6D SCFTs and Integrable Spin Chains

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based on 2007.07262 w/ F. Baume and J.J. Heckman

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Motivation

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(Why am I talking about 6D SCFTs at a string pheno seminar?)

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QFT + symmetries

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QFT + symmetries

Supersymmetry + conformal symmetry

$\Longrightarrow$ questions become tractable

Motivation

What kind of questions?

- correlation functions $\langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n) \rangle = ?$

- in CFT correlation functions of local operators are controlled by the scaling dimensions of operators

part of the "CFT data"

Motivation

Why 6D?

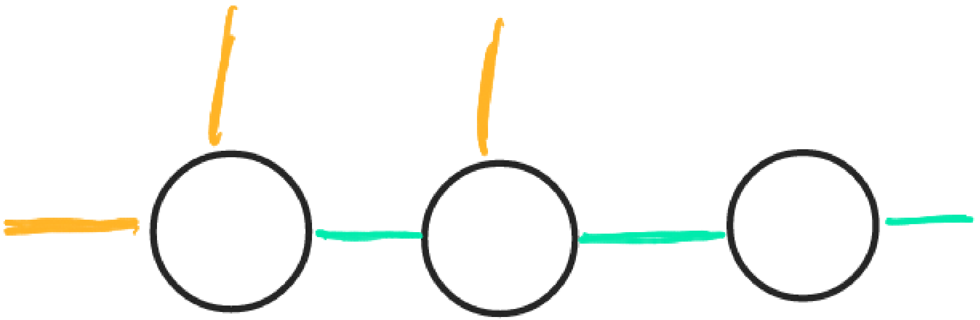
A vast number of 6D $\mathcal{N}=(1,0)$ SCFTs have geometric realisations from string theory. [Heckman, Morrison, Rudelius, Vafa]

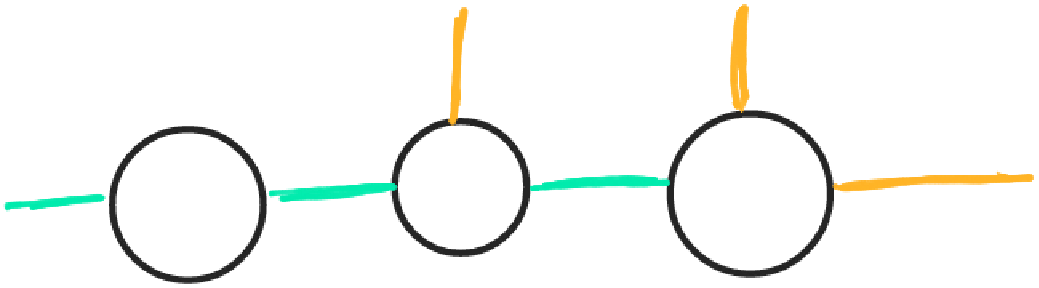
- Many theories and lots of information, but unclear how to read off operator dimensions from this construction.

Contents

6D SCFTs

In [Heckman, Morrison, Rudelius, Vafa] it was shown that 6D SCFTs from F-theory take the form of (generalized) linear quivers, with conformal matter as the links between nodes [del Zotto, Heckman, Tomasiello, Vafa] with some non-linear decorations at the ends.





Since conformal matter behaves as a building block

$\longrightarrow$ what do we know the spectrum?

(SU(N), SU(N)) conformal matter

$\longrightarrow$ bifundamental hypermultiplet

$su(2)_R$ 2 index

contains scalar $\phi^\alpha = (X, Y^+)$ $\boxed{s = \frac{1}{2}}$

$su(2)_R$ 3 index

(SO(K), SO(K)) $\longrightarrow$ for $K>4$ two half-hypers in $SO(K) \times Sp(K-4)$

contains scalar $\psi^i = (\phi^\alpha \otimes \phi^\alpha) = (X^{(1)}, X^{(0)}, X^{(-1)})$ $\boxed{s = 1}$

Similarly an analysis of the other strongly coupled conformal matter reveals bifundamental scalar fields

$$\varphi^{s} = \left(X^{(s)}, X^{(s-1)}, \ldots, X^{(-s)}\right)$$

transforming in the spin s rep of $SU(2)_R$.

