Vanishing Yukawa couplings in heterotic compactifications

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Based on work with L. Anderson, J. Gray, M. Larfors and R. Schneider
2103.10454

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Seminar Series on String Phenomenology

Setup

- Compactify $E_8 \times E_8$ heterotic string on Calabi-Yau three-fold, X, with a bundle, $V \to X$;
- Structure group of V embedded in E_8 and its commutant is gauge group of effective theory;
- Will assume V an $SU(3)$ bundle $\rightsquigarrow E_6$ gauge symmetry;

- Effective $N = 1$ supersymmetric gauge theory.
- Interactions of chiral supermultiplets, Φ, governed by superpotential $\int d^2\theta W(\Phi) + h.c$;

Yukawa coupling I

- Perturbative Yukawa couplings correspond to cubic term in the (perturbative) superpotential;
- Physical Yukawa couplings depend on correct normalisation of fields $\implies$ explicit Calabi-Yau metric dependence;
- Whether coupling vanishes or not is normalisation independent;
- Implications for physics, e.g. top quark mass .

Yukawa coupling II

The holomorphic Yukawa coupling

- Matter fields correspond to bundle-valued harmonic one-forms $\Phi^I \leftrightarrow \nu_I \in \mathrm{Harm}^{0,1}(V)$;
- From 10d action, deduce cubic term $W \supset \lambda(\nu_I, \nu_J, \nu_K)\Phi^I \Phi^J \Phi^K$, with

$$\lambda(\nu_1, \nu_2, \nu_3) = \int_X \Omega \wedge \mathrm{tr}(\nu_1 \wedge \nu_2 \wedge \nu_3)\,;$$

- $\Omega \in \Omega^{3,0}$, holomorphic three-form;
- tr = map induced from projection $V^{\otimes 3} \to \Lambda^3 V \to \mathcal{O}_X$;

Yukawa coupling III

$$\lambda(\nu_1, \nu_2, \nu_3) = \int_X \Omega \wedge \mathrm{tr}(\nu_1 \wedge \nu_2 \wedge \nu_3)\,;$$

Key properties

- Depends only on cohomology classes $[\nu_i]$;
- Composition of map $H^1(X, V)^{\otimes 3} \to H^3(X, \mathcal{O}_X)$, and $\int_X \Omega \wedge - : H^3(X, \mathcal{O}_X) \to \mathbb{C}$, isomorphism!

Implications

- $\lambda(\nu_1, \nu_2, \nu_3) = 0$ if and only if the cohomology class $[\mathrm{tr}(\nu_1 \wedge \nu_2 \wedge \nu_3)] = 0$.
- Computing $[\mathrm{tr}(\nu_1 \wedge \nu_2 \wedge \nu_3)]$ is very hard!

Outline

Motivation for sheaf cohomology

Decomposing cohomology

Need a mechanism to split up cohomology group that is compatible with Yukawa computation.

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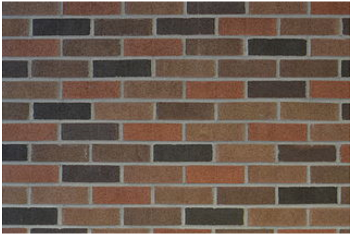

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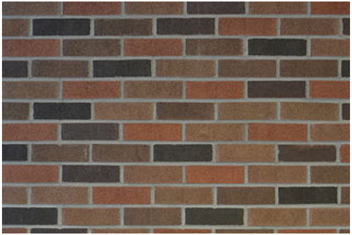

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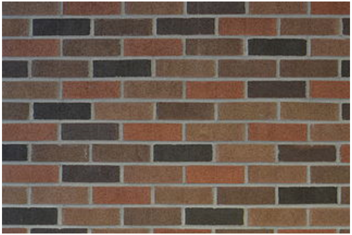

We "decompose" before taking cohomology $\rightsquigarrow$ gives controlled splitting.

Images: Public domain, via Wikimedia Commons

Examples of sheaves

- For X a complex manifold, define $\mathcal{O}_X(U) = \{\text{holomorphic maps } U \to \mathbb{C}\}$ with the natural restriction maps: structure sheaf.
- For any vector bundle, V, define $\mathcal{V}(U) = \Gamma(U, V|_U)$: sheaf of sections.
- For any abelian group, A, can define $\underline{A}(U) = \{\text{locally constant functions } U \to A\}$: constant sheaf.

