A model with perfect competition where AI *decreases* GDP

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I previously made a model of a non-competitive economy where AI decreases GDP (available here), and at the time I thought that the lack of competition was the essential ingredient to get this effect. It turns out this is not the case: AI can cause GDP to drop in a competitive economy (assuming there are no mistakes in my calculation). If you find a mistake, please contact me at casey.oday.barkan@gmail.com.

Model

1. **Production:** There are two available technologies, the *old* technology and the *automation* technology, and their production functions are $f_{\text{old}}(K, L) = A_{\text{old}} K^{\alpha} L^{1-\alpha}$ and $f_{\text{auto}}(K) = A_{\text{auto}} K$. We will assume A_{auto} is zero initially and increases as AI technology improves, and we will compute the equilibrium as a function of A_{auto} without modeling the dynamics of how A_{auto} increases in time. The total production function is determined by allocating capital between the two technology so as to maximize output for the given inputs:
$$F(K, L) = \max_{0 \leq K_{\text{old}} \leq K} \{ f_{\text{old}}(K_{\text{old}}, L) + f_{\text{auto}}(K - K_{\text{old}}) \} \tag{1}$$
2. **Capital supply:** Fixed value of K
3. **Labor supply:** $L^s(W)$, with a "reservation wage" W_{min}, where $L^s(W) = 0$ when $W < W_{\text{min}}$.
4. **Competitive markets:** Factor inputs are paid their marginal products, i.e. $W = F_L(K, L)$ and $R = F_K(K, L)$. Because $F(K, L)$ is constant returns to scale, this condition on W and R ensures firms' profits are zero, as is the case in competitive equilibrium.

Specifying parameters

To simplify things so that the equilibrium can be calculated by hand, let's make the following choices:

- $A_{\text{old}} = 1$
- $K = 1$
- $\alpha = \frac{1}{2}$
- $L^s(W) = 1 - \frac{1}{W}$ (this means that $W_{\text{min}} = 1$). Written differently, $W = \frac{1}{1 - L^s}$.

Computing the general equilibrium

Start by solving the maximization problem in Equation 1 to find the production function. We want to find the K_{old} that maximizes $\sqrt{K_{\text{old}} L} + A_{\text{auto}}(1 - K_{\text{old}})$. The solution is:

$$K_{\text{old}} = \begin{cases} \frac{L}{4A_{\text{auto}}^2} & \text{if } \frac{L}{4A_{\text{auto}}^2} < 1 \\ 1 & \text{if } \frac{L}{4A_{\text{auto}}^2} \geq 1 \end{cases} \tag{2}$$

The reason for the two cases is that $0 \leq K_{\text{old}} \leq 1$. So when A_{auto} is small, $K_{\text{old}} = 1$ and the automation technology (AI) isn't used at all. Only when A_{auto} surpasses a threshold will AI begin to be utilized for automated production. Let's call this threshold A_1, and it satisfies $\frac{L}{4A_1^2} = 1$. We will compute it's numerical value a bit later. Note that there will be another threshold where the marginal product of labor drops below W_{min}, in which case $L = 0$ and $K_{\text{old}} = 0$. Call this second threshold A_2, and we will compute it's numerical value later as well. Above A_2, all production is done with AI.

Computing the equilibrium in the different ranges of A_{auto} :

- $\boldsymbol{A_{\text{auto}} < A_1}$:

  $F(K, L) = \sqrt{L}$, so $W = \frac{1}{2\sqrt{L}}$. Labor market clearing condition requires $W = \frac{1}{1-L}$, so we L must satisfy $2\sqrt{L} = 1 - L$. The solution to this is $L = 3 - 2\sqrt{2} \approx 0.17$. Hence, $W = \frac{1}{1-L} = \frac{1}{2\sqrt{2}-2} \approx 1.2$, and therefore $F = \sqrt{L} = \sqrt{3 - 2\sqrt{2}} \approx 0.4$. Now we can compute the threshold $A_1 = \frac{\sqrt{L}}{2} = \sqrt{\frac{3}{4} - \frac{\sqrt{2}}{2}} \approx 0.2$. Note that all these quantities are independent of A_{auto} because AI is not utilized in this regime.

