The Swampland at the Boundary of Field Space

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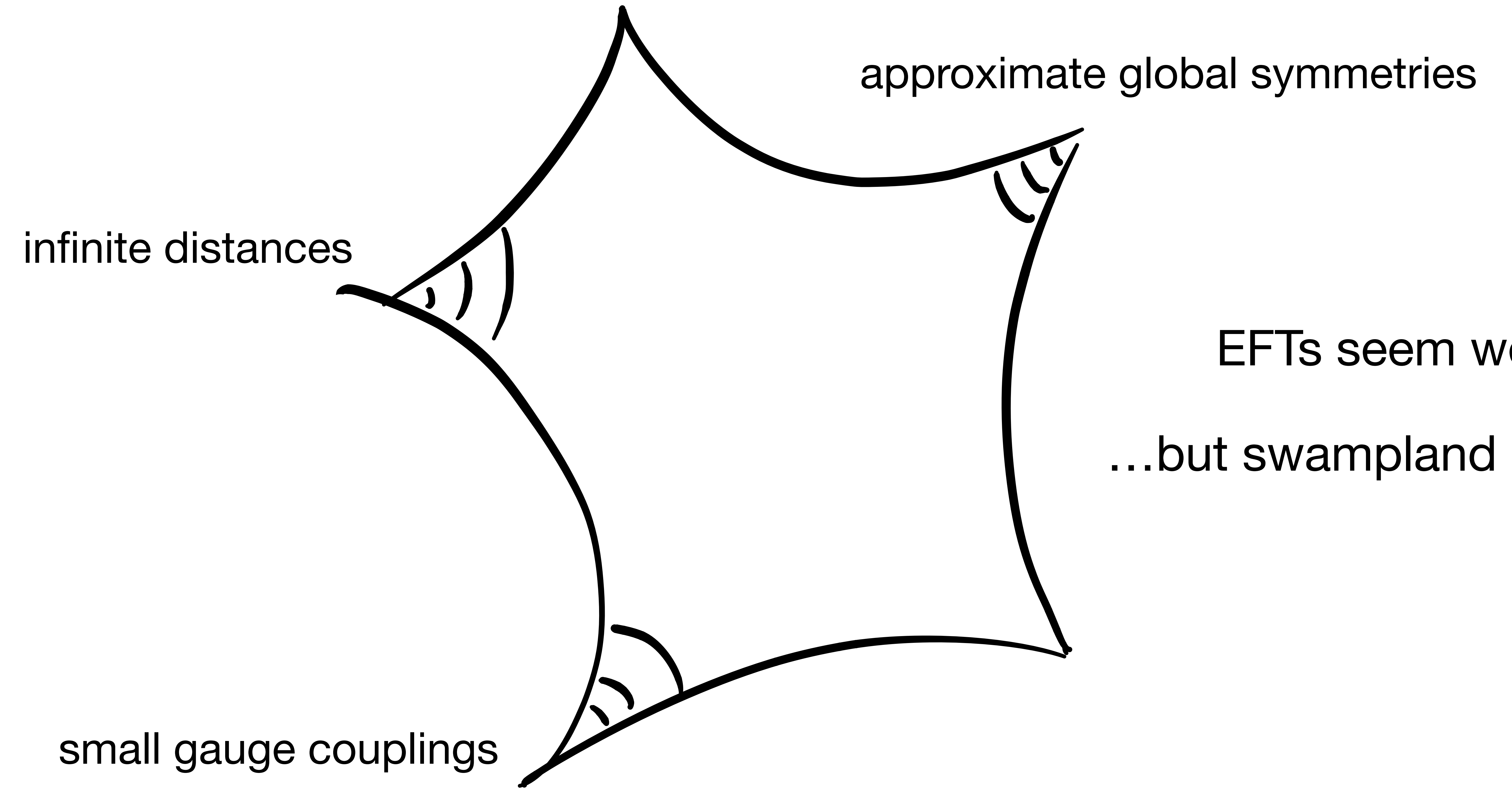
Based on: work in progress, with **B. Bastian** and **T.W. Grimm**

String Phenomenology
Seminar Series

Introduction

Why look at the boundary of field space?



EFTs seem well-controlled in asymptotic regimes…

…but swampland conjectures predict they must break down!

