String Phenomenology seminar series — 04 May 2021

Probing inflation with the stochastic gravitational wave background and an opportunity for string pheno

Lukas Witkowski

arXiv:2012.02761 + work in progress

with Jacopo Fumagalli and Sébastien Renaux-Petel

European Research Council
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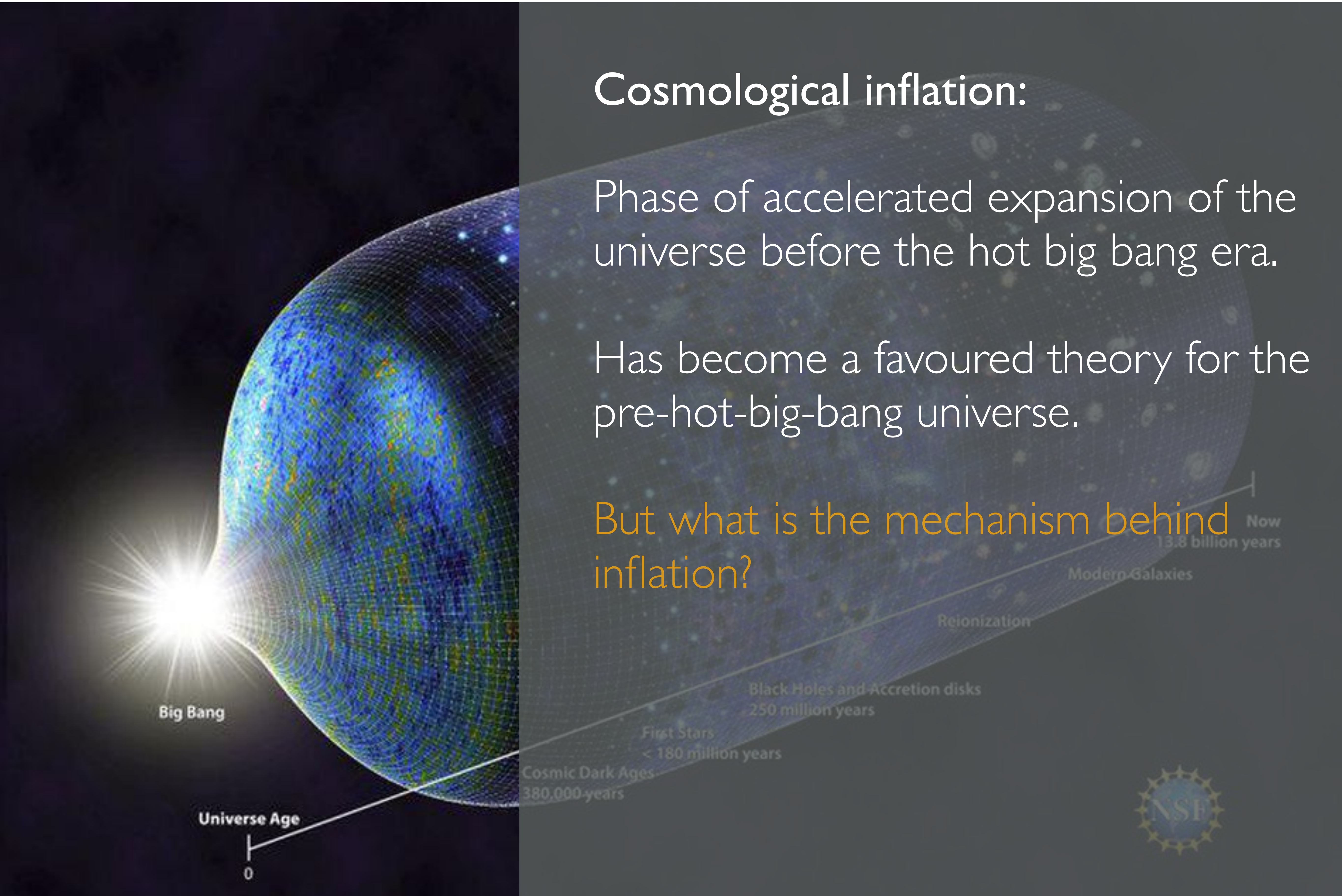
This talk

Start with

String Pheno

arrive at

Intro: Inflation and small-scale features



Cosmological inflation:

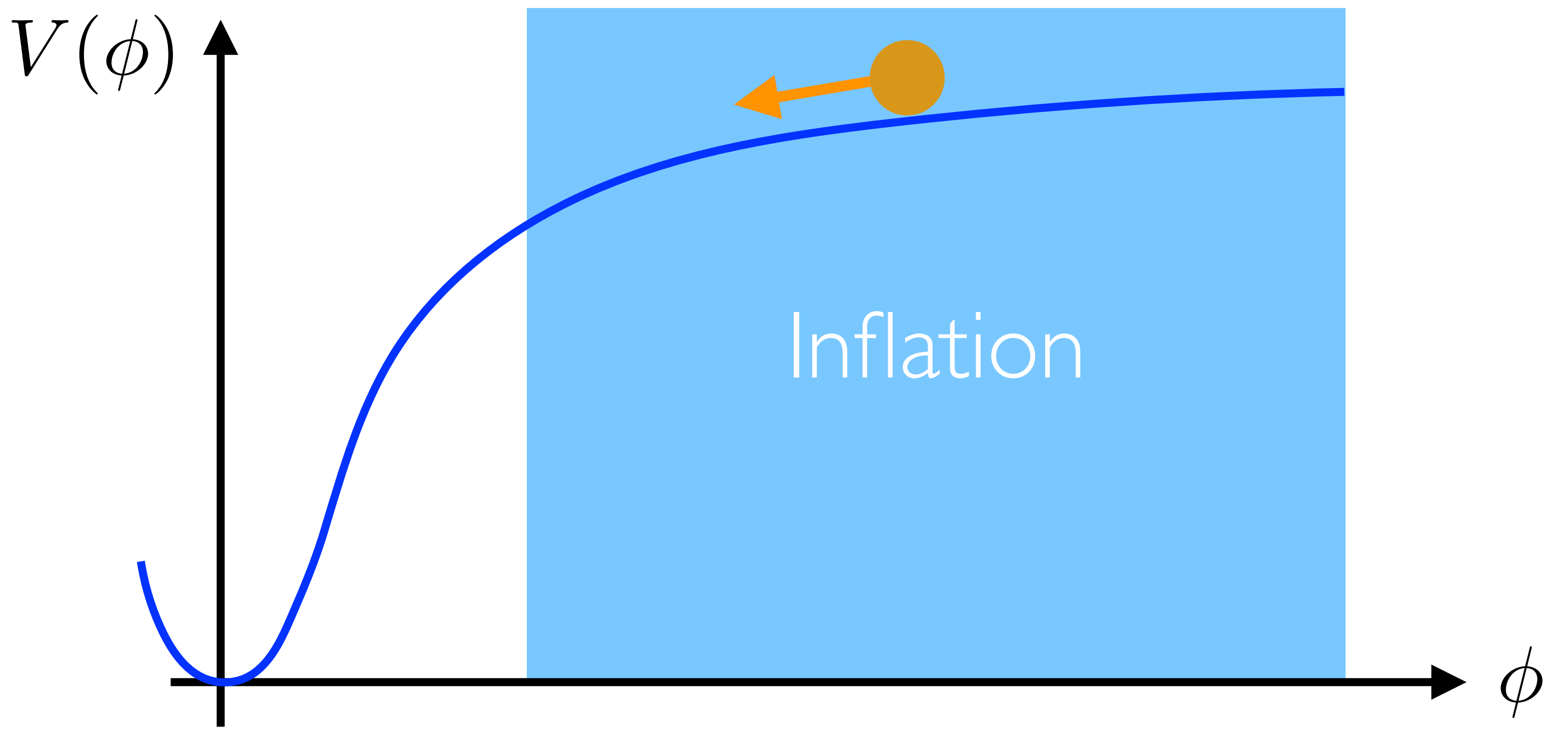
Phase of accelerated expansion of the universe before the hot big bang era.

Has become a favoured theory for the pre-hot-big-bang universe.

But what is the mechanism behind inflation?

Intro: Inflation and small-scale features

Standard paradigm for theoretical modelling of inflation:
Single-field slow-roll inflation

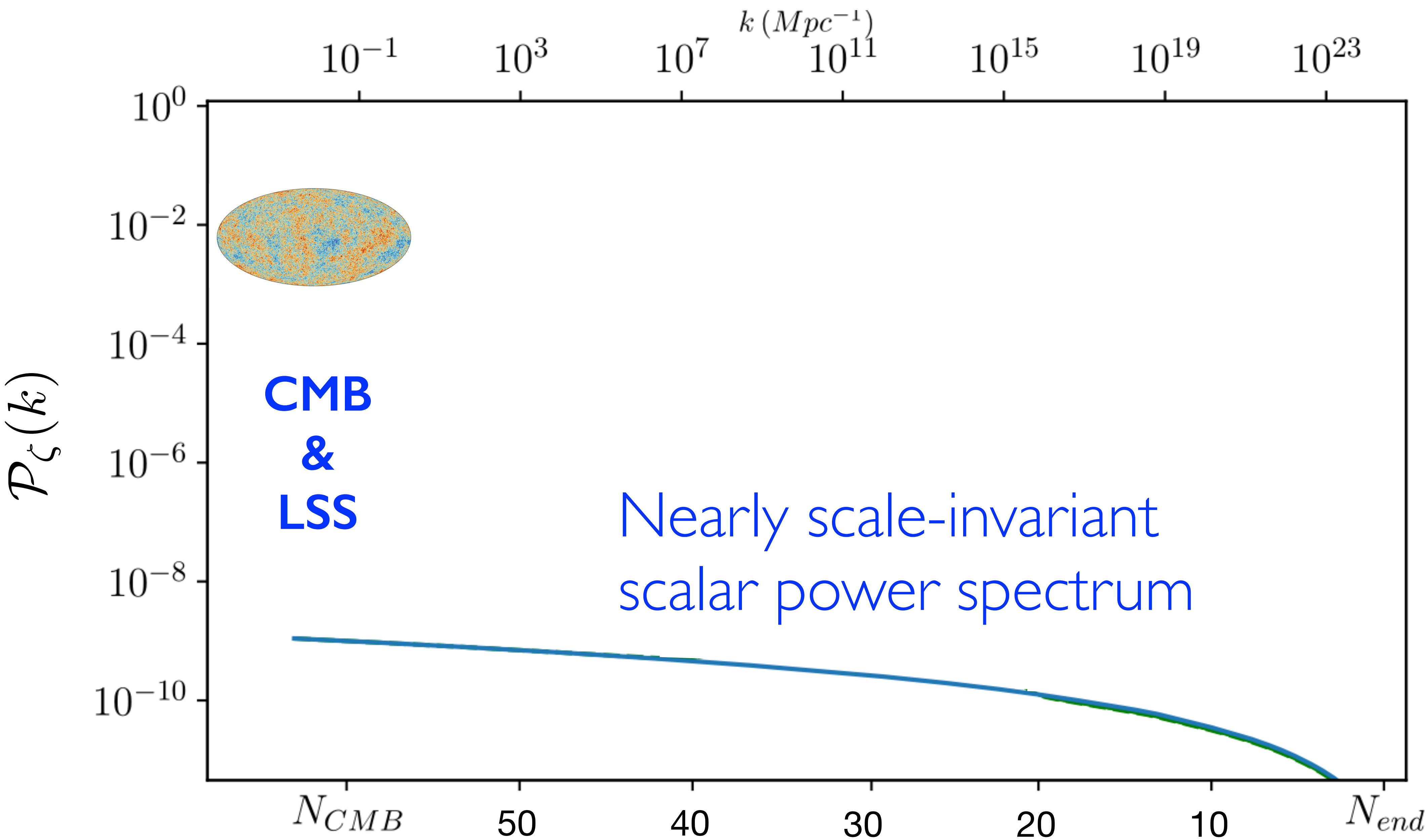


Predictions: quantum fluctuations with

- **nearly scale-invariant** power spectrum
- obeying ≈ **Gaussian statistics**.

