COBORDISMS, ANOMALIES, AND SWAMPLAND IN NINE DIMENSIONS

Miguel Montero
Harvard

StringPheno Summer series

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Reason: Doesn't tell us how strongly the symmetry is broken

[See however Fichet-Saraswat, Daus-Hebecker-Leondhart-MarchRussell '20]

THE GIST OF THIS TALK

[MM, Cumrun Vafa, to appear]

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Gauge anomaly

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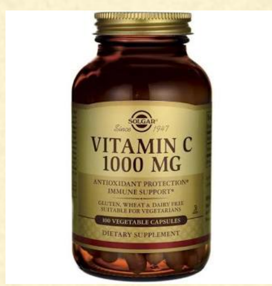

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Swampland (for the low-energy EFT!)

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[Similar ideas: Kim-Shiu-Vafa '19, Kim-Tarazi-Vafa '19, Katz-Kim-Tarazi-Vafa '20]

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Please interrupt me!

- We are used to **topological symmetries** in field theory.

$$Q_m = \int F$$

(these must also be broken/gauged in gravity!)

- In a theory with dynamical gravity, there can also be **topological symmetries** involving **geometry**

For instance, take 5d gravity and on a t=0 slice

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