$\Longrightarrow$ sparked many detailed studies of asymptotic string compactifications

[Kläwer, Palti ’16; Palti ’17, Grimm, Palti, Valenzuela ’18; Blumenhagen, Kläwer, Schlechter, Wolf ’18; Lee, Lerche, Weigand ’18/’19; Grimm, Li, Palti ’18; Corvilain, Grimm, Valenzuela ’18; Joshi, Klemm ’19; Font, Herráez, Ibáñez ’19; Marchesano, Wiesner ’19; Grimm, DH ’19; Erkinger, Knapp ’19; Heidenreich, Reece, Rudelius ’19; Grimm, Li, Valenzuela ’19; Baume, Marchesano, Wiesner ’19; Cecotti ’20; Andriot, Cribiori, Erkinger ’20; Gendler, Valenzuela ’20; Lanza, Marchesano, Martucci, Valenzuela ’20; Heidenreich, Rudelius ’20;…]

Introduction

Important to make bounds in Swampland Conjectures precise

- Swampland Distance Conjecture [Ooguri, Vafa '06] $M \sim e^{-\lambda \Delta\phi}$

- Weak Gravity Conjecture [Arkani-Hamed, Motl, Nicolis, Vafa '06] $\frac{Q}{M} \geq \frac{Q}{M}\Big|_{\text{extremal}}$

- De Sitter Conjecture [Obied, Ooguri, Spodyneiko, Vafa '18], [Ooguri, Palti, Shiu, Vafa '18] $\|\nabla V\| \geq cV$

Goal: determine bounds in asymptotic string compactifications

Use **asymptotic Hodge theory** as framework $\Longrightarrow$ captures parametrical growth **and** leading coefficients

(think of Kähler potential, gauge kinetic functions, masses, flux potentials, …)

Outline

- Part I: Overview of asymptotic Hodge theory

- Part II: Bounds for the Weak Gravity Conjecture

- Part III: Bootstrap at boundaries in CY moduli spaces

Part I: Overview of asymptotic Hodge theory

Field space of Calabi-Yau compactifications

Consider complex structure moduli space

$h^{2,1}$ complex structure moduli

$\Longrightarrow$ encoded in a single holomorphic (3,0)-form $\Omega(t^i)$

Kähler metric $\quad K_{i\bar{j}} = \partial_i \partial_{\bar{j}} K\,, \qquad K(t^i, \bar{t}^i) = -\log\; i \int_{Y_3} \Omega(t^i) \wedge \bar{\Omega}(\bar{t}^i)$

Expand Ω in an integral basis of three-forms as $\Omega = \mathbf{\Pi}^I \gamma_I$

$\Longrightarrow$ periods $\mathbf{\Pi}^I(t^i)$ given by $\mathbf{\Pi}^I(t^i) = \int_{\Gamma_I} \Omega(t^i)$

$\boxed{\text{How do the periods } \mathbf{\Pi}^I(t^i) \text{ behave close to the boundary?}}$

