The Calabi-Yau Landscape: Beyond the Lampposts

Mehmet Demirtas
Cornell University

String Pheno Series, 2020

Based on works with (various subsets of):
Manki Kim, Cody Long, Liam McAllister, Jakob Moritz, Mike Stillman, Andres Rios Tascon

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- de-Sitter solutions?
- Super-Planckian field ranges?
- Quintessence?
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Can answer for: Weakly coupled compactifications of superstring theories.

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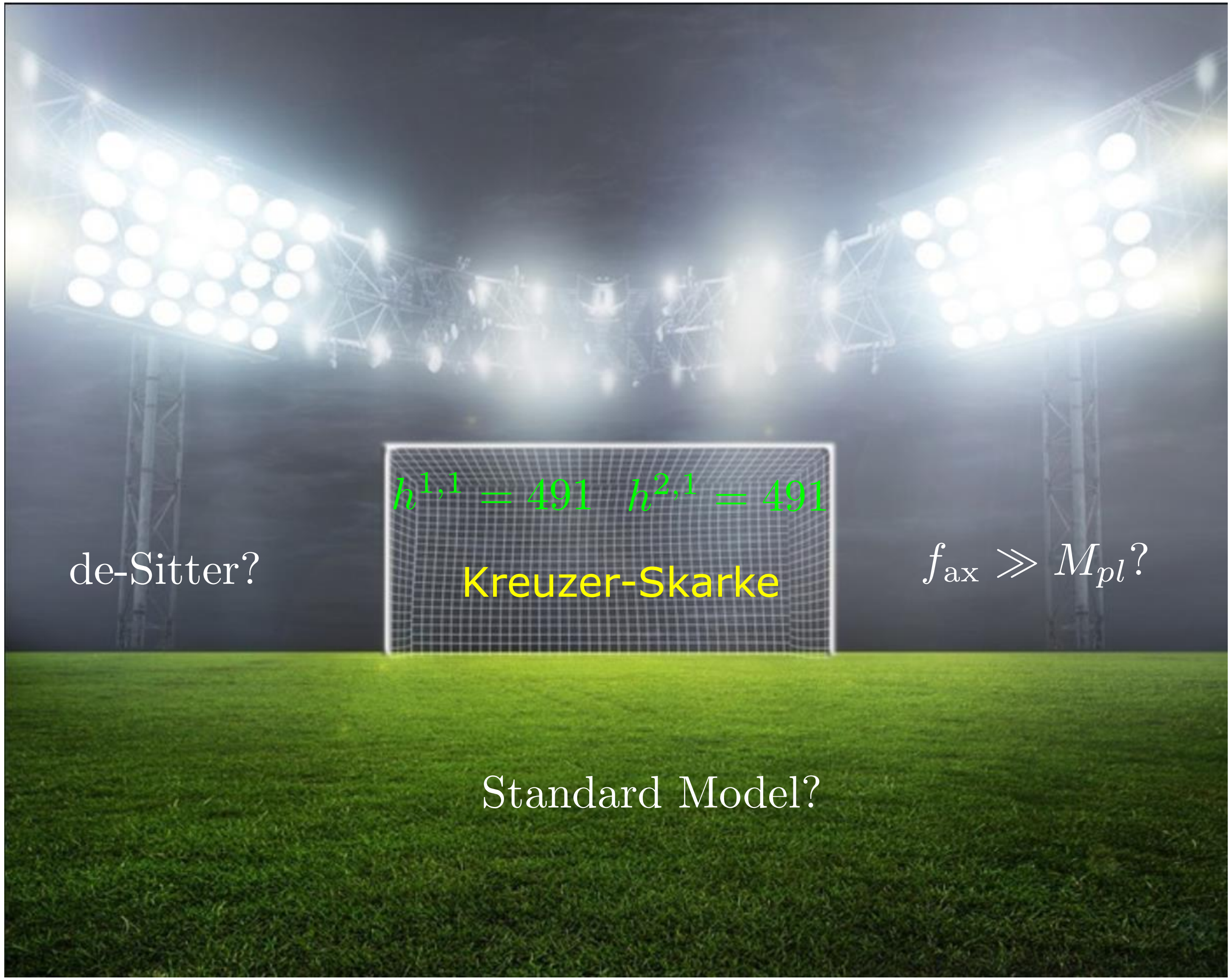
However: this is an *exponentially small* fraction of the String Landscape.

- Number of (known) topologically inequivalent CY manifolds increases exponentially with $h^{1,1}$.
  [MD, McAllister, Rios Tascon, hep-th/2008.01730]
- Number of flux vacua in type IIB (F-Theory) compactifications increases exponentially with $h^{2,1}$ ($h^{3,1}$).
  [Denef, Douglas, hep-th/0404116]
  [Denef, Douglas, hep-th/0411183]
  [Taylor, Wang, hep-th/1511.03209]

We can now construct CY threefolds with largest known Hodge numbers and compute relevant topological data.



Outline

I. CY_3's from Triangulations

II. Holomorphic Cycles
   Application: Ultralight Axions

III. 3-cycles
   Application: Towards KKLT

A Quick Review

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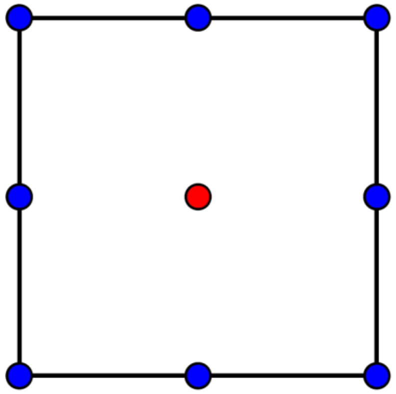
1. Take a 4D reflexive lattice polytope

   Reflexive: the only interior point of the polytope (and its dual) is the origin.



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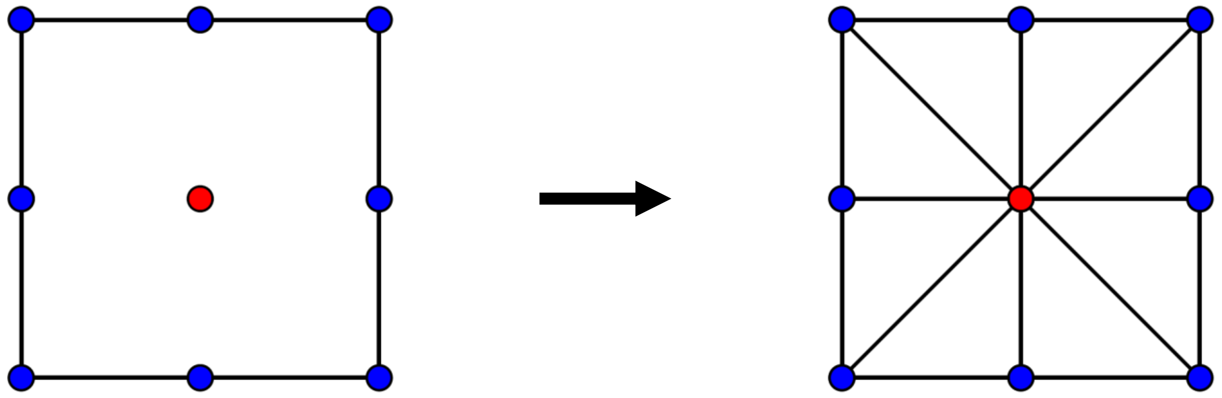

