Conifold Vacua with Small Flux Superpotential

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based on 2009.xxxxx with Mehmet Demirtas, Liam McAllister, and Jakob Moritz.

Opening Remarks

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- This is a serious problem. We should resolve this problem.

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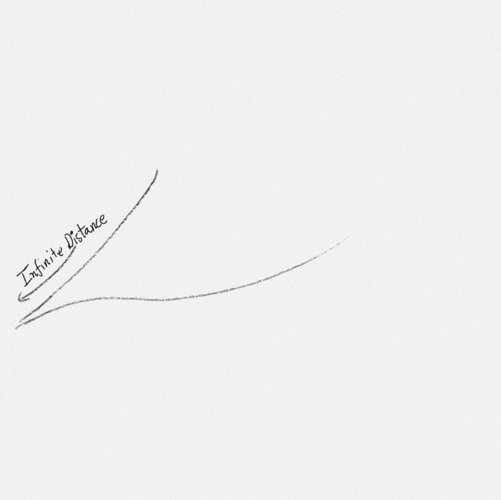

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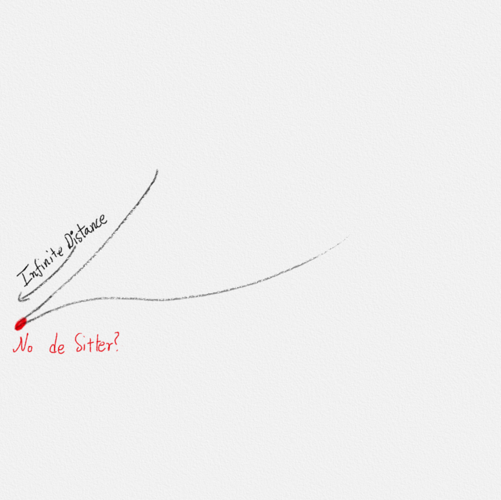

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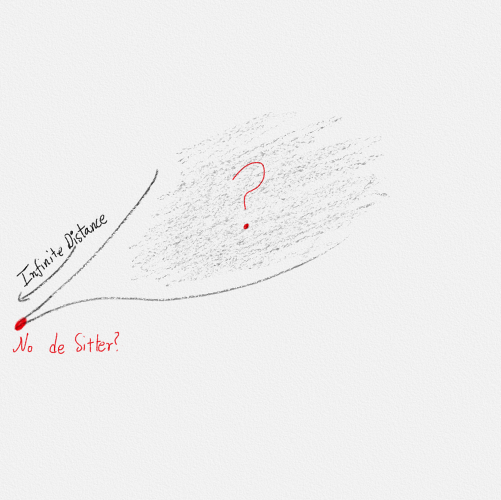

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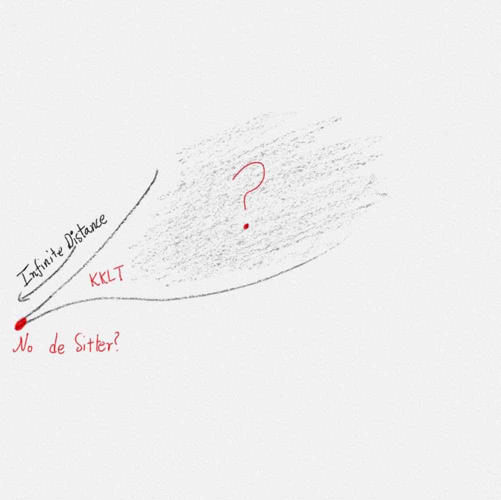

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Modules for KKLT

In the context of KKLT scenario, in a given compactification one needs

- Exponentially small $W_0 := \langle W_{flux} \rangle$.

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- At least one complex structure modulus stabilized near a conifold singularity.

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- Uplift to a meta-stable de Sitter vacuum.

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- Analytic computation of prepotential near conifold singularities
- Mechanism to find small W_0 near conifold singularities.
- Method to find orientifolds for CYs with $h^{1,1} = \mathcal{O}(100)$.

See also a nice work of Blumenhagen, Álvarez-García, Brinkmann, Schlechter on this subject which will appear tonight on arxiv simultaneously with ours.

Review on Flux Vacua

- Consider type IIB on an O3/O7 orientifold of a CY threefold $\tilde{X}$.

