8D String Universality and Non-Simply-Connected Gauge Groups

Based on 2008.10605

with Mirjam Cvetič, Markus Dierigl and Ling Lin

Hao Y. Zhang

University of Pennsylvania

Sep. 29, 2020

Section I: String Universality

I
Goal: String Universality
Focusing on 8D
Non-simply-connected Gauge Groups

II
F-theory on elliptic K3: 8D Vacua
Mordell-Weil Group and Arithmetics

III
Higher Form Center Symmetry
B_4 Periodicity and Mixed Anomaly
Anomaly-free 8D Theories

IV
Comparison: Strong Evidence for
8D String Universality
With G Non-simply-connected

V
Future Directions

String Universality in 10D

String Universality in 10D

String universality conjecture: All consistent theories of quantum gravity should come as low energy limits of string theory.

String Universality in 10D

String universality conjecture: All consistent theories of quantum gravity should come as low energy limits of string theory.
10D $\mathcal{N} = 1$ supergravities [Green, Schwarz, Witten, Vol.2]:

$$E_8 \times E_8, \quad SO(32), \quad E_8 \times U(1)^{248}, \quad U(1)^{496} \tag{1}$$

The first two are realized in heterotic string theories. Long time puzzle: the last two theories are neither realized in string theory nor ruled out.

String Universality in 10D

String universality conjecture: All consistent theories of quantum gravity should come as low energy limits of string theory.
10D $\mathcal{N} = 1$ supergravities [Green, Schwarz, Witten, Vol.2]:

$$E_8 \times E_8, \quad SO(32), \quad E_8 \times U(1)^{248}, \quad U(1)^{496} \tag{1}$$

The first two are realized in heterotic string theories. Long time puzzle: the last two theories are neither realized in string theory nor ruled out.
Solution: they are found inconsistent via the following non-stringy approaches:

- Via SUSY and Abelian gauge invariance [Adams, DeWolfe, Taylor '10]
- Unitarity bound: rank $\leq$ 26-d $= 16$ [Kim, Shiu, Vafa '19]

String Universality in 10D

String universality conjecture: All consistent theories of quantum gravity should come as low energy limits of string theory.
10D $\mathcal{N} = 1$ supergravities [Green, Schwarz, Witten, Vol.2]:

$$E_8 \times E_8, \quad SO(32), \quad E_8 \times U(1)^{248}, \quad U(1)^{496} \tag{1}$$

The first two are realized in heterotic string theories. Long time puzzle: the last two theories are neither realized in string theory nor ruled out.
Solution: they are found inconsistent via the following non-stringy approaches:

- Via SUSY and Abelian gauge invariance [Adams, DeWolfe, Taylor '10]
- Unitarity bound: rank $\leq$ 26-d $=$ 16 [Kim, Shiu, Vafa '19]

Recent progress [Montero, Vafa '20]: e.g., 8D supergravity theories need to have rank 18, 10 or 2. This explains the modulo-8 rank pattern in 8D string vacua without referring to string theory.

Global Structure of Gauge Groups

Global Structure of Gauge Groups

Global structure of standard model gauge group - candidates:

$$(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_n, \quad n = 1, 2, 3, 6 \tag{2}$$

Global Structure of Gauge Groups

Global structure of standard model gauge group - candidates:

$$(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_n, \quad n = 1, 2, 3, 6 \tag{2}$$

$SU(2)$ vs $SO(3) \simeq SU(2)/\mathbb{Z}_2$: e.g., **2** rep. allowed/forbidden.

Global Structure of Gauge Groups

Global structure of standard model gauge group - candidates:

$$(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_n, \quad n = 1, 2, 3, 6 \tag{2}$$

$SU(2)$ vs $SO(3) \simeq SU(2)/\mathbb{Z}_2$: e.g., **2** rep. allowed/forbidden.

Visible at the level of non-local objects: Wilson/'t Hooft lines, etc.

Global Structure of Gauge Groups

Global structure of standard model gauge group - candidates:

$$(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_n, \quad n = 1, 2, 3, 6 \tag{2}$$

$SU(2)$ vs $SO(3) \simeq SU(2)/\mathbb{Z}_2$: e.g., **2** rep. allowed/forbidden.

Visible at the level of non-local objects: Wilson/'t Hooft lines, etc.

Focus on 8D theories: 8D string vacua exhibits intricate patterns in gauge groups global structure. Can also be understood in F-theory via arithmetic structure of elliptic K3s.

Global Structure of Gauge Groups

Global structure of standard model gauge group - candidates:

$$(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_n, \quad n = 1, 2, 3, 6 \tag{2}$$

$SU(2)$ vs $SO(3) \simeq SU(2)/\mathbb{Z}_2$: e.g., **2** rep. allowed/forbidden.

Visible at the level of non-local objects: Wilson/'t Hooft lines, etc.

Focus on 8D theories: 8D string vacua exhibits intricate patterns in gauge groups global structure. Can also be understood in F-theory via arithmetic structure of elliptic K3s.

