Math Handbook

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1 LaTeX

1.1 Font

1.1.1 Font sizes

texblog.org	\Huge
texblog.org	\huge
texblog.org	\LARGE
texblog.org	\Large
texblog.org	\large
texblog.org	\normalsize
texblog.org	\small
texblog.org	\footnotesize
texblog.org	\scriptsize
texblog.org	\tiny

Table 1: Font sizes

1.2 TikZ

1.2.1 Drawing lines

TODO: TikZ

All drawing commands inside a tikzpicture environment:

All drawing commands inside tikzpicture

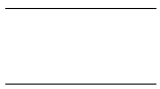
```
\begin{tikzpicture}
    \draw (1,0) -- (0,0) -- (0,1);
\end{tikzpicture}
```

Connected path closed with `--cycle` operation

```
\draw (1,0) -- (0,0) -- (0,1) -- cycle;
```

Move-to operation

```
\draw (0,0) -- (2,0) (0,1) -- (2,1);
```



Connect points by first horizontal then vertical

```
\draw (0,0) -| (1,1);
```

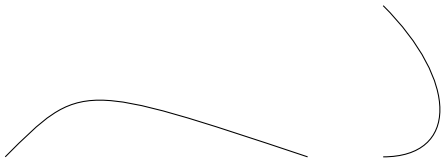
Connect points by first vertical then horizontal

```
\draw (0,0) |- (1,1);
```

1.2.2 Drawing curved paths

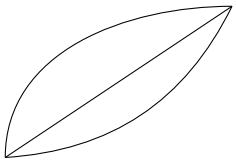
Bezier curve using `controls` command

```
\draw (0,0) .. controls (1,1) .. (4,0)
(5,0) .. controls (6,0) and (6,1) .. (5,2);
```



User-defined paths using the `to` operation

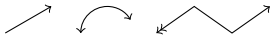
```
\draw (0,0) to (3,2);
\draw (0,0) to[out=90,in=180] (3,2);
\draw (0,0) to[bend right] (3,2);
```



1.2.3 Special curves

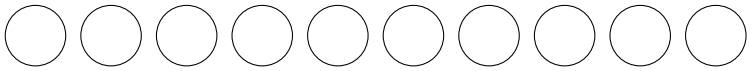
Arrow tips to lines

```
\draw [->] (0,0) -- (30:20pt);
\draw [<->] (1,0) arc (180:30:10pt);
\draw [<<->] (2,0) -- ++(0.5,10pt) -- ++(0.5,-10pt) -- ++(0.5,10pt);
```



Loop using `foreach` command

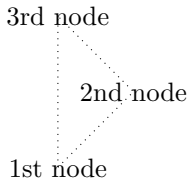
```
\foreach \x in {0,...,9}
\draw (\x,0) circle (0.4);
```



1.2.4 Nodes

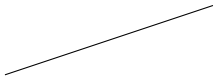
Draw text along path with node command

```
\draw[dotted]
(0,0) node {1st node}
-- (1,1) node {2nd node}
-- (0,2) node {3rd node}
-- cycle;
```



Connect nodes using nodes' labels as coordinates (Option 1)

```
\path (0,0) node(x) {}
(3,1) node(y) {};
\draw (x) -- (y);
```

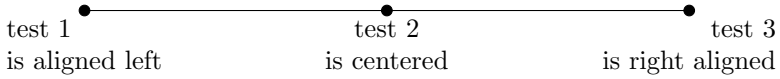


Connect nodes using nodes' labels as coordinates (Option 2)

```
\coordinate (x) at (0,0);
\coordinate (y) at (3,1);
\draw (x) -- (y);
```

Multi-line text inside a node

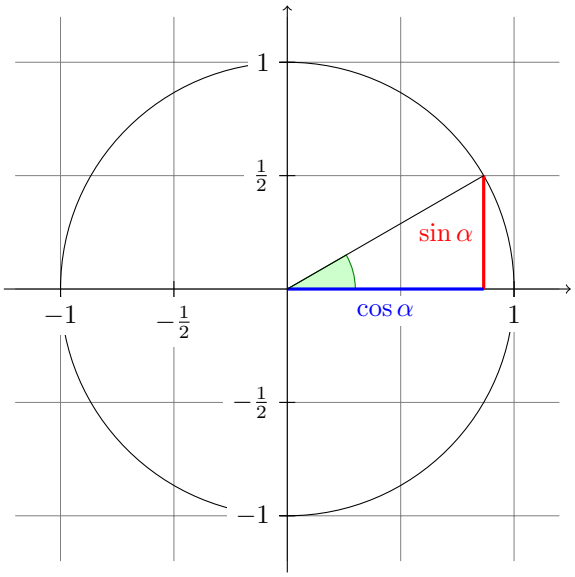
```
\filldraw
(0,0) circle (2pt) node[align=left,   below] {test 1\\is aligned left} --
(4,0) circle (2pt) node[align=center, below] {test 2\\is centered}     --
(8,0) circle (2pt) node[align=right,  below] {test 3\\is right aligned};
```



