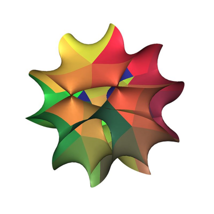
[hep-th/2003.13682]

Geometric Unification of Higgs Bundle Vacua Part – II

Thomas Rochais

with Cvetic, Heckman, Torres, Zoccarato



Summer Series on String Phenomenology

Introduction:

Several canonical approaches to realizing 4D $\mathcal{N} = 1$ vacua from strings:

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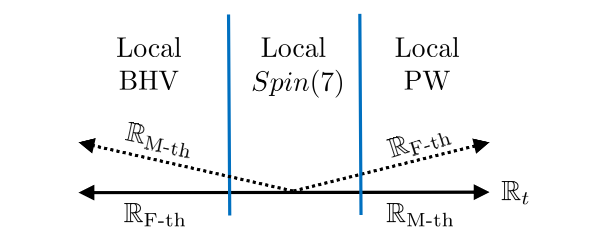
Outline

- Introduction
- Solution building techniques for bulk BPS equations of motion
- Supersymmetric Interfaces
- Interpolating BHV – PW solutions
- Conclusions and Outlook

The F-/M-theory picture

- [Beasley, Heckman, Vafa: 0802.3391]
  BPS equations for F-theory on Calabi-Yau manifolds
  $\Rightarrow$ BHV system
- [Pantev, Wijnholt: 0905.1968]
  BPS equations for M-theory on G_2-manifolds involve Higgs bundles
  $\Rightarrow$ PW system
- [Braun, Cizel, Hübner, Schäfer-Nameki: 1812.06072]
  Local picture of M-theory on G_2 and TCS construction
- [Barbosa, Cvetic, Heckman, Lawrie, Torres, Zoccarato: 1906.02212]
  Added T-branes onto G_2 manifolds

Spin(7) as the glue between G_2 and CY_4



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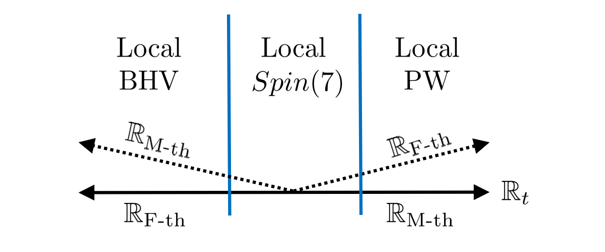

- F-theory on a non-compact elliptically fibered Calabi-Yau fourfold (left)

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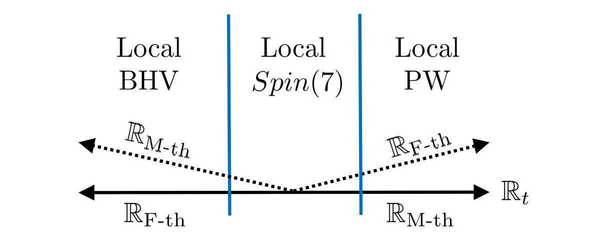

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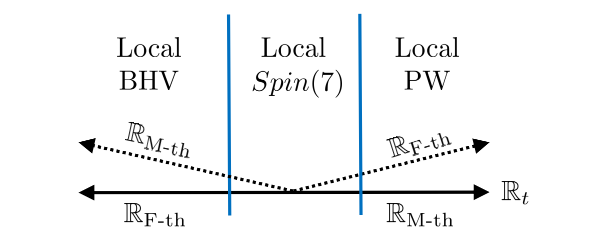

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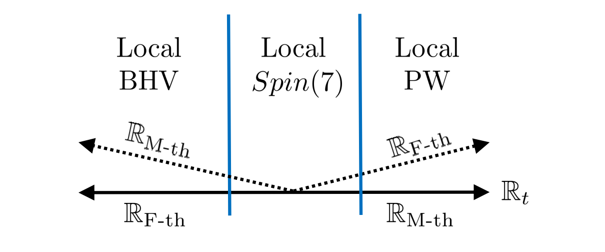