- Part II: Connection to F-theory arithmetics
- Part III: Connection to higher form symmetries

Section II: 8D Stringy Vacua and Arithmetics

I
Goal: String Universality
Focusing on 8D
Non-simply-connected Gauge Groups

II
F-theory on elliptic K3: 8D Vacua
Mordell-Weil Group and Arithmetics

III
Higher Form Center Symmetry
B_4 Periodicity and Mixed Anomaly
Anomaly-free 8D Theories

IV
Comparison: Strong Evidence for
8D String Universality
With G Non-simply-connected

V
Future Directions

F-theory on Elliptic K3s

F-theory on Elliptic K3s

F-theory: Strong coupling version of type IIB, $SL(2,\mathbb{Z})$ geometrized. [Vafa '96; Morrison Vafa '96], review: [Weigand '18]

F-theory on Elliptic K3s

F-theory: Strong coupling version of type IIB, $SL(2, \mathbb{Z})$ geometrized. [Vafa '96; Morrison Vafa '96], review: [Weigand '18]

F-theory 8D SUGRA vacua corresponds to 24 7-branes of specific (p, q) types locating on the base $\mathbb{P}^1$, whose overall monodromy is identity.

F-theory on Elliptic K3s

F-theory: Strong coupling version of type IIB, $SL(2, \mathbb{Z})$ geometrized. [Vafa '96; Morrison Vafa '96], review: [Weigand '18]

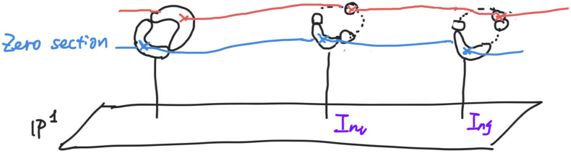
F-theory 8D SUGRA vacua corresponds to 24 7-branes of specific (p, q) types locating on the base $\mathbb{P}^1$, whose overall monodromy is identity.

Elliptically fibered K3 perspective: Gauge group can be read off via Kodaira's classification. Generically 24 I_1 fibers, collision gives enhancement.

Mordell-Weil Torsion in F-theory and Global Structure of Gauge Group

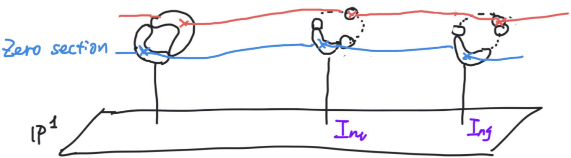
Mordell-Weil Torsion in F-theory and Global Structure of Gauge Group

Mordell-Weil(MW) group of an elliptic fibration consists of all of its global sections. The group law is the fiberwise addition law of cubic curves. See [Cvetič, Lin '18] for a review in the F-theory context.



Mordell-Weil Torsion in F-theory and Global Structure of Gauge Group

Mordell-Weil(MW) group of an elliptic fibration consists of all of its global sections. The group law is the fiberwise addition law of cubic curves. See [Cvetič, Lin '18] for a review in the F-theory context.

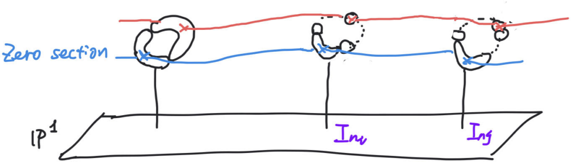


As a finitely generated Abelian group, one has:

$$MW(Y) = \mathbb{Z}^r \oplus \bigoplus_i \mathbb{Z}_{n_i} \tag{3}$$

Mordell-Weil Torsion in F-theory and Global Structure of Gauge Group

Mordell-Weil(MW) group of an elliptic fibration consists of all of its global sections. The group law is the fiberwise addition law of cubic curves. See [Cvetič, Lin '18] for a review in the F-theory context.



As a finitely generated Abelian group, one has:

$$MW(Y) = \mathbb{Z}^r \oplus \bigoplus_i \mathbb{Z}_{n_i} \quad (3)$$

- Each free section gives a $U(1)$ gauge symmetry [Morrison, Park '12], and a quotient $\frac{G_{nA} \times U(1)}{Z(G)}$ at the presence of G_{nA} [Cvetič. Lin '17].
- Each ℓ-torsional section gives a $\mathbb{Z}_\ell$ global quotient: Main focus. [Aspinwall, Morrison '98][Mayrhofer, Morrison, Till, Weigand '14][Hajouji, Oehlmann '19].

Geometric Properties in Elliptic K3s

Geometric Properties in Elliptic K3s

G	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[2, 2]	[4, 2]	[6, 2]	[3, 3]	[4, 4]	Total
$\lvert\mathcal{P}^G\rvert$	2746	732	85	41	6	10	1	1	61	5	1	3	1	3693

Figure: A complete list of ADE fiber types in elliptic K3s [Shimada '00]. $[n]$: a $\mathbb{Z}_n$ MW torsion, $[n, m]$: a $\mathbb{Z}_n \times \mathbb{Z}_m$ MW torsion.

