Holography, 1-Form Symmetries & Confinement

based on 2104.12764 w. M. VAN BEEST
D. GOULD
S. SCHÄFER-NAMEKI

FABIO APRUZZI - OXFORD

Motivation:

* Symmetries & Anomalies constrain strongly coupled Dynamycs
* In $N=1$ $SU(N)$ sym (or Adj QCD) Confinement has precise notion
  1-form symm preserving vacuum

$$\langle W_f(L)\rangle \xrightarrow{L\to\infty} 0 \quad (\text{CONFINING}) \qquad \langle W_f(L)\rangle \xrightarrow{L\to\infty} \neq 0 \quad (\text{NOT CONFINING})$$

GOAL: Study Confinement in terms of 1-form symmetries & Anomalies

HOW: Using Holography to determine Symmetries & Anomalies

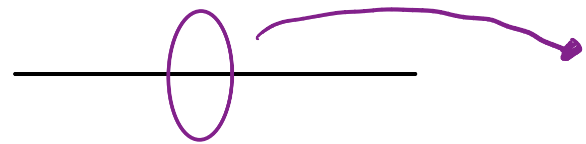
Set up: Cascade $SU((k+1)M) \times SU(kM) \longrightarrow SU(M)$
dual Klebanov-Strassler solution in IIB

1. 1-Form symmetries

[GAIOTTO – KAPUSTIN – SEIBERG – WILLETT]

$$U_g(S^{d-q-1})\mathcal{V}(C^q) = g(\mathcal{V})\mathcal{V}(C^q)$$

WILSON LINE IN GAUGE THEORIES:

$\mathcal{V}(C^1) = \oint e^{iA} \longrightarrow$  U_g Topological Operator

- $\Gamma^{(1)} = Z(G) \longrightarrow$ **Center of The Gauge Group G**

  (NO MATTER)

G	$Z(G)$
$SU(N)$	$\mathbb{Z}_N$
$Sp(N)$	$\mathbb{Z}_2$
$Spin(N)$, N odd	$\mathbb{Z}_2$
$Spin(4N+2)$	$\mathbb{Z}_4$
$Spin(4N)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
E_6	$\mathbb{Z}_3$
E_7	$\mathbb{Z}_2$

MATTER BREAKS $\Gamma^{(1)}$ by screening

- $\Gamma^{(1)}$ acts on Fund. W.L.

  $W_F \longrightarrow \omega\, W_F \qquad (\omega^N = 1 \text{ for } \mathbb{Z}_N)$

- Backgrounds $B_2 \in H^2(M, Z(G))$

  MAKES instanton density $Tr(F^2)$ fractional

Line operators in Gauge Theory

$\mathfrak{g}$ = Lie Algebra (simple)

labelled by $(\lambda_e, \lambda_m) \in L = Z(G) \oplus Z(G)$ [AHARONY - SEIBERG - TACHIKAWA]

Dirac Quantization = Mutual locality $\lambda_e \lambda_m' - \lambda_m \lambda_e' = 0$

(λ_e, λ_m) & (λ_e', λ_m')

Different choices Ex. $SU(2) = \mathfrak{g}$ $L = \mathbb{Z}_2^e \oplus \mathbb{Z}_2^m$

1) $(1,0)$ = WL $\Rightarrow$ $\Gamma^{(1)} = \mathbb{Z}_2^e$ $\Rightarrow$ $G = SU(2)$

2) $(0,1)$ = 'T L $\Rightarrow$ $\Gamma^{(1)} = \mathbb{Z}_2^m$ $\Rightarrow$ $G = SO(3)_+$

3) $(1,1)$ $\Rightarrow$ $SO(3)_-$

GLOBAL FORM !

