

Resolving Space-time Singularities in flux compactifications & KKLT

based on: 2102.05281 with Federico Carta

(also: 1902.01412 with F.Carta & Alexander Westphal)

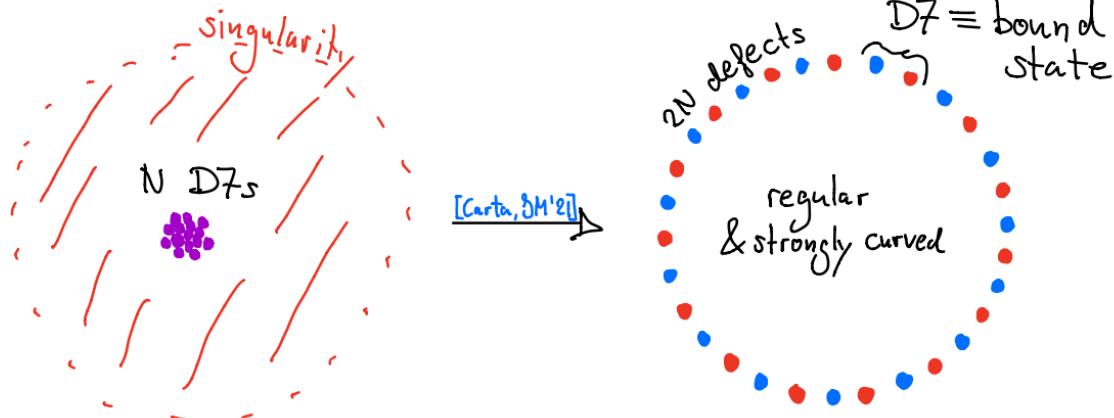
Jakob Moritz (Cornell)

preview:

issue: in KKLT loci hosting negative D3-charge surrounded by large singularity.

[Carta, JM, Westphal '19], [Gao, Hebecker, Jungmans '20]

today: for special case of D7 on K3



Outline

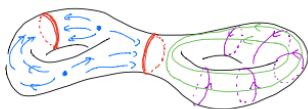
- ① Motivation: The singularity & why it matters
 - D. Jungmans' talk
 - [Carta, JM, Westphal '19]
 - [Gao, Hebecker, Jungmans '20]
- ② Resolution of singularities
- ③ Comments on instanton expansion in HKLT:

I

Motivation

The setup:

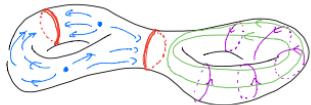
- type IIB flux compactifications à la GKP
[Giddings, Kachru, Polchinski '01]
i.e. $\mathbb{R}^{1,3} \times_w CY_3$ + F_3 & H_3 fluxes
+ seven-branes (& D3 branes)



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i.e. $\mathbb{R}^{1,3} \times_w CY_3$ + F_3 & H_3 fluxes



+ seven-branes (& D3 branes)

$$ds^2 = \frac{1}{t} e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \underbrace{\sum_{ij} g_{ij}^{CY} dy^i dy^j}_{\text{unit volume Ricci-flat metric}}$$

warp factor

volume modulus: $t := \int d^6 y \sqrt{g^{CY}} e^{-4A}$

The setup: (continued)

- conformal factor e^{-4A} sourced by D3-charge:

$$-\nabla_{\text{cy}}^2 e^{-4A} = \oint_{\text{D3}}$$

[GKP'01],
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→ as $t \rightarrow \infty$:

$$\text{Vol}_{6d} \rightarrow t^{3/2}$$

$$ds^2 \rightarrow \frac{1}{t^{3/2}} \eta_{\mu\nu} dx^\mu dx^\nu + t^{\frac{1}{2}} g_{ij}^{\text{cy}} dy^i dy^j$$

(except near localized sources!)