- The no-scale 4d EFT consists of the following data

$$W_{flux} = \int_{\tilde{X}} (F - \tau H) \wedge \Omega \,,$$

$$\mathcal{K} = -\log\left(-i \int_{\tilde{X}} \Omega \wedge \overline{\Omega}\right) - \log(-i(\tau - \bar{\tau}))\,.$$

Gukov, Vafa, Witten 99, Giddings, Kachru, Polchinski 01

- VEVs of complex structure moduli are then dynamically determined.

- The game is to pick the fluxes such that the complex structure moduli are stabilized with the desired properties.
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- How do we compute W_{flux}?

- $\int_\gamma \Omega$ can be efficiently computed by mirror symmetry.
  Greene, Plesser 90, Candelas, De La Ossa, Green, Parkes 90, Batyrev 93, Hosono, Klemm, Theisen, Yau 93, Candelas, Font, Katz, Morrison 94, Hosono, Klemm, Theisen, Yau 94
- Consider a symplectic basis $\{\gamma_a, \gamma^b\}$ of $H_3(\tilde{X}, \mathbb{Z})$. $a, b : 0, \dots, h^{2,1}(\tilde{X})$.
- Mirror symmetry relates $\int_\gamma \Omega$ at LCS with $\int_{\Sigma_{(2d)}} e^J$ for a $\Sigma_{(2d)} \in H_{2d}(X, \mathbb{Z})$.

$\tilde{X}$	$\int_{\gamma_0} \Omega$	$\int_{\gamma_i} \Omega$	$\int_{\gamma^i} \Omega$	$\int_{\gamma^0} \Omega$
X	1	$\int_{\Sigma_{(2),i}} J$	$\int_{\Sigma^i_{(4)}} J^2/2 + \dots$	$\int_X J^3/6 + \dots$
Π	1	z^i	$\partial_{z^i} \mathcal{F}$	$2\mathcal{F} - z^i \partial_{z^i} \mathcal{F}$

- The mirror map can compute the non-perturbative corrections to volumes.

Review on Flux Vacua with small W_0

- At LCS prepotential splits as, $\mathcal{F}(z) = \mathcal{F}_{\text{cubic}}(z) + \mathcal{F}_{\text{inst}}(z)$,

$$\mathcal{F}_{\text{inst}} = -\frac{1}{(2\pi i)^3} \sum_{\Sigma_{(2)}} n_{\Sigma_{(2)}} \mathrm{Li}_3(q^{\Sigma_{(2)}}) \,,$$

$$q^{\Sigma_{(2)}} = \exp\left(2\pi i \int_{\Sigma_{(2)}} (iJ - B)\right) = \exp\left(2\pi i z_{\Sigma_{(2)}}\right) \,.$$

- Non-perturbative corrections are encoded in GV-invariants $n_{\Sigma_{(2)}}$.
  Gopakumar, Vafa 98

- $n_{\Sigma_{(2)}}$ counts number of D2-brane BPS states wrapped on $\Sigma_{(2)}$ in X.

- Likewise, flux superpotential enjoys the same splitting

$$W_{flux}(z) = W_{\text{cubic}}(z) + W_{\text{inst}}(z) \,,\ W_{\text{inst}} = \mathcal{O}\left(e^{2\pi i z}\right) \,.$$

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- **Trick** : Choose fluxes $F,\ H$ such that there exists a perturbative moduli space $W_{\text{cubic}} = dW_{\text{cubic}} = 0$. Demirtas, Kim, McAllister, Moritz 19

- Remaining light moduli will then be stabilized by W_{inst}, à la racetrack.

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  such that $p \in \mathcal{K}(X)$ and $k_i p^i = 0$.

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  Then, given the fluxes

$$\vec{F} = (F_0(m), F_i(m), 0, m^i), \quad \vec{H} = (0, k_i, 0, 0)$$

  we find $dW_{\text{cubic}}(z) = W_{\text{cubic}} = 0$ along $\vec{z} = \vec{p}\tau$.

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  We are left with the perturbative moduli space parametrized by τ.

