoulakis [2014] give a rigorous analysis of *clustering*: two nodes that have a common neighbour are more likely to be connected. They define the *global clustering coefficient* for a graph $\mathcal{G}$ as

$$C(\mathcal{G}) = \frac{3T(\mathcal{G})}{\Lambda(\mathcal{G})},$$

where $T(\mathcal{G})$ is the number of triangle in graph $\mathcal{G}$ and $\Lambda(\mathcal{G})$ is the number of incomplete triangles. The parameter, $C(\mathcal{G})$, measures how likely two nodes that share a neighbour are to be themselves adjacent.

Under these conditions, for an appropriate setting of hyperparameters, the global clustering coefficient approaches zero and the degree of nodes follows the power law. This induces a tree like structure. A graph is said to have *strong clustering* when it has a higher global clustering coefficient.

3.2 Application To Routing Packets In Networks

The problem of routing packets refers to the problem of finding the smallest path from one node (source) to another (destination) under the condition that each node has access to its own coordinates, the coordinates of its neighbours and the coordinates of the destination. Krioukov et al. [2010] show how for networks with smaller t can be navigated using *greedy forwarding*. Greedy forwarding refers to the idea that each node forwards the packet to the neighbour closest to the destination in the underlying coordinate space. For real world networks, like the Internet, embedded into the hyperbolic space, traversal via greedy forwarding results in *hierarchical paths* which are also optimal [Krioukov et al., 2010]. A hierarchical path is characterised by two segments: a segment of nodes with increasing degrees and another segment of nodes with decreasing degrees. A toy example for 2-dimensional Poincaré ball can be seen in Figure 1.
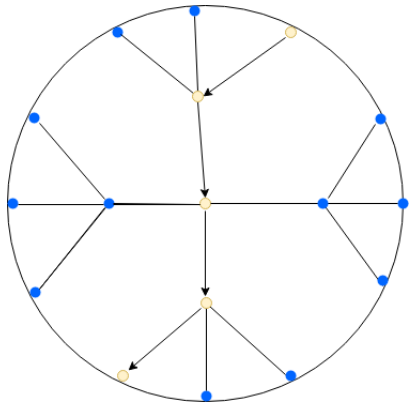

Figure 1: Example of a hierarchical path in a toy network embedded into the Poincaré ball. The degrees of nodes first increase and then decrease.

3.3 Applications To Learning Text And Graph Embeddings

Recently, Nickel and Kiela [2017] introduced the idea of learning the *embeddings* of nodes in a graph us-

ing stochastic Riemannian optimization methods. To compute the Poincaré embedding of a graph $\mathcal{G} = (\mathcal{S}, \mathcal{D})$, where $\mathcal{S}$ is the set of nodes and $\mathcal{D}$ is the set of edges, is to find a set of embeddings $\{\boldsymbol{\theta}_i\}_{i=1}^{|\mathcal{S}|}$ for the set of symbols $\{s_i\}_{i=1}^{|\mathcal{S}|}$, which maps nodes of the graph to the Poincaré ball $\mathcal{B}^d$ by optimizing a problem-specific loss function. This results in $\boldsymbol{\theta}_i \in \mathcal{B}^d$, for all i.

The problem specific loss function is denoted by $\mathcal{L}(\Theta)$, where $\Theta = \{\boldsymbol{\theta}_i\}_{i=1}^{|\mathcal{S}|}$. Nickel and Kiela [2017] solve the optimisation task of minimising the loss function by taking steps in the direction of the *natural gradient* in the Riemannian manifold $\mathcal{B}^d$. The natural gradient for a function $f : \mathcal{B}^d \times B^d \to \mathbb{R}$ can be obtained as

$$\widetilde{\nabla}_{\boldsymbol{\theta}} f(\boldsymbol{x}, \boldsymbol{\theta}) = \mathbf{G}_p(\boldsymbol{\theta})^{-1} \nabla_{\boldsymbol{\theta}} f(\boldsymbol{x}, \boldsymbol{\theta}),$$

where $\nabla_{\boldsymbol{\theta}} f(\boldsymbol{x}, \boldsymbol{\theta})$ represents the partial derivative of the function f with respect to $\boldsymbol{\theta}$ in the euclidean space and $\mathbf{G}_p$ being the Reimannian metric tensor as in equation 2. The direction of steepest descent of $\mathcal{L}(\Theta)$ in the Riemmanian manifold, for a single embedding $\boldsymbol{\theta}$, $\mathcal{B}^d$ is given by its natural gradient [Amari, 1998]. The embeddings are constrained to stay within the Poincaré ball using the projection