Predictions consistent with experimental data from **CMB** and **LSS**.

Intro: Inflation and small-scale features



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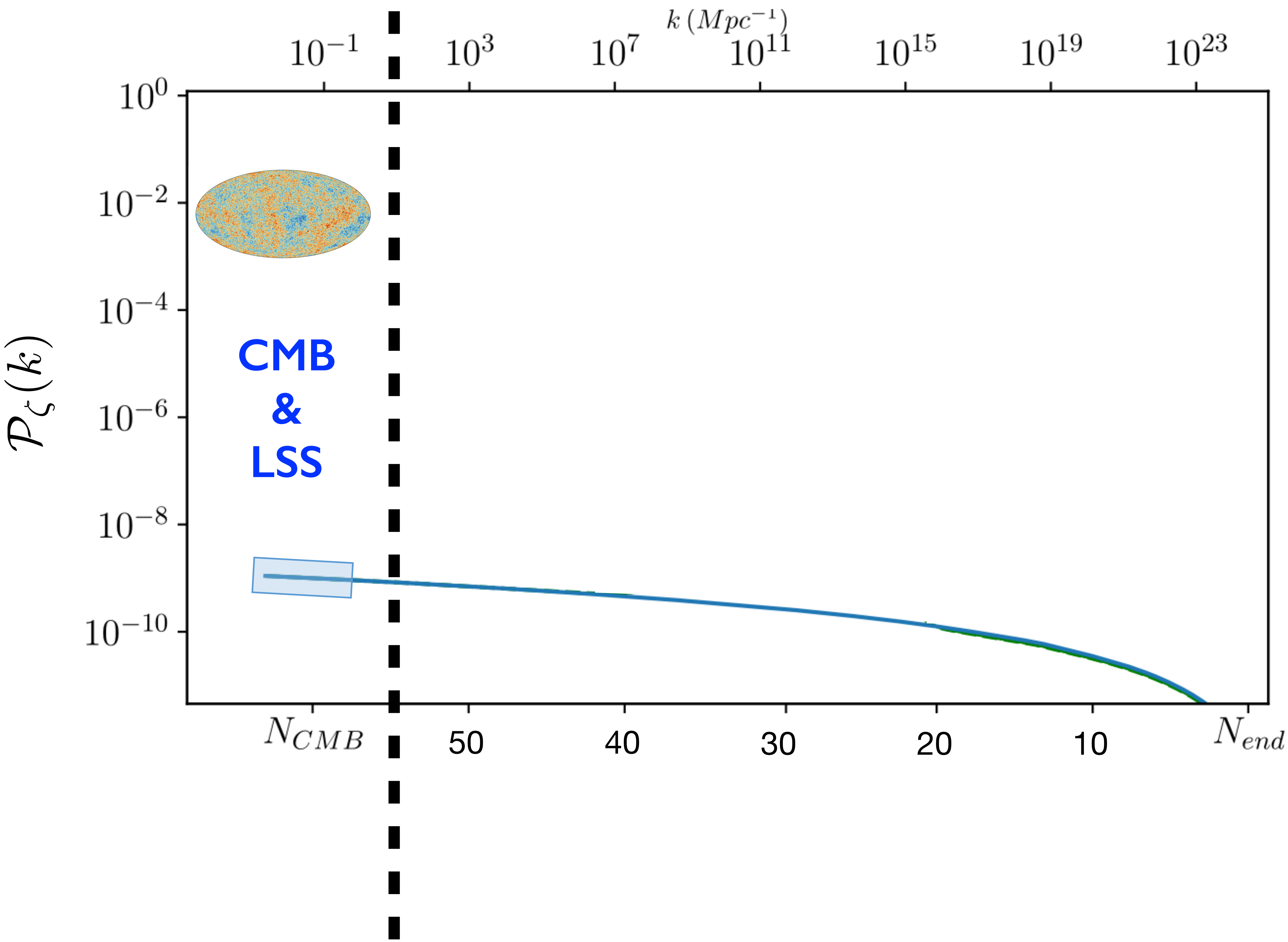

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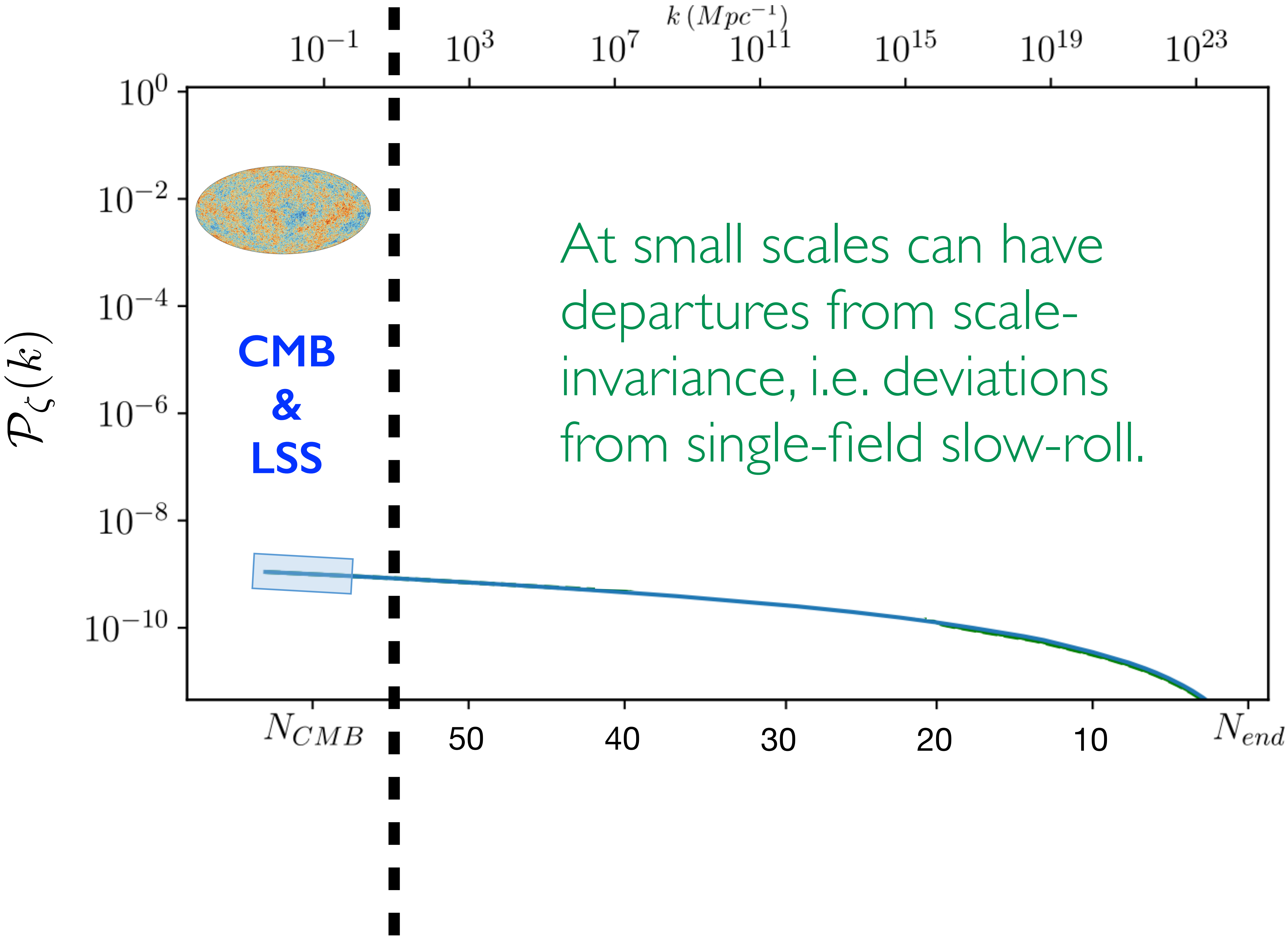

Intro: Inflation and small-scale features

Particularly interesting from the observational point of view:

Sharp transitions during inflation ("sharp features"):

sharp = transition occurs in less than one e-fold

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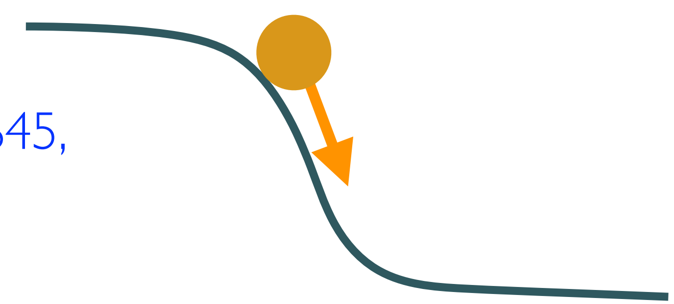
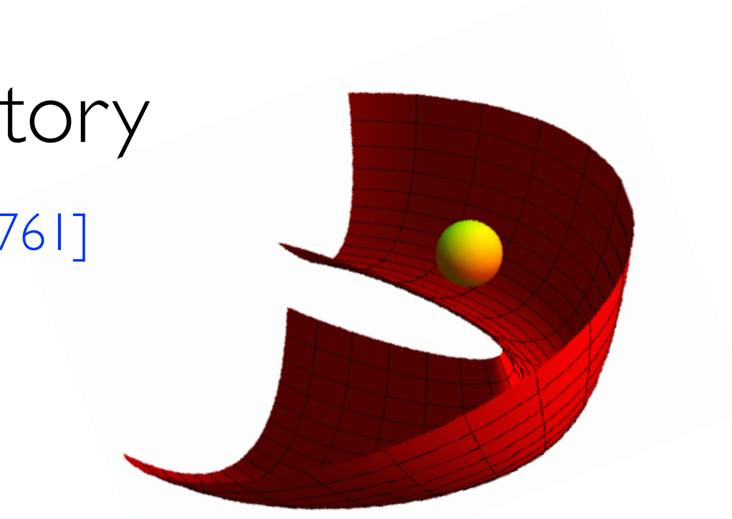
E.g.
- A short period of violation of slow-roll, for example due to a step in the inflaton potential.



