A New Spin on the Weak Gravity Conjecture

or: *WGC on and Beyond the Horizon*

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Based on 2011.05337 (JHEP) with L. Aalsma, G. Loges, and G. Shiu

Outline

1. Motivation: swampland, (mild) WGC and horizons **"on the horizon"**
2. Covariant condition for mild WGC **"beyond the horizon"**
3. BTZ and holographic c-theorem **"beyond the horizon as RG"**
4. 5D black string and "*Total Landscaping Principle*" **"many horizons"**
5. Conclusion

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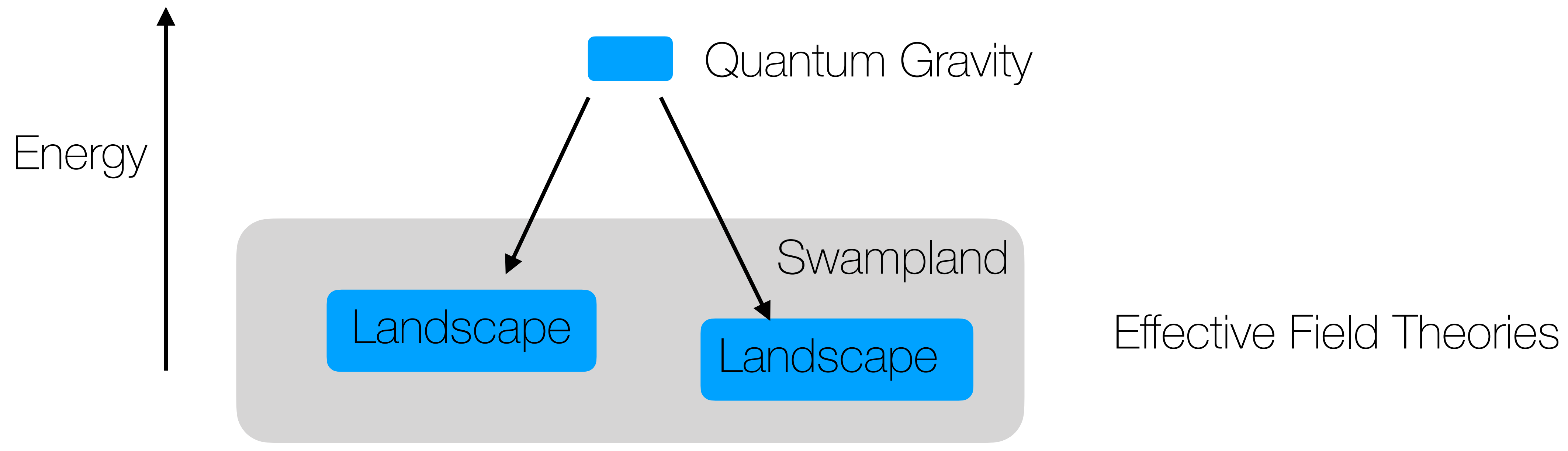
Motivation: Swampland

- It is well-known that consistent UV physics constrains IR EFTs.
- When the UV is quantum gravity, this is the swampland program. **[Vafa; Ooguri, Vafa; …]**
- To separate landscape from swampland we use precision tools like supersymmetry and black holes.

