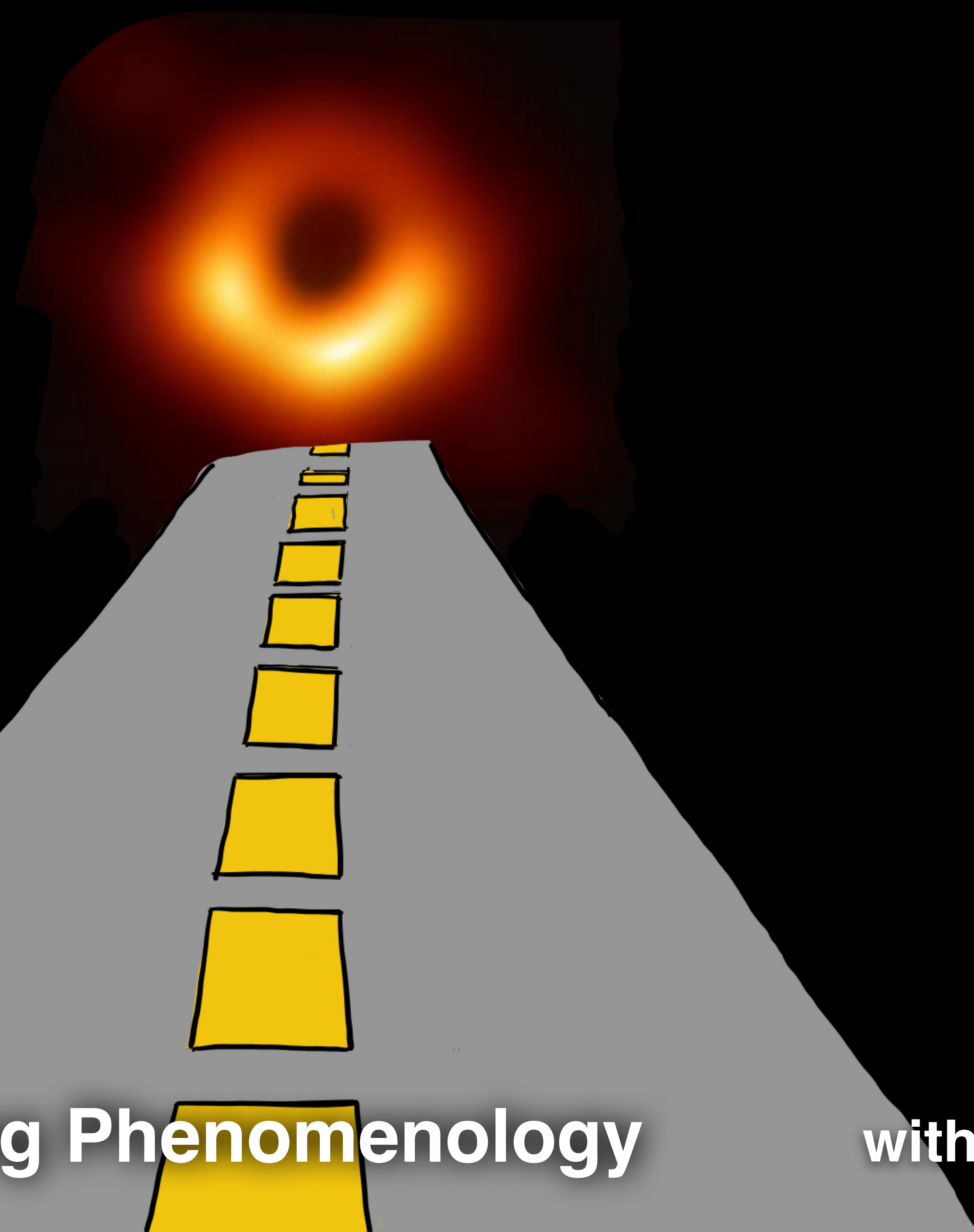
# BPS States and Extremal Black Holes at Infinite Distance



