```cpp
// Problem #    : snip
// Created on   : 2018-10-21 10:03:54
#include <bits/stdc++.h>
#define FR(i, n) for (int i = 0; i < (n); ++i)
using namespace std;

typedef long long ll;
typedef pair<int, int> ii;
typedef vector<int> vi;

// Binomial Coefficient
// Non-DP method, for when bottom-up DP with:
// C(n, k) = C(n - 1, k - 1) + C(n - 1, k), is not feasible
ll C(int n, int k) {
   if (k == 0 || k == n) return 1;
   k = min(k, n - k);  // Since C(n, k) == C(n, n - k)
   ll ans = 1;
   for (ll i = 1; i <= k; i++) {
      ans *= (n - k + i) / i;
   }
   return ans;
}

// Count # of digits in number
int countDigits(long long n) {
   return n > 0 ? (int)log10((double)n) + 1 : 1;
}

// Dijkstra SSSP
vector< vector<ii> > g; vi D; int n;
void dijkstra(int s) {
   D.assign(n, -1);
   priority_queue<ii, vector<ii>, greater<ii> > pq;
   pq.emplace(0, s);
   while (!pq.empty()) {
      int u = pq.top().second, d = pq.top().first; pq.pop();
      if (D[u] != -1) continue;
      D[u] = d;
      for (auto &i : g[u]) {
	int v = i.second, w = i.first;
	pq.emplace(d + w, v);
      }
   }
}

// Prime factorization, n >= 0
// Complexity: O(log(n))
vi factor(int n) {
   vi f;
   if (n < 2) return vi();  // vi(1, n), if want 'n' for n < 2
   while (~n & 1) n >>= 1, f.push_back(2);
   for (ll p = 3; p * p <= n; p += 2)
      while (n % p == 0) n /= p, f.push_back((int)p);
   if (n > 1) f.push_back(n);
   return f;
}
```

```
// Compute nth Fibonacci number with matrix exponentiation
// Time complexity: O(log(n))
// Warning: 46th Fibonacci number (i.e. fib(46)) is largest
// that will fit into signed 32-bit integer; use long long if need
// longer.
int f[1000];
int fib(int n) {
   if (n < 2) return n;
   if (f[n]) return f[n];
   int k = (n + 1) / 2;
   f[n] = (n & 1) ? fib(k) * fib(k) + fib(k - 1) * fib(k - 1)
      : (2 * fib(k - 1) + fib(k)) * fib(k);
   return f[n];
}

// Floyd-warshall's APSP, initially g[i][j] is weight of path from i -> j
// for direct connections given; otherwise INF (0x3f3f3f3f ~ 1 bil).
// 'g' is represented as an adjacency matrix.  This algorithm is generally
// useable as long as number of vertices, V <= 400 (approx.)
void floydWarshall_APSP(vector<vi> &g, int V) {
   FR(k, V) FR(i, V) FR(j, V) g[i][j] = min(g[i][j], g[i][k] + g[k][j]);
}

// Least Common Multiple - O(log n), where n = max(a, b) (as in gcd)
int LCM(int a, int b) {
   return a * (b / __gcd(a, b));
}

// Longest Common Subsequence
const int MM = 20;
int lcsAndPath(int A[MM], int a, int B[MM], int b, int ans[MM]) {
   int L[MM + 1][MM + 1];
   for (int i = a; i >= 0; i--)
      for (int j = b; j >= 0; j--)
     if (i == a || j == b)
        L[i][j] = 0;
     else if (A[i] == B[i])
        L[i][j] = 1 + L[i + 1][j + 1];
     else
        L[i][j] = max(L[i + 1][j], L[i][j + 1]);

   int i = 0, j = 0, k = 0;
   while (i < a && j < b) {
      if (A[i] == B[j])
     ans[k++] = A[i], i++, j++;
      else if (L[i + 1][j] > L[i][j + 1])
     i++;
      else if (L[i + 1][j] < L[i][j + 1])
     j++;
      else
     j++; // tiebreaker
   }
   return L[0][0]; // len, ans has actual values as the path
}
```

```
// LIS - O(n log(n))
vi LIS(vi &A) {
   int n = A.size(), len = 1;
   // Uncomment to find for descending subsequence
   // reverse(A.begin(), A.end());
   vi last(n + 1), pos(n + 1), pred(n);
   if (n == 0) return vi();
   last[1] = A[pos[1] = 0];
   for (int i = 1; i < n; i++) {
      int j = upper_bound(begin(last) + 1, begin(last) + len + 1, A[i]) -
     last.begin();
      // Uncomment (and comment above line) for STRICTLY asc (desc)
      // int j = lower_bound(begin(last) + 1, begin(last) + len + 1, A[i]) -
      //    last.begin();
      pred[i] = (j - 1 > 0) ? pos[j - 1] : -1;
      last[j] = A[pos[j] = i];
      len = max(len, j);
   }
   // Uncomment if only need length, and end here
   // return len;
   int start = pos[len];
   vi S(len);
   for (int i = len - 1; i >= 0; i--) {
      S[i] = A[start]; start = pred[start];
   }
   return S;
}

// LIS - O(n^2)
// TODO: TEST THIS FUNCTION (may need n+1 size, etc.)
