Anomalies of 2d SCFTs from wrapped D3 branes in F-theory

Based on: arXiv:1904.05361 and arXiv:2006.07380
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See also: 1808.05228

30th March: String Pheno Seminars
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Motivation

Side note: All these black holes are asymptotically flat. Surprisingly, given AdS/CFT, the first computation of the entropy of asymptotically AdS black holes was performed in 2015.

- The first microscopic counting of the Bekenstein—Hawking entropy of a black hole was performed in Strominger—Vafa. (5d BHs in type II on $K3 \times S^1$)

- Extended to 5d BHs in M-theory on CY_3 [Vafa] and 4d BHs in M-theory on $\mathrm{CY}_3 \times S^1$ [MSW].

- More recently black strings arising in F-theory have been considered.

- The holy grail of this program is to understand the Bekenstein—Hawking entropy both macroscopically (from gravity) and a matching microscopic (field theory) computation.

- Since (extremal) black objects have AdS near-horizon region AdS/CFT goes some way to explaining the entropy. Task is to identify the dual SCFT.

- Swampland Constraints.

Black Strings in Type IIB/F-theory

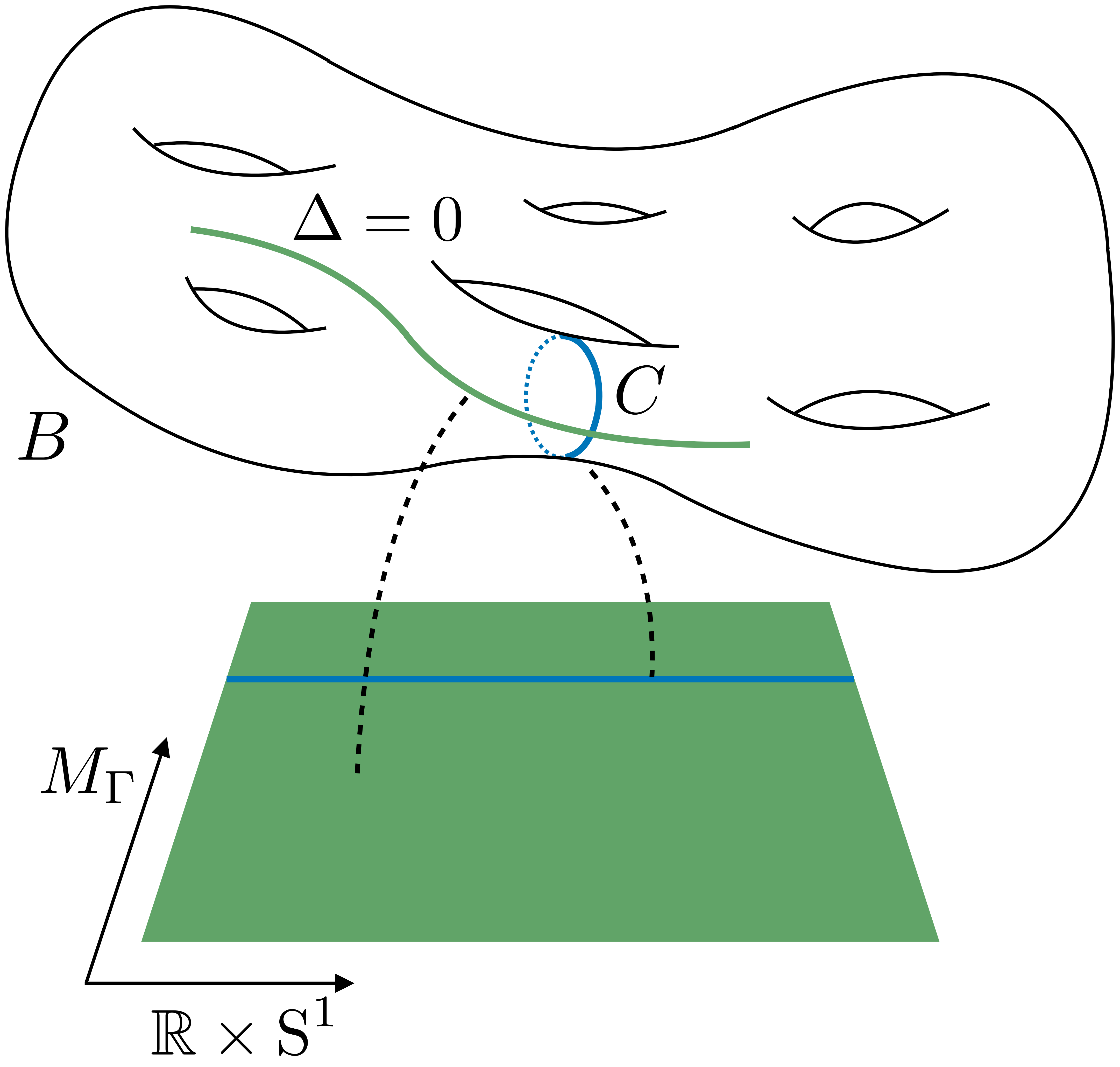
- Can construct Black objects by wrapping or intersecting branes.
- We will consider black strings constructed by wrapping D3 branes on curves inside the base of a singular elliptically fibered Calabi—Yau threefold, probed by ALE and ALF spaces.
- These have near-horizon geometries of the form $\mathrm{AdS}_3 \times S^3/\Gamma \times B$.
- A 2d $\mathcal{N} = (0{,}4)$ SCFT lives on the black strings, from which we can microscopically compute the entropy of the black string.
- We identify the anomalies of these SCFTs macroscopically.
- Classical contributions must be supplemented by both higher derivative corrections and one-loop corrections.