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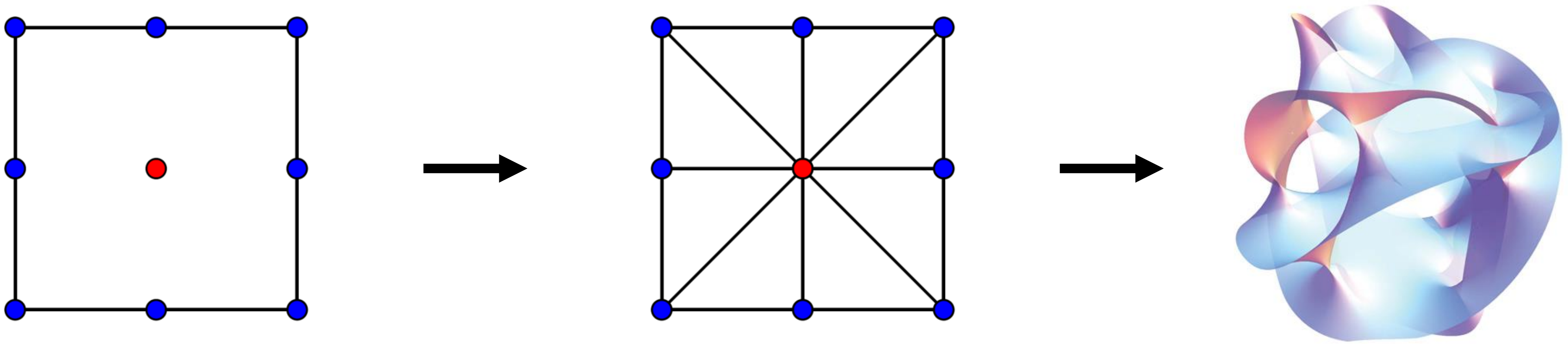
This triangulation defines a fan, which describes a toric variety V that has a CY hypersurface X.



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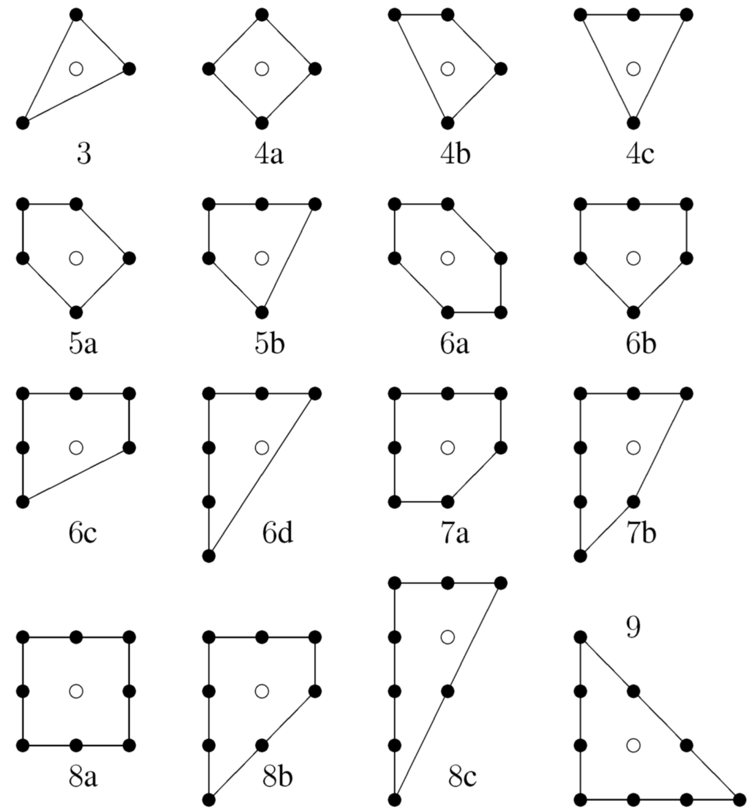

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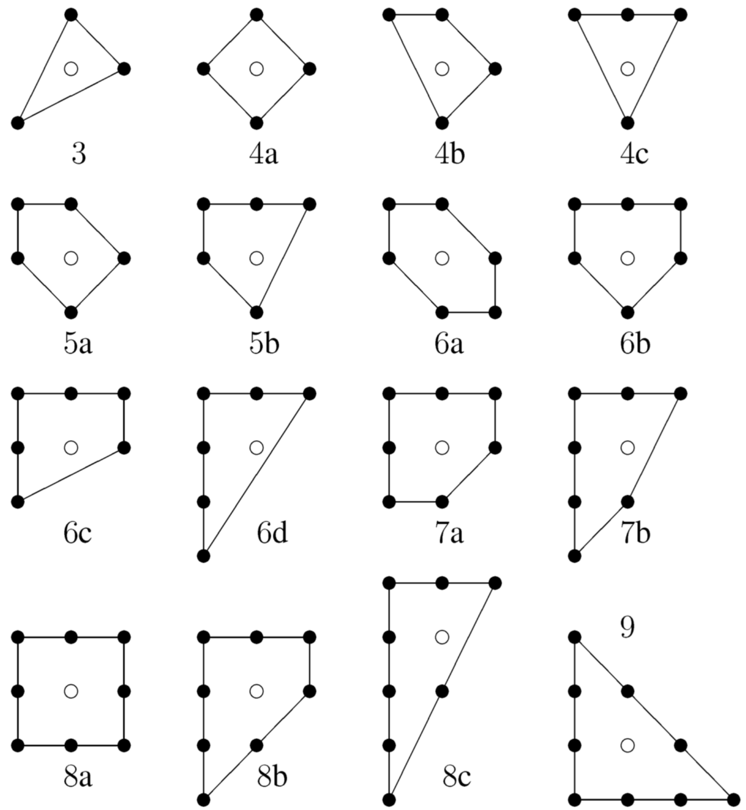