Geometric Properties in Elliptic K3s

G	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[2, 2]	[4, 2]	[6, 2]	[3, 3]	[4, 4]	Total
$\lvert\mathcal{P}^G\rvert$	2746	732	85	41	6	10	1	1	61	5	1	3	1	3693

Figure: A complete list of ADE fiber types in elliptic K3s [Shimada '00]. $[n]$: a $\mathbb{Z}_n$ MW torsion, $[n, m]$: a $\mathbb{Z}_n \times \mathbb{Z}_m$ MW torsion.

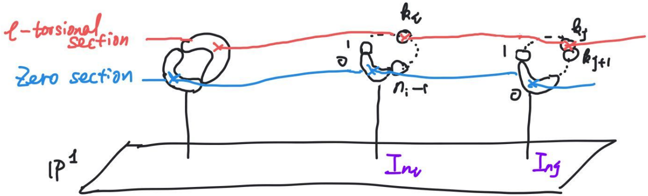


Figure: k_i: # of component in I_{n_i} fiber intersected by the ℓ-torsional section.

Arithmetic Constraint on Global Structure of Gauge Groups

Arithmetic Constraint on Global Structure of Gauge Groups

For a K3 with only I_n fibers and a single ℓ-torsional section, the global gauge group is $(\bigotimes_i SU(n_i))/\mathbb{Z}_\ell$.

Arithmetic Constraint on Global Structure of Gauge Groups

For a K3 with only I_n fibers and a single ℓ-torsional section, the global gauge group is $(\bigotimes_i SU(n_i))/\mathbb{Z}_\ell$.

But what is the embedding? Now $G = \bigotimes_i SU(n_i)$, and this $(k_1, \ldots, k_s)$ is precisely the generator of the embedded $\mathbb{Z}_\ell$ quotient into the overall center $Z(G) = \bigotimes_i Z(G_i)$.

Arithmetic Constraint on Global Structure of Gauge Groups

For a K3 with only I_n fibers and a single ℓ-torsional section, the global gauge group is $(\bigotimes_i SU(n_i))/\mathbb{Z}_\ell$.

But what is the embedding? Now $G = \bigotimes_i SU(n_i)$, and this $(k_1, \ldots, k_s)$ is precisely the generator of the embedded $\mathbb{Z}_\ell$ quotient into the overall center $Z(G) = \bigotimes_i Z(G_i)$.

Not all embedding are allowed. [Miranda, Persson '88 '89 '91]:

$$q\left(k_1, \ldots, k_s\right) = \sum_{i=1}^{s} \frac{n_i - 1}{2n_i} k_i^2 \equiv 0 \mod \mathbb{Z} \tag{4}$$

Arithmetic Constraint on Global Structure of Gauge Groups

For a K3 with only I_n fibers and a single ℓ-torsional section, the global gauge group is $(\bigotimes_i SU(n_i))/\mathbb{Z}_\ell$.

But what is the embedding? Now $G = \bigotimes_i SU(n_i)$, and this $(k_1, \ldots, k_s)$ is precisely the generator of the embedded $\mathbb{Z}_\ell$ quotient into the overall center $Z(G) = \bigotimes_i Z(G_i)$.

Not all embedding are allowed. [Miranda, Persson '88 '89 '91]:

$$q\left(k_1, \ldots, k_s\right) = \sum_{i=1}^{s} \frac{n_i - 1}{2n_i} k_i^2 \equiv 0 \mod \mathbb{Z} \tag{4}$$

e.g. $G = SU(7)^3/\mathbb{Z}_7$ with a $\mathbb{Z}_7$ torsional section:
$(k_1, k_2, k_3) = (1, 2, 3)$ is an allowed intersection pattern, corresponding to an allowed $\mathbb{Z}_7$ center embedding.
However, $(k_1, k_2, k_3) = (1, 1, 1)$ is not allowed.

Arithmetic Constraint on Global Structure of Gauge Groups

For a K3 with only I_n fibers and a single ℓ-torsional section, the global gauge group is $(\bigotimes_i SU(n_i))/\mathbb{Z}_\ell$.

But what is the embedding? Now $G = \bigotimes_i SU(n_i)$, and this $(k_1, \ldots, k_s)$ is precisely the generator of the embedded $\mathbb{Z}_\ell$ quotient into the overall center $Z(G) = \bigotimes_i Z(G_i)$.

Not all embedding are allowed. [Miranda, Persson '88 '89 '91]:

$$q(k_1, \ldots, k_s) = \sum_{i=1}^{s} \frac{n_i - 1}{2n_i} k_i^2 \equiv 0 \mod \mathbb{Z} \tag{4}$$

e.g. $G = SU(7)^3/\mathbb{Z}_7$ with a $\mathbb{Z}_7$ torsional section:
$(k_1, k_2, k_3) = (1, 2, 3)$ is an allowed intersection pattern, corresponding to an allowed $\mathbb{Z}_7$ center embedding.
However, $(k_1, k_2, k_3) = (1, 1, 1)$ is not allowed.

Any field-theoretic reasons behind this?