For inline

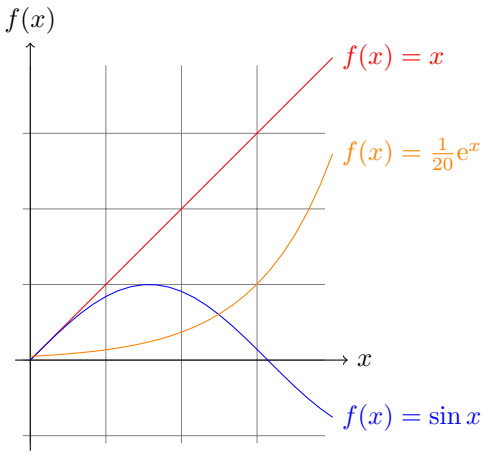
```
\tikz[]{\draw[red, dashed, very thick, rotate=30] (1,0) -- (0,0) -- (0,1);}
```

Unit circle on graph



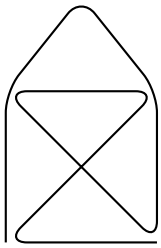
Functions

```
\begin{tikzpicture}[domain=0:4]
    \draw[very thin,color=gray] (-0.1,-1.1) grid (3.9,3.9);
    \draw[->] (-0.2,0) -- (4.2,0) node[right] {$x$};
    \draw[->] (0,-1.2) -- (0,4.2) node[above] {$f(x)$};
    \draw[color=red]    plot (\x,\x)             node[right] {$f(x) =x$};
    \draw[color=blue]   plot (\x,{sin(\x r)})    node[right] {$f(x) = \sin x$};
    \draw[color=orange] plot (\x,{0.05*exp(\x)}) node[right] {$f(x) = \frac{1}{20} \mathrm e^x$};
\end{tikzpicture}
```



Shape with rounded corners

```
\begin{tikzpicture}
    \draw[thick,rounded corners=8pt] (0,0) -- (0,2) -- (1,3.25)
    -- (2,2) -- (2,0) -- (0,2) -- (2,2) -- (0,0) -- (2,0);
\end{tikzpicture}
```



1.3 Letters

TODO: Letters

1.4 Source code

TODO: Source code listings

1.5 Comments on equations

1.5.1 Comment on parts

$$z = \overbrace{\underbrace{x}_{\text{real}} + i \underbrace{y}_{\text{imaginary}}}^{\text{complex number}}$$

1.5.2 Comments longer than formula

$$y = a + f(\underbrace{\qquad bx \qquad}_{\geq 0 \text{ by assumption}}) = a + f(\underbrace{bx}_{\geq 0 \text{ by assumption}})$$

2 Math

2.0.1 Limits

$$\lim_{a \to \infty} \frac{1}{a} \tag{1}$$

$$\lim_{a \to \infty} \frac{1}{a} \tag{2}$$

2.0.2 Log base conversion

$$\frac{log_x n}{log_x B} = log_B n$$

2.0.3 Ceiling integer division

$$\left\lfloor \frac{n}{d} \right\rfloor = \frac{n + d - 1}{d}$$

2.0.4 Bit shift equivalent of multiply by 10

```
// (x << 3) + (x << 1) ≡ x * 10
int x, y;
// ...
x = (x << 3) + (x << 1);
y = y * 10;
assert(x == y);
```

2.1 C++

2.1.1 Logarithm base 2

```
// log2(n)
log2(n) = 31 - __builtin_clz(n);
```

2.1.2 Add value, update average

```
// avg_{n+1} = (sum_n(n+1) + kn - sum) / (n(n+1))
int n, sum;
// ...
double avg = sum / n;
while ((int)(avg + 0.5) < k) {
    avg = sum * n + sum + k * n - sum;
    avg /= n * n + n;
    sum += k;
    n++;
}
```

2.1.3 Binomial coefficient

```
// C(n,k) = C(n-1,k-1) + C(n-1,k)
typedef long long ll;
ll binom(int n, int k) {
    if (k == 0 || k == n) return 1;
    k = min(k, n - k); // Since C(n,k) == C(n,n-k)
    ll ans = 1LL;
    for (ll i = 1; i <= k; i++) {
        ans = ans * (n - k + i) / i;
    }
}
ll choose(int n, int k, ll p = 1e9+7) {
    if (n < k) return 0;
    k = min(k, n - k);
    ll num = 1, den = 1;
    for (int i = 0; i < k; i++) num = num * (n - i) % p;
    for (int i = 1; i <= k; i++) den = den * i % p;
    return num * powmod(den, p - 2, p) % p;
}
ll multichoose(int n, int k, ll p = 1e9+7) {
    return choose(n + k - 1, k, p);
}
```