[Adams et al. astro-ph/0102236, Chen et al. astro-ph/0611645, Bean et al. 0802.0491, Hazra et al. 1005.2175, Adshead et al. 1110.3050, Kefala et al. 2010.12483, Inomata et al. 2104.03972 + and more]

- A sharp turn in the inflationary trajectory

[Palma et al. 2004.06106, Fumagalli, Renaux-Petel, **LW** 2012.02761]



Intro: Inflation and small-scale features

Particularly interesting from the observational point of view:

Sharp transitions during inflation ("sharp features"):

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Scalar power spectrum:

$$\mathcal{P}_\zeta(k) = \underline{\overline{\mathcal{P}}(k)}\Big(1 + A_{\rm lin}\cos\big(\underline{\omega_{\rm lin}k + \phi_{\rm lin}}\big)\Big) \quad \text{with} \quad \omega_{\rm lin} \simeq \frac{2}{k_{\rm f}}$$

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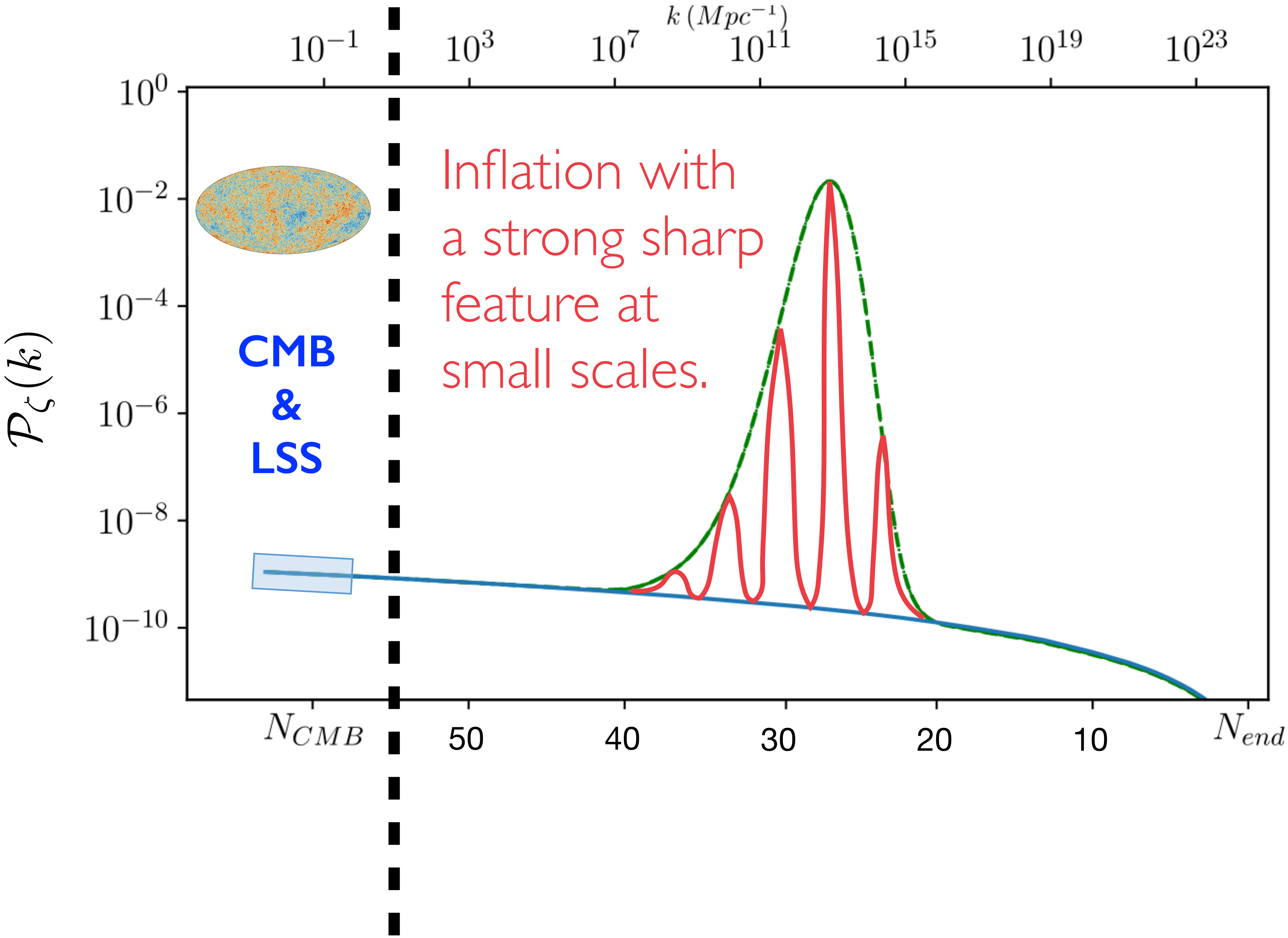
- The signature of a **sharp** transition is a k -periodic sinusoidal modulation of $\mathcal{P}_\zeta(k)$ with frequency $\omega_{\rm lin} \simeq 2/k_{\rm f}$.

- If the transition is sufficiently "**strong**", e.g. $\epsilon_{\rm step} \gg \epsilon_{\text{slow-roll}}$, or the dimensionless rate of turn $\eta_\perp \gg 1$, then the amplitude of fluctuations is enhanced and $\overline{\mathcal{P}}(k)$ exhibits a peak.

Intro: Inflation and small-scale features



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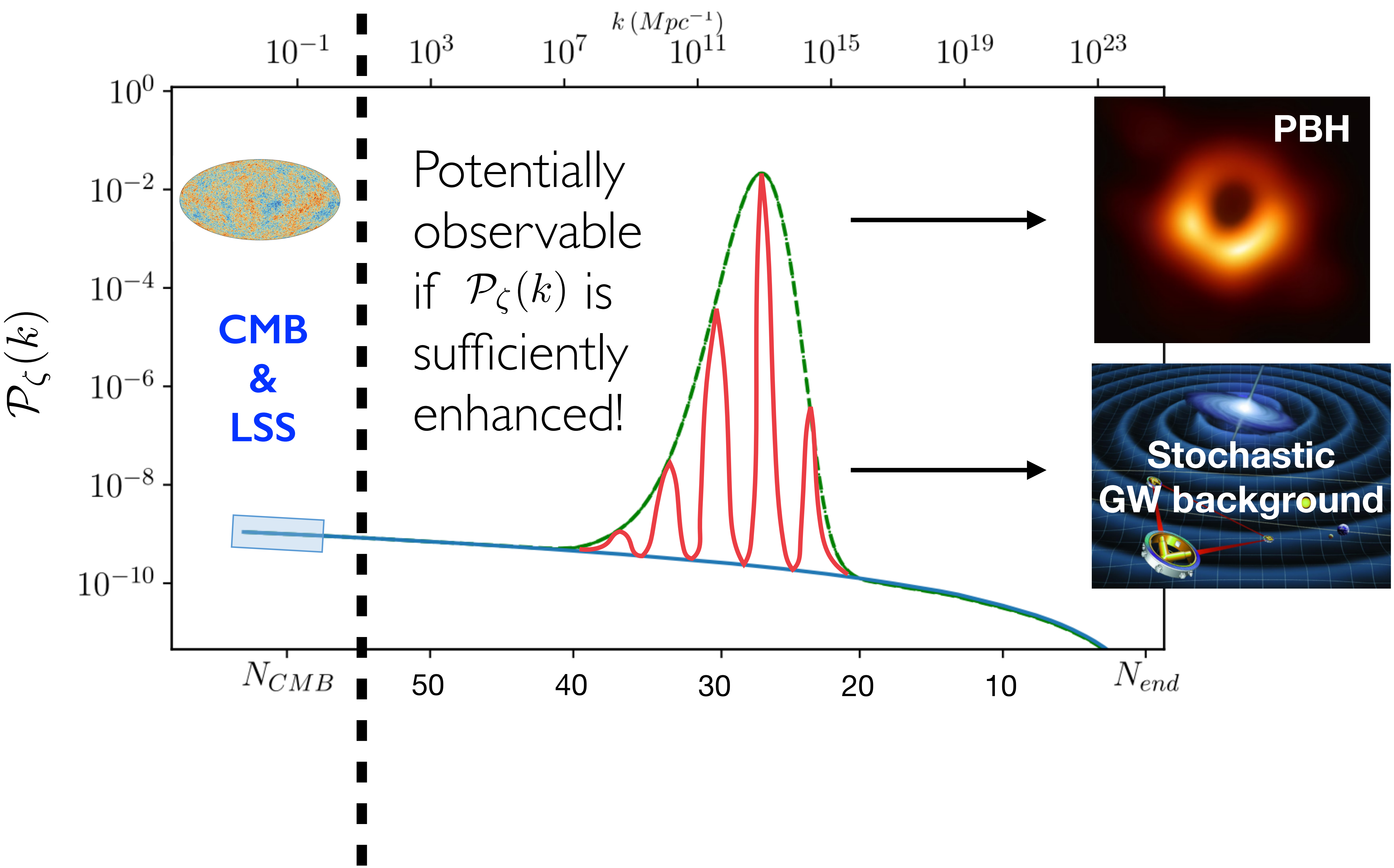

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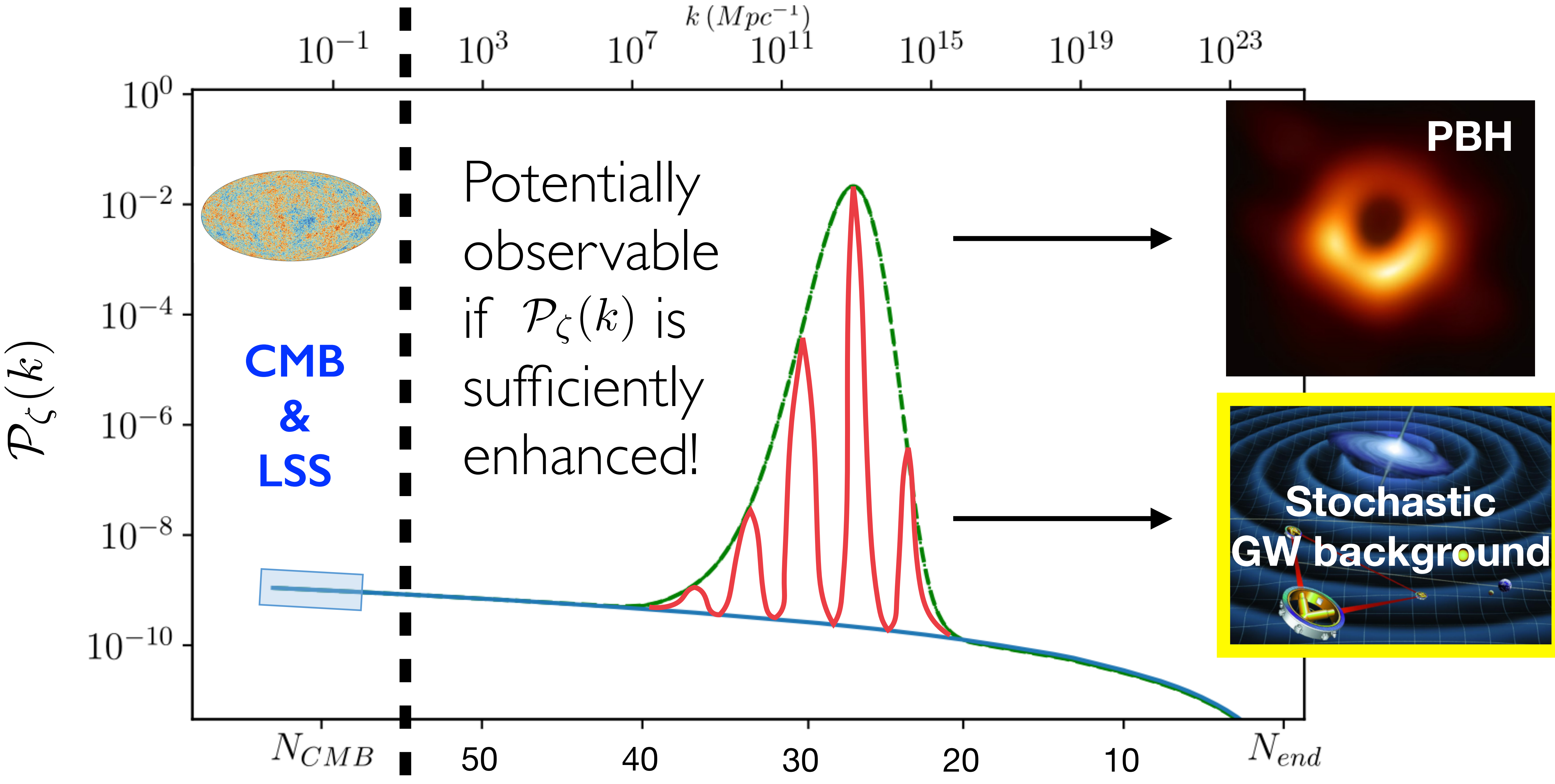


Oscillations in $\mathcal{P}_\zeta(k)$ induce an oscillation in the SGWB! [Fumagalli, Renaux-Petel, LW 2012.02761]