Asymptotic limits in moduli space

Parametrize boundary via $t^i = x^i + iy^i \to i\infty \quad (i = 1,\ldots,n)$

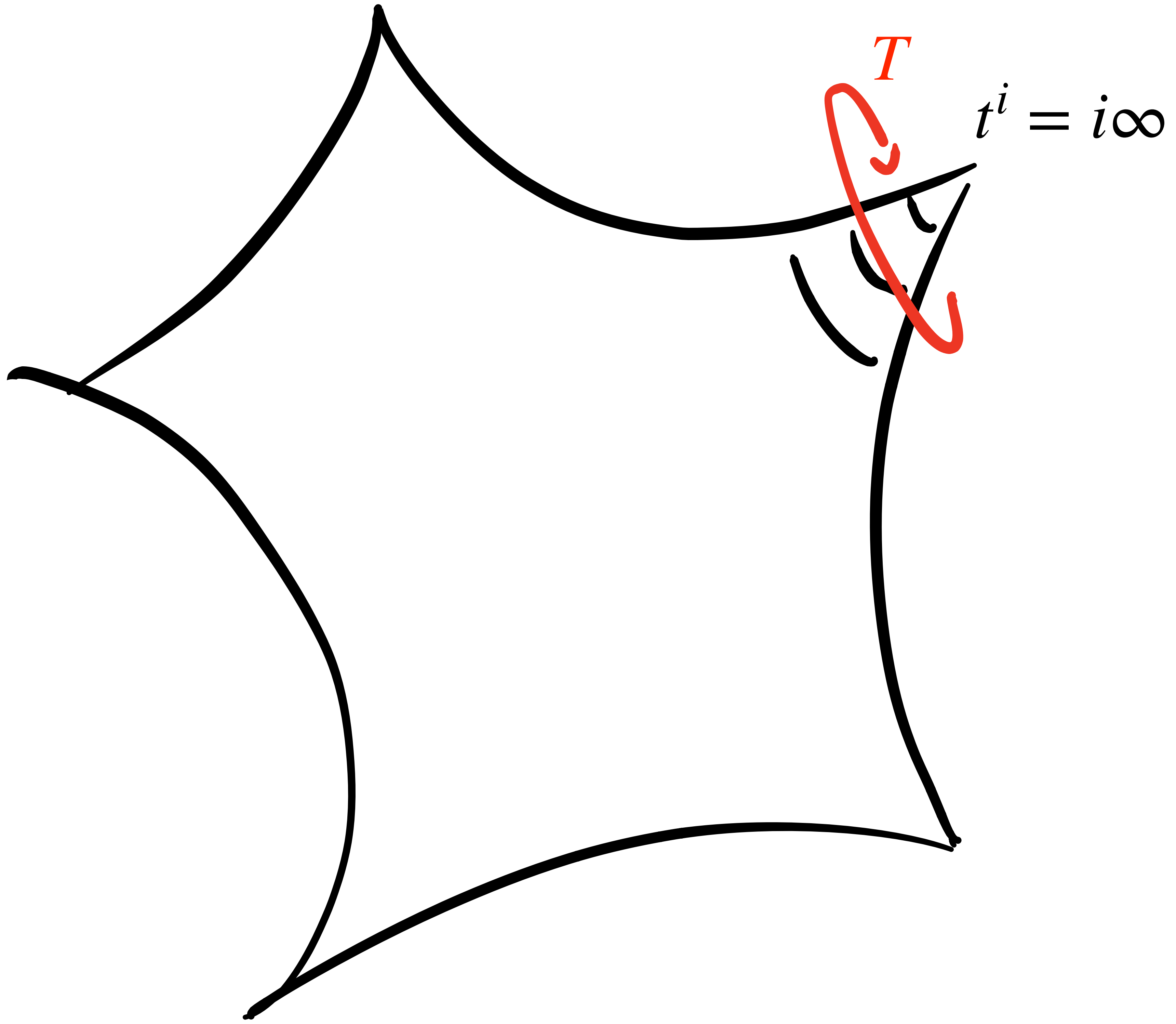
$\Longrightarrow$ shifting axions $x^i \to x^i + 1$ induces monodromies $\mathbf{\Pi}(t^i + 1) = T_i \cdot \mathbf{\Pi}(t^i)$

Behavior of $\mathbf{\Pi}(t^i)$ close to boundary [Schmid '70]

$$\mathbf{\Pi}(t^i) = e^{t^i N_i}(\mathbf{a}_0 + \underbrace{\mathbf{a}_i e^{2\pi i t^i} + \ldots}_{\text{“instanton” part}})$$

“perturbative” part

- Log-monodromy matrices $N_i = \log T_i$ are nilpotent
- Terms $\mathbf{a}_0, \mathbf{a}_i, \ldots$ depend on moduli not sent to limit



Aside: asymptotic behavior of Kähler metric

Study limits with $y^1 \gg y^2 \gg \ldots \gg y^n \gg 1$ (think of as expansion in $\frac{y^{i+1}}{y^i}, \frac{1}{y^n}$ around boundary)

Use asymptotic behavior of periods $\mathbf{\Pi}^I(t^i)$ to describe Kähler potential

$$K = -\log\, (y^1)^{d_1}(y^2)^{d_2-d_1}\cdots(y^n)^{d_n-d_{n-1}}\,,$$ with $0 \leq d_i \leq d_{i+1} \leq 3$ $((N_1 + \ldots + N_i)^{d_i}\mathbf{a}_0 \neq 0)$

discrete data that characterizes boundary

Compute Kähler metric from asymptotic Kähler potential

$$K_{i\bar{i}} = \frac{d_i - d_{i-1}}{(y^i)^2} \implies$$ **degenerate** metric when $d_i = d_{i-1}$

Should be careful with interchanging order of limits and derivatives!

