

# A model with perfect competition where AI *decreases* GDP

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I previously made a model of a non-competitive economy where AI decreases GDP (available [here](#)), and at the time I thought that the lack of competition was the essential ingredient to get this effect. It turns out this is not the case: AI can cause GDP to drop in a competitive economy (assuming there are no mistakes in my calculation). If you find a mistake, please contact me at [casey.oday.barkan@gmail.com](mailto:casey.oday.barkan@gmail.com).

## Model

1. **Production:** There are two available technologies, the *old* technology and the *automation* technology, and their production functions are  $f_{\text{old}}(K, L) = A_{\text{old}} K^\alpha L^{1-\alpha}$  and  $f_{\text{auto}}(K) = A_{\text{auto}} K$ . We will assume  $A_{\text{auto}}$  is zero initially and increases as AI technology improves, and we will compute the equilibrium as a function of  $A_{\text{auto}}$  without modeling the dynamics of how  $A_{\text{auto}}$  increases in time. The total production function is determined by allocating capital between the two technologies so as to maximize output for the given inputs:

$$F(K, L) = \max_{0 \leq K_{\text{old}} \leq K} \{f_{\text{old}}(K_{\text{old}}, L) + f_{\text{auto}}(K - K_{\text{old}})\} \quad (1)$$

2. **Capital supply:** Fixed value of  $K$ .
3. **Labor supply:**  $L^s(W)$ , with a “reservation wage”  $W_{\min}$  (this is a wage below which people do not supply labor, perhaps because they don’t have enough income to survive).  $L^s(W) = 0$  when  $W < W_{\min}$ .
4. **Competitive markets:** Factor inputs are paid their marginal products, i.e.  $W = F_L(K, L)$  and  $R = F_K(K, L)$ . Because  $F(K, L)$  is constant returns to scale, this condition on  $W$  and  $R$  ensures firms’ profits are zero, as is the case in competitive equilibrium.

## Specifying parameters

To simplify things so that the equilibrium can be calculated by hand, let’s make the following choices:

- $A_{\text{old}} = 1$
- $K = 1$
- $\alpha = \frac{1}{2}$
- $L^s(W) = 1 - \frac{1}{W}$  (this means that  $W_{\min} = 1$ ). Written differently,  $W = \frac{1}{1-L^s}$ . This can be derived by maximizing the utility function  $u(C, L) = \sqrt{(2+C)(2-L)}$ , where  $C = WL$  is consumption (note that this assumes laborers do not own capital, so their income is only from labor).

## Computing the general equilibrium

Start by solving the maximization problem in Equation 1 to find the production function. We want to find the  $K_{\text{old}}$  that maximizes  $\sqrt{K_{\text{old}} L} + A_{\text{auto}}(1 - K_{\text{old}})$ . The solution is:

$$K_{\text{old}} = \begin{cases} \frac{L}{4A_{\text{auto}}^2} & \text{if } \frac{L}{4A_{\text{auto}}^2} < 1 \\ 1 & \text{if } \frac{L}{4A_{\text{auto}}^2} \geq 1 \end{cases} \quad (2)$$

The reason for the two cases is that  $0 \leq K_{\text{old}} \leq 1$ . So when  $A_{\text{auto}}$  is small,  $K_{\text{old}} = 1$  and the automation technology (AI) isn’t used at all. Only when  $A_{\text{auto}}$  surpasses a threshold will AI begin to be utilized for automated production. Let’s call this threshold  $A_1$ , and it satisfies  $\frac{L}{4A_1^2} = 1$ . We will

compute it's numerical value a bit later. Note that there will be another threshold where the marginal product of labor drops below  $W_{\min}$ , in which case  $L = 0$  and  $K_{\text{old}} = 0$ . Call this second threshold  $A_2$ , and we will compute it's numerical value later as well. Above  $A_2$ , all production is done with AI.

Computing the equilibrium in the different ranges of  $A_{\text{auto}}$  :

- $A_{\text{auto}} < A_1$ :

$F(K, L) = \sqrt{L}$ , so  $W = \frac{1}{2\sqrt{L}}$ . Labor market clearing condition requires  $W = \frac{1}{1-L}$ , so we  $L$  must satisfy  $2\sqrt{L} = 1 - L$ . The solution to this is  $L = 3 - 2\sqrt{2} \approx 0.17$ . Hence,  $W = \frac{1}{1-L} = \frac{1}{2\sqrt{2}-2} \approx 1.2$ , and therefore  $F = \sqrt{L} = \sqrt{3 - 2\sqrt{2}} \approx 0.4$ . Now we can compute the threshold  $A_1 = \frac{\sqrt{L}}{2} = \sqrt{\frac{3}{4} - \frac{\sqrt{2}}{2}} \approx 0.2$ . Note that all these quantities are independent of  $A_{\text{auto}}$  because AI is not utilized in this regime.