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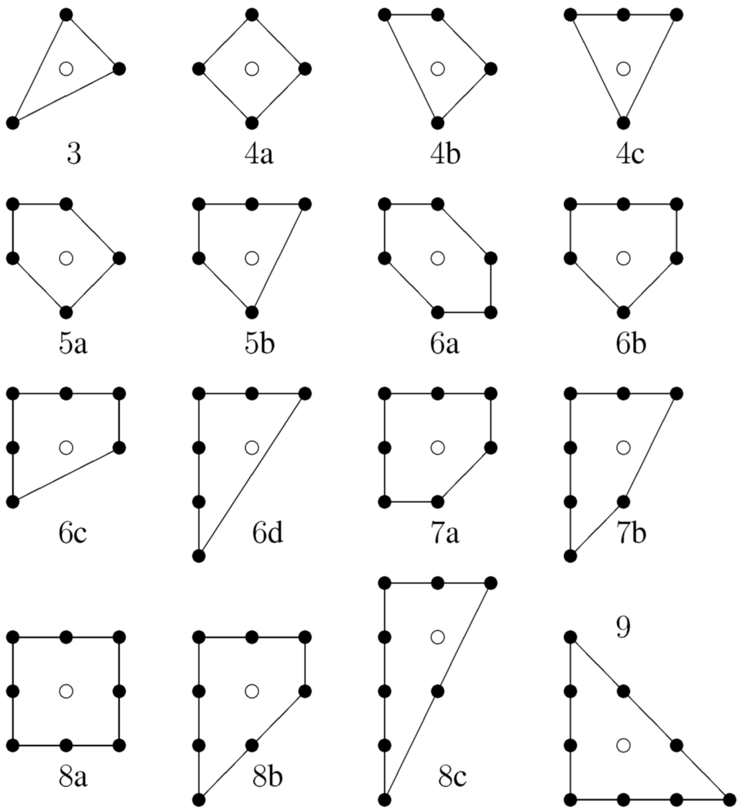

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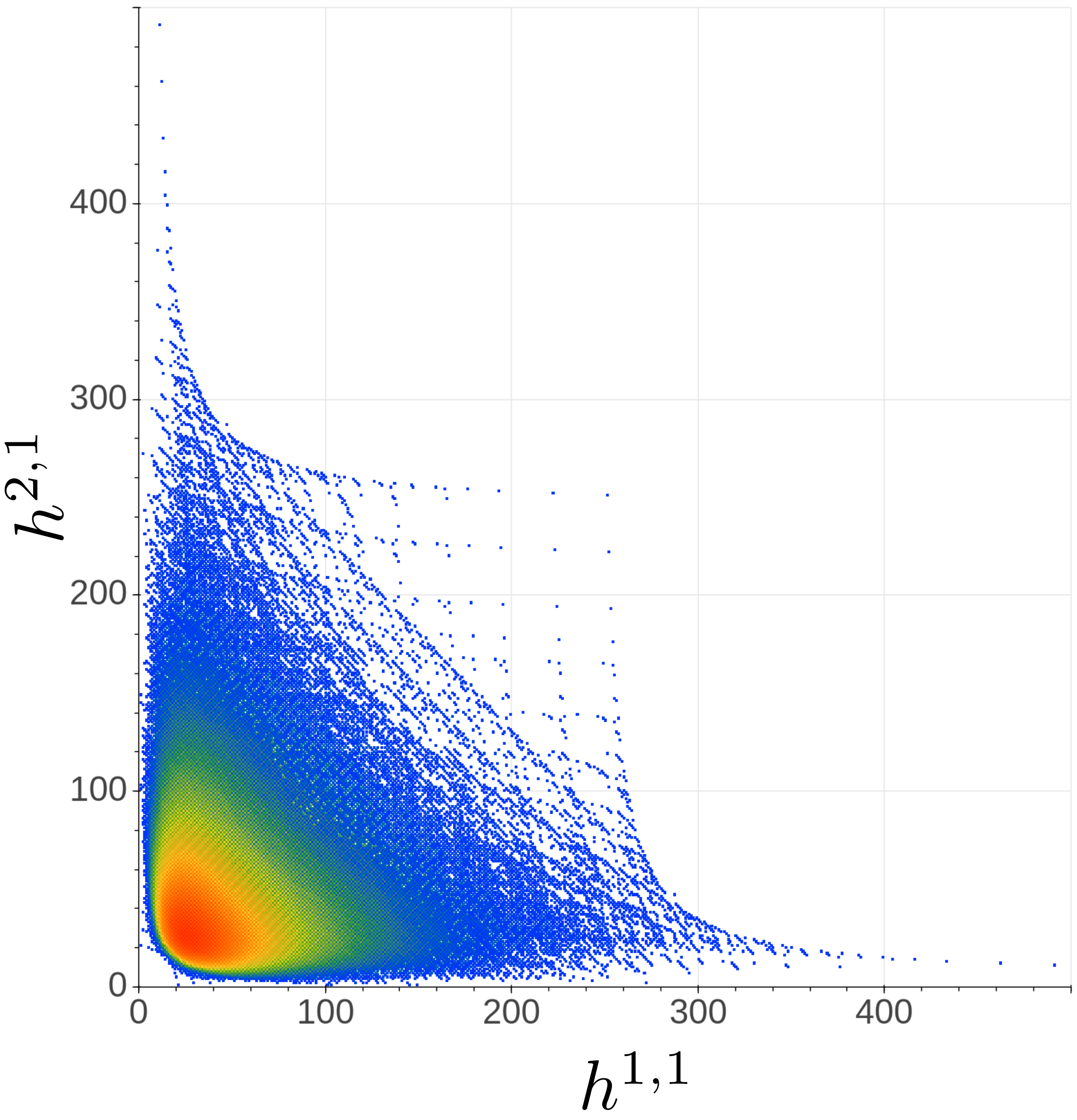

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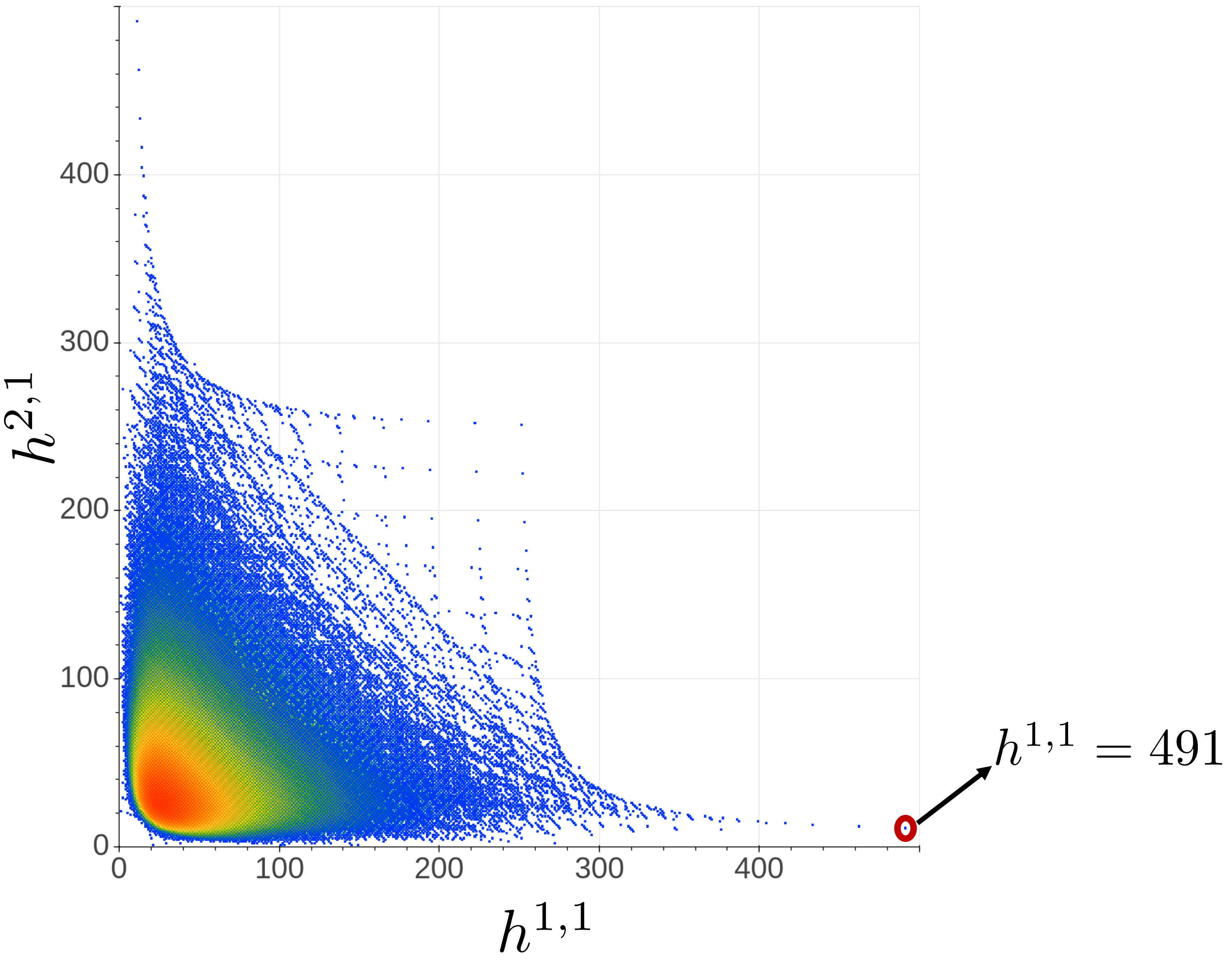

