PROVING THE WGC IN TREE LEVEL STRING THEORY

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TO APPEAR (very soon...)

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SEMINAR ~~SUMMER~~ SERIES ON STRING PHENOMENOLOGY

WGC

FROM IN-STABILITY OF SUBEXTREMAL BLACK HOLES

$\exists$ STATE SUCH THAT

$$\boxed{\frac{Q}{M} \geq \underbrace{\left(\frac{Q}{M}\right)}_{\text{EXTREMAL BLACK HOLE}}}$$

ARE THERE SUCH STATES?

RFC

LONG RANGE FORCE ARGUMENTS

$$F_{12} = K_{ab} \frac{Q_{1a} Q_{2b}}{r^{D-2}} - \frac{G_N m_1 m_2}{r^{D-2}} - \frac{g^{ij} \mu_{1i} \mu_{2j}}{r^{D-2}}$$

$$\mu_{1i} = \frac{\partial m_1}{\partial \phi_i}$$ SCALAR CHARGE.

$$\boxed{F_{11} \geq 0}$$

IN THE ABSENCE OF MODULI, THE CONTENT OF THE TWO CONJECTURES IS THE SAME.

SOME REFINEMENTS

WGC SUPEREXTREMAL STATE + MODULAR INVARIANCE $\Rightarrow$

SUBLATTICE WGC

$\exists$ SUPEREXTREMAL STATE AT EACH SITE OF A SUBLATTICE $\Gamma_0 \subset \Gamma$ OF THE CHARGE LATTICE

SUBLATTICE RFC.

AS ABOVE, REPLACE

SUPEREXTREMAL $\rightarrow$ SELF-REPULSIVE

NOTATION: SELF-REPULSIVE INCLUDES ZERO SELF-FORCE LIMITING CASE

CONNECTION BETWEEN WGC & RFC

WHEN THERE ARE MODULI:

ASSUME THERE $\exists$ A SELF-REPULSIVE PARTICLE SPECIES

+

IF SELF-REPULSIVE EVERYWHERE IN MODULI SPACE

SATISFIES A TYPE OF BOGOMOL'NYI BOUND [HEIDENREICH TO APPEAR]

$\Downarrow$

SUPEREXTREMAL $\sim$ WGC ✓

GOAL

DOES THERE ALWAYS EXIST A STATE THAT IS <u>SELF-REPULSIVE</u> OR HAS <u>ZERO</u> SELF-FORCE?

RFC ✓

IF THIS HOLDS EVERYWHERE IN MODULI SPACE, THEN THE STATE IS SUPEREXTREMAL.

WGC ✓

THE CANDIDATES

MODULAR INVARIANCE ALLOWS/FORCES TO HAVE STATES RELATED BY SPECTRAL FLOW:

$$Q \to Q + \rho \qquad \left(\tilde{Q} \to \tilde{Q} + \tilde{\rho} \right)$$

$$T = \Delta - \frac{1}{2} Q^2 \qquad \bar{T} = \tilde{\Delta} - \frac{1}{2} \tilde{Q}^2$$

LIGHTEST LEVEL-MATCHED STATES

$$\frac{\alpha'}{4} m^2 = \max \left(T_{min} + \frac{1}{2} Q^2 , \tilde{T}_{min} + \frac{1}{2} \tilde{Q}^2 \right)$$

THE CANDIDATES

OUR CANDIDATES ARE STATES RELATED BY SPECTRAL FLOW TO THE GRAVITON:

$$\Delta - \frac{1}{2}Q^2 = 0$$

- SHOWN TO BE EXTREMAL IN ORBIFOLD EXAMPLES

WE TAKE RFC POINT OF VIEW
WANT TO COMPUTE THE 'SELF-FORCE'?

- HOW DO WE SET IT UP?