Basic definitions

Exact sequences of sheaves

A complex of sheaves

$$\cdots \mathcal{F}_2 \xrightarrow{f_2} \mathcal{F}_1 \xrightarrow{f_1} \mathcal{F}_0 \to \cdots$$

is called exact if the sequence of groups obtained by restricting to smaller and smaller sets, is exact. Roughly:

- $\ker(f_i(U)) = \mathrm{Im}(f_{i+1}(U))$ for all “very small” U

Example

$$0 \to \underline{\mathbb{Z}} \xrightarrow{2\pi i\cdot} \mathcal{O}_{\mathbb{C}^*}(-,\mathbb{C}) \xrightarrow{\exp} \mathcal{O}_{\mathbb{C}^*}(-,\mathbb{C}^*) \to 0$$

Global sections

Exactness & global sections

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- Pass to cohomology $\rightsquigarrow$ long exact sequence of groups:

$$\cdots \to H^{i-1}(\mathcal{O}^*_{\mathbb{C}^*}) \xrightarrow{\delta} H^i(\mathbb{Z}) \to H^i(\mathcal{O}_{\mathbb{C}^*}) \to H^i(\mathcal{O}^*_{\mathbb{C}^*}) \to \cdots$$

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- Defined using special exact sequence of sheaves:

$$0 \to \mathcal{F} \to \mathcal{I}^0 \to \mathcal{I}^1 \to \cdots$$

$$H^i(X, \mathcal{F}) = \frac{\ker(\mathcal{I}^i(X) \to \mathcal{I}^{i+1}(X))}{\mathrm{Im}(\mathcal{I}^{i-1}(X) \to \mathcal{I}^i(X))}.$$

Godement resolution

- We will use "Godement resolutions"
- Behaves well with tensor product ("universal exactness"):

$$
\begin{array}{ccc}
\mathcal{G}^\bullet(\mathcal{F}) \otimes \mathcal{G}^\bullet(\mathcal{F}') & \xrightarrow{u} & \mathcal{G}^\bullet(\mathcal{F} \otimes \mathcal{F}') \\
\uparrow & & \uparrow \\
\mathcal{F} \otimes \mathcal{F}' & \xrightarrow{\mathrm{Id}} & \mathcal{F} \otimes \mathcal{F}'
\end{array}
$$

- (Also for $\mathcal{F}, \mathcal{F}'$ complexes of sheaves)

Cup product

Definition

- Use Godement resolution to define cohomology $H^i(X, V) = \frac{\ker(\mathcal{G}^i(V)(X) \to \mathcal{G}^{i+1}(V)(X))}{\mathrm{Im}(\mathcal{G}^{i-1}(V)(X) \to \mathcal{G}^i(V)(X))}$;
- Cup product is a map $\smile: H^i(\mathcal{F}) \otimes H^j(\mathcal{F}') \to H^{i+j}(\mathcal{F} \otimes \mathcal{F}')$;
- Defined by $[\nu] \otimes [\mu] \mapsto [u(\nu \otimes \mu)]$;
- Same operation as wedge product.

$$\begin{tikzcd} \mathcal{G}^\bullet(\mathcal{F}) \otimes \mathcal{G}^\bullet(\mathcal{F}') \arrow[r, "u"] & \mathcal{G}^\bullet(\mathcal{F} \otimes \mathcal{F}') \\ \mathcal{F} \otimes \mathcal{F}' \arrow[u] \arrow[r, "\mathrm{Id}"] & \mathcal{F} \otimes \mathcal{F}' \arrow[u] \end{tikzcd}$$

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Redefinition of Yukawa coupling

Class appearing in Yukawa coupling,

$$[\mathrm{tr}(\nu_1 \wedge \nu_2 \wedge \nu_3)]$$

is expressed as composition of

- Two cup products $\smile \otimes \smile: H^1(X, V)^{\otimes 3} \to H^3(X, V^{\otimes 3})$;
- Map $\mathrm{tr} : H^3(X, V^{\otimes 3}) \to H^3(X, \mathcal{O}_X)$

Outline

Known vanishing theorems I

(p, q)-vanishing theorem

Braun, He, Ovrut 0601204, ...

- Can be applied when X is elliptically fibred, $E \to X \xrightarrow{p} B$;
- Use Leray spectral sequence to decompose $H^1(X, V) \hookrightarrow H^0(B, R^1 p_* V) \oplus H^1(B, p_* V)$;
- Only allowed couplings are of form

$$H^1(B, p_* V) \otimes H^1(B, p_* V) \otimes H^0(B, R^1 p_* V)$$

- Assuming $p_* V, R^1 p_* V$ are locally free.

Stability walls

Anderson, Gray, Ovrut 1001.2317, ...

Known vanishing theorems I

(p, q)-vanishing theorem

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Stability walls

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- $SU(3)$ bundle splits $V \to K \oplus F$ at stability wall;
- Enhanced effective gauge group $E_6 \to E_6 \times U(1)$;
- Splits cohomology $H^1(V) = H^1(K) \oplus H^1(F)$;
- Summands have definite $U(1)$ charge.
- Only couplings that have **zero charge** are allowed at this locus.

