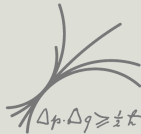


Small W_0 near the Conifold

Max Brinkmann

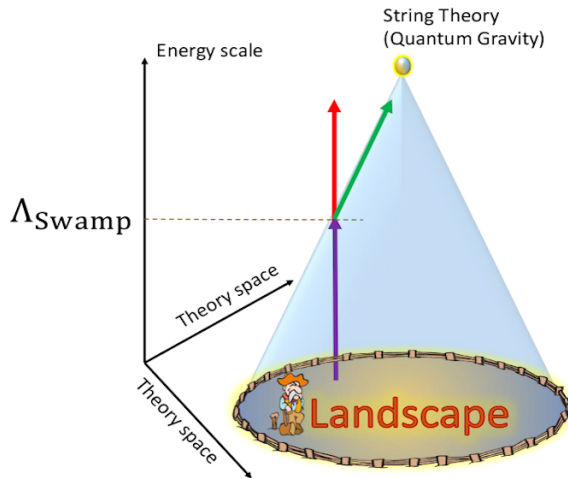
String Phenomenology Seminar Series

06.10.2020



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FÜR PHYSIK

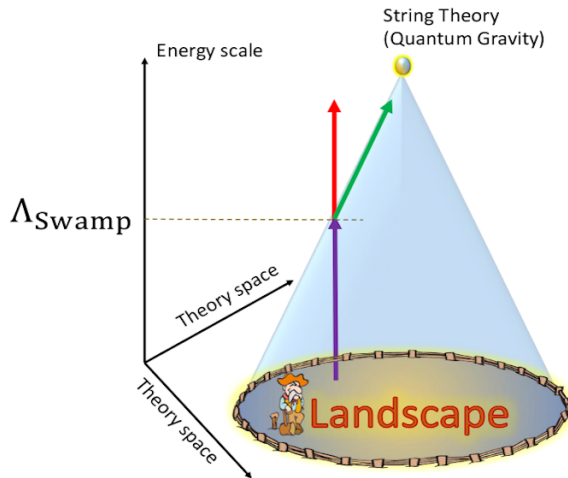
The Big Picture



[adapted from Eran Palti's review, 1903.06239]

Where does **dS** lie in this picture?

The Little Picture



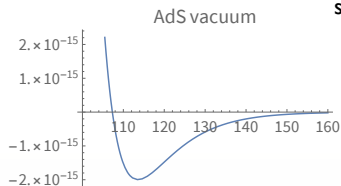
[adapted from Eran Palti's review, 1903.06239]

Where does **KKLT** lie in this picture?

KKLT – a recipe for dS

CS moduli stabilized
at $|W_0| \ll 1$

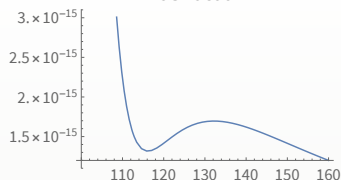
nonpert. effects



strong warping
(conifold)

small $\overline{D3}$ uplift

dS vacuum



KKLT – a recipe for dS

[Kachru,Kalosh,Linde,Trivedi '03]

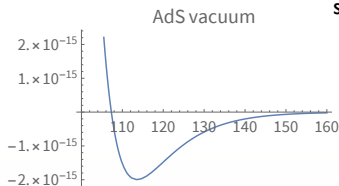
CS moduli stabilized

at $|W_0| \ll 1$

[statistics: Douglas et al '04/'05]

[Demirtas,Kim,McAllister,Moritz '19]

nonpert. effects



[Sethi '17]

[Moritz,Retolaza,Westphal '17]

[Gautason, Van Hemelryck, Van Riet '19]

[Kallosh,Linde,McDonough,Scalisi '18]

[Hamada,Hebecker,Shiu,Soler '18]

[Carta,Moritz,Westphal '19]

[Blumenhagen,M.B.,Makridou '20]

strong warping

(conifold)

[Klebanov,Strassler '00]

[Giddings,Kachru,Polchinski '02]

[Douglas,Shelton,Torroba '07]

