Inference in Hybrid Bayesian Networks

state-of-the-art

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Hybrid Bayesian Networks

- Hybrid models are used for representing uncertainty in domains containing not only discrete variables, but also continuous such as distance, temperature or location.

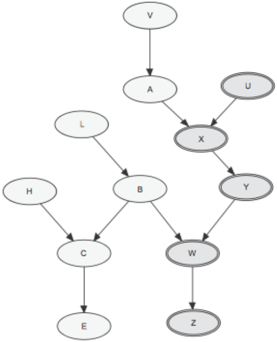

Figure : Hybrid Bayesian Network

Challenges in HBNs

- Continuous and discrete variables
- Accuracy
- Performance

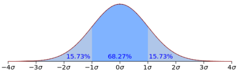

Figure : probability density function

Probability in HBNs

- Heskes and Zoeter (2003): Generalized Belief propagation to approximate inference in HBNs.
  mean : $E[x]$, *covariance* : $E[(x - E[x])(y - E[y])]$
- Schrempf and Hanebeck, 2004 considered that using first two moments is a drawback

The drawback of using only the first two moments to describe a continuous density lies in the fact that there exist many densities having identical first moments. This can be seen in figure 1 for the functions $f_1(x) = N(x, 0, 1)$ and $f_2(x) = 0.5N(x, -\sqrt{0.5}, \sqrt{0.5}) + 0.5N(x, \sqrt{0.5}, \sqrt{0.5})$ where $N(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp -\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}$ is a Gaussian density with mean μ and deviation σ. Both densities (f_1, f_2) have mean 0 and variance 1 which are the first two moments.

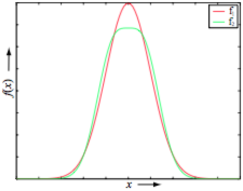

Figure 1. Two distinct densities $f_1(x) = N(x, 0, 1)$ and $f_2(x) = 0.5N(x, -\sqrt{0.5}, \sqrt{0.5}) + 0.5N(x, \sqrt{0.5}, \sqrt{0.5})$ with identical means and variances.

- Deterministic Variables: Barry R. Cobb and Prakash P. Shenoy, 2004

Figure : conditionally deterministic variable

- Continuous variables not normally distributed with *Mixtures of Truncated Exponentials (MTE)* potentials: Barry R. Cobb and Prakash P. Shenoy, 2005

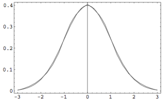

Figure : MTE approximation over on the standard normal distribution

- Discrete and continuous variables with *Mixture of Gaussians (MoG)* BNs: Barry R. Cobb and Prakash P. Shenoy, 2006

HLBP

- *Hybrid Loopy Belief Propagation(HLBP)*: Changhe Yuan and Marek Druzdzel, 2006

Algorithm: HLBP

1. Initialize the messages that evidence nodes send to themselves and their children as indicating messages with fixed values, and initialize all other messages to be uniform.
2. **while** (stopping criterion not satisfied)

   Recompute all the messages using Monte Carlo integration methods.

   Normalize discrete messages.

   Approximate all continuous messages using MGs.

   end while
3. Calculate $\lambda(x)$ and $\pi(x)$ messages for each variable.
4. Calculate the posterior probability distributions for all the variables by sampling from the product of $\lambda(x)$ and $\pi(x)$ messages.

Figure : the Hybrid Loopy Belief Propagation algorithm

- *Direct Message Passing for Hybrid Bayesian Network (DMP-HBN)*: Wei Sun and KC Chang, 2010

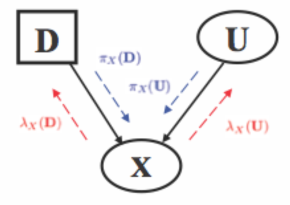

Figure : continuous node X has discrete parent D and continuous parent U

Conclusions and future research

In this short state-of-the-art we have reviewed some of the most important research papers on inference algorithms in Hybrid Bayesian Networks published to date.

The quality of these algorithms, quality as accuracy and performance, is constantly evolving and improving being actually this aspect the main line of research.