Appendix to the SIS Susceptibles vs. Infected Model

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0.1 SIS Problem

0.2 Variables

- $g_S =$ Growth term of the Susceptibles
- $g_I =$ Growth term of the Infected
- $c_S =$ Competition term of the Susceptibles
- $c_I =$ Competition term of the Infected
- $N =$ the total, fixed size of the population
- $I =$ number of infected, initially
- $S =$ number of susceptibles
- $\gamma = \text{Recovery Rate} = \dfrac{1}{\text{Infection Duration}}$
- $\beta =$ Infection Rate

0.3 Assumptions

- $N = 10000; I = 120; S = 9880$
- $\gamma = \dfrac{1}{6}; \beta = 0.85$
- $g_S = -\dfrac{\beta S I}{N}$
- $g_I = \dfrac{\beta S I}{N}$
- $c_S, c_I = \gamma I$
- $\frac{dS}{dt} = -\dfrac{\beta S I}{N} + \gamma I = -\beta I\Big(1 - \frac{I}{N}\Big) + \gamma I$
- $\frac{dI}{dt} = \dfrac{\beta S I}{N} - \gamma I = \beta I\Big(1 - \frac{I}{N}\Big) - \gamma I$
- $S, I \geq 0$

0.3.1 Packages

```
In [1]: import numpy as np
        import sympy as sp
        from sympy.solvers import solve
        from matplotlib import pyplot as plt
        import math as m
```

0.3.2 Variables

```
In [2]: I = sp.symbols('I')
        gS, gI, cS, cI = -0.85*I*(1 - I/10000), 0.85*I*(1 - I/10000), (1/6)*I, (1/6)*I
        dSdt, dIdt = gS + cS, gI - cI
        sp.init_printing()
```

0.3.3 Functions

```
In [3]: def susceptibles(infected):
            """Rate of Change (differential) for susceptibles"""
            return (-0.85*infected*(1 - infected/10000)) + (1/6)*infected


        def infected(infected):
            """Rate of Change (differential) for infected"""
            return 0.85*infected*(1 - infected/10000) - (1/6)*infected


        def reproductiveNumber(beta, gamma):
            """basic reproductive number, when (\beta)/(\gamma) > 1, the disease
            will spread and approach the second steady state; otherwise, it
            will eventually reach the disease-free state."""
            return beta/gamma

        def criticalThreshold(r0):
            """ critical vaccination threshold, which is equal to 1-(1/ R0)"""
            return  (1-(1/r0))
```

0.3.4 Equilibrium Points

```
In [4]: solve([dSdt, dIdt], I)
```

Out[4]:

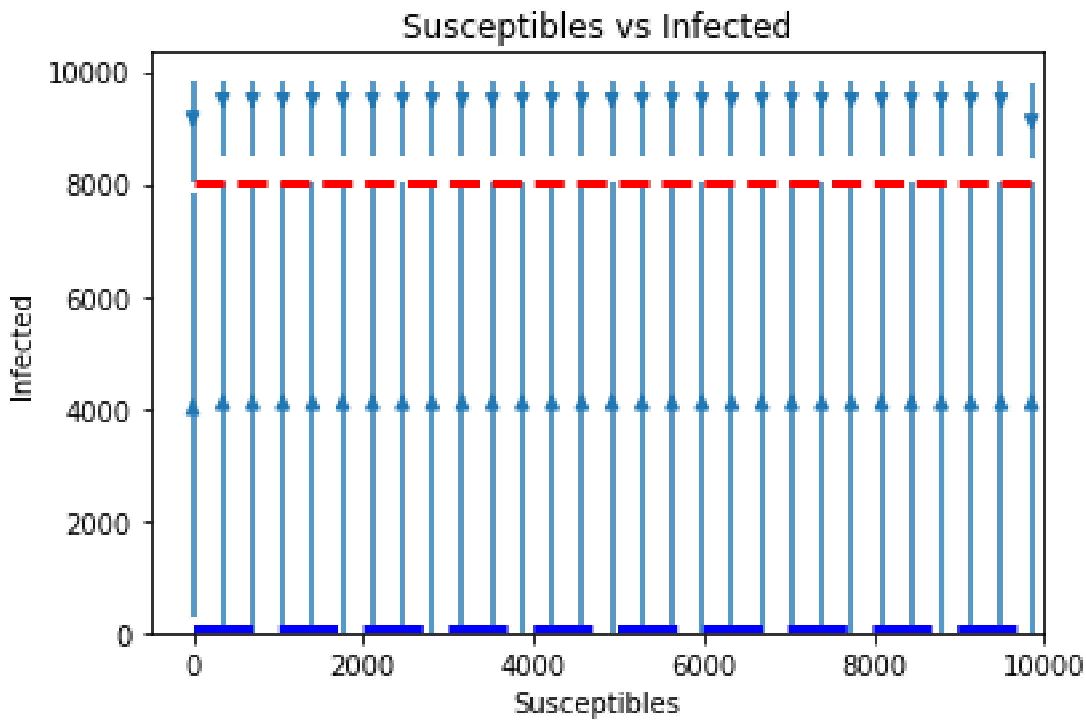
$$[(0.0),\quad (8039.21568627451)]$$

0.3.5 Plot

```
In [5]: x, y = np.linspace(0, 10000), np.linspace(0, 10000)
        X, Y = np.meshgrid(x, y)
```

```
U, V = 0*susceptibles(Y), infected(Y)
fig, ax1 = plt.subplots()
ax1.streamplot(X, Y, U, V)
ax1.plot([0, 10000], [solve([dSdt, dIdt], I)[1][0], solve([dSdt, dIdt], I)[1][0]],
         c= 'r', ls= '--', lw= 3)

ax1.plot([0, 10000], [solve([dSdt, dIdt], I)[0][0], solve([dSdt, dIdt], I)[0][0]],
         c= 'b', ls= '--', lw= 6)
ax1.set(xlabel= 'Susceptibles', ylabel= 'Infected',
        title= 'Susceptibles vs Infected');
```



0.3.6 Reproductive Number

To calculate this value, we'll use the following formula:

- $R_0 = \beta\zeta$
- $\gamma = \text{Recovery Rate} = \dfrac{1}{\text{Infection Duration}}$
- $\beta = \text{Infection Rate}$

```
In [6]: beta, gamma = 0.85, 1/6
        r0 = reproductiveNumber(beta, gamma)
        r0
```

Out[6]:

5.1000000000000005

0.3.7 Critical Threshold

```
In [7]: criticalThreshold(r0)
```

Out[7]:

0.803921568627451