Structure at the boundary

[Cattani, Kaplan, Schmid '86]

Two key structures emerge in strict asymptotic regime $y^1 \gg \ldots \gg y^n \gg 1$

Boundary Hodge decomposition

Decompose space of three-forms as

$$H^3(Y_3, \mathbb{C}) = H^{3,0}_\infty \oplus H^{2,1}_\infty \oplus H^{1,2}_\infty \oplus H^{0,3}_\infty$$

Operator C_∞ that acts as

$C_\infty w^{p,q} = i^{p-q} w^{p,q}$ for $w^{p,q} \in H^{p,q}_\infty$

sl(2)-splitting

n commuting $sl(2,\mathbb{R})$-triples

$$(N_i^-, N_i^+, Y_i)$$

Decompose space of three-forms as

$$H^3(Y_3, \mathbb{R}) = \oplus_{\ell_1 \cdots \ell_n} V_{\ell_1 \cdots \ell_n}$$

eigenspaces of weight operators Y_i

Behavior of the "Hodge norm"

Hodge norm for three-form $w \in H^3(Y_3, \mathbb{Q})$

$$\|w\|^2 = \int_{Y_3} \bar{w} \wedge \star w$$

moduli-dependence

Ubiquitous in string compactifications $\Longrightarrow$ this talk:

- **Physical charge** for gauge field $C_4 = A \wedge w$
- **Scalar potential** for fluxes $F_3 = w$ or $H_3 = w$

Hodge norm for $w \in V_{\ell_1 \cdots \ell_n}$ in strict asymptotic regime

$$\|w\|^2 = (y^1)^{\ell_1 - 3} (y^2)^{\ell_2 - \ell_1} \cdots (y^n)^{\ell_n - \ell_{n-1}} \|w\|^2_\infty$$

$$\|w\|^2_\infty = \int_{Y_3} \bar{w} \wedge (C_\infty w)$$

- $sl(2,\mathbb{R})$-algebras determine parametrical scaling
- Boundary Hodge structure fixes leading coefficient

Part II: Bounds for the Weak Gravity Conjecture

Weak Gravity Conjecture (WGC) [Arkani-Hamed, Motl, Nicolis, Vafa ’06]

Predicts existence of a **superextremal particle** for gravitational theories with a U(1) gauge field:

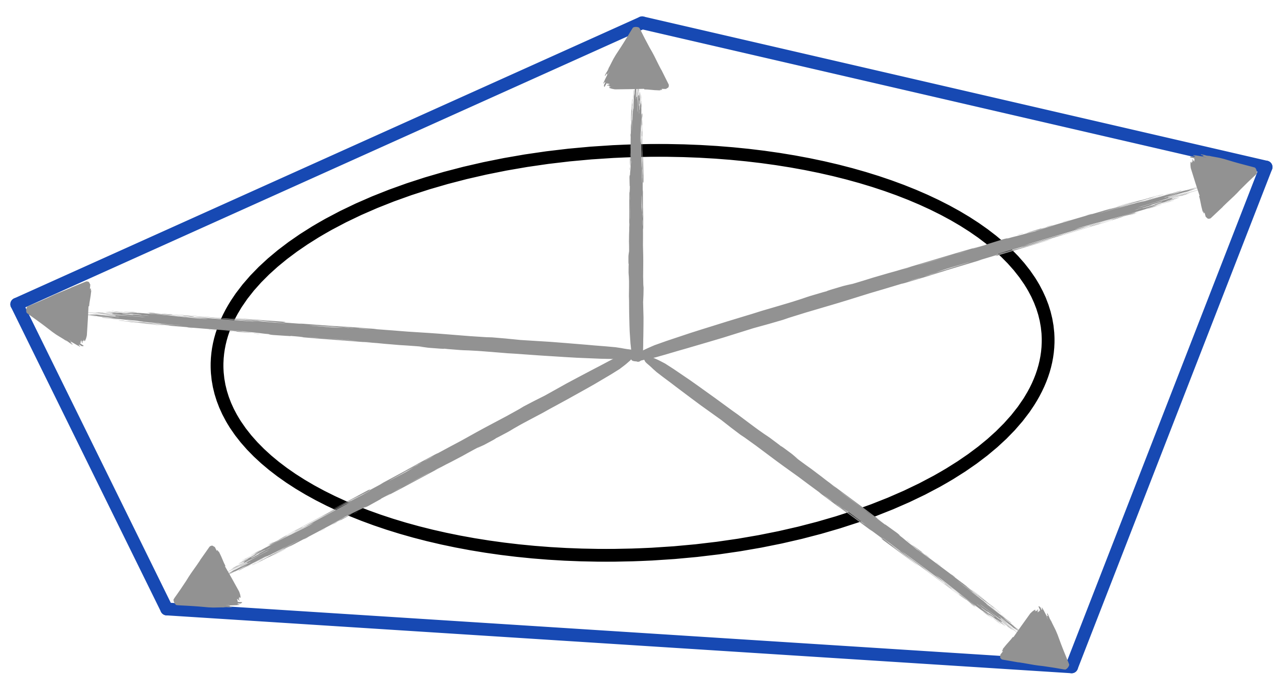
$$\frac{Q}{M} \geq \left.\frac{Q}{M}\right|_{\text{extremal}}$$

