ALGORITHMICALLY SOLVING THE TADPOLE PROBLEM

[arXiv:2010.10519, arXiv:2103.03250]

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Seminar Series on String Phenomenology

March 23, 2021

MOTIVATION

- Flux compactifications:
  Huge landscape of vacua: $10^{\text{(big number)}}$
  [Ashok, Douglas '03], [Denef, Douglas '04], [Taylor, Wang '15]
- CYs with large Hodge numbers: many choices for fluxes
- Constraints on fluxes:
  - Integer quantization
  - Tadpole cancelation
- Which properties have the consistent vacua satisfying these constraints?

EXPLORING THE LANDSCAPE

- Systematically
  - Extensive/random scan over flux configurations
  - only possible for very special examples
    (for example [Betzler, Plauschinn '19] for toroidal orbifolds)
- Statistically [Ashok, Douglas '03], [Denef, Douglas '04], [Taylor, Wang '15]
- Analytically
  - Explore symmetries or mathematical structure
  - Related: Swampland program
- Algorithmically
  - Use modern Big Data / AI / Machine Learning algorithms

AI APPROACH TO THE LANDSCAPE

Machine Learning / AI:

a lot of recent activity for String Theory related problems

(see e.g. [Ruehle '20])

Here: Use algorithms inspired by

biological evolution / genetics

to search for / generate string vacua with specific properties.

(see for example also [Blåbäck, Danielson, Dibitetto '13; Damian et al. '13; Abel, Rizos '14; Ruehle '17; Cole, Schachner, Shiu '19; AbdusSalam, et al '20; Cabo Bizet et al. '20])

Specifically: Flux configurations with full moduli stabilization.

FLUX COMPACTIFICATION

- M-theory on CY_4 / F-theory:

$$\mathscr{L}_{kin} \sim \int G_4 \wedge \star \overline{G}_4 \longrightarrow$$ „$\star$" depends on CY-metric $\longrightarrow$ Potential for moduli

- Superpotentials [Gukov, Vafa, Witten '99; Haack, Louis '01]:

$$W \sim \int_{CY} G_4 \wedge \Omega \qquad \hat{W} \sim \int_{CY} G_4 \wedge J \wedge J$$

- F-term equations: $D_i W = D_a \hat{W} = 0 \quad (i = 1, \ldots, h^{2,1};\ a = 1, \ldots, h^{1,1})$

➡ complex structure sector: $h^{2,1}$ equations for $h^{2,1}$ moduli

- But: requires knowledge of the period integrals $\int \chi_i = \int D_i \Omega$

THE TADPOLE PROBLEM

[Bena, Blåbäck, Graña, SL '20]

- Tadpole cancellation: (M-theory on CY_4 / F-theory)

$$\frac{1}{2}\int G_4 \wedge G_4 = \frac{\chi(CY_4)}{24}$$

- Scaling behavior at large $h^{3,1} \gg h^{1,1} \sim h^{2,1} \sim \mathcal{O}(1)$:

$$\frac{\chi(CY_4)}{24} \propto \frac{1}{4} h^{3,1}$$

- Similar linear scaling for flux induced charge?

$$\boxed{Q_{D3}^{\text{flux}} = \frac{1}{2}\int G_4 \wedge G_4 \gtrsim \alpha \times h^{3,1}}$$

WHAT IS α ?

- Examples from literature indicate $\alpha \approx 0.4$!

- Use Big Data / AI algorithms to systematically search for flux configurations which
  - ★ stabilize all moduli
  - ★ at a generic point in moduli
  - ★ with as small charge $Q_{\text{flux}} = \frac{1}{2}\int G_4 \wedge G_4$ as possible?