[Blumenhagen,Herschmann,Wolf '16]

small $\overline{D3}$ uplift

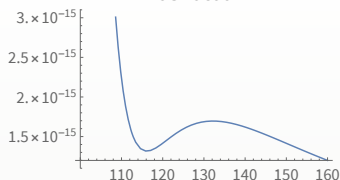
[Bena,Graña,Halmagry '10]

[Michel,Mintun,Polchinski,Puhm,Saad '14]

[Bena,Dudas,Graña,Lüst '18]

[Blumenhagen,Kläwer,Schlechter '19]

dS vacuum

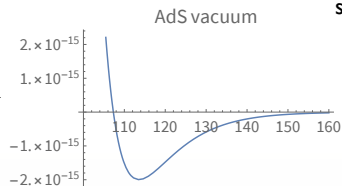


[...and many, many more!]

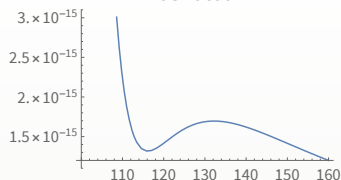
KKLT – a recipe for dS



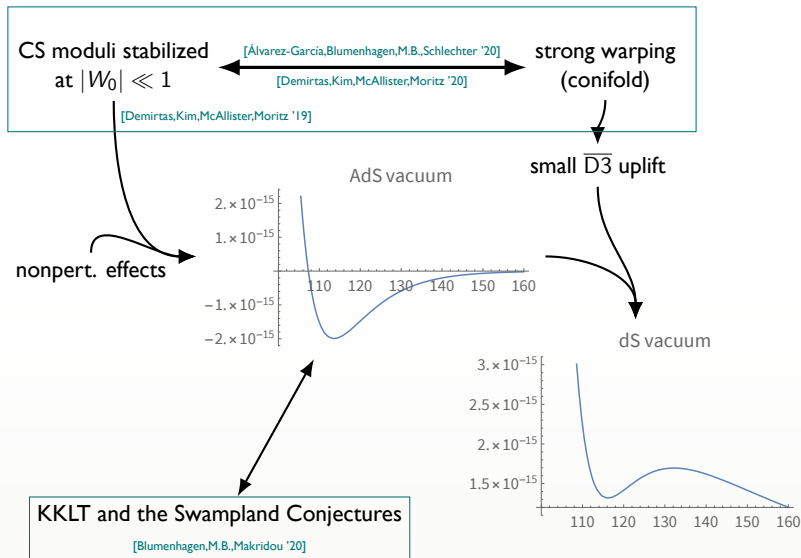
nonpert. effects



small $\overline{D3}$ uplift



KKLT – a recipe for dS



A recipe for $|W_0| \ll 1$

Idea: natural hierarchies

Small values can naturally be generated by perturbative mechanisms, if the leading term vanishes analytically.

The prepotential of the complex structure moduli splits into a classical and a nonperturbative part, if they are

- at weak string coupling,
- at large complex structure,
- expressed in terms of the mirror variables.

A recipe for $|W_0| \ll 1$

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The prepotential of the complex structure moduli splits into a classical and a nonperturbative part, if they are

- at weak string coupling,
- at large complex structure,
- expressed in terms of the mirror variables.

To-Do List

1. Find fluxes that solve pert. F-term eq. for vanishing superpotential.
2. Generate a small nonpert. potential for the remaining flat direction.

A recipe for $|W_0| \ll 1$

Setup

- Orientifold X of a CY 3-fold.
- Wrapped by D7-branes carrying $-Q_{D3}$.
- Period vector $\Pi = \begin{pmatrix} U^a \\ \mathcal{F}_a \end{pmatrix}$ w/ sympl. pairing $\Sigma = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.
- Prepotential \mathcal{F} s.th. $\mathcal{F}_a = \partial_{U^a} \mathcal{F}$.
- Inhomogeneous coordinates: $U^0 = 1$, $\mathcal{F}_0 = 2\mathcal{F} - U^i \mathcal{F}_i$.

The prepotential splits into $\mathcal{F}(U) = \mathcal{F}_{\text{pert}}(U) + \mathcal{F}_{\text{inst}}(U)$.

