Memorandum

To: Dr. Tom Bensky

From: Colin Barrett, Kristen Guaderrama, Matthew Hui, Ilan Izakov

Date: October 22, 2019

Re: Brinell Hardness by Strain and Temperature Design and Analysis

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The purpose of this memo is to describe the findings from the analyses we ran, and the statistical methods used to find them.

We hope that the information provided within helps inform you the effects of temperature and strain on Brinell hardness of carbon steel coils while answering your research question:

> *"In carbon steel coils, what combination of temperature and strain will produce coils with a Brinell hardness between 340 and 360?"*

This memo is organized into five sections: Page

I. Abstract of Key Findings

When comparing temperature and strain and how it affects Brinell hardness, we were able to fit a model which found significant effects for both explanatory variables. We found a quadratic non-linear relationship with temperature (Figure 1) and a linear relationship through strain (Figure 2). In order to fit our data and optimize the response of getting a Brinell hardness of 350, we used the response surface optimizer in Minitab. The optimizer produced this model equation:

$$\text{Predicted Brinell Hardness} = 5538 - 9.359(\text{Temperature }^\circ\text{C}) + 2.236\,(\text{Strain }\%) + 0.004109(\text{Temperature }^\circ\text{C})^2$$

It also gave us suggested settings of temperature of 1150°C and strain of approximately 63.12% to get the predicted Brinell Hardness of 350. However, we won't be getting that score every time we put it at the setting it is just the predicted value of the model that was created using the data collected. Using these settings, we are 95% confident that the average Brinell hardness for carbon steel coils running through the hot strip mill will fall between 340.91 HB and 359.09 HB. For a single carbon steel coil that goes through these specifications in the hot mill strip, we are 95% confident that we will have a Brinell hardness between 319.66 HB and 380.33 Hb. We provided other options and our reasoning in the results and discussion section on page 7 if you would like to refer to that as well.

Figure 1. (Quadratic) Distribution of Brinell Hardness by Temperature

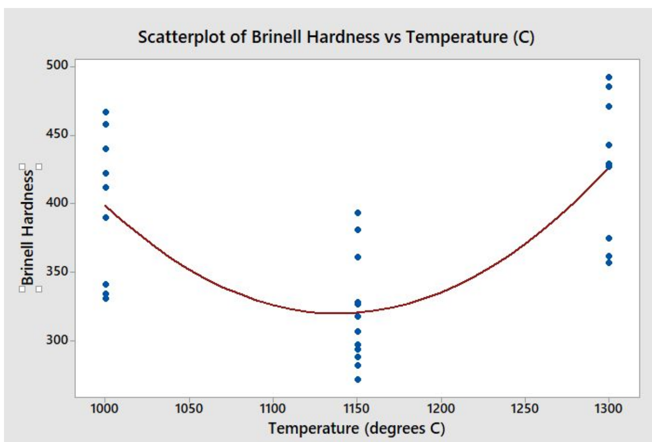
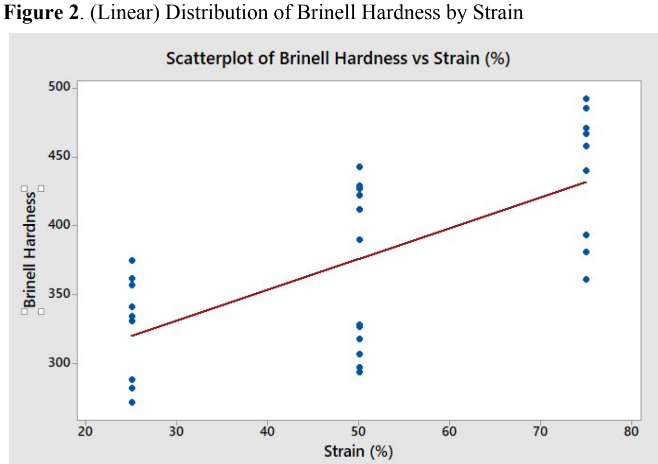


Figure 2. (Linear) Distribution of Brinell Hardness by Strain



II. Background and Data

After our initial consulting session, our team came to an understanding that you were seeking assistance on a research project for California Steel Industries from Fontana CA, and that there were two primary tasks for us in this project. The first task was to **create an experimental design**, in order for your company to collect the data necessary in answering your question. The second task was to analyze this data, in order to **find the optimal conditions for both temperature and strain, in order to produce a coil with a Brinell hardness score of 350.** We understand these coils are made to be used in industrial applications such as heavy machinery. The variables that were provided to us for analysis are described below.