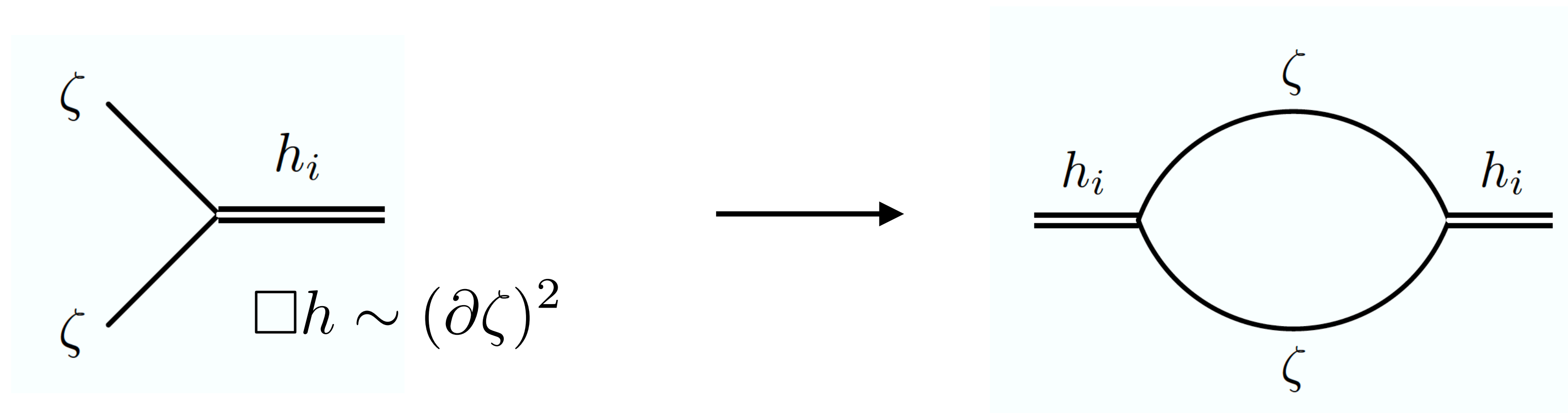
Outline

I. Scalar-induced GWs due to a sharp feature

II. Implications for string phenomenology

Scalar-induced GWs

Scalar fluctuations source GWs at 2nd order:



[Acquaviva et al. 2002; Mollerach, Harari, Matarrese 2003; Ananda, Clarkson, Wands 2006; Baumann et al. 2007 …]

Energy density per $\log(k)$-interval of post-inflationary GWs:

$$\boxed{\Omega_{\rm GW}(k) = c_g \Omega_{\rm r,0} \int_0^{\frac{1}{\sqrt{3}}} \mathrm{d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} \mathrm{d}s\, \mathcal{T}_{\rm RD}(d,s)\, \mathcal{P}_\zeta\left(\frac{\sqrt{3}k}{2}(s+d)\right) \mathcal{P}_\zeta\left(\frac{\sqrt{3}k}{2}(s-d)\right)}$$

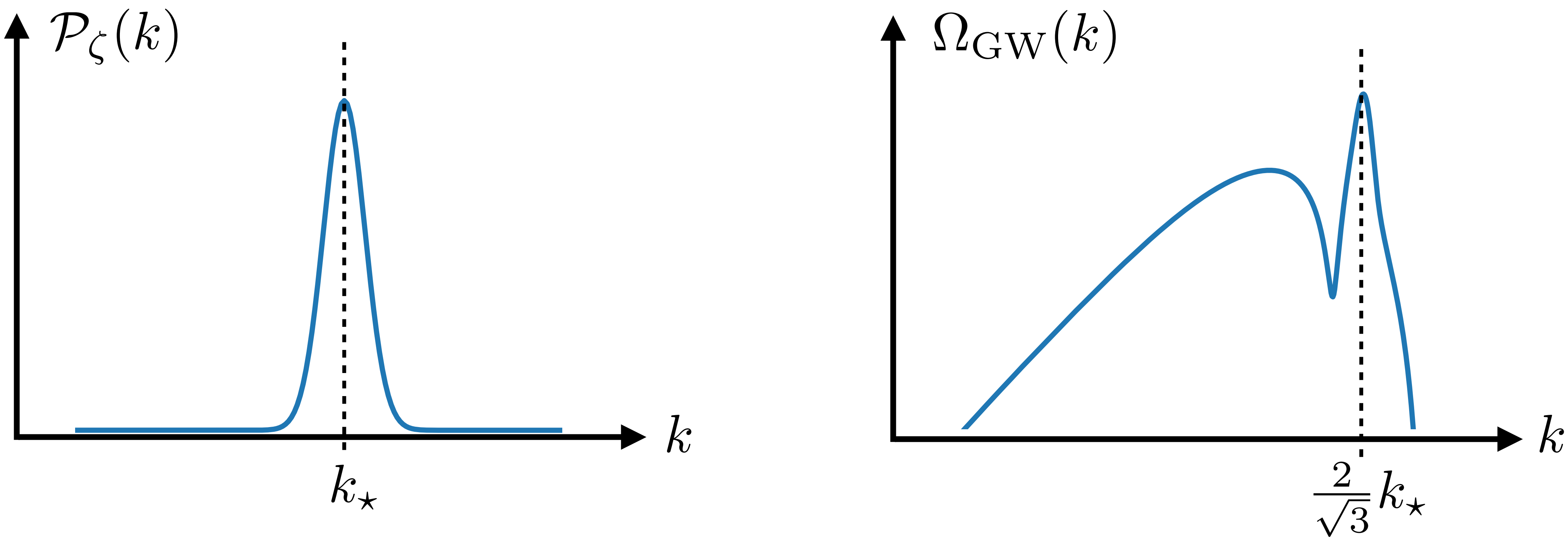
Caveats:

- Also have contribution **sourced during inflation**, which is slow-roll suppressed, but can become important in some models.
- Will assume **radiation-domination** during horizon re-entry.

Scalar-induced GWs

How does an oscillations in $\mathcal{P}_\zeta(k)$ manifest itself in $\Omega_{\rm GW}(k)$?

Consider a single narrow peak in $\mathcal{P}_\zeta(k)$:



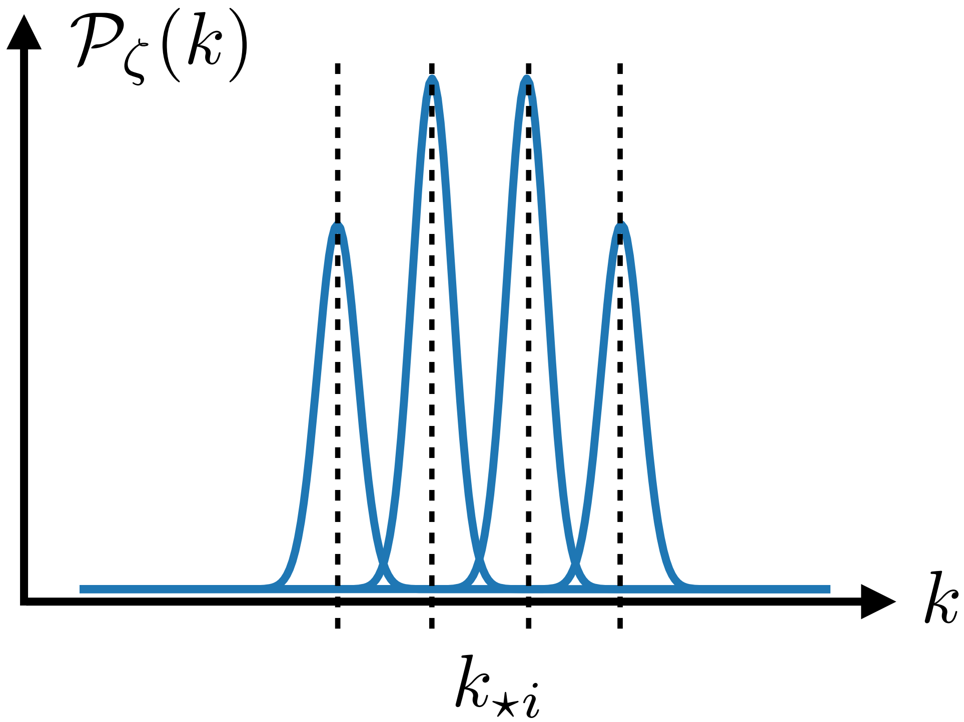
principal peak from resonant amplification

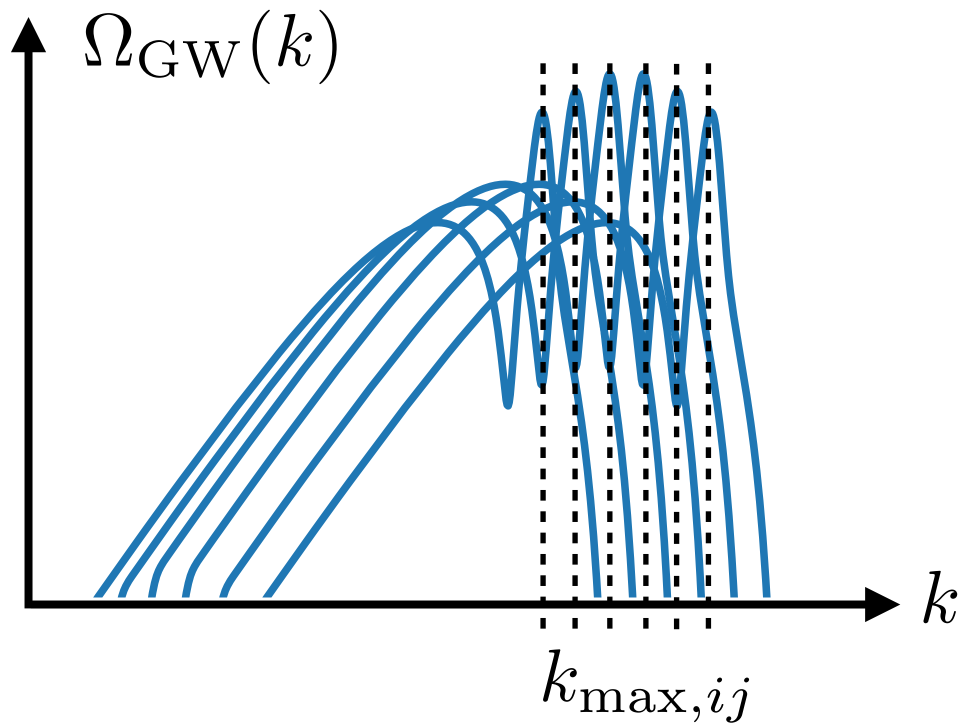
Scalar-induced GWs

How does an oscillations in $\mathcal{P}_\zeta(k)$ manifest itself in $\Omega_{\rm GW}(k)$?