Including the axio-dilaton:

$$K = -\log(-i \Pi^\dagger \Sigma \Pi) - \log(S + \bar{S}),$$
$$W = (F + iSH)^T \cdot \Sigma \cdot \Pi.$$

A recipe for $|W_0| \ll 1$

Perturbative Prepotential

$$\mathcal{F}_{\text{pert}}(U) = -\frac{1}{3!}\mathcal{K}_{abc}U^aU^bU^c + \frac{1}{2}a_{ab}U^aU^b + b_aU^a + \xi$$

with

- \mathcal{K}_{abc} the triple intersection numbers of the mirror CY,
- $\xi = -\frac{\zeta(3)\chi}{2(2\pi i)^3}$, and χ the Euler number of the mirror CY,
- a_{ab} , b_a rational numbers related to the mirror CY.

A recipe for $|W_0| \ll 1$

Lemma: existence of a perturbatively flat vacuum

If a pair $(\vec{M}, \vec{K}) \in \mathbb{Z}^n \times \mathbb{Z}^n$ satisfies:

$$-\frac{1}{2}\vec{M} \cdot \vec{K} \leq Q_{D3},$$

$$N_{ab} = \mathcal{K}_{abc} M^c \text{ is invertible,}$$

$$\vec{K}^T N^{-1} \vec{K} = 0,$$

$\vec{p} = N^{-1} \vec{K}$ lies in the Kähler cone,

$a \cdot \vec{M}$ and $b \cdot \vec{M}$ are integer-valued.

Then:

The tadpole bound is satisfied,

$\partial W = 0$ has solution $\vec{U} = \vec{p}S$,

$W = 0$ in that solution,

the minimum is trustable

and the fluxes are integer valued.

[Demirtas, Kim, McAllister, Moritz '19]

The appropriate choice of fluxes is given by

$$F = (\vec{b} \cdot \vec{M}, a \cdot \vec{M}, 0, \vec{M})^T,$$

$$H = (0, \vec{K}, 0, 0)^T,$$

s.th. the superpotential is hom. of deg. 2 in the U^i .

A recipe for $|W_0| \ll 1$

Instanton Contributions

$$\mathcal{F}_{\text{inst}}(U) \sim \sum_{\vec{q}} e^{2\pi i \vec{q} \cdot \vec{U}}$$

with the sum running over effective curves.

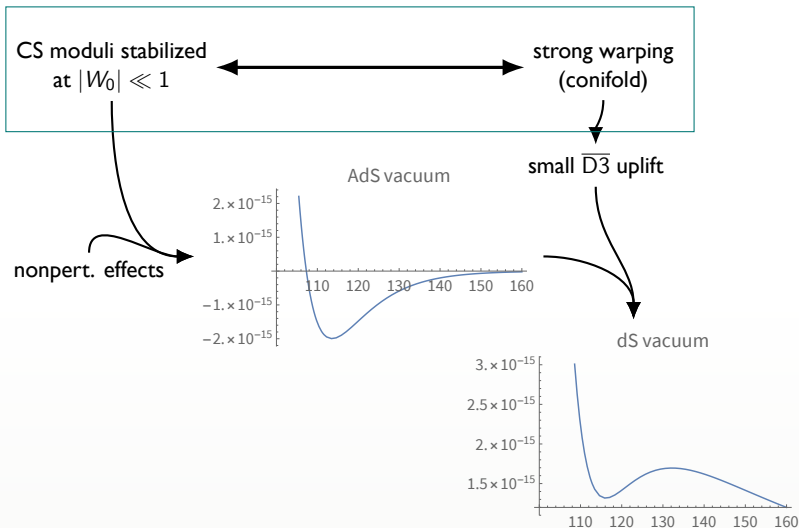
Stabilizing the axio-dilaton: Racetrack

In the perturbative minimum: $\vec{U} = \vec{p}S$.

$$W_{\text{eff}}(S) \sim M^a \partial_a \mathcal{F}_{\text{inst}} \sim \sum_{\vec{q}} e^{2\pi i S \vec{p} \cdot \vec{q}}$$

This stabilizes S to exponentially small values if the two leading instanton contributions satisfy $\vec{p} \cdot \vec{q}_1 \approx \vec{p} \cdot \vec{q}_2$.

What's next?



Periods at the Conifold

The hardest part is finding the periods/prepotential at the conifold!