Section III: Higher Form Symmetry and Mixed Anomalies

I
Goal: String Universality
Focusing on 8D
Non-simply-connected Gauge Groups

II
F-theory on elliptic K3: 8D Vacua
Mordell-Weil Group and Arithmetics

III
Higher Form Center Symmetry
B_4 Periodicity and Mixed Anomaly
Anomaly-free 8D Theories

IV
Comparison: Strong Evidence for
8D String Universality
With G Non-simply-connected

V
Future Directions

Higher-form Global Symmetry

Higher-form Global Symmetry

The operators charged under a q-form global symmetry are of dimension q. They have to be Abelian for $q \geq 1$. [Gaiotto, Kapustin, Seiberg, Willett '14] Here we focus on the class of center higher form symmetries.

Higher-form Global Symmetry

The operators charged under a q-form global symmetry are of dimension q. They have to be Abelian for $q \geq 1$. [Gaiotto, Kapustin, Seiberg, Willett '14] Here we focus on the class of center higher form symmetries.

- A continuous example: 4D $U(1)$ gauge theory: an *electric* 1-form global $U(1)$ whose charged objects are the Wilson lines. (We ignore the magnetic symmetry for our purpose.)

Higher-form Global Symmetry

The operators charged under a q-form global symmetry are of dimension q. They have to be Abelian for $q \geq 1$. [Gaiotto, Kapustin, Seiberg, Willett '14] Here we focus on the class of center higher form symmetries.

- A continuous example: 4D $U(1)$ gauge theory: an *electric* 1-form global $U(1)$ whose charged objects are the Wilson lines. (We ignore the magnetic symmetry for our purpose.)
- A discrete example: 4D $SU(N)$ theory has a center $Z(SU(N)) = \mathbb{Z}_N$-valued 1-form electric global symmetry charging the discrete Wilson lines. Main focus.

Gauging 1-form Global Symmetry

Gauging 1-form Global Symmetry

For G gauge theories without any matter charged under a subgroup $\Gamma \subset Z(G)$ of the center, it has a Γ-valued 1-form global symmetry.

Gauging 1-form Global Symmetry

For G gauge theories without any matter charged under a subgroup $\Gamma \subset Z(G)$ of the center, it has a Γ-valued 1-form global symmetry.

To gauge this 1-form global symmetry, one first couple it to a 2-form Γ-valued background gauge field C, reducing the electric global symmetry: $Z(G) \to Z(G)/\Gamma$.

Gauging 1-form Global Symmetry

For G gauge theories without any matter charged under a subgroup $\Gamma \subset Z(G)$ of the center, it has a Γ-valued 1-form global symmetry.

To gauge this 1-form global symmetry, one first couple it to a 2-form Γ-valued background gauge field C, reducing the electric global symmetry: $Z(G) \to Z(G)/\Gamma$.
One then sum over all possible C-configurations in the partition function, giving rise to a G/Γ partition function in the 0-form sector.

Gauging 1-form Global Symmetry

For G gauge theories without any matter charged under a subgroup $\Gamma \subset Z(G)$ of the center, it has a Γ-valued 1-form global symmetry.

To gauge this 1-form global symmetry, one first couple it to a 2-form Γ-valued background gauge field C, reducing the electric global symmetry: $Z(G) \to Z(G)/\Gamma$.
One then sum over all possible C-configurations in the partition function, giving rise to a G/Γ partition function in the 0-form sector.

In the end, the G bundle is twisted into a G/Γ bundle with a non-trivial Γ-valued second Stiefel-Whitney class.

$\mathbb{Z}_N$ 1-form Symmetry and Mixed Anomaly in 4D Field Theories

$\mathbb{Z}_N$ 1-form Symmetry and Mixed Anomaly in 4D Field Theories

In 4d Yang-Mills theories, there is a mixed anomaly between the 1-form global symmetry and the θ-periodicity [Córdova, Freed, Lam, Seiberg '19].

$\mathbb{Z}_N$ 1-form Symmetry and Mixed Anomaly in 4D Field Theories

In 4d Yang-Mills theories, there is a mixed anomaly between the 1-form global symmetry and the θ-periodicity [Córdova, Freed, Lam, Seiberg '19].

$\frac{i\theta}{8\pi^2}\int \mathrm{Tr}(F \wedge F)$ under a 1-form symmetry transformation and $\theta \to \theta + 2\pi$ gains:

$$\mathcal{A}(\theta, C) = 2\pi i \alpha_G \int_{M_4} \mathcal{P}(C) \mod \mathbb{Z} \tag{5}$$

$\mathbb{Z}_N$ 1-form Symmetry and Mixed Anomaly in 4D Field Theories

In 4d Yang-Mills theories, there is a mixed anomaly between the 1-form global symmetry and the θ-periodicity [Córdova, Freed, Lam, Seiberg '19].

$\frac{i\theta}{8\pi^2}\int \mathrm{Tr}(F \wedge F)$ under a 1-form symmetry transformation and $\theta \to \theta + 2\pi$ gains:

$$\mathcal{A}(\theta, C) = 2\pi i \alpha_G \int_{M_4} \mathcal{P}(C) \mod \mathbb{Z} \tag{5}$$

α_G: G-dependent fractional group theory constant. e.g., $\alpha_G = \frac{N-1}{2N}$ for $G = SU(N)$

$\mathbb{Z}_N$ 1-form Symmetry and Mixed Anomaly in 4D Field Theories

In 4d Yang-Mills theories, there is a mixed anomaly between the 1-form global symmetry and the θ-periodicity [Córdova, Freed, Lam, Seiberg '19].