Review on Flux Vacua with small W_0

- The perturbative moduli space is lifted by type IIA worldsheet instantons

$$W_{eff}(\tau) = \sum_i A_i e^{2\pi i p_i \tau} \, .$$

- p_i is a rational number determined by the fluxes and K_{ijk}.
- The prefactor A_i is determined by GV invariants and the fluxes.
- Resulting $W_0 \simeq \mathcal{O}\left(\left|\langle e^{2\pi i z}\rangle\right|\right)$ is *exponentially* suppressed.
- In Demirtas et al. 19, $W_0 \simeq 10^{-8}$ was found in $\mathbb{P}_{[1,1,1,6,9]}[18]$.

Towards Conifold Vacua

- We should find small W_0 near to conifold singularities.
- Near a conifold singularity, as $z_{cf} \equiv z_1 \to 0$

$$\partial_{z_{cf}} \mathcal{F} = n_{cf} \tfrac{z_{cf}}{2\pi i} \log(2\pi i z_{cf}) + f(z_{cf}, z), \;\; f(0,z) = \mathcal{O}(1)\,.$$

$$W_{flux} = W_{cf} + W_{bulk}\,,$$

$$W_{cf} = M\left(\frac{n_{cf}}{2\pi i} z_{cf} \log(z_{cf}) + f(z_{cf}, z)\right) - \tau K z_{cf}\,, \;\; z_{cf} \simeq \exp(-2\pi K/g_s M)\,,$$

with $R_{tip} \simeq \sqrt{g_s M}\,.$

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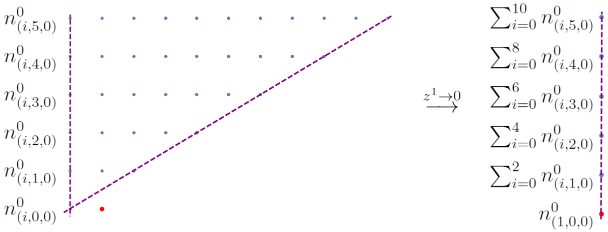
with $R_{tip} \simeq \sqrt{g_s M}$.

- The challenge is to compute the holomorphic piece $f(z_{cf}, z)$ accurately, while retaining the structure of LCS prepotential as much as possible to cancel $Mf(z_{cf}, z)$ against W_{bulk}.

Prepotential near $z_{cf} \simeq 0$

As an illustration of the method, we present a CY X with $h^{1,1} = 3,\ h^{2,1} = 99$.



2d projection of the lattice of effective curves.
GV invariants of marked curves are non-vanishing.

- Collapsing $z_1 \to 0$ generates $n^0_{(1,0,0)}$ massless BPS states wrapped on $\Sigma_{(1,0,0)}$.
- $z_1 \to 0$ in X is mirror dual to $z_{cf} \to 0$ in $\tilde{X}$ with $z_{cf} \equiv z_1$ and $n_{cf} = n_{(1,0,0,)}$.
- One can determine $f(z_{cf}, z)$ by re-summing the instanton series.

Prepotential near $z_{cf} \simeq 0$

- At LCS, $\partial_{z_1}\mathcal{F}(z_1, z^i) = \partial_{z_1}\mathcal{F}_{\text{cubic}}(z_1, z^i) + \partial_{z_1}\mathcal{F}_{\text{inst}}(z_1, z^i)$

$$(2\pi i)^2 \partial_{z_1}\mathcal{F}_{\text{inst}}(z_1, z^i) = -n_{cf}\text{Li}_2(e^{2\pi i z_1}) - \sum_{\Sigma \neq (1,0,0)} n_\Sigma \int_{(1,0,0)} [\Sigma] \text{Li}_2(q^\Sigma)\,.$$

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  This expansion behaves well near $z_1 = z_{cf} = 0$.

- Near $z_{cf} = 0$,

$$\begin{aligned}\partial_{z_{cf}}\mathcal{F}_{\text{inst}}(z_{cf}, z^i) =& n_{cf}\left(\frac{1}{24} + \frac{z_{cf}}{2\pi i}\left(\log(-2\pi i z_{cf}) - 1\right)\right)\\ &- \sum_{\Sigma \neq (1,0,0)} n_\Sigma \int_{(1,0,0)} [\Sigma]\mathrm{Li}_2(q^\Sigma|_{z_{cf}=0}) + \mathcal{O}\left(z_{cf}^2, z_{cf}\, e^{2\pi i z}\right)\,.\end{aligned}$$

W_{flux} near $z_{cf} = 0$

Let us work with the expansion scheme $z_{cf} \ll \mathcal{O}(e^{2\pi i z^i})$

- Then W_{flux} enjoys the following expansion

$$W_{flux}(z_{cf}, z^i, \tau) = W^{(0)}(z^i, \tau) + W^{(1)}(z_{cf}, z^i, \tau) z_{cf} + \mathcal{O}(z_{cf}^2)\,,$$

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- Treating z_{cf} as an exponentially suppressed variable, we can confirm that the structure of W_{flux} hasn't changed much from W_{flux} at LCS.