Variables:

Temperature: 9 reps at 1000 °C, 12 reps at 1150 °C, and 9 reps at 1300 °C

Strain: 9 reps at 25%, 12 reps at 50%, and 9 reps at 75%

Brinell Hardness (response): Brinell hardness ranged from 272 to 467 BH. Descriptive statistics for brinell hardness are available in table 1 and table 2 below.

Table 1. Numerical summaries of Brinell hardness by temperature

Variable	Temperature (°C)	N	Mean	StDev	Minimum	Median	Maximum
Brinell Hardness	1000	9	399.4	53.4	331.0	412.0	467.0
	1150	12	320.7	39.3	272.0	312.5	393.0
	1300	9	426.8	52.0	357.0	429.0	492.0

Table 2. Numerical summaries of Brinell hardness by strain

Variable	Strain (%)	N	Mean	StDev	Minimum	Median	Maximum
Brinell Hardness	25	9	326.9	37.5	272.0	334.0	375.0
	50	12	366.2	58.9	294.0	359.0	443.0
	75	9	438.7	48.3	361.0	458.0	492.0

When our team was considering which statistical methods we should recommend to you, we considered a few different things. Mainly, we wanted to create a design that would allow us to come up with an optimization of temperature and strain that was most likely to create a steel coil with a Brinell hardness score of 350. These methods are described in greater detail in the next section of this memo.

Next we would like to document our understanding of how the data was collected. After our initial meeting, we provided an experimental design for a sample of 30 carbon steel coils at three temperature levels (1000 °C, 1150 °C, and 1300 °C) and three strain levels (25%, 50%, and 75%). These coils were run in ascending order by temperature, with strain level randomly assigned (within temperature blocks) to each coil. Brinell hardness was measured after the coils were run according to the treatment specifications then allowed time to cool. The random assignment of strain level serves to ensure that any differences we may find in Brinell hardness are not attributed to minor, unintentional changes in temperature level throughout testing.

Lastly, we would like to discuss our recommendations on the generalizations that can be made from the analyses ran. <u>We recommend only generalizing conclusions to steel coils composed of the same chemical makeup, constructed by similar hot strip mills to the one used in this experiment.</u> We recommend this because the experiment that was run used a specific type of steel, with particular characteristics. The experiment also used the same hot strip mill for all runs, so the results only apply to strips produced in hot strip mills similar to this one.

III. Statistical Methods

The experimental design strategy we recommended to you for this experiment is known as a **response surface design,** more specifically a **central composite design**. The goal of a response surface design is to allow a full investigation of any potential effects of explanatory variables on the response and to make optimization predictions based on the effects identified. Our hope is that using this design will mean that even though we only have 30 steel coils and are only setting them at a few different different temperature and strain settings, we will still have a good chance at finding a good combination of settings to achieve the ideal Brinell hardness.

This particular response surface design was also designed to be a central composite design, meaning that in addition to the linear effects it will also investigate **second order effects.** This means our model will consider effects that are both linear and nonlinear in nature, and that we can investigate potential **interactions** between the explanatory variables. An interaction in our context would mean that the effect of temperature on hardness would depend on the strain, so perhaps a change in temperature might have a larger effect at a high strain than it would at a low strain. To summarize, the goal of using this model is to investigate all potential linear and quadratic effects of strain or temperature on hardness, as well as investigating whether there is an interaction (as defined above) between strain and hardness. Though all of these potential effects are explored, those that are not found to be significant are then excluded from the final model.

So the final model takes the form of a **regression model**, in which we use our **explanatory variables x1,x2** (hardness and temperature) to predict a **continuous response variable ŷ** (predicted Brinell Hardness). The general form of the regression model we are using is:

$$\widehat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 (x_1)^2 + \beta_4 (x_2)^2 + \beta_4 x_1 x_2$$

Where in our case:

$\widehat{y}$ = predicted Brinell Hardness

x_1 = temperature
x_2 = strain
β_i = Coefficients for each effect

That is the generalized and unreduced form, but in the results section below you will find the actual final model equation including the coefficients that were calculated from the data.

After we had a final model we were happy with, we then used the **response surface optimizer** tool in Minitab to identify combinations of temperature and strain that would most likely achieve the target hardness of 350.