Black String setup: 10d viewpoint

- D3 branes wrap a curve in the base B of an elliptically fibered Calabi—Yau threefold.
- D3 branes are on $\mathbb{R} \times S^1 \times C$.
- Normal to the D3 brane is a 4d ALE/ALF space M_Γ.
- 7-branes wrap the discriminant locus $\Delta = 0$ and are extended along $\mathbb{R} \times S^1 \times \Delta \times M_\Gamma$.
- This gives rise to a black string along $\mathbb{R} \times S^1$.

As a 10d solution of type IIB supergravity this suffers from problems.
The theory contains not only D7 branes but more general (p, q) 7-branes. Type IIB does not contain these non-perturbative branes.

This is not really a problem if we use F-theory. Two options:
1. Use M-F duality to study the M-theory dual.
2. Use 6d SUGRA from F-theory on a CY_3.



6d SUGRA from F-theory on CY_3

- Reduce F-theory on a singular Calabi—Yau threefold. [Ferrara-Minasian-Sagnotti, Grimm-Bonetti]
- Contains gravity, vector, tensor and hyper-multiplets.
- Matter content organised in representations of $SU(2) \times SU(2)$.
- Multiplicities determined by the data of the three-fold. (Equivalent to anomaly free condition in EFT.)
- Singularities give rise to stacks of 7-branes with non-abelian gauge theories on their worldvolumes. In the SCFT they become flavour symmetries.

Black String Solutions

- The theory above admits black string solutions of the form

$$\mathrm{d}s_6^2 = 2H^{-1}(\mathrm{d}u + \beta)\left(\mathrm{d}v + \omega + \frac{1}{2}F(\mathrm{d}u + \beta)\right) + H\,\mathrm{d}s^2(M_\Gamma)$$

  - M_Γ non-compact with a Ricci-flat metric and H a harmonic function on M_Γ. β, ω, F similarly determined given M_Γ.

  - Choices of M_Γ classified by their asymptotic properties, the AL(E,F,G,H) spaces. Near the centre all look locally like $\mathbb{C}^2/\Gamma$ with $\Gamma \subset \mathrm{SU}(2)$.

  - Supported by non-trivial tensor multiplets with (charge shifted) charges

$$Q^\alpha = q^\alpha - \frac{1}{4}p_1(M_\Gamma)c^\alpha$$

  - The microscopic charge q^α is shifted due to string-like sources (further shift possible by instantons on 7-branes).

Central Charges and Levels from SUGRA

- We can compute from SUGRA the various levels and central charges of the black strings. However there are some subtleties.

- *Classical contributions* come from reducing the 6d theory to a 3d Chern—Simons theory. One needs to reduce at asymptotic infinity not on the NH geometry.

- The near-horizon geometry takes the same form AdS$_3 \times S^3/\Gamma$, irrespective of the asymptotics.

- There is black string hair outside the horizon, which contributes to the microscopic degeneracy.

- One-loop contributions from integrating out massive modes contribute to the degeneracy. These are *quantum corrections*.

Classical Contributions

- Reduce 6d Pseudo-action on the compact space around the black string at asymptotic infinity.
- Gauge the isometries of the solution: either $\mathrm{U}(1)_L \times \mathrm{SU}(2)_R$ or just $\mathrm{SU}(2)_R$
- Study variation of resultant Chern—Simons 3d action from the 6d one
- $S^{(6)} \supset \int_{M_6} \left[-\frac{1}{4} g_{\alpha\beta} G^{\alpha} \wedge \star G^{\beta} - \frac{1}{8} \eta_{\alpha\beta} c^{\alpha} B^{\beta} \wedge \mathrm{tr} \mathscr{R} \wedge \mathscr{R} \right]$
- $k_L = k_R = \frac{1}{2} |\Gamma| Q \cdot Q$ (2-deriv)

Classical Contributions

4-derivative

- The 2-derivative contributions do not distinguish between ALE and ALF, these are the leading order contributions to the levels/entropy.

- Higher derivative corrections are needed to see the asymptotic differences, (the blue terms from before).