- IN AdS/CFT specified by Boundary condition of $S_{5d} \supset N \int B_2\, dC_2$

$SU(N)$ vs $SU(N)/\mathbb{Z}_N$ vs $SU(N)/\mathbb{Z}_K$ $\mathbb{Z}_K \subset \mathbb{Z}_N$

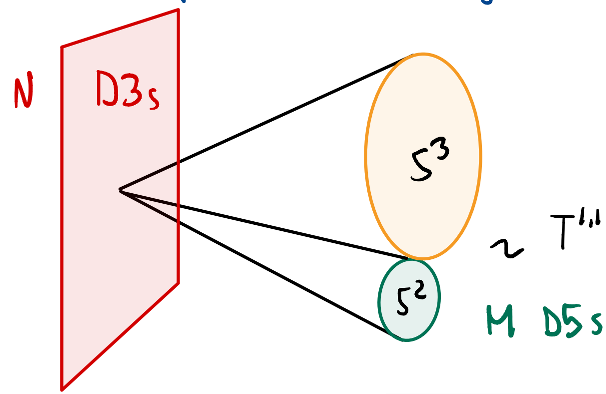
[WITTEN '98]

2. Holography : KS solution

[Klebanov - Strassler ...]

string theory : D3 probing IIB conifold $C(T^{1,1})$

Calabi-Yau cone

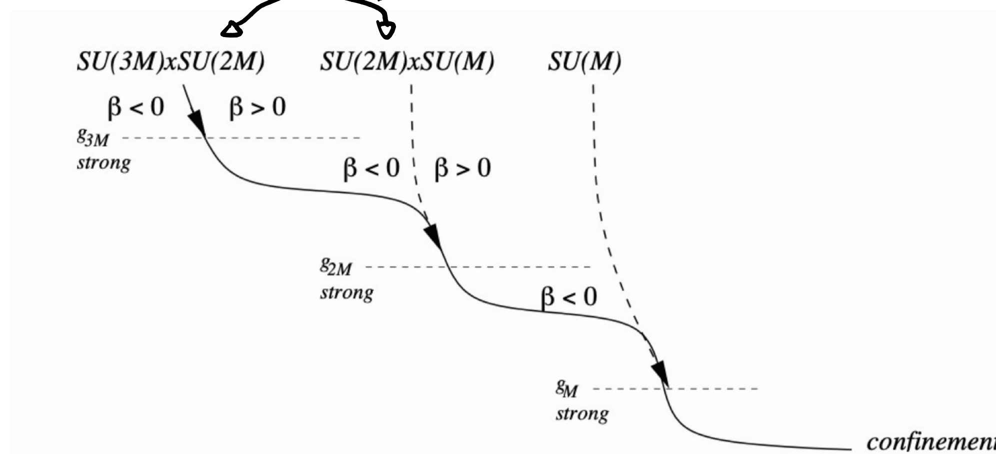


N D3 + M D5

$\Downarrow$

Fractional D3s

Seiberg Duality



$N = kM$

$SU([k+1]M) \rightleftarrows SU(kM)$

Holographic Solution:

$$r_s \sim r_0 \exp\left(-\frac{N}{M^2} - \frac{1}{4}\right)$$

$r \gg 1$ 2 Regimes $r < r_K$

UV KS SOLUTION:

$$ds_{10}^2 = \frac{r^2 dx^2}{R(r)^2} + \frac{R^2(r)}{r^2} dr^2 + R^2(r)\, ds^2_{T^{1,1}}$$

$$R(r) \sim \ln\left(\frac{r}{r_s}\right)^{1/4} \qquad \frac{N}{M^2} \gg 1$$

$$\int_{S^3} F_3 = M \qquad \int_{S^2} B = \mathcal{L}(r) \sim M \ln(r/r_0)$$

$$\int_{T^{1,1}} F_5 = N + M\mathcal{L}(r) = K(r) \gg 1$$

$$r_j = r_0\, e^{-\frac{j}{M}} \qquad \boxed{K(r_j) = (K - j)\, M}$$

IR KS SOLUTION:

$$K(r_K) \sim 0 \qquad \left(SU(n) \text{ at strong coupling}\right)$$

$$ds_{10}^2 \sim e^{2T/3} dx_4^{10} + \frac{M}{2} d\tau^2 + \frac{1}{2} d\Omega_{S^3} + \frac{1}{4}\tau^2 d\Omega_{S^2}$$