IV. Results and Discussion

Model Results

When fitting the data to the full model laid out in the “Statistical Methods” section, the interaction between temperature and strain, and also the squared strain term were not found to be statistically significant; the variation in Brinell hardness associated with these terms could easily be attributed to chance alone. As a result, we chose to use the following simpler model including only the effects found to have a significant impact on Brinell Hardness.:

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 (x_1)^2$$

The final model with calculated coefficients (model created in Minitab 18) is as follows:

Table 3. Final Model

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
14.0797	95.92%	95.45%	94.67%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	5538	306	18.07	0.000
Temperature (degrees C)	-9.359	0.537	-17.43	0.000
Strain (%)	2.236	0.133	16.84	0.000
Temperature (degrees C)*Temperature (degrees C)	0.004109	0.000233	17.62	0.000

As an equation, this model can be written as:

Est. Brinell Hardness = 5538 - 9.359(Temperature °C) + 2.236 (Strain %) + 0.004109(Temperature °C)^2

The inclusion of the “(Temperature °C)^2 term indicates that there is a nonlinear relationship between temperature and Brinell hardness. In other words, the effect of temperature on Brinell hardness is not constant. The negative coefficient of the linear term combined with the positive coefficient of the squared term means at the lower range of our data, increasing temperature leads to decreased hardness, but at the higher end of the data, increasing temperature increases hardness.

So for example, at 1000 degrees, each additional 10 degrees of temperature is predicted to lower the Brinell hardness by 10.991. But at 1300 degrees, our model found each additional 10 degrees of temperature is predicted increased the hardness by 12.833. Again, this only describes the relationship between temperature and hardness in the range we tested, this quadratic relationship might not hold at higher or lower temperatures.

Meanwhile our other explanatory variable, strain, was found to have a simple linear relationship with Brinell hardness where each 1% increase in strain is predicted to increase hardness by 2.24 HB.

Optimization Results

Setting the desired value for Brinell hardness at 350 HB, we were able to find an optimal combination of temperature and strain levels likely to achieve this target. Results from this optimization indicated that a **temperature of 1150°C** and **strain of approximately 63.12%** is most likely to produce a carbon steel coil with a Brinell hardness of 350 HB. Furthermore, we can be 95% confident that the average Brinell hardness for carbon steel coils (such as those used in this experiment) run through the hot strip mill at this temperature and strain specification will fall between 340.91 HB and 359.09 HB. Additionally, as indicated by the prediction interval in Table 5, we can be 95% confident that the Brinell hardness of a single carbon steel coil at these specifications will have a Brinell hardness between 319.66 HB and 380.33 HB.

So 1150 degrees and 63.12% strain is a potential solution, but it is not the only potential combination that might be around the target hardness. One of the main reasons it is our main solution is because we had extra replication in the center of our experiment, where the temperature was held at 1150 degrees. This was by design in order to better understand the variance and make better predictions around the center of the model. However we can find alternate settings that might also be optimal by holding either strain or temperature at a particular value.

In your response to our email inquiry, you mentioned to us that if there were multiple options for optimizing hardness, there might be incentives to pick certain combinations. For example, holding the strain at a high percentage might enable faster processing, or holding the temperature at a lower value may reduce costs. So to address this, below we have included a table of alternative options that might optimize the result. If certain combinations of temperature and strain proved particularly appealing, future experiments could be designed to investigate around those values further. Solution 1 is our most informed prediction, solution 2 is another low variance suggestion, solution 3 provides a potential low temperature treatment, and solution 4 provides a potential high strain (high speed) treatment.

Figure 3. Surface Plot of Brinell Hardness vs Temperature (x-axis), Strain (y-axis)

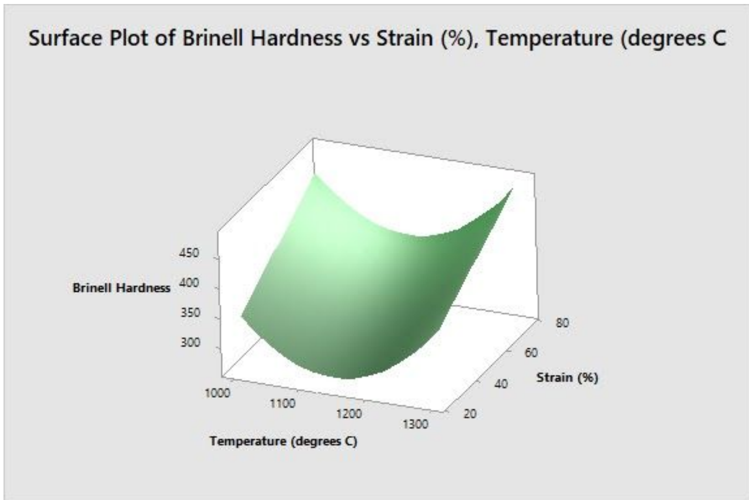


Table 4. Table of Potential Optimizations

Solution	Temperature (°C)	Strain (%)	Brinell Hardness Predicted
1	1150	63.12	350
2 (Alternate)	1075	55.84	350
3 (Low Temp)	1000	27.72	350
4 (High Strain)	1083.39	70.46	378.573