Known vanishing theorems II

Koszul complexes

Blesneag, Buchbinder, Candelas, Lukas 1512.05322, ...

- Applicable when $X \hookrightarrow \mathcal{A} = \mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_k}$ and $V = \mathcal{V}|_X$ for $\mathcal{V}$ a sum of line bundles on ambient space;
- Exists canonical resolution, "Koszul complex":

$$0 \to \Lambda^{\bullet}\mathcal{N}^* \otimes \mathcal{V} \to V \to 0$$

- Induces decomposition

$$H^1(X, V) \hookrightarrow \bigoplus_{\tau=1}^{\dim \mathcal{A}-2} H^{\tau}(\mathcal{A}, \Lambda^{\tau-1}\mathcal{N}^* \otimes \mathcal{V})$$

- Call τ the "type" of class;
- Coupling $\lambda(\nu_1, \nu_2, \nu_3)$ vanishes unless $\tau_1 + \tau_2 + \tau_3 \geq \dim \mathcal{A}$.

Towards new vanishing theorems

Setup

Anderson, Gray, Larfors, M.M., Schneider, 2103.10454

- Start with a resolution

$$0 \to \mathcal{F}_1 \xrightarrow{f} \mathcal{F}_0 \xrightarrow{\pi} V \to 0$$

- Use Godement resolution:

$$0 \to \mathcal{G}^\bullet(\mathcal{F}_1) \to \mathcal{G}^\bullet(\mathcal{F}_0) \to \mathcal{G}^\bullet(V) \to 0$$

Towards new vanishing theorems

$$0 \to \mathcal{G}^\bullet(\mathcal{F}_1) \to \mathcal{G}^\bullet(\mathcal{F}_0) \to \mathcal{G}^\bullet(V) \to 0$$

Coboundary map

$$\begin{tikzcd}
0 \arrow[r] & \Gamma(\mathcal{G}^2(\mathcal{F}_1)) \arrow[r] & \Gamma(\mathcal{G}^2(\mathcal{F}_0)) \arrow[r] & \Gamma(\mathcal{G}^2(V)) \arrow[r] & 0 \\
0 \arrow[r] & \Gamma(\mathcal{G}^1(\mathcal{F}_1)) \arrow[r] \arrow[u] & \Gamma(\mathcal{G}^1(\mathcal{F}_0)) \arrow[r] \arrow[u] & \Gamma(\mathcal{G}^1(V)) \arrow[r] \arrow[u] & 0 \\
 & & & \nu \arrow[u, dashed] &
\end{tikzcd}$$

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$$0 \to \mathcal{G}^\bullet(\mathcal{F}_1) \to \mathcal{G}^\bullet(\mathcal{F}_0) \to \mathcal{G}^\bullet(V) \to 0$$

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 & & \hat{\nu} \arrow[r, mapsto] & \nu &
\end{tikzcd}$$

Towards new vanishing theorems

$$0 \to \mathcal{G}^\bullet(\mathcal{F}_1) \to \mathcal{G}^\bullet(\mathcal{F}_0) \to \mathcal{G}^\bullet(V) \to 0$$

Coboundary map

$$\begin{tikzcd}
 & & \mathfrak{d}\hat{\nu} \arrow[d, dashed] & & \\
0 \arrow[r] & \Gamma(\mathcal{G}^2(\mathcal{F}_1)) \arrow[r] & \Gamma(\mathcal{G}^2(\mathcal{F}_0)) \arrow[r] & \Gamma(\mathcal{G}^2(V)) \arrow[r] & 0 \\
0 \arrow[r] & \Gamma(\mathcal{G}^1(\mathcal{F}_1)) \arrow[r] \arrow[u] & \Gamma(\mathcal{G}^1(\mathcal{F}_0)) \arrow[r] \arrow[u] & \Gamma(\mathcal{G}^1(V)) \arrow[r] \arrow[u] & 0 \\
 & & \hat{\nu} \arrow[r, mapsto, red] \arrow[uuu, bend right] & \nu &
\end{tikzcd}$$

Towards new vanishing theorems

$$0 \to \mathcal{G}^\bullet(\mathcal{F}_1) \to \mathcal{G}^\bullet(\mathcal{F}_0) \to \mathcal{G}^\bullet(V) \to 0$$