// ---> Ascending
void LIS_n2_asc(int n) {
   vi asc(n, 0);
   for (int i = n - 1; i >= 0; i--) {
      asc[i] = 1;
      for (int j = i + 1; j < n; j++)
     if (A[i] < A[j]) asc[i] = max(asc[i], asc[j] + 1);
   }
   return asc[0];
}
// <--- Descending
void LIS_n2_desc(int n) {
   vi desc(n, 0);
   for (int i = n - 1; i >= 0; i--) {
      desc[i] = 1;
      for (int j = i + 1; j < n; j++)
     if (A[i] > A[j]) desc[i] = max(desc[i], desc[j] + 1);
   }
   return desc[0];
}
```

```
// Maximal Matching (Hopcroft-Karp)
// https://en.wikipedia.org/wiki/Hopcroft–Karp_algorithm
// time complexity: O(E * sqrt(V))
class MaxMatching {
public:
   static tuple<int, vi> max_matching(const vector<vi> &g) {
      int m = 0, n = g.size();

      for (auto &gg : g) for (int u : gg) m = max(m, u + 1);
      vi A(m, -1), D(n), used(n);

      for (int i = 0, f = 0;; i += f, f = 0) {
        vi vis(n);
        bfs(g, used, A, D);
        FR(u, n) if (!used[u] && dfs(g, vis, used, A, D, u)) f++;
        if (!f) return make_tuple(i, A);
      }
   }

   static void bfs(const vector<vi> &g, vi &used, vi &A, vi &D) {
      int n = g.size(), q = 0;
      fill(begin(D), end(D), -1);
      vi Q(n);
      FR(u, n) if (!used[u]) Q[q++] = u, D[u] = 0;
      FR(i, q) {
     int u = Q[i];
     for (int v : g[u]) {
        int w = A[v];
        if (w >= 0 && D[w] < 0) D[w] = D[u] + 1, Q[q++] = w;
     }
      }
   }

   static bool dfs(const vector<vi> &g,
          vi &vis, vi &used,
          vi &match, vi &D, int u) {
      vis[u] = 1;
      for (int v : g[u]) {
     int w = match[v];
     if (w < 0 || (!vis[w] && D[w] == D[u] + 1 &&
              dfs(g, vis, used, match, D, w))) {
        match[v] = u;
        used[u] = true;
        return true;
     }
      }
      return false;
   }
};


// Modular Exponentiation
// Compute x^n mod m
int modexp(int x, int n, int m) {
   if (n == 0) return 1;
   if (n & 1) return ((x % m) * modexp(x, n - 1, m)) % m;
   int y = modexp(x, n / 2, m);
   return (y * y) % m;
}
```

```
// Palindrome
// Check if a string is a palindrome
bool palindrome(string s) {
   return equal(s.begin(), next(s.begin(), s.size() / 2), s.rbegin());
}

// RMQ - O(n log(n)), O(1) lookup
class RMQ {
public:
   vector<int> A; vector< vector<int> > M;
   RMQ(const vector<int> &B) {
      A = B; int n = A.size();
      int m = 31 - __builtin_clz(n) + 1;
      M.assign(m, vector<int>(n));
      FR(j, n) M[0][j] = j;
      for (int i = 1; (1 << i) <= n; i++) {
     for (int j = 0; (j + (1 << i)) <= n; j++) {
        M[i][j] = (A[M[i - 1][j]] <=
              A[M[i - 1][j + (1 << (i - 1))]])
           ? M[i - 1][j]
           : M[i - 1][j + (1 << (i - 1))];
     }
      }
   }
   int query(int L, int R) {
      int k = 31 - __builtin_clz(R - L + 1);
      return (A[M[k][L]] <= A[M[k][R - (1 << k) + 1]])
     ? M[k][L]
     : M[k][R - (1 << k) + 1];
   }
};

// Sieve + optimized prime testing
ll sz; bitset<10000010> p; vi primes;
void sieve(ll m) {
   sz = m + 1;
   p.set(); p[0] = p[1] = 0;
   for (ll i = 2; i <= sz; i++)
      if (p[i]) {
     for (ll j = i * i; j <= sz; j += i) p[j] = 0;
     primes.push_back((int)i);
      }
}
bool isPrime(ll x) {
   if (x <= sz) return p[x];
   for (int i = 0; i < (int)primes.size(); i++)
      if (x % primes[i] == 0) return false;
   return true;
}
```

```
// Union Find (Disjoint Set)
class UF {
public:
   vi p, r;
   UF(int n) {
      p.assign(n, 0); iota(begin(p), end(p), 0);
      r.assign(n, 0);
   }
   int find(int n) { return (p[i] == i ? i : (p[i] = find(p[i]))); }
   bool same(int i, int j) { return find(i) == find(j); }
   void merge(int i, int j) {
      if (!same(i, j)) {
     int x = find(i), y = find(j);
     if (r[x] > r[y]) p[y] = x;
     else {
        p[x] = y;
        if (r[x] == r[y]) r[y]++;
     }
      }
   }
};

int main() {

   ios_base::sync_with_stdio(false);
   cin.tie(NULL);




}
```