We find

G	$SU(N)$	$SO(N)$	E_6	E_7	E_8
s	$1/2$	1	$3/2$	2	3

At leading order triplet of D-terms related to quadratic terms in the Xs.

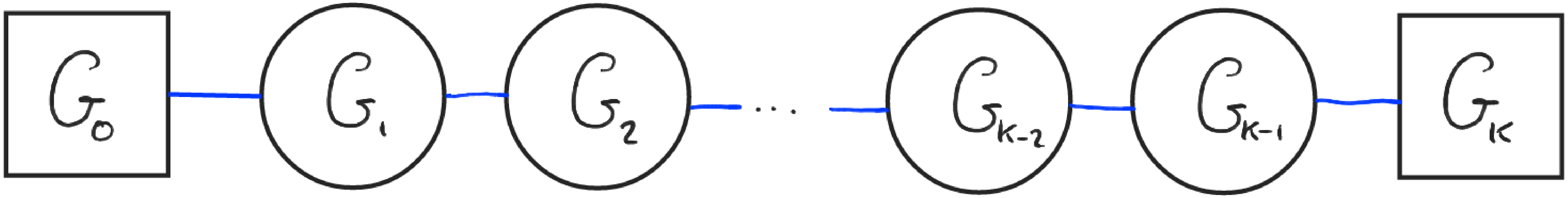
$$D^R_{i,a} = \frac{1}{s} \times \left(\mathrm{Tr}_{i+1}(X_i^{\dagger(m_i)} S_R^{(m_i,n_j)} T_{i,a} X_i^{(n_j)}) - \mathrm{Tr}_{i-1}(X_{i-1}^{(m_i)} S_R^{(m_i,n_j)} T_{i,a} X_{i-1}^{\dagger(n_j)}) \right) + \cdots$$

for $s = 1/2$ this is just the minimal coupling between 6D hyper and vector

for $s > 1/2$ we expect higher order correction. (M5-brane fractionation).

An Integrable Sector

Rank K (G, G) conformal matter has generalised quiver



 = minimal (G, G) conformal matter

As we have seen:

at each — there are bifundamental modes

$$X_i^{(m_i)} \qquad -s \leq m_i \leq s$$

where

G	$SU(N)$	$SO(N)$	E_6	E_7	E_8
s	$1/2$	1	$3/2$	2	3

Construct operators

$$\mathcal{O} = \underbrace{\mathcal{O}^{(m_1,\dots,m_k)} = X_1^{(m_1)} X_2^{(m_2)} \cdots X_k^{(m_k)}}_{\text{bifundamental of flavor } G_0 \times G_k}$$

$$m_i = s: \quad \underbrace{\mathcal{O}_{pure} = X_1^{(s)} \cdots X_k^{(s)}}_{\text{scalar of } \mathcal{D}\text{-type superconformal multiplet}}$$

$\Rightarrow$ ½ - BPS $\Rightarrow$ no anomalous dimension $\Rightarrow \Delta(\mathcal{O}_{pure}) = 4s(k+1)$.

R-charge of $\mathcal{O}_{pure}$:

$$R[\mathcal{O}_{pure}] = (K+1)S$$

Consider $\mathcal{O}$ with "small" number of "impurities"

$$X_i^{(m_i)} \quad \text{for } m_i \neq S.$$

long quivers

$$R[\mathcal{O}] \text{ large when } \underbrace{K \gg 1}$$

Treat $1/R$ as expansion parameter.

Let's start with (A_N, A_N) conformal matter

then $\textcolor{blue}{\text{———}}$ = bifundamental hypermultiplet

$\Longrightarrow$ we can be very explicit!