$\frac{i\theta}{8\pi^2}\int \mathrm{Tr}(F \wedge F)$ under a 1-form symmetry transformation and $\theta \to \theta + 2\pi$ gains:

$$\mathcal{A}(\theta, C) = 2\pi i \alpha_G \int_{M_4} \mathcal{P}(C) \mod \mathbb{Z} \tag{5}$$

α_G: G-dependent fractional group theory constant. e.g., $\alpha_G = \frac{N-1}{2N}$ for $G = SU(N)$

$\mathcal{P}(C)$: Pontryagin square of $C \in H^2(X, \mathbb{Z}_N)$, an integral element of the 4-th cohomology. ($\mathcal{P}(C)$ can be roughly understood as $C \wedge C$ when promoted to differential forms.)

$\mathbb{Z}_N$ 1-form Symmetry and Mixed Anomaly in 8D Supergravity

$\mathbb{Z}_N$ 1-form Symmetry and Mixed Anomaly in 8D Supergravity

In 8D $\mathcal{N} = 1$, gravity multiplet contains B_2 [Salam, Sezgin '85]. It can be dualized to B_4 [Awada, Townsend '85] with coupling ($G = \bigotimes_i G_i$):

$$\sum_i \int_{M_8} B_4 \wedge \frac{1}{2}\mathrm{Tr}(F_i \wedge F_i) \tag{6}$$

$\mathbb{Z}_N$ 1-form Symmetry and Mixed Anomaly in 8D Supergravity

In 8D $\mathcal{N}=1$, gravity multiplet contains B_2 [Salam, Sezgin '85]. It can be dualized to B_4 [Awada, Townsend '85] with coupling ($G=\bigotimes_i G_i$):

$$\sum_i \int_{M_8} B_4 \wedge \frac{1}{2}\mathrm{Tr}(F_i \wedge F_i) \tag{6}$$

A gauged $\Gamma \subset Z(G) = \bigotimes_i Z(G_i)$ should be coupled to $C^{(i)}$'s each valued in $Z(G_i)$, giving an instanton contribution:

$$I_4(G/\Gamma) = \sum_i \alpha_{G_i} \mathcal{P}(C^{(i)}) \mod \mathbb{Z} \tag{7}$$

$\mathbb{Z}_N$ 1-form Symmetry and Mixed Anomaly in 8D Supergravity

In 8D $\mathcal{N} = 1$, gravity multiplet contains B_2 [Salam, Sezgin '85]. It can be dualized to B_4 [Awada, Townsend '85] with coupling ($G = \bigotimes_i G_i$):

$$\sum_i \int_{M_8} B_4 \wedge \frac{1}{2}\mathrm{Tr}(F_i \wedge F_i) \tag{6}$$

A gauged $\Gamma \subset Z(G) = \bigotimes_i Z(G_i)$ should be coupled to $C^{(i)}$'s each valued in $Z(G_i)$, giving an instanton contribution:

$$I_4(G/\Gamma) = \sum_i \alpha_{G_i} \mathcal{P}(C^{(i)}) \mod \mathbb{Z} \tag{7}$$

Together with the integral period shift $B_4 \to B_4 + b_4$, we obtain:

$$\mathcal{A}(b_4, C^{(i)}) = 2\pi i \sum_i \alpha_{G_i} \int_{M_8} b_4 \cup \mathcal{P}(C^{(i)}) \mod \mathbb{Z} \tag{8}$$

$\mathbb{Z}_N$ 1-form Symmetry and Mixed Anomaly in 8D Supergravity

In 8D $\mathcal{N}=1$, gravity multiplet contains B_2 [Salam, Sezgin '85]. It can be dualized to B_4 [Awada, Townsend '85] with coupling ($G = \bigotimes_i G_i$):

$$\sum_i \int_{M_8} B_4 \wedge \frac{1}{2}\mathrm{Tr}(F_i \wedge F_i) \tag{6}$$

A gauged $\Gamma \subset Z(G) = \bigotimes_i Z(G_i)$ should be coupled to $C^{(i)}$'s each valued in $Z(G_i)$, giving an instanton contribution:

$$I_4(G/\Gamma) = \sum_i \alpha_{G_i} \mathcal{P}(C^{(i)}) \mod \mathbb{Z} \tag{7}$$

Together with the integral period shift $B_4 \to B_4 + b_4$, we obtain:

$$\mathcal{A}(b_4, C^{(i)}) = 2\pi i \sum_i \alpha_{G_i} \int_{M_8} b_4 \cup \mathcal{P}(C^{(i)}) \mod \mathbb{Z} \tag{8}$$

In 6D theories, an analogous anomaly comes from a GS term $\int_{M_6} \Omega_{ij} B^i \wedge \mathrm{Tr}(F^j \wedge F^j)$ [Apruzzi, Dierigl, Lin '20].