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- M-theory on a non-compact G_2 space (right)
- In the 4D effective field theory, this involves an interpolating profile in a direction $\mathbb{R}_t$
- The interpolating profiles are captured by a local BHV system in the F-theory region and a local PW system in the M-theory region

The BPS equations

M-theory on *Spin*(7)

$$\boxed{D_A\phi_{SD} = 0, \quad F_{SD} + \phi_{SD} \times \phi_{SD} = 0}$$

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Kähler four-manifold

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$$M_4 = Q_3 \times S^1$$

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PW system

$$D_A\phi_{PW} + D_A *_3 \phi_{PW} = 0$$

$$D_A * \phi_{PW} = 0$$

$$F - [\phi_{PW}, \phi_{PW}] + *_3(D_\theta A - d_3 A_\theta) = 0$$

Solution building techniques

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$$\phi_{\mathrm{SD}} = \phi_\alpha(dx\wedge d\theta - dt\wedge dy) + \phi_\beta(dt\wedge dx + dy\wedge d\theta) + \phi_\gamma(dt\wedge d\theta + dx\wedge dy)$$

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- Assuming a flat metric, and expanding along the cylinder's t direction:

$$A_i(x, y, \theta, t) = \sum_{k=0}^{\infty} A_i^{(k)}(x, y, \theta) t^k$$

$$\phi_i(x, y, \theta, t) = \sum_{k=0}^{\infty} \phi_i^{(k)}(x, y, \theta) t^k$$

- Temporal gauge: $A_t(x, y, \theta, t) = 0$

Building BHV equations

- Expanding the BHV equations gives:

$$\partial_x \phi_\beta^{(j)} - \partial_y \phi_\alpha^{(j)} + \sum_{n=0}^{j} \left(\left[A_x^{(j-n)}, \phi_\beta^{(n)} \right] - \left[A_y^{(j-n)}, \phi_\alpha^{(n)} \right] \right) = 0$$

$$\partial_x \phi_\alpha^{(j)} + \partial_y \phi_\beta^{(j)} + \sum_{n=0}^{j-1} \left(\left[A_x^{(j-n)}, \phi_\alpha^{(n)} \right] + \left[A_y^{(j-n)}, \phi_\beta^{(n)} \right] \right) = 0$$

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- Along with recursion relations:

$$\begin{aligned}
(j+1) A_\theta^{(j+1)} &= -F_{xy}^{(j)} + [\phi_\alpha, \phi_\beta]^{(j)} \\
(j+1) A_x^{(j+1)} &= -F_{y\theta}^{(j)} \\
(j+1) A_y^{(j+1)} &= F_{x\theta}^{(j)} \\
(j+1) \phi_\alpha^{(j+1)} &= -D_\theta^{(j)} \phi_\beta^{(j)} \\
(j+1) \phi_\beta^{(j+1)} &= D_\theta^{(j)} \phi_\alpha^{(j)}
\end{aligned}$$

Building BHV solutions

- It is sufficient to solve the zeroth order differential equations

$$D_x^{(0)}\phi_\beta^{(0)} - D_y^{(0)}\phi_\alpha^{(0)} = 0, \qquad D_x^{(0)}\phi_\alpha^{(0)} + D_y^{(0)}\phi_\beta^{(0)} = 0$$

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Building PW solutions

[Barbosa, Cvetic, Heckman, Lawrie, Torres, Zoccarato: 1906.02212]

- It is sufficient to solve the zeroth order differential equations

$$F_{ab}^{(0)} - [\phi_a^{(0)}, \phi_b^{(0)}] = 0, \quad D_a^{(0)}\phi_b^{(0)} - D_b^{(0)}\phi_a^{(0)} = 0$$

- $\phi_t^{(0)}$ is free, and sets the "trajectory" of the solution
- And then one can simply propagate through the recursion relations

$$(j+1)g^{tt}\phi_t^{(j+1)} = -g^{ab}\left(\partial_a\phi_b^{(j)} + \sum_{m=0}^{j}[A_a^{(j-m)}, \phi_b^{(m)}]\right)$$

$$(j+1)A_a^{(j+1)} = \sum_{m=0}^{j}[\phi_t^{(j-m)}, \phi_a^{(m)}]$$

$$(j+1)\phi_a^{(j+1)} = \partial_a\phi_t^{(j)} + \sum_{m=0}^{j}[A_a^{(j-m)}, \phi_t^{(m)}] = 0$$