THE EFT

- U(1) GAUGE THEORY COUPLED TO GRAVITY + MODULI

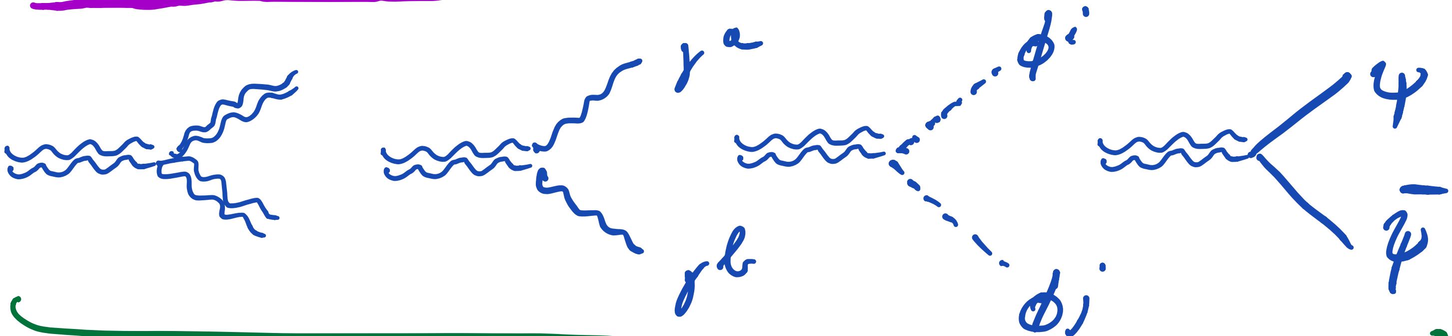
GRAVITON $g_{\mu\nu}$

GAUGE FIELD A_μ

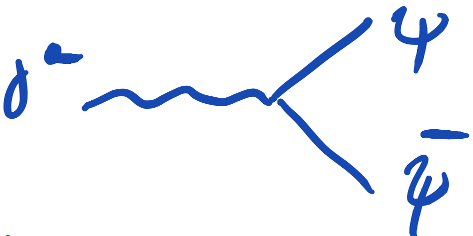
CHARGED STATES ψ

MODULI ϕ_i

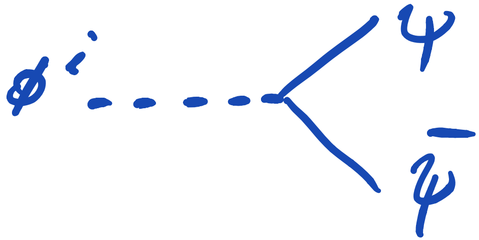
Tree Level Diagrams



WAVEFUNCTION RENORMALIZATION



CHARGE



SCALAR CHARGE

WHAT TO COMPUTE

UNIVERSAL TERMS, IF NEEDED, CAN BE COMPUTED IN ONE EXAMPLE

$\psi\Psi\phi_i$: PARAMETRIZES THE VARIATION IN SELF-FORCE OF THE CHARGED STATES.

COMPUTE SCALAR CHARGE, DETERMINE DEPENDENCE ACROSS MODULI SPACE.

$\downarrow$

THIS IS WHAT WE WANT!

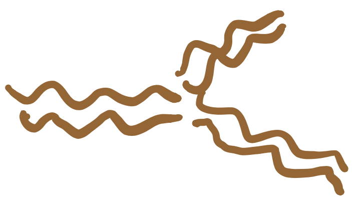
WORLD SHEET THEORY

BOSONIC

$$\underbrace{(\text{FREE BOSON})^D}_{\text{UNIVERSAL}} \otimes \underbrace{(\text{UNITARY CFT})}_{\substack{\text{ONLY NEED}\\ \text{CURRENT/VIRASORO ALG.}\\ \text{\& EXACTLY MARGINAL OPS.}}}$$

E.G. GRAVITON VERTEX OP.

$$\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1}|0,k\rangle \otimes \underbrace{|1\rangle}_{\substack{\text{GROUND STATE IN}\\ \text{INTERNAL PART.}}}$$

$\longrightarrow$  IS UNIVERSAL.