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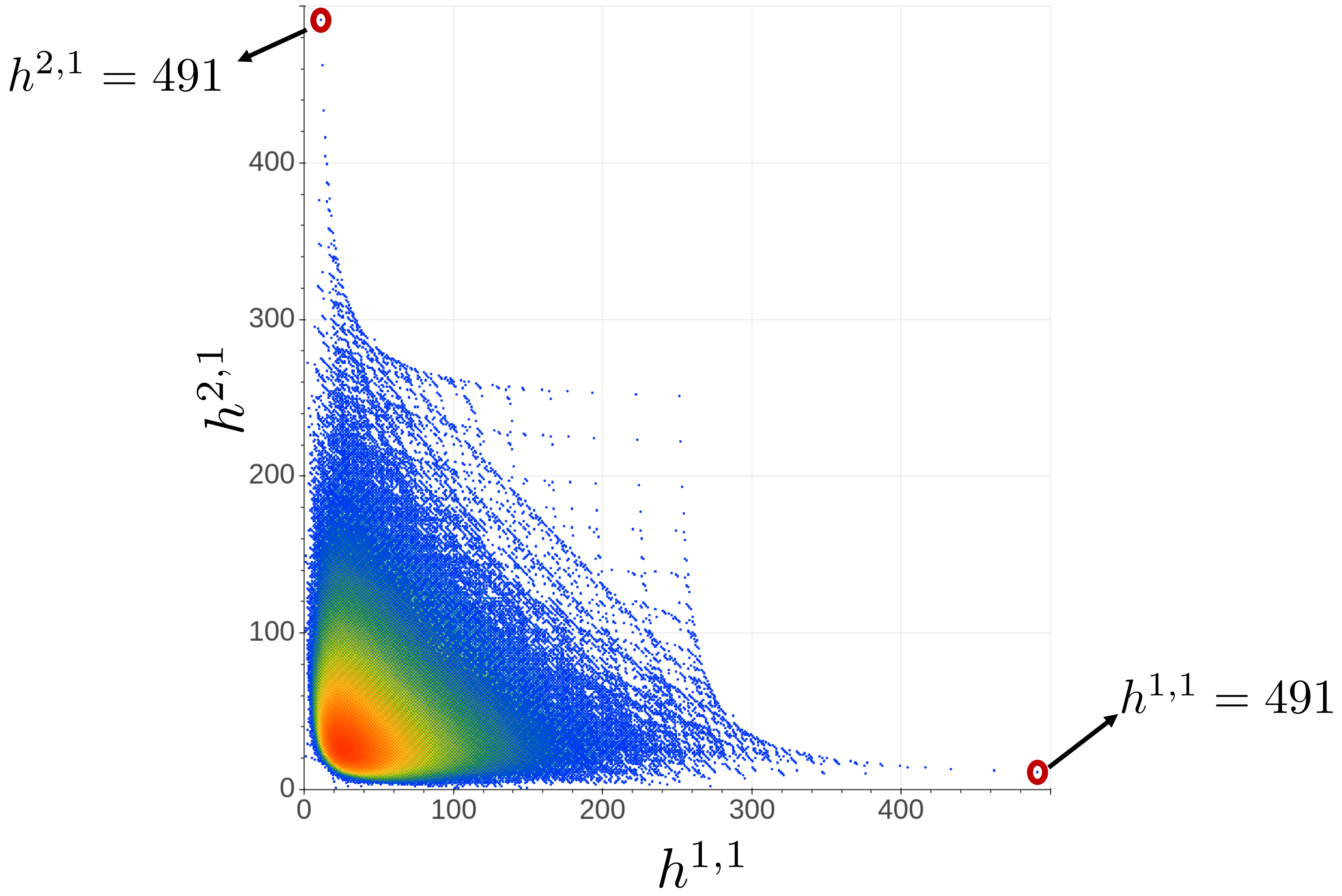