Building full *Spin*(7) solutions

- It is sufficient to solve the zeroth order differential equations

$$D_x^{(0)}\phi_\beta^{(0)} - D_y^{(0)}\phi_\alpha^{(0)} + D_\theta^{(0)}\phi_\gamma^{(0)} = 0$$

Building full $Spin(7)$ solutions

- It is sufficient to solve the zeroth order differential equations

$$D_x^{(0)}\phi_\beta^{(0)} - D_y^{(0)}\phi_\alpha^{(0)} + D_\theta^{(0)}\phi_\gamma^{(0)} = 0$$

- Before simply propagating through the recursion relations

$$jA_\theta^{(j)} = -\partial_x A_y^{(j-1)} + \partial_y A_x^{(j-1)} - \sum_{n=0}^{j-1}\left(\left[A_x^{(j-1-n)}, A_y^{(n)}\right] - \left[\phi_\alpha^{(j-1-n)}, \phi_\beta^{(n)}\right]\right)$$

$$jA_x^{(j)} = -\partial_y A_\theta^{(j-1)} + \partial_\theta A_y^{(j-1)} - \sum_{n=0}^{j-1}\left(\left[A_y^{(j-1-n)}, A_\theta^{(n)}\right] - \left[\phi_\gamma^{(j-1-n)}, \phi_\alpha^{(n)}\right]\right)$$

$$jA_y^{(j)} = \partial_x A_\theta^{(j-1)} - \partial_\theta A_x^{(j-1)} + \sum_{n=0}^{j-1}\left(\left[A_x^{(j-1-n)}, A_\theta^{(n)}\right] + \left[\phi_\gamma^{(j-1-n)}, \phi_\beta^{(n)}\right]\right)$$

$$j\phi_\gamma^{(j)} = -\partial_x \phi_\alpha^{(j-1)} - \partial_y \phi_\beta^{(j-1)} - \sum_{n=0}^{j-1}\left(\left[A_x^{(j-1-n)}, \phi_\alpha^{(n)}\right] + \left[A_y^{(j-1-n)}, \phi_\beta^{(n)}\right]\right)$$

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$$\phi_\alpha^{(j)} = \frac{1}{j}\begin{cases} (-1)^{j/2}\left(\partial_x^2 + \partial_y^2 + \partial_\theta^2\right)^{j/2}\phi_\alpha^{(0)}\,, & j \text{ even} \\ (-1)^{(j-1)/2}\left(\partial_x^2 + \partial_y^2 + \partial_\theta^2\right)^{(j-1)/2}\left(\partial_x\phi_\gamma^{(0)} - \partial_\theta\phi_\beta^{(0)}\right)\,, & j \text{ odd} \end{cases}$$

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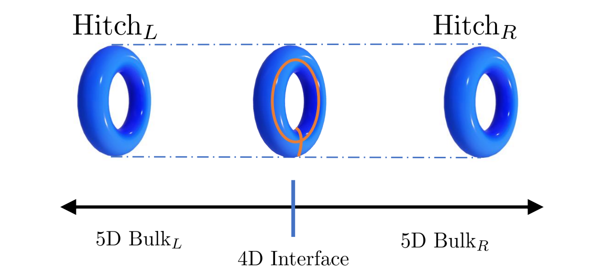
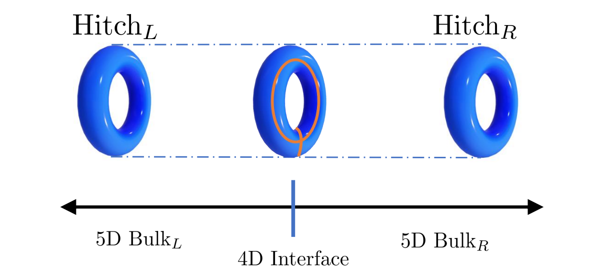
5D interfaces