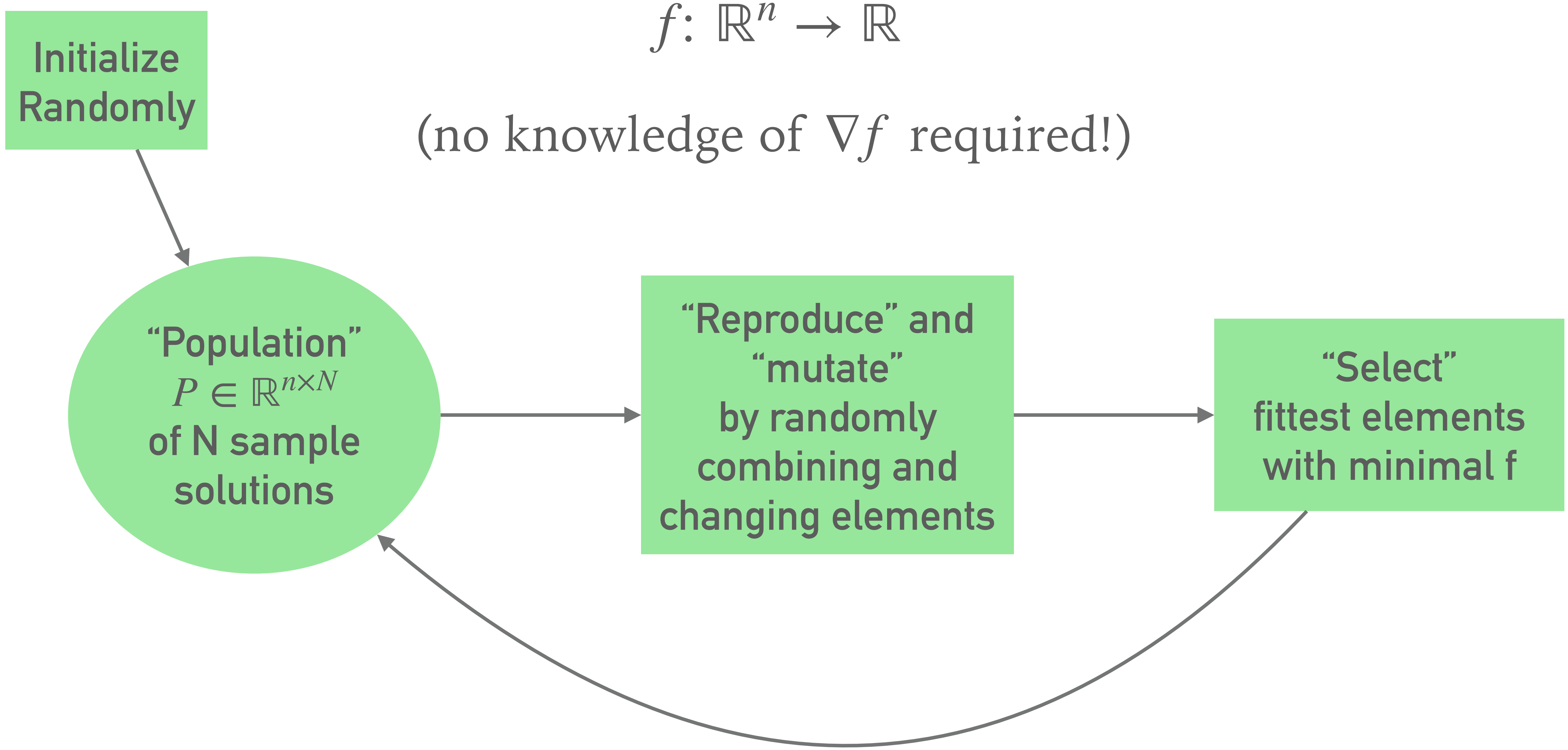
➜ Differential Evolution

DIFFERENTIAL EVOLUTION

- Global optimization algorithm inspired by biological evolution/genetics
- Goal: Find global minimum of *fitness function*

$$f\colon \mathbb{R}^n \to \mathbb{R}$$

(no knowledge of ∇f required!)



M-THEORY ON K3 X K3

- Challenge: design suitable fitness function!
  Requires knowledge of W and $\hat{W}$… still very difficult

- Well-known playground for flux compactification:
  [Dasgupta, Rajesh, Sethi '99; Aspinwall, Kallosh '05]

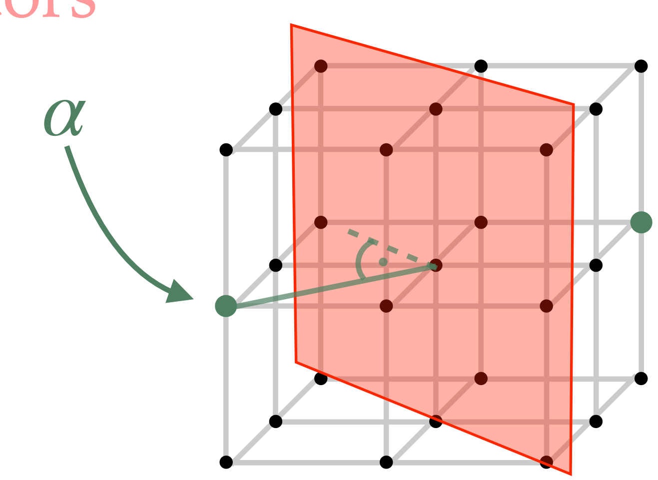
$$\mathrm{K3} \times \mathrm{K3}$$

- [Braun Hebecker, Ludeling, Valandro '08]:
  - Stabilize all moduli (Kähler + complex str.) by fluxes
  - No knowledge of period maps necessary!

