Tackling the SDC in AdS with CFTs

Florent Baume

arXiv:2011.03583 w/ J. Calderón (IFT Madrid)

String Pheno Seminars, 2021-03-23

FONDS NATIONAL SUISSE
SCHWEIZERISCHER NATIONALFONDS
FONDO NAZIONALE SVIZZERO
SWISS NATIONAL SCIENCE FOUNDATION

UNIVERSITY of PENNSYLVANIA

The Swamp

Learn Quantum Gravity from the IR and vice-versa

What properties EFTs must have to be consistently embedded in QG/ST?

The Swamp

Learn Quantum Gravity from the IR and vice-versa

What properties EFTs must have to be consistently embedded in QG/ST?

Swampland Programme:

- Weak Gravity Conjecture (WGC)
  [Arkani-Hamed, Motl, Nicolis, Vafa ’06]

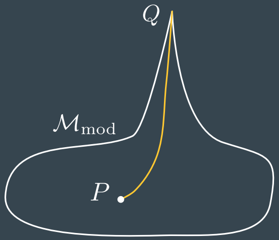
- Swampland Distance Conjecture (SDC)
  [Ooguri, Vafa ’06]

- . . .

The Swampland Distance Conjecture [Ooguri, Vafa '06]

$\exists$ infinite tower of massless states at infinite-distance points

$$\frac{m}{M_{\text{Pl}}} \sim e^{-\alpha_G \cdot \text{dist}_G} \qquad \text{as } \text{dist}_G \to \infty$$



$$ds^2 = G_{ij}\, d\phi^i d\phi^j$$

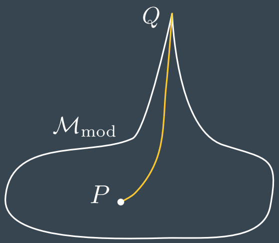
ϕ^i massless

The Swampland Distance Conjecture

[Ooguri, Vafa '06]

$\exists$ infinite tower of massless states at infinite-distance points

$$\frac{m}{M_{\text{Pl}}} \sim e^{-\alpha_G \cdot \text{dist}_G} \qquad \text{as } \text{dist}_G \to \infty$$



Well studied in flat space

Here: consider AdS spaces

$$ds^2 = G_{ij}\, d\phi^i d\phi^j$$

ϕ^i massless

Fields may be stabilised $\quad m_{\phi^k} \neq 0 \qquad \mathcal{M}_{\text{mod}}^{\text{AdS}} \subset \mathcal{M}_{\text{mod}}$

Two Possible Directions

AdS$_5$ for concreteness from now on

Apparent dichotomy:

- "Decompactification": change AdS scale $L \sim \mathrm{Vol}(X_5)$

$$ADC: \qquad m/M_{\mathrm{Pl}} \sim (LM_{\mathrm{Pl}})^{-\alpha} \sim e^{-\mathrm{dist}}$$

[Lüst, Palti, Vafa '19]

- "Equi-dimensional": AdS scale fixed

$$\frac{m}{M_{\mathrm{Pl}}} \sim e^{-\mathrm{dist}} \quad ?$$

What happens at infinite-distance points when varying the moduli (after stabilisation)?

AdS/CFT

[Maldacena '97]

Idea: study the SDC via AdS/CFT correspondence

AdS$_5$	CFT$_4$
$\Psi(X) \in \{\phi(X)\,, \psi(X), A_M(X)\ \ldots\}$	$\mathcal{O}(x)\,, \partial_\mu(x)\,, \partial_\mu\partial_\nu\mathcal{O}(x)\ \ldots$

$$\phi(X)\big|_{z=\infty} = \mathcal{O}(x) \qquad X = (x, z)$$

Dictionary:

$$m^2L^2 = \Delta(\Delta - 4)\,, \qquad \ell = 0$$

$$m^2L^2 = (\Delta + \ell - 2)(\Delta - \ell - 2)\,, \qquad \ell > 0$$

Holography and the Swampland

- No Global Symmetries

  [Harlow, Ooguri '18]

- Weak Gravity Conjecture

  [Montero '18]

  [Aalsma, Cole , Loges, Shiu '20]

  See Cole's talk

- The Swampland and positivity

  [Conlon, Revello '20]