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Summer Series on String Phenomenology

2004.10768
with Irene Valenzuela

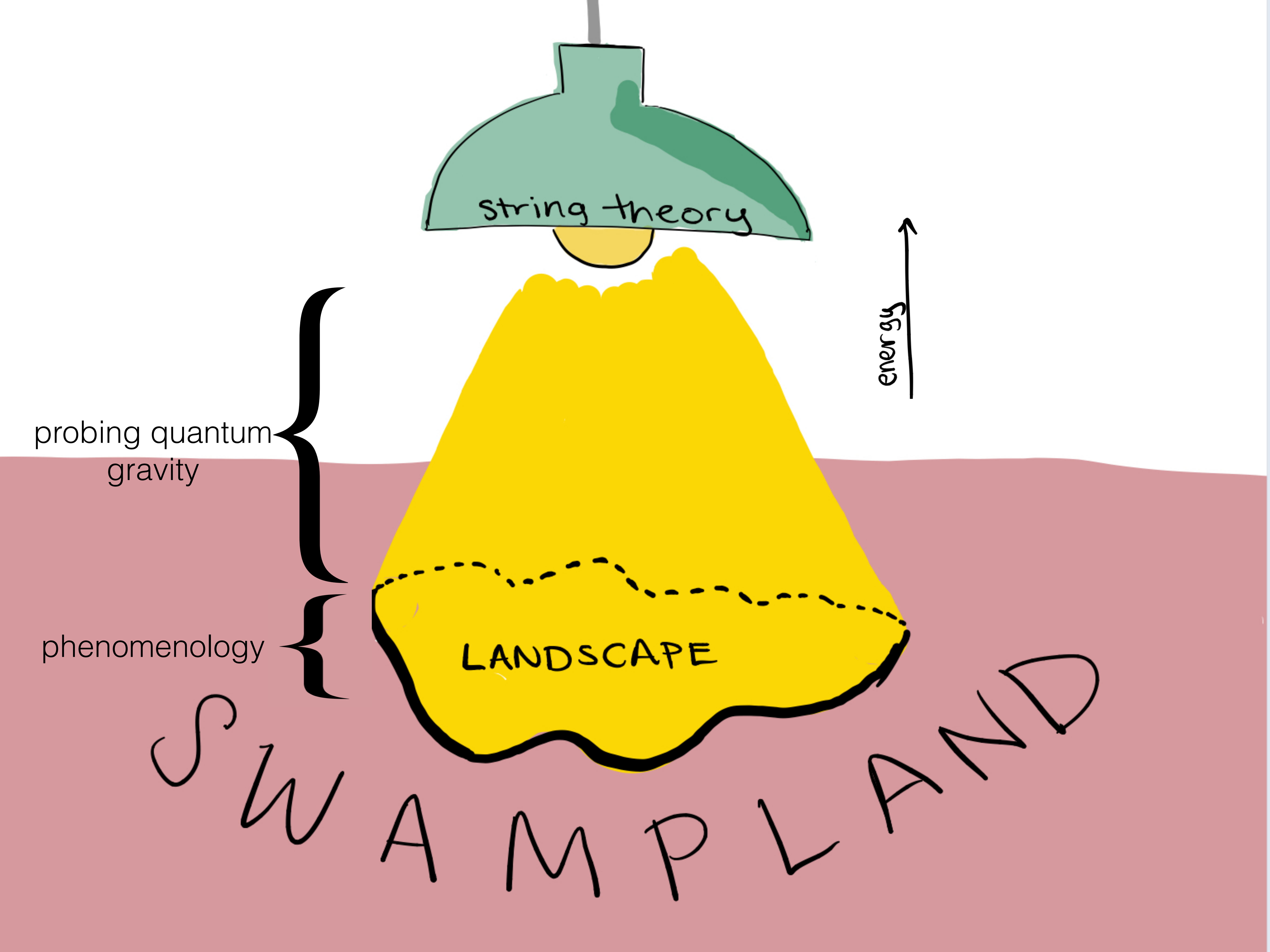
# The general goal

In N=2 compactifications of Type IIB string theory,

1. Verify that the Weak Gravity Conjecture is saturated by the BPS D3-brane states that satisfy the Swampland Distance Conjecture.