- $A_1 \leq A_{\text{auto}} < A_2$ :

$K_{\text{old}} = \frac{L}{4A_{\text{auto}}^2} > 1$ , so  $F(K, L) = \sqrt{K_{\text{old}}L} + A_{\text{auto}}(1 - K_{\text{old}}) = \frac{L}{4A_{\text{auto}}} + A_{\text{auto}}$ . Wage  $W = F_L(K, L) = \frac{1}{4A_{\text{auto}}}$ . From labor supply,  $L = 1 - \frac{1}{W} = 1 - 4A_{\text{auto}}$ , and therefore  $F = \frac{1}{4A_{\text{auto}}} - 1 + A_{\text{auto}}$ .

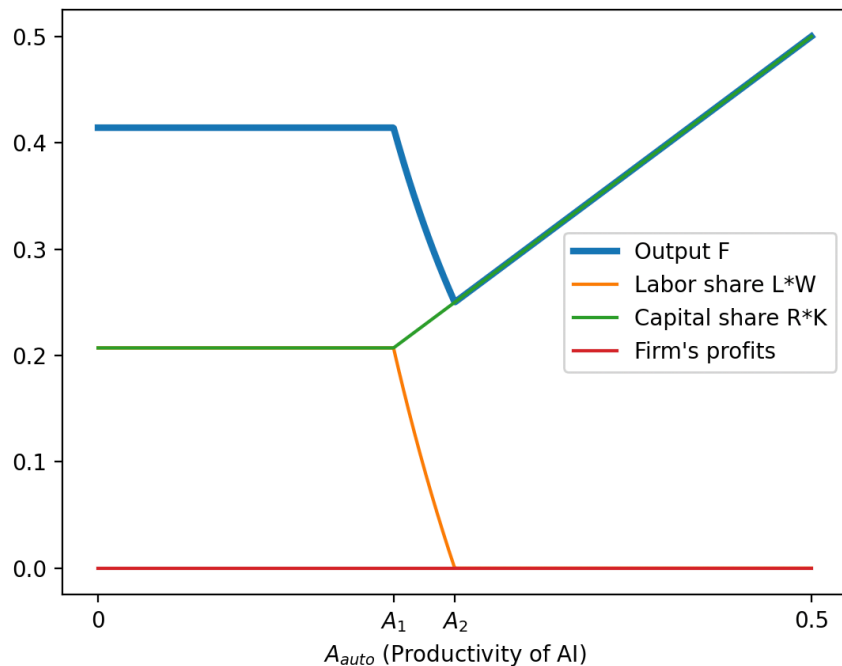
We can now compute  $A_2$ , which is the value of  $A_{\text{auto}}$  at which  $W = W_{\min}$  and  $L = 0$ .  $W_{\min} = 1$ , so  $1 = \frac{1}{4A_2}$ , therefore  $A_2 = \frac{1}{4}$ .

- $A_{\text{auto}} \geq A_2$ :

Now  $L = 0$  and  $K_{\text{old}} = 0$ , so  $F = A_{\text{auto}}$ .

## Plotting the equilibrium as a function of $A_{\text{auto}}$

Here is output  $F$ , capital & labor share, and firm's profits, plotted as a function of  $A_{\text{auto}}$ . **The output of the economy is decreasing for  $A_1 < A_{\text{auto}} < A_2$ .** Note that this plot requires computing  $R$ , which is done below.



### Computing rent $R$

- $A_{\text{auto}} < A_1$ :

$$F = \sqrt{KL}, \text{ so } R = F_K(K, L) = \frac{1}{2} \sqrt{\frac{L}{K}}. K = 1, \text{ so } R = \frac{\sqrt{L}}{2}.$$

- $A_1 \leq A_{\text{auto}} < A_2$ :

In this intermediate regime, capital is utilized for both technologies, so its marginal product for the two technologies must be equal. It's marginal product for the automation technology is  $A_{\text{auto}}$ , so  $R = A_{\text{auto}}$ .

- $A_{\text{auto}} \geq A_2$ :

$$F = A_{\text{auto}} K, \text{ so } R = A_{\text{auto}}.$$