Singularities near seven-branes:

- Wrapped seven-branes (on 4-cycles) carry negative D3-charge: (let's assume all of it)

$$Q_{D3}^7 = -\frac{1}{24} \int_{\Sigma_4} c_2(\Sigma_4) \propto - \int_{\Sigma_4} R \wedge R < 0$$

[Green, Harvey, Moore '96]
[Cheung, Yin '97]

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\Rightarrow solution close to Σ_4 :

$$e^{-4A} \approx \frac{\left| S_{D3}^{\Sigma_4} \right|}{2\pi} \log(r/r^*) < 0$$

at small radii $r < r^*$

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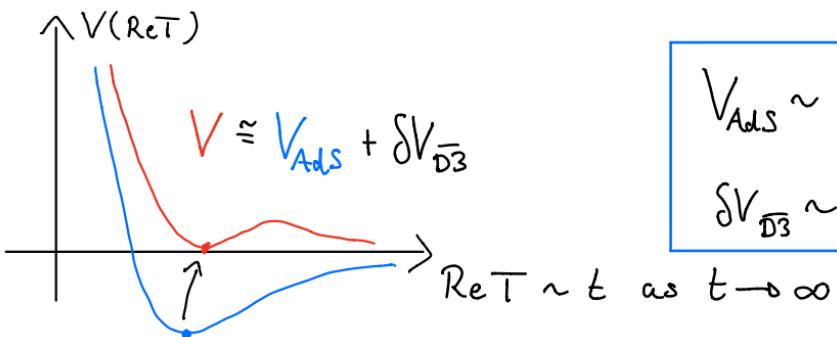
at large volume: $r^* \rightarrow \# e^{-\frac{2\pi}{|Q_{D3}^7|} \text{Vol}(\Sigma_4)}$

($t \rightarrow \infty$)

!

Intermediate volumes in KKLT

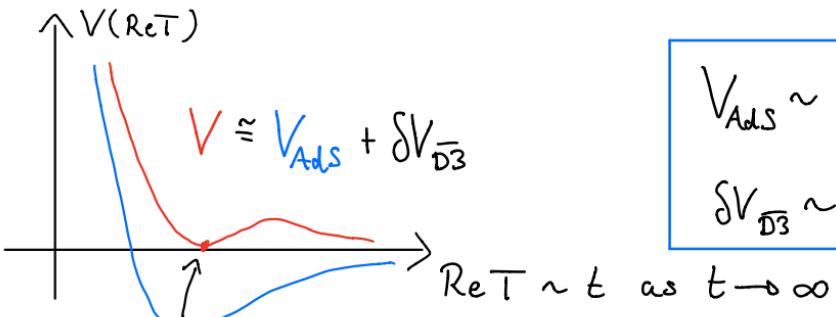
- KKLT uplift to dS from $\overline{D3}$ in KS-throat:
[Kachru, Kallosh, Linde, Trivedi '03] [Klebanov, Strassler '00], [Kachru, Pearson, Verlinde '01], ...



$$V_{AdS} \sim e^{-\# Re(T_0)}$$
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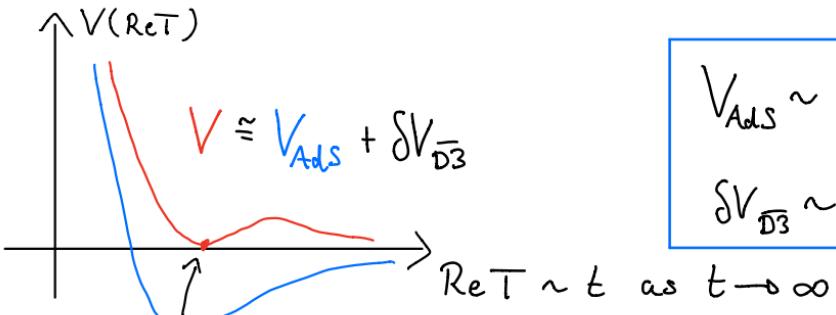
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- For small enough uplift: $\frac{Q_{D3}^{\text{throat}}}{R_{IR}^4} \gtrsim Re(T_0) \sim \text{Vol}(\Sigma_4)$
- For SUGRA control: $R_{IR}^4 \gtrsim 1$