2. Demonstrate that in the asymptotic regimes of moduli space, the black hole extremality bound is equal to a force-cancellation condition. Palti ’17; Heidenreich, Reece, Rudelius ’19

3. Calculate a lower bound on the exponential rate at which these BPS states become massless.


string theory
energy
probing quantum gravity
phenomenology
LANDSCAPE
SWAMPLAND

# The Weak Gravity Conjecture (WGC)

Arkani-Hamed, Motl, Nicolis, Vafa '06

Consider a gravitational theory containing a U(1) gauge field.

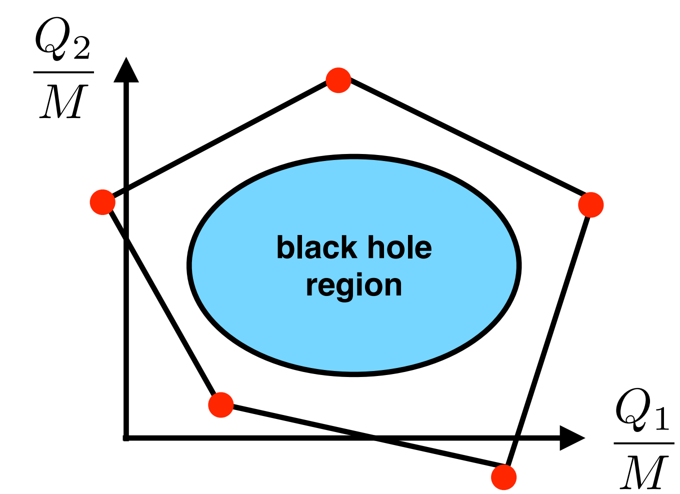
Then to be a consistent theory of quantum gravity, there should exist a particle with

$$\frac{Q}{M} \geq \left(\frac{Q}{M}\right)_{\text{extremal}}$$

extremality bound of the black hole solutions

## With multiple U(1) gauge fields:

The convex hull of the charge-to-mass ratios of states in the theory must contain the black hole region.





Kats, Motl, Padi '07; Giveon, Gorbonos, Stern '10; Cheung, Remmen '14; de la Fuente, Saraswat, Sundrum '14; Nakayama, Nomura '15; Bachlechner, Long, McAllister '15; Brown, Cottrell, Shiu, Soler '15; Heidenreich, Reece, Rudelius '15; Kooner, Parameswaran, Zavala '15; Ibáñez, Montero, Uranga, Valenzuela, '15; Hebecker, Rompineve, Westphal '15; Montero, Uranga, Valenzuela '15; Hebecker, Mangat, Rompineve, Witkowski '15; Harlow '15; Rudelius, '15; Heidenreich, Reese, Rudelius '16; Junghans '16; Brown, Cottrell, Shiu, Soler '16; Danielsson, Dibitetto '16; Baume, Palti '16; Klaewer, Palti '16; Saraswat '16; Conlon, Krippendorf '16; Heidenreich, Reece, Rudelius '16; Benjamin, Dyer, Fitzpatrick, Kachru '16; Cottrell, Shiu, Soler '16; Heidenreich, Reece, Rudelius '17; Fisher, Mogni '17; Montero, Shiu, Soler '17; Herraez, Ibáñez '17; Palti '17; Lüst, Palti '17; Cottrell, Shiu, Soler '17; Hebecker, Henkenjohann, Witkowski '17; Ibáñez, Montero '17; Ibáñez, Martin-Lozano, Valenzuela '17; Hamada, Shiu '17; Hebecker, Soler '17; Montero, Uranga, Valenzuela '17; Giombi, Perlmutter '17; Cottrell, Montero '17; Hod '17; Furuuchi '17; Crisford, Horowitz, Santos '18; Yu, Wen '18; Cheung, Liu, Remmen '18; Andriolo, Junghans, Noumi, Shiu '18; Lee, Lerche, Weigand '18; Moritz, Van Riet '18; Bonnefoy, Dudas, Lüst '18; Urbano '18; Reece '18; Hebecker, Junghans, Schachner '18; Vittmann '18; Gonzalo, Herráez, Ibáñez '18; Gonzalo, Ibáñez '18; de Rham, Heisenberg, Tolley '18; Heidenreich, Reece, Rudelius '18; Craig, Garcia Garcia, Koren '18; Hamada, Noumi, Shiu '19, Bellazzini, Lewandowski, Serra '19; Chen, Huang, Noumi, Wen '19; Montero '19; Lee, Lerche, Weigand '19; Gonzalo, Ibáñez '19; Lee, Lerche, Weigand '19; Heidenreich, Reece, Rudelius '19; Buratti, García-Valdecasas, Uranga '19; Jones, McPeak '19; Loges, Noumi, Shiu '20; Andriolo, Huang, Noumi, Ooguri, Shiu '20; Grimm, Van De Heisteeg '20; Heidenreich, Long, McAllister, Rudelius, Stout '20; Demirtas, Long, McAllister, Stillman '19;

