Can an increase in productivity cause a decrease in GDP? Insights from a model economy with AI automation

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Abstract

It is widely assumed that increases in economic productivity necessarily lead to economic growth. In this report, it is shown that this is not always the case. An idealized model of an economy is presented in which a new technology allows capital to be utilized autonomously without labor input. This is motivated by the possibility that advances in artificial intelligence (AI) will give rise to AI agents that act autonomously in the economy. The economic model involves a single profit-maximizing firm which is a monopolist in the product market and a monopsonist in the labor market. The new technology causes the firm to replace labor with capital in such a way that its profit increases while total production decreases. The model is not intended to capture the full structure of a real economy, but rather to illustrate how basic economic mechanisms can give rise to counterintuitive and undesirable outcomes.

1 Introduction

It is often stated that economic productivity gains fueled by AI will inevitably lead to economic growth (Crafts, 2021; Brynjolfsson and Unger, 2023). This report presents a counter-example to this common assumption using a simple economic model. In the model, an increase in the productivity of AI automation technology causes both mass labor displacement and a decrease in total economic output. The decrease in total output is due to the producing firm's profit maximization: with the new technology, the firm maximizes profit by reallocating capital and reducing labor in such a way that total production declines. This counterintuitive result illustrates the importance of mathematical modeling for probing common hypotheses and assumptions about the economic impact of AI.

While mathematical models can illuminate fundamental economic mechanisms, real-world empirical data is typically indispensable for making accurate predictions. However, AI automation may bring unprecedented changes to the economy, such as mass labor displacement, and there simply does not exist empirical data to inform us how these changes will restructure the economy and society. Without empirical data, mathematical modeling is positioned to play a crucial role in predicting the impact of AI automation.

The model presented here involves an economy with two markets: a product market and a labor market (labor is assumed to be undifferentiated and all laborers receive the same wage). There is a single firm which is a monopolist in the product market and a monopsonist in the labor market. Initially, the firm's production utilizes a non-AI technology, called the *old* technology. The old technology requires both capital[1] input and labor input. Then, a *new* technology is introduced which allows capital to be productively utilized with no labor input. We suppose that the new technology gradually improves—in other words, the productivity of capital employed with the new technology gradually increases. The model predicts how the economy responds to this increase in productivity. The model makes predictions of total production (i.e. GDP), labor employment, and capital allocation between the technologies, as a function of the productivity of capital employed with the new technology. The model's predictions depend upon parameters that specify labor supply, available capital stock, and labor productivity. For some (but not all) parameter values, the model predicts that an increase in capital productivity decreases GDP.

Both the labor market and product market in our model have zero competition on the firm side, which contrasts with the perfectly competitive markets used in common general equilibrium models (such as the Ramsey model (Barro and Sala-i-Martin, 2004) and heterogeneous agent

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[1] *Capital* refers to the physical machinery used in production, which includes compute resources used by AI.

models (Heer and Maussner, 2009)). This zero-competition assumption could describe a scenario where one AI company makes a breakthrough allowing it to dominate the market, or where one large tech company comes to dominate the market through acquisitions and mergers. However, it is not likely that a single AI company will gain true monopolist-monopsonist power. It remains a crucial and interesting open question how the results in this report would change for a model economy with competitive markets.

The model also differs from dynamic general equilibrium models in that there is no investment or capital accumulation. Whereas dynamic general equilibrium models describe agents' consumption and investment decisions as optimal control problems in which agents choose their investment to optimize future consumption, our model describes all agent behavior via an aggregate labor supply function. Of course, investment behavior is a key determinant of the impact of labor displacement: investment in AI capital provides passive income which could offset losses in labor income. Moreover, our model does not include a financial sector or financial frictions. Yet, financial crisis is likely to ensue if GDP declines alongside mass labor displacement (financial crisis could be spurred by pension fund withdrawals or bank runs). For this reason, the model's predictions, as dire as they are, may *underestimate* the true economic hardship caused by AI automation. These shortcomings of the model illustrate the need for future work on more sophisticated models.