DEFORMED CONIFOLD

$$\int_{S^3} F_3 = M$$

3.5d Bulk Topological Couplings

Anomalies of QFT $\longleftrightarrow$ Topological Couplings SUGRA

$Z[\text{bckground}] \to e^{i\,\text{Anomaly}} Z[\text{background}]$ $\qquad$ $\text{Anomaly} = \delta S_{5d}\big|_{\text{boundary}}$

CASCADE : $SU(N-(l-1)M) \times SU(N-lM)$ with Bifund.

Matter breaks $\mathbb{Z}_{N-(l-1)M} \times \mathbb{Z}_{N-lM}$ to $\mathbb{Z}_{GCD(M,N)}$

(bifund are unchanged)

- if $N = KM$ $\quad GCD(M,N) = M$

$SU(M)$ at strong coupling with $\mathbb{Z}_M$ 1-form symm

GOAL IS TO SEE THIS FROM HOLOGRAPHY!

Strategy: Reduce IIB sugra on $T^{1,1} \simeq S^2 \times S^3$ [CASSANI - FAEDO]

UV KS SOLUTION AT $r = r_0 + r'$ $r' \ll 1$ $g > K$

$$F_3 = F_3^{backgr} + dC_2 + (dC_0 + 2MA)\,\omega_2 + \dots \qquad F_5 = F_5^{backgr} + da_2 \wedge \omega_2 + \dots$$

$$H_3 = H_3^{bcgr} + db_2 + \dots$$

- Reeb vector dual to $U(1)_R$ $d\beta \to d\beta - A$ (Hopf-fiber of S^3)
- $\omega_2 \leftrightarrow S^2$, $\omega_3 \leftrightarrow S^3$
- $C_0 \sim C_0 + 2\pi$

stückelberg coupling

$U(1)_R \to \mathbb{Z}_{2M}$

dual to chiral or ABJ anomaly

$\Downarrow$ BIANCHI ID ✓ $F = * F_5$ ✓

DOMINANT at $r' \ll 1$

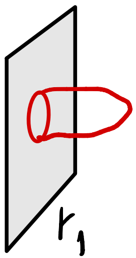
$$S_{5d} \supset \int \frac{1}{2} |dC_0 + 2MA|^2 + \boxed{M\, b_2\, dC + M^2 b_2^2 A}$$

$C = pC_2 - q a_2$

(p,q) coprime

Boundary Conditions $\boxed{M b_2 dC}$ & 1-Form symmetries : $T^{(1)}$

(1) b_2 Dirichlet , C Neumann , $SU((k-l+1)M)\times SU((k-l)M)$ $\qquad M\, db_2 = 0$



δC varies at r_1

F1-strings $\Rightarrow$ WL

M F1 screened by baryon vertex

Bianchi ids:

$$\int_{T^{1,1}} dF_7 = (k-l)\, M H_3$$

$$\int_{S^3} dF_5 = M H_3$$

$$T^{(1)} = \mathbb{Z}_M$$

(2) $C \leftrightarrow b_2$ $\qquad G = \dfrac{SU((k-l+1)M)\times SU((k-l)M)}{\mathbb{Z}_M}$ $\qquad$ M D3/D1 screened by B.V.

$$T^{(1)} = \mathbb{Z}_M^m$$

(3) $M = pq$ $\qquad G = \dfrac{SU((k-l+1)M)\times SU((k-l)M)}{\mathbb{Z}_p}$ $\qquad T^{(1)} = \mathbb{Z}_q \times \mathbb{Z}_p$

Mixed Anomaly

$$\boxed{\omega = M^2 b_2^2 A}$$

Choice of B.C. (1) + $U(1)_a \to \mathbb{Z}_{2M}$ $\Longrightarrow$ $\begin{cases} A \to \mathbb{Z}_{2M} \text{ background} & \oint A \in \frac{\mathbb{Z}}{2M} \\ b_2 \to \Gamma^{(1)} \text{ background} & \oint b_2 \in \frac{\mathbb{Z}}{M} \end{cases}$