  See Revello's talk

- SDC, see also:

  [Perlmutter, Rastelli, Vafa, Valenzuela '20]

Conformal dimension bounded by unitarity

$$\Delta \geq 1\,, \qquad \ell = 0$$

$$\Delta \geq \ell + 2\,, \qquad \ell > 0$$

$$[D, \mathcal{O}] = \Delta \mathcal{O}$$

Saturated by free fields or currents:

ℓ	0	1	2	> 2 (HS)
Δ	1	3	4	$\ell + 2$
Field	free ϕ	J_μ	$T_{\mu\nu}$	$J_{\mu_1 \ldots \mu_\ell} \sim \bar{\phi}\, \partial_{\mu_1} \ldots \partial_{\mu_\ell} \phi$

Conformal dimension bounded by unitarity

$$\Delta \geq 1\,, \qquad \ell = 0$$
$$\Delta \geq \ell + 2\,, \qquad \ell > 0$$

$$[D, \mathcal{O}] = \Delta \mathcal{O}$$

Saturated by free fields or currents:

ℓ	0	1	2	> 2 (HS)
Δ	1	3	4	$\ell + 2$
Field	free ϕ	J_μ	$T_{\mu\nu}$	$J_{\mu_1 \ldots \mu_\ell} \sim \bar{\phi}\, \partial_{\mu_1} \ldots \partial_{\mu_\ell} \phi$

Maldacena–Zhiboedov Theorem '11

$\exists$ HS $\quad \partial^{\mu_i} J_{\mu_1 \ldots \mu_i \ldots \mu_\ell} = 0 \quad \Leftrightarrow \quad$ (partially) free theory

One current $\Rightarrow$ Tower of currents.

Back to AdS/CFT

$$m^2 L^2 = \Delta(\Delta - 4)\,, \qquad\qquad \ell = 0$$

$$m_\phi = 0 \quad \text{(moduli)} \qquad \Leftrightarrow \qquad \Delta_{\mathcal{O}} = 4 \quad \text{(marginal operator)}$$

$$(\mathcal{M}_{\text{mod}}, G_{ij}(\phi)) \quad \Leftrightarrow \quad (\mathcal{M}_{\text{CFT}}, \chi_{ij}(\lambda))$$

Back to AdS/CFT

$$m^2 L^2 = \Delta(\Delta - 4)\,, \qquad \qquad \ell = 0$$

$$m_\phi = 0 \quad \text{(moduli)} \qquad \Leftrightarrow \qquad \Delta_{\mathcal{O}} = 4 \quad \text{(marginal operator)}$$

$$(\mathcal{M}_{\text{mod}}, G_{ij}(\phi)) \quad \Leftrightarrow \quad (\mathcal{M}_{\text{CFT}}, \chi_{ij}(\lambda))$$

$\mathcal{M}_{\text{CFT}}$: conformal manifold, deformations leaving theory conformal

$$\mathcal{L} \to \mathcal{L} + \lambda^i \mathcal{O}_i \qquad \qquad \beta^i = 0 \qquad \Delta_{\mathcal{O}_i} = 4$$

Endowed with Zamolodchikov metric

$$\left\langle \mathcal{O}_i(x) \mathcal{O}_j^\dagger(y) \right\rangle = \frac{\chi_{ij}(\lambda)}{|x-y|^8}$$

Summary

The moduli space maps to the conformal manifold

$$(\mathcal{M}_{\text{mod}}, G_{ij}(\phi)) \longleftrightarrow (\mathcal{M}_{\text{CFT}}, \chi_{ij}(\lambda))$$

$$(LM_{\text{Pl}})^3 \, G_{ij}(\phi) \sim \chi_{ij}(\lambda)$$

Reformulation

SDC in $\mathcal{M}_{\text{mod}}$ $\Leftrightarrow$ DC in $\mathcal{M}_{\text{CFT}}$

HS conserved currents $\Leftrightarrow$ free theory

Simplest Example: AdS$_5 \times S^5 \leftrightarrow \mathcal{N} = 4$ SYM

Gravity side: type IIB on S^5 with N units of F_5-flux

$$L^4 = R^4 = g_s N M_s^{-4} \qquad\qquad L M_{\mathrm{Pl}} \sim N^{2/3} = \text{const}$$