2 Results

The conceptual setup of the model is described here, and the mathematical details are given in appendix A. The single firm chooses the quantity of labor and capital to employ in production in order to maximize profit. Because the firm is the sole producer, its production is equal to the economy's total output, or GDP. In the labor market, a labor supply function $w(L)$ determines the wage w that the firm must offer to procure a quantity L of labor. In production, the firm has access to two technologies: the *old* technology, which requires both capital and labor input, and the *new* technology, which requires only capital input. The productivity of capital employed by the new technology is quantified by a parameter a_{new}. We assume that, initially, $a_{\mathrm{new}} = 0$ (meaning the new technology is unproductive), then a_{new} increases as the new technology develops. The firm chooses how to allocate capital between the two technologies in order to maximize profit. The equilibrium production, profit, labor employment, and capital allocation, at any value of a_{new}, is computed via the profit-maximization problem described in appendix A.

The key effect that drives both labor displacement and the drop in GDP is the firm's realloaction of capital from the old technology to the new, which occurs once a_{new} is sufficiently high. As one would expect intuitively, when $a_{\mathrm{new}} = 0$ all capital is allocated to the old technology, and when $a_{\mathrm{new}} \to \infty$ all capital is allocated to the new technology. Counter-intuitively, however, a gradual increase in a_{new} does not result in a gradual transition to the new technology. Rather, there is an abrupt transition that occurs once a_{new} reaches a critical threshold.

Figure 1 shows the firm's profit-maximizing capital allocation and labor employment as a function of a_{new}. An abrupt transition occurs when a_{new} reaches a value of approximately 12. Below this transition, all production is done using the old technology and labor is fully employed. There is a narrow transitional period in which capital is employed by both technologies, but for $a_{\mathrm{new}} > 14$ all production is done using the new technology. With no capital allocated to the old technology, labor becomes unproductive, and labor employment drops to zero. This is a concerning

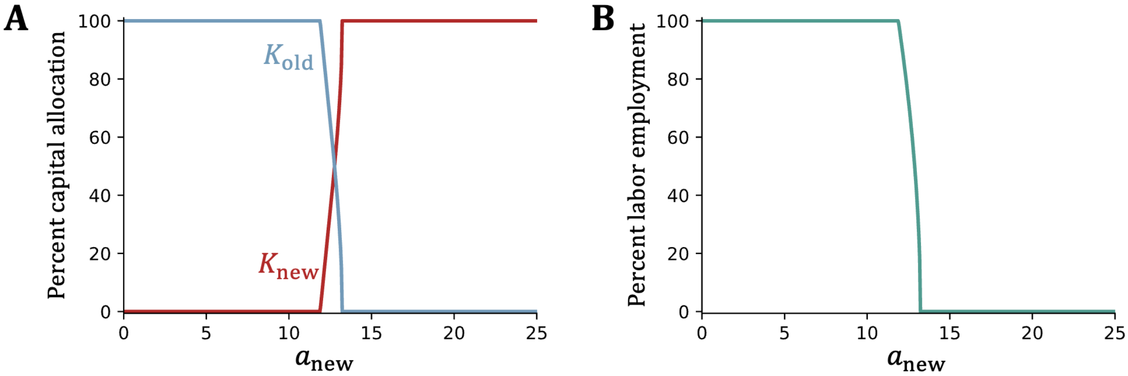

Figure 1: (A) Capital allocation as a function of a_{new}. Blue (red) curve shows percent of capital allocated to the old (new) technology. (B) labor employment as a function of a_{new}.

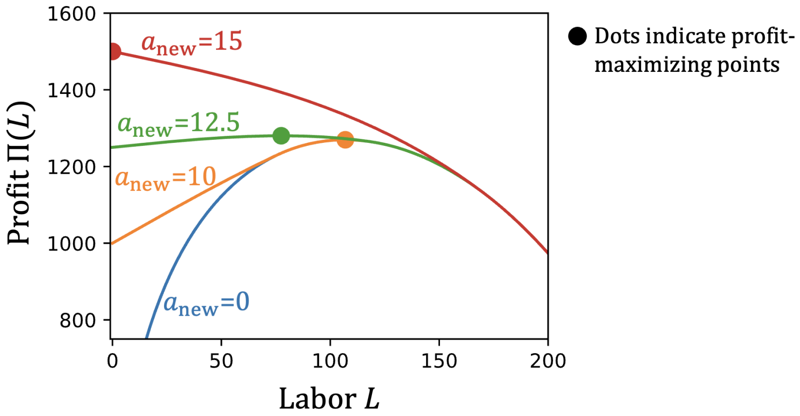

Figure 2: Firm chooses labor L to maximize its profit function $\Pi(L)$. The function $\Pi(L)$ is shown for four values of a_{new}, and the maximum points are indicated by dots.

prediction: as AI automation technology develops, the model predicts no change in the economy until a massive and abrupt transition occurs. In other words, AI technology can become quite advanced with minor societal impact, until the technology reaches a critical level, at which point a profound restructuring of the economy occurs.