Mixed $\Gamma^{(0)} = \mathbb{Z}_{2M} \,/\, \Gamma^{(1)} = \mathbb{Z}_M$ anomaly

$$Z[b_2] \xrightarrow{\mathbb{Z}_{2M}} Z[b_2] \exp\left[2\pi i \frac{1}{2M} \int B_2^2\right]$$

$$B_2 \sim M b_2 \qquad \oint B_2 \in \mathbb{Z}$$

IR STRONGLY COUPLED PHYSICS CONSTRAINED BY

- 't HOOFT ANOMALY MATCHING
- TQFT PROPERTIES OF GAPPED CONFINING VACUA

$\boxed{\text{Anomaly: } \omega = M^2 b_2^2 A}$

[CORDOVA - OHMORI]

IR: 4D GAPPED CONFINING VACUUM :

MODELED BY TQFT

- $\omega \neq 0$ $\Rightarrow$ NO TQFT w. $\mathbb{Z}_M^{(1)}$ & $\mathbb{Z}_{2M}^{(0)}$
- TQFT $\Rightarrow$ $\omega = 0$ on "SPIN manifold"

$\downarrow$

proven by using properties of unitary TQFT

Consequences:

- $\omega = 0$ when $\mathbb{Z}_{2M} \to \mathbb{Z}_2 \Rightarrow \chi_{SB}$!
- χ_{SB} is a necessary condition for IR TQFT
- spontaneous χ_{SB} $\Rightarrow$ M VACUA
- Domain walls between the M vacua : D5s on S^3

QUESTION: WHAT IS IR TQFT THAT MODEL THIS?

IR 5d Action & 4d TQFT : Anomaly Matching

$$S_{5d} = \int \frac{1}{2} \left| dc_0 + 2MA \right|^2 + M \frac{dc_0}{2} b_2^2$$

"2 deriv." "1 deriv." both important

IR Regimes $\tau \to 0$ in deform. conifold

THE DOMAIN WALLS ARE D5 on S^3 with $C_6 = C_3 \wedge \omega_3$

& $F_3 = * F_7 \implies * \frac{1}{2} \left(dc_0 + 2MA \right) = dc_3$

$$S_{5d} = \int dc_0 \, dc_3 + M \frac{dc_0}{2} b_2^2 \xrightarrow[\boxed{M db_2 = 0}]{\delta C_0} S_{4d} = \int c_0 \, dc_3 + \frac{M}{2} c_0 b_2^2$$

symmetries : - $\Gamma^{(1)} = \mathbb{Z}_M^{(1)}$ b_2 is background

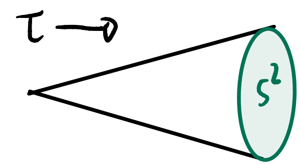
- $c_0 \to c_0 + 2\pi$ $\mathbb{Z}_2^{(0)}$ symmetry

- Realises $\mathbb{Z}_{2M} \to \mathbb{Z}_2$ spontaneous breaking

- DOMAIN-WALLS: $ds^2_{10} \sim e^{2/3\tau} dx^2 + \frac{1}{2} d\tau^2 + \frac{1}{2} d\Omega^2_{S^3} + \tau^2 d\Omega_{S^2}$

IR KS-SOLUTION, WARPED DEFORMED CONIFOLD

K D5 ON S^3 source $\int_B F_3 \sim \langle c_0 \rangle \int_{S^2} \omega_2 \sim K \int_{S^2} \omega_2$ $\quad \langle c_0 \rangle \sim K$

