# A model with perfect competition where AI *decreases* GDP

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I previously made a model of a non-competitive economy where AI decreases GDP (available here), and at the time I thought that the lack of competition was the essential ingredient to get this effect. It turns out this is not the case: AI can cause GDP to drop in a competitive economy (assuming there are no mistakes in my calculation). If you find a mistake, please contact me at casey.oday.barkan@gmail.com.

## Model

1. **Production:** There are two available technologies, the *old* technology and the *automation* technology, and their production functions are $f_{\text{old}}(K, L) = A_{\text{old}} K^{\alpha} L^{1-\alpha}$ and $f_{\text{auto}}(K) = A_{\text{auto}} K$. We will assume $A_{\text{auto}}$ is zero initially and increases as AI technology improves, and we will compute the equilibrium as a function of $A_{\text{auto}}$ without modeling the dynamics of how $A_{\text{auto}}$ increases in time. The total production function is determined by allocating capital between the two technologies so as to maximize output for the given inputs:

$$F(K, L) = \max_{0 \le K_{\text{old}} \le K} \{ f_{\text{old}}(K_{\text{old}}, L) + f_{\text{auto}}(K - K_{\text{old}}) \} \tag{1}$$

2. **Capital supply:** Fixed value of $K$.

3. **Labor supply:** $L^s(W)$, with a "reservation wage" $W_{\text{min}}$ (this is a wage below which people do not supply labor, perhaps because they don't have enough income to survive). $L^s(W) = 0$ when $W < W_{\text{min}}$.

4. **Competitive markets:** Factor inputs are paid their marginal products, i.e. $W = F_L(K, L)$ and $R = F_K(K, L)$. Because $F(K, L)$ is constant returns to scale, this condition on $W$ and $R$ ensures firms' profits are zero, as is the case in competitive equilibrium.

## Specifying parameters

To simplify things so that the equilibrium can be calculated by hand, let's make the following choices:

- $A_{\text{old}} = 1$
- $K = 1$
- $\alpha = \frac{1}{2}$
- $L^s(W) = 1 - \frac{1}{W}$ (this means that $W_{\text{min}} = 1$). Written differently, $W = \frac{1}{1 - L^s}$. This can be derived by maximizing the utility function $u(C, L) = (2 + C)(2 - L)$, where $C = WL$ is consumption (note that this assumes laborers do not own capital, so their income is only from labor).

## Computing the general equilibrium

Start by solving the maximization problem in Equation 1 to find the production function. We want to find the $K_{\text{old}}$ that maximizes $\sqrt{K_{\text{old}} L} + A_{\text{auto}}(1 - K_{\text{old}})$. The solution is:

$$K_{\text{old}} = \begin{cases} \frac{L}{4A_{\text{auto}}^2} & \text{if } \frac{L}{4A_{\text{auto}}^2} < 1 \\ 1 & \text{if } \frac{L}{4A_{\text{auto}}^2} \ge 1 \end{cases} \tag{2}$$

The reason for the two cases is that $0 \leq K_{\text{old}} \leq 1$. So when $A_{\text{auto}}$ is small, $K_{\text{old}} = 1$ and the automation technology (AI) isn't used at all. Only when $A_{\text{auto}}$ surpasses a threshold will AI begin to be utilized for automated production. Let's call this threshold $A_1$, and it satisfies $\frac{L}{4A_1^2} = 1$. We will compute it's numerical value a bit later. Note that there will be another threshold where the marginal product of labor drops below $W_{\text{min}}$, in which case $L = 0$ and $K_{\text{old}} = 0$. Call this second threshold $A_2$, and we will compute it's numerical value later as well. Above $A_2$, all production is done with AI.

Computing the equilibrium in the different ranges of $A_{\text{auto}}$ :

- $\boldsymbol{A_{\text{auto}} < A_1}$:
  $F(K, L) = \sqrt{L}$, so $W = \frac{1}{2\sqrt{L}}$. The labor market clearing condition requires $W = \frac{1}{1-L}$, so $L$ must satisfy $2\sqrt{L} = 1 - L$. The solution to this is $L = 3 - 2\sqrt{2} \approx 0.17$. Hence, $W = \frac{1}{1-L} = \frac{1}{2\sqrt{2}-2} \approx 1.2$, and therefore $F = \sqrt{L} = \sqrt{3 - 2\sqrt{2}} \approx 0.4$. Now we can compute the threshold $A_1 = \frac{\sqrt{L}}{2} = \sqrt{\frac{3}{4} - \frac{\sqrt{2}}{2}} \approx 0.2$. Note that all these quantities are independent of $A_{\text{auto}}$ because AI is not utilized in this regime.