Table 5. Table of Inference Intervals for Potential Optimizations

Solution	Temperature (°C)	Strain (%)	95% Prediction Interval	95% Confidence Interval
1	1150	63.12	(319.66, 380.33)	(340.91, 359.09)
2 (Alternate)	1075	55.84	(320.10, 379.90)	(342.49, 357.51)
3 (Low Temp)	1000	27.72	(318.53, 380.74)	(338.23, 361.04)
4 (High Strain)	1083.39	70.46	(348.34, 409.16)	(369.42, 388.08)

Suggestions and Limitations

Lastly, we would like to offer a few suggestions for future experiments, and reiterate the scope to which we believe the results of the current experiment can be generalized.

Since we ran a response surface design with such a broad range of levels for each factor, it is hard to get a precise prediction of optimization. If greater accuracy is desired, we would recommend using the results provided from the first experiment to run another experiment with narrower levels of temperature and strain. This new experiment could be centered around one of the potential optimal points determined by this experiment. This will allow a more accurate understanding of the true effects of temperature and strain on hardness, along with a more precise prediction of an optimal setting.

Furthermore, running a larger experiment with a larger sample size would also help to get a better estimate as well as a better sense of the true nature of the effects.

We suggest these results only be generalized to carbon steel coils of similar characteristics to the ones used in this experiment, produced using a similar hot strip mill process as was used in our experiment. Furthermore, the results should only be generalized to steel coils produced by temperatures and strains within the range we investigated, as we do not have any information outside the ranges run in this experiment (ie. results cannot be generalized to temperatures much below 1000 or above 1300 degrees).

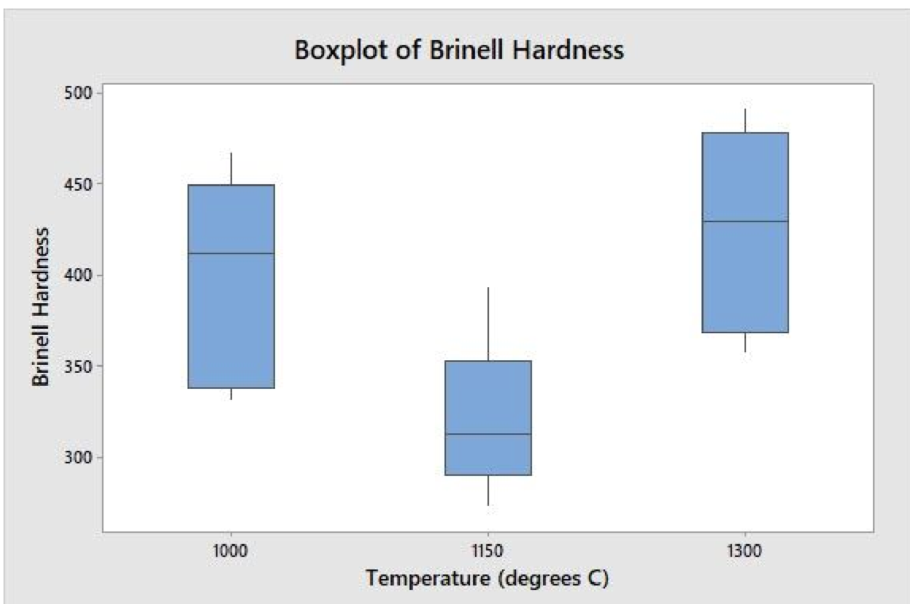
V. Technical Output

Descriptive Statistics: Brinell Hardness

Statistics

Variable	Temperature (degrees C)	N	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Brinell Hardness	1000	9	399.4	17.8	53.4	331.0	337.5	412.0	449.0
	1150	12	320.7	11.3	39.3	272.0	289.5	312.5	352.8
	1300	9	426.8	17.3	52.0	357.0	368.5	429.0	478.0