$$\text{proj}(\boldsymbol{\theta}) = \begin{cases} \boldsymbol{\theta}/\|\boldsymbol{\theta}\| - \epsilon & \text{if } \|\boldsymbol{\theta}\| \geq 1 \\ \boldsymbol{\theta} & \text{otherwise} \end{cases},$$

where ϵ is a small constant. The update for a single embedding is of the form

$$\boldsymbol{\theta}' \leftarrow \text{proj}\left(\boldsymbol{\theta} - \eta \frac{(1 - \|\boldsymbol{\theta}\|^2)^2}{4} \nabla_{\boldsymbol{\theta}} \mathcal{L}(\Theta)\right), \quad (4)$$

where η is the *learning rate*. For the task of embedding hypernymy relations in the WORDNET dataset Nickel and Kiela [2017] define the loss function $\mathcal{L}(\Theta)$ as

$$\mathcal{L}(\Theta) = \sum_{(u,v) \in \mathcal{D}} \log \frac{e^{-d_p(\boldsymbol{\theta}_u, \boldsymbol{\theta}_v)}}{\sum_{v' \in \mathcal{N}(u)} e^{-d_p(\boldsymbol{\theta}_u, \boldsymbol{\theta}_{v'})}}, \quad (5)$$

where $\mathcal{N}(u) = \{v | u, v \notin \mathcal{D}\}$ and $\boldsymbol{\theta}_u, \boldsymbol{\theta}_v, \boldsymbol{\theta}_{v'}$ are the mappings of nodes u, v, v' onto the Poincaré ball $\mathcal{B}^d$.

Following Nickel and Kiela [2017], models were proposed which worked towards exploiting the underlying geometry of the hyperbolic space in downstream applications of text embeddings [Dhingra et al., 2018, Tay et al., 2018], further improvements to learn better embeddings were also explored [Ganea et al., 2018, Nickel and Kiela, 2018] and Sala et al. [2018] dealt with the theoretical aspects of learning effective embeddings and introduced a PCA style algorithm for learning embeddings in the hyperbolic space.

4 LEARNING HYPERBOLIC STATE REPRESENTATIONS FOR HIERARCHICAL REINFORCEMENT LEARNING

Finding meaningful options to split the learning problem into smaller ones has been a long standing problem in hierarchical reinforcement learning [Bacon et al., 2017, Konidaris and Barto, 2009, Machado et al., 2017, Şimşek and Barto, 2008, Thrun and Schwartz, 1995, Bacon, 2013]. We first describe a notion of graphs, nodes, edges and neighbourhood befitting our purpose of dividing a task into smaller sub-tasks. We then explain how we apply the approach similar to routing a packet in a network to learning these sub-tasks.

We define the N-neighbourhood of a state s for a given sequence of states visited by the agent, $H = \{s_0, s_1, ... s_t\}$, as the set of states which occur within N-steps following or preceding the state s within the given sequence H. A state is excluded from its own neighbourhood for reasons which will become apparent in the following paragraphs. We denote this set using $\Gamma_N(H, s)$. For distinct sequences of states visited by the agent $\{H_j\}_{j=1}^M$ we find the N-neighbourhood of the state by the union: $\Gamma_{M,N}(s) = \cup_{j=1}^M \Gamma_N(H_j, s)$. This notion of N-neighbourhood is symmetric: meaning $s' \in \Gamma_{M,N}(s) \iff s \in \Gamma_{M,N}(s)$, for $s, s' \in \mathcal{S}$.

In addition to the N-neighbourhood we also define $\mathrm{Count}(s, H, s')$ as the number of times a state s' occurs in the N-neighbourhood of state s for a given sequence H. This gives us the probability of two states being in the same neighbourhood, for the given M sequences as:

$$\Pr(s' \in \Gamma_{M,N}(s) | \{H_i\}_{i=1}^M) = \frac{\sum_{i=1}^M \mathrm{Count}(s, H_i, s')}{\sum_{s'' \in \mathcal{S} \backslash s} \sum_{i=1}^M \mathrm{Count}(s, H_i, s'')}. \quad (6)$$