Method 1: Resumming with GV-nilpotent curves

- + Elegantly avoid complete computation.
- Only special cases: conifold cycle mirror to shrinking curve.
- No full solution, only good approximation.

[Demirtas, Kim, McAllister, Moritz '20]

Method 2: analytic computation of transition matrices

- + Fairly general method
- + Complete solution, can check approximations numerically
- Fairly involved computations

[Álvarez-García, Blumenhagen, M.B., Schlechter '20; Lorenz' talk]

Periods at the Conifold

Transition Matrix

Periods in a symplectic basis hard to compute. Solving the Picard-Fuchs equations $\mathcal{D}_i \omega = 0$ in any local basis is easy.

$$\Pi = m \cdot \omega$$

At LCS, monodromies fix the transition matrix m uniquely. At the conifold monodromies are not enough.

We can rewrite the fundamental period at the LCS as a hypergeometric function, which allowed us to analytically determine the transition matrix.

Periods at the Conifold

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With the analytic transition matrix we can efficiently compute the periods at the conifold to very high order (30+), as well as the prepotential and mirror maps. (*Lorenz' talk*)

$|W_0| \ll 1$ at conifold

Setup

- n-parameter CY orientifold
- One modulus close to the conifold, others at LCS

Notation: $X^0 = 1$, $X^i = (U^\alpha, Z)^T$, $(i = 1, \dots, n)$, $(\alpha = 1, \dots, n-1)$

Prepotential

$$\mathcal{F}_{\text{pert}} = -\frac{1}{3!} \mathcal{K}_{ijk} X^i X^j X^k + \frac{1}{2} A_{ij} X^i X^j + B_i X^i + C - \frac{Z^2 \log Z}{2\pi i},$$

$$\mathcal{F}_{\text{inst}} = \frac{1}{(2\pi i)^3} \sum_{\vec{c}} a_{\vec{c}} \prod_{i=1}^n q_i^{n_i},$$

$|W_0| \ll 1$ at conifold

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$$\mathcal{F}_{\text{inst}} = \frac{1}{(2\pi i)^3} \sum_{\vec{c}} a_{\vec{c}} \prod_{i=1}^n q_i^{n_i},$$

Note that while the q_i are still simply exponentials in the LCS moduli, the conifold modulus enters linearly!

$|W_0| \ll 1$ at conifold

To-Do List

1. Find fluxes that solve pert. eq. for vanishing superpotential at $Z = 0$.
2. Stabilize conifold modulus at leading order to $W_Z = O(Z)$.
3. Include instanton corrections for the remaining flat direction.

$|W_0| \ll 1$ at conifold

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Choosing Fluxes

Want the superpotential to be degree 2 in the LCS moduli:

$$F = \begin{pmatrix} B_i M^i \\ (A_{i\alpha} M^i, M^n)^T \\ 0 \\ \vec{M} \end{pmatrix}, \quad H = \begin{pmatrix} 0 \\ \vec{K} \\ 0 \\ 0 \end{pmatrix}, \quad \vec{M}, \vec{K} \in \mathbb{Z}^n$$

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$$\Rightarrow B_i M^i, A_{i\alpha} M^i \in \mathbb{Z}$$

$$\Rightarrow B_i, A_{i\alpha} \text{ must be rational}$$

$|W_0| \ll 1$ at conifold

Step I: first order, $Z=0$

Stabilizing the LCS moduli at the conifold locus:

$$\begin{aligned} W = \frac{1}{2} N_{\alpha\beta} U^\alpha U^\beta + i S K_\alpha U^\alpha = 0 & \quad \left| \quad (N^{-1})^{\alpha\beta} K_\alpha K_\beta = 0 \right. \\ \partial_\alpha W = 0 & \quad \left| \quad U^\alpha = p^\alpha S \right. \end{aligned}$$

with $N_{\alpha\beta} = \mathcal{K}_{i\alpha\beta} M^i$ and $p^\alpha = -i(N^{-1})^{\alpha\beta} K_\beta$.

