

# Appendix to the SIS Susceptibles vs. Infected Model

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## 0.1 SIS Problem

## 0.2 Variables

- $g_S$  = Growth term of the Susceptibles
- $g_I$  = Growth term of the Infected
- $c_S$  = Competition term of the Susceptibles
- $c_I$  = Competition term of the Infected
- $N$  = the total, fixed size of the population
- $I$  = number of infected, initially
- $S$  = number of susceptibles
- $\gamma$  = Recovery Rate =  $\frac{1}{\text{Infection Duration}}$
- $\beta$  = Infection Rate

## 0.3 Assumptions

- $N = 10000; I = 120; S = 9880$
- $\gamma = \frac{1}{6}; \beta = 0.85$
- $g_S = -\frac{\beta SI}{N}$
- $g_I = \frac{\beta SI}{N}$
- $c_S, c_I = \gamma I$
- $\frac{dS}{dt} = -\frac{\beta SI}{N} + \gamma I = -\beta I \left(1 - \frac{I}{N}\right) + \gamma I$
- $\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I = \beta I \left(1 - \frac{I}{N}\right) - \gamma I$
- $S, I \geq 0$

### 0.3.1 Packages

```
In [2]: import numpy as np
import sympy as sp
from sympy.solvers import solve
from matplotlib import pyplot as plt
import math as m
```

### 0.3.2 Variables

```
In [3]: I = sp.symbols('I')
gS, gI, cS, cI = -0.85*I*(1 - I/10000), 0.85*I*(1 - I/10000), (1/6)*I, (1/6)*I
dSdt, dIdt = gS + cS, gI - cI
sp.init_printing()
```

### 0.3.3 Functions

```
In [4]: def susceptibles(Infected):
    """Rate of Change (differential) for susceptibles"""
    return (-0.85*Infected*(1 - Infected/10000)) + (1/6)*Infected

def infected(Infected):
    """Rate of Change (differential) for infected"""
    return 0.85*Infected*(1 - Infected/10000) - (1/6)*Infected

def reproductiveNumber(beta, gamma):
    """basic reproductive number, when (\beta)/(\gamma) > 1, the disease
    will spread and approach the second steady state; otherwise, it
    will eventually reach the disease-free state."""
    return beta/gamma

def criticalThreshold(r0):
    """ critical vaccination threshold, which is equal to 1-(1/ R0)"""
    return (1-(1/r0))
```

### 0.3.4 Equilibrium Points

```
In [5]: solve([dSdt, dIdt], I)
```

Out[5]:

$[(0.0), (8039.21568627451)]$

### 0.3.5 Plot

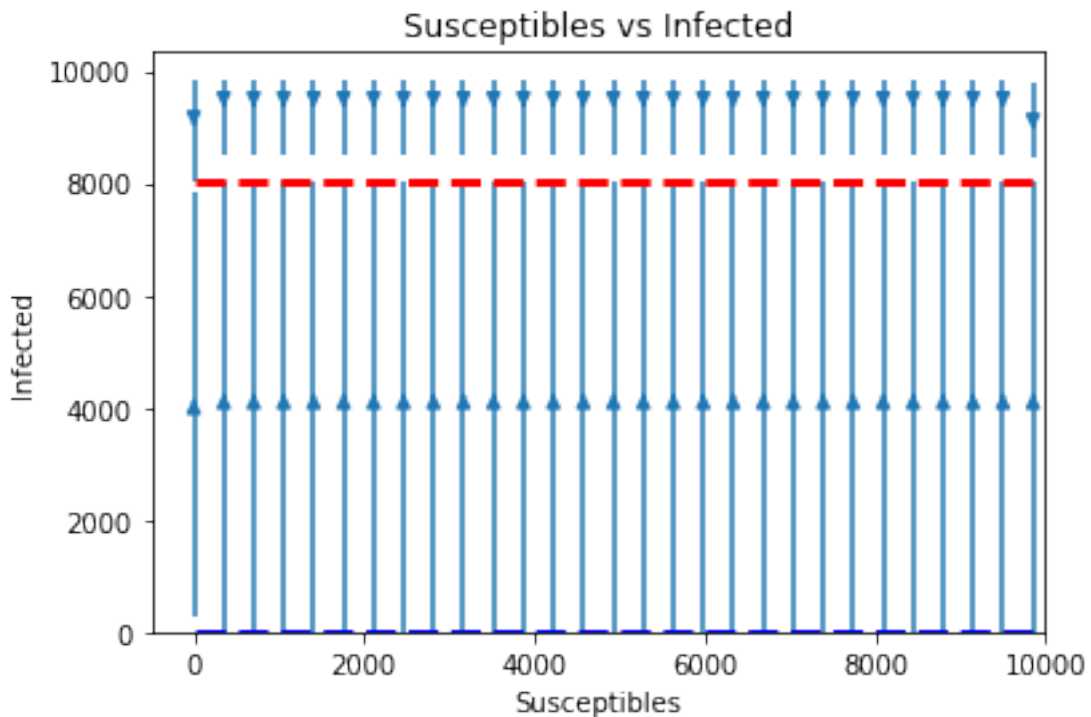
```
In [10]: x, y = np.linspace(0, 10000), np.linspace(0, 10000)
X, Y = np.meshgrid(x, y)
```

```

U, V = 0 * susceptibles(X), infected(Y)
fig, ax1 = plt.subplots()
ax1.streamplot(X, Y, U, V)
ax1.plot([0, 10000], [solve([dSdt, dIdt], I)[1][0], solve([dSdt, dIdt], I)[1][0]],
         c= 'r', ls= '--', lw= 3)

ax1.plot([0, 10000], [solve([dSdt, dIdt], I)[0][0], solve([dSdt, dIdt], I)[0][0]],
         c= 'b', ls= '--', lw= 3)
ax1.set(xlabel= 'Susceptibles', ylabel= 'Infected',
        title= 'Susceptibles vs Infected');

```



### 0.3.6 Reproductive Number

To calculate this value, we'll use the following formula:

- $R_0 = \frac{\beta}{\gamma}$
- $\gamma = \text{Recovery Rate} = \frac{1}{\text{Infection Duration}}$
- $\beta = \text{Infection Rate}$

```

In [6]: beta, gamma = 0.85, 1/6
        r0 = reproductiveNumber(beta, gamma)
        r0

```

Out [6] :

5.1000000000000005

### 0.3.7 Critical Threshold

In [9]: criticalThreshold(r0)

Out [9] :

0.803921568627451