need to know black hole extremality bound!

$\Longrightarrow$ compute charge-to-mass ratios for states in string compactifications

Multiple U(1) gauge fields [Cheung, Remmen ’14]

Convex hull of charge-to-mass vectors contains the black hole extremality region



BPS states in 4d $\mathcal{N} = 2$ supergravities

Charge-to-mass spectrum of electric BPS states has an intricate structure

$\implies$ **ellipsoid** with two non-degenerate directions [Gendler, Valenzuela '20]

$\implies$ $\mathcal{N} = 2$ supersymmetry constrains radii via $\gamma_1^{-2} + \gamma_2^{-2} = 1$

In string compactifications: determine physical charge and mass from underlying Calabi-Yau geometry

$\implies$ BPS states arise from D3-branes wrapping 3-cycle q of Y_3

$$Q^2 = \frac{1}{2}\int_{Y_3} q \wedge \star q \qquad M = e^{K/2}\left|\int_{Y_3} q \wedge \Omega\right|$$

Asymptotic charge-to-mass ratios

Study "single-charge" states $q \in V_{\ell_1 \ell_2 \cdots \ell_n}$

Ω at boundary: $\Omega_\infty = e^{i(N_1^- + \ldots + N_n^-)}\,\tilde{a}_0$

$\Longrightarrow$ useful to examine coupling to **asymptotic graviphoton:** $\displaystyle\int_{Y_3} q \wedge \Omega_\infty$

Single-charge states with vanishing coupling $\Longrightarrow$ charge-to-mass ratio diverges

(e.g. charges related to a_1)

Single-charge states with non-vanishing coupling $\Longrightarrow$

(charges of the form $q = N_i^- \tilde{a}_0,\ N_i^- N_j^- \tilde{a}_0,\ \ldots$)

$$\boxed{\left(\frac{Q}{M}\right)^{-2} = 2^{1-d_n} \prod_{i=1}^{n} \begin{pmatrix} \Delta d_i \\ (\Delta d_i - \Delta \ell_i)/2 \end{pmatrix} \times \begin{cases} 1 \text{ for } d_n = 3\,, \\ \frac{1}{2} \text{ for } d_n \neq 3\,. \end{cases}}$$

$\Delta d_i = d_i - d_{i-1},\ \Delta \ell_i = \ell_i - \ell_{i-1}$

discrete data associated with boundary and charges

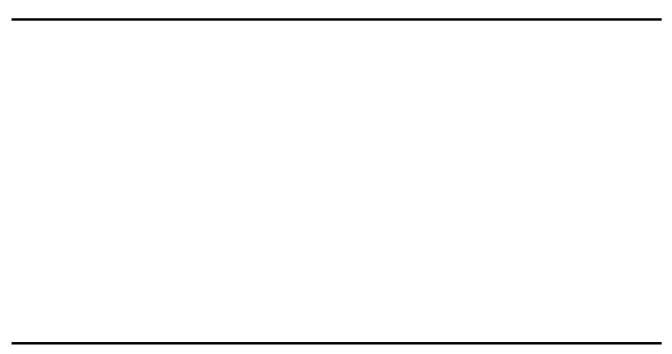
$\Longrightarrow$ resolves issue with $d_i = d_{i-1}$

