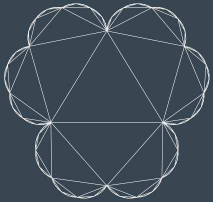
Insights from Non-Ambient Flops



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Based on 2010.06597 and 2011.xxxxx

with Andrei Constantin and Andre Lukas

String Phenomenology Seminar Series
November 24th 2020

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- How to perform them
- What one can learn from them

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- To analyse limits of the physical moduli space, important to understand extended Kähler cone
- Understanding flops can allow one to immediately write down data required for model-building

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- New tool: All line bundle cohomology from CICY data

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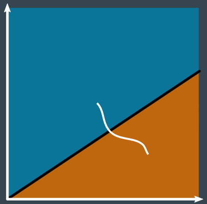
Isomorphism in codimension 1, so

$$H^{1,1}(X) \cong H^{1,1}(X')$$

Manifold moves to new Kähler cone,

$$\mathcal{K}(X) \to \mathcal{K}(X')$$



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But could have chosen to pair as $\{yx_0 = vx_1\,,\ zx_0 = ux_1\}$

These two choices give spaces related by a flop

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'Favourable' if $h^{1,1}(X) = h^{1,1}(\mathcal{A})$
'Kähler-favourable' if $\mathcal{K}(X) = \mathcal{K}(\mathcal{A})$ ($\Rightarrow$ favourable)

For simplicity, below will only discuss Kähler-favourable CICYs

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(Allows to count shrinking curves: compare Euler characteristic)

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x_0	x_1	y_0	y_1	y_2	y_3	y_4	(Eq. 1)	(Eq. 2)
1	1	0	0	0	0	0	1	1
0	3	1	1	1	1	1	4	4

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New CY is isomorphic to original if $\vec{v}_i$ are equal

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$$\left[\begin{array}{c|ccccc} \mathbb{P}^2[x] & 1 & 1 & 1 & 0 & \ldots \\ \vdots & \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vdots & \ddots \end{array}\right] \equiv \left\{ \begin{pmatrix} F_1 & F_2 & F_3 \\ G_1 & G_2 & G_3 \\ H_1 & H_2 & H_3 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \vec{0} \right\}$$

Flopped space given by taking transpose of matrix of polynomials

$$X \equiv \{M\vec{x} = 0\} \subset \mathbb{P}^2 \times \ldots \qquad X_{\text{flop}} \equiv \{M^T\vec{x} = 0\} \subset \mathcal{A}$$

New ambient space is toric with appropriate weight system

New CY is isomorphic to original if $\vec{v}_i$ are equal

Other rows giving flops: $(\mathbb{P}^1|2\;0\ldots0)$, $(\mathbb{P}^n|2\;1\ldots1\;0\ldots0)$

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So non-ambient flops not only possible, but generic

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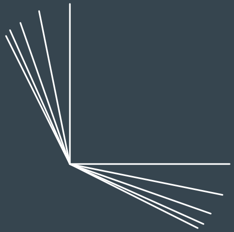

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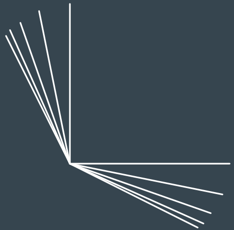

Note: novel

Can't occur from ambient flops

(Num. triangulations always finite)

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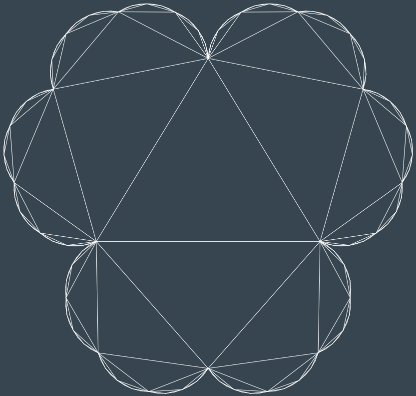

Depiction of extended Kähler cones of CICYs #7447 and #7862

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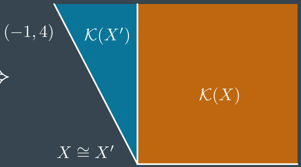

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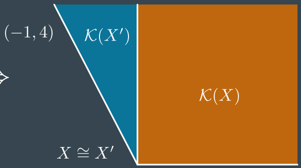

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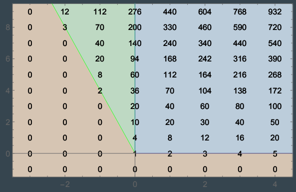

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gCICY on the right recently used for model-building [2010.09763]

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Result is generalised toric complete intersections

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- Cohomology: relation of flops to higher cohomologies - read them off too?

Thanks for listening

Questions?