Com. Elec. Formula Sheet

Decibel Formulas

Relative Power Gain

$$A_P = \frac{P_O}{P_I}$$

where P_O and P_I are defined as the following:

$$P_I = \frac{V_I^2}{R_I}$$

$$P_O = \frac{V_O^2}{R_O}$$

Relative Voltage Gain

$$A_V = \frac{V_O}{V_I}$$

Relative Power Gain in dB

$$A_P(db) = 10\log_{10} A_P$$

Given that $R_O = R_I$

If $R_O \neq R_I$ then the general form is given by the following:

$$A_P(db) = 10\log_{10}\left(\frac{\frac{V_O^2}{R_O}}{\frac{V_I^2}{R_I}}\right)$$

Relative Voltage Gain in dB

$$A_V(db) = 20\log_{10}\left(\frac{V_O}{V_I}\right) = 20\log_{10} A_V$$

If $R_O \neq R_I$ then the general form is given by the following:

$$A_V(db) = 20\log_{10}\left(\frac{V_O}{V_I}\right) - 10\log_{10}\left(\frac{R_O}{R_I}\right)$$

Special Case

If $R_O \neq R_I$ then the general form is given by the following:

$$A_V(db) = 10\log_{10}\left(\frac{V_O^2}{V_I^2}\right) - 10\log_{10}\left(\frac{R_O}{R_I}\right)$$

$$= 20\log_{10}\left(\frac{V_O}{V_I}\right) - 10\log_{10}\left(\frac{R_O}{R_I}\right)$$

Absolute Power Gain dBm

$$A_{P(dBm)} = 10\log_{10}\left(\frac{P}{1\text{ mW}}\right),\ \text{dBm}$$

Absolute Power Gain dBw

$$A_{P(dBw)} = 10\log_{10}\left(\frac{P}{1\text{ W}}\right),\ \text{dBw}$$

Signal-to-Noise Ratio

$$\text{SNR } = 10\log_{10}\left(\frac{\text{Signal Power}}{\text{Noise Power}}\right)$$

And given that $R_O = R_I$,

$$\text{SNR } = 10\log_{10}\left(\frac{V_S^2}{V_N^2}\right)$$

$$\text{SNR } = 20\log_{10}\left(\frac{V_S}{V_N}\right)\text{dB}$$

Impulse Noise

$$dB_S = 20\log_{10}\left(\frac{P}{0.0002\bar{\mu}}\right),\ \text{where P is sound pressure in } \bar{\mu}$$

$$\bar{\mu} = 1\frac{\text{dyne}}{\text{cm}^2} = 10^{-6} \text{ of atmospheric pressure at sea level}$$

Gaussian (White) Noise

$$P_n = kT\Delta f$$

$$k = \text{ Boltzmann's Constant } (1.38 * 10^{-23}) \text{ J/K}$$

$$T = \text{ resistor temperature in Kelvin } (K)$$

$$\Delta f = \text{ system bandwidth.}$$

Gaussian (White) Noise Formulas

Using the above proportionality we can relate bandwidth to noise, shown below:
Given the noise can be represented as e_n then we can say the following:

$$P_n = \frac{V_n^2}{R} = kT\Delta f,\ \text{where } V_n = \frac{e_n}{2}$$

This is true by Ohm's law. By solving in terms of e_n we get the following:

$$\frac{V_n^2}{R} = kT\Delta f \text{ where } V_n = \frac{e_n}{2}$$

$$\frac{\left(\frac{e_n}{2}\right)^2}{R} = \frac{\left(\frac{e_n^2}{4}\right)}{R} = kT\Delta f$$

$$\left(\frac{e_n^2}{4}\right) = kT\Delta f R$$

$$e_n = \sqrt{4kT\Delta f R}$$

Noise Ratio

$$NF = 10\log_{10}\frac{\frac{S_i}{N_i}}{\frac{S_o}{N_o}} = 10\log_{10} NR$$

$$NR = \frac{\frac{S_i}{N_i}}{\frac{S_o}{N_o}} \text{ is the Noise Ratio}$$

$$\frac{S_i}{N_i} = \text{ input SNR}$$

$$\frac{S_o}{N_o} = \text{ output SNR}$$

Reactance Noise Effects

$$\Delta f_{eq} = \frac{\pi}{2} BW$$

$BW = 3$ dB; bandwidth for $RC, LC,$ or RLC circuits.

Noise Created by Amplifiers in Cascade

$$NR = NR_1 + \frac{NR_2 - 1}{P_{G_1}} + ... + \frac{NR_n - 1}{P_{G_1} * P_{G_2} * P_{G_{(n-1)}}}$$

$NR =$ overall noise ratio of n stages.

$P_G =$ power gain ratio

Equivalent Noise Temperature

$$T_{eq} = T_0(NR - 1)$$

where $T_0 = 290$ K, a reference temperature in Kelvin.