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- Remaining light moduli will then be stabilized by $W^{(0)}_{\text{inst}}$ and $W^{(1)} z_{cf}$, à la racetrack.

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  a, b, c run $1 \ldots h^{2,1}$ and i, j, k run $2 \ldots h^{2,1}$.

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Let us work with the expansion scheme $z_{cf} \ll \mathcal{O}(e^{2\pi i z^i})$

- The effective flux superpotential is then

$$W_{eff}(z_{cf}, \tau) = M \frac{n_{cf}}{(2\pi i)^2} \left(\log(-2\pi i z_{cf}) - 1\right) - K' \tau z_{cf} + \sum_i A_i(m,k) e^{2\pi i p_i \tau}$$

$$M := -m^1, \quad K' = K_1 - M^a K_{1ai} p^i \, .$$

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$$M := -m^1, \quad K' = K_1 - M^a K_{1ai} p^i \,.$$

- Solving F-term equations, we obtain

$$\langle W_{eff} \rangle \simeq \mathcal{O}(|\langle e^{2\pi i z^i} \rangle|) \,,$$

$$z_{cf} \simeq \frac{1}{2\pi} \exp\left(\frac{2\pi i K' \tau}{n_{cf} M}\right)$$

Hence finding small W_0 near $z_{cf} = 0$. Note that z_{cf} is set by τ.

- For vacua with $z_{cf}^{2/3} = e^{2A_{tip}} \simeq W_0$, $m_{z_{cf}} \gg m_\tau$.

Orientifold of $\tilde{X}$

- $\tilde{X}$ has $h^{1,1}_{\text{toric}}(\tilde{X}) = 97$, $h^{1,1}(\tilde{X}) = 99$, and $h^{2,1}(\tilde{X}) = h^{2,1}_{\text{toric}}(\tilde{X}) = 3$.

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  *We call the divisors that are inherited from the ambient variety **toric**.

- The defining equation for $\tilde{X}$ is

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&- \psi_4 x_3^6 x_4^{12} x_7^6 x_8^5 x_9^4 - \psi_5 x_5^3 x_6^6 x_7^3 - \psi_6 x_3^6 x_5^6 x_8 x_9^2 - \psi_7 x_3^6 x_4^6 x_5^3 x_7^3 x_8^3 x_9^3 \\
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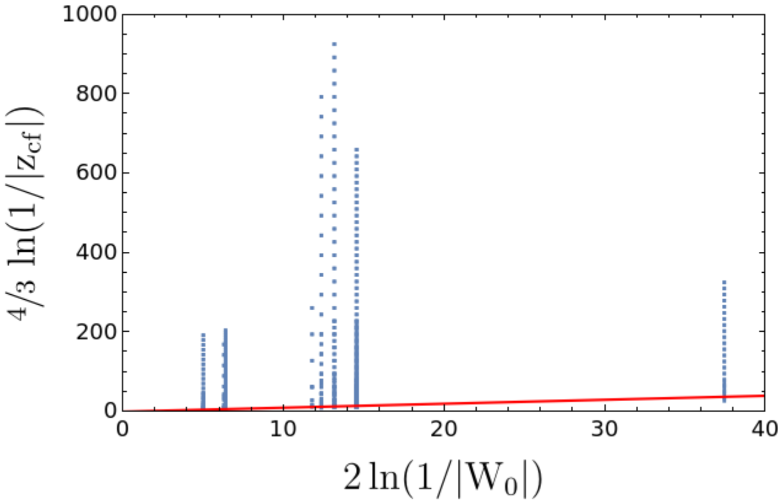
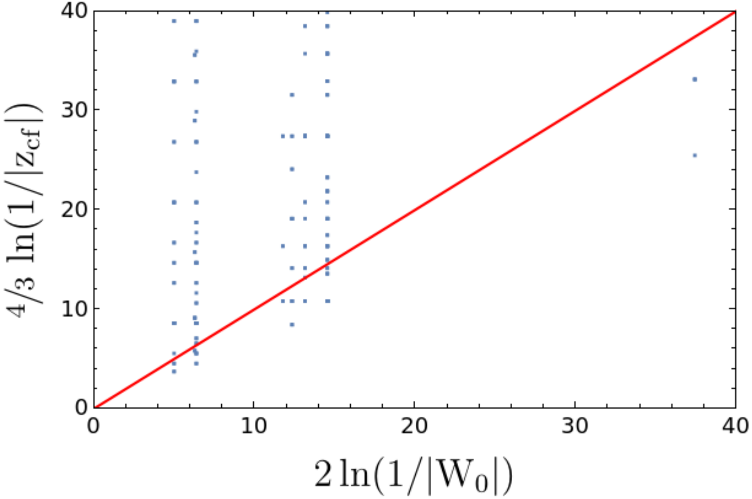
- $\tilde{I}$ acts trivially on $x_8 = f = 0$. Similarly, $\tilde{I}$ acts trivially on $x_9 = f = 0$. Hence we obtain $h_{+}^{1,1} = 99$.
- Placing 4 D7-branes on each O7-plane, we obtain $Q_{D3} = \frac{\chi_f}{4} = 52$. For more exciting details on the orientifolding, see our paper tonight!