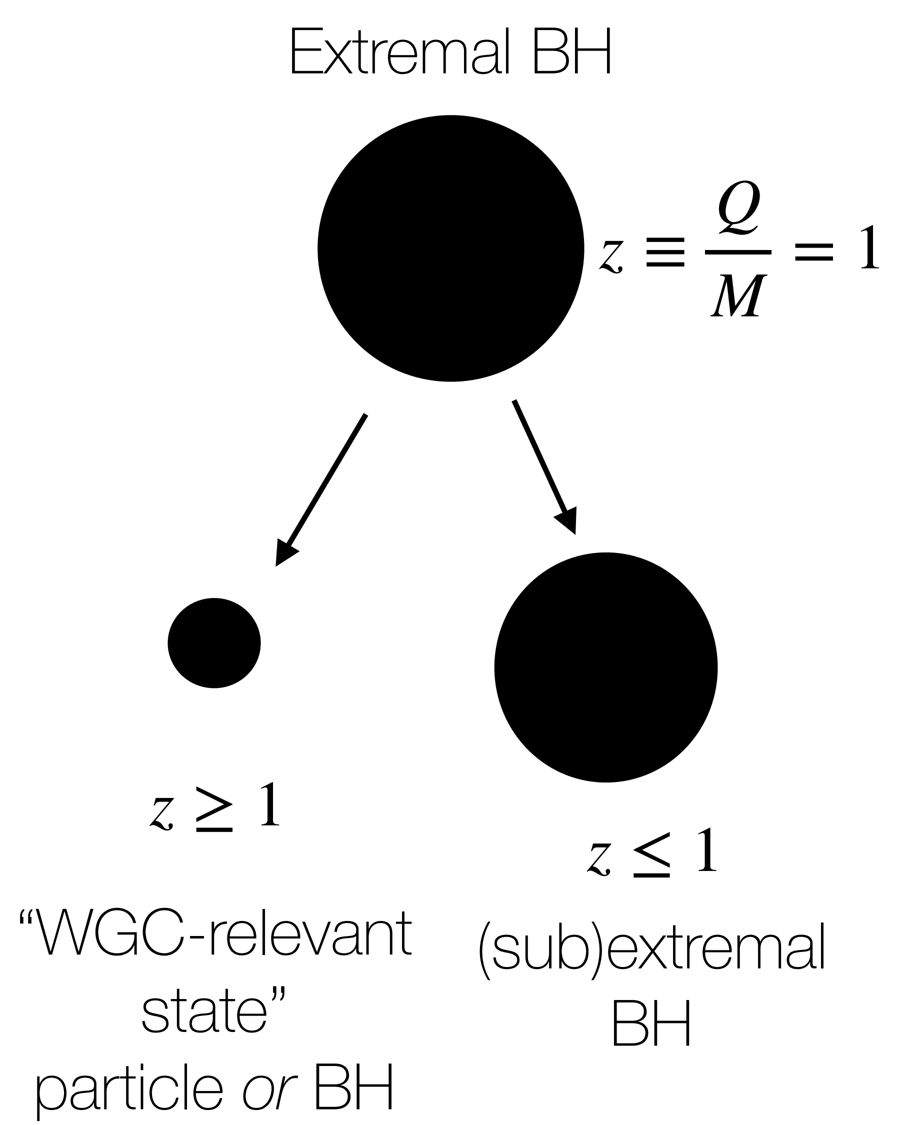


Weak Gravity Conjecture

- The Weak Gravity Conjecture **[Arkani-Hamed, Motl, Nicolis, Vafa]** quantifies quantum gravity's objection to global symmetry limits.

- WGC requires the *kinematic* possibility of extremal black hole decay.

    - Can be achieved by modifications to black hole extremality bound. ("Mild form." Can be strengthened in some circumstances **[Montero, Shiu, Soler; Heidenreich, Reece, Rudelius; Lee, Lerche, Weigand; Aalsma, AC, Shiu]**)



WGC and Positivity

- Higher-derivative terms modify the extremality bound of Reissner-Nordström black holes.

$$\mathscr{L} = \mathscr{R} - \frac{1}{4}F_{ab}F^{ab} + \frac{a_1}{4}\left(F_{ab}\underbrace{F^{ab}\right)^2 + \frac{a_2}{2}F}_{\text{leading higher-derivative terms}}{}_{ab}F_{cd}W^{abcd}$$

$$\Delta z\big|_{T=0} \sim \frac{2a_1 - a_2}{Q^2} \geq 0 \quad \text{for WGC}$$

[Kats, Motl, Padi]

- Unitarity and causality constrain Wilson coefficients, but more UV info needed to prove WGC. [Hamada,Noumi,Shiu ’18; Bellazzini,Lewandowski,Serra ’19; Loges,Noumi,Shiu ’19; Alberte,de Rham, Jaitly,Tolley ’20 x 2;…]

  - *What is the minimal set of assumptions needed to prove mild WGC, and what are “deep reasons” it should be true?*

WGC at the Horizon

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2^2$$

- In Schwarzschild gauge, horizon location(s) given by zero(s) of $f(r)$.