What mechanism causes the abrupt transition from the old technology to the new? This can be understood by examining the firm's profit-maximizing decision. The firm's profit as a function of labor employed, $\Pi(L)$ (given by Eq. 6 in section A), is shown in Fig. 2 for four values of a_{new}. For each value of a_{new}, the firm selects L in order to maximize $\Pi(L)$; the maximum points of $\Pi(L)$ are indicated by dots in Fig. 2. As a_{new} increases, the firm's ability to earn profits with the new technology, which requires no labor, increases. Below the transition, this increase has no effect on the optimal L because the old technology remains more profitable than the new. But, once a_{new} is high enough, the optimal L jumps quickly to 0.

Figure 3A shows the firm's profit as functions of a_{new}. Below the transition, changes in a_{new} have no effect on the firm's profit because the firm does not utilize the new technology. Above the transition, profit increases linearly with a_{new} because a_{new} is, by definition, the productivity under the new technology.

The transition causes a large decrease in GDP. As the firm transitions its production from the old to the new technology, it reduces its expenditure on labor by such an extent that its profit goes up while it's production goes down. Mathematically, the firm's profit function is $\Pi(L) = f(L) - w(L)L - rK$ (equation 6), where $f(L)$ is the revenue from production, $w(L)$ is the wage, and rK is the expenditure on capital. The transition causes a decrease in revenue $f(L)$ but a larger decrease in labor cost $w(L)L$, so that $\Pi(L)$ increases. Figure 3B shows total output as a function of a_{new}. GDP drops by 45% when the transition to the new technology occurs.

Note that the 45% drop in GDP is substantially larger than the drop in GDP during the Great Depression, which, in the United States, was 30%. Such a large drop in GDP would likely induce a financial crisis with sweeping consequences. The model predicts that as a_{new} continues to increase, total output rises and eventually surpasses its pre-transition level. However, if financial crisis

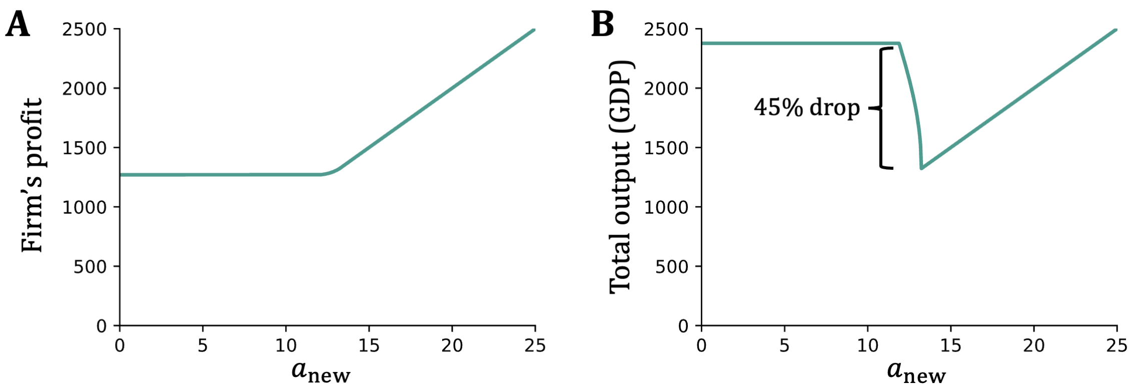

Figure 3: Firm's profit (panel A) and total economic output (panel B) as functions of a_{new}.

occurs, the technological progress that increases a_{new} could be hindered, and additional economic downturn could ensue.