Variable	Temperature (degrees C)	Maximum
Brinell Hardness	1000	467.0
	1150	393.0
	1300	492.0

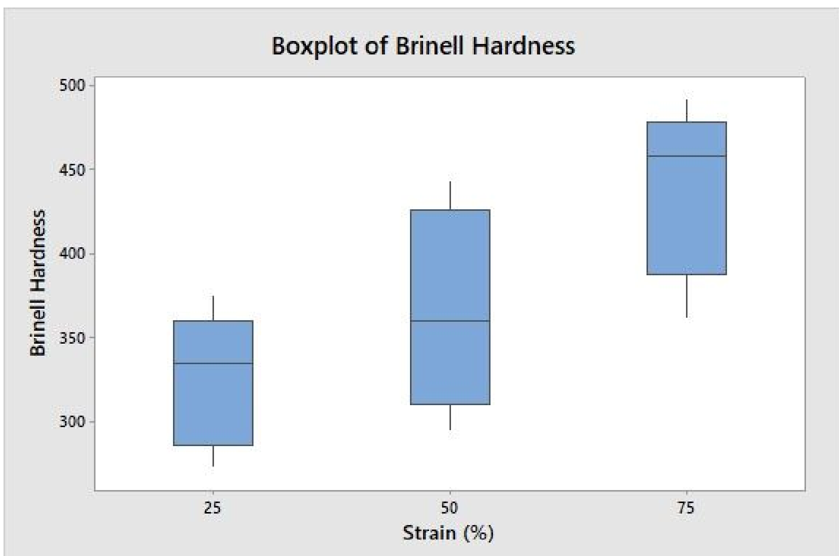


Descriptive Statistics: Brinell Hardness

Statistics

Variable	Strain (%)	N	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Brinell Hardness	25	9	326.9	12.5	37.5	272.0	285.0	334.0	359.5
	50	12	366.2	17.0	58.9	294.0	309.8	359.0	425.8
	75	9	438.7	16.1	48.3	361.0	387.0	458.0	478.0

Variable	Strain (%)	Maximum
Brinell Hardness	25	375.0
	50	443.0
	75	492.0



Model Including All Terms

Response Surface Regression: Brinell Hardness versus ... C), Strain (%)

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	5	121130	24226.0	113.09	0.000
Linear	2	59586	29793.1	139.07	0.000
Temperature (degrees C)	1	3362	3362.0	15.69	0.001
Strain (%)	1	56224	56224.2	262.45	0.000
Square	2	61542	30770.9	143.64	0.000
Temperature (degrees C)*Temperature (degrees C)	1	59555	59555.1	278.00	0.000
Strain (%)*Strain (%)	1	11	10.7	0.05	0.825
2-Way Interaction	1	2	2.1	0.01	0.922
Temperature (degrees C)*Strain (%)	1	2	2.1	0.01	0.922
Error	24	5141	214.2		
Lack-of-Fit	3	1877	625.7	4.03	0.021
Pure Error	21	3264	155.4		
Total	29	126271			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
14.6364	95.93%	95.08%	93.87%

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	320.05	5.05	63.38	0.000	
Temperature (degrees C)	13.67	3.45	3.96	0.001	1.00
Strain (%)	55.89	3.45	16.20	0.000	1.00
Temperature (degrees C)*Temperature (degrees C)	92.24	5.53	16.67	0.000	1.03
Strain (%)*Strain (%)	1.24	5.53	0.22	0.825	1.03
Temperature (degrees C)*Strain (%)	-0.42	4.23	-0.10	0.922	1.00

Regression Equation in Uncoded Units

Brinell Hardness = 5524 - 9.332 Temperature (degrees C) + 2.17 Strain (%)
+ 0.004099 Temperature (degrees C)*Temperature (degrees C)
+ 0.00198 Strain (%)*Strain (%)
- 0.00011 Temperature (degrees C)*Strain (%)

Model Including Only Significant Terms

Response Surface Regression: Brinell Hardness versus Temperature (degrees C), Strain (%)

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	3	121117	40372.4	203.65	0.000
Linear	2	59586	29793.1	150.29	0.000
Temperature (degrees C)	1	3362	3362.0	16.96	0.000
Strain (%)	1	56224	56224.2	283.62	0.000
Square	1	61531	61531.0	310.39	0.000
Temperature (degrees C)*Temperature (degrees C)	1	61531	61531.0	310.39	0.000
Error	26	5154	198.2		
Lack-of-Fit	5	1890	378.0	2.43	0.069
Pure Error	21	3264	155.4		
Total	29	126271			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
14.0797	95.92%	95.45%	94.67%