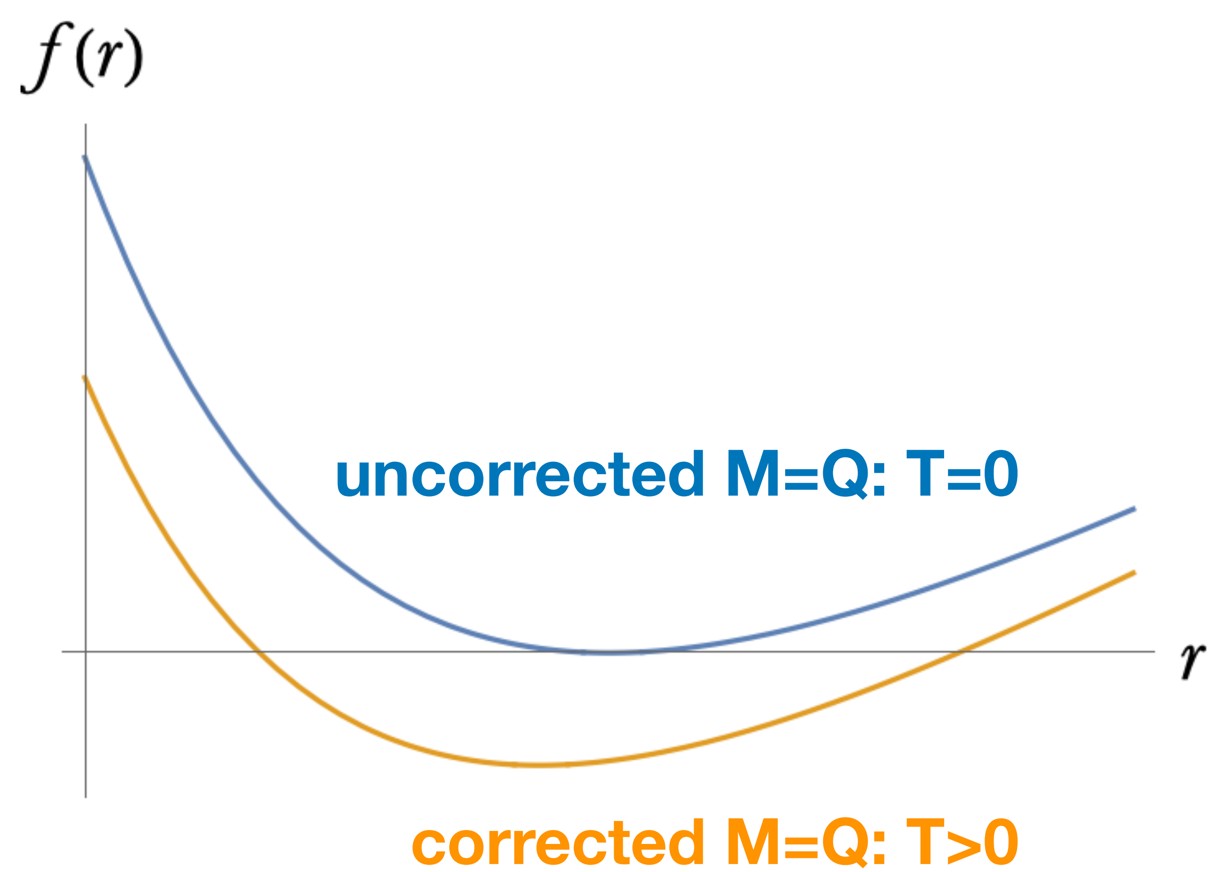
- Mild WGC:

  - Fix charges: $\Delta S|_{z=1} \geq 0$

    [Hamada, Noumi, Shiu]:

    $\Delta S \sim \Delta A_{BH} \propto \sqrt{a_i}$

  - Fix T: $\Delta z|_{T=0} \geq 0$



$\Delta f(r_H) < 0$

WGC-satisfying corrections

see also [Cheung,Liu,Remmen; Goon,Penco]

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Covariant condition

- **Iyer-Wald** formalism describes off-shell variations of Hamiltonians associated with surface charges. One can derive the useful formula:

Correction to horizon/entropy

Killing vector

(💪) $$\left(\int_{S_\infty^{d-2}} - \int_{S_{\rm hor}^{d-2}}\right)\delta \mathbf{H}_\xi = \int_\Sigma d^{d-1}x\sqrt{h}\,\delta T^{\rm eff}_{ab} n^a \xi^b$$

Variation of asymptotic charge

Stress tensor of corrections

- We evaluate this on an extremal black hole background. Valid for any stationary spacetime.