[Fumagalli, Renaux-Petel, **LW** 2012.02761]

One of our strategies: Model the oscillation as a series of peaks:





$$k_{\max,ij} = \frac{1}{\sqrt{3}}(k_{\star i} + k_{\star j})\,, \quad \text{with} \quad k_{\max,ij} > |k_{\star i} - k_{\star j}|$$ [Cai et al. 1901.10152]

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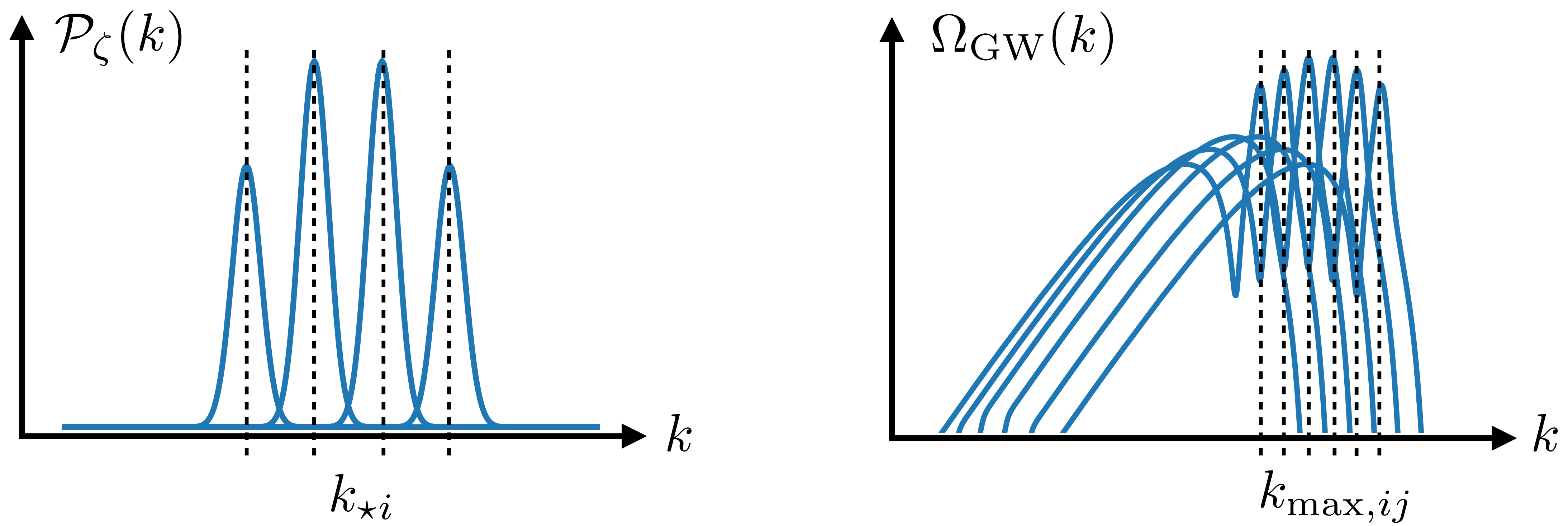

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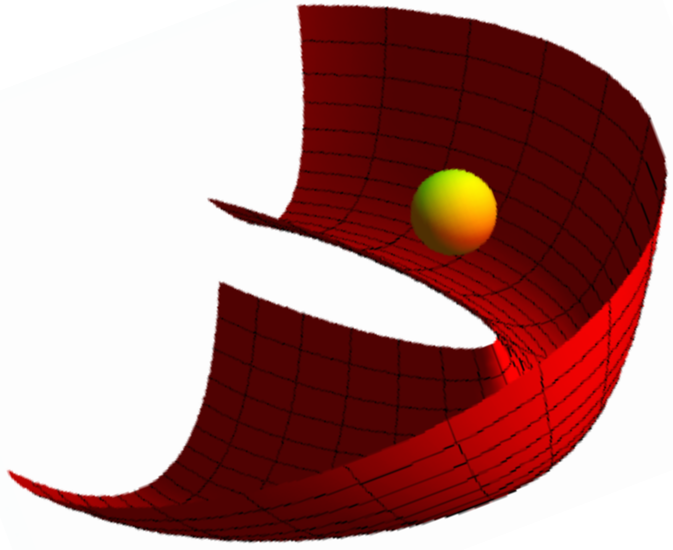
Sharp feature: $\Delta k \longrightarrow \dfrac{\Delta k}{\sqrt{3}}$

GWs from a sharp feature

Consider explicit realisation of a sharp feature in terms of a sharp turn in the inflationary trajectory in multi-field inflation:

[Palma et al. 2004.06106]
[Fumagalli, Renaux-Petel, LW 2012.02761]

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}G_{IJ}(\phi^K)\partial_\mu\phi^I\partial^\mu\phi^J - V(\phi^I)\right)$$



The turn is described by three parameters:

δ : duration of turn in e-folds

$\eta_\perp$: dimensionless rate of turning

k_f : scale that leaves horizon during time of turn

GWs from a sharp feature

Consider explicit realisation of a **sharp feature** in terms of a **sharp turn** in the inflationary trajectory in multi-field inflation:

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Amplified part of the power spectrum:

$$\mathcal{P}_\zeta(k) = \mathcal{P}_0 \underbrace{\frac{\eta_\perp^2 k_\mathrm{f}^2}{4(2\eta_\perp k_\mathrm{f} - k)k}\, e^{2\sqrt{(2\eta_\perp k_\mathrm{f} - k)k}\frac{\delta}{k_\mathrm{f}}}}_{\text{Envelope with an exponentially enhanced peak}} \underbrace{\left(1 - \cos\left(\frac{2k}{k_\mathrm{f}} + \arctan\left(\frac{k}{\sqrt{(2\eta_\perp k_\mathrm{f} - k)k}}\right)\right)\right)}_{\text{Rapid order one sinusoidal modulations}}$$

Envelope with an exponentially enhanced peak

Rapid order one sinusoidal modulations $\omega_\mathrm{lin} \simeq \dfrac{2}{k_\mathrm{f}}$

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Envelope with an exponentially enhanced peak

Rapid order one sinusoidal modulations $\omega_\mathrm{lin} \simeq \dfrac{2}{k_\mathrm{f}}$

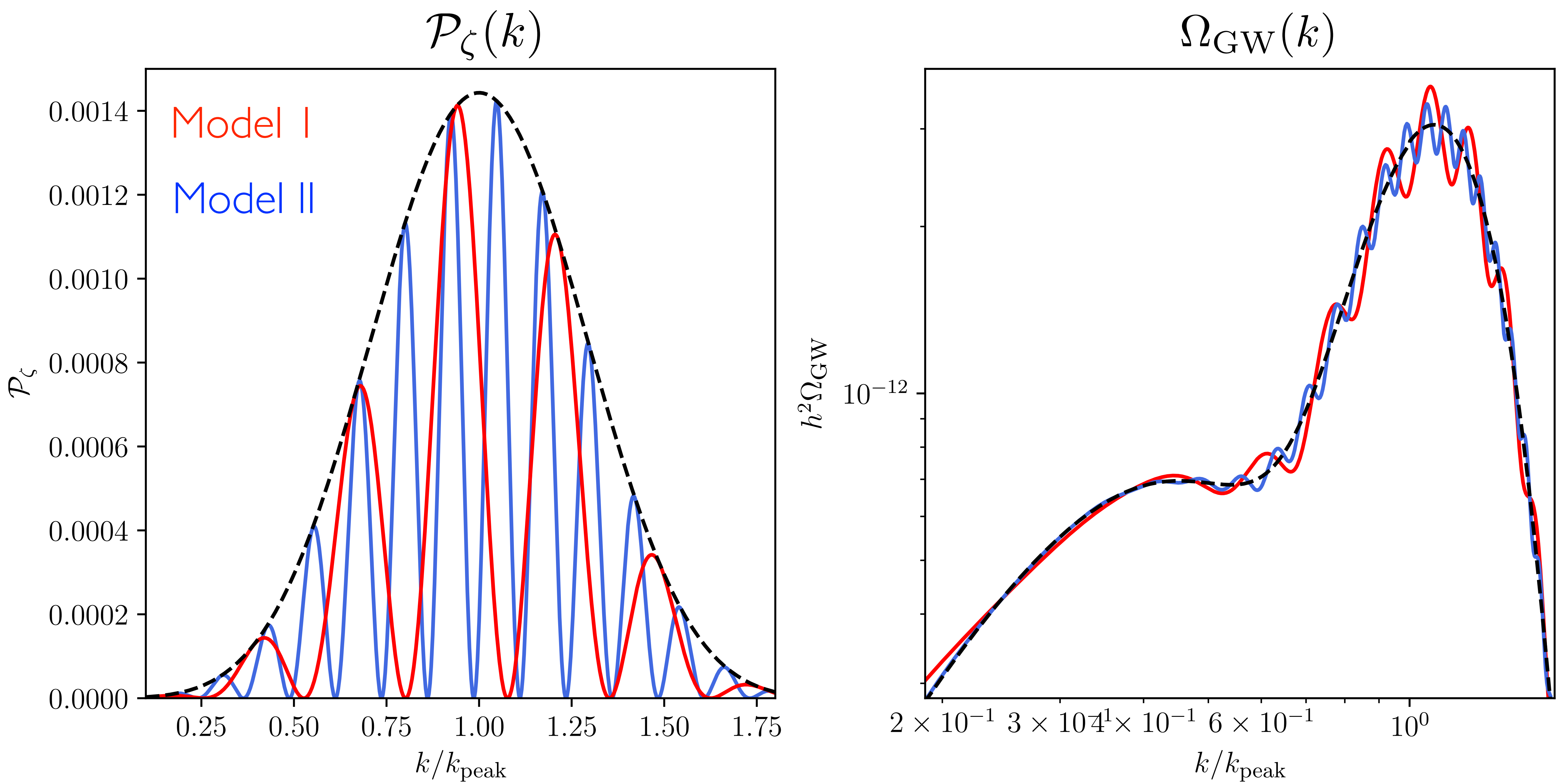
This takes indeed the form of a sharp feature:

$$\mathcal{P}_\zeta(k) = \underbrace{\overline{\mathcal{P}}(k)}_{}\Big(1 + A_\mathrm{lin} \underbrace{\cos\left(\omega_\mathrm{lin} k + \phi_\mathrm{lin}\right)}_{}\Big)$$

GWs from a sharp feature

Consider two example models (I and II) and compute the corresponding (post-inflationary contribution to the) GW spectrum:

[Fumagalli, Renaux-Petel, **LW** 2012.02761]



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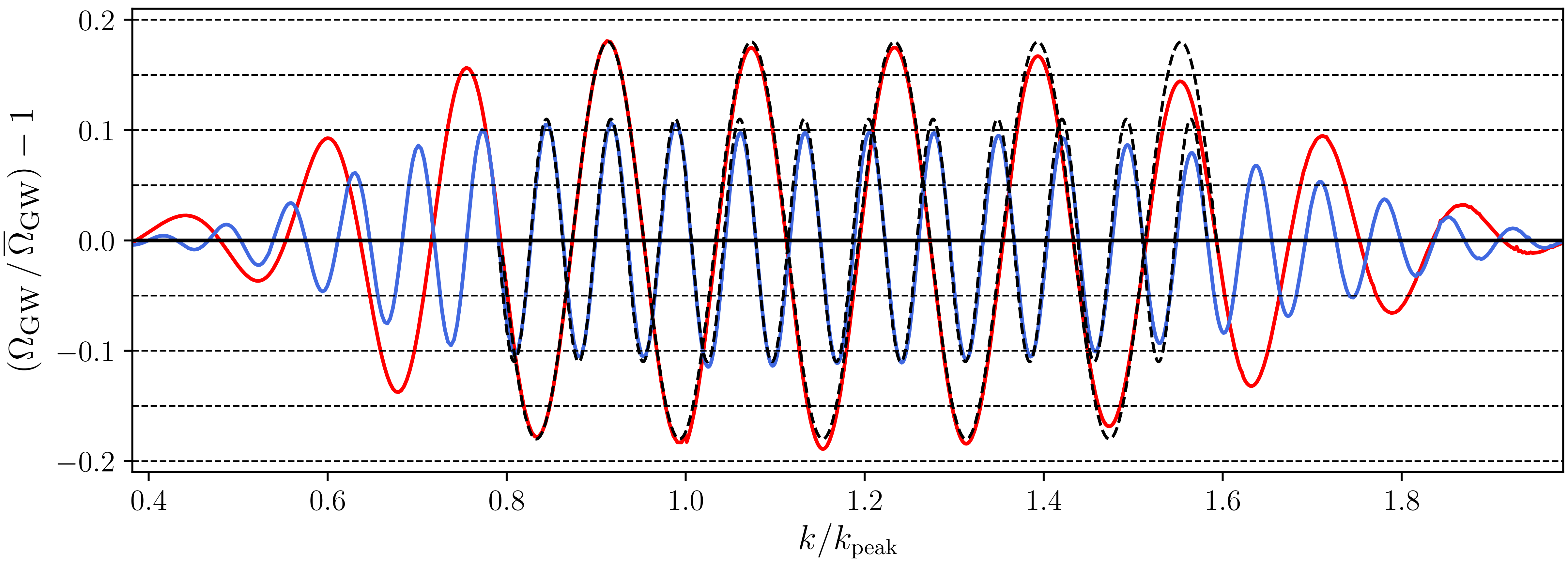
$\overline{\Omega}_{\rm GW}(k)$: smoothed GW spectrum



Have sinusoidal oscillations in $\Omega_{\rm GW}(k)$ over the principal peak.