An interpolating PW system: $(C = T^2) \times \mathbb{R}$

5D interfaces



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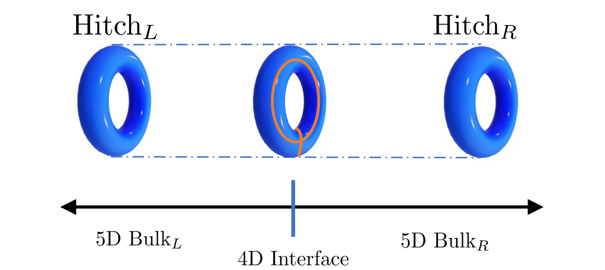


An interpolating PW system: $(C = T^2) \times \mathbb{R}$

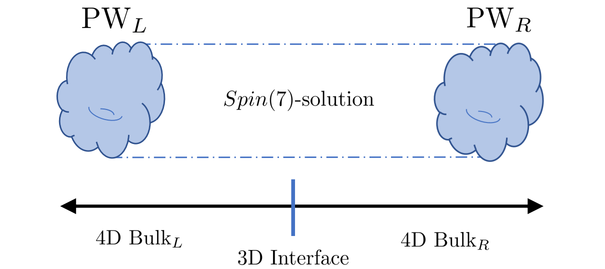
- $\phi_{\mathrm{PW}} = \phi^L + \phi^R$
- An interpolating solution with $\phi^{L,R} \to 0$ for $t \to \pm\infty$:

$$\phi^L = \mathrm{Re}\left[f_1^L(u)\frac{-\tanh(u)+1}{2}du + f_2^L(v)\frac{-\coth(v)+1}{2}dv\right]$$

$$\phi^R = \mathrm{Re}\left[f_1^R(u)\frac{\tanh(u)+1}{2}du + f_2^R(v)\frac{\coth(v)+1}{2}dv\right]$$

holomorphic in
$u = t + ix$ and $v = t + iy$

4D interfaces



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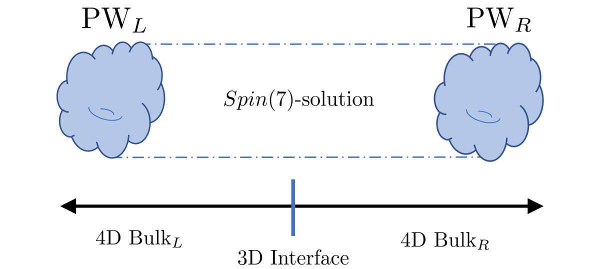

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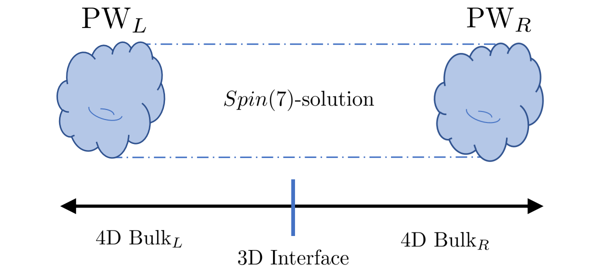

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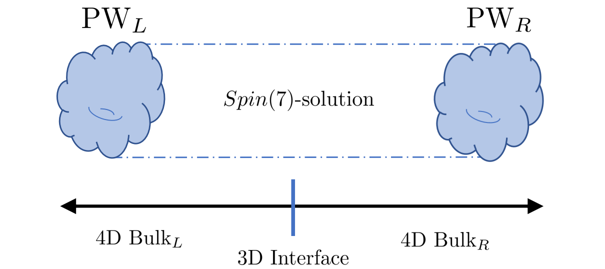

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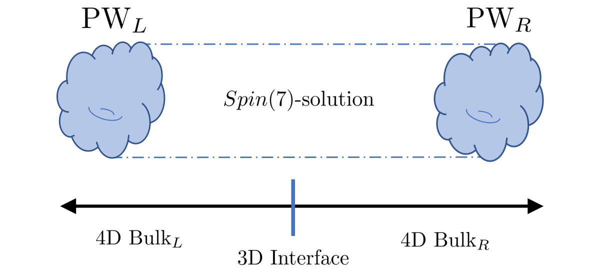