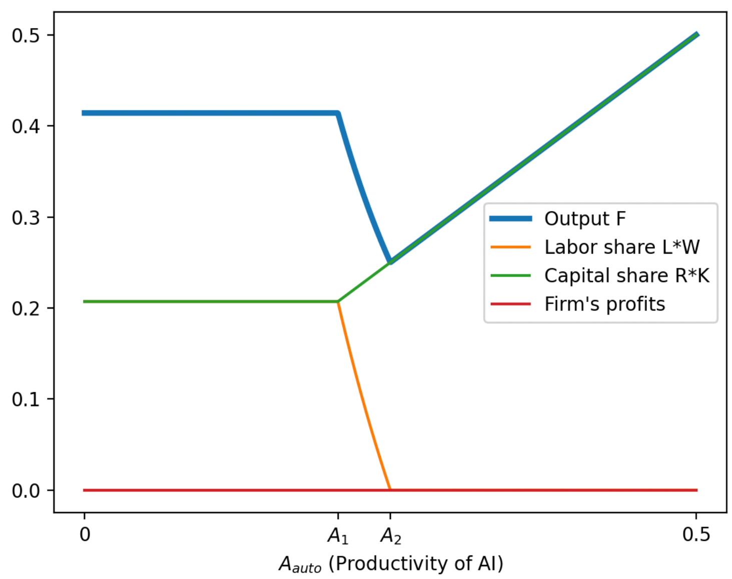
- $\boldsymbol{A_1 \leq A_{\text{auto}} < A_2}$:
  $K_{\text{old}} = \frac{L}{4A_{\text{auto}}^2} < 1$, so $F(K, L) = \sqrt{K_{\text{old}} L} + A_{\text{auto}}(1 - K_{\text{old}}) = \frac{L}{4A_{\text{auto}}} + A_{\text{auto}}$. Wage $W = F_L(K, L) = \frac{1}{4A_{\text{auto}}}$. From labor supply, $L = 1 - \frac{1}{W} = 1 - 4A_{\text{auto}}$, and therefore $F = \frac{1}{4A_{\text{auto}}} - 1 + A_{\text{auto}}$.

  We can now compute $A_2$, which is the value of $A_{\text{auto}}$ at which $W = W_{\text{min}}$ and $L = 0$. $W_{\text{min}} = 1$, so $1 = \frac{1}{4A_2}$, therefore $A_2 = \frac{1}{4}$.

- $\boldsymbol{A_{\text{auto}} \geq A_2}$:
  Now $L = 0$ and $K_{\text{old}} = 0$, so $F = A_{\text{auto}}$.

## Plotting the equilibrium as a function of $A_{\text{auto}}$

Here is output $F$, capital & labor share, and firm's profits, plotted as a function of $A_{\text{auto}}$. **The output of the economy is *decreasing* for $A_1 < A_{\text{auto}} < A_2$.** Note that this plot requires computing $R$, which is done below.



**Computing rent $R$**

- $\boldsymbol{A_{\mathrm{auto}} < A_1}$:
  $F = \sqrt{KL}$, so $R = F_K(K, L) = \frac{1}{2}\sqrt{\frac{L}{K}}$. $K = 1$, so $R = \frac{\sqrt{L}}{2}$.

- $\boldsymbol{A_1 \leq A_{\mathrm{auto}} < A_2}$:
  In this intermediate regime, capital is utilized for both technologies, so its marginal product for the two technologies must be equal. It's marginal product for the automation technology is $A_{\mathrm{auto}}$, so $R = A_{\mathrm{auto}}$.

- $\boldsymbol{A_{\mathrm{auto}} \geq A_2}$:
  $F = A_{\mathrm{auto}}K$, so $R = A_{\mathrm{auto}}$.