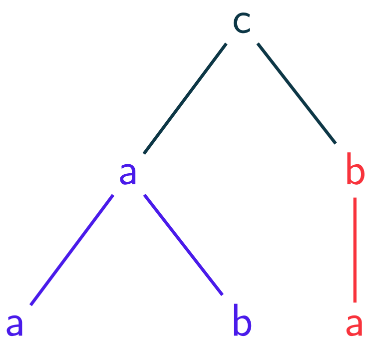
Progress on weak validation of streamed trees

Corentin Barloy

University of Lille

Processing streamed trees

XML encoding of trees:



$\langle c \rangle$

$\quad\langle a \rangle$

$\qquad\langle a \rangle\langle / a \rangle$

$\qquad\langle b \rangle\langle / b \rangle$

$\quad\langle / a \rangle$

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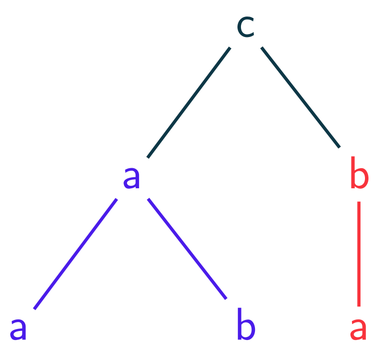

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Validation: Does a XML document belongs to a given regular language of trees?

Weak validation

Weak validation: the document is well-formed.

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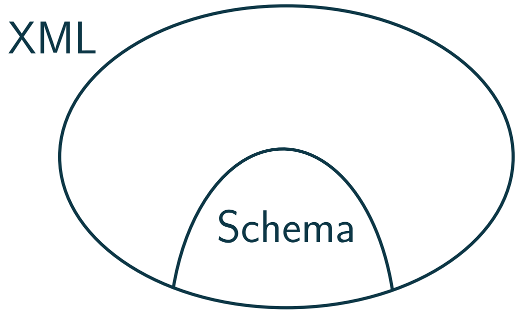

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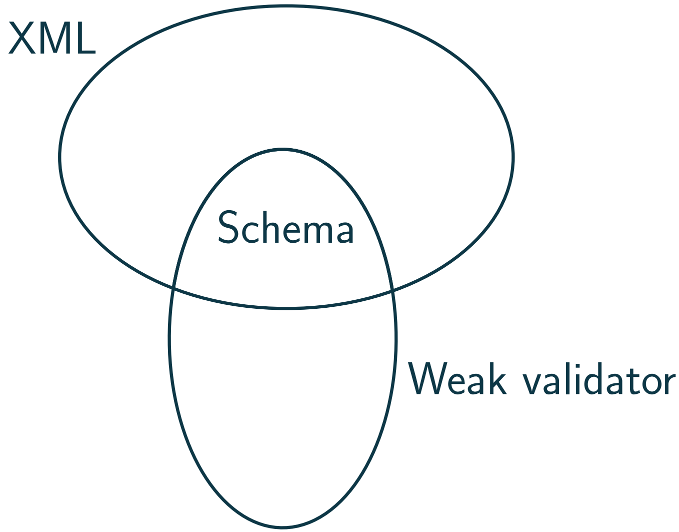

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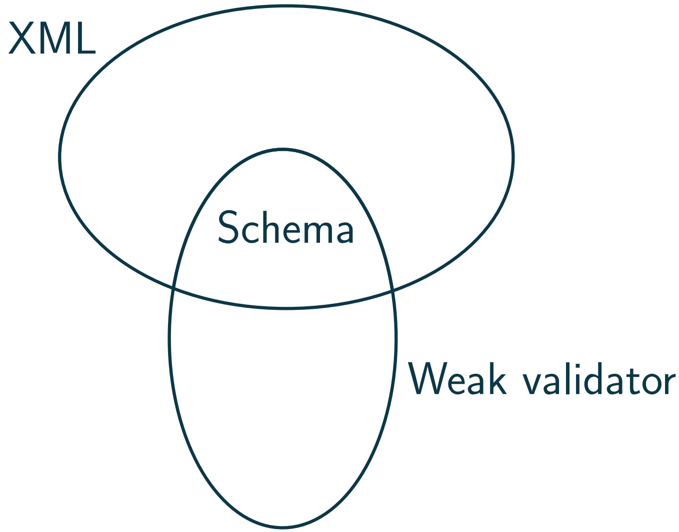

Question

What is the space complexity of weakly validating documents in streaming?