WS CORRELATORS

WE WANT TO COMPUTE
3-POINT FUNCTIONS IN THE EFT

LIKE THE VERTEX OPERATORS, THESE
WILL FACTORIZE

$$\langle A\, B\, C \rangle_{EFT} \simeq \underbrace{\langle A\, B\, C \rangle_{EXT}} \times \underline{\langle A\, B\, C \rangle_{INT}}$$

EXTERNAL PART
IS FREE THEORY
↓
UNIVERSAL CONTRIBUTIONS

FOCUS
↓
INTERESTING
STRUCTURE FROM
INTERNAL CFT

Wavefunction Renormalization Terms

These all contain 1 graviton:

- Vertex operator:

$$g_{\mu\nu\,EFT} \sim \alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1}|0,k\rangle_{EXT} \otimes |1\rangle_{INT}$$

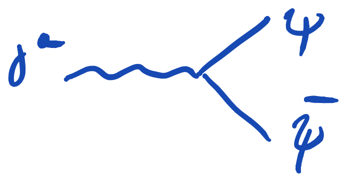
In terms of internal CFT correlators, these are just 2-point functions.

$$\propto \langle \phi^i | \phi^j \rangle$$

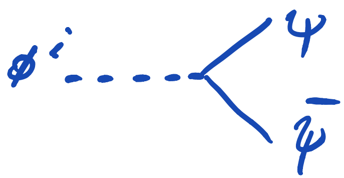
$\sim$ 2-point function normalization

MORE INTERESTING 3-POINT FUNCTIONS

INTERESTING PIECES ARE GENUINE INTERNAL CFT 3-POINT FUNCTIONS



$\longrightarrow$ THIS SHOULD GIVE CHARGE NORMALIZATION OF CHARGED STATES



$\longrightarrow$ MAPS TO INTERACTIONS BETWEEN CHARGED STATES AND MODULI

MORE INTERESTING 3-POINT FUNCTIONS

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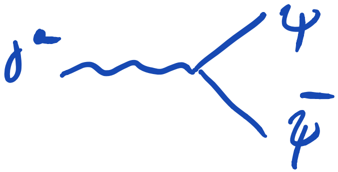
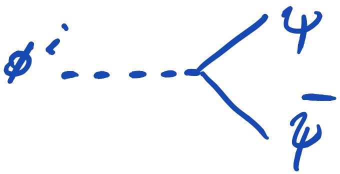


→ THIS SHOULD GIVE CHARGE NORMALIZATION OF CHARGED STATES



→ MAPS TO INTERACTIONS BETWEEN CHARGED STATES AND MODULI

FIRST ONLY DEPENDS ON CONSERVED CURRENT PART. ANOTHER UNIVERSAL PIECE.

VERTEX OPERATORS

A GENERIC STATE IS:

$$C^{\mu\nu}|0,k\rangle \otimes |\xi\rangle$$

$$(\#,\#') + \left(\frac{\alpha'}{4}k^2, \frac{\alpha'}{4}k^2\right) + (h_\xi, \tilde{h}_\xi) = (1,1).$$

EFT	WS DESCRIPTION
$g_{\mu\nu}$	$\alpha^\mu_{-1}\tilde{\alpha}^\nu_{-1}|0,k\rangle \otimes |1\rangle$
A_μ	$\alpha^\mu_{-1}|0,k\rangle \otimes |\tilde{\lambda}\rangle$, $\tilde{\alpha}^\mu_{-1}|0,k\rangle \otimes |\lambda\rangle$
ψ	$\alpha^\mu_{-1}\tilde{\alpha}^\nu_{-1}|0,k\rangle \otimes \underbrace{|1_Q\rangle}_{\text{SPECTRAL FLOW FROM GRAVITON}}$
ϕ	$|0,k\rangle \otimes |\chi\rangle$

STATE - OPERATOR MAP

CFT STATES	OPERATORS	WEIGHTS
$\lvert 1 \rangle$	$\hat{1}$	$(0,0)$
$\lvert \lambda \rangle$	$J^a(z)$	$(1,0)$
$\lvert \tilde{\lambda} \rangle$	$\tilde{J}^b(\bar{z})$	$(0,1)$
$\lvert 1_Q \rangle$	$\hat{Q}(z,\bar{z})$	$\left(\frac{1}{2}Q_L^2, \frac{1}{2}Q_R^2\right)$
$\lvert \chi \rangle$	$\chi_{1,1}(z,\bar{z})$	$(1,1)$

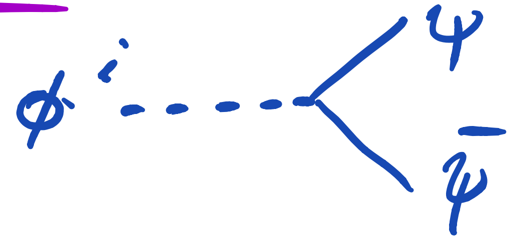
WE HAVE TWO OPERATORS OF WEIGHT (1,1)

$$\lambda^{ab}(z,\bar{z}) = J^a(z)\tilde{J}(\bar{z}) \quad \& \quad \mathcal{O}_{1,1}(z,\bar{z})$$