# The Swampland Distance Conjecture (SDC) Ooguri, Vafa '07

Consider a gravitational theory with a moduli space.

Then, as an infinite geodesic distance $\Delta\phi$ in moduli space is traversed, there must exist a tower of states with mass

$$M \sim M_0 e^{-\lambda \Delta\phi}$$

"mass decay rate"

canonically normalized modulus

Grimm, Palti, Valenzuela '18; Blumenhagen '18; Blumenhagen, Valenzuela, Wolf '17; Scalisi, Valenzuela '18; Corvilain, Grimm, Valenzuela '18; Blumenhagen, Kläwer, Schlechter, Wolf '18; Joshi, Klemm '19; Buratti, Calderon, Uranga '19; Kehagias, Riotto '19; Font, Herráez, Ibáñez '19; Klaewer, Palti '17; Blumenhagen, Klaewer, Schlechter '19; Hebecker, Junghans, Schachner '19; Valenzuela '17; Baume, Palti '16; Ooguri, Shiu, Palti, Vafa '18; Heidenreich, Reece, Rudelius '18; Lee, Lerche, Weigand '18; Lee, Lerche, Weigand '19; Palti '17; Grimm, Li, Palti '19; Marchesano, Wiesner '19; Baume, Marchesano, Wiesner '19; Cecotti '20; Gonzalo, Ibáñez, Uranga '19; Palti '15; Hebecker, Henkenjohann, Witkowski '17; Cicoli, Ciupke, Mayrhofer, Shukla '18; Erkinger, Knapp '19;

We will check, and precisely relate, these conjectures in Calabi-Yau compactifications of type IIB.

# Black Hole Extremality Bound

Consider a Reissner-Nordström black hole with charge $Q$ and mass $M$.

When the charge is large enough that the two horizons coincide, the black hole is called "extremal": a larger charge would result in a naked singularity.

The extremality bound for a Reissner-Nordstrom BH in 4D is

$$\left(\frac{Q}{M}\right)_{\mathrm{extremal}} = 1$$

If your theory includes massless scalars, **this bound changes.** Horowitz, Strominger '91

$$\mathcal{L} \supset e^{\vec{\alpha}\cdot\vec{\phi}} F_{\mu\nu} F^{\mu\nu} \qquad \Longrightarrow \qquad \left(\frac{Q}{M}\right)_{\mathrm{extremal}} = \sqrt{1 + \frac{1}{2}|\vec{\alpha}|^2}$$

# Checking the Weak Gravity Conjecture

$$\boxed{\frac{Q}{M}} \geq \boxed{\left(\frac{Q}{M}\right)_{\text{extremal}}}$$

Two ingredients:

- Charge to mass ratios of states
- Black hole extremality bound(s)

Then one can compare these data and test the conjecture.