A LATTICE PROBLEM

- Do not study K3xK3 directly…
  … instead solve a related lattice problem:

- Input data: even lattice Λ with inner product $d \in \Lambda^\star \otimes \Lambda^\star$
  (of indefinite signature)
- Search space: all matrices $G \in \Lambda \otimes \Lambda$ such that
  (I) GdG^Td and G^TdGd diagonalizable with non-negative eigenvalues
  (II) d has definite signature on all eigenspaces
  (III) no root $\alpha \in \Lambda$ orthogonal to positive norm eigenvectors
- Target: $$Q_{\min}(\Lambda) = \frac{1}{2}\min_G \mathrm{tr}(GdG^Td) = ?$$



RELATION TO K3 X K3

Relation to K3 x K3: [Braun, Hebecker, Ludeling, Valandro '08]

$$\Lambda = H^2(K3, \mathbb{Z}) = (-E_8) \oplus (-E_8) \oplus U \oplus U \oplus U$$

(I) $GdG^T d$ and $G^T dGd$ diagonalizable with non-negative eigenvalues
$\rightarrow$ Minkowski vacuum

(II) d has definite signature on all eigenspaces
$\rightarrow$ all moduli stabilized

(III) no root $\alpha \in \Lambda$ orthogonal to positive norm eigenvectors
$\rightarrow$ K3 is smooth

$$Q_{\text{flux}} = \frac{1}{2} \int G_4 \wedge G_4 = \frac{1}{2} \text{tr}(GdG^T d)$$

DESIGN OF THE FITNESS FUNCTION

Population consists of "flux" matrices $x \in \mathbb{R}^{D \times D}$

1. Round to the closest integer: $N = \mathrm{round}(x)$.
2. Assign "penalties" $p_k(N)$ whenever one of (I) - (III) is violated.
3. Compute $Q = \mathrm{tr}(NdN^T d)$.

Fitness function:

$$f(x) = \sum_k w^k p_k(N) + w^Q Q(N)$$

weights

(determined "experimentally")

DIFFERENTIAL EVOLUTION FOR K3 X K3

- Implementation of Differential Evolution:
  in **Julia** using **BlackBoxOptim.jl** [Feldt et al.] and **bbsearch.jl** [Blåbäck]
- Challenges:
  - HUGE search spaces!
  - Finding roots orthogonal to eigenvectors
    (lattice vectors of minimal length) is NP-hard!

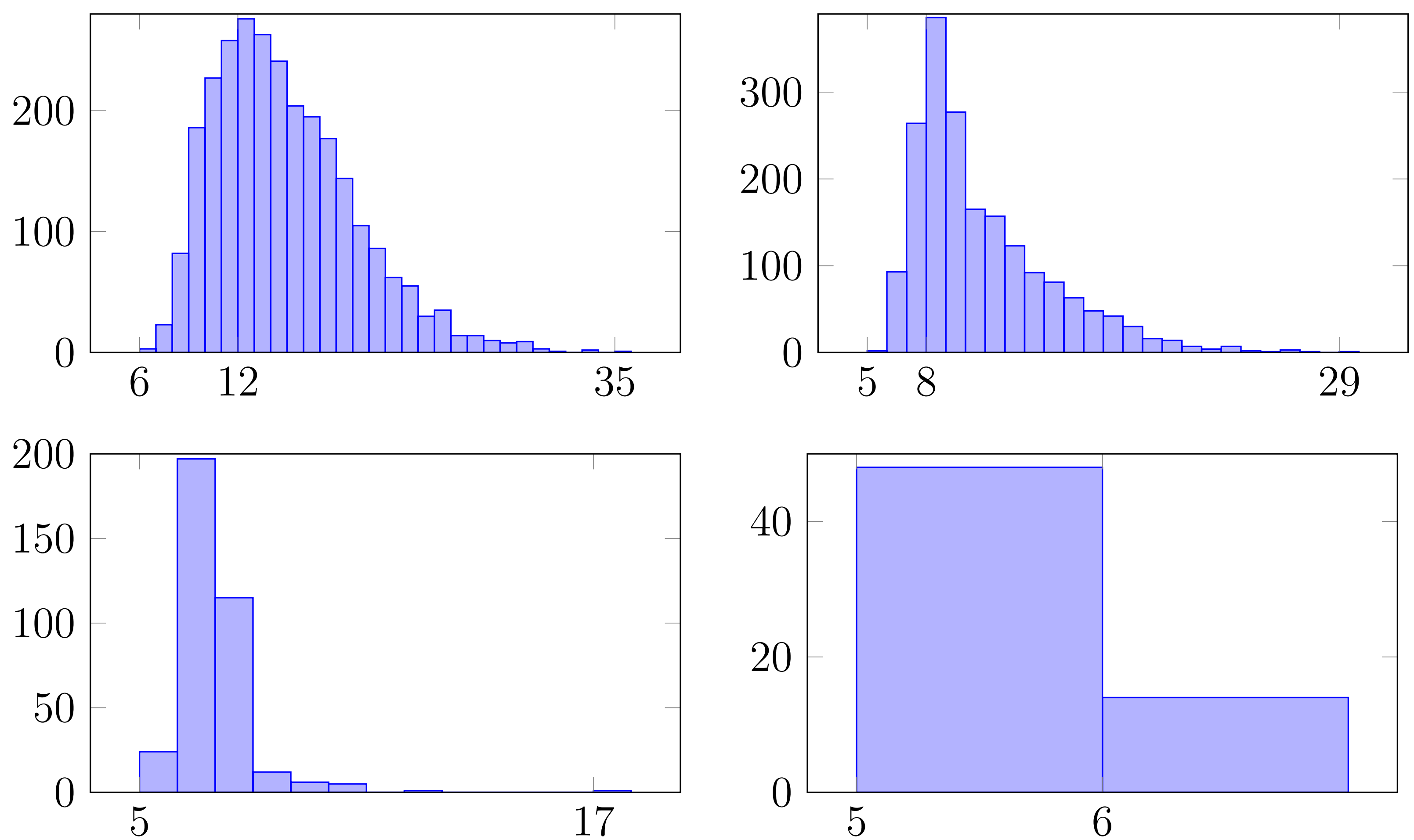
➡ Slow convergence.