For simplicity focus on single impurity case

$$\mathcal{O}_i = X_0 \cdots X_{i-1} Y_i^\dagger X_{i+1} \cdots X_K$$

If gauge coupling is switched off the $\Delta_i = 2(K+1)$

If gauge coupling is on $\mathcal{O}_i$ will mix with $\mathcal{O}_j$.

$$\langle \mathcal{O}_i^\dagger(x)\,\mathcal{O}_j(0)\rangle = \frac{1}{|x|^{2\Delta_i}}\left(\delta_{ij} - \gamma_{ij}\log\left(|x|^2\Lambda^2\right) + \dots\right)$$

anomalous dimension matrix.

Using triplet of D-terms we find, at one loop,

$$\langle \mathcal{O}_i^\dagger(x)\,\mathcal{O}_{i-1}(0)\rangle = \frac{1}{|x|^{2\Delta_0}}\left(1 + \frac{2g_i^2\hat{C}_i}{(4\pi^3)^2}\int d^6z\,\frac{|x|^{4\Delta_{kk}}}{|x-z|^{4\Delta_{kk}}\,|z|^{4\Delta_{kk}}}\right)$$

Evaluating the integral we find

$$\langle \mathcal{O}_i^\dagger(x)\,\mathcal{O}_{i-1}(0)\rangle = \frac{1}{|x|^{2\Delta_0}}\left(1+\frac{g_i^2\hat{C}_i}{16\pi^3}\log(|x|^2\Lambda^2)+\dots\right)$$

Working out all correlators we find

$$\gamma_{i,i} = \frac{1}{16\pi^3}\left(g_i^2\hat{C}_i + g_{i+1}^2\hat{C}_{i+1}\right)$$

$$\gamma_{i,i+1} = \gamma_{i+1,i} = -\frac{g_{i+1}^2\hat{C}_{i+1}}{16\pi^3}$$

We can write this as 1D lattice Laplacian

$$\underline{\gamma} = \lambda_A \begin{bmatrix} 1 & -1 & & & & & \\ -1 & 2 & -1 & & & 0 & \\ & -1 & 2 & -1 & & & \\ & & -1 & 2 & -1 & & \\ & 0 & & -1 & \ddots & -1 & \\ & & & & -1 & 2 & -1 \\ & & & & & -1 & 1 \end{bmatrix} \quad \text{with} \quad \lambda_A = \frac{g_{eff}^2 \hat{C}_A}{16\pi^3}.$$

Operator mixing by such a lattice Laplacian can be written as a 1D spin chain with $s = \pm 1/2$ on each of $K+1$ sites and Hamiltonian

$$H = -\lambda_A \sum_i \left(2\vec{S}_i \cdot \vec{S}_{i+1} - 1/2\right)$$

This is exactly the Hamiltonian for the <u>integrable</u> Heisenberg spin chain $\Longrightarrow$ we have found <u>integrability</u>!

Spin Chain State		6D SCFT Operator
$\vert \uparrow ... \uparrow\uparrow\uparrow ... \uparrow\rangle$	$\longleftrightarrow$	$X_0 ... X_{i-1} X_i X_{i+1} ... X_N$
$\vert \uparrow ... \downarrow\uparrow\uparrow ... \uparrow\rangle$	$\longleftrightarrow$	$X_0 ... Y^\dagger_{i-1} X_i X_{i+1} ... X_N$
$\vert \uparrow ... \uparrow\downarrow\uparrow ... \uparrow\rangle$	$\longleftrightarrow$	$X_0 ... X_{i-1} Y^\dagger_i X_{i+1} ... X_N$
$\vert \uparrow ... \uparrow\uparrow\downarrow ... \uparrow\rangle$	$\longleftrightarrow$	$X_0 ... X_{i-1} X_i Y^\dagger_{i+1} ... X_N$

Extending Integrability

The AdS/CFT duals of 6D rank k conformal matter are $AdS_7 \times S^4/\Gamma$ $\quad \Gamma$ finite $\subset SU(2)$.

Our protected operators are not sensitive to the different Γ in the dual $\longrightarrow$ reasonable to expect integrability to persist.