Simplest Example: AdS$_5 \times S^5 \leftrightarrow \mathcal{N} = 4$ SYM

Gravity side: type IIB on S^5 with N units of F_5-flux

$$L^4 = R^4 = g_s N M_s^{-4} \qquad\qquad L M_{\mathsf{Pl}} \sim N^{2/3} = \text{const}$$

One modulus: $\tau = C_0 + \frac{i}{g_s}$

$$G_{\tau\bar{\tau}} = \frac{1}{\mathrm{Im}(\tau)^2} \qquad\qquad \mathrm{dist}(P,Q) = \int_P^Q ds \sim \log \mathrm{Im}\tau$$

Infinite-distance points: $\tau = 0\,, i\infty$

Simplest Example: AdS$_5 \times S^5 \leftrightarrow \mathcal{N} = 4$ SYM

CFT side: $\mathcal{N} = 4$ super-Yang–Mills with $\mathcal{G} = SU(N)$

$$\Phi = \{\phi\,,\Psi\,,A_\mu\} \qquad\qquad \mathcal{O} = \mathrm{Tr}_{\mathcal{G}}(\phi^n)$$

$$\mathcal{L} = \int d^4x \left(\frac{1}{g^2_{\mathrm{YM}}}F^2 + i\theta F\wedge F + \cdots\right)$$

Simplest Example: AdS$_5 \times S^5 \leftrightarrow \mathcal{N}=4$ SYM

CFT side: $\mathcal{N}=4$ super-Yang–Mills with $\mathcal{G}=SU(N)$

$$\Phi = \{\phi\,,\Psi\,,A_\mu\} \qquad\qquad \mathcal{O} = \mathrm{Tr}_{\mathcal{G}}(\phi^n)$$

$$\mathcal{L} = \int d^4x \left(\frac{1}{g^2_{\mathrm{YM}}} F^2 + i\theta F\wedge F + \cdots\right)$$

Only one free parameter:

$$\tau_{\mathrm{YM}} = \frac{\theta}{2\pi} + \frac{i}{g^2_{\mathrm{YM}}} = \tau_{IIB}$$

Simplest Example: AdS$_5 \times S^5 \leftrightarrow \mathcal{N}=4$ SYM

CFT side: $\mathcal{N}=4$ super-Yang–Mills with $\mathcal{G}=SU(N)$

$$\Phi = \{\phi\,, \Psi\,, A_\mu\} \qquad\qquad \mathcal{O} = \mathrm{Tr}_{\mathcal{G}}(\phi^n)$$

$$\mathcal{L} = \int d^4x \left(\frac{1}{g^2_{\mathrm{YM}}} F^2 + i\theta F \wedge F + \cdots \right)$$

Metric protected against corrections:

$$\chi_{\tau\bar\tau} = (N^2-1)\,\frac{1}{\mathrm{Im}(\tau)^2}$$

As expected

$$G_{\tau\bar\tau} \propto \chi_{\tau\bar\tau}$$

A Tower of Higher-Spin Currents

CFT side, infinite point at $g_{\text{YM}} \to 0 \quad \Rightarrow \quad$ perturbation!

$$\Delta \sim \Delta_{\text{free}} + \eta \, g_{\text{YM}}^{\beta} \qquad\qquad \forall \, \mathcal{O} \, , \quad \Delta_{\text{free}} = [\mathcal{O}]$$

A Tower of Higher-Spin Currents

CFT side, infinite point at $g_{\text{YM}} \to 0 \quad \Rightarrow \quad$ perturbation!

$$\Delta \sim \Delta_{\text{free}} + \eta \, g_{\text{YM}}^{\beta} \qquad\qquad \forall\, \mathcal{O}\,, \quad \Delta_{\text{free}} = [\mathcal{O}]$$

Using $\chi \sim \text{Im}(\tau)^{-2}$

$$\Delta \sim \Delta_{\text{free}} + \eta \, e^{-\alpha_\chi \text{dist}_\chi}$$

A Tower of Higher-Spin Currents

CFT side, infinite point at $g_{\text{YM}} \to 0 \quad \Rightarrow \quad$ perturbation!