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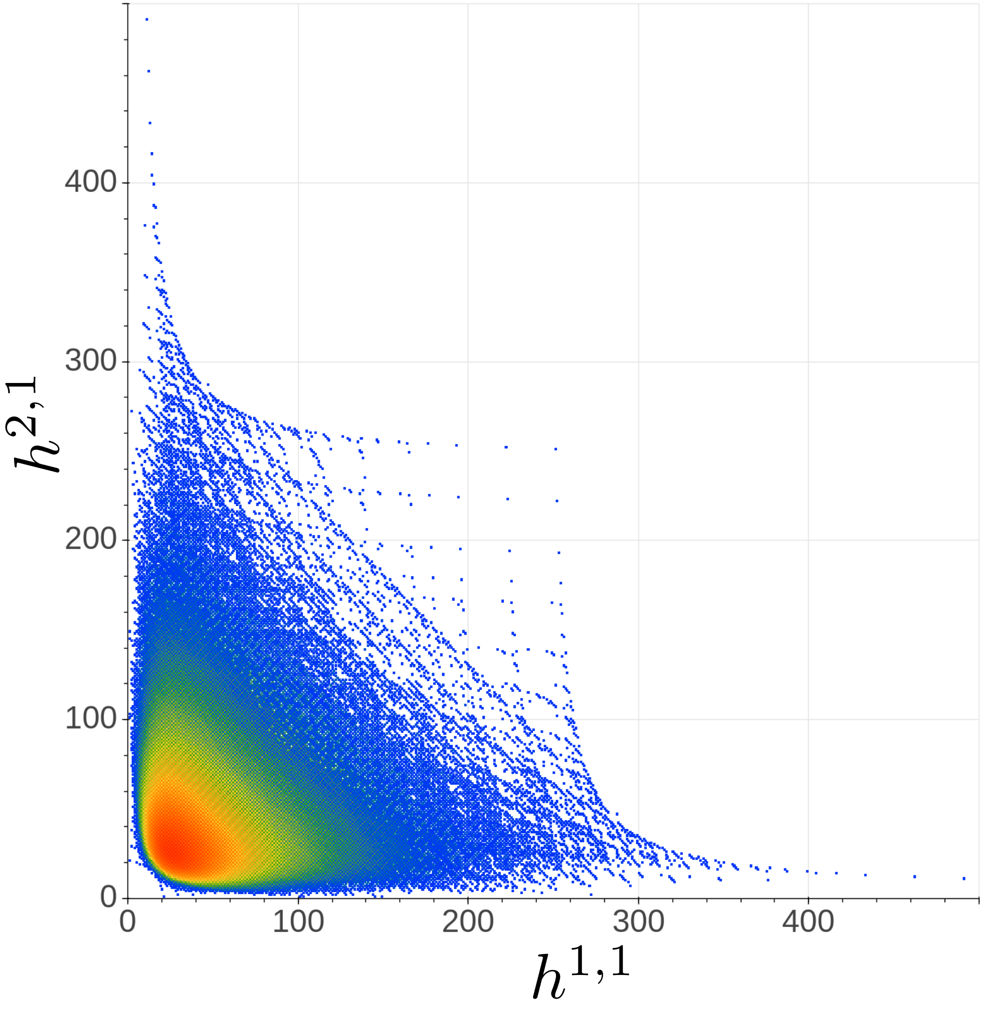

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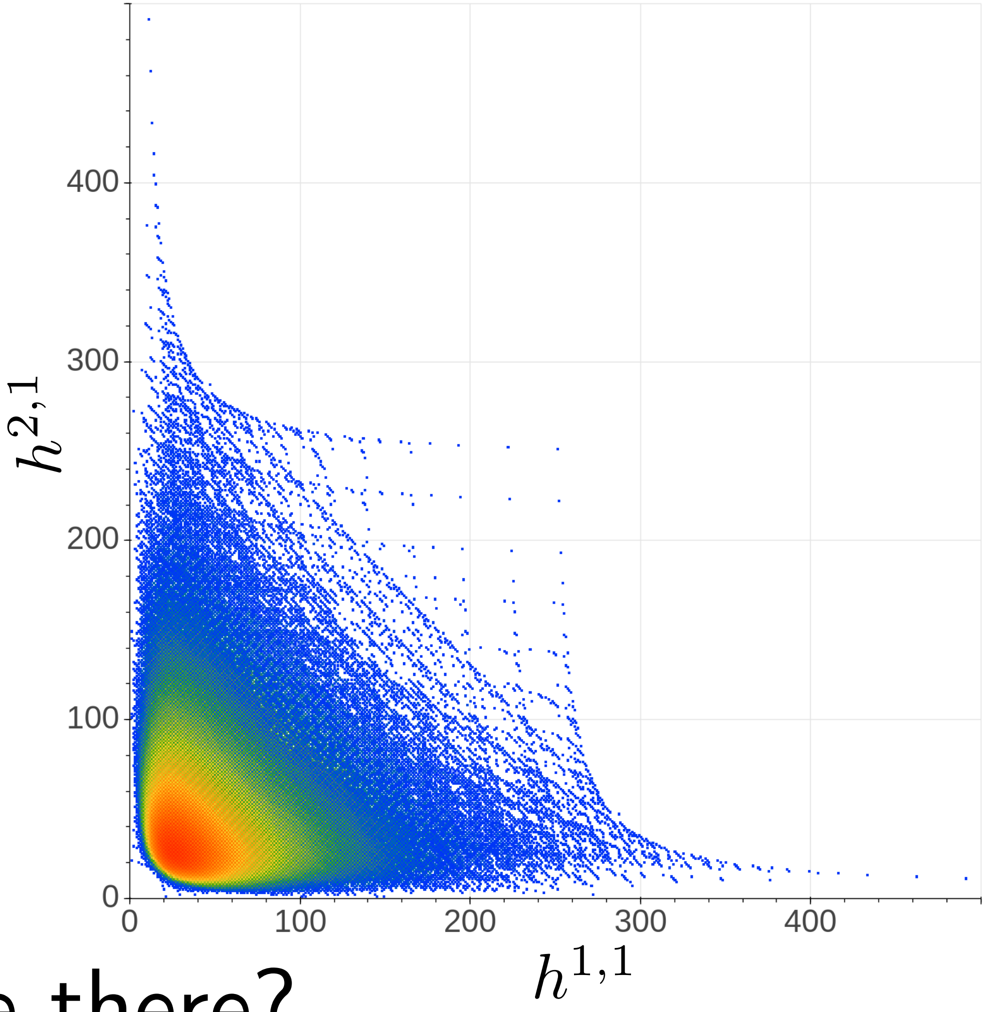

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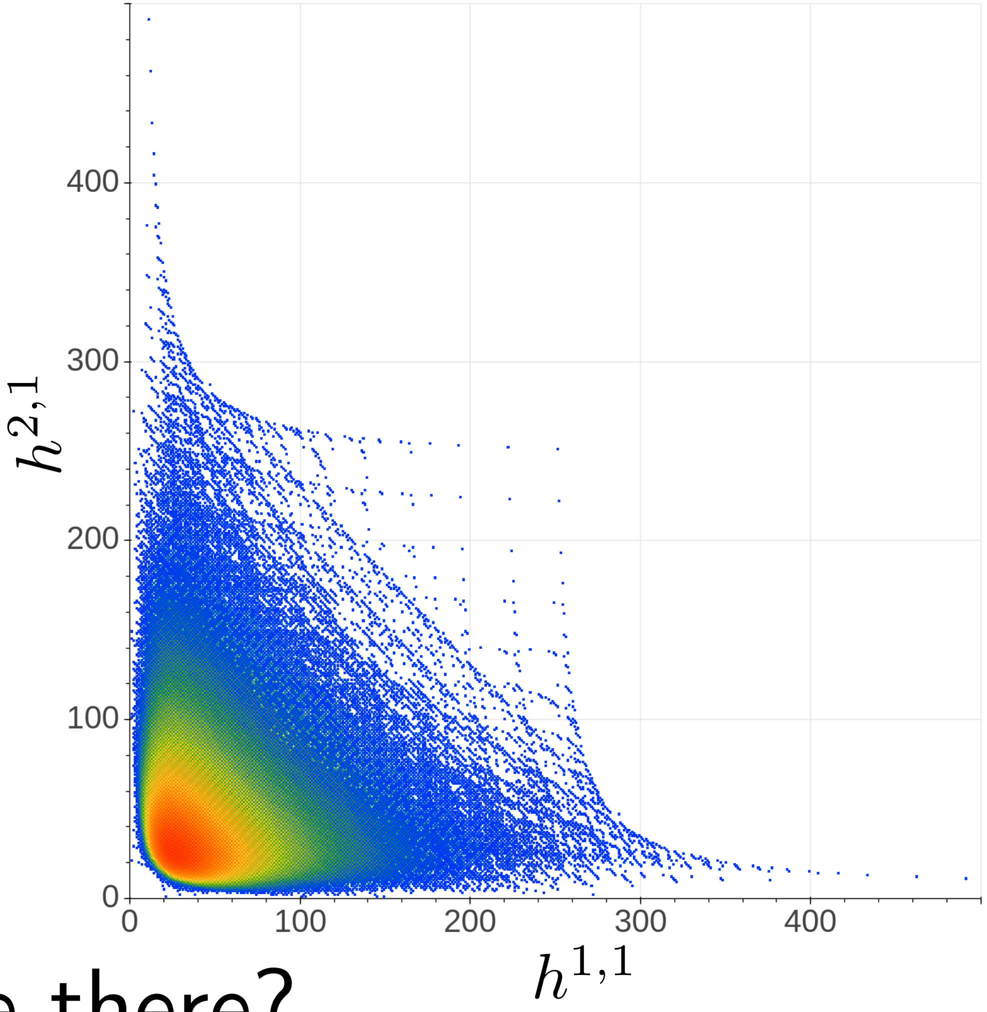