<u>Assumption</u>: operator mixing in this sector is controlled by the

1D <u>integrable</u> open Heisenberg XXX_s spin chain

also for $s > 1/2$.

dilatation operator $\leftarrow$ $\underbrace{\text{spin chain Hamiltonian}}$

integrability fixes this Hamiltonian uniquely! [Babujian]

$$H_s = -\lambda_s \sum_{i=0}^{k-1} Q_{2s}(\vec{S}_i \cdot \vec{S}_{i+1})$$

(Recall: G => s.)

where $\vec{S}_i$ = spin s operator

Q_{2s} = degree 2s polynomial

$$Q_{2s}(x) = -2\sum_{l=0}^{2s}\sum_{k=l+1}^{2s}\frac{1}{k}\prod_{\substack{j=0\\ j\neq l}}^{2s}\frac{x-x_j}{x_l-x_j}, \quad \text{with} \;\; x_l = \frac{1}{2}l(l+1) - s(s+1)$$

$$Q_{A_k}(x) = -\frac{1}{2} + 2x$$

$$Q_{D_k}(x) = +\frac{1}{2}x - \frac{1}{2}x^2$$

$$Q_{E_6}(x) = -\frac{3}{4} - \frac{1}{8}x + \frac{1}{27}x^2 + \frac{2}{27}x^3$$

$$Q_{E_7}(x) = -\frac{1}{2} + \frac{13}{24}x + \frac{43}{432}x^2 - \frac{5}{216}x^3 - \frac{1}{144}x^4$$

$$Q_{E_8}(x) = -\frac{148}{125} - \frac{1687}{9000}x + \frac{1297}{18000}x^2 + \frac{593}{20250}x^3 + \frac{79}{97200}x^4 - \frac{77}{243000}x^5 - \frac{1}{48600}x^6$$

For $s > 1/2$ there are higher order spin-spin interaction terms.

$\longrightarrow$ consistent w/ "fractionation" of M5-branes for (D,D) and (E,E) conformal matter.

Bethe Ansatz

Consider operator $\mathcal{O}$ with I impurities:

$$e^{ip_j} = \frac{\mu_j + is}{\mu_j - is}$$

momenta rapidity

Energy / anomalous dimension

$$\underbrace{\Delta - \Delta_0 = E_G = \lambda_s \sum_{j=1}^{I} \left(\frac{i}{\mu_j + is} - \frac{i}{\mu_j - is} \right)}$$

Bethe ansatz equations: $\left(\frac{\mu_j + is}{\mu_j - is} \right)^{2(k+1)} = -\prod_{l \neq j} \frac{(\mu_j - \mu_l + i)}{(\mu_j - \mu_l - i)} \frac{(\mu_j + \mu_l + i)}{(\mu_j + \mu_l - i)}$

Decoupling: $\sum_i p_i = 0 \Rightarrow \prod_{j=1}^{I} \frac{\mu_j + is}{\mu_j - is} = 1$

Example: $G = SU(N)$, $I = 2$

Operators like

$$XX\cdots Y^\dagger X \cdots XY^\dagger X \cdots X$$

Two momenta p_1, p_2

$\longrightarrow$ decoupling equation $\Rightarrow$ $\mu_1 = -\mu_2 = \mu$

BAE becomes: $$\left(\frac{\mu + i/2}{\mu - i/2}\right)^{2(K+1)} = \frac{2\mu + i}{2\mu - i} = \frac{\mu + i/2}{\mu - i/2}$$

$$\Rightarrow p_1 = -p_2 = \frac{2\pi m}{2K+1} \quad \text{for } m = 0, \ldots, K-1$$

Anomalous dimensions: $$\Delta - \Delta_0 = \lambda_{1/2} \times 8 \sin^2\left(\frac{\pi m}{2K+1}\right).$$

Conclusions

1) Uncovered one-loop integrability in certain large R-charge sectors of 6D SCFTs.

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→ as a byproduct: we determined the anomalous dimension of an interesting class of "nearly protected" operators.

Future

- how far does this integrable structure extend?
  - higher loops?
  - different operator sectors/impurities?
  - connections with AdS/CFT?
  - ...

Work to appear w/ Baume and Heckman

Thank
you