$|W_0| \ll 1$ at conifold

Step 1: first order, $Z=0$

Stabilizing the LCS moduli at the conifold locus:

$$\begin{array}{l|l} W = \frac{1}{2} N_{\alpha\beta} U^\alpha U^\beta + i S K_\alpha U^\alpha = 0 & (N^{-1})^{\alpha\beta} K_\alpha K_\beta = 0 \\ \partial_\alpha W = 0 & U^\alpha = p^\alpha S \end{array}$$

with $N_{\alpha\beta} = \mathcal{K}_{i\alpha\beta} M^i$ and $p^\alpha = -i(N^{-1})^{\alpha\beta} K_\beta$.

$$\Rightarrow N_{\alpha\beta} = \mathcal{K}_{i\alpha\beta} M^i \text{ invertible}$$

$|W_0| \ll 1$ at conifold

Step 2: Stabilize Z

Integrating out the LCS moduli U^α :

$$W_{\text{pert}}(S, Z) = -\frac{mZ \log(Z)}{2\pi i} + m_1 Z + n_1 SZ + O(Z^2),$$

$$D_Z W = \partial_Z W + \partial_Z K \cdot O(Z)$$

$|W_0| \ll 1$ at conifold

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$$D_Z W = \partial_Z W + \partial_Z K \cdot O(Z)$$

$$\Rightarrow Z_0 = \zeta_0 e^{-2\pi p^Z S}, \quad W_Z = \frac{mZ}{2\pi i} + O(Z^2)$$

$|W_0| \ll 1$ at conifold

Step 3: Instanton Contributions and Racetrack

The CS moduli are stabilized to $\log(Z) \sim U^\alpha \sim S$.

$$W_{\text{eff}}(S) = -M^i \partial_i \mathcal{F}_{\text{inst}} + \frac{mZ}{2\pi i} \sim \sum a_n e^{c_n S}.$$

That this can be stabilized to small W_0 has to be checked case by case.

$|W_0| \ll 1$ at conifold - Example

Example: 3-parameter CY $\mathbb{P}_{1,1,2,8,12}[24]$

$$\mathcal{K}_{111} = 8, \mathcal{K}_{112} = 2, \mathcal{K}_{113} = 4, \mathcal{K}_{123} = 1, \mathcal{K}_{133} = 2,$$

$$A_{33} = \left(\frac{1}{2} + \frac{3 - 2 \log(2\pi)}{2\pi i} \right), \quad B = \left(\frac{23}{6}, 1, \frac{23}{12} \right)^T.$$

The leading instanton contributions are

$$\mathcal{F}_{\text{inst}} = -\frac{5i q_{U^1}}{36\pi^3} - \frac{493 q_{U^1}^2}{10368\pi^3} + \frac{5i q_{U^1} q_Z}{36\pi^3} + \dots,$$

$$q_{U^1} = 864 e^{2\pi i U^1}, \quad q_{U^2} = \frac{4}{(\frac{\pi}{i} Z)^4} e^{2\pi i U^2}, \quad q_Z = (\frac{\pi}{i} Z)$$

$|W_0| \ll 1$ at conifold - Example

First Checks

$B_i M^i, A_{i\alpha} M^i \in \mathbb{Z} \Rightarrow B_i, A_{i\alpha}$ must be rational ✓

After steps 1 and 2: $Z_0 \ll 1$ for $\Re(S) \gg 1$ ✓

Choosing Fluxes

The fluxes must be chosen to satisfy

- $B_i M^i, A_{i\alpha} M^i \in \mathbb{Z}$,
- $N_{\alpha\beta} = \mathcal{K}_{i\alpha\beta} M^i$ invertible,
- $(N^{-1})^{\alpha\beta} K_\alpha K_\beta = 0$,
- Instanton series: $1 > |q_i| \sim |e^{f(\vec{M}, \vec{K})S}| \leftrightarrow f(\vec{M}, \vec{K}) < 0$.

A simple search finds many reasonably small fluxes satisfying these.