How many CY_3 hypersurfaces are there?

- Not known.
- We recently proved an upper bound of 10^{428}. [MD, McAllister, Rios Tascon, hep-th/2008.01730]

Holomorphic Cycles

Notation:

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- No general algorithm for computing $\mathcal{M}(X)$ in hypersurfaces.
  - Can compute $\mathcal{M}(X)$ on a case-by-case basis.
  - Can compute $\mathcal{M}(V) \supset \mathcal{M}(X)$.

Holomorphic Cycles

- Volumes of 2-cycles C, 4-cycles D, and X itself

$$\mathrm{Vol}(C) = \int_C J \qquad \mathrm{Vol}(D) = \frac{1}{2}\int_D J \wedge J \qquad \mathrm{Vol}(X) = \frac{1}{6}\int_X J \wedge J \wedge J$$

are determined by the Kähler form J and the intersection numbers:

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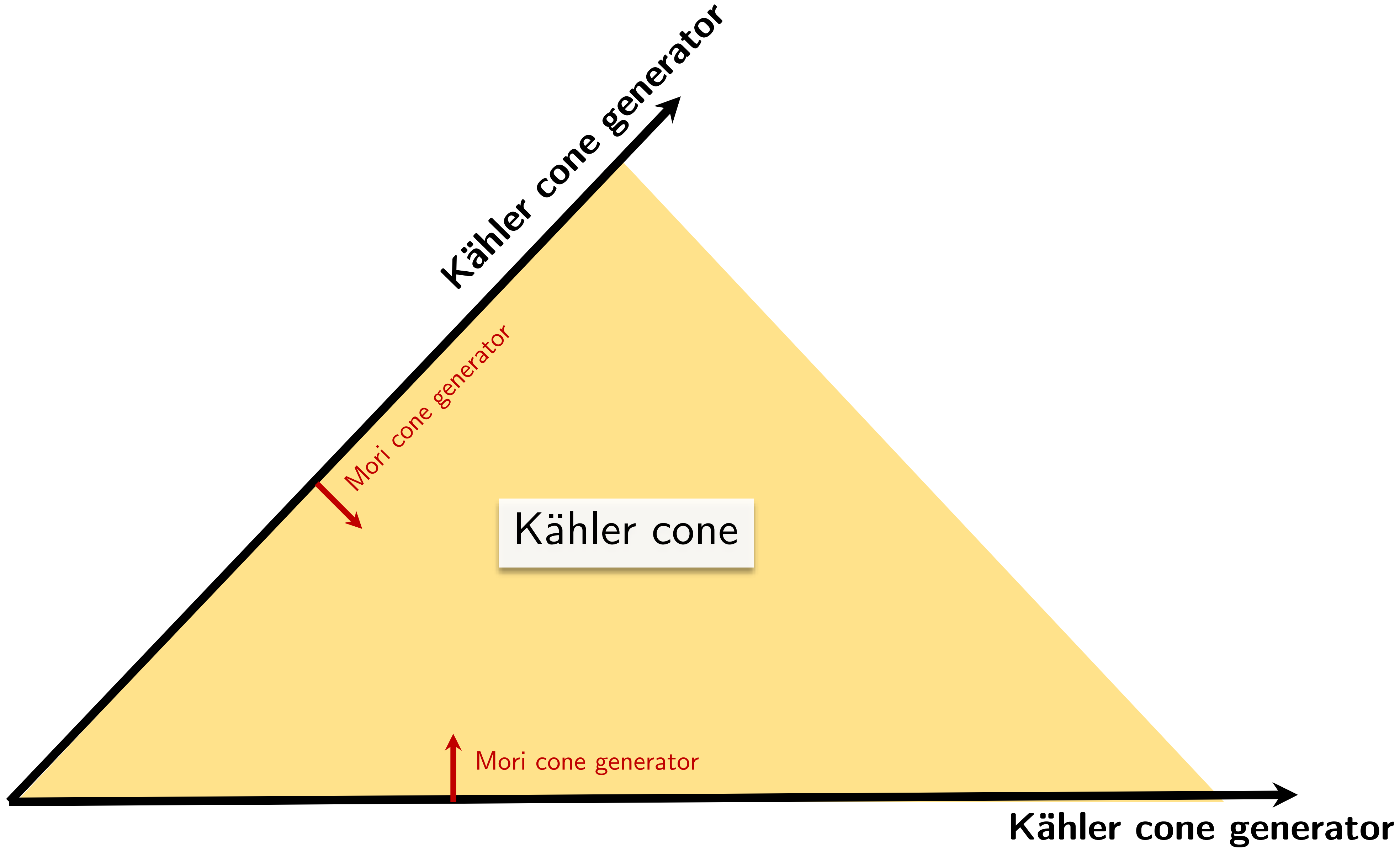
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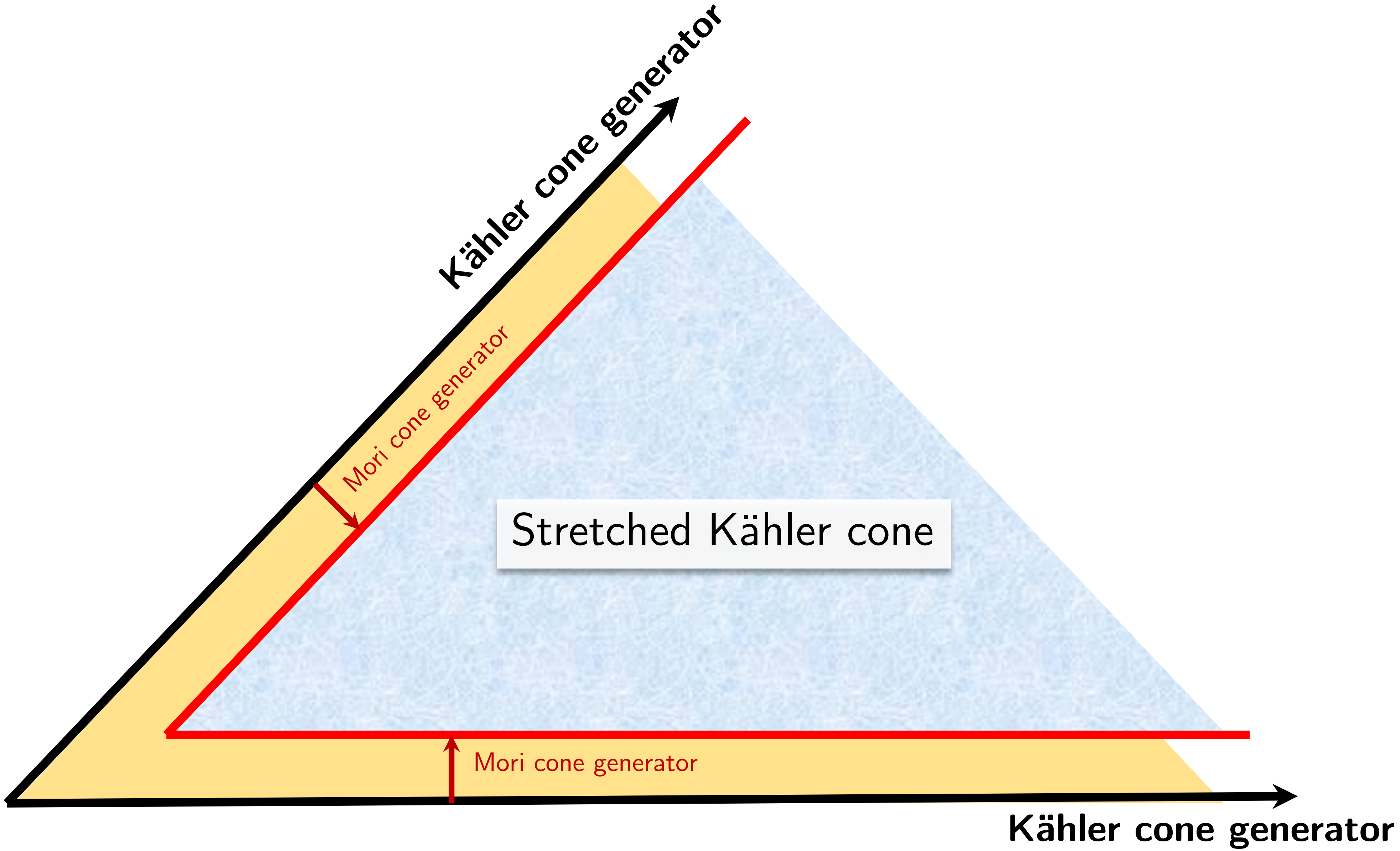
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estimate for the convergence of the worldsheet instanton expansion and the control of the α' expansion. [Candelas, De La Ossa, Green, Parkes, '90]