3 Conclusions and Discussion

An essential question is, who wins and who loses from the transition to the new technology? The winners are the owners of the firm and the owners of capital (the owners of capital may or may not be the same people as the owners of the firm), whose profits increase due to the transition. The losers are the laborers, whose employment drops to zero during the transition. In the absence of redistributive policy, the displaced laborers must either subsist through non-market activities or starve. Redistributive policies like universal basic income (UBI) are widely proposed as a means to support displaced laborers. If AI automation leads to an increase in GDP, redistributive policies could let everyone benefit from AI automation. However, if GDP decreases, it is not possible for everyone to benefit from the transition.

Extensions of the model to alternative scenarios and more realistic contexts will be illuminating. The model in this work is intended to illustrate a counterintuitive outcome in the simplest possible setting, but its simplicity prevents it from accounting for important aspects of the real economy. Important ingredients that the model leaves out are competitition in markets, investment and capital accumulation, and heterogeneous agents (both heterogeneous laborers and investors). Extending this model to a heterogeneous agent general equilibrium model would allow for the study of wealth distributions, which is indispensible for understanding the impact of AI on inequality, and for investigating the effects of redistributive policies. The interplay between capital ownership, investment, and wealth distributions is of crucial importance: if capital ownership is broadly distributed through the population, UBI may not be necessary. In fact, Sam Altman has proposed redistribution of AI ownership as an alternative to UBI (Altman, 2021). Redistribution of ownership has the advantage over UBI that it provides passive income in perpetuity, so even if redistributive polices are revoked, recipients will continue to benefit.

There is no historical precedent for the profound changes that AI will likely bring to our society. Hence, we cannot rely on historical economic data to predict our economic future. As an alternative, quantitative models provide a way to probe hypotheses and intuitions about economic mechanisms. This report represents an early step in exploring the impacts that AI automation and labor displacement may bring.

A Appendix: Model details and methods

The model involves a single firm that is a monopolist in the product market and monopsonist in the labor market. Quantities are measured in real terms (rather than nominal terms) in units of the price of the product (i.e. units are set so price $p = 1$). There is no savings or capital accumulation, so total consumption rate equals total production rate (this is sometimes called a 'hand-to-mouth' assumption). Hence, the market clearing condition in the product market is

$$f = wL + rK + \Pi \tag{1}$$

where f is production, and the terms on the right-hand-side represent consumption: wL is consumption by laborers (w is the wage and L is labor employed), rK is consumption by capital owners (r is the rental rate of capital, and K is capital employed), and Π are firm's profits, which are consumed by the firm's owners.

The production rate is determined by capital and labor inputs as well as by the technology available to the firm. With the old technology, the firm has a production function $f_{\text{old}}(K, L) = a_{\text{old}} K^{\alpha} L^{1-\alpha}$, and with the new technology the production function is $f_{\text{new}}(K) = a_{\text{new}} K$. Note that both production functions are of the Cobb-Douglas form, which is standard in economic models (Barro and Sala-i-Martin, 2004). These production functions have the following key properties: with the old technology, capital requires labor input to be productively utilized, whereas with the new technology, capital is productive without labor input. We assume that $a_{\text{new}} = 0$ initially, and we study how the economy changes as a_{new} increases.

The firm's total production function, $f(K, L)$, is determined by allocating capital between the two technologies so as to maximize output. Mathematically,

$$f(K, L) = \max_{K_{\text{old}} \in [0, K]} \{ f_{\text{old}}(K_{\text{old}}, L) + f_{\text{new}}(K - K_{\text{old}}) \} \tag{2}$$

The K_{old} that solves this maximization problem can be found by differentiating the expression on the right-hand-side with respect to K_{old}, setting the result to zero, and solving for K_{old}. This yields the optimal capital allocation $K^*_{\text{old}}(K, L)$, given by

$$K^*_{\text{old}}(K, L) = \min \left\{ L \left(\frac{\alpha \, a_{\text{old}}}{a_{\text{new}}} \right)^{\frac{1}{(1-\alpha)}}, K \right\} \tag{3}$$

The minimum operation in Eq. 3 ensures that the constraint $K_{\text{old}} \in [0, K]$ is satisfied. The firm's production function can then be rewritten as

$$f(K, L) = a_{\text{old}} K^*_{\text{old}}(K, L)^{\alpha} L^{1-\alpha} + a_{\text{new}} \left(K - K^*_{\text{old}}(K, L) \right) \tag{4}$$

According to the market clearing condition in the product market (equation 1), the firm's profit as a function of K and L is

$$\Pi(K, L) = f(K, L) - w(L)L - r(K)K \tag{5}$$

where $w(L)$ is the wage the firm must offer to procure a quantity of labor L. $w(L)$ is called the *labor supply* function. $r(K)$ is the rental rate procure K units of capital (we don't explicitly include the market for rented capital in the model, but if this were included, $r = r(K)$ would be the market clearing condition). Note that equation 5 automatically incorporates the market clearing condition in the labor market, $w = w(L)$. The form of labor supply $w(L)$ is specified below.