- ALE: $$k_L^{4d} = -\,k_R^{4d} = \frac{1}{2}c\cdot Q\,, \quad (c_L - c_R)^{4d} = 6c\cdot Q$$

- ALF: $$k_L^{4d} = 0\,, \quad k_R^{4d} = c\cdot Q\,, \quad (c_L - c_R)^{4d} = 6c\cdot Q$$

Total Classical Contributions

- The total contribution from the classical contributions is
- ALF

$$k_L^{\text{class}} = \frac{|\Gamma|}{2}\left(C - \frac{1}{4}p_1(M_\Gamma)c_1(B)\right)^2$$

$$k_R^{\text{class}} = \frac{|\Gamma|}{2}\left(C - \frac{1}{4}p_1(M_\Gamma)c_1(B)\right)^2 + c_1(B)\cdot\left(C - \frac{1}{4}p_1(M_\Gamma)c_1(B)\right)$$

$$(c_L - c_R)^{\text{class}} = 6c_1(B)\cdot\left(C - \frac{1}{4}p_1(M_\Gamma)c_1(B)\right)$$

For Taub-NUT ($\Gamma = \mathbb{Z}_n$) we have field theory to compare to. Something is missing.

Non-abelian flavour symmetries

- Similar computations give the levels for the non-abelian flavour symmetries.
- These are left-moving non-abelian current algebras.
- These arise due to singularities in the Calabi—Yau threefold over divisors S_i .
- We find

$$k_{G_i}^{\text{class}} = S_i \cdot \left(C - \frac{1}{4} p_1(M_\Gamma) c_1(B) \right)$$

Quantum Corrections

- So far we have computed the classical contributions. For Taub-NUT (ALF) we have field theory to compare to: missing (subleading) terms.
- We need to also think about contributions from integrating out massive KK modes following [Grimm, het Lam, Mayer, Vandoren].
- These modes arise from the ones which can lead to anomalies in 6d.
- Gravitino, spin-$\frac{1}{2}$ fermions from tensor, vector and hypermultiplets, and (anti-) self dual two-forms.
- In 3d these give rise to massive spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ fermions plus massive chiral vectors.
- However only ALF has quantum corrections, ALE does not!

Huh? Why only for ALF?

- We reduced on the spherical part at asymptotic infinity to 3d.
- ALE and ALF have very different asymptotic properties. The spherical part in both is topologically S^3/Γ, however they approach

ALE: $\mathbb{R}^4/\Gamma$ The radius of the sphere becomes infinitely large.

ALF: $(\mathbb{R}^3 \times S^1)/\Gamma$ There is a finite radius circle (Hopf) at infinity.

- The KK modes have masses inversely proportional to the radius.
- For ALE they all become massless and so do not contribute to anomaly.
- For ALF they remain massive and so must be taken into account.

Computing the Quantum corrections

- We use spectrum of [Boer 98] and truncate to $\mathcal{N} = (1{,}0)$.
- Spectrum in reps of $\mathfrak{so}(4) = \mathfrak{su}(2)_L \oplus \mathfrak{su}_R(2)_R$ with a mass, e.g. $(i\gamma \cdot \partial \pm m)\psi$
- Take infinite towers and project out states which are not invariant under Γ.
- To find contribution either compute one-loop Feynman diagram or use Index theorem and anomaly inflow.
- For a spin-$\frac{1}{2}$ field anomaly is cancelled off by the counterterm

$$\pi \mathrm{sgn}(m) \int_{M_3} Q_{\frac{1}{2}}\left(\{A^i\}, \omega\right)$$

Where $\mathrm{d}Q_{\frac{1}{2}}\left(A^i, \omega\right) = \hat{A}(M_3) \wedge \mathrm{ch}(F)\Big|_{4\text{-form}}$

Regularizing the infinite sums

- We can now sum the contribution of all the chiral fields to the anomaly.
- We will encounter an infinite sum over all the massive KK states.
- Need to regularise. We use zeta-function regularization.
- The total contribution (classical plus quantum) to the levels is (ALF A-series here)

$$k_L = \frac{1}{2}mC^2 - \frac{1}{2}m^2 c_1(B)\cdot C$$

$$k_R = k_L + \frac{1}{6}m^3 c_1(B)^2 + c_1(B)\cdot C - \frac{1}{6}mc_1(B)^2 + m$$

$$k_{G_i} = C\cdot S_i$$

$$c_L - c_R = 6c_1(B)\cdot C - mc_1(B)^2 + 6m$$

Microscopic computation

- Ideally we would like to match to microscopic computations however only partial results exist.
- Can try to look at the spectrum of the wrapped D3 branes. [Lawrie, Schafer-Nameki, Weigand]
- Need to perform a non-abelian topological duality twist [Martucci], unknown.
- For Taub-NUT this can be dualized to MSW bypassing this problem.
- For the other cases dualities are much more subtle, e.g. Orientifolds.
- All admit an M2-brane picture but this is challenging.
- Can we use (suitably generalised) anomaly polynomial of type IIB in [Bah,Bonetti,Minasian, Weck]?

Conclusions

- We have computed holographically the current algebra levels and central charges of 2d $\mathcal{N} = (0,4)$ SCFTs.
- SCFT lives on black strings in F-theory.
- One-loop corrections from integrating out massive modes is necessary.
- Where the field theory result exists we have an exact match.
- For the other cases this serves as a prediction for the SCFT.
- A consistency check for the non-abelian topological duality twist or anomaly inflow for F-theory a la [BBMW].
- Described string defects coupled to tensor fields: possible swampland constraints using [Kim, Shiu, Vafa] + ….

Thank you for your attention.