Monty Hall

Sebastian Liu

Problem

You are on a game show, and in front of you there are three doors: behind one door there is a car, and behind the other two there are goats. Before revealing what is behind the door that you selected, the game show host reveals that behind another door, there is a goat. The host then asks you if you would like to switch your selected door. Should you switch? Does switching increase your chances of winning the car?

An Initial Thought

An initial thought might be that switching makes no difference. Why? Because initially, your picking the winning door is $\frac{1}{3}$. When a goat door is opened, that has nothing to do with the chances of you picking the winning door or not. After all, how does opening a non-winning door increase your chances of winning or losing? Therefore, there is no need to switch, since not switching and switching both yield a winning chance of $\frac{1}{3}$, or 33%

Why Switching is Better

The easiest way to see that switching indeed does make a difference and increases your chances of winning is to enumerate the possible scenarios. Without loss of generality, suppose we call the doors **Door 1**, **Door 2** and **Door 3**. Further suppose that your choice is **Door 1**. That these assumptions are in fact without a loss of generality is left to the reader to justify.

1. Winning Door: **Door 1**
   (a) **No Switch: Win**
   (b) Switch: Lose
2. Winning Door: **Door 2**
   (a) No Switch: Lose
   (b) **Switch: Win**
3. Winning Door: **Door 3**
   (a) No Switch: Lose
   (b) **Switch: Win**

It follows that switching yields a success of $\frac{2}{3}$ or 66%. The reason behind this, however, is perhaps not entirely clear. But let's consider the case where the winning door is **Door 1** and you correctly guess **Door 1**. Initially, it might seem as though your guessing **Door 1** and the host opening **Door 2** is in fact a different scenario than your guessing **Door 1** and the host opening **Door 3**. After all, those two doors are different. However, these two events should be thought of as equivalent. All that is required in the problem is that the host open a non-winning door. With this in mind, the fact that switching in fact is better might seem more intuitive.