Space complexity

Segoufin, Vianu:

What can be weakly validated in constant space?

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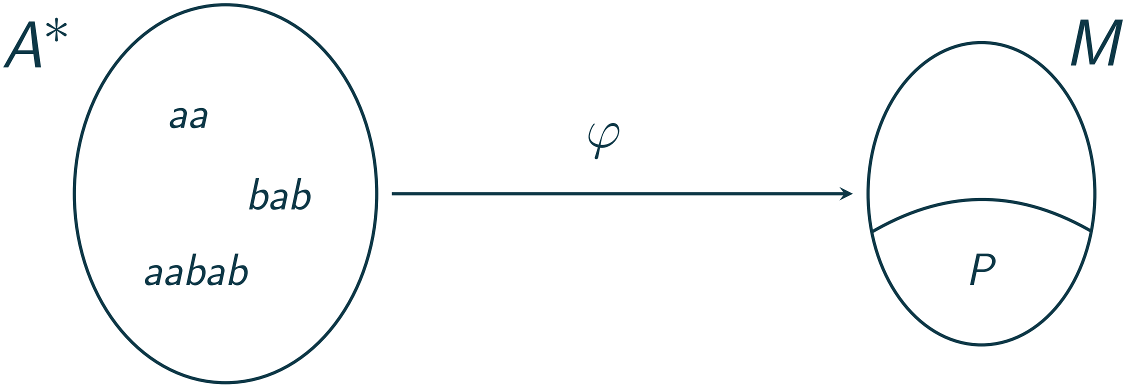
Does the trichotomy holds in general?

Monoïds



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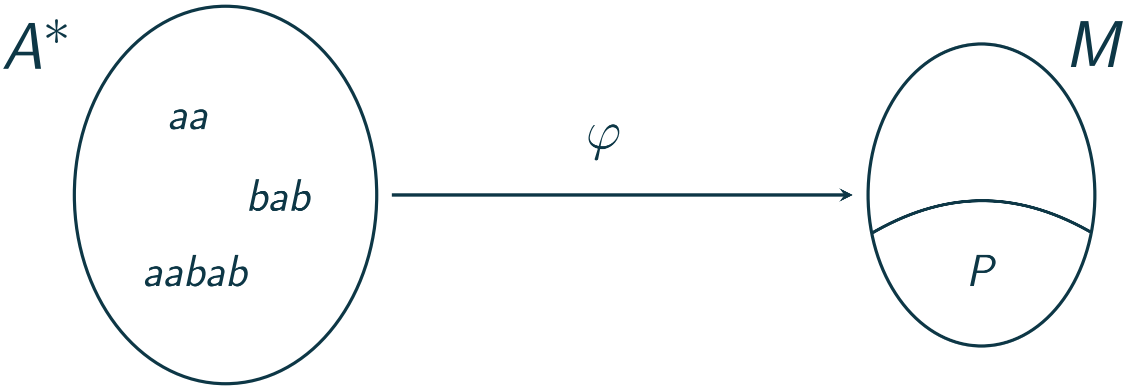

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Syntactic monoïd:

- Congruence on A^*
- Computed from the minimal automaton
- Smallest monoïd recognizing $\mathcal{L}$