- Computationally convenient: don't need explicit form of corrected solution.

- Non-covariant version of argument appeared in **[Kats, Motl, Padi]**

[Aalsma, AC, Loges, Shiu]

Example: Electric Charge

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2^2, \quad f(r) = \frac{(r-r_-)(r-r_+)}{r^2}$$

uncorrected solution

$$A = \left(-\frac{Q}{4\pi r} + \Phi_+\right) dt$$

gauge subtleties, see [Elgood, Meessen, Ortín; Elgood, Ortín, Pereñíguez]

$$\delta H_{\partial_t} = \int_{S^2_\infty} \delta \mathbf{H}_{\partial_t} = \delta M_4, \quad \delta H_\lambda = \int_{S^2_\infty} \delta \mathbf{H}_\lambda = -\Phi_+ \delta Q$$

asymptotic charges

$$\delta \mathbf{H}_{\partial_t} = -\frac{1}{8\pi G_4}\left[\frac{\rho'(r)}{\rho(r)}\delta f(r) + \frac{2f(r)}{\rho(r)}\delta\rho'(r) - \frac{f'(r)}{\rho(r)}\delta\rho(r)\right]\rho(r)^2 \sin\theta \, d\theta \wedge d\phi$$

$$\delta \mathbf{H}_\lambda = -\left[\Phi(r)\delta\Phi'(r) + \frac{2\Phi(r)\Phi'(r)}{\rho(r)}\delta\rho(r)\right]\rho(r)^2 \sin\theta \, d\theta \wedge d\phi \, ,$$

- Evaluating (💪) with fixed charges we find

$$-\frac{r_+ \delta f(r_+)}{2G_4} = -\int_\Sigma d^3x \sqrt{h}\, \delta T^{\text{eff}}_{ab} n^a \xi^b \geq 0 \iff \Delta S_{z=1} \geq 0$$

[Aalsma, AC, Loges, Shiu]

Example: Electric Charge

- For the theory

$$\mathscr{L} = \mathscr{R} - \frac{1}{4}F_{ab}F^{ab} + \frac{a_1}{4}(F_{ab}F^{ab})^2 + \frac{a_2}{2}F_{ab}F_{cd}W^{abcd}$$

one computes

fixing charges

$$\delta f(r_+) + \#\cancel{\delta M} + \#\cancel{\delta Q} \sim \int_\Sigma d^3x\sqrt{h}\,\delta T^{\mathrm{eff}}_{ab}n^a\xi^b \sim -(2a_1 - a_2) \leq 0 \quad \text{for WGC}$$

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BTZ with corrections

- Consider the three-dimensional Lagrangian

$$\mathscr{L} = \mathscr{R} + \frac{2}{\ell^2} + \alpha_1 \ell \mathscr{R}^2 + \alpha_2 \ell R_{ab} R^{ab}$$

AdS$_3$ Gravity **CFT$_2$**

$$\left.\frac{J_3}{\ell M_3}\right|_{T=0} = 1 + \frac{48\pi G_3(3\alpha_1 + \alpha_2)}{\ell} \iff c = \frac{3\ell}{2G_3}\left(1 - \frac{48\pi G_3(3\alpha_1 + \alpha_2)}{\ell}\right)$$

- A positive extremality correction $\Delta z\big|_{T=0} \geq 0$ corresponds to a negative central charge correction $\Delta c \leq 0$.

Positivity from Holographic RG

- For 2D QFTs, the c-function decreases along RG flows: $c_{IR} \leq c_{UV}$ [Zamolodchikov ’86].
- For relevant deformations, a **spinning WGC** follows.