We will now obtain these data for the effective theories arising from compactifying type IIB string theory on Calabi-Yau threefolds.

# The Lagrangian

Consider a Calabi-Yau threefold with $h^{2,1}$ complex structure moduli and $h^{1,1}$ Kähler moduli.

Compactifying type IIB string theory on this manifold results in an effective theory for the $h^{2,1}$ complex scalar fields and $h^{2,1}+1$ U(1) gauge fields.

The effective action for this sector is

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} - K_{i\bar{j}} \partial_\mu T^i \partial^\mu \bar{T}^{\bar{j}} + \mathrm{Im}(\mathcal{N})_{IJ} F^I_{\mu\nu} F^{J,\mu\nu} + \mathrm{Re}(\mathcal{N})_{IJ} F^I_{\mu\nu} (\star F)^{J,\mu\nu} \right]$$

$K_{i\bar{j}}$, $\mathcal{N}_{IJ}$ are specified functions of the prepotential, $F$, and the complex structure moduli $T^i$.

# BPS states

This theory also contains states that carry electric and magnetic charge under the U(1)s. Among these, the BPS states are D3-branes.

A tower of these D3 branes was shown to satisfy the SDC.
Grimm, Palti, Valenzuela '18

The BPS D3 branes have mass equal to the central charge of the state:

$$M^2 = |Z(\vec{q}, F, T^i)|^2$$

and have charge given by

$$|Q|^2 = -\frac{1}{2}\vec{q}^T \mathrm{Im}(\mathcal{N})^{-1}\vec{q} = |Z|^2 + K^{i\bar{j}} D_i Z D_{\bar{j}} \overline{Z}$$

These states satisfy a no-force condition.

We will show that they are also extremal in the asymptotic limits of moduli space.

# BPS charges

Not all charges in the quantized charge lattice can support BPS states.

In order to be a BPS black hole, it must satisfy

$$\partial_i |Z|\Big|_{horizon} = 0$$

Ferrara, Kallosh, Strominger '95
Denef '00

This imposes **restrictions** on the charges, independent of the moduli:

$$f(\vec{q}) > 0$$

We will now analyze the charge-to-mass ratios of **BPS states.**

# The structure of BPS states

Because the charge and the mass are given by a complex central charge $Z$, the charge-to-mass ratios form a **degenerate ellipsoid** with 2 non-degenerate directions**.**

Trivial fact: $\dfrac{|Z|^2}{M^2} = 1$

Plug in $Z = e^{\mathcal{K}/2}(\vec{q}^{\,T}\eta\vec{\Pi})$ and $\vec{z} = \dfrac{|Q|}{M}\hat{Q}$ .

Obtain $\vec{z}^{\,T}\mathbb{A}\vec{z} = 1$

$$\mathbb{A} = -2\frac{G^T\eta\,\mathrm{Re}(\vec{\Pi}\vec{\Pi}^\dagger)\eta^T G}{\mathrm{Im}(\overline{X}^I\partial_I F)}$$

$\mathbb{A}$ has 2 non-zero eigenvalues.