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	320.67	4.06	78.90	0.000	
Temperature (degrees C)	13.67	3.32	4.12	0.000	1.00
Strain (%)	55.89	3.32	16.84	0.000	1.00
Temperature (degrees C)*Temperature (degrees C)	92.44	5.25	17.62	0.000	1.00

Regression Equation in Uncoded Units

Brinell Hardness = 5538 - 9.359 Temperature (degrees C) + 2.236 Strain (%)
+ 0.004109 Temperature (degrees C)*Temperature (degrees C)

Response Optimization: Brinell Hardness

Parameters

Response	Goal	Lower	Target	Upper	Weight	Importance
Brinell Hardness	Target	272	350	492	1	1

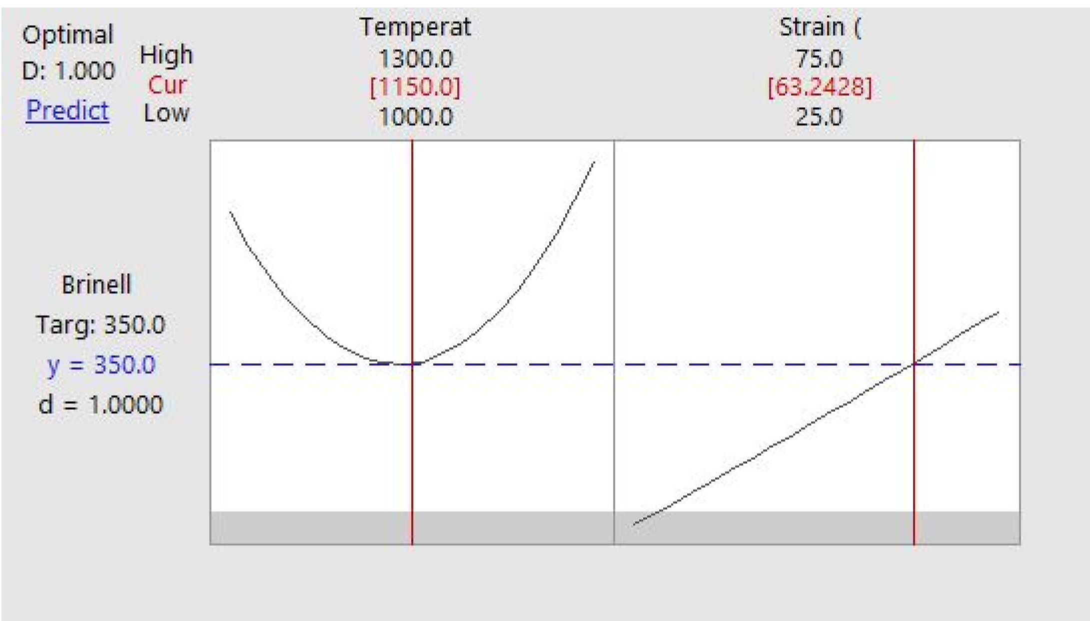
Solutions

Solution	Temperature (degrees C)	Strain (%)	Brinell Hardness Fit	Composite Desirability
1	1150.00	63.1213	350.000	1.00000
2	1283.37	25.0000	350.016	0.99989
3	1000.00	25.2776	344.176	0.92533
4	1083.39	70.4633	378.573	0.79878
5	1207.13	70.6434	385.433	0.75047

Multiple Response Prediction

Variable	Setting
Temperature (degrees C)	1150
Strain (%)	63.1213

Response	Fit	SE Fit	95% CI	95% PI
Brinell Hardness	350.00	4.42	(340.91, 359.09)	(319.66, 380.34)



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Brinell Hardness	Target	272	350	492	1	1

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2	1283.37	25.0000	350.016	0.99989
3	1000.00	25.2776	344.176	0.92533
4	1083.39	70.4633	378.573	0.79878
5	1207.13	70.6434	385.433	0.75047

Multiple Response Prediction

Variable	Setting
Temperature (degrees C)	1150
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Brinell Hardness	350.00	4.42	(340.91, 359.09)	(319.66, 380.34)

Response Optimization: Brinell Hardness

Parameters

Response	Goal	Lower	Target	Upper	Weight	Importance
Brinell Hardness	Target	272	350	492	1	1