- Add additional local search (*"Spider"*)
- Smaller lattices converge much faster.

EXAMPLE 1

$$\Lambda = U \oplus U \oplus U \quad (D = 6)$$

$$d(U) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

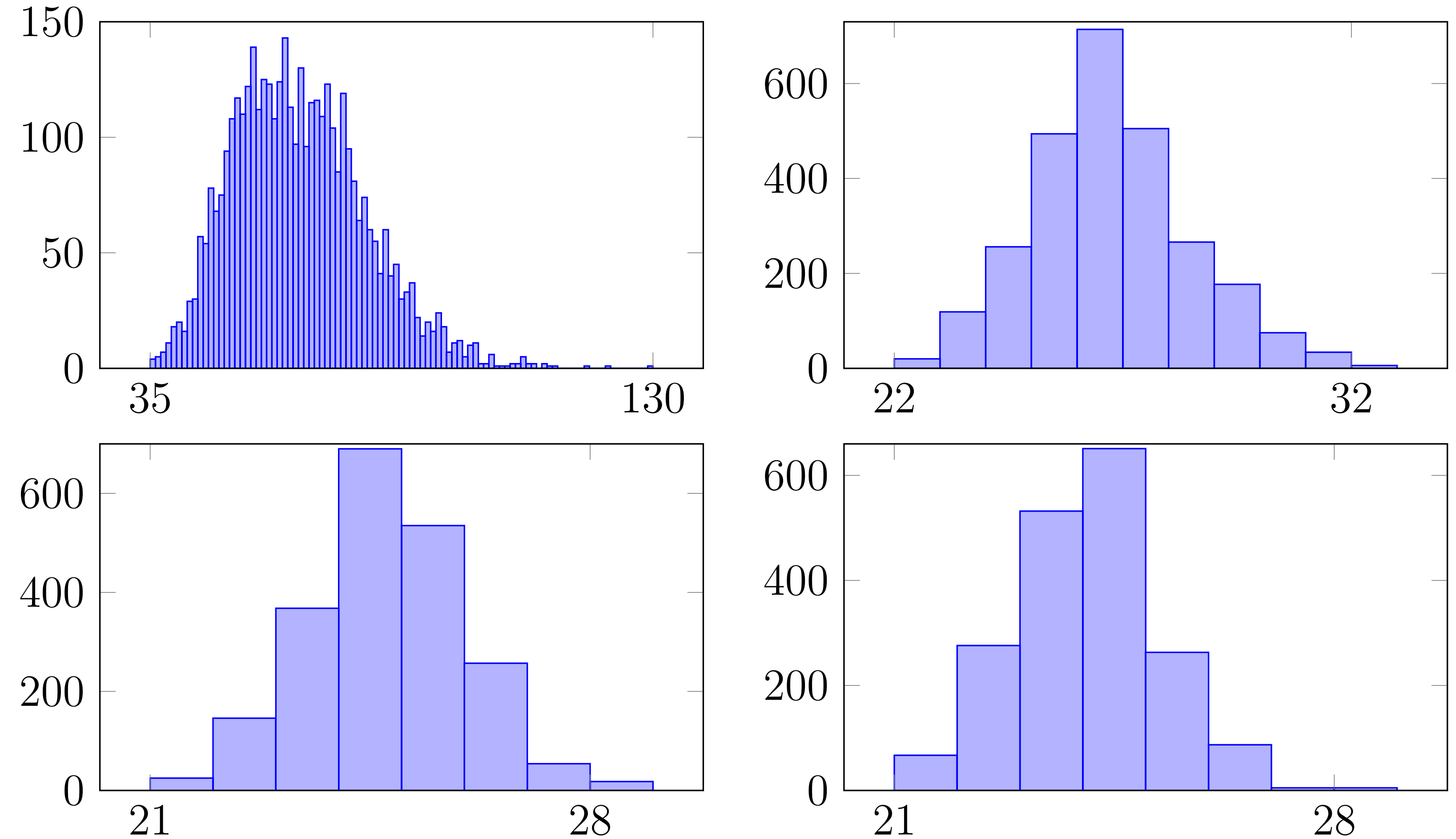


Snapshot of the distribution of Q(N) after 60 seconds, 2 minutes, 5 minutes, and 30 minutes.

(population size: 8 x 500)

EXAMPLE 2

$$\Lambda = E_8 \oplus E_8 \oplus U \quad (D = 18)$$



Snapshot of the distribution of Q(N) after 20 minutes, 12 hours, 24 hours, and 36 hours.

(population size: 15 x 1000)

RESULTS

All lattices we analyzed:

lattice Λ	$D = \dim(\Lambda)$	$Q_{\min}(\Lambda)$
$3\,U$	6	5
$A_4 \oplus U$	6	6
$D_4 \oplus U$	6	6
$A_4 \oplus 2\,U$	8	7
$D_4 \oplus 2\,U$	8	6
$E_6 \oplus U$	8	9
$A_4 \oplus 3\,U$	10	9
$D_4 \oplus 3\,U$	10	9

lattice Λ	$D = \dim(\Lambda)$	$Q_{\min}(\Lambda)$	
$E_8 \oplus U$	10	10	
$E_8 \oplus 2\,U$	12	12	
$E_8 \oplus 3\,U$	14	13	
$2\,E_6 \oplus 2\,U$	16	14	
$2\,E_8 \oplus U$	18	20	
$2\,E_8 \oplus 2\,U$	20	21	
$2\,E_8 \oplus 3\,U$	22	25	$\mathrm{K3} \times \mathrm{K3}$

The always exists a non-trivial

$$Q_{\min}(\Lambda) \sim D$$

INTERPRETATION

- For small lattices: very quick and reliable convergence to