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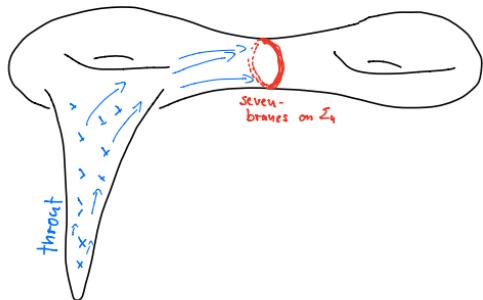
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$$\Rightarrow \boxed{\text{Vol}(\Sigma_4) \lesssim Q_{D3}^{\text{throat}} \text{ at KKLT min.}}$$

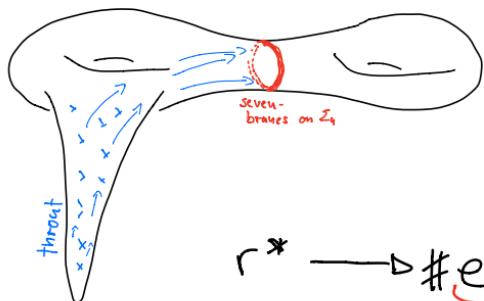
Intermediate volumes in KKLT (continued)

- But Q_{D3}^{throat} cancelled by negative charge on seven-brane stack: $Q_{D3}^{\text{throat}} \leq |Q_{D3}^7|$
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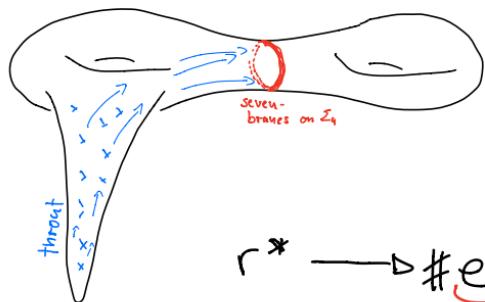
\Rightarrow lose exp. control over
Seven-brane singularity:

$$r^* \rightarrow \# e^{-\frac{2\pi}{|Q_{D3}^7|} \text{Vol}(\Sigma_4)} \quad \mathcal{O}(1)$$

[Carta, JM, Westphal '19]
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$$r^* \rightarrow \# e^{-\frac{2\pi}{|Q_{D3}^7|} \text{Vol}(\Sigma_4)} \Theta(1)$$

[Carta, JM, Westphal '19]
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\Rightarrow in KKLT, scale of D7-singularity is KK-scale

II

Resolution of singularities

Analogy with F-theory:

- e^{-4A} near wrapped D7s:
- $\frac{1}{g_s}$ near O7-plane;

$$e^{-4A} \approx \left| \frac{\beta_{D3}^{\Sigma_4}}{2\pi} \right| \log(r/r^*) < 0$$

at $r < r^*$

$$\frac{1}{g_s} \approx \frac{4}{2\pi} \log(r/r^*) < 0$$

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- F-theory:
O7 = bound state of
two (p_{17}) 7-branes
[Sen '96], [Banks, Douglas, Seiberg '96]

?

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Resolution of O7 via gauge instantons:

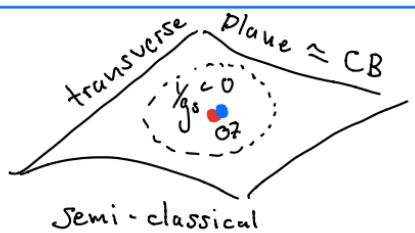
- $\frac{1}{g_s} \approx \frac{4\pi}{g_{YM}^2}$ on 4d D3 world volume
- D3 probing O7: \rightarrow Cartan $U(1) \subset Sp(2) \cong SU(2)$ YM

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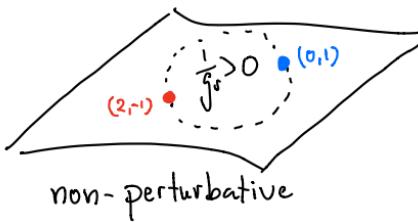
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\rightarrow Coulomb branch of YM \simeq D3 position in \mathbb{C} -plane transverse to O7

