A/D Com. Formula Sheet

American Wire Gauge (AWG/SLIC):

Definition:

Subscriber Loop Interface Circuit:

1. Located at the central office.
2. Supports various functions defined under the BORSCHT anagram.
3. This is done through the Subscriber Loop Lines. These are lines connecting the central office to the patrons homes.

Objective:

1. To use the largest possible AWG to support certain requirements. Basically finding where company and consumer interest align. Not to expensive for the company while also providing good service to the consumer.

Method:

Assumptions:

1. Minimum Current: 20 mA
2. Maximum Current: 120 mA
3. Central Office Resistance: 400 Ω
4. Telephone Resistance: 400 Ω

Example 1:

Find AWG wire to support the following:

1. 25 mA current.
2. Distance: 5 km
3. Max Attenuation Unloaded: 7 dB

Answer:

Find Required DC Resistance:

$$V = IR$$

$$48V = 25mA(400 + 400 + x)\Omega$$

$$x\Omega = \left(\frac{48V}{25mA}\right) - 800\Omega$$

$$x\Omega = 1120\Omega$$

If the following is true then that is the AWG to use:

- miles $*$ Attenuation (dB/mile unloaded) $< 7dB$
- miles $*$ Round Trip Loop Resistance $< 1120\Omega$

Example 2:

Goal: Support the following system

1. 24 mA DC minimum current.
2. 400Ω telephone and CO Resistances.
3. Attenuation: $< 7dB$
4. Using AWG 19 wire

Answer:

Find Required DC Resistance:

$$V = IR$$

$$48V = 24mA(400 + 400 + x)\Omega$$

$$x\Omega = \left(\frac{48V}{24mA}\right) - 800\Omega$$

$$x\Omega = 1200\Omega$$

Look at table for AWG 19: Properties

- Round Trip Loop Resistance: $\frac{85\Omega}{mi}$
- Attenuation (dB/mile unloaded): $\frac{1.12 \text{ dB}}{mi}$

Find Resistance Constraint:

$$xmi = \frac{1200\Omega}{80\frac{\Omega}{mi}} = 14.1mi = 22.7km$$

Find Attenuation Constraint:

$$xmi = \frac{9dB}{1.12\frac{dB}{mi}} = 8.04mi = 12.49km$$

Max Distance is the smaller of the two, as both conditions must be satisfied

Fourier Series

Type 1

- **Function**

$$f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$

- **Fourier Series Equations**

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} fx()\sin(nx)dx$$

- "U" substitution: Let $u = nx$; $\frac{du}{dx} = ndx$

Type 2

- **Function**

$$f(x) = \begin{cases} -1 & -\frac{1}{2}T < x < 0 \\ 1 & 0 < x < \frac{1}{2}T \end{cases}$$

- **Generalized Fourier Series Equations**

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos(\frac{n\pi x}{L})dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin(\frac{n\pi x}{L})dx$$

- "U" substitution: Let $1 = \frac{\frac{2n\pi}{T}}{\frac{2n\pi}{T}}$, $u = \frac{2n\pi x}{T}$, $du = \frac{2n\pi}{T}$

Fourier Transforms

Type 1

- **Function**

$$f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$

- **Fourier Series Equations**

$$c_0 = a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$c_n = \frac{a_n - ib_n}{2} = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}dx$$

$$c_{-n} = \frac{a_n + ib_n}{2} = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{inx}dx$$

- "U" substitution: Let $u = nx$; $\frac{du}{dx} = ndx$

Finding Voltage Spectral Densities

$$VPD = A\tau * \text{sinc}(f\tau) = A\tau * \frac{\sin f\tau\pi}{f\tau\pi}$$

```
# Pulse width (Tau) = 3 microseconds ()
# Amplitude (Amp) = 5 Volts
# Frequency (Freq) = 30, 100, 3000
# Voltage Spectral Density =
# Amp * Tau * (sin(freq*Tau*pi)/(freq*Tau*pi))

def voltageSpectralDensity(freq,
                           Tau= 3*m.pow(10, -3),
                           Amp= 5):
    """Calculates the Voltage Spectral Density
    given Pulse Width (Tau), Amplitude (Amp),
    Frequency."""
    sinc = lambda K: m.sin(K*m.pi)/(K*m.pi)
    return Amp*Tau*sinc(freq*Tau)
```

One of the HW Questions

A sine wave is described by 5sin(300t+27), where t is the time in seconds. Determine the waveform (a) amplitude, (b) rms value, (3) frequency, (4) periodic time, and (5) time lag or lead.

1. Amplitude: 5 Volts
2. RMS value: $\frac{1}{\sqrt{2}} * 5 \approx 0.7071 * 5 \approx 3.53$ Volts
3. Frequency: $\omega = 2\pi f = 300$ so $f = \frac{300}{2\pi} \approx 47.75$
4. Period: $T = \frac{1}{f} \approx \frac{1}{47.75} \approx 0.021$
5. Time: Leading by $27°$

Bode Plot

Method:

We are given a transfer function

$$H(s) = \frac{K(s+z_1)}{s(s+p_1)}$$

where:

- $K =$ constant
- $z_1 =$ the zeros of the transfer function
- $p_1 =$ the poles of the transfer function

We rewrite it by factoring both the numerator and denominator into the standard form, giving us

$$H(s) = \frac{Kz_1(\frac{s}{z_1}+1)}{sp_1(\frac{s}{p_1}+1)}$$

Then we do the following:

- Find K (dB) given K $K_{dB} = 20\log_{10}(K)$
- Find the value of the zeros z_n; where $n = 1, 2, 3, ...$
- Find the value of the poles p_n; where $n = 1, 2, 3, ...$

To plot the bode plot we follow these instructions

- Effect of Constant Terms: Constant terms such as K contribute a straight horizontal line of magnitude 20 log10(K)
- Effect of Individual Zeros: Each occurrence of this causes a positively sloped line passing through the value of 'z' with a rise of 20 db over a decade.
- Effect of Individual Poles: Each occurrence of this causes a negatively sloped line passing through the value of 'p' with a rise of 20 db over a decade.

Transfer Function:

Transfer Function for Low Pass Filters

$$|H(j\omega)| = \left|\frac{V_{OUT}(j\omega)}{V_{IN}(j\omega)}\right| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_C}\right)^{2n}}} = \frac{1}{\sqrt{1+\left(\frac{f}{f_C}\right)^{2n}}}$$

where $f_C = \frac{1}{2\pi RC} Hz$

Convolution

- Convolution of two rectangular pulses of unequal duration will be a trapezoid
- Convolution of two rectangular pulses of equal duration will be a triangle

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Official Steps

1. Given the function

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

we take the wave representing

$$h(t-\tau)$$

and flip it across the y-axis. We then start the wave at the point on the x-axis closest to $-\infty$

2. The magnitudes of $x(\tau)$ and $h(t-\tau)$ is the amplitude of the waves

Unofficial Idea/Steps

- basically we use the above function for output.
  we only calculate the integral when both square waves intersect.
- The magnitudes of $x(\tau)$ and $h(t-\tau)$ is the amplitude of the waves

---