Note that this probability is also symmetric. For our problem of embedding the state set $\mathcal{S} = \{s_i\}_{i=1}^{|\mathcal{S}|}$ into the hyperbolic space, using a set parametrized by $\Theta = \{\boldsymbol{\theta}_i\}_{i=1}^{|\mathcal{S}|}$ where each $\boldsymbol{\theta}_i \in \mathcal{B}^d$, we define the loss as follows:

$$\mathcal{L}(\Theta) = \sum_{s,s' \in \mathcal{S}} w_{s,s'} \log \frac{e^{|r - d_p(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{s'})|}}{\sum_{s'' \notin \Gamma_{M,N}(s)} e^{|r - d_p(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{s''})|}}, \quad (7)$$

where $w_{s,s'} \propto \Pr(s' \in \Gamma_{M,N}(s) | \{H_i\}_{i=1}^M)$ (equation 6), r, M and N are hyper-parameters and d_p is the Poincaré distance.

We optimize for the loss, $\mathcal{L}(\Theta)$, using equation 4. Post the optimization we connect states based on step func-

tion over the Fermi-Dirac distribution (equation 3):

$$\mathrm{step}(s, s', \Theta, h) = \begin{cases} 1 & \text{if } 1/\left(e^{(d_p(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{s'}) - r)/t} + 1\right) > h, \\ 0 & \text{otherwise} \end{cases},$$

where h is the threshold, r is the hyper-parameter same as above and t is the temperature hyper-parameter. We compute the value of the step function for all the node pairs s, s' such that $s' \in \Gamma_{M,N}(s)$. This gives us a set of edges $\mathcal{D}' = \{(s, s') : \mathrm{step}(s, s', \Theta, h) = 1 \text{ and } s' \in \Gamma_{M,N}(s)\}$. We also define the set of edges in the embedding space corresponding to $\mathcal{D}'$ as $\mathcal{D}'_\Theta = (\boldsymbol{\theta}_s, \boldsymbol{\theta}_{s'}) : (s, s') \in \mathcal{D}'$

4.1 Finding Options Through Learned Embeddings

We posit that an agent tasked with reaching a goal state follows a sequence of hierarchical paths. Given the effectiveness of hyperbolic spaces for routing in computer networks, we invoke the idea of finding a *semi-hierarchical path*, which we define as the segment of a hierarchical path that has an increasing set of degrees. More specifically, we find states which are the end points of a semi-hierarchical path and consequently have a higher degree. The idea is to route the agent to states which are end points of semi-hierarchical paths so that it is closer to other nodes.

Given the set of edges inferred by exploiting the underlying geometry of hyperbolic spaces, $\mathcal{D}'$, we split the task of learning an option, o into two parts: finding the goal state s_o and finding the initiation set $\mathcal{I}_o$. We choose a set of goal states by seeking out the top K states sorted in the decreasing order of their degrees (given the set of edges $\mathcal{D}'$) - this number K can be a hyperparameter or it can be calculated using heuristics. These states along with the goal state(s) of the MDP constitute the terminal states of options, we denote this set using $\mathcal{T}$. Meaning that for each state, $s_o \in \mathcal{T}$, the corresponding option has a simple termination function, β_o, which maps to 1 for s_o and 0 elsewhere. We define the initiation set of an option o, $\mathcal{I}_o$, as the set of all the states that have a semi-hierarchical path to s_o in the embedding graph: $(\Theta, \mathcal{D}'_\Theta)$. This path is obtained by using greedy forwarding in the graph $(\Theta, \mathcal{D}'_\Theta)$ starting at $\boldsymbol{\theta}_s$ and ending at $\boldsymbol{\theta}_{s_o}$.

Effectively, we try to route the agent towards states with higher degree that connect to other chunks of the environment, thereby increasing the probability of reaching the goal. In doing so we hope that the agent will be able to split the task of reaching the goal state effectively into sub-tasks of reaching these terminal states.

The loss defined in equation 7 forces the nodes that are more likely to co-occur within a neighbourhood closer

together and pushes apart the nodes that are unlikely to be so. Instead of summing over all the s'' in the denominator of the expression in equation 7 we sample from $\mathcal{S} \setminus \Gamma_{M,N}(s)$. We obtain M distinct sequences, $\{H_i\}_{i=1}^{M}$, by doing random walks in the environment. Thereby we incorporate the transition probabilities between states in our loss function $\mathcal{L}(\Theta)$.