Asymptotic shape of the electric charge-to-mass spectrum

Can compute radii of electric charge-to-mass spectrum from charge-to-mass ratios [Gendler, Valenzuela, '20]

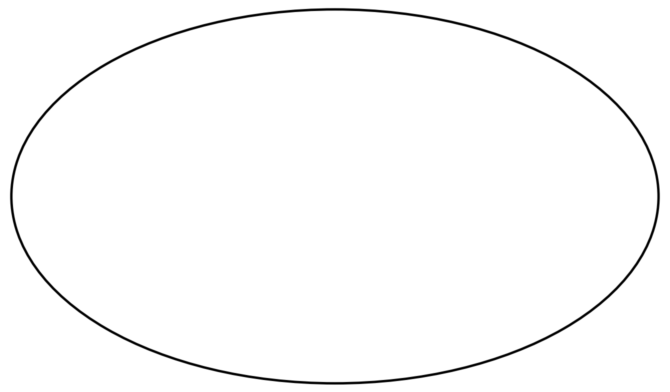
$\Longrightarrow$ use derived formula for charge-to-mass ratios

finite distance boundaries

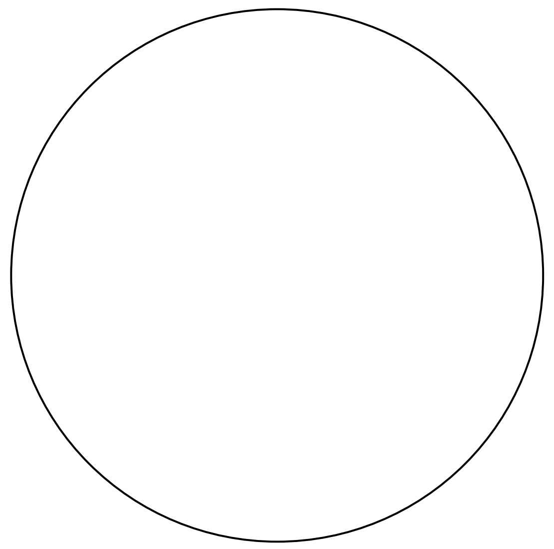


$\gamma_1^{-2} = 1,\ \gamma_2^{-2} = 0$

infinite distance boundaries



$\gamma_1^{-2} = \frac{3}{4},\ \gamma_2^{-2} = \frac{1}{4}$



$\gamma_1^{-2} = \gamma_2^{-2} = \frac{1}{2}$

Lower bound for infinite distance singularities $\frac{Q}{M} \geq \frac{2}{\sqrt{3}}$

Recover $\mathcal{N} = 2$ constraint via $\gamma_1^{-2} + \gamma_2^{-2} = \sum \left(\frac{Q}{M}\right)^{-2} = \prod_{i=1}^{n} \sum_{\Delta\ell_i} 2^{-\Delta d_i} \binom{\Delta d_i}{(\Delta d_i - \Delta\ell_i)/2} = 1$

Bounds for dS conjecture

de Sitter Conjecture [Obied, Ooguri, Spodyneiko, Vafa ’18], also [Ooguri, Palti, Shiu, Vafa ’18]

Scalar potentials are constrained by $\|\nabla V\| \geq cV$

$\Longrightarrow$ test by studying potentials in flux compactifications see also [Grimm, Li, Valenzuela ’19], [Andriot, Cribiori, Erkinger ’20] [Lanza, Marchesano, Martucci, Valenzuela ’20]

This talk: consider Type IIB flux potential $V = \frac{1}{4}e^{4\phi}\int_{Y_3} F_3 \wedge \star F_3 + \frac{1}{4}e^{2\phi}\int_{Y_3} H_3 \wedge \star H_3 - \frac{1}{2}e^{3\phi}\int_{Y_3} F_3 \wedge H_3$