[see also Braglia, Chen, Hazra 2012.05821]

GWs from a sharp feature

$$\mathcal{P}_\zeta(k) = \overline{\mathcal{P}}(k)\left(1 + A_{\rm lin}\cos\left(\omega_{\rm lin}k + \phi_{\rm lin}\right)\right)$$

[Fumagalli, Renaux-Petel, **LW** 2012.02761]

$$\omega_{\rm lin}^{\rm GW} = \sqrt{3}\,\omega_{\rm lin}$$

$$\Omega_{\rm GW}(k) \underset{k\sim\frac{2}{\sqrt{3}}k_{\rm peak}}{=} \overline{\Omega}_{\rm GW}\left(1+\mathcal{A}_{\rm lin}\cos\left(\omega^{\rm GW}_{\rm lin}k+\varphi_{\rm lin}\right)\right)$$

Overall shape determined by envelope of power spectrum

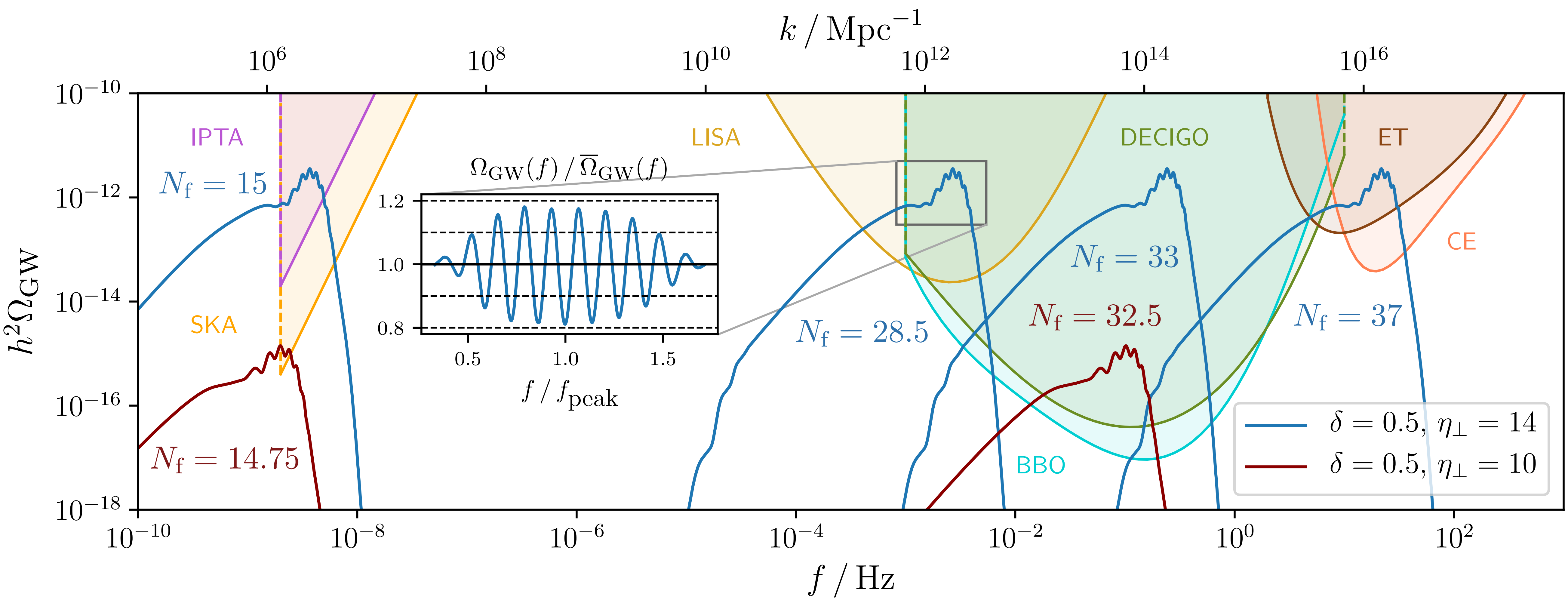
Periodic structure in k $\longrightarrow$ Periodic structure in k

Averaging-out effect: at best $\mathcal{A}_{\rm lin} \sim 10\%$ even with $A_{\rm lin} = 1$.

$\mathcal{A}_{\rm lin}$ also decreases as $\omega_{\rm lin}k_{\rm peak}$ is increased for fixed $\overline{\mathcal{P}}(k)$.

Scalar-induced GWs

Potentially detectable in the upcoming generation of GW observatories:



$N_{\rm f} =$ time of feature in numbers of e-folds after horizon exit of CMB modes

GWs from a resonant feature

Resonant feature: caused by some components of the background oscillating with a frequency larger than the Hubble scale (as e.g. in monodromy inflation)

→ $\log(k)$-periodic modulation in $\mathcal{P}_\zeta(k)$.

$$\mathcal{P}_\zeta(k) = \underline{\overline{\mathcal{P}}(k)}\Big(1 + \underline{A_{\log}}\,\underline{\cos\left(\omega_{\log}\log(k/k_{\rm ref})\right.} + \phi_{\log}\big)\Big)$$

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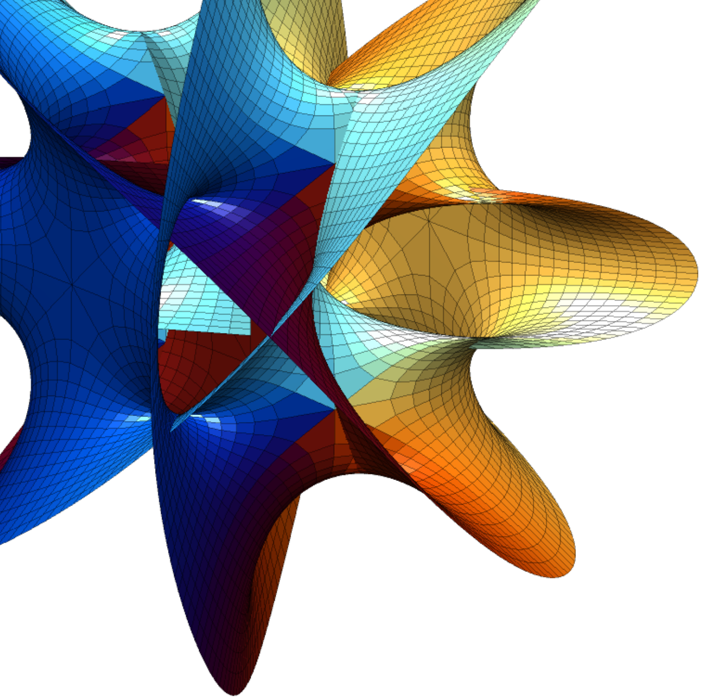
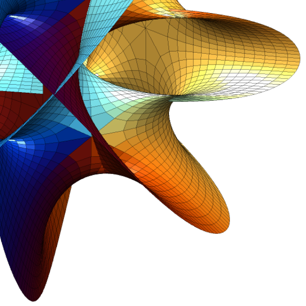
[Fumagalli, Renaux-Petel, **LW** 2012.02761 and in preparation]

$$\Omega_{\rm GW}(k) \underset{k \sim \frac{2}{\sqrt{3}} k_{\rm peak}}{=} \overline{\Omega}_{\rm GW}(k)\Big[1 + \mathcal{A}_{\log,1} \cos\big(\omega_{\log} \log(k/k_{\rm ref}) + \phi_{\log,1}\big) + \mathcal{A}_{\log,2} \cos\big(2\omega_{\log} \log(k/k_{\rm ref}) + \phi_{\log,2}\big)\Big] .$$



II. Implications for string phenomenology



Implications for string pheno

General remark: many models of inflation in string compactifications aim at reproducing single-field slow-roll for 50 to 60 e-folds.

This is not strictly required by current data permitting deviations from single-field slow-roll at small scales.

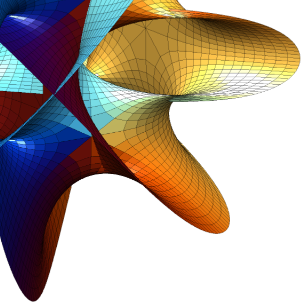
$\longrightarrow$ Embrace the complexities of inflation in string compactifications (multiple fields, complicated potentials)

[Although this is also hampered by technical difficulties.]

Implications for string pheno

Objection: Can we confidently speak about inflation in string compactifictions giving the status of the field?

- Open questions regarding **theoretical control in KKLT** including the uplift.
- **Attainability of (quasi-) de Sitter** in string theory (cf. dS conjecture).



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History: Questions in inflation have been a driver for theoretical developments in string compactifications:

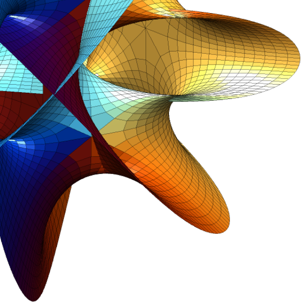
- Possibility of **large-field inflation in string theory**?

  The task of constructing or ruling out models of large field inflation was one driver behind the development and sharpening of the **weak gravity conjecture** and the **swampland distance conjecture**.

Implications for string pheno

Continue to use questions related to inflation to learn both about cosmology and string compactifications.

Suggestion: study the possibility of sharp features in string theory.

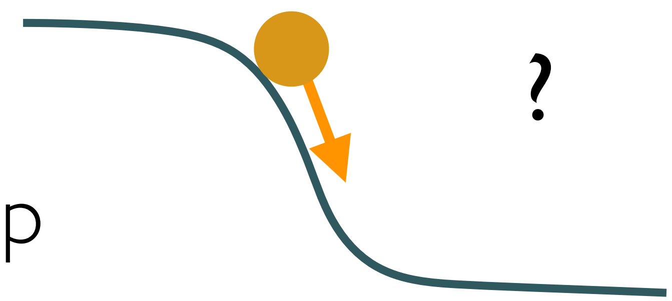


Implications for string pheno

Continue to use questions related to inflation to learn both about cosmology and string compactifications.

Suggestion: study the possibility of sharp features in string theory.

1.) Are sharp steps in the potential possible?



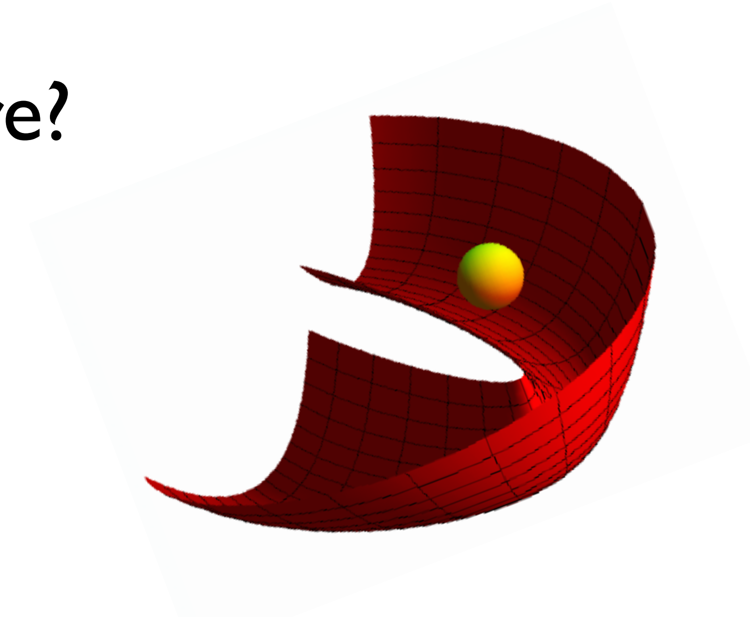
Is there a bound on the steepness of the step linking two flat plateaus?