Variable Ranges

Variable	Values
Temperature (degrees C)	(1000, 1150)
Strain (%)	(25, 75)

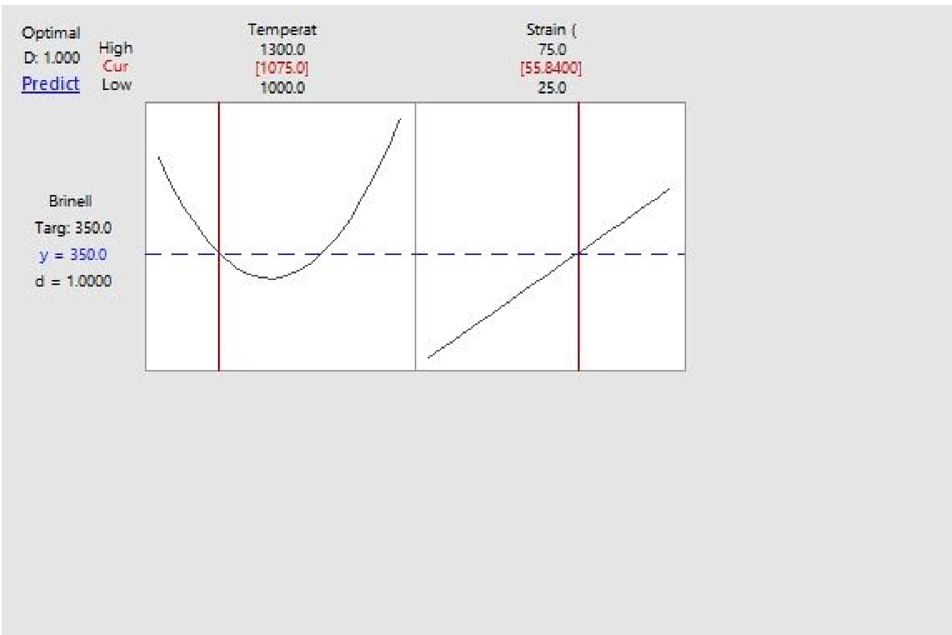
Solutions

Solution	Temperature (degrees C)	Strain (%)	Brinell Hardness Fit	Composite Desirability
1	1075.00	55.8400	350.000	1.00000
2	1150.00	63.1213	350.000	1.00000
3	1133.55	63.7915	351.111	0.99218
4	1000.00	25.2776	344.176	0.92533
5	1083.39	70.4633	378.573	0.79878

Multiple Response Prediction

Variable	Setting
Temperature (degrees C)	1075
Strain (%)	55.84

Response	Fit	SE Fit	95% CI	95% PI
Brinell Hardness	350.00	3.65	(342.49, 357.51)	(320.10, 379.90)



Regression Analysis: Brinell Hardness versus ... (degrees C), Strain (%)

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	121117	40372.4	203.65	0.000
Temperature (degrees C)	1	60248	60247.7	303.91	0.000
Strain (%)	1	56224	56224.2	283.62	0.000
Temperature (degrees C)*Temperature (degrees C)	1	61531	61531.0	310.39	0.000
Error	26	5154	198.2		
Lack-of-Fit	5	1890	378.0	2.43	0.069
Pure Error	21	3264	155.4		

Total	29	126271

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
14.0797	95.92%	95.45%	94.67%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	5538	306	18.07	0.000	
Temperature (degrees C)	-9.359	0.537	-17.43	0.000	588.78
Strain (%)	2.236	0.133	16.84	0.000	1.00
Temperature (degrees C)*Temperature (degrees C)	0.004109	0.000233	17.62	0.000	588.78

Regression Equation

Brinell Hardness = 5538 - 9.359 Temperature (degrees C) + 2.236 Strain (%)
+ 0.004109 Temperature (degrees C)*Temperature (degrees C)

Response Optimization: Brinell Hardness

Parameters

Response	Goal	Lower	Target	Upper	Weight	Importance
Brinell Hardness	Target	272	350	492	1	1

Variable Ranges

Variable	Values
Temperature (degrees C)	1000
Strain (%)	27.72

Solution

Solution	Temperature (degrees C)	Strain (%)	Brinell Hardness Fit	Composite Desirability
1	1000	27.72	349.636	0.995337

Multiple Response Prediction

Variable	Setting
Temperature (degrees C)	1000
Strain (%)	27.72

Response	Fit	SE Fit	95% CI	95% PI
Brinell Hardness	349.64	5.55	(338.23, 361.04)	(318.53, 380.74)