AdS$_3$ Gravity	**CFT$_2$**
$\left.\dfrac{J_3}{\ell M_3}\right\vert_{T=0} = 1$ $\xrightarrow{\text{couple to light matter}}$	**CFT**$_{\mathrm{UV}}(c)$ $\xrightarrow{\text{relevant deformation}}$
$\Big\downarrow$ solve EoM	$\Big\downarrow$ RG flow
$\left.\dfrac{J_3}{\ell M_3}\right\vert_{T=0} = 1 + \#(c - c')$	**CFT**$_{\mathrm{IR}}(c')$
NEC implies $\displaystyle\int_\Sigma d^2x\sqrt{h}\,\delta T_{ab}\xi^a n^b \leq 0$	c-theorem: $c' \leq c$

A Spinning Weak Gravity Conjecture

- Do we need a spinning Weak Gravity Conjecture?
  - BH thermodynamics arguments insensitive to nature of charge. However, Penrose process allows extraction of $\boldsymbol{J}$.
  - Generalization of Repulsive Force Conjecture **[Palti; Heidenreich, Reece, Rudelius]**
  - Relate to constraints on “microscopic” HS particles? cf. **[Kaplan, Kundu; …]**
  - BH angular momentum related to electric charge via NH-limits and dimensional reduction.

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Boosted Black String

- When BTZ arises as a near-horizon limit, entropy can be computed via Cardy formula. **[Strominger '98]**
- How does spinning WGC for BTZ relate to charged WGC?

Boosted and Charged 5D Black String

reduce on string direction

near-horizon limit

4D Charged Black Hole $\longleftrightarrow$ **BTZ x S^2**

entropies match

Boosted Black String

[Aalsma, AC, Loges, Shiu]

$$\mathscr{L}_5 = \mathscr{R} - \frac{3}{4}F_{ab}F^{ab} + \alpha_1\left(F_{ab}F^{ab}\right)^2 + \alpha_2 F_{ab}F_{cd}W^{abcd} + \alpha_3 R_{\mathrm{GB}}$$