Assume that there is a fixed total capital stock available in the economy, denoted K_{max}. This is consistent with our assumption of zero savings, and also assumes negligible depreciation rate. Then, the rental rate is $r_{\text{max}} = r(K_{\text{max}})$. With these assumptions, the firm's profit function simplifies to a function of only L:

$$\Pi(L) = f(K_{\text{max}}, L) - w(L)L - r_{\text{max}} K_{\text{max}} \tag{6}$$

Assume that labor supply $w(L)$ is inelastic for high wages. Inelastic labor supply is a standard assumption, used in the classic Solow-Swan model (Barro and Sala-i-Martin, 2004) and adopted by many subsequent models (such as the Ramsey model and subsequent general equilibrium models (Heer and Maussner, 2009)). However, we assume that the quantity of supplied labor drops to zero below a critical wage w_{min}, meaning that labor supply becomes elastic at low wages. In other words, laborers will not work for a wage less than w_{min}. This could be because people turn to non-market activities to subsist (e.g. gardening, gathering, hunting) or, more darkly, because starvation occurs for wages less than w_{min}. To model the transition from inelastic labor supply at high w to the vanishing of labor supply at w_{min}, we assume the following form for $w(L)$:

$$w(L) = \frac{w_{\text{min}}}{\cos(\pi L / 2L_{\text{max}})} \tag{7}$$

where $0 \leq L \leq L_{\text{max}}$. Figure 4 shows this labor supply curve and, for comparison, a fully inelastic labor supply curve.

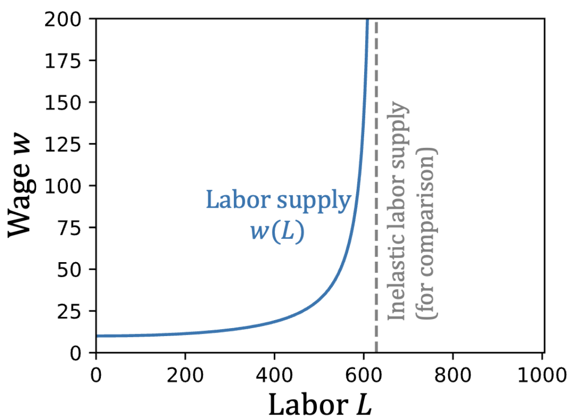

Figure 4: Labor supply curve $w(L)$ (blue), and an inelastic labor supply curve (grey) for comparison.

The firm chooses labor L to maximize profit, solving the optimization problem $\Pi^* = \max_{L \geq 0} \Pi(L)$. This can be solved numerically very efficiently, and python code will be provided soon. Letting L^* denote the profit-maximizing labor quantity, the economy's output is $f(K_{\text{max}}, L^*)$, and capital allocation is $K_{\text{old}} = K^*_{\text{old}}(K_{\text{max}}, L^*)$ and $K_{\text{new}} = K_{\text{max}} - K_{\text{old}}$. Note that all of these quantities depend on a_{new}, as well as the other parameters of the model.

References

Altman, Sam (2021). "Moore's law for everything", *March*, Vol. 16, p. 2021.

Barro, Robert and Sala-i-Martin, Xavier (2004). *Economic Growth*, 2nd Edition. The MIT Press.

Brynjolfsson, Erik and Unger, Gabriel (2023). "The Macroeconomics of Artificial Intelligence", *Finance & Development Magazine*,

Crafts, Nicholas (2021). "Artificial intelligence as a general-purpose technology: an historical perspective", *Oxford Review of Economic Policy*, Vol. 37, (3).

Heer, Burkhard and Maussner, Alfred (2009). *Dynamic general equilibrium modeling*, Springer.