Anomaly-free 8D Supergravity Theories

Anomaly-free 8D Supergravity Theories

$$\mathcal{A}(b_4, C^{(i)}) = 2\pi i \sum_i \alpha_{G_i} \int_{M_8} b_4 \cup \mathcal{P}(C^{(i)}) \mod \mathbb{Z} \tag{9}$$

According to [Cordova, Ohmori '19], such mixed anomaly cannot be cancelled. It also gives a fractional electric B_4 charge, which is not consistent [Apruzzi, Dierigl, Lin '20]. So it must vanish modulo integer.

Anomaly-free 8D Supergravity Theories

$$\mathcal{A}(b_4, C^{(i)}) = 2\pi i \sum_i \alpha_{G_i} \int_{M_8} b_4 \cup \mathcal{P}(C^{(i)}) \mod \mathbb{Z} \qquad (9)$$

According to [Cordova, Ohmori '19], such mixed anomaly cannot be cancelled. It also gives a fractional electric B_4 charge, which is not consistent [Apruzzi, Dierigl, Lin '20]. So it must vanish modulo integer.

$\bigoplus_i SU(N_i)$ case: assuming we have a gauged $\mathbb{Z}_\ell$ factor with a $\mathbb{Z}_\ell$-valued 2-form C field. We further set $C^{(i)} = k^i C$, namely the $\mathbb{Z}_\ell$ quotient of $G/\mathbb{Z}_\ell$ is generated by $(k_1, \ldots, k_s)$ in $Z(G)$. Then one has a constraint:

$$\sum_i \frac{N_i - 1}{2N_i} k_i^2 \in \mathbb{Z} \qquad (10)$$

Anomaly-free 8D Supergravity Theories

$$\mathcal{A}(b_4, C^{(i)}) = 2\pi i \sum_i \alpha_{G_i} \int_{M_8} b_4 \cup \mathcal{P}(C^{(i)}) \mod \mathbb{Z} \tag{9}$$

According to [Cordova, Ohmori '19], such mixed anomaly cannot be cancelled. It also gives a fractional electric B_4 charge, which is not consistent [Apruzzi, Dierigl, Lin '20]. So it must vanish modulo integer.

$\bigoplus_i SU(N_i)$ case: assuming we have a gauged $\mathbb{Z}_\ell$ factor with a $\mathbb{Z}_\ell$-valued 2-form C field. We further set $C^{(i)} = k^i C$, namely the $\mathbb{Z}_\ell$ quotient of $G/\mathbb{Z}_\ell$ is generated by $(k_1, \ldots, k_s)$ in $Z(G)$. Then one has a constraint:

$$\sum_i \frac{N_i - 1}{2N_i} k_i^2 \in \mathbb{Z} \tag{10}$$

This is exactly the same as the arithmetic condition for elliptic K3s with only I_n singularities!

IV: Results and Conclusions

I
Goal: String Universality
Focusing on 8D
Non-simply-connected Gauge Groups

II
F-theory on elliptic K3: 8D Vacua
Mordell-Weil Group and Arithmetics

III
Higher Form Center Symmetry
B_4 Periodicity and Mixed Anomaly
Anomaly-free 8D Theories

IV
Comparison: Strong Evidence for
8D String Universality
With G Non-simply-connected

V
Future Directions

Constraints on G with Non-trivial $\pi_1(G)$

Constraints on G with Non-trivial $\pi_1(G)$

(a) Product-of-SU gauge groups. Assuming there is a non-trivial $\mathbb{Z}_N$ gauged 1-form symmetry, also taking into account the rank $\leq 26 - d = 18$ unitarity bound [Kim, Shiu, Vafa '19]:

Constraints on G with Non-trivial $\pi_1(G)$

(a) Product-of-SU gauge groups. Assuming there is a non-trivial $\mathbb{Z}_N$ gauged 1-form symmetry, also taking into account the rank $\leq 26 - d = 18$ unitarity bound [Kim, Shiu, Vafa '19]:

- For $N = p^k \geq 7$ power of prime, all anomaly free theories are realized via string theory.

Constraints on G with Non-trivial $\pi_1(G)$

(a) Product-of-SU gauge groups. Assuming there is a non-trivial $\mathbb{Z}_N$ gauged 1-form symmetry, also taking into account the rank $\leq 26 - d = 18$ unitarity bound [Kim, Shiu, Vafa '19]:

- For $N = p^k \geq 7$ power of prime, all anomaly free theories are realized via string theory.
  - Exactly one anomaly free theory for $p^k = 7, 8$ resp.
  - No anomaly free theories for $9 \leq p^k \leq 19$.

Constraints on G with Non-trivial $\pi_1(G)$

(a) Product-of-SU gauge groups. Assuming there is a non-trivial $\mathbb{Z}_N$ gauged 1-form symmetry, also taking into account the rank $\leq 26 - d = 18$ unitarity bound [Kim, Shiu, Vafa '19]:

- For $N = p^k \geq 7$ power of prime, all anomaly free theories are realized via string theory.
  - Exactly one anomaly free theory for $p^k = 7, 8$ resp.
  - No anomaly free theories for $9 \leq p^k \leq 19$.
- N not a power of prime. If one assume that $\mathbb{Z}_N$ acts faithfully on one simple G_i factor, then all such $10 \leq N \leq 18$ give no anomaly-free theories.