$$\Delta \sim \Delta_{\text{free}} + \eta \, g_{\text{YM}}^{\beta} \qquad \forall \, \mathcal{O} \, , \quad \Delta_{\text{free}} = [\mathcal{O}]$$

Using $\chi \sim \text{Im}(\tau)^{-2}$

$$\Delta \sim \Delta_{\text{free}} + \eta \, e^{-\alpha_\chi \text{dist}_\chi}$$

Back to gravity side

$$m^2 L^2 = (\Delta + \ell - 2)(\Delta - \ell - 2) \sim \; e^{-\alpha_G \text{dist}_G} \qquad J^{\mu_1 \dots \mu_\ell} = \bar{\phi} \partial^\ell \phi$$

Summary $\quad \text{AdS}_5 \times S^5 \quad \leftrightarrow \quad \mathcal{N} = 4$ SYM

- Tower of HS fields

$$m_\ell^2 \sim e^{-\alpha_G \text{dist}_G}$$

- No scalar tower ($\mathcal{O} = \text{Tr}\varphi^k$)

$$m^2 L^2 = \Delta(\Delta - 4) \sim k^2 + e^{-\alpha_G \text{dist}_G}$$

- stringy origin: tensionless F1 ($g_s = 0$)

- $\alpha_G \sim \left(\frac{c_T}{\text{dim}\mathcal{G}}\right)^{1/2} = \mathcal{O}(1)$

- SDC: $\checkmark$

  $\longrightarrow$ Follows from Maldacena–Zhiboedov!

Generalisation

- $\mathcal{N} = 4$ very constrained
- What about $\mathcal{N} < 4$? Next-to-simplest cases: $\mathcal{N} = 2$

$\mathcal{O}$ marginal iif:

- $\Delta = 4$
- Preserves $\mathcal{N} = 2$:
  - Annihilated by $Q\,,\bar{Q}$
  - $U(2)_R$ singlet

Generalisation

- $\mathcal{N} = 4$ very constrained
- What about $\mathcal{N} < 4$? Next-to-simplest cases: $\mathcal{N} = 2$

$\mathcal{O}$ marginal iif:

- $\Delta = 4$
- Preserves $\mathcal{N} = 2$:
  - Annihilated by $Q\,,\bar{Q}$
  - $U(2)_R$ singlet

Superconformal representation, unique candidate:

$$\Delta\mathcal{L} = \int d^4\theta\,\mathcal{E}_2 + c.c. \;\sim\; (F^2 + F\wedge F)$$

[Dolan–Osborn '03]

Infinite-distance points

Richer network of infinite-distance points, but same behaviour:

$$\mathcal{G} = \mathcal{G}_1 \times \cdots \times \mathcal{G}_n \qquad\qquad n = \mathsf{dim}(\mathcal{M}_{\mathsf{CFT}})$$

Operator dimensions:

$$\Delta = \Delta_{\mathsf{free}} + \gamma(\tau_1, \ldots, \tau_n)$$

Infinite-distance points

Richer network of infinite-distance points, but same behaviour:

$$\mathcal{G} = \mathcal{G}_1 \times \cdots \times \mathcal{G}_n \qquad\qquad n = \mathsf{dim}(\mathcal{M}_{\mathsf{CFT}})$$

Operator dimensions:

$$\Delta = \Delta_{\mathsf{free}} + \gamma(\tau_1, \ldots, \tau_n)$$

Generically not all operators become free:

$$\gamma(\tau_1, \ldots, i\infty, \tau_n) \neq 0$$

Infinite-distance points

Richer network of infinite-distance points, but same behaviour:

$$\mathcal{G} = \mathcal{G}_1 \times \cdots \times \mathcal{G}_n \qquad\qquad n = \text{dim}(\mathcal{M}_{\text{CFT}})$$