Seiberg-Witten:
solution



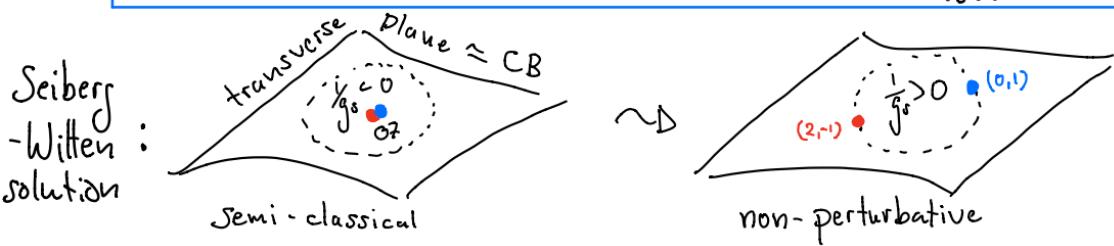
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splitting $O7 \rightarrow (2,-1) + (0,1)$ $\leftarrow \Rightarrow$ semi-classical singularity
via Seiberg-Witten sol.

$\frac{1}{g_s} \approx \frac{4\pi}{g_{YM}^2}$ on 4d D3 world volume

[Banks, Douglas, Seiberg '96]

The analogy:

O7 + D3
in 10d

?

gauge theory:

$N=2 \text{ SU}(2)$

coupling:

$$\bar{c} = c_0 + i \frac{1}{g_s}$$

singularity:

$$\frac{1}{g_s} < 0 \text{ near O7}$$

O7 = duality defect
in 10d

duality group:

$$SL(2, \mathbb{Z})_{\mathbb{Z}}$$

BPS dyons:

stretched $(p_{1,2})$ -strings

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$$\bar{c}' := \int_{K3} C_4 + i \int_{K3} f_{\text{d7}} \bar{g}_{\text{d7}} e^{-4A} - \bar{c}$$

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D7 on K3 = duality defect
in 6d $(2,0)$

$$O(5, 21; \mathbb{Z}) \supseteq SL(2, \mathbb{Z})_{\bar{c}'}$$

stretched $(p'_1 q'_1)$ -strings

Resolution of wrapped D7s:

- Singularities in field theory \leftrightarrow mutually local defects coincide
- Singularities in classical YM-theory

\leftrightarrow coinciding roots of char. pol.:

$$W(x) = \det(x\mathbb{1} - \underline{\Phi}) = \prod_{i=1}^N (x - e_i)$$

adjoint scalar

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W-boson mass: $m_{ij} \propto |e_i - e_j|$

$\Rightarrow e_i = \text{pos. of } i\text{-th D7 in } \mathbb{C}$

W-bosons = stretched F-string

Resolution of wrapped D7s: (continued)

- Singularities in quantum YM-theory [Seiberg, Witten '94], [Argyres, Faraggi '94], [Klemm, Lerche, Yankielowicz, Theisen '95], ...

\leftrightarrow coinciding roots of $\omega(x)^2 - \Lambda^{2N} \equiv \omega_+(x)\omega_-(x)$

with $\omega_{\pm}(x) := \omega(x) \mp \Lambda^N \equiv \prod_{i=1}^N (x - e_i^{\pm})$

i.e. singularities when $e_i^+ = e_j^+$ or $e_i^- = e_j^-$, if j

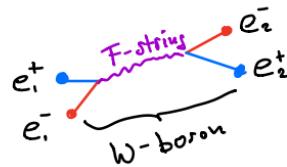
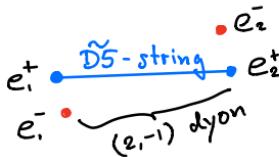
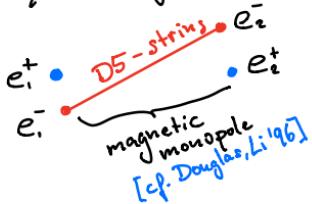
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- Our proposal:

D7 at $e_i \equiv$ bound state of mutually non-local
 (p^i, q^i) branes at $\{e_i^+, e_i^-\}$

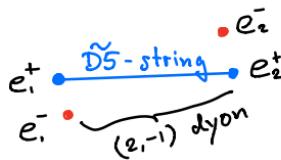
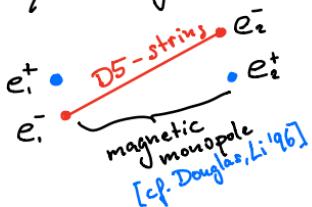
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- dyons for $SU(2)$:



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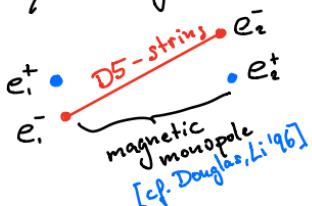
D7-monodromy: $\bar{c}_{D7} \rightarrow \frac{a \bar{c}_{D7} + b}{c \bar{c}_{D7} + d}$, $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})'$

$$M_{D7} = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix}$$

reproduces
field theory
monodromies

Resolution of wrapped D7s: (continued)

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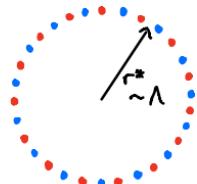
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- origin of CB:

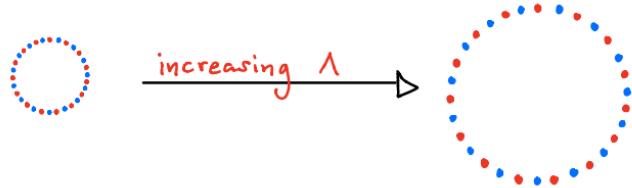
bulk interpretation
of $U(V_R \rightarrow \mathbb{Z}_{2N})$
by instantons



"radial running of e^{-4A}
halted by non-perturbative
splitting of D7-branes"

Comments:

- as K3 - volume modulus driven to strong coupling

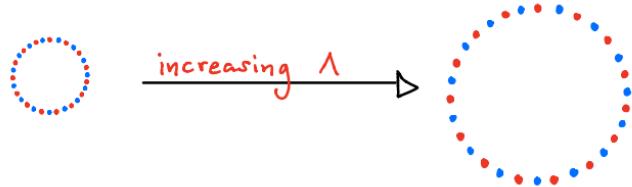


non-perturbative
defects spread
out in transverse plane.

\Rightarrow semi-classical vanishing locus $e^{-4A} = 0$
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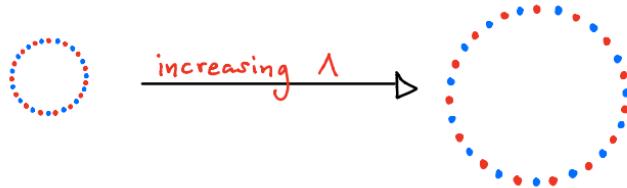
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- we show: "interior region" has $\mathcal{O}(1)$ local K3 volume.

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- we show: "interior region" has $O(1)$ local K3 volume.

- aside: defects drawn in same color are mutually local
 \rightarrow can be stacked on top of each other

comparison
 \Rightarrow
with field theory

n mutually local defects

host (A_1, A_{n-1}) Argyres-Douglas

J_i SCFTs [Argyres, Douglas '95]