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- $h^{1,1} = \mathcal{O}(100)$: Only recently.

[MD, Long, McAllister, Stillman, hep-th/1808.01282]
[Halverson, Long, Nelson, Salinas, hep-th/1909.05257]
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2018	491	30s
2019	491	3s

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- The dual polytope
- Faces, dual faces of the polytope
- GLSM charge matrix
- Stanley-Reisner ideal
- Second Chern class
- Mori cone of the ambient variety
- Stretched Kähler cone
- Volumes of cycles
- ... many more!

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Aside: Some of these quantities can be predicted using Machine Learning.

- Achieved using a deep neural net. High precision even at $h^{1,1} = 491$.
- $50\mu s$ per prediction. A *further* speed-up of a factor of $\sim 10,000$. [MD, McAllister, Rios Tascon, hep-th/2008.01730]

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Construct Ensembles of Geometries → Detect Patterns → Study Consequences

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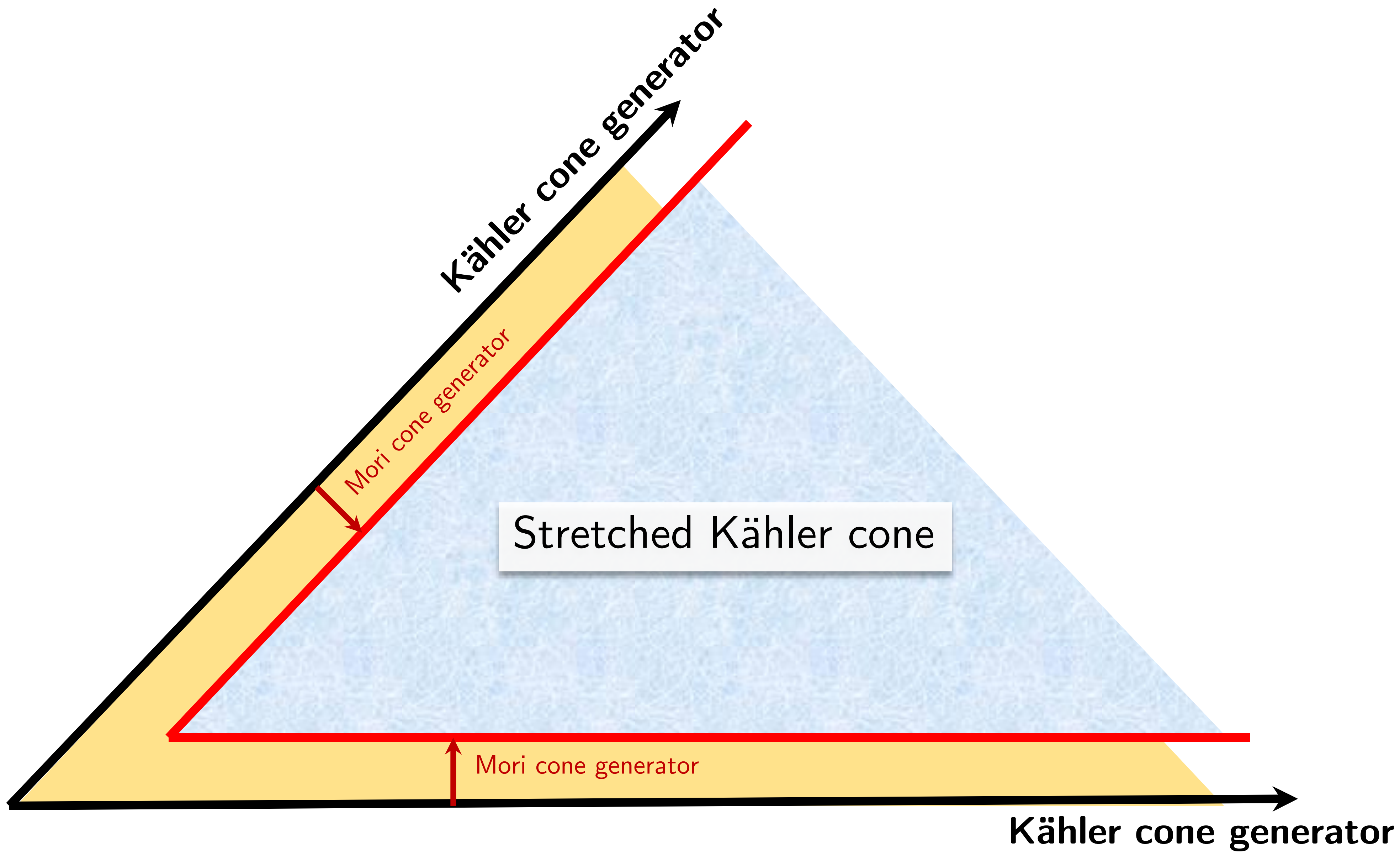
Pattern: At large $h^{1,1}$, **Kähler cones** are **narrow.** [MD, Long, McAllister, Stillman, hep-th/1808.01282]

- Stretched Kähler cone is far away from the origin.