Constraints on G with Non-trivial $\pi_1(G)$

(a) Product-of-SU gauge groups. Assuming there is a non-trivial $\mathbb{Z}_N$ gauged 1-form symmetry, also taking into account the rank $\leq 26 - d = 18$ unitarity bound [Kim, Shiu, Vafa '19]:

- For $N = p^k \geq 7$ power of prime, all anomaly free theories are realized via string theory.
  - Exactly one anomaly free theory for $p^k = 7, 8$ resp.
  - No anomaly free theories for $9 \leq p^k \leq 19$.
- N not a power of prime. If one assume that $\mathbb{Z}_N$ acts faithfully on one simple G_i factor, then all such $10 \leq N \leq 18$ give no anomaly-free theories.

(b) Single-factor gauge groups.

Constraints on G with Non-trivial $\pi_1(G)$

(a) Product-of-SU gauge groups. Assuming there is a non-trivial $\mathbb{Z}_N$ gauged 1-form symmetry, also taking into account the rank $\leq 26 - d = 18$ unitarity bound [Kim, Shiu, Vafa '19]:

- For $N = p^k \geq 7$ power of prime, all anomaly free theories are realized via string theory.
  - Exactly one anomaly free theory for $p^k = 7, 8$ resp.
  - No anomaly free theories for $9 \leq p^k \leq 19$.
- N not a power of prime. If one assume that $\mathbb{Z}_N$ acts faithfully on one simple G_i factor, then all such $10 \leq N \leq 18$ give no anomaly-free theories.

(b) Single-factor gauge groups.

- $\frac{SU(16)}{\mathbb{Z}_2}, \quad \frac{SU(18)}{\mathbb{Z}_3}, \quad \frac{\text{Spin}(32)}{\mathbb{Z}_2}$: F-theory on K3

Constraints on G with Non-trivial $\pi_1(G)$

(a) Product-of-SU gauge groups. Assuming there is a non-trivial $\mathbb{Z}_N$ gauged 1-form symmetry, also taking into account the rank $\leq 26 - d = 18$ unitarity bound [Kim, Shiu, Vafa '19]:

- For $N = p^k \geq 7$ power of prime, all anomaly free theories are realized via string theory.
  - Exactly one anomaly free theory for $p^k = 7, 8$ resp.
  - No anomaly free theories for $9 \leq p^k \leq 19$.
- N not a power of prime. If one assume that $\mathbb{Z}_N$ acts faithfully on one simple G_i factor, then all such $10 \leq N \leq 18$ give no anomaly-free theories.

(b) Single-factor gauge groups.

- $\frac{SU(16)}{\mathbb{Z}_2}, \quad \frac{SU(18)}{\mathbb{Z}_3}, \quad \frac{\mathrm{Spin}(32)}{\mathbb{Z}_2}$: F-theory on K3
- $\frac{Sp(4)}{\mathbb{Z}_2}, \quad \frac{Sp(8)}{\mathbb{Z}_2}, \quad \frac{SU(8)}{\mathbb{Z}_2}, \quad \frac{SU(9)}{\mathbb{Z}_3}, \quad \frac{\mathrm{Spin}(16)}{\mathbb{Z}_2}$: 8D CHL

Constraints on G with Non-trivial $\pi_1(G)$

(a) Product-of-SU gauge groups. Assuming there is a non-trivial $\mathbb{Z}_N$ gauged 1-form symmetry, also taking into account the rank $\leq 26 - d = 18$ unitarity bound [Kim, Shiu, Vafa '19]:

- For $N = p^k \geq 7$ power of prime, all anomaly free theories are realized via string theory.
  - Exactly one anomaly free theory for $p^k = 7, 8$ resp.
  - No anomaly free theories for $9 \leq p^k \leq 19$.
- N not a power of prime. If one assume that $\mathbb{Z}_N$ acts faithfully on one simple G_i factor, then all such $10 \leq N \leq 18$ give no anomaly-free theories.

(b) Single-factor gauge groups.

- $\frac{SU(16)}{\mathbb{Z}_2}$, $\frac{SU(18)}{\mathbb{Z}_3}$, $\frac{\mathrm{Spin}(32)}{\mathbb{Z}_2}$: F-theory on K3
- $\frac{Sp(4)}{\mathbb{Z}_2}$, $\frac{Sp(8)}{\mathbb{Z}_2}$, $\frac{SU(8)}{\mathbb{Z}_2}$, $\frac{SU(9)}{\mathbb{Z}_3}$, $\frac{\mathrm{Spin}(16)}{\mathbb{Z}_2}$: 8D CHL
- $\frac{Sp(12)}{\mathbb{Z}_2}$, $\frac{Sp(16)}{\mathbb{Z}_2}$: Unknown

Conclusions

Conclusions

We have provided strong evidence for string universality for non-simply-connect non-Abelian gauge groups G/Z in eight dimensions.