- MAKE SURE ($\to$ IMPOSE) NO KINETIC MIXING

$$\mathcal{O}_{1,1} \in \chi_{1,1} \qquad \langle \lambda^{ab}(z,\bar{z})\, \mathcal{O}_{1,1}(z',\bar{z}') \rangle = 0$$

INTERACTION WITH MODULI

$\langle 1_Q | \mathcal{O}_{1,1}(z,\bar{z}) | 1_Q \rangle \sim$ 

• IN THE PRESENCE OF CONSERVED CURRENTS, SUGAWARA DECOMPOSITION FACTORIZES ALGEBRA

$$L_m = \hat{L}_m + L^j_m$$

$\phi \sim \mathcal{O}_{1,1}$ IS A NEUTRAL CURRENT PRIMARY, THEREFORE:

$$[L_m, \mathcal{O}_{1,1}(z,\bar{z})] = [\hat{L}_m, \mathcal{O}_{1,1}(z,\bar{z})]$$

INTERACTION WITH MODULI

MOREOVER, THE $|1_Q\rangle$ STATES HAVE CONSTRAINED WEIGHTS $\sim \left(\frac{1}{2}Q_L^2, \frac{1}{2}Q_R^2\right)$

WE EXPECT, THANKS TO SUGAWARA, FACTORIZATION:

$$\langle 1_Q | \mathcal{O}_{1,1}(z,\bar z) | 1_{Q'}\rangle \sim$$

$$\langle 1_Q | 1_{Q'}\rangle \langle \mathcal{O}_{1,1}(z,\bar z)\rangle$$

ONLY DEPENDENT ON CURRENT PART OF ALGEBRA

ONLY DEPENDENT ON "HAT" (ORTHOGONAL TO CURRENT) PART OF VIRASORO ALGEBRA.

THE (TOO) QUICK ANALYSIS

$$\langle 1_Q | \mathcal{O}_{1,1}^{(2,\bar{2})} | 1_Q \rangle$$

$$\Downarrow$$

FROM ALGEBRA:

$$\langle 1_Q | \cdot \left(\mathcal{O}_{1,1} = [L_0, \mathcal{O}_{1,1}] = [\hat{L}_0, \mathcal{O}_{1,1}] = L_0 \mathcal{O}_{1,1} - \mathcal{O}_{1,1} L_0 \right) \cdot | 1_Q \rangle$$

$$\langle \ldots \rangle \propto \hat{L}_0 | 1_Q \rangle =$$

$$(L_0 - L_0^j) | 1_Q \rangle =$$

$$\left(\tfrac{1}{2} Q_L^2 - \tfrac{1}{2} Q_L^2 \right) | 1_Q \rangle = 0 \; ?$$

SCALING DIMENSION

CHARGE EIGENVALUE

THE (TOO) QUICK ANALYSIS

$$\langle 1_Q | \theta_{1,1}(z,\bar{z}) | 1_Q \rangle$$

$$\downarrow$$

FROM ALGEBRA:

$$\langle 1_Q | \cdot \theta_{1,1} = [L_0, \theta_{1,1}] = [\hat{L}_0, \theta_{1,1}] \cdot | 1_Q \rangle$$

$$= L_0 \theta_{1,1} - \theta_{1,1} L_0$$

$\langle \ldots \rangle$

WAIT!

SHIFTING VIRASORO MODES

THE CORRELATION FUNCTION WAS:

$$\langle 1_Q | \underbrace{\mathcal{O}_{1,-1}(z,\bar{z})}_{\S} | 1_Q \rangle$$

$$[L_0(z), \mathcal{O}_{1,1}(z,\bar{z})] = [\hat{L}_0(z), \mathcal{O}_{1,1}(z,\bar{z})] \propto \hat{L}_\delta(z) | 1_Q \rangle$$

WE NEED TO CHECK THAT SHIFTED L_m DECOMPOSE APPROPRIATELY. .

VIRASORO MODES L_m AND CURRENT MODES L_m^j DEFINED BY CONTOUR INTEGRALS.