- $\boldsymbol{A_1 \leq A_{\text{auto}} < A_2}$:

  $K_{\text{old}} = \frac{L}{4A_{\text{auto}}^2} > 1$, so $F(K, L) = \sqrt{K_{\text{old}} L} + A_{\text{auto}}(1 - K_{\text{old}}) = \frac{L}{4A_{\text{auto}}} + A_{\text{auto}}$. Wage $W = F_L(K, L) = \frac{1}{4A_{\text{auto}}}$. From labor supply, $L = 1 - \frac{1}{W} = 1 - 4A_{\text{auto}}$, and therefore $F = \frac{1}{4A_{\text{auto}}} - 1 + A_{\text{auto}}$.

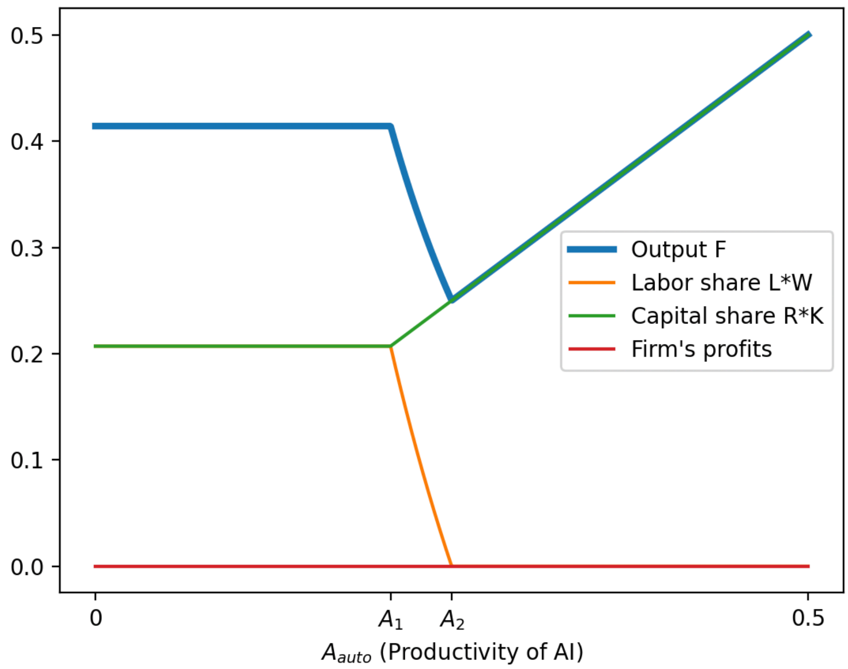
  We can now compute A_2, which is the value of A_{auto} at which $W = W_{\text{min}}$ and $L = 0$. $W_{\text{min}} = 1$, so $1 = \frac{1}{4A_2}$, therefore $A_2 = \frac{1}{4}$.

- $\boldsymbol{A_{\text{auto}} \geq A_2}$:

  Now $L = 0$ and $K_{\text{old}} = 0$, so $F = A_{\text{auto}}$.

Plotting the equilibrium as a function of A_{auto}

Here is output F, capital & labor share, and firm's profits, plotted as a function of A_{auto}. **The output of the economy is *decreasing* for $\boldsymbol{A_1 < A_{\text{auto}} < A_2}$.** Note that this plot requires computing R, which is done below.



Computing rent R

- $\boldsymbol{A_{\text{auto}} < A_1}$:

  $F = \sqrt{KL}$, so $R = F_K(K, L) = \frac{1}{2}\sqrt{\frac{L}{K}}$. $K = 1$, so $R = \frac{\sqrt{L}}{2}$.

- $\boldsymbol{A_1 \leq A_{\text{auto}} < A_2}$:

  In this intermediate regime, capital is utilized for both technologies, so its marginal product for the two technologies must be equal. It's marginal product for the automation technology is A_{auto}, so $R = A_{\text{auto}}$.

- $\boldsymbol{A_{\text{auto}} \geq A_2}$:

  $F = A_{\text{auto}} K$, so $R = A_{\text{auto}}$.