

Com. Elec. Formula Sheet

Decibel Formulas

Relative Power Gain

$$A_P = \frac{P_O}{P_I}$$

where P_O and P_I are defined as the following:

$$P_I = \frac{V_I^2}{R_I}$$

$$P_O = \frac{V_O^2}{R_O}$$

Relative Voltage Gain

$$A_V = \frac{V_O}{V_I}$$

Relative Power Gain in dB

$$A_P(db) = 10 \log_{10} A_P$$

$$\text{Given that } R_O = R_I$$

If $R_O \neq R_I$ then the general form is given by the following:

$$A_P(db) = 10 \log_{10} \left(\frac{V_O^2}{V_I^2} \frac{R_I}{R_O} \right)$$

Relative Voltage Gain in dB

$$A_V(db) = 20 \log_{10} \left(\frac{V_O}{V_I} \right) = 20 \log_{10} A_V$$

If $R_O \neq R_I$ then the general form is given by the following:

$$A_V(db) = 20 \log_{10} \left(\frac{V_O}{V_I} \right) - 10 \log_{10} \left(\frac{R_O}{R_I} \right)$$

Special Case

If $R_O \neq R_I$ then the general form is given by the following:

$$\begin{aligned} A_V(db) &= 10 \log_{10} \left(\frac{V_O^2}{V_I^2} \right) - 10 \log_{10} \left(\frac{R_O}{R_I} \right) \\ &= 20 \log_{10} \left(\frac{V_O}{V_I} \right) - 10 \log_{10} \left(\frac{R_O}{R_I} \right) \end{aligned}$$

Absolute Power Gain dBm

$$A_{P(dbm)} = 10 \log_{10} \left(\frac{P}{1 \text{ mW}} \right), \text{ dBm}$$

Absolute Power Gain dBw

$$A_{P(dbw)} = 10 \log_{10} \left(\frac{P}{1 \text{ W}} \right), \text{ dBw}$$

Signal-to-Noise Ratio

$$\text{SNR} = 10 \log_{10} \left(\frac{\text{Signal Power}}{\text{Noise Power}} \right)$$

$$\text{And given that } R_O = R_I,$$

$$\text{SNR} = 10 \log_{10} \left(\frac{V_S^2}{V_N^2} \right)$$

$$\text{SNR} = 20 \log_{10} \left(\frac{V_S}{V_N} \right) \text{dB}$$

Impulse Noise

$$dB_S = 20 \log_{10} \left(\frac{P}{0.0002 \bar{\mu}} \right), \text{ where } P \text{ is sound pressure in } \bar{\mu}$$

$$\bar{\mu} = 1 \frac{\text{dyne}}{\text{cm}^2} = 10^{-6} \text{ of atmospheric pressure at sea level}$$

Gaussian (White) Noise

$$P_n = kT\Delta f$$

$$k = \text{Boltzmann's Constant } (1.38 * 10^{-23}) \text{ J/K}$$

$$T = \text{resistor temperature in Kelvin } (K)$$

$$\Delta f = \text{system bandwidth.}$$

Gaussian (White) Noise Formulas

Using the above proportionality we can relate bandwidth to noise, shown below:

Given the noise can be represented as e_n

then we can say the following:

$$P_n = \frac{V_n^2}{R} = kT\Delta f, \text{ where } V_n = \frac{e_n}{2}$$

This is true by Ohm's law. By solving in terms of e_n we get the following:

$$\frac{V_n^2}{R} = kT\Delta f \text{ where } V_n = \frac{e_n}{2}$$

$$\left(\frac{e_n}{2} \right)^2 = \left(\frac{e_n^2}{4} \right) = kT\Delta f$$

$$\left(\frac{e_n^2}{4} \right) = kT\Delta f$$

$$e_n = \sqrt{4kT\Delta f R}$$

Noise Ratio

$$NF = 10 \log_{10} \frac{\frac{S_i}{N_i}}{\frac{S_o}{N_o}} = 10 \log_{10} NR$$

$$NR = \frac{\frac{S_i}{N_i}}{\frac{S_o}{N_o}} \text{ is the Noise Ratio}$$

$$\frac{S_i}{N_i} = \text{input SNR}$$

$$\frac{S_o}{N_o} = \text{output SNR}$$

Reactance Noise Effects

$$\Delta f_{eq} = \frac{\pi}{2} BW$$

$$BW = 3 \text{ dB; bandwidth for } RC, LC, \text{ or } RLC \text{ circuits.}$$

Noise Created by Amplifiers in Cascade

$$NR = NR_1 + \frac{NR_2 - 1}{P_{G_1}} + \dots + \frac{NR_n - 1}{P_{G_1} * P_{G_2} * P_{G_{(n-1)}}}$$

$$NR = \text{overall noise ratio of } n \text{ stages.}$$

$$P_G = \text{power gain ratio}$$

Equivalent Noise Temperature

$$T_{eq} = T_0(NR - 1)$$

where $T_0 = 290 \text{ K}$, a reference temperature in Kelvin.