Conclusions

We have provided strong evidence for string universality for non-simply-connect non-Abelian gauge groups G/Z in eight dimensions.

- We observe intricate patterns and restrictions regarding global structure of gauge groups in 8D string vacua.

Conclusions

We have provided strong evidence for string universality for non-simply-connect non-Abelian gauge groups G/Z in eight dimensions.

- We observe intricate patterns and restrictions regarding global structure of gauge groups in 8D string vacua.
  - This corresponds to patterns in arithmetic structures of elliptic fibrations in F-theory.

Conclusions

We have provided strong evidence for string universality for non-simply-connect non-Abelian gauge groups G/Z in eight dimensions.

- We observe intricate patterns and restrictions regarding global structure of gauge groups in 8D string vacua.
  - This corresponds to patterns in arithmetic structures of elliptic fibrations in F-theory.
- Key supergravity constraint: the mixed anomaly bewteen center 1-form symmery and large gauge transformation of the B_4 supergravity field should vanish.

Conclusions

We have provided strong evidence for string universality for non-simply-connect non-Abelian gauge groups G/Z in eight dimensions.

- We observe intricate patterns and restrictions regarding global structure of gauge groups in 8D string vacua.
  - This corresponds to patterns in arithmetic structures of elliptic fibrations in F-theory.
- Key supergravity constraint: the mixed anomaly bewteen center 1-form symmery and large gauge transformation of the B_4 supergravity field should vanish.
  - Such a constraint nicely matches with arithmetic properties of elliptic K3 surfaces in the pure I_n singularity case.

Conclusions

We have provided strong evidence for string universality for non-simply-connect non-Abelian gauge groups G/Z in eight dimensions.

- We observe intricate patterns and restrictions regarding global structure of gauge groups in 8D string vacua.
  - This corresponds to patterns in arithmetic structures of elliptic fibrations in F-theory.
- Key supergravity constraint: the mixed anomaly bewteen center 1-form symmery and large gauge transformation of the B_4 supergravity field should vanish.
  - Such a constraint nicely matches with arithmetic properties of elliptic K3 surfaces in the pure I_n singularity case.
- Via this constraint we can ruled out a large family of apparantly consistent 8D $\mathcal{N}=1$ theories (with G non-simply-connected) that cannot be realized via string theory.

Conclusions

We have provided strong evidence for string universality for non-simply-connect non-Abelian gauge groups G/Z in eight dimensions.

- We observe intricate patterns and restrictions regarding global structure of gauge groups in 8D string vacua.
  - This corresponds to patterns in arithmetic structures of elliptic fibrations in F-theory.
- Key supergravity constraint: the mixed anomaly bewteen center 1-form symmery and large gauge transformation of the B_4 supergravity field should vanish.
  - Such a constraint nicely matches with arithmetic properties of elliptic K3 surfaces in the pure I_n singularity case.
- Via this constraint we can ruled out a large family of apparantly consistent 8D $\mathcal{N} = 1$ theories (with G non-simply-connected) that cannot be realized via string theory. $\rightarrow$ Towards string universality in 8D.

V: Future Directions

I
Goal: String Universality
Focusing on 8D
Non-simply-connected Gauge Groups

II
F-theory on elliptic K3: 8D Vacua
Mordell-Weil Group and Arithmetics

III
Higher Form Center Symmetry
B_4 Periodicity and Mixed Anomaly
Anomaly-free 8D Theories

IV
Comparison: Strong Evidence for
8D String Universality
With G Non-simply-connected

V
Future Directions

Future Directions

Future Directions

- One can seek for new insights into the realization of 8D CHL string gauge groups with global quotients. (See also [Font, Fraiman, Graña, Núñez, Parra '20])

Future Directions

- One can seek for new insights into the realization of 8D CHL string gauge groups with global quotients. (See also [Font, Fraiman, Graña, Núñez, Parra '20])
- Incorporating the $U(1)$s will introduce additional global quotients $(G \times U(1))/Z(G)$ in F-theory [Cvetič, Lin '17]. One could seek for a field-theoretic explanation for this, possibly via unHiggsing.

Future Directions

- One can seek for new insights into the realization of 8D CHL string gauge groups with global quotients. (See also [Font, Fraiman, Graña, Núñez, Parra '20])
- Incorporating the $U(1)$s will introduce additional global quotients $(G \times U(1))/Z(G)$ in F-theory [Cvetič, Lin '17]. One could seek for a field-theoretic explanation for this, possibly via unHiggsing.
- Finally, we can compare this analysis with other swampland constraints. [McNamara, Vafa '19][Montero, Vafa '20]. In particular, the results obtained in [Montero, Vafa '20] regarding non-simply-connected gauge groups in 8D is consistent with ours.

Thanks!

terima kasih
σας ευχαριστώ
આભાર
salamat
ačiū
dank u
Thank you
Danke
謝謝
شكرا
спасибо
ありがとう
감사합니다
merci
aitäh
ขอบคุณ
Tak
शुक्रिया
obrigado