In summary, we form a model of the environment by doing M random walks and embed the learnt neighbourhood relationship between states into the hyperbolic space, thus forming a model of the environment. We use this model along with the underlying geometry of the hyperbolic state to define meaningful options.

5   EXPERIMENTS

We demonstrate results in two distinct domains: four rooms and pinball domain. One is discrete and the other continuous. Both the domains share the common characteristic that they are best solved by breaking the task of reaching a goal into sub-tasks.

5.1   Four rooms

The four rooms domain [Sutton et al., 1999] is a discrete action gridworld which contains four rooms with doors between adjacent rooms. The agent begins at a random state and has to reach a target state, which we fix to be in the bottom right half of the grid. There are four primitive actions available to the agent at each state: move up, down, right or left. These actions are stochastic in the sense that with $2/3$ probability the agent moves in the corresponding direction and with $1/3$ probability it moves in one of the other three directions uniformly randomly. In all the cases, if the agent bumps into a wall, it remains in its current state. The agent receives a reward of $+1$ on reaching the goal and 0 otherwise. Figure 2 shows a diagram of the four rooms domain.

We perform 20 random walks in the four rooms domain and embed the states into the hyperbolic space using equation 7 learnt using Riemannian optimisation [Nickel and Kiela, 2017]. We obtain the initiation sets, $\mathcal{I}_o$ and termination functions, β_o, for options as described in section 4.1.

We obtain two dimensional embeddings in the Poincaré ball, $\mathcal{B}^2$, using a learning rate, η, of 1.0 and $r = 3.5$. We use $M = 20$ and $N = 8$ for doing random walks and obtaining the N-neighbourhood. We infer the set of edges using the Fermi-Dirac distribution using $t = 1$ and a threshold $h = 0.73$. As can be seen from Figure 3, the embeddings are clearly split into 4 distinct clusters. Each cluster connected through a bridge node. Learning the optimal set of intra-option

policies, π_o, using 5 different goal states: each one of the bridge nodes and the goal state. Each options terminates upon reaching the corresponding goal state. We learn the option-value function, $q_{\mathcal{O}}$, using equation 1 [Sutton et al., 1999]. We learn the intra-option policies, π_o, using the intra-option policy gradient theorem as introduced by Bacon et al. [2017]. We set the learning rate for the intra-options policies to 0.1, ϵ to 0.01, the learning rate for $q_{\mathcal{O}}$ as 0.5 and the discount factor, γ, to 0.99. The learning curve, as demonstrated by the number of steps taken averaged over 350 runs, in Figure 4 shows that the options learned from hyperbolic embeddings performs better. More importantly, the number of option switches goes down significantly owing to the nature of options learnt.
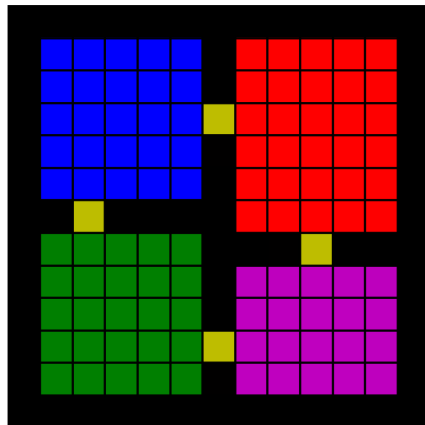

Figure 2: Four rooms domains. We colour code each room and the bottleneck states with different colours: blue, red, magenta, green and yellow.