extremality bounds disagree

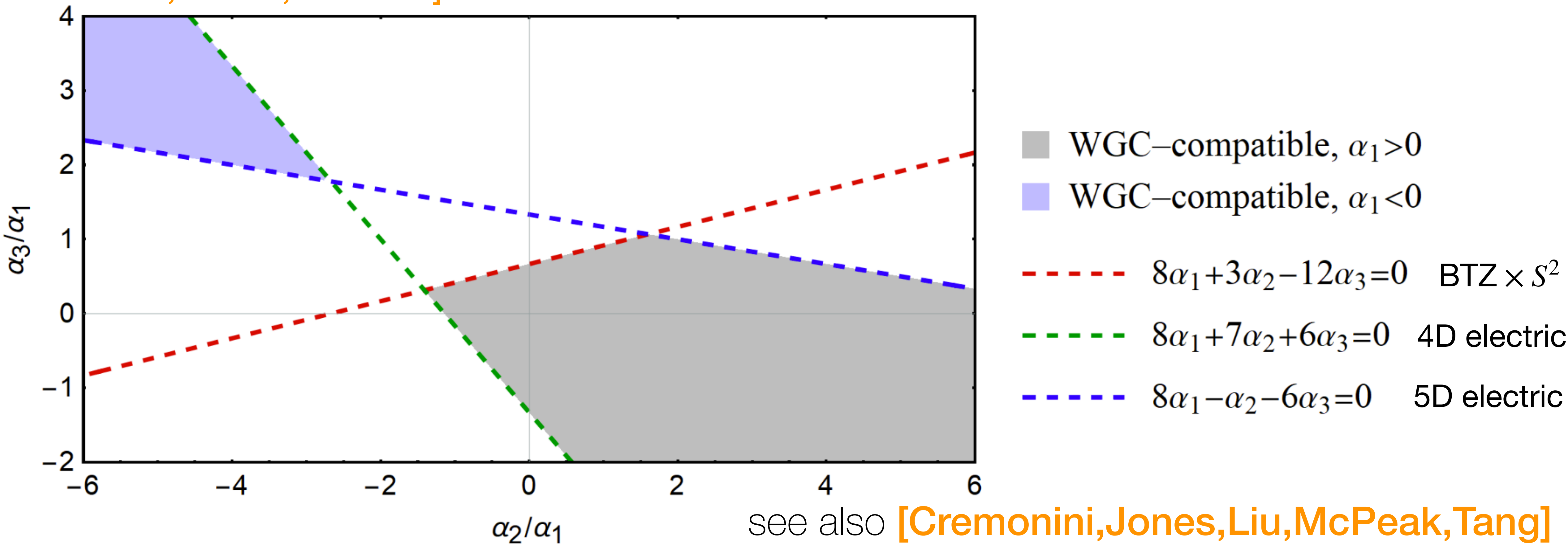
extremal entropies agree

$\mathrm{BTZ}\times S^2$	$T=0$	$z = 1 + \frac{8\alpha_1+3\alpha_2-12\alpha_3}{2}$	
		$S = 2\pi Q\sqrt{\frac{M_3}{G_3}}\left(1 - \frac{8\alpha_1+3\alpha_2-12\alpha_3}{2}\right)$	$= \frac{\pi Q^2}{G_4}\left(1 - \frac{8\alpha_1+3\alpha_2-12\alpha_3}{2}\right)$
	$z=1$	$T = \sqrt{\frac{G_3 J_3(8\alpha_1+3\alpha_2-12\alpha_3)}{\pi Q^3}}$	
		$S = 2\pi Q\sqrt{\frac{M_3}{G_3}}\left(1 + \sqrt{\frac{8\alpha_1+3\alpha_2-12\alpha_3}{2}}\right)$	$= \frac{\pi Q^2}{G_4}\left(1 + \sqrt{\frac{8\alpha_1+3\alpha_2-12\alpha_3}{2}}\right)$
4D	$T=0$	$z = 1 + \frac{2a_1+a_2}{10}$	$= 1 + \frac{8\alpha_1+7\alpha_2+6\alpha_3}{40}$
		$S = \frac{\pi Q^2}{G_4}(1 - 4a_1 + 4a_3)$	$= \frac{\pi Q^2}{G_4}\left(1 - \frac{8\alpha_1+3\alpha_2-12\alpha_3}{2}\right)$
	$z=1$	$T = \frac{\pi}{Q}\sqrt{\frac{2(2a_1+a_2)}{5}}$	$= \frac{\pi}{Q}\sqrt{\frac{8\alpha_1+7\alpha_2+6\alpha_3}{10}}$
		$S = \frac{\pi Q^2}{G_4}\left(1 + \sqrt{\frac{2(2a_1+a_2)}{5}}\right)$	$= \frac{\pi Q^2}{G_4}\left(1 + \sqrt{\frac{8\alpha_1+7\alpha_2+6\alpha_3}{10}}\right)$

Total Landscaping

[Aalsma, AC, Loges, Shiu]

- The extremal entropy of the 4D BH and BTZ agree, but their extremality bounds do not.
- The spinning and charged WGCs therefore give complementary information.
- "***Total Landscaping Principle***": enforce consistency of every compactification/NH-limit.
  cf. [Andriolo, Junghans, Noumi, Shiu; Heidenreich, Reece, Rudelius]



see also [Cremonini,Jones,Liu,McPeak,Tang]

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Summary

- We reformulated the mild WGC as a covariant condition on the effective stress tensor of corrections

$$\int_\Sigma d^{d-1}x\sqrt{h}\ \delta T^{\text{eff}}_{ab}\ \xi^a n^b \leq 0.$$

- We checked a few examples. For BTZ black holes, a **spinning WGC** can be interpreted in terms of a **holographic c-theorem**.

- Boosted black string led to ***Total Landscaping Principle***: apply conjectures to every compactification and NH-limit.

Future work

- Holographic RG for higher-dimensional black holes. Relate to Hamilton-Jacobi description of holographic RG **[de Boer, Verlinde, Verlinde]**.

- Consider quantum corrections. cf. **[Charles]**

Thanks!

Backup slides