$$\eta = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

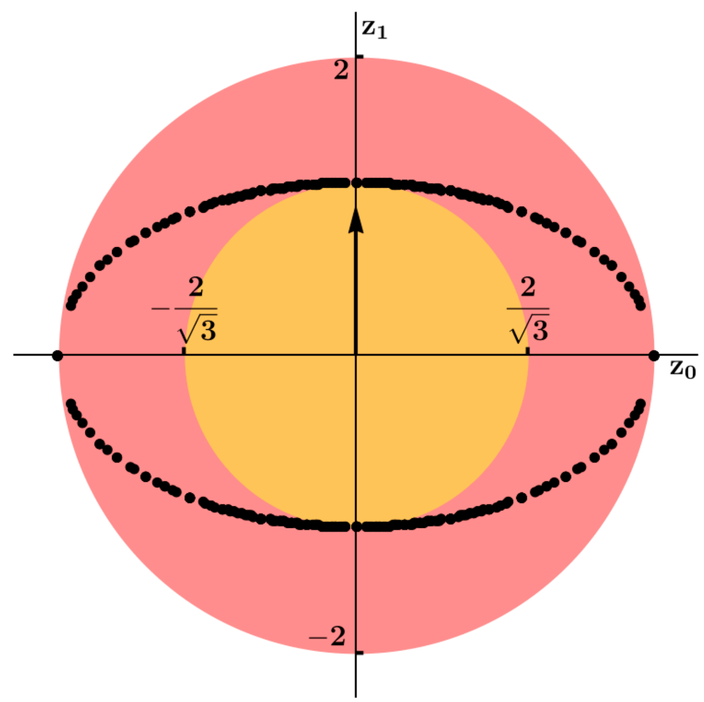
$$\vec{\Pi} = \begin{pmatrix} X^I \\ \partial_I F \end{pmatrix}$$

$$G^T G = -\frac{1}{2}\mathrm{Im}(\mathcal{N})$$

# Example: BPS charge-to-mass ratios

- Prepotential: $F = -\dfrac{(X^1)^3}{X^0}$ can be thought of as an infinite distance limit
- 1 modulus, 2 gauge fields
- All charges of the form $(q_0, q_1, 0, 0)$ are BPS charges

$$\left(\frac{Q}{M}\right)_{BPS} = \frac{2}{\sqrt{3}}\sqrt{\frac{3q_0^2 + 6q_0q_1\theta + q_1^2\left(3\theta^2 + t^2\right)}{q_0^2 + 2q_0q_1\theta + q_1^2\left(\theta^2 + t^2\right)}}, \qquad T = \theta + it$$




note: not the unit ball

$$\mathbb{A} = \begin{pmatrix} 1/4 & 0 \\ 0 & 3/4 \end{pmatrix}, \qquad \left(\frac{Q}{M}\right)_1 = 2 \;\&\; \left(\frac{Q}{M}\right)_2 = \frac{2}{\sqrt{3}}$$

# Example: black hole extremality bound

1. Calculate the gauge kinetic matrix, $\mathrm{Im}(\mathcal{N})$

2. Canonically normalize the modulus

$$\mathrm{Im}(\mathcal{N}) = \begin{pmatrix} -e^{\sqrt{6}t} & 0 \\ 0 & -3e^{\sqrt{\frac{2}{3}}t} \end{pmatrix}$$

Recall $\left(\frac{Q}{M}\right)_{\text{extremal}} = \sqrt{1 + \frac{1}{2}|\vec{\alpha}|^2}$

$$\left(\frac{Q}{M}\right)_1 = \sqrt{1 + \frac{1}{2} \times 6} = 2$$

$$\left(\frac{Q}{M}\right)_2 = \sqrt{1 + \frac{1}{2} \times \frac{2}{3}} = \frac{2}{\sqrt{3}}$$

**These states saturate WGC bound.**

# BPS states in higher dimensional moduli spaces

- Degenerate directions

- Attractor mechanism forbids some charges

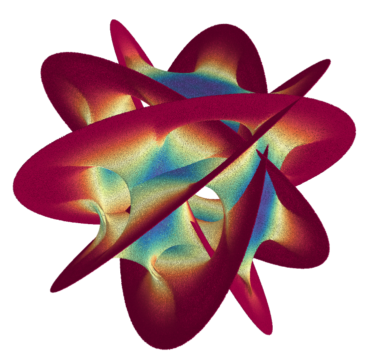
- One always finds: in the infinite distance limit,

$$\boxed{\text{extremal} = \text{BPS}}$$

# A general formula for D3 brane charge-to-mass ratios

- Computing the prepotential, $F$ needed for repeating the previous calculations is hard, in general.