5.2   Pinball

Konidaris and Barto [2009] introduced the pinball domain for RL. The agent is given a ball at a fixed location in a maze with obstacles and the objective is to guide the ball to reach a target position. The state set is continuous in four dimensions with the components representing the position and velocity of the ball in x and y directions. There are 5 discrete actions available: increase or decrease the velocity of the ball in either direction, or do nothing. Taking the *null* action gets a reward of -1 while all the other actions get a reward of -5. Upon reaching the target the agent receives a reward of 10000. The collisions with the obstacles are elastic and the drag coefficient is 0.995. We use linear function approximation over the Fourier basis [Konidaris et al., 2011] of order 3 to estimate the option-value and action-value functions. We also use the setup for learning intra-option policies, π_o, as
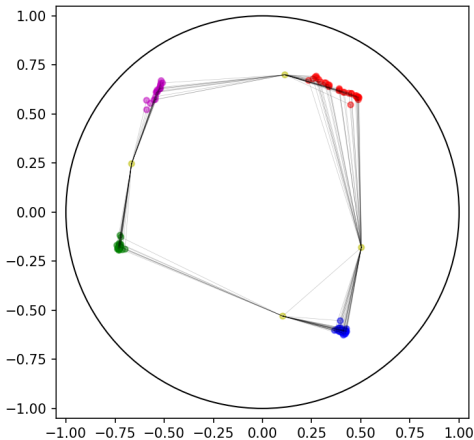
Figure 3: Four rooms states embedded into the hyperbolic space, colour coded similar to Figure 2. The edges represent the edges in $\mathcal{D}'$ (restricted to those of the salient states, $s \in \mathcal{T}$).
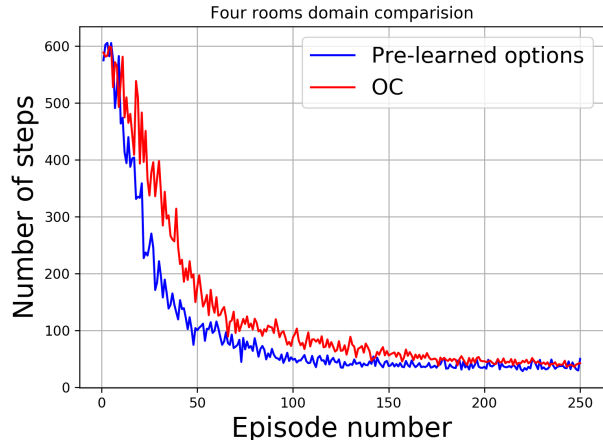

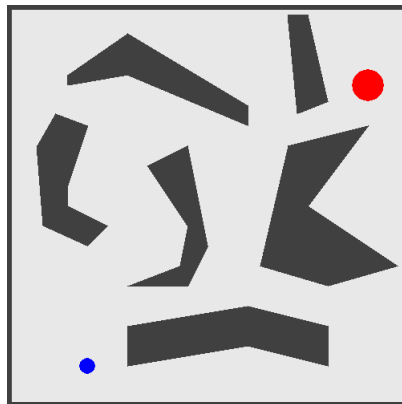Figure 4: Four rooms comparison of option critic and pre-learned options. Averaged over 350 runs.

Figure 5: The Pinball Environment. The starting point of the blue ball is the bottom left corner and the goal is the red dot on the top right corner.

described by Bacon et al. [2017].

Since pinball is a continuous domain we first discretize the state set in order to embed it into the hyperbolic space, $\mathcal{B}^2$. We achieve this by running the k-means clustering algorithm with 100 clusters over the x, y positions of the ball in the pinball environment. We take the learned clusters and find the trajectories in order to obtain the probability as in equation 6. This gives us the embedded graph and the set of options as described in section 4.1. We change one detail when defining options: instead of applying a simple termination function, β_o, with values 1 or 0 we terminate based on the cumulative distribution function of the Gaussian distribution over the distance r_{s_o} of the agent in the pinball domain from the termination state s_o. We scale the Gaussian distribution for all options by a scaling factor σ.

We report the results in Figure 6. We were unable to reproduce the results provided by Bacon et al. [2017], the temperature hyper-parameter for Boltzmann distribution is not reported. The challenge our method faces with continuous domains is evident. The spikes can also be attributed to running fewer number of trials. The reported results are for the configuration where learning rate of the critic is 0.00075, the learning rate for intra-option policies is 0.001. The scaling factor for the Gaussian distribution, σ, is 1.0. The agent terminates after 10000 steps. The hyperbolic embeddings are learnt using a learning rate, η, of 1.0, the size of the sample of negative set is 15, r is set to 2.5, t is set to 1 and the threshold for equation 6 is 0.73. The values of M and N are 20 and 10 respectively.

6   RELATED WORK IN HIERARCHICAL REINFORCEMENT LEARNING

Finding meaningful options can take various meanings and has inspired research in distinct directions. We provide a summary of the approaches that have been applied in the past and comment what our approach has to offer in comparison.