Equivalent Noise Resistance

Sometimes used by Manufacturers to represent the noise generated by a device with a fictitious resistance. The following represents this:

$$R_{eq} = \sqrt{4kT\Delta f R}$$

Modulation Index

$$m = \frac{E_i}{E_c}$$

$$\%m = \frac{E_i}{E_c} * 100\%$$

$$\%m = \frac{B - A}{B + A} * 100\%$$

$$B = \text{ AM Waveform}$$

$$A = \text{ The Minimum Peak-to-Peak value}$$

Overmodulation

$$\%m = \frac{B - O}{B + O} * 100\%$$

$$B = \text{ AM Waveform}$$

$$O = \text{ The Minimum Peak-to-Peak value} \leq 0$$

Amplitude Modulation/Mixing in Frequency Domain

Carrier Signal

$$e_c = E_C \sin\omega_c t$$

where $e_i =$ is the instantaneous value of the carrier

$E_C =$ is the maximum peak value of the carrier when unmodulated

$\omega = 2\pi f$ ("f" is the carrier frequency)

$t =$ is a unit of measure

Information Signal

$$e_i = E_I \sin\omega_i t$$

where $e_i =$ is the instantaneous value of the information

$E_i =$ is the maximum peak value of the intelligence when unmodulated

$\omega = 2\pi f$ ("f" is the carrier frequency)

$t =$ is a unit of measure

AM Modulated Waveform

$$e = E_c \sin\omega_c t + \frac{mE_c}{2}\cos(\omega_c - \omega_i)t - \frac{mE_c}{2}\cos(\omega_c + \omega_i)t$$

1. $E_c \sin\omega_c t$ **relates to the carrier** (1)
2. $\frac{mE_c}{2}\cos(\omega_c - \omega_i)t$ **relates to the the lower sideband at** $f_c - f_i(2)$
3. $\frac{mE_c}{2}\cos(\omega_c + \omega_i)t$ **relates to the the upper sideband at** $f_c + f_i(3)$

Power Distribution in Carriers and Sidebands

$$E_{SF} = \frac{mE_C}{2}$$

$E_{SF} =$ side frequency amplitude

$m =$ modulation index

$E_C =$ carrier amplitude

Total Transmitted Power

$$P_t = P_c\left(1+\frac{m^2}{2}\right)$$

$$P_t = \text{ Total Transmitted Power (sidebands and carrier)}$$

$$m = \text{ modulation index}$$

$$P_c = \text{ carrier power}$$

Total Transmitted Current

$$I_t = \text{ Total Transmitted Current (sidebands and carrier)}$$

$$I = \text{ modulation index}$$

$$I_c = \text{ carrier current}$$

Frequency Modulation

How FM Generator Works? "The Concept of Deviation"

$$f_{OUT} = f_C + ke_i$$

$$f_{OUT} = \text{ instantaneous output frequency}$$

$$f_C = \text{ output carrier frequency}$$

$$k = \text{ deviation constant [kHz/V]}$$

$$e_i = \text{ modulating (intelligence) input}$$

Quick Facts

1. Deviation constant defines how much carrier frequency will deviate for input voltage level.
2. Deviation constant dependent on system design.
3. Knowing deviation on either side of carrier is essential for determining occupied bandwidth of modulated signal.

Direct FM

Direct FM involves messing w/ the frequency component of a sinusoidal wave:

$$f \text{ in } \omega(A_P\sin(\omega t+\theta) = A_P\sin(2\pi ft+\theta))$$

Indirect FM

Indirect FM involves messing w/ the phase angle component of a sinusoidal wave:

$$\theta \text{ in the sinusoid}(A_P\sin(\omega t+\theta) = A_P\sin(2\pi ft+\theta))$$

FM IN THE FREQUENCY DOMAIN

$$e = A\sin(\omega_c t + m_f\sin\omega_i t)$$

$$e = \text{ instantaneous voltage}$$

$$A = \text{ peak value of original carrier wave}$$

$$\omega_c = \text{ carrier carrier angular velocity } (2\pi f_c)$$

$$\omega_i = \text{ modulating intelligence signal angular velocity } (2\pi f_i)$$

Modulation Index

$$m_f = \text{FM Modulation Index} = \frac{\delta}{f_i}$$

$\delta =$ maximum frequency shift caused by the intelligence signal (deviation)
either above or below the carrier; therefore, deviation written as 3 kHz, for example, has
$\delta =$ 3 kHz (not 6 kHz) in the above.
$f_i =$ of the intelligence (modulating) signal

FM Spectrum Analyzer

Remember that each Bessel table entry represents the ratio $\frac{V_2}{V_1}$ for its respective carrier J_0 or sideband J_1 and above signal component, for a given modulation index.

$$P_{dB} = 20\log_{10}\frac{V_2}{V_1}$$

Power Distribution

Carson's Rule Approximation

$$BW \cong 2(\delta_{max} + f_{i_{max}})$$

Percent of Modulation and Deviation Ratio

$$DR = \frac{\text{max possible freq deviation}}{\text{max input freq}} = \frac{f_{dev(max)}}{f_{i(max)}}$$

Fourier Series

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\cos(nx)dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\sin(nx)dx$$

A0

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} 0dx + \frac{1}{2\pi}\int_{0}^{\pi} 0dx$$

$$= 0 + \frac{1}{2\pi} = \frac{1}{2}$$

An

For $n \geq 1$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\cos(nx)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} 0dx + \frac{1}{\pi}\int_{0}^{\pi}\cos(nx)dx$$

$$= 0 + \frac{1}{\pi}\frac{\sin(nx)}{n}\int_{0}^{\pi} = \frac{1}{n\pi}(\sin(n\pi) - \sin(0)) = 0$$

Bn

For $n \geq 1$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\sin(nx)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} 0dx + \frac{1}{\pi}\int_{0}^{\pi}\sin(nx)dx$$

$$= -\frac{1}{\pi}\frac{\cos(nx)}{n}\int_{0}^{\pi} = -\frac{1}{n\pi}(\cos(n\pi) - \cos(0)) =$$

$$0 \text{ if } n \text{ is even; } \frac{2}{n\pi} \text{ if } n \text{ is odd}$$