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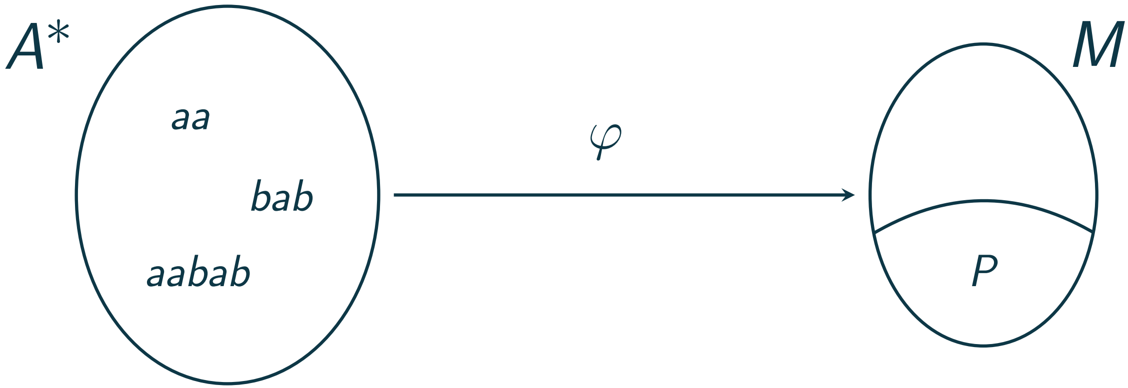

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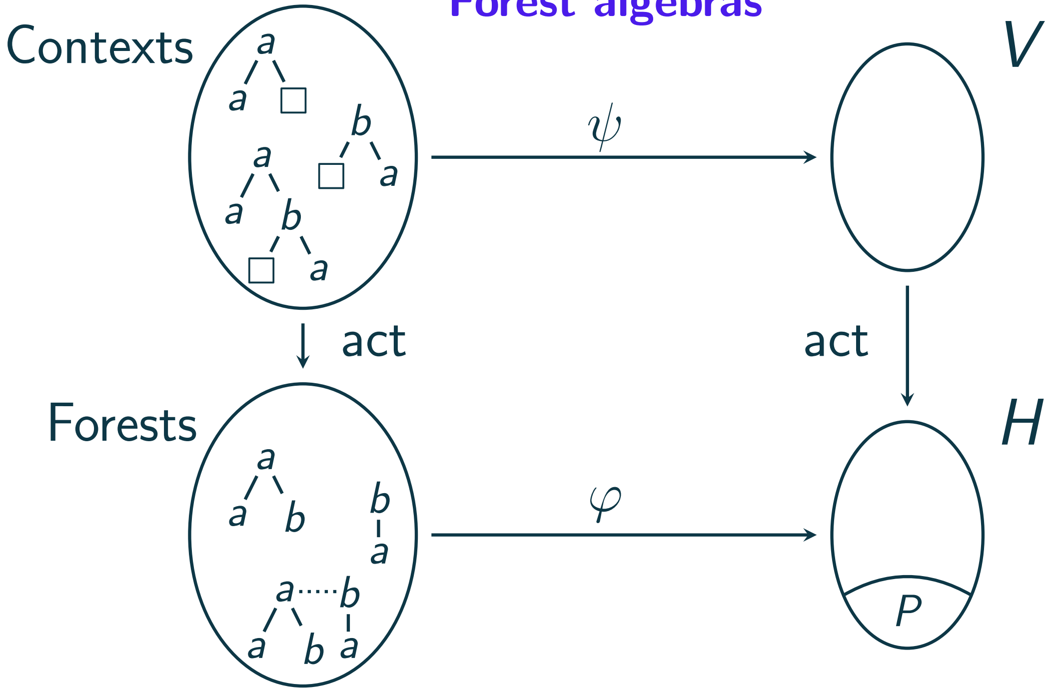
Varieties: membership to a class can be read on the syntactic monoïd.

Equational theory.