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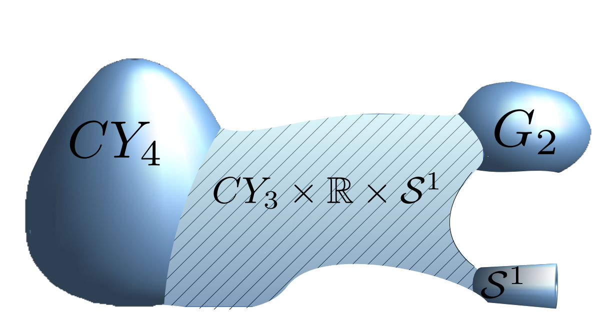
$$\Rightarrow \boxed{\phi_{\mathrm{PW}} = \phi^{(1)}_{\mathrm{Hit}} + \phi^{(2)}_{\mathrm{Hit}} + \phi^{(3)}_{\mathrm{Hit}}}$$

- Solutions can be summed, producing an interpolating *Spin*(7) solution!

Generalized Connected Sums

[Braun, Schäfer-Nameki: 1803.10755]



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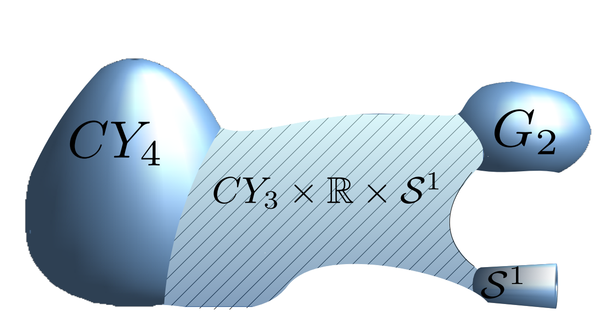

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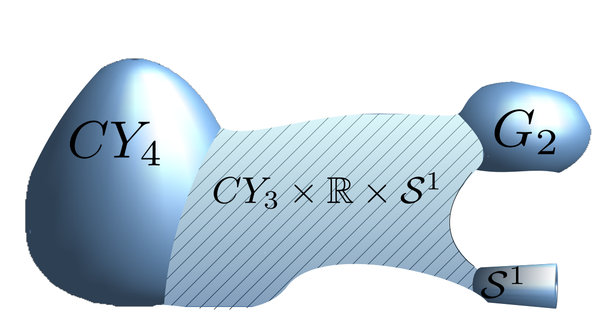

- Calabi-Yau Block: Non-compact CY_4 with a region $X^{cyl} \simeq (\mathbb{R} \times S^1) \times Z_3^{cpt}$
- G_2 Block: Non-compact G_2 manifold Y with an S^1, s.t. outside a compact submanifold $Y \simeq CY_3 \times I$

Generalized Connected Sums and Local Models

- BHV Building Block:

- PW Building Block:

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  - Supersymmetric configurations on a three-cycle $C \times S^1$ inside a G_2 manifold
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Donaldson gluing

[Kovalev: math/0012189]

- Consider four manifold M_4, so the total space of the bundle of self-dual two forms is a local G_2 space with associative three-form:

$$\Phi_{G_2} = dy^{123} - dy^1\left(dx^{14} + dx^{23}\right) - dy^2\left(dx^{24} + dx^{31}\right) - dy^3\left(dx^{34} + dx^{12}\right)$$

Donaldson gluing

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- **BHV block:** We need a Kähler surface and a non-compact CY_3 given by the total space of the canonical bundle: $\mathcal{O}(K_{M_4}) \to M_4$. Letting y_1, y_2 be coordinates in the normal bundle direction:

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Letting x_1, x_2, x_3 be the local coordinates:

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  The associative three-form on the G_2 manifold $K3 \times \mathbb{R}_t \times \mathbb{R}_{\tilde{t}} \times \mathbb{R}_\psi$ is