Potential bound on V' reminiscent of "dS vacua conjecture".

Implications for string pheno

II.) Are there bounds on the field space curvature?

In models with strong sharp turns the maximal enhancement of fluctuations depends on the total angle of turn.

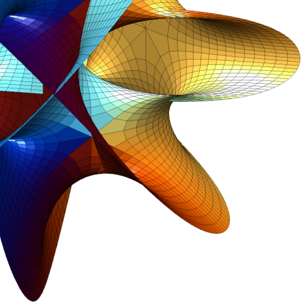


$$\frac{\mathcal{P}_{\rm max}}{\mathcal{P}_{\rm CMB}} \gtrsim 10^7 \quad \Rightarrow \quad \Delta\theta \gtrsim 4\pi$$ [Palma et al. 2004.06106]

Only possible in **curved spacetime**.

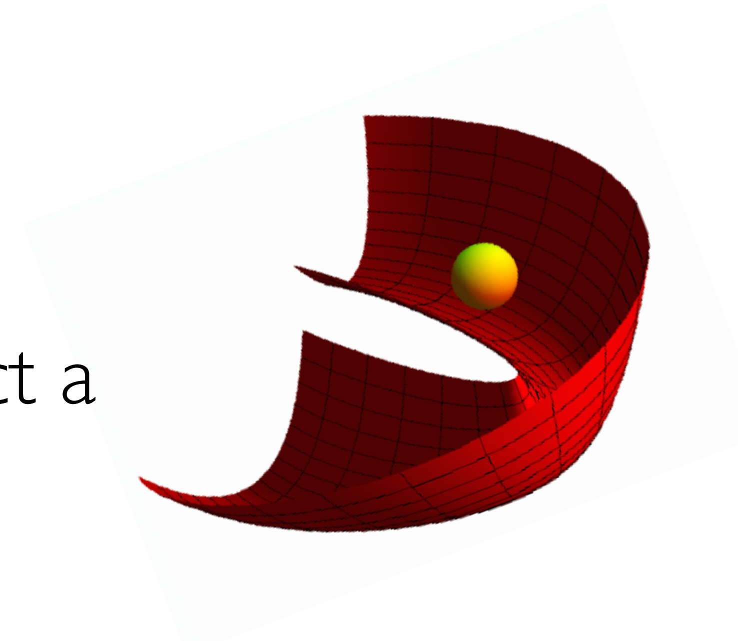
String theory moduli spaces are curved: $K = 2\ln\mathcal{V}$ etc.
This is inherited by the lower-dimensional moduli spaces once heavier moduli spaces are fixed.

Are there bounds on the curvature?

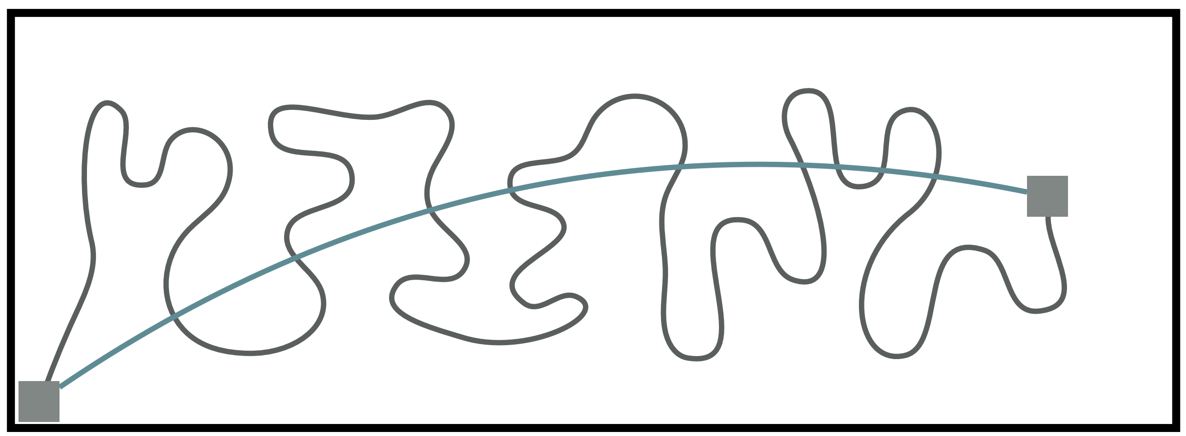


Implications for string pheno

III.) Are there bounds on the rate of turn?



Consider several strong sharp turns in a row.
Can connect several turns in a row to construct a long trajectory in a finite moduli space.

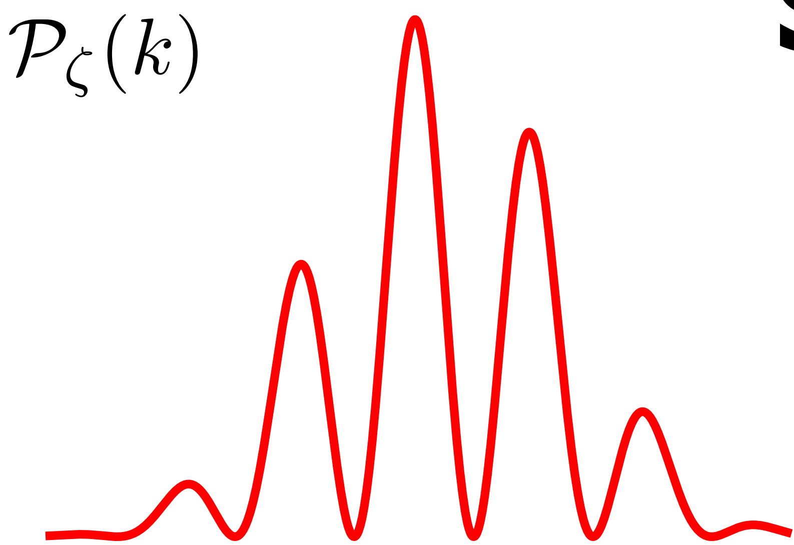


Cf. axion alignment mechanism.

Can imagine violating the Refined swampland distance conjecture.

Is there a bound on the rate of turn?

Summary



Oscillations in $\mathcal{P}_\zeta(k)$ are a key signature of departures of inflation from single-field slow-roll

Find corresponding modulation in the scalar-induced contribution to the SGWB.

Sharp feature: $\omega_{\rm lin} \longrightarrow \omega_{\rm lin}^{\rm GW} = \sqrt{3}\omega_{\rm lin}$

Resonant feature: $\omega_{\rm log} \longrightarrow \omega_{\rm log}^{\rm GW} = \omega_{\rm log},\ 2\omega_{\rm log}$

Summary

- Expect string theory to place bounds on inflation models with sharp features:

  $\longrightarrow$ Bounds on steps in the inflaton potential?

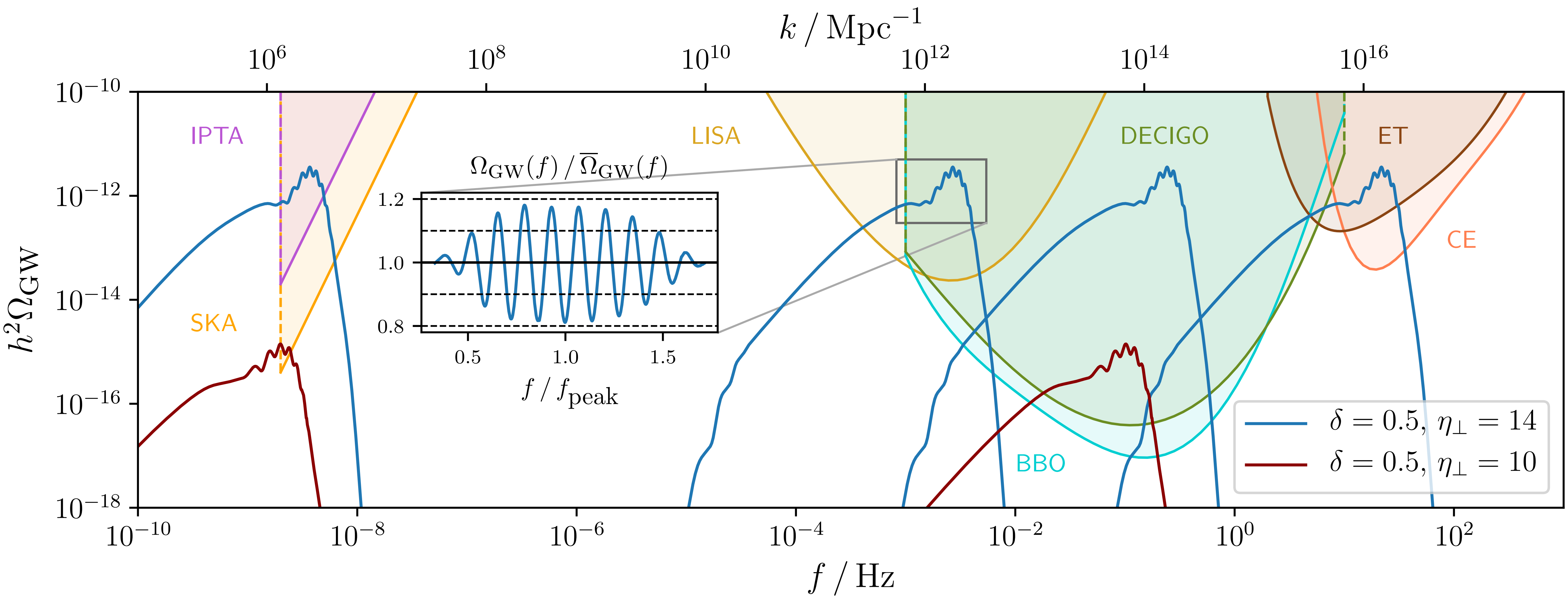
  $\longrightarrow$ Bounds on the field space curvature?

  $\longrightarrow$ Bounds on the rate of turn?

- This will require addressing the difficult question of how the potential in string theory compactifications is constrained.

  This is still largely open except to some extent for instanton contributions in the case of axion potentials.

  However, this is changing, see the initial work on constraining departures from geodesics. [Calderón-Infante, Uranga, Valenzuela 2012.00034]



Many thanks for listening!