Learning Options: When Sutton et al. [1999] introduced the options framework for RL, the problem of automatically finding initiation sets and terminations was left open. McGovern and Barto [2001] applied the concept of *diverse density* to find useful sub-goals. Stolle and Precup [2002] introduced the idea of bottleneck states, to RL. They obtain the bottleneck states by figuring out the most visited states. They generate the initiation set using the average number of times
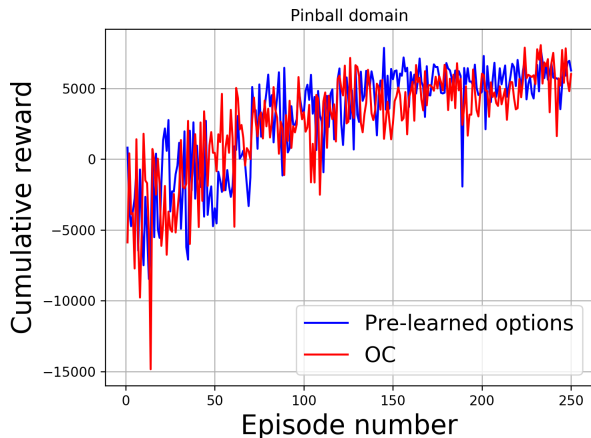
Pinball domain

Figure 6: Pinball domain comparison

other states occur in trajectories containing these bottleneck states.

Skill characterization based on betweenness: Şimşek and Barto [2008] introduce a charecterization of skills based on graphical representations. They capture the bottleneck concept by introducing a betweenness measure [Freeman, 1977] to states of an MDP. The betweenness measure captures the fraction of successful trajectories that go through a state. They intuitively claim that states with higher betweenness measure tend to be better candidates for sub-goals. Application to discrete domains shows improvements. The betweenness measure can be calculated in $O(nm)$ time complexity and $O(n+m)$ space complexity, for an MDP with n states and m edges between these states.

Skill Chaining: Konidaris and Barto [2009] introduce skill chaining, wherein they produce successive chains of skills, under the options framework, to reach the goal state. Each individual skill terminates upon reaching its successor skill's activation state. They also propose forming more general skill trees that can be learned in a similar way. They show improvements on the pinball domain using skill chaining.

Deep Hierarchical RL: With the increased interest Deep RL, owing to the results in the Atari arcade environment [Mnih et al., 2013], various frameworks of deep hierarchical RL that perform well on complicated tasks have been introduced [Kulkarni et al., 2016, Vezhnevets et al., 2017, Bacon et al., 2017, Aytar et al., 2018, Frans et al., 2017, Hausman et al., 2018]. Aytar et al. [2018] learn neural embeddings from YouTube videos of hard exploration games from the Atari environment. They apply these embeddings to guide the agent at the time of learning. Kulkarni et al. [2016] provide a framework to learn *intrinsic goals* in the Atari environment by learning *entity rela-*

tionships between pairs of objects in an unsupervised manner. Their agent learns by optimizing directly for actions targeted towards getting the agent to the *sub-goal* (or *intrinsic goal*) states by providing *intrinsic rewards*. Hausman et al. [2018] learn *general, versatile* and *indentifiable* skills for simulated robot learning tasks. They do so by embedding skills in a latent space and combining reinforcement learning and variational inference.

Our contribution includes embedding the state set into the hyperbolic space forming a neighbourhood graph which can be traversed in $O(|S| \text{avg}(|\Gamma_{M,N}(s))||)$ time using greedy forwarding to form meaningful options or sub-tasks. Due to the strong clustering induced by the choice of hyper-parameters the *depth* of the graph is lower. We enforce global topology which in turn enables the agent to take decision at any state without the complete knowledge of the environment. Our method paves way for applying neural network based state representations. The task of finding a function f which maps from the euclidean space, $\mathbb{R}^d$ to the hyperbolic space, $\mathcal{B}^d$, is subject to further research. It poses a challenge since it is impossible to map $\mathcal{B}^d$ to $\mathbb{R}^d$ while preserving the distance between all the points, meaning they are not isometric.

7 Discussion And Future Work

We have shown how embedding states of an MDP into the hyperbolic space can help find meaningful sub-tasks. We have applied the idea of routing to hierarchical reinforcement learning. We introduce a new loss for learning state embeddings and apply the Riemannian optimisation method as introduced by Nickel and Kiela [2017]. This allows us to scale our results up to larger state sets.

Future directions of work include applying hyperbolic embeddings to neural network state representations that can tackle much more complex tasks. To accomplish this we will also need general functional and probabilistic abstractions of greedy routing. For more complex tasks the embeddings will evolve as the agent discovers new regions of the state set. Adding the weight coefficient in our loss function (equation 7) will be similar to performing stochastic natural gradient descent on the states encountered within a neighbourhood.

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