$|W_0| \ll 1$ at conifold - Example

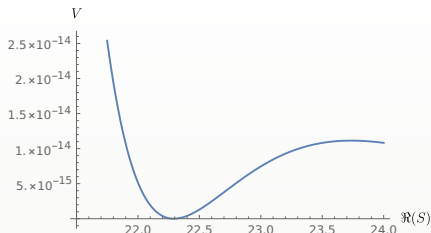
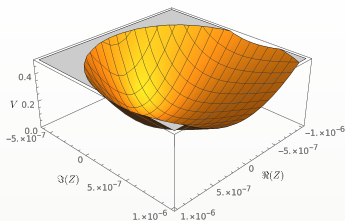
A concrete solution

Choosing $\vec{M} = (-24, 120, 24)^T$, $\vec{K} = (-9, 3, -4)^T$:

$$\langle U^1 \rangle = 2.79 i, \quad \langle U^2 \rangle = 8.36 i, \quad \langle Z \rangle = 1.36 \cdot 10^{-6} i, \quad \langle S \rangle = 22.3,$$

$$|q_i| = (2 \cdot 10^{-5}, 0.2, 4 \cdot 10^{-6}),$$

$$W_0 = -3.10 \cdot 10^{-6}.$$



$|W_0| \ll 1$ at conifold - Example

Masses

Full access to the Periods allows us to compute the masses.

$$\{m^2\} = \underbrace{\{6 \cdot 10^{14}\}}_Z, \underbrace{\{1 \cdot 10^3, 3 \cdot 10^2\}}_{U^i}, \underbrace{\{2 \cdot 10^{-11}\}}_S M_{\text{pl}}^2.$$

Note that $m_S^2 \approx |W_0|^2 \approx m_{\text{KKLT}}^2$:

The axio-dilaton cannot be integrated out before KKLT starts!

$|W_0| \ll 1$ at conifold - Example

Masses

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$$\{m^2\} = \underbrace{\{6 \cdot 10^{14}, 1 \cdot 10^3, 3 \cdot 10^2\}}_Z \underbrace{\{2 \cdot 10^{-11}\}}_{U^i} M_{\text{pl}}^2.$$

Note that $m_S^2 \approx |W_0|^2 \approx m_{\text{KKLT}}^2$:

The axio-dilaton cannot be integrated out before KKLT starts!

Search Results

- Inexhaustive search over fluxes
- Semianalytic computation, no numerical checks
- $O(10^4)$ vacua with $|W_0| \leq 10^{-6}$
- More detailed search described by [Demirtas, Kim, McAllister, Moritz '20]

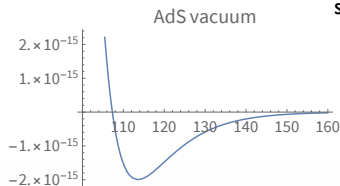
KKLT and the Swampland Conjectures

CS moduli stabilized
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strong warping
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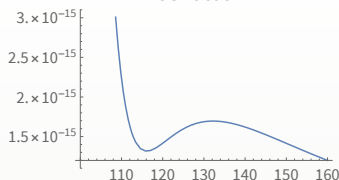


nonpert. effects



small $\overline{D3}$ uplift

dS vacuum



**KKLT and the Swampland:
The AdS Distance Conjecture**

[Blumenhagen, M.B., Makridou '20]

AdS distance conjecture (ADC)

For an AdS vacuum the limit $\Lambda \rightarrow 0$ is at *infinite distance* in field space and there is a *tower of light states* with

$$m_{\text{tower}} = c_{\text{AdS}} |\Lambda|^\alpha$$

for $\alpha > 0$. Supersymmetric vacua have $\alpha = 1/2$.

- Usually this tower is assumed to be the KK-tower.
- Satisfied by most treelevel vacua.
- DGKT are counterexamples for the strong version!
- Strong version forbids scale separation.
- Is SUSY the wrong distinguishing feature for strong version?

KKLT AdS minimum

$$A(2a\tau + 3) = -3W_0 e^{a\tau}, \quad \Lambda = -\frac{a^2 A^2}{6\tau} e^{-2a\tau}.$$

ADC: what is the real KK scale?

- $\Lambda \rightarrow 0$ when $\tau \rightarrow \infty$, at infinite distance in field space. ✓
- Naive KK scale: $m_{\text{KK}} \sim 1/\tau \gg |\Lambda|^\alpha$. ADC violated!?
- Warped throat: KK modes near tip redshifted [Blumenhagen, Kläwer, Schlechter '19]

$$m_{\text{KK}}^2 \sim \frac{1}{\log^2 |\Lambda|} |\Lambda|^{\frac{1}{3}}.$$

- Up to log-corrections, ADC satisfied with $\alpha = 1/6$. ✓
- Since $\alpha < 1/2$, scale separation is possible.