2.1.4 Catalan numbers

```
typedef long long ll;
ll catalan(int n, ll p = 1e9+7) {
    return choose(2 * n, n, p) * powmod(n + 1, p - 2, p) % p;
}
ll powmod(ll x, ll n, ll m) {
    ll a = 1, b = x;
    for (; n > 0; n >>= 1) {
        if (n & 1) a = mulmod(a, b, m);
        b = mulmod(b, b, m);
    }
    return a % m;
}
ll mulmod(ll x, ll n, ll m) {
    ll n = 0, b = x % m;
    for (; n > 0; n >>= 1) {
        if (n & 1) a = (a + b) % m;
        b = (b << 1) % m;
    }
    return a % m;
}
```

2.1.5 Count number of digits in a number

```
// digits = ⌊log10(n)⌋ + 1
int countDigits(long long n) {
    return n > 0 ? (int)log10((double)n) + 1 : 1;
}
```

2.1.6 Enumerate combinations of N elements in K in lexical order

```
typedef long long ll;
ll catalan(int n, ll p = 1e9+7) {
    return choose(2 * n, n, p) * powmod(n + 1, p - 2, p) % p;
}
ll powmod(ll x, ll n, ll m) {
    ll a = 1, b = x;
    for (; n > 0; n >>= 1) {
        if (n & 1) a = mulmod      b = mulmod(b, b, m);
    }
    return a % m;
}
ll mulmod(ll x, ll n, ll m) {
    ll n = 0, b = x % m;
    for (; n > 0; n >>= 1) {
        if (n & 1) a = (a + b) % m;
        b = (b << 1) % m;
    }
    return a % m;
}
```

2.1.7 Prime factorization

```
typedef vector<int> vi;
vi factor(int n) {
    vi f;
    if (n < 2) return vi();
    while (~n & 1) n /= 2, f.push_back(2);
    for (long long p = 3; p * p <= n; p += 2)
        while (n % p == 0) n /= p, f.push_back((int)p);
    if (n > 1) f.push_back(n);
    return f;
}
```

2.1.8 Fibonacci

```
// Complexity: O(log(n))
// Compute x^n mod m
int modexp(int x, int n, int m) {
  if (n == 0) return 1;
  if (n & 1) return ((x % m) * modexp(x, n - 1, m)) % m;
  int y = modexp(x, n / 2, m);
  return (y * y) % m;
}
```

2.1.9 Modular Exponentiation

```
// Complexity: O(log(n))
// Compute x^n mod m
int modexp(int x, int n, int m) {
  if (n == 0) return 1;
  if (n & 1) return ((x % m) * modexp(x, n - 1, m)) % m;
  int y = modexp(x, n / 2, m);
  return (y * y) % m;
}
```

2.1.10 Sieve + Optimized primality testing

```cpp
// Sieve + optimized prime testing
typedef long long ll;
typedef vector<int> vi;

ll sz;
bitset<10000010> p; // 10^7 + 10
vi primes;
void sieve(ll m) {
    sz = m + 1;
    p.set();
    p[0] = p[1] = 0;
    for (ll i = 2; i <= sz; i++) {
        if (p[i]) {
            for (ll j = i * i; j <= sz; j += i) {
                p[j] = 0;
            }
            primes.push_back((int)i);
        }
    }
}
bool isPrime(ll x) {
    if (x <= sz) return p[x];
    for (int i = 0; i < (int)primes.size(); i++) {
        if (x % primes[i] == 0) return false;
    }
    return true;
}
```

2.1.11 Base conversion

```cpp
// Base conversion
// Complexity: O(N), N digits
// Given digits of int x in base a, return x's digits in base b.

typedef vector<int> vi;

// x : digit representation of number
// a : base of x
// b : desired base
// returns => vector<int> digits of number in base b.
// Note: vec[0] stores the most significant digit.
vi convert_base(const vi &x, int a, int b) {
    unsigned long long base10 = 0;
    FR(i, x.size()) base10 += x[i] * pow(a, x.size() - i - 1);
    int N = ceil(log(base10 + 1) / log(b));
    vi bb;
    for (int i = 1; i <= N; i++)
        bb.emplace_back((int)(base10 / pow(b, N - i)) % b);
    return bb;
}

// x : number
// b : desired base
// returns => vector<int> digits of number in base b
vi base_digits(int x, int b = 10) {
    vi bb;
    while (x != 0) bb.emplace_back(x % b), x /= b;
    reverse(begin(bb), end(bb));
    return bb;
}

int main() {
    // consider 123_5, (i.e. 123 in base 5)
    vi x{1, 2, 3}; int a = 5;
    vi z = convert_base(x, a, 10); // 123_5 = 38_10, z = {3, 8}
    vi y = convert_base(x, a, 3); // 123_5 = 1102_3, y = {1, 1, 0, 2}
}
```