Operator dimensions:

$$\Delta = \Delta_{\text{free}} + \gamma(\tau_1, \ldots, \tau_n)$$

Vector multiplet sector:

$$J_i^{\mu_1 \ldots \mu_\ell} = \bar{\phi}_i \partial^\ell \phi_i \qquad\qquad \phi_i \in \{\phi_i, A_i^\mu\}$$

$$\chi = \frac{1}{\text{Im}(\tau_i)^2} + \mathcal{O}(\text{Im}\tau_i^{-3}) \qquad \text{near } \tau_i = i\infty$$

$$\gamma \sim e^{-\alpha_\chi \, \text{dist}_\chi} \quad \Rightarrow \quad m_\ell \sim e^{-\alpha_G \, \text{dist}_G}$$

Infinite-distance points

Richer network of infinite-distance points, but same behaviour:

$$\mathcal{G} = \mathcal{G}_1 \times \cdots \times \mathcal{G}_n \qquad\qquad n = \text{dim}(\mathcal{M}_{\text{CFT}})$$

Operator dimensions:

$$\Delta = \Delta_{\text{free}} + \gamma(\tau_1, \ldots, \tau_n)$$

Vector multiplet sector:

$$J_i^{\mu_1 \ldots \mu_\ell} = \bar{\phi}_i \partial^\ell \phi_i \qquad\qquad \phi_i \in \{\phi_i, A_i^\mu\}$$

$$\chi = \frac{1}{\text{Im}(\tau_i)^2} + \mathcal{O}(\text{Im}\tau_i^{-3}) \qquad \text{near } \tau_i = i\infty$$

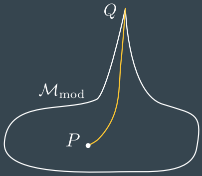
$$\gamma \sim e^{-\alpha_\chi \, \text{dist}_\chi} \quad \Rightarrow \quad m_\ell \sim e^{-\alpha_G \, \text{dist}_G}$$

$\longrightarrow$ tower of massless HS fields in the bulk.

Is it exhaustive?

For $\mathcal{N} = 2$ CFT$_4$:

(tower of HS fields $\Leftrightarrow$ free) $\Rightarrow$ dist $= \infty$



Are there cases infinite-distance points that are not free?

Beyond free points

No general argument. Relevant candidates:

Beyond free points

No general argument. Relevant candidates:

- Flavour enhancement (new current J_F^μ)

  representation theory: $J_F^\mu \in \left\{ J_F^\mu \, , \, J^{\mu_1 \ldots \mu_\ell} \right\}$

Beyond free points

No general argument. Relevant candidates:

- Flavour enhancement (new current J_F^μ)

  representation theory: $J_F^\mu \in \left\{ J_F^\mu \,, J^{\mu_1 \dots \mu_\ell} \right\}$

  $\longrightarrow$ free points

Beyond free points

No general argument. Relevant candidates:

- Flavour enhancement (new current J_F^μ)

  representation theory: $J_F^\mu \in \{ J_F^\mu \, , J^{\mu_1 \ldots \mu_\ell} \}$

  $\longrightarrow$ free points

- Tower of scalars?

  new $\Delta_{\mathcal{O}} = 4 \quad \Rightarrow \quad$ gauge enhancement, rk$\mathcal{G} = \infty$

  $\longrightarrow$ no known examples

Beyond free points

No general argument. Relevant candidates:

- Flavour enhancement (new current J_F^μ)

  representation theory: $J_F^\mu \in \{ J_F^\mu \, , J^{\mu_1 \dots \mu_\ell} \}$

  $\longrightarrow$ free points

- Tower of scalars?

  new $\Delta_{\mathcal{O}} = 4 \quad \Rightarrow \quad$ gauge enhancement, rk$\mathcal{G} = \infty$

  $\longrightarrow$ no known examples

- Strong-coupling points:

$$\mathcal{G}_i \, , \quad \tau_i \to 0 \qquad \xleftrightarrow{\text{S-dual}} \qquad \mathcal{G}_i^\vee \, , \quad \tilde{\tau}_i \to i\infty$$

Beyond free points

No general argument. Relevant candidates:

- Flavour enhancement (new current J_F^μ)

  representation theory: $J_F^\mu \in \left\{ J_F^\mu\,, J^{\mu_1\ldots\mu_\ell} \right\}$

  $\longrightarrow$ free points

- Tower of scalars?

  new $\Delta_{\mathcal{O}} = 4 \quad \Rightarrow \quad$ gauge enhancement, rk$\mathcal{G} = \infty$