Equivalent Noise Resistance

Sometimes used by Manufacturers to represent the noise generated by a device with a fictitious resistance. The following represents this:

$$R_{eq} = \sqrt{4kT\Delta f R}$$

Modulation Index

$$m = \frac{E_i}{E_c}$$

$$\%m = \frac{E_i}{E_c} * 100\%$$

$$\%m = \frac{B - A}{B + A} * 100\%$$

$$B = \text{AM Waveform}$$

$$A = \text{The Minimum Peak-to-Peak value}$$

Overmodulation

$$\%m = \frac{B - O}{B + O} * 100\%$$

$$B = \text{AM Waveform}$$

$$O = \text{The Minimum Peak-to-Peak value} \leq 0$$

Amplitude Modulation/Mixing in Frequency Domain

Carrier Signal

$$e_c = E_C \sin \omega_c t$$

where $e_i =$ is the instantaneous value of the carrier

$E_C =$ is the maximum peak value of the carrier when unmodulated

$$\omega = 2\pi f \text{ ("f" is the carrier frequency)}$$

$$t = \text{is a unit of measure}$$

Information Signal

$$e_i = E_I \sin \omega_i t$$

where $e_i =$ is the instantaneous value of the information

$E_i =$ is the maximum peak value of the intelligence

when unmodulated

$$\omega = 2\pi f \text{ ("f" is the carrier frequency)}$$

$$t = \text{is a unit of measure}$$

AM Modulated Waveform

$$e = E_c \sin \omega_c t + \frac{mE_c}{2} \cos(\omega_c - \omega_i)t - \frac{mE_c}{2} \cos(\omega_c + \omega_i)t$$

1. $E_c \sin \omega_c t$ relates to the carrier (1)

2. $\frac{mE_c}{2} \cos(\omega_c - \omega_i)t$ relates to the the lower sideband at $f_c - f_i$ (2)

3. $\frac{mE_c}{2} \cos(\omega_c + \omega_i)t$ relates to the the upper sideband at $f_c + f_i$ (3)

Power Distribution in Carriers and Sidebands

$$E_{SF} = \frac{mE_C}{2}$$

$$E_{SF} = \text{side frequency amplitude}$$

$$m = \text{modulation index}$$

$$E_C = \text{carrier amplitude}$$

Total Transmitted Power

P_t = P_c \left(1 + \frac{m^2}{2}\right)

P_t = Total Transmitted Power (sidebands and carrier)

m = modulation index

P_c = carrier power

Total Transmitted Current

I_t = Total Transmitted Current (sidebands and carrier)

I = modulation index

I_c = carrier current

Frequency Modulation

How FM Generator Works? ”The Concept of Deviation”

f_{OUT} = f_C + k e_i

f_{OUT} = instantaneous output frequency

f_C = output carrier frequency

k = deviation constant [kHz/V]

e_i = modulating (intelligence) input

Quick Facts

- 1. Deviation constant defines how much carrier frequency will deviate for input voltage level.
- 2. Deviation constant dependent on system design.
- 3. Knowing deviation on either side of carrier is essential for determining occupied bandwidth of modulated signal.

Direct FM

Direct FM involves messing w/ the frequency component of a sinusoidal wave:

f \text{ in } \omega(A_P \sin(\omega t + \theta) = A_P \sin(2\pi f t + \theta))

Indirect FM

Indirect FM involves messing w/ the phase angle component of a sinusoidal wave:

\theta \text{ in the sinusoid } (A_P \sin(\omega t + \theta) = A_P \sin(2\pi f t + \theta))

FM IN THE FREQUENCY DOMAIN

e = A \sin(\omega_c t + m_f \sin \omega_i t)

e = instantaneous voltage

A = peak value of original carrier wave

\omega_c = carrier carrier angular velocity (2\pi f_c)

\omega_i = modulating intelligence signal angular velocity (2\pi f_i)

Modulation Index

m_f = FM Modulation Index = \frac{\delta}{f_i}

\delta = maximum frequency shift caused by the intelligence signal (deviation) either above or below the carrier; therefore, deviation written as 3 kHz, for example, has \delta = 3 kHz (not 6 kHz) in the above. f_i = of the intelligence (modulating) signal

FM Spectrum Analyzer

Remember that each Bessel table entry represents the ratio \frac{V_2}{V_1} for its respective carrier J_0 or sideband J_1 and above signal component, for a given modulation index.

P_{dB} = 20 \log_{10} \frac{V_2}{V_1}

Power Distribution

Carson’s Rule Approximation

BW \cong 2(\delta_{max} + f_{i_{max}})

Percent of Modulation and Deviation Ratio

DR = \frac{\text{max possible freq deviation}}{\text{max input freq}} = \frac{f_{dev(max)}}{f_{i(max)}}

Fourier Series

a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx

a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \cos(nx) dx

A0

b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin(nx) dx

a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx

= \frac{1}{2\pi} \int_{-\pi}^0 0 dx + \frac{1}{2\pi} \int_0^{\pi} 0 dx = 0 + \frac{1}{2\pi} = \frac{1}{2}

An

For n ≥ 1

a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \cos(nx) dx

= \frac{1}{\pi} \int_{-\pi}^0 0 dx + \frac{1}{\pi} \int_0^{\pi} \cos(nx) dx

= 0 + \frac{1}{\pi} \frac{\sin(nx)}{n} \bigg|_0^{\pi} = \frac{1}{n\pi} (\sin(n\pi) - \sin(0)) = 0

Bn

For n ≥ 1

b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin(nx) dx

= \frac{1}{\pi} \int_{-\pi}^0 0 dx + \frac{1}{\pi} \int_0^{\pi} \sin(nx) dx

= -\frac{1}{\pi} \frac{\cos(nx)}{n} \bigg|_0^{\pi} = -\frac{1}{n\pi} (\cos(n\pi) - \cos(0)) =

0 if n is even; \frac{2}{n\pi} if n is odd