WE CAN COMPUTE THESE AT ARBITRARY POSITION.

THE IMPROVED ANALYSIS

SUMMARY. SHIFT AND CURRENT DECOMPOSITION ~ COMMUTE

$$\begin{cases} L_0(z) = L_0 - zL_{-1} \\ L_0^j(z) = L_0^j - zL_{-1}^j \end{cases} \Rightarrow \hat{L}_0(z) = \hat{L}_0 - z\hat{L}_{-1}$$

BUT $\hat{L}_0 |1_a\rangle = \hat{L}_{-1} |1_a\rangle = 0$

ALSO WHEN SHIFTED $\hat{L}_0(z) |1_a\rangle = 0$

$$\Downarrow$$

$$\langle 1_a | \theta_{1,1}(z,\bar{z}) | 1_a\rangle = \underline{0} \checkmark$$

GENERALIZING TO THE SUPERSTRING

1- SUPERCONFORMAL PRIMARIES ARE NOW STATES WITH DIFFERENT WEIGHTS

2- AMPLITUDES / GHOST CONTRIBUTIONS DEPEND ON INTERACTION PICTURE

3- SUGAWARA CURRENT DECOMPOSITION HAS MORE STRUCTURE.

HOW MUCH OF WHAT WE DISCUSSED CAN WE CARRY OVER?

SUPERCONFORMAL PRIMARIES

1 - SUPERCONFORMAL PRIMARIES ARE NOW STATES OF WEIGHT $\left(\frac{1}{2}, \frac{1}{2}\right)$ TYPE II

$\left(1, \frac{1}{2}\right)$ HETEROTIC

BUILDING BLOCKS IN INTERNAL THEORY ARE NOW

TYPE II

CFT STATES	OPERATORS	WEIGHTS
$\|1\rangle$	$\hat{1}$	$(0,0)$
$\|\lambda\rangle$	$J^a(z)$	$\left(\frac{1}{2}, 0\right)$
$\|\tilde{\lambda}\rangle$	$\tilde{J}(\bar{z})$	$\left(0, \frac{1}{2}\right)$
$\|1_Q\rangle$	$\hat{Q}(z,\bar{z})$	$\left(\frac{1}{2} Q_L^2, \frac{1}{2} Q_R^2\right)$
$\|\chi\rangle$	$\mathcal{O}_{\frac{1}{2},\frac{1}{2}}(z,\bar{z})$	$\left(\frac{1}{2}, \frac{1}{2}\right)$

PHYSICAL AMPLITUDES

2 - WE CAN AVOID GHOSTS BY WORKING IN THE (0,0) PICTURE

$$|\psi\rangle \longrightarrow G_{-\frac{1}{2}}\tilde{G}_{-\frac{1}{2}}|\psi\rangle = |\psi\rangle^0$$

BUT WITH OUR FACTORIZED DESCRIPTION OF THE WORLD-SHEET:

$$\left(G^{EXT}_{-\frac{1}{2}} \oplus G^{INT}_{-\frac{1}{2}}\right)\left(\tilde{G}^{EXT}_{-\frac{1}{2}} \oplus \tilde{G}^{INT}_{-\frac{1}{2}}\right)\left(|\xi\rangle_{EXT} \otimes |\xi\rangle_{INT}\right)$$

$\rightarrow$ GENERATES A CONSIDERABLE PROLIFERATION OF STATES

$\sim$ 4x EFT FIELDS

PHYSICAL AMPLITUDES

LUCKILY, WE ONLY NEED TO COMPUTE 3-POINT FUNCTIONS.

PHYSICAL CORRELATOS =

$${}^{0}\langle A | B^{0} | C \rangle^{0} \equiv {}^{-1}\langle A | \underline{B^{0}} | C^{-1} \rangle$$

SUBSCRIPTS INDICATE THE "PICTURE"