Forest algebras



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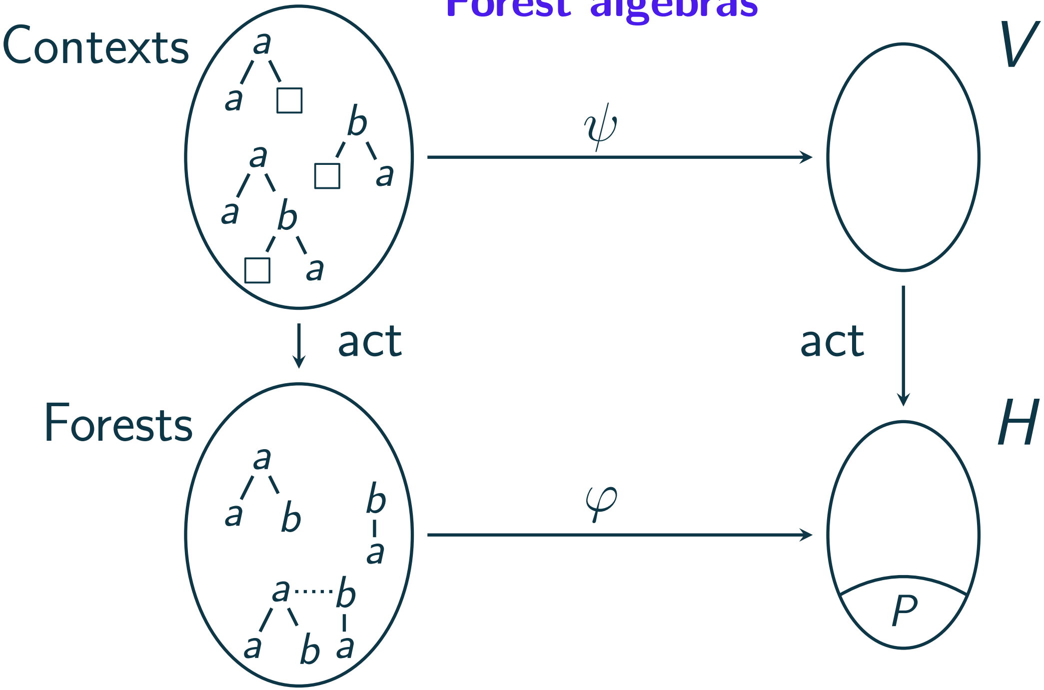

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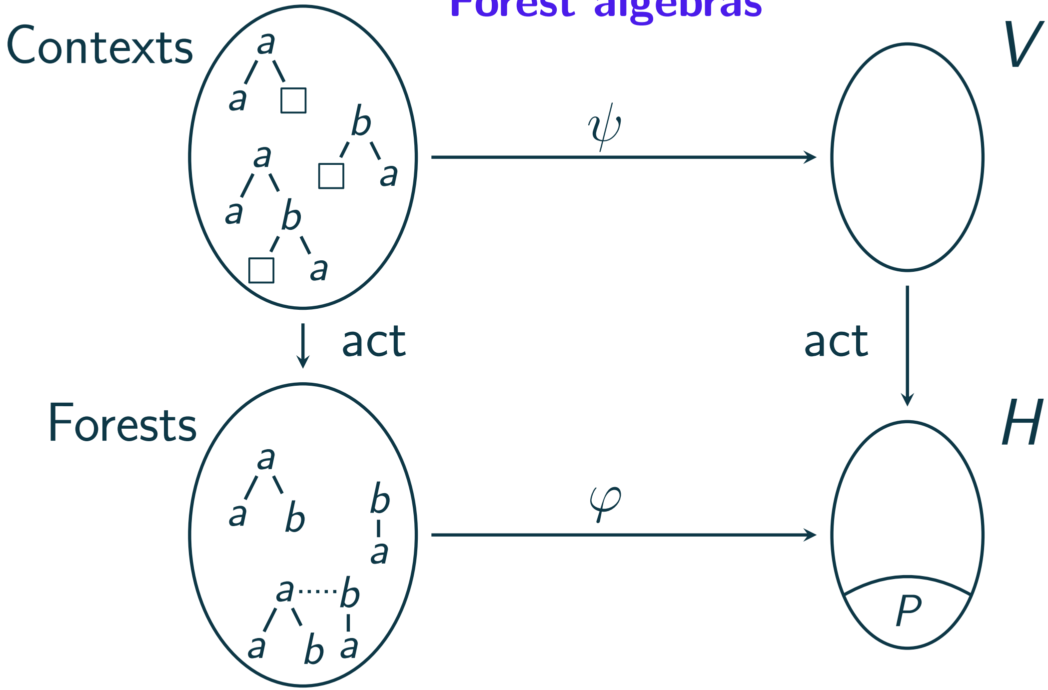