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Mori cone generator
Stretched Kähler cone
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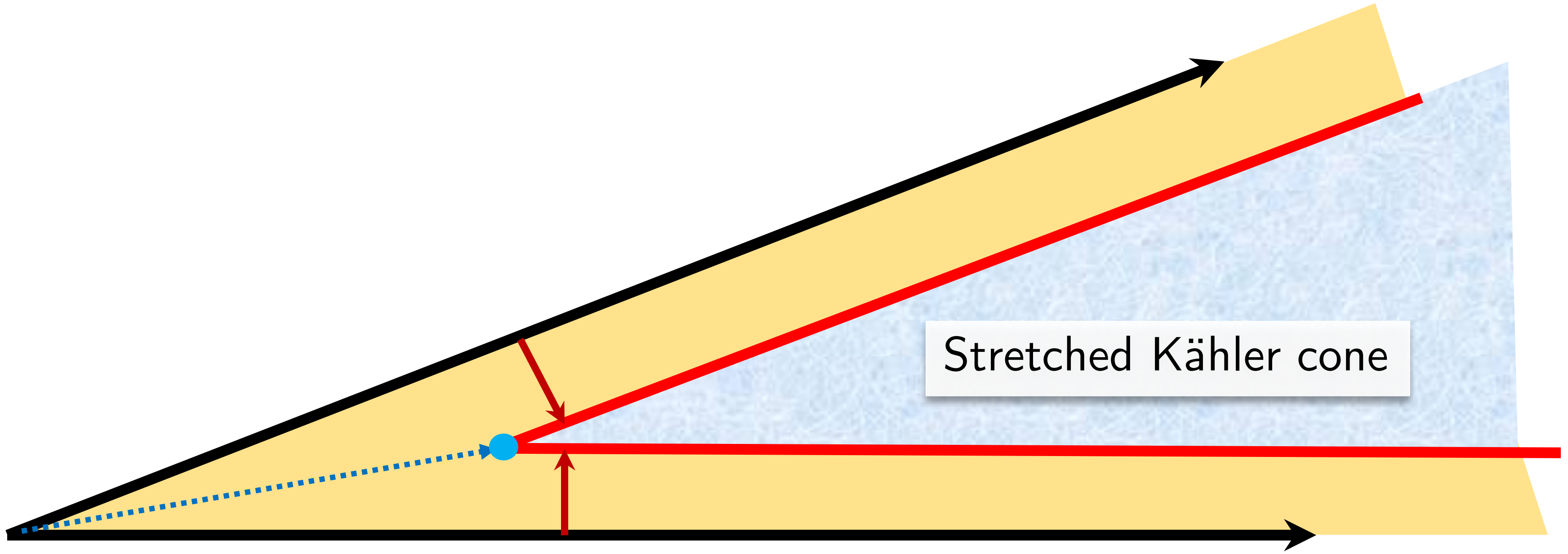
Kähler cone generator


Stretched Kähler cone

What is generic in the Calabi-Yau hypersurface landscape?

Method:

Construct Ensembles of Geometries → Detect Patterns → Study Consequences

Pattern: At large $h^{1,1}$, **Kähler cones** are **narrow.** [MD, Long, McAllister, Stillman, hep-th/1808.01282]

- Stretched Kähler cone is far away from the origin.
- **Volumes** of effective 2-cycles, 4-cycles and the CY itself are **large.**

Consequences:

Consider type IIB compactified on an O3/O7 orientifold of X.

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C_4 axions: $\theta_i := \int_{D_i} C_4 \,,$

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- Implications for fitting warped throats in compactifications

[Carta, Moritz, Westphal, hep-th/1902.01412]

Periods of 3-cycles

To study flux compactifications, we need to compute the periods of 3-cycles.

- Pick a basis of $H_3(X, \mathbb{Z})$, $\{A^i, B_j\}$ such that

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- Can be written in terms of a prepotential $\mathcal{F}$:

$$\vec{\Pi}(\vec{u}) = \begin{pmatrix} 2\mathcal{F} - u^a \partial_a \mathcal{F} \\ \partial_a \mathcal{F} \\ 1 \\ u^a \end{pmatrix}$$

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- Around a large complex structure point,

$$\mathcal{F}(\vec{u}) = \mathcal{F}_{\text{poly}}(\vec{u}) + \mathcal{F}_{\text{exp}}(\vec{u})$$

$$\mathcal{F}_{\text{poly}}(\vec{u}) = -\frac{1}{3!}\tilde{\kappa}_{abc}u^a u^b u^c + \frac{1}{2}a_{ab}u^a u^b + b_a u^b + \frac{\zeta(3)\chi(\tilde{X})}{2(2\pi i)^3}\,, \qquad \tilde{X}\text{: mirror of } X$$

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- Coefficients of $\mathcal{F}_{\text{poly}}(\vec{u})$ depend on the geometric data of holomorphic cycles in $\tilde{X}$.

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- Existing methods allow for computing n_C^0 when $h^{2,1} \lesssim 5$.
  - Also, hardly any results unless $\mathcal{M}(\tilde{X})$ is smooth and simplicial.

[Greene, Plesser, ‘90]
[Candelas, De La Ossa, Green, Parkes, ‘90]
[Batyrev, alg-geom/9310003]
[Hosono, Klemm, Theisen, Yau, hep-th/9308122]
[Hosono, Klemm, Theisen, Yau, hep-th/9406055]
... and more

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  - Also, hardly any results unless $\mathcal{M}(\tilde{X})$ is smooth and simplicial.
- **Now: can compute n_C^0 *systematically* for $h^{2,1} = \mathcal{O}(10)$.** [MD, Kim, McAllister, Moritz, Rios Tascon, work in progress]
  - No requirements on $\mathcal{M}(\tilde{X})$,
  - Up to $h^{2,1} = \mathcal{O}(100)$ for some curves!

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Ultimate Goal: An **explicit** construction of a dS vacuum.

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   [Ashok, Douglas, hep-th/0307049], [Denef, Douglas, hep-th/0404116], [Denef, Douglas, hep-th/0411183]

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THANK YOU!