Express gradient in terms of charge-to-mass ratios

$$\frac{\|\nabla V\|^2}{V^2}\bigg|_{F_3=0} = 2\left(\left(\frac{Q}{M}\right)^2 + 3\right) \qquad \frac{\|\nabla V\|^2}{V^2}\bigg|_{H_3=0} = 2\left(\left(\frac{Q}{M}\right)^2 + 15\right)$$

Lower bound for infinite distance singularities: $c \geq \sqrt{\frac{26}{3}}$

Bounds for the Swampland Distance Conjecture

Swampland Distance Conjecture [Ooguri, Vafa '06]

An infinite tower of states must become exponentially light for large field excursions

$$M \sim e^{-\lambda \Delta\phi}$$

Complex structure moduli space for Type IIB on Y_3 $\implies$ towers of wrapped D3-brane states

[Grimm, Palti, Valenzuela '18; Grimm, Li, Palti '18]

related work in [Blumenhagen, Kläwer, Schlechter, Wolf '18], [Lee, Lerche, Weigand '18/'19], [Corvilain, Grimm, Valenzuela '18], [Font, Herráez, Ibáñez '19]

A bound for λ [Lee, Lerche, Weigand '18], [Gendler, Valenzuela '20]

$$\lambda^2 = \left| \frac{\nabla_i M}{M} u^i \right|^2 = \frac{1}{2}\left(\left(\frac{Q}{M} \right)^2 - 1 \right)$$

Plug in obtained charge-to-mass ratios

$$\implies \lambda \geq \frac{1}{\sqrt{6}}$$

also [Grimm, Palti, Valenzuela '18; Gendler, Valenzuela '20]
related arguments in [Andriot, Cribiori, Erkinger '20]

Part III: Bootstrap for boundaries in CY moduli spaces

Bootstrap for boundaries in CY moduli spaces

Main idea: constrain form of periods $\mathbf{\Pi}^I(t^i)$ based on general principles

$\Longrightarrow$ holomorphicity, symmetry, positivity

- **holomorphicity:** expand holomorphic part of period vector $\quad \mathbf{\Pi}(t^i) = e^{t^i N_i}(\mathbf{a}_0 + e^{2\pi i t^i}\mathbf{a}_i + \dots)$
- **symmetry:** Kähler transformations and coordinate redefinitions $\quad \mathbf{\Pi}(t^i) \to e^{f(t^i)}\mathbf{\Pi}(t^i)$
- **positivity:** use control via boundary Hodge structure $\quad \|w\|^2_\infty = \int_{Y_3} \bar{w} \wedge (C_\infty w) > 0$

E.g. at one-modulus conifold point

$$e^{-K} = i\int_{Y_3} \mathbf{\Pi} \wedge \bar{\mathbf{\Pi}} = c_1 + c_2 t e^{2\pi i t} + \dots\ , \qquad \text{with } \ c_1 = i\int_{Y_3} a_0 \wedge \bar{a}_0 > 0\,, \ \ c_2 = \int_{Y_3} a_1 \wedge (N\bar{a}_1) > 0$$

Bootstrap for two-moduli boundaries

Aim: construct generic models for two-moduli Calabi-Yau compactifications

$\Longrightarrow$ constrained expressions for Kähler potentials, gauge kinetic functions, masses, flux potentials, …

Starting point: classification of two-moduli singularities [Kerr, Pearlstein, Robles '17]

Result: four classes of two-moduli Calabi-Yau compactifications

$\Longrightarrow$ "instanton" corrections cannot be ignored (aside from "large complex structure" class)

(e.g. needed for non-degenerate Kähler metric)

Goals

- Learn about structure of polynomially and exponentially suppressed corrections
- Test swampland conjectures further into interior of moduli space
- Construct and test models of axion monodromy inflation
- …

Summary

- Asymptotic Hodge theory provides structure to control both parametrical scaling and leading coefficients

  $\Longrightarrow$ sl(2)-splitting and boundary Hodge decomposition

- Demonstrated by computing charge-to-mass ratios in IIB CY compactifications

  $\Longrightarrow$ bounds for WGC, dSC, SDC

Outlook

- Study polynomially **and** exponentially suppressed corrections
- Extend to F-theory fourfold setups (also [Grimm, Li, Valenzuela '19])
- …