$B =$ 

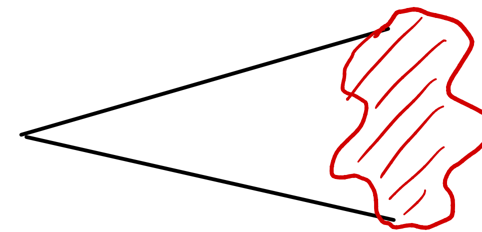
- ANOMALY $A b_2^2 M^2$ matched by IR $K \to K+1$ ACTION of $\mathbb{Z}_{2M}^{(0)}$ ON IR THEORY

Conclusions & Outlook

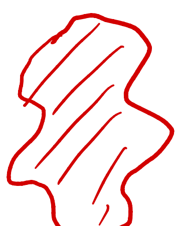
* 1-form symmetry & 0-1 mixed anomaly constrain IR dynamics of $\mathcal{N}=4$ $SU(N)$ SYM

* We derived 1-form symm, anomaly & IR TQFT from Holography

* Framework which can be generalised for many set up in string th engineering



* G_2 in M-th $\downarrow$ 4d $\mathcal{N}=1$ SYM



$= S^3 \times S^3/\Gamma$, CY_3 in M-th $\downarrow$ 5d SCFT

$= Y^{p,q}$

Thank you!

4. LITTLE STRING THEORY & MALDACENA-NUNEZ

IIB M NS5 $\Rightarrow$ $(1,1)$ 6d SYM $SU(N)$ IR of LST

Mixed anomaly $I_8 \supset 2N\, c_2(R)\, I_4(F)$ ⟶ instanton density of $SU(N)$

c_2 2nd Chern class

$$I_6 = \int_{S^2} I_8$$

twisted reduction on S^2

$$F_{R_{6d}} = \Big(\underset{U(1)_R}{dA_R} + \tfrac{1}{2}\underset{\text{spin connection}}{A}\Big)^2$$

$$I_6 = 2M\, dA_R \wedge c_2(F_G) \qquad dI_5 = I_6$$

ABJ anomaly: trivializes when $2N\, dA_R = 0$

$U(1)_R \to \mathbb{Z}_{2M}$ $\quad \oint A_R \in \frac{\mathbb{Z}}{2M}$

descent $I_5 = 2M A_c I_4(F)$

1-form symm background

$I_4(F_G) \to I_4(F_{G/Z(G)})$ Fractional

Outline:

1. 1-Form symmetries & th. definition of confinement
2. Holography: Review of Klebanov-Strassler
3. 5d bulk topological couplings in supergravity
   - Boundary conditions & 1-Form symmetry
   - 't Hooft Anomaly & χ_{SB}
   - IR Gapped vacuum & 4d TQFT
4. LITTLE STRING THEORY PERSPECTIVE

- **Background 1-Form Symmetry** $\longrightarrow$ **Pair:** $(C_2, C), \quad NC_2 = dC$

$$SU(N), \quad \Gamma^{(1)} = \mathbb{Z}_N$$

C is a *U(1)* Connection

- **Instanton Density Shift:** $\longrightarrow$ $I_4 \to c_2(F') + \dfrac{N-1}{2N} dC \wedge dC$

I_4, F **Invariant under transformation of** $\Gamma^{(1)}$

- $\int_{M_4 \subset 6d} c_2(F') \in \mathbb{Z} \quad \oint dC \in \mathbb{Z}$ **Instanton Density takes <u>Fractional Values</u>**

Theoretical definition of confinement in Adj QCD

A confining vacuum in SYM is when $\Gamma^{(1)} \neq 0$

$\langle W_F \rangle$ order parameter for $\Gamma^{(1)}$

$$\langle W_F \rangle \sim e^{-TV(r)}$$

$$\begin{cases} \langle W_F \rangle \sim e^{-TL} & \text{Area} \\ \langle W_F \rangle \sim e^{-T/L} & \text{perimeter} \end{cases}$$

L = length of line

Area $\Rightarrow \langle W_F \rangle \xrightarrow{L \to \infty} 0$

symmetry preserved

Perimeter $\Rightarrow \langle W_F \rangle \xrightarrow{L \to \infty} \neq 0$ symm spont. broken