Extra Slides

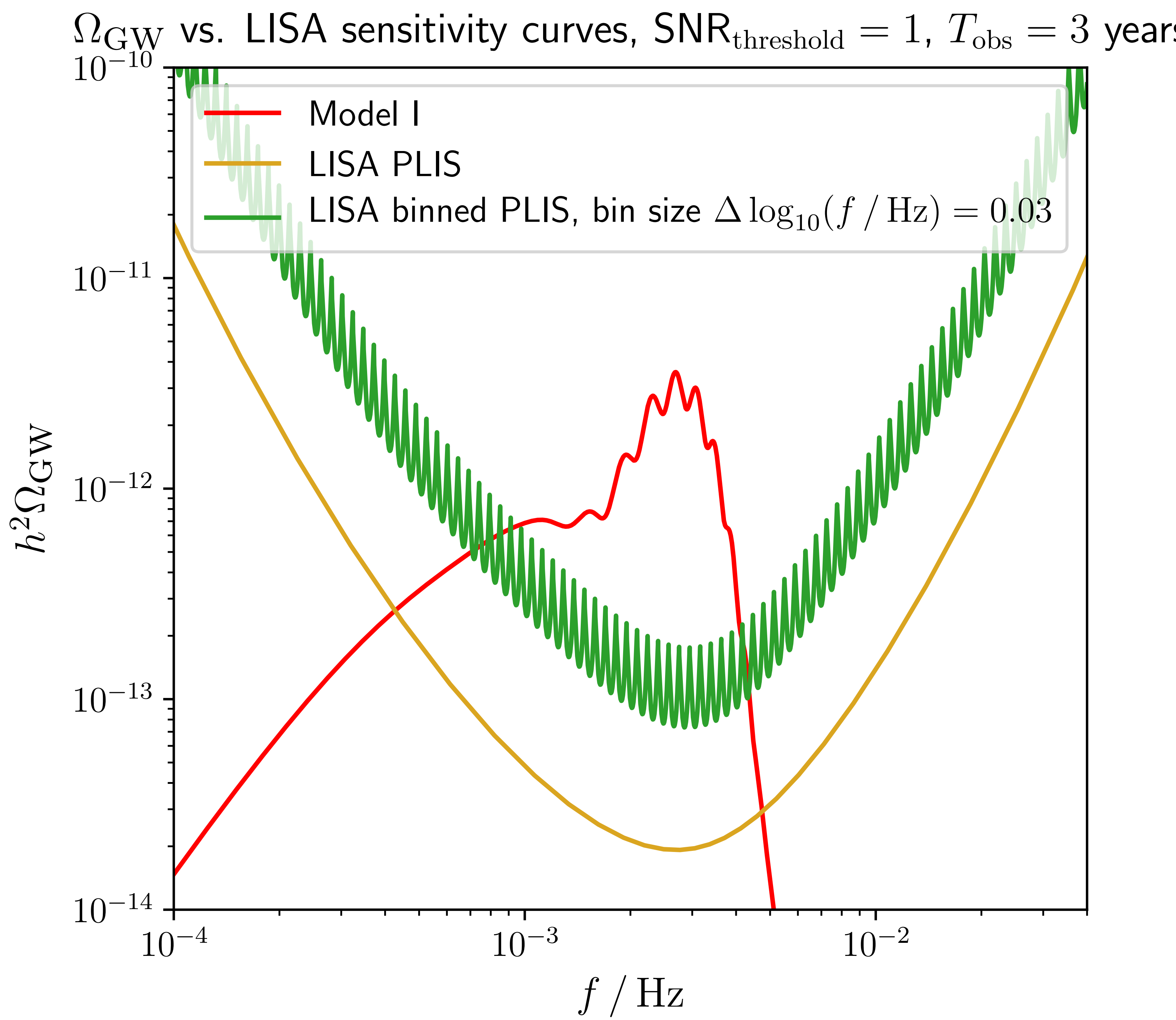
A sharp turn as a sharp feature

Signal detectable?
Can get a first idea by comparing with the power-law-integrated sensitivity (PLIS) curves:
[Thrane, Romano 1310.5300]

LISA PLIS

LISA binned PLIS
[Caprini et al. 1906.09244]

Oscillations appear resolvable with LISA if the overall magnitude of $\Omega_{\rm GW}(k)$ is sufficiently high.

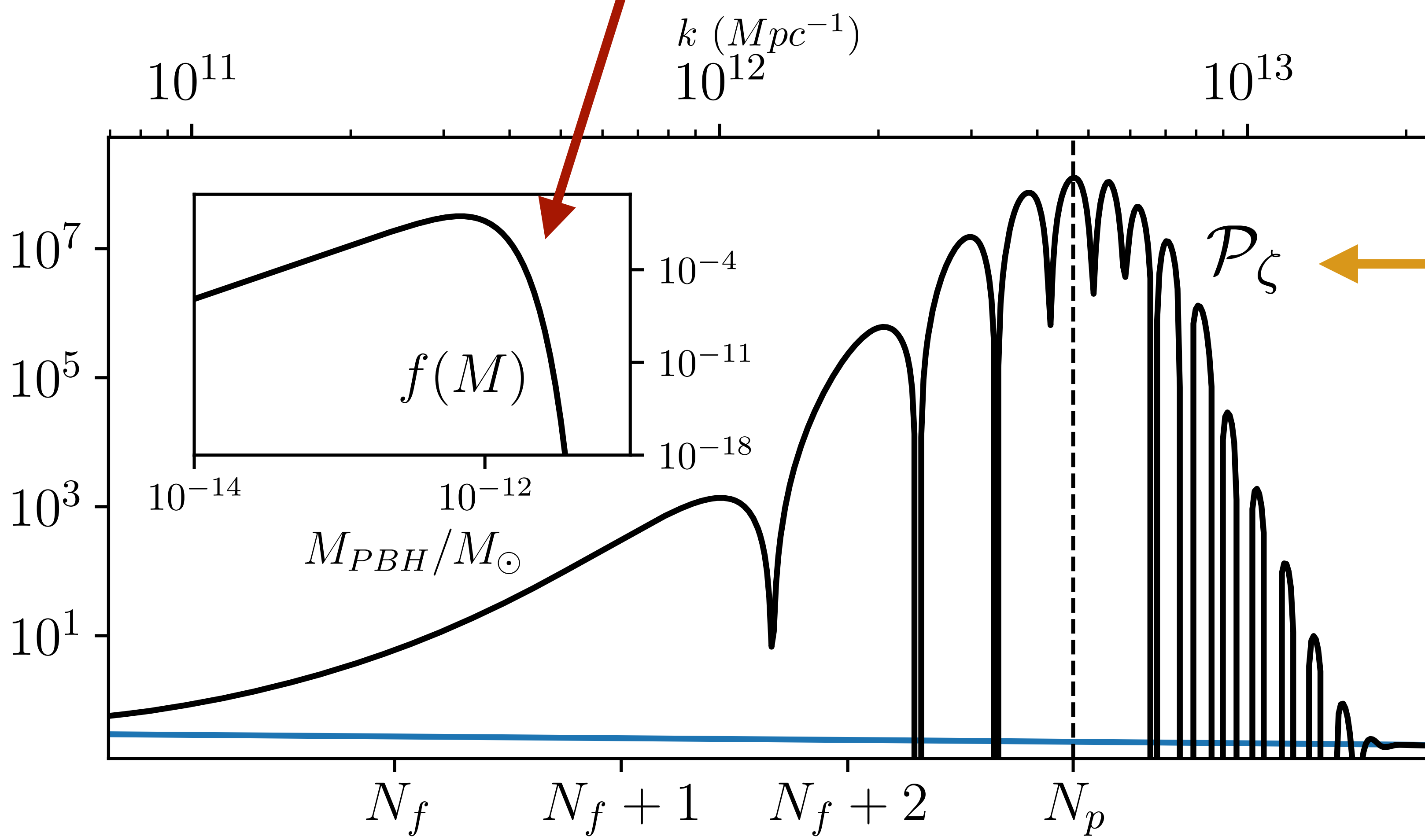


Definite answer requires dedicated analysis, to see if 10% modulations can be resolved.

PBH abundance

PBH mass function $f(M)$ for a sharp feature model

[Fumagalli, Renaux-Petel, Ronayne, **LW** 2004.08369]



Scalar power spectrum with oscillations

Here: compute $f(M)$ assuming the fluctuations obey Gaussian statistics.

Oscillations washed out in $f(M)$ as a result of smoothing and the integration over the formation time.

Primordial Features

Bunch-Davies

Sharp feature

$$\hat{\zeta}_k(\tau) = \zeta_k^{\rm BD}(\tau)\hat{a}(\boldsymbol{k}) + {\rm h.c.}(-\boldsymbol{k})$$

$$\zeta_k^{\rm BD}(\tau) = \left(\frac{k^3}{2\pi^2}\right)^{-1/2} \mathcal{P}_0^{1/2} e^{-ik\tau}(1+ik\tau)$$

$$\mathcal{P}_0 = \frac{H^2}{8\pi^2 \epsilon M_{\rm Pl}^2}$$

N

$N_{\rm f} \sim \log(k_{\rm f})$

Primordial Features

Bunch-Davies

Sharp feature

Excited state

$$\hat{\zeta}_k(\tau) = \zeta_k^{\rm BD}(\tau)\hat{a}(\boldsymbol{k}) + \text{h.c.}(-\boldsymbol{k}) \longrightarrow \hat{\zeta}_k(\tau) = \left[\alpha_k\,\zeta_k^{\rm BD}(\tau) + \beta_k\,\zeta_k^{*\rm BD}(\tau)\right]\hat{a}(\boldsymbol{k}) + \text{h.c.}(-\boldsymbol{k})$$

$$\zeta_k^{\rm BD}(\tau) = \left(\frac{k^3}{2\pi^2}\right)^{-1/2}\mathcal{P}_0^{1/2}e^{-ik\tau}(1+ik\tau)$$

quantisation: $|\alpha_k|^2 - |\beta_k|^2 = 1$

$$\mathcal{P}_0 = \frac{H^2}{8\pi^2\epsilon M_{\rm Pl}^2}$$

matching: $\boxed{\text{ph}\left(\frac{\beta_k}{\alpha_k}\right) \sim e^{2ik/k_{\rm f}}}$

$N_{\rm f} \sim \log(k_{\rm f})$ $\quad N$

Primordial Features

Bunch-Davies

Sharp feature

Excited state

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$$\mathcal{P}_0 = \frac{H^2}{8\pi^2\epsilon M_{\rm Pl}^2}$$

$N_{\rm f} \sim \log(k_{\rm f})$ $\longrightarrow N$

$$\boxed{\mathcal{P}_\zeta(k) \sim \mathcal{P}_0(k)\left(|\alpha_k|^2 + |\beta_k|^2 + 2|\alpha_k||\beta_k|\cos\left(\frac{2k}{k_{\rm f}}\right)\right)}$$

Primordial Features

Why peaks in $\mathcal{P}_\zeta(k)$ and oscillations go together (for sharp features)

$$\mathcal{P}_\zeta(k) \sim \mathcal{P}_0(k)|\alpha_k|^2\left(1 + \frac{|\beta_k|^2}{|\alpha_k|^2} + 2\frac{|\beta_k|}{|\alpha_k|}\cos\left(\frac{2k}{k_{\rm f}}\right)\right) \qquad |\alpha_k|^2 - |\beta_k|^2 = 1$$

Primordial Features

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Want a boosted power spectrum:

$$\mathcal{P}_\zeta(k) \gg \mathcal{P}_0(k)$$

$$\Rightarrow |\alpha_k| \gg 1$$

Quantisation condition implies:

$$\frac{|\beta_k|}{|\alpha_k|} \to 1$$

Have order one oscillations in $\mathcal{P}_\zeta(k)$

A peak in $\mathcal{P}_\zeta(k)$ and oscillations go together

Backreaction & perturbative control

Large enhancement of fluctuations can induce **strong backreaction** on background dynamics or lead to **loss of perturbative control**.

May not be fatal for the mechanism, but needs to be taken into account and will certainly affect the phenomenology of results.

For the sharp turn model:

No excessive backreaction: $$\eta_\perp^4 e^{2\delta\eta_\perp} \lesssim 10^{11} \left(\frac{10^{-9}}{\mathcal{P}_0}\right) ,$$

Perturbative control: $$\eta_\perp^4 e^{2\delta\eta_\perp} \lesssim 10^{9} \left(\frac{10^{-9}}{\mathcal{P}_0}\right) .$$

The perturbativity bound is more stringent, but a more rigorous computation than this estimate is required.