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Varieties

Equations

Monoïdal forest algebras

A forest algebra (V, H) is monoïdal iff:

For all $v \in V$, there exists $x, y \in H$ such that for all $h \in H$

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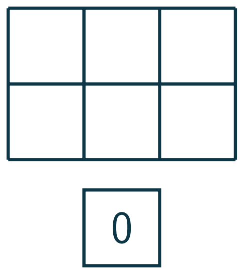
Solved for algebras with H that is 0-simple :



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$\mathcal{V}$ is the variety of all forests satisfying:

“A lot of”

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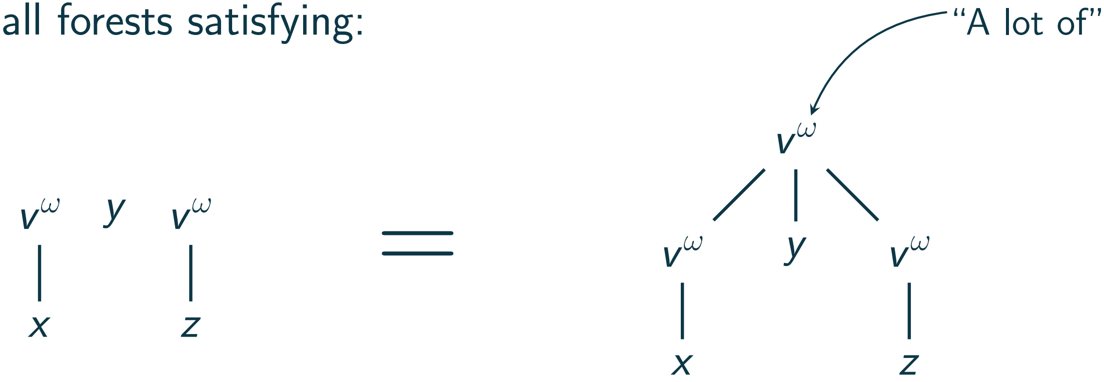

Fact

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Proof

Mimics a pumping argument.

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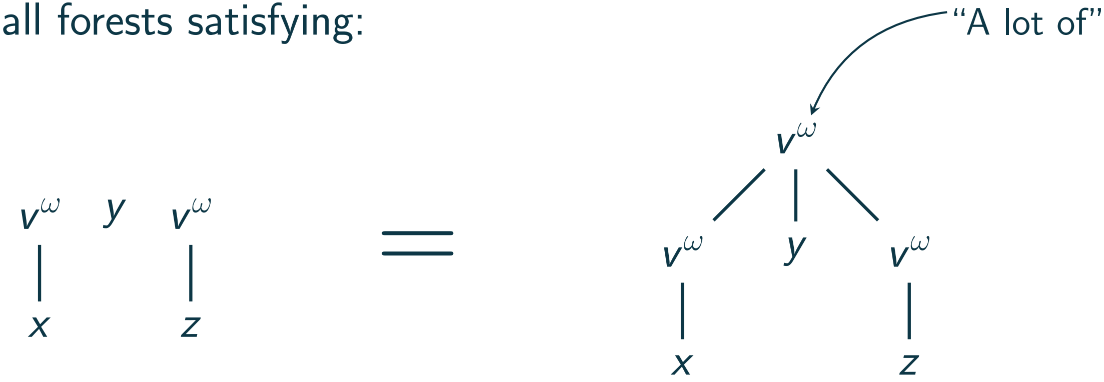

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Do we need more equations?

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Thanks!