Response Optimization: Brinell Hardness

Parameters

Response	Goal	Lower	Target	Upper	Weight	Importance
Brinell Hardness	Target	272	350	492	1	1

Variable Ranges

Variable	Values
Temperature (degrees C)	1083
Strain (%)	70.4633

Solution

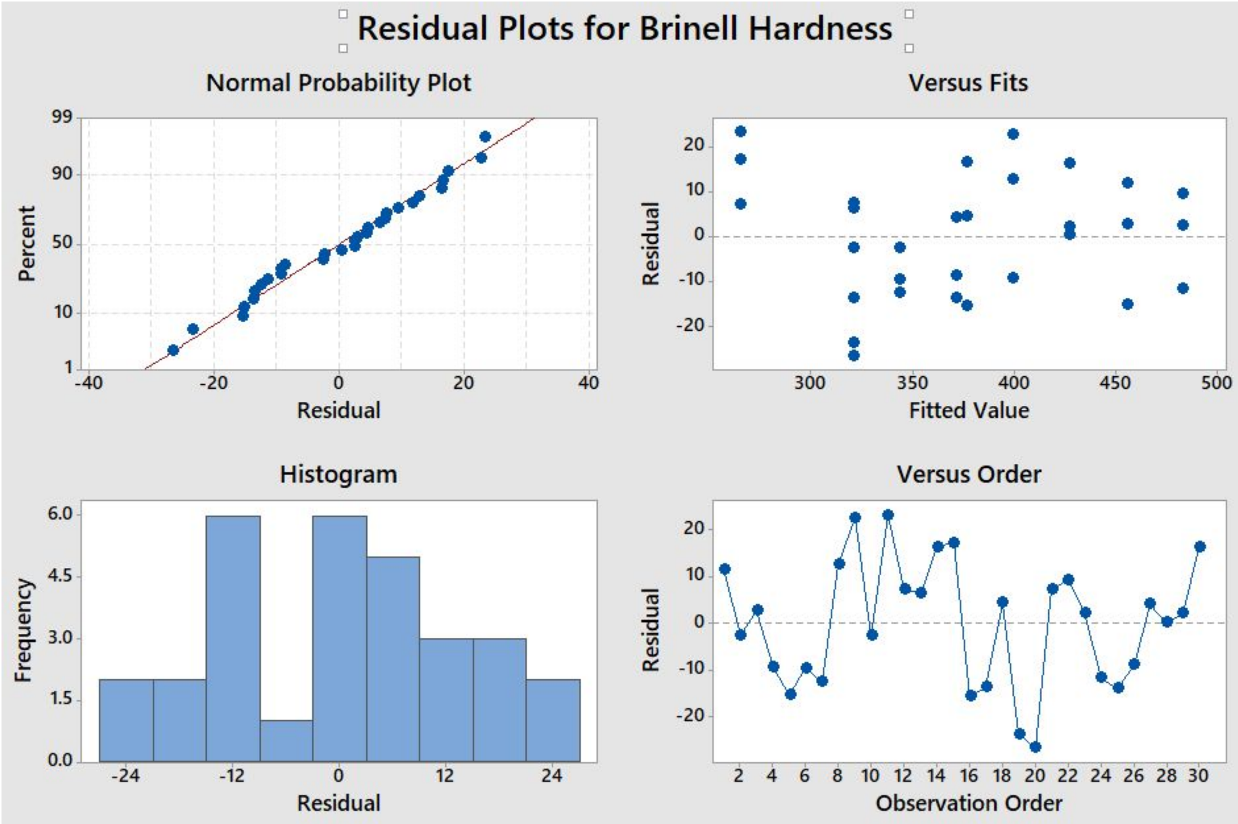
Solution	Temperature (degrees C)	Strain (%)	Brinell Hardness Fit	Composite Desirability
1	1083	70.4633	378.753	0.797516

Multiple Response Prediction

Variable	Setting
Temperature (degrees C)	1083
Strain (%)	70.4633

Response	Fit	SE Fit	95% CI	95% PI
Brinell Hardness	378.75	4.54	(369.42, 388.08)	(348.34, 409.16)

Assumptions Checking:


Residual Plots for Brinell Hardness
Normal Probability Plot
Percent
Residual
Versus Fits
Residual
Fitted Value
Histogram
Frequency
Residual
Versus Order
Residual
Observation Order