Coboundary map

$$\begin{tikzcd}
 & & \mathfrak{d}\hat{\nu} \arrow[r] & 0 & \\
0 \arrow[r] & \Gamma(\mathcal{G}^2(\mathcal{F}_1)) \arrow[r] & \Gamma(\mathcal{G}^2(\mathcal{F}_0)) \arrow[r] & \Gamma(\mathcal{G}^2(V)) \arrow[r] & 0 \\
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Coboundary map

$$\begin{tikzcd}
 & \delta\nu \arrow[r, mapsto, red] & \mathfrak{d}\hat{\nu} & & \\
0 \arrow[r] & \Gamma(\mathcal{G}^2(\mathcal{F}_1)) \arrow[r] & \Gamma(\mathcal{G}^2(\mathcal{F}_0)) \arrow[r] & \Gamma(\mathcal{G}^2(V)) \arrow[r] & 0 \\
0 \arrow[r] & \Gamma(\mathcal{G}^1(\mathcal{F}_1)) \arrow[r] \arrow[u] & \Gamma(\mathcal{G}^1(\mathcal{F}_0)) \arrow[r] \arrow[u] & \Gamma(\mathcal{G}^1(V)) \arrow[r] \arrow[u] & 0 \\
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\end{tikzcd}$$

Cohomology decomposition

Cohomological type

We will say a cohomology class, $[\nu]$ is

- Type 1 if $\mathfrak{d}\hat{\nu} = 0$;
- Type 2 else.
- Induces $H^1(V) \hookrightarrow H^1(\mathcal{F}_0) \oplus H^2(\mathcal{F}_1)$

$$\begin{tikzcd}
 & \delta\nu \arrow[r, mapsto] & \mathfrak{d}\hat{\nu} & & \\
0 \arrow[r] & \Gamma(\mathcal{G}^2(\mathcal{F}_1)) \arrow[r] & \Gamma(\mathcal{G}^2(\mathcal{F}_0)) \arrow[r] & \Gamma(\mathcal{G}^2(V)) \arrow[r] & 0 \\
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\end{tikzcd}$$

Yukawa contribution

- Represent classes in $H^1(V)$ by sections

$$(\hat{\nu}, \delta\nu) \in \Gamma(\mathcal{G}^1(\mathcal{F}_0)) \oplus \Gamma(\mathcal{G}^2(\mathcal{F}_1))\,;$$

- Type 1 represented by $\hat{\nu} \in \mathcal{G}^1(\mathcal{F}_0)$;
- (Type 1)^3 coupling represented by

$$[\hat{\nu}_1 \otimes \hat{\nu}_2 \otimes \hat{\nu}_3] \in \mathcal{G}^3(\Lambda^3 \mathcal{F}_0)\,;$$

- If $H^3(\Lambda^3 \mathcal{F}_0) = 0$, then the coupling must be trivial.

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$$(\hat{\nu}, \delta\nu) \in \Gamma(\mathcal{G}^1(\mathcal{F}_0)) \oplus \Gamma(\mathcal{G}^2(\mathcal{F}_1))\,;$$

- Similarly, (type 1)^2(type 2) coupling is represented by

$$\begin{bmatrix} \hat{\nu}_1 \otimes \hat{\nu}_2 \otimes \hat{\nu}_3 \\ \hat{\nu}_1 \otimes \hat{\nu}_2 \otimes \delta\nu_3 \end{bmatrix} \in \Gamma(\mathcal{G}^3(\Lambda^3\mathcal{F}_0)) \oplus \Gamma(\mathcal{G}^4(\Lambda^2\mathcal{F}_0 \otimes \mathcal{F}_1))\,.$$

- If $H^4(\Lambda^2\mathcal{F}_0 \otimes \mathcal{F}_1) = 0$ and $H^3(\Lambda^3\mathcal{F}_0) = 0$, then (type 1)^2(type 2) couplings vanish.

Vanishing theorem

Theorem 2103.10454

- If $H^3(\Lambda^3\mathcal{F}_0) = 0$, then the coupling between three type 1 fields vanishes;
- If in addition $H^4(\mathcal{F}_1 \otimes \Lambda^2\mathcal{F}_0) = 0$, then the coupling between two type 1s and a type 2 vanishes.

Vanishing theorem

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2103.10454

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Higher type couplings?

- Rely on exact sequence:

$$?? \rightarrow \Lambda^2\mathcal{F}_0 \otimes \mathcal{F}_1 \rightarrow \Lambda^3\mathcal{F}_0 \rightarrow \Lambda^3 V \rightarrow 0\,;$$

- Higher type needs the unknown terms.

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Theorem 2103.10454

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- If in addition $H^4(\mathcal{F}_1 \otimes \Lambda^2\mathcal{F}_0) = 0$, then the coupling between two type 1s and a type 2 vanishes.

- Argument can be extended to complex of **arbitrary** length, still only these couplings are trustworthy;
- Can sometimes weaken second condition.

Summary and Outlook

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- New theorems constraining Yukawa couplings;
- Wide range of applicability;
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Thanks for listening!