$$Q_{\mathrm{min}}(\Lambda) \sim D$$

➜ *seems to be universal behavior*

- More challenging: Determine the actual value of $Q_{\mathrm{min}}(\Lambda)$ (in particular for larger lattices)
- Problem: Given a putative $Q_{\mathrm{min}}(\Lambda)$, what is the probability that the absence of $Q < Q_{\mathrm{min}}(\Lambda)$ is just a statistical effect?
- Requires knowledge over distribution of Q and quantitative performance of search algorithm.

K3 X K3

- Result:
  - $\mathcal{O}(10^5)$ matrices with $Q_{D3}^{\text{flux}} = 25$
  - 0 matrices with $Q_{D3}^{\text{flux}} \leq 24$
- Remember: $$\frac{\chi(K3xK3)}{24} = 24$$

➡ Moduli stabilization at generic (smooth) point in moduli space not possible!

- tadpole conjecture constant: $$\alpha = \frac{min(Q_{D3}^{\text{flux}})}{\#\text{moduli}} = \frac{25}{57} \approx 0.44$$

CONCLUSION

[Bena, Blåbäck, Graña, SL ’20, ’21]

- M-theory on K3 x K3:
  - stabilization of all moduli
  - generic point in moduli space (no orbifold singularity)
  - fluxes with arbitrary small M2-charge ($Q \lesssim 24$)

$\rightarrow$ cannot have all three!

fluxes with small charge	$\leftrightarrow$	*additional light d.o.f*

THANK YOU!