1000
800
600
400
200
0
0
10
20
30
40
$4/3 \ln(1/|z_{cf}|)$
$2\ln(1/|W_0|)$

Statistics of Conifold Vacua



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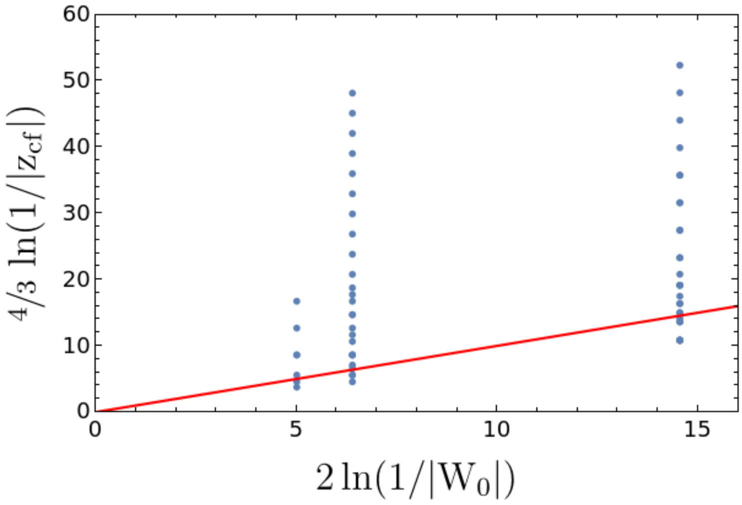

Interesting Conifold Vacua

$\vec{M}$	$\vec{K}$	$\lvert W_0\rvert$	$\lvert z_{cf}\rvert$	$\frac{\lvert z_{cf}\rvert^{2/3}-\lvert W_0\rvert}{\lvert W_0\rvert}$	$g_s M$
$(4,-8,10)$	$(-6,3,-4)$	7.4×10^{-9}	5.4×10^{-14}	-0.8	0.6
$(8,-12,6)$	$(-5,1,-2)$	6.9×10^{-4}	1.4×10^{-5}	-0.2	1.0
$(-8,4,12)$	$(5,1,-4)$	4.1×10^{-2}	5.2×10^{-3}	-0.3	2.8
$(-14,6,27)$	$(4,1,-2)$	1.4×10^{-3}	5.3×10^{-5}	0.03	0.9

- $g_s M$ is at the borderline of the regime of validity of 10d SUGRA near the tip.
- One can engineer generic D7-brane configurations to enlarge the tadpole bound to achieve larger $g_s M$.
- SUSY breaking in KS gauge theory?

Conclusions

- We have found :
  a method to find $\mathcal{F}_{cf}$, small W_0, and orientifolds with $h^{1,1} = \mathcal{O}(100)$.
- We should explore more generic conifold singularities.
- Light complex structure moduli are inevitable in our construction.
- We should compute one-loop Pfaffian to stabilize Kähler moduli.
- Quest to find KKLT de Sitter vacua must continue!