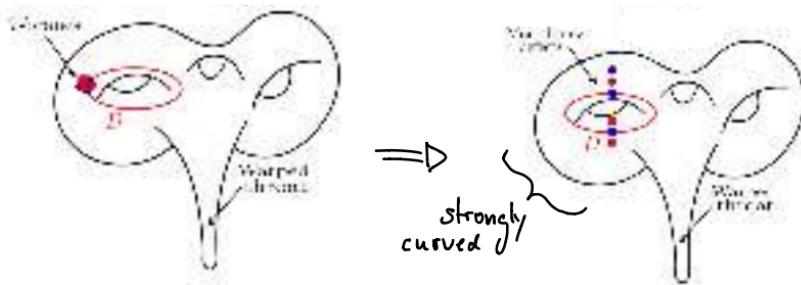
III

Instanton expansion in KKLT

KKLT dS vacua with strongly curved bulk:

- have learnt: Near KKLT minimum, spreading of non-perturbative defects is macroscopic

→ strongly curved 'inside region' occupies $O(1)$ -fraction of CY



D = divisor
hosting leading
ED3 instanton

Q: Can this threaten the KKLT vacua?

KKLT dS vacua with strongly curved bulk:

- size of strongly curved region depends (exponentially!) on Kähler moduli;
 - affects 4d EFT via non-perturbative corrections $\mathcal{O}(e^{-\frac{2\pi}{|Q|} T})$
(+ perhaps perturbative)

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- superpotential: (A) $W(T) = W_0 + e^{-\frac{2\pi}{c} T} + \dots$ with $c \sim |Q|$?
 - yes, if $c = \text{dual Coxeter } \# \text{ of confining sector}$
 $\Rightarrow e^{-\frac{2\pi}{|Q|} T} \sim W_0 \ll 1 \Rightarrow \text{no problem.}$

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 - yes, if $c = \text{dual Coxeter } \# \text{ of confining sector}$
 $\Rightarrow e^{-\frac{2\pi}{|Q|} T} \sim W_0 \ll 1 \Rightarrow \text{no problem.}$
- (B) $W(T) = W_0 + e^{-\frac{2\pi}{c} T} + \dots$ with $c \ll |Q|$
 $\Rightarrow \text{no dangerous corr. to superpotential.}$

KKLT dS vacua with strongly curved bulk:

- Kähler potential? We don't know...

[thanks to D. Jangheus
for discussion]

But KKLT AdS vacua are highly robust against Kähler potential modifications:

$$\text{CS} + \bar{c}: 0 = D_i w = \partial_i w + (\partial_i K) w_0 \approx \partial_i w$$

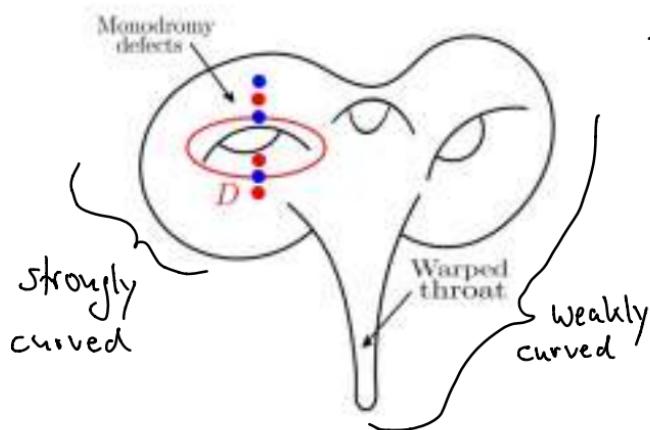
$$\text{Kähler mod.: } 0 = D_T w \Leftrightarrow e^{-\frac{2\pi i}{c}} = \frac{\partial_T K}{\partial_T K - \frac{2\pi i}{c}} w_0 \ll 1$$

unless corr. to K are fine-tuned:

$$\left. \partial_T K \right|_{T=T_0 \approx \mathcal{O}(1)} - \frac{2\pi i}{c} \sim |w_0| \ll 1$$

KKLT dS vacua with strongly curved bulk:

- the uplift: Unless divisor D 'hangs' all the way to bottom of throat, throat remains in SUGRA regime:



⇒ expect no significant modifications to uplift potential

Conclusions:

- reviewed issue of large singular region in KKLT
[Carta, JH, Westphal '19], [Gao, Hebecker, Junghans '20]

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Thanks!