  $\longrightarrow$ no known examples

- Strong-coupling points:

$$\mathcal{G}_i\,, \quad \tau_i \to 0 \qquad \overset{\text{S-dual}}{\longleftrightarrow} \qquad \mathcal{G}_i^\vee\,, \quad \tilde{\tau}_i \to i\infty$$

  $\longrightarrow$ free points in disguise

Example: IIB on AdS$_5 \times S^5/\mathbb{Z}_K$ [Lawrence, Nebraskov, Vafa '98]

moduli from blow-up cycles

$$\tau_a = c_2 + \tau b_2\,, \qquad b_a = \int_{\Sigma_a} B_2$$

Example: IIB on AdS$_5 \times S^5/\mathbb{Z}_K$ [Lawrence, Nebraskov, Vafa '98]

moduli from blow-up cycles

$$\tau_a = c_2 + \tau b_2\,, \qquad b_a = \int_{\Sigma_a} B_2$$

CFT: Quiver with $\quad \mathcal{G} = SU(N)^K$

$$\mathcal{G}_a\,, \tau_a \to 0 \qquad \overset{\text{S-dual}}{\longleftrightarrow} \qquad \mathcal{G}_a^\vee = SU(K)\,,\ \tilde{\tau}_a \to i\infty$$

Driven by axions!

Example: IIB on AdS$_5 \times S^5/\mathbb{Z}_K$ [Lawrence, Nebraskov, Vafa '98]

moduli from blow-up cycles

$$\tau_a = c_2 + \tau b_2\,, \qquad b_a = \int_{\Sigma_a} B_2$$

CFT: Quiver with $\mathcal{G} = SU(N)^K$

$$\mathcal{G}_a\,, \tau_a \to 0 \quad \overset{\text{S-dual}}{\longleftrightarrow} \quad \mathcal{G}_a^\vee = SU(K)\,,\ \tilde{\tau}_a \to i\infty$$

Generalises to zoo of theories in class $\mathcal{S}$ dual to M-theory on $AdS_5 \times Y_6$. [Gaiotto, Maldacena, '09]

Can study which ones have decoupling limits

[Genish, Narovlansky '18]

Decomposability

[Beem, Lemos, Liendo, Rastelli, van Rees, '14]

Proving

$$\text{dist} = \infty \quad \Rightarrow \quad \text{free } (\Leftrightarrow \text{ HS tower})$$

is related to:

Conjecture: Decomposability of $\mathcal{N} = 2$ SCFTs

For any $\mathcal{N} = 2$ SCFT, $\mathcal{T}$, with dim$\mathcal{M} = n$:

$$\mathcal{T} = \mathcal{T}_1 \oplus \cdots \oplus \mathcal{T}_k \qquad \text{dim}\mathcal{M}_{\mathcal{T}_i} = 0$$

Gluing by gauging n flavour groups.

Would mean that can only have free infinite-distance point (up to dual).

Conclusions

SDC in AdS

- Infinite-distance points $\Leftrightarrow$ tower of HS massless states
- Sector decouples in dual CFT

- What about less SUSY/dimensions?
  (e.g. $4D\,\mathcal{N} = 1$ has D-terms marginal deformation)

  see [Perlmutter, Rastelli, Vafa, Valenzuela '20]

Conclusions

SDC in AdS

- Infinite-distance points $\Leftrightarrow$ tower of HS massless states
- Sector decouples in dual CFT

- What about less SUSY/dimensions?
  (e.g. $4D\,\mathcal{N}=1$ has D-terms marginal deformation)

  see [Perlmutter, Rastelli, Vafa, Valenzuela '20]

- $\mathcal{N}=2$: geometric proof on either side?

Conclusions

SDC in AdS

- Infinite-distance points $\Leftrightarrow$ tower of HS massless states
- Sector decouples in dual CFT

- What about less SUSY/dimensions?
  (e.g. $4D\,\mathcal{N}=1$ has D-terms marginal deformation)

  see [Perlmutter, Rastelli, Vafa, Valenzuela '20]
- $\mathcal{N}=2$: geometric proof on either side?
- CFT arguments start bleeding to the Swampland

  Bootstrap programme similar to Swampland programme