ONLY NEED TO TRANSFORM THE MIDDLE OPERATOR

$$\mathcal{O}_{\frac{1}{2},\frac{1}{2}} \rightarrow \begin{cases} \mathcal{O}_{\frac{1}{2},\frac{1}{2}}, & \mathcal{O}_{1,\frac{1}{2}} = G^{INT}_{\frac{1}{2}} \mathcal{O}_{\frac{1}{2},\frac{1}{2}} \\ \mathcal{O}_{\frac{1}{2},1} = \tilde{G}^{INT}_{\frac{1}{2}} \mathcal{O}_{\frac{1}{2},\frac{1}{2}}, & \mathcal{O}_{1,1} = G^{INT}_{\frac{1}{2}} \tilde{G}^{INT}_{\frac{1}{2}} \mathcal{O}_{\frac{1}{2},\frac{1}{2}} \end{cases}$$

SUPER-SUGAWARA DECOMPOSITION

3 - STRESS TENSOR(S) IN SUPERSTRING.

$$T_F = \bar{\bar{T}}_F + T_F^{(j,\psi)}$$

$$T_B = \bar{\bar{T}}_B + T_B^{(j,\psi)} = \bar{T}_B + T_B^{j} + T_B^{\psi}$$

BOSONIC + FERMIONIC CURRENTS

WE HAVE TO KEEP TRACK + COMPUTE SHIFTED VERSIONS OF MANY OPERATORS + CHECK CONSISTENCY OF ALGEBRA

$G_n^{(j,\psi)}$ $\quad \overset{\psi}{\dot{L}}_m$ $\quad \hat{G}_n^{\cdot}$ $\quad L_m$ $\quad L'_m$ $\quad \hat{L}_m$ $\quad G_n$

all shifted of course!

WHAT CORRELATORS SURVIVE

SO, CAN WE RUN THE SAME ARGUMENT WE USED IN THE BOSONIC CASE?

BOSONIC CASE: $\langle 1_Q | \mathcal{O}_{1,1}(z,\bar{z}) | 1_{Q'} \rangle$

$\{|1_Q\rangle\}$, $\mathcal{O}_{1,1}$ WERE CONFORMAL PRIMARIES IN COMPLETELY SEPARATE SECTORS.

SUPERSTRING. CORRELATORS THAT SURVIVE:

$$\langle 1_Q | \mathcal{O}_{\frac{1}{2},\frac{1}{2}} | 1_{Q'} \rangle \qquad \langle 1_Q | \mathcal{O}_{1,\frac{1}{2}} | 1_{Q'} \rangle$$

$$\langle 1_Q | \mathcal{O}_{\frac{1}{2},1} | 1_{Q'} \rangle \qquad \langle 1_Q | \mathcal{O}_{1,1} | 1_{Q'} \rangle$$

WHAT CORRELATORS SURVIVE

$$\langle 1_Q | \mathcal{O}_{x,y}(z,\bar{z}) | 1_{Q'} \rangle$$

ACT WITH VIRASORO MODES

SUPER-SUGAWARA DECOMPOSITION

$\otimes$

SHIFT IN THE COORDINATES

LOTS OF ALGEBRA

$$\sim \langle 1_Q | 1_{Q'} \rangle \langle \mathcal{O}_{x,y} \rangle = 0$$

THE RESULT

$\langle 1_Q | \theta_{x,y}(z,\bar{z}) | 1_Q \rangle = 0 \quad \Rightarrow$

THE $|1_Q\rangle$ STATES ARE NOT CHARGED UNDER THE MODULUS CORRESPONDING TO $\theta_{\frac{1}{2},\frac{1}{2}}$ (+ DESCENDANTS)

→ $|1_Q\rangle$ STATES DO NOT HAVE SCALAR CHARGE CONTRIBUTION

THE RESULT

1 - $|1 Q\rangle$ STATES CONNECTED TO GRAVITON BY SPECTRAL FLOW.

2 - COMPUTED IN AN EXAMPLE TO HAVE ZERO SELF-FORCE

3 - VANISHING CONTRIBUTION TO SCALAR CHARGE → NO DEPENDENCE ON MODULI

4 - 2 HOLDS EVERYWHERE IN MODULI SPACE (THANKS TO 3) + ~ BOGOMOL'NYI BOUND

$\Downarrow$

SUPEREXTREMAL STATES

WGC ✓

Thanks!

REFS.

ARKANI-HAMED ET AL. HEP-TH/0601001

HEIDENREICH ET AL. 1606.08437

MONTERO ET AL. 1606.08438

HEIDENREICH ET AL. 1906.02206

HEIDENREICH 2006.09378

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