- The tools of **Mixed Hodge Structures** give us a way to compute the leading behavior of $F$ near singularities in the moduli space. Schmid '73; Cattani, Kaplan '82; Cattani, Kaplan, Schmid '86; Kerr, Pearlstein, Robles '19; Grimm, Palti, Valenzuela '18; Corvilain, Grimm, Valenzuela '19; Grimm, Li, Palti '19; Font, Herráez, Ibáñez '19; Grimm, Van de Heisteeg '20; Grimm, Li, Valenzuela '19; Cecotti '20



CY by Geoffrey Fatin

integers $\{\vec{\ell}, \vec{d}\}$ characterizing singular limits

NG, Valenzuela '20

- A general expression: $$\boxed{\left(\frac{Q}{M}\right)^2\Bigg|_{q^I} = 1 + \sum_{i=1}^{h^{2,1}} \frac{(\ell_i^I - \ell_{i-1}^I)^2}{d_i - d_{i-1}} + \mathcal{O}\left(\frac{t_i}{t_{i+1}}\right)}$$

- In any general infinite distance limit, extremal=BPS.

# The mass decay rate and extremal black holes

The **mass decay rate** is

$$\lambda \equiv \left| \frac{\nabla_i M}{M} u_i \right|$$

where $u_i$ is a unit vector pointing along a geodesic trajectory.

For a **single modulus**, since extremal=BPS at infinite distance, we have

$$\left(\frac{Q}{M}\right)_{\mathrm{BPS}} = \left(\frac{Q}{M}\right)_{\mathrm{extremal}}$$

$$\implies 1 + 2\left|\frac{\nabla M}{M}\right|^2 = 1 + \frac{1}{2}|\vec{\alpha}|^2$$

$$\implies \lambda = \frac{1}{2}|\vec{\alpha}|$$ Lee, Lerche, Weigand '18

# Mass decay rate for higher-dimensional moduli spaces

Path dependence issues obscure the identification of the mass decay rate.

Nonetheless, we can place a lower bound on the decay rate for **some states:**

NG, Valenzuela ’20

$$\boxed{\lambda \geq \frac{\nabla_1 M}{M} = \frac{1}{\sqrt{2}} \frac{|\ell_1 - 3|}{\sqrt{d_1}}}$$

related to extremality bound of black hole associated to fastest-growing modulus

The weakest, but most general bound for a Calabi-Yau is then:

$$\ell_1 = 2,\ d_1 = 3 \qquad \Longrightarrow \qquad \lambda \geq \frac{1}{\sqrt{6}}$$

see also Andriot, Cribiori, Erkinger ‘20

# A unifying tower

The Weak Gravity Conjecture and the Swampland Distance Conjecture both predict light states in asymptotic regimes of moduli space. **Both statements have ambiguities.**

There appears to be, in this context, a **single, unambiguous statement:**

In any infinite field distance limit with a vanishing gauge coupling, there exists an infinite tower of charged states satisfying

$$\left(\frac{\mathcal{Q}}{M}\right) \geq \left(\frac{\mathcal{Q}}{M}\right)\bigg|_{\text{extremal}} = \left(\frac{\mathcal{Q}}{M}\right)\bigg|_{\text{BPS}}$$

and the gauge coupling decreases exponentially in terms of the geodesic field distance.

# Summary

- Computed charge-to-mass ratios of BPS states
- Verified that these states saturate the WGC bound
- Gave general expressions for these charge-to-mass ratios using infinite distance data
- Found a general lower bound on SDC mass decay rate in terms of these black hole extremality bounds