Other quantum vacua: LVS

We find very similar behavior for the large volume scenario:

LVS AdS minimum

$$V_{\text{LVS}} = \lambda \frac{\sqrt{\tau_s} e^{-2a\tau_s}}{\mathcal{V}} - \mu \frac{\tau_s e^{-a\tau_s}}{\mathcal{V}^2} + \frac{\nu}{\mathcal{V}^3}, \quad \Lambda \sim -\frac{e^{-3a\tau_s}}{\tau_s} \left(1 + O\left(\frac{1}{\tau_s}\right)\right).$$

ADC: satisfied with $\alpha = 2/9$ up to log-corrections ✓

- $\Lambda \rightarrow 0$ as $\tau_s \rightarrow \infty$. ✓
- For LVS, no constraint on W_0 to be small, naive KK-scale justified:

$$m_{\text{KK}}^2 \sim \frac{1}{\mathcal{V}^{\frac{4}{3}}} \sim \frac{1}{\tau_s^{\frac{2}{3}}} e^{-\frac{4}{3}a\tau_s} \sim \frac{1}{\log^{\frac{2}{9}} |\Lambda|} |\Lambda|^{\frac{4}{9}}.$$

Log-Corrections to Swampland Conjectures

ADC - classical

The limit $\Lambda \rightarrow 0$ is at *infinite distance* in field space and there is a *tower of light states* with

$$m_{\text{tower}} = c_{\text{AdS}} |\Lambda|^\alpha$$

for $\alpha > 0$. Supersymmetric vacua have $\alpha = 1/2$.

Log-Corrections to Swampland Conjectures

ADC - quantum

The limit $\Lambda \rightarrow 0$ is at *infinite distance* in field space and there is a *tower of light states* with

$$m_{\text{tower}} = c_{\text{AdS}} |\Lambda|^\alpha \frac{1}{\log |\Lambda|^\beta}$$

for $\alpha, \beta > 0$. Vacua *without dilute flux limit* have $\alpha = 1/2$.

Summary

- KKLT is a recipe for dS which has never actually been cooked.
- Understanding the interplay between ingredients is important.
- We have shown that small $|W_0|$ can be found near a conifold.
- Progress has shown no big holes in the argument yet.
- However, KKLT violates AdS and dS swampland conjectures.
- Intrinsically quantum in nature – can this be the reason?

Thank you for your attention!



MAX-PLANCK-GESELLSCHAFT

IIA flux vacua - DGKT

- Closed string moduli stabilized via RR, H_3 fluxes
- Dilute flux limit exists

Example: isotropic 6-torus; moduli S , $T = T_k$, $U = U_k$

- $W = if_0 T^3 - 3if_4 T + ih_0 S + 3ih_1 U$
- KK scales: $m_{\text{KK},i}^2 \sim |\Lambda|^{7/9}$ **(weak) ADC ✓**

This minimum is supersymmetric, strong ADC is violated!

IIA flux vacua - Freund-Rubin

- Backreaction of fluxes on the geometry important
- Encoded by geometric fluxes
- No dilute flux limit

Example: isotropic 6-torus; moduli S , $T = T_k$, $U = U_k$

- $W = f_6 + 3f_2 T^2 - \omega_0 ST - 3\omega_1 UT$
- (naive) KK scales: $m_{\text{KK},1}^2 \sim \frac{|\Lambda|}{\omega_1^2}$, $m_{\text{KK},2}^2 \sim \frac{|\Lambda|}{\omega_1 \omega_2}$
- Looks like ω_i allow for scale separation!

IIA flux vacua - Freund-Rubin

- Backreaction of fluxes on the geometry important
- Encoded by geometric fluxes
- No dilute flux limit

Example: isotropic 6-torus; moduli S , $T = T_k$, $U = U_k$

- $W = f_6 + 3f_2 T^2 - \omega_0 ST - 3\omega_1 UT$
- Must take backreaction into account! [Font, Herráez, Ibáñez '19]
- KK scales: $m_{\text{KK},i}^2 \sim |\Lambda|$ **strong ADC** ✓

Difference between these models are flux backreactions on the geometry, i.e geometric fluxes. Is this the relevant feature for distinguishing strong/weak ADC cases?