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- The gluing is then achieved by:

$$\boxed{\begin{aligned} &\mathrm{Im}\left(\Omega_{\mathrm{K3,PW}}\right) = -\mathrm{Im}\left(\Omega_{\mathrm{K3,BHV}}\right) \\ &\mathrm{Re}\left(\Omega_{\mathrm{K3,PW}}\right) = J_{\mathrm{K3,BHV}} \\ &J_{\mathrm{K3,PW}} = \mathrm{Re}\left(\Omega_{\mathrm{K3,BHV}}\right) \\ &t_{\mathrm{PW}} = -t_{\mathrm{BHV}} \\ &\psi_{\mathrm{PW}} = \tilde{t}_{\mathrm{BHV}} \\ &\tilde{t}_{\mathrm{PW}} = \psi_{\mathrm{BHV}} \end{aligned}}$$

Abelian BHV – PW Interpolation

- Local $Spin(7)$ system: $d\phi_{SD} = 0$
- $\Rightarrow$ decompose $\phi_{SD} = \phi_{SD,BHV} + \phi_{SD,PW}$
- To recover the local geometric gluing of BHV and PW blocks: $\lim_{t\to\infty} \phi_{SD,BHV} = 0$, $\lim_{t\to-\infty} \phi_{SD,PW} = 0$

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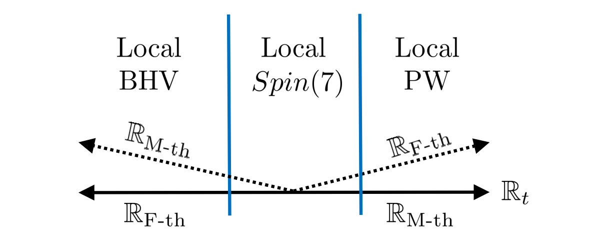
$$\partial_u f = \mathrm{Re}\left[f_1(u)\frac{\tanh(u)+1}{2}\right], \quad \partial_v f = \mathrm{Re}\left[f_2(v)\frac{\coth(v)+1}{2}\right]$$

holomorphic in
$u = t + ix$ and $v = t + iy$

Conclusions



- Spin(7)-manifolds unification of M-theory on G_2 and F-theory on CY_4
- BPS equations of motion in Spin(7) can reduce to PW/BHV solutions and interesting interfaces result from interpolating between the two

Outlook

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- Expand on the interactions amongst matter fields in the resulting 3D $\mathcal{N} = 1$ theories

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Thank You!

Spectral covers and localization of matter

- For local *Spin*(7), ambient geometry: total space of the bundle of self-dual two-forms over M

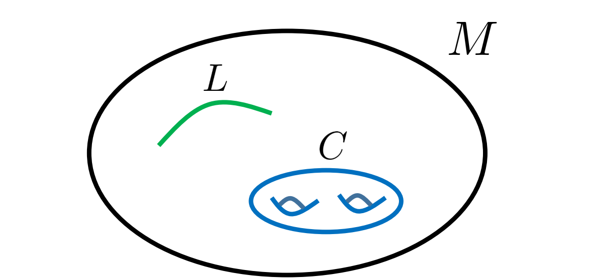
  Pick a section v of $\Omega^2_+(M)$

- Gauge group: $G = SU(N)$, fundamental representation, then spectral equations: $\det\left(v\mathbb{I}_N - \phi_{N\times N}\right) = 0$

- Abelian profile for $\phi_{SD} \Rightarrow \phi_{\mathrm{SD}} = \mathrm{diag}\left(\lambda_1, \ldots, \lambda_N\right)$
  So spectral cover: $\prod_{i=1}^{N}\left(v - \lambda_i\right) = 0$

- This means that the spectral cover is the union of N sheets

Spectral covers and localization of matter



- Matter localized on codimension two subspace
  $\Rightarrow$ two components of the triplet become identical with the third one being zero
  $\Rightarrow$ localized matter from BHV solutions

- Matter localized on codimension three subspace
  $\Rightarrow$ three components of a pair of eigenvalues must coincide with no component being identically zero
  $\Rightarrow$ localized matter from PW solutions