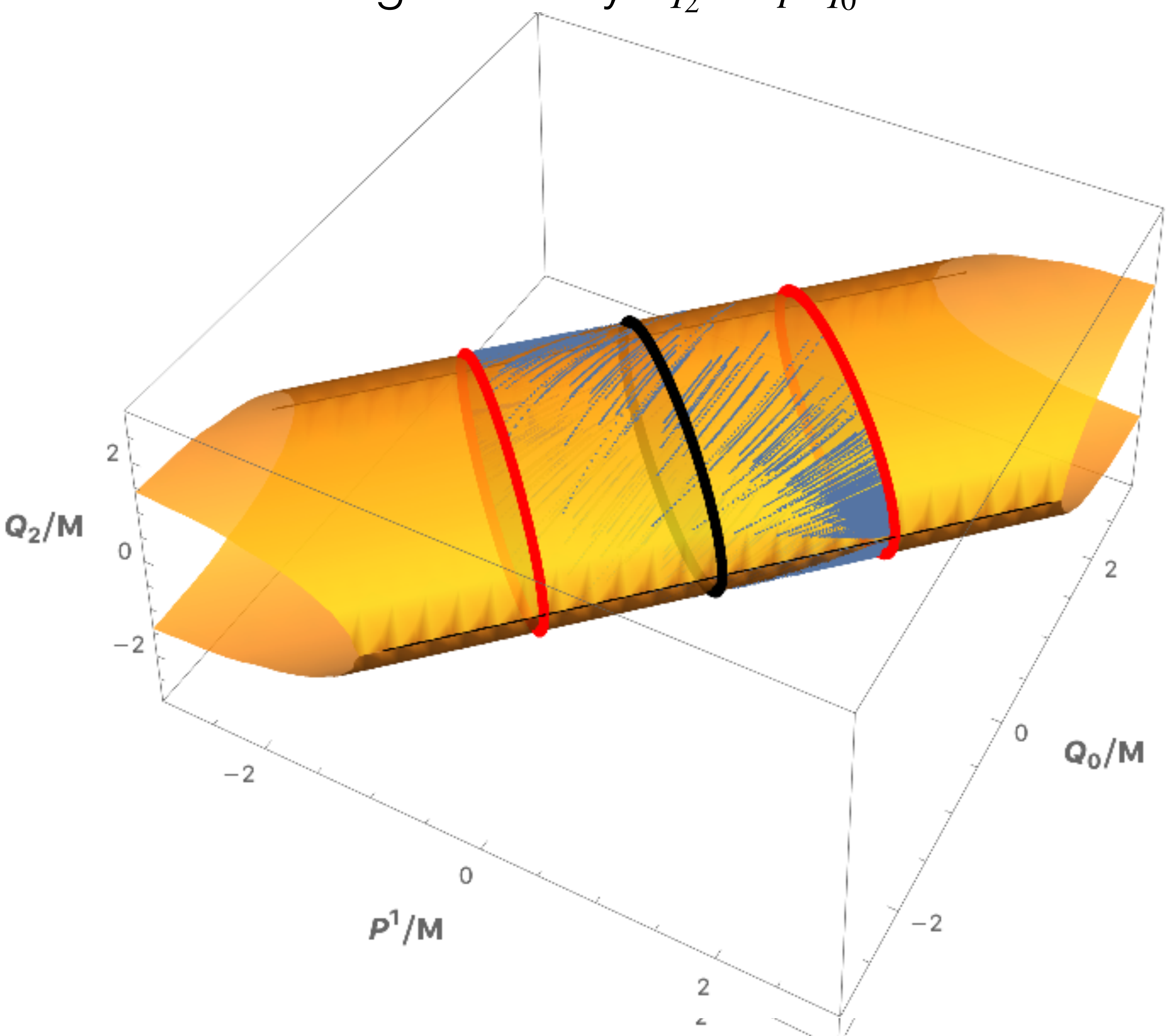
Thanks!

# Example 2: BPS charge-to-mass ratios

- Prepotential: $F = -\frac{1}{6X_0} T S^2$

- 2 moduli, 3 gauge fields

- BPS charges satisfy $3q_2^2 > 2p^1 q_0$




$$\mathbb{A} = \begin{pmatrix} 1/4 & -1/4 & 0 \\ -1/4 & 1/4 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$$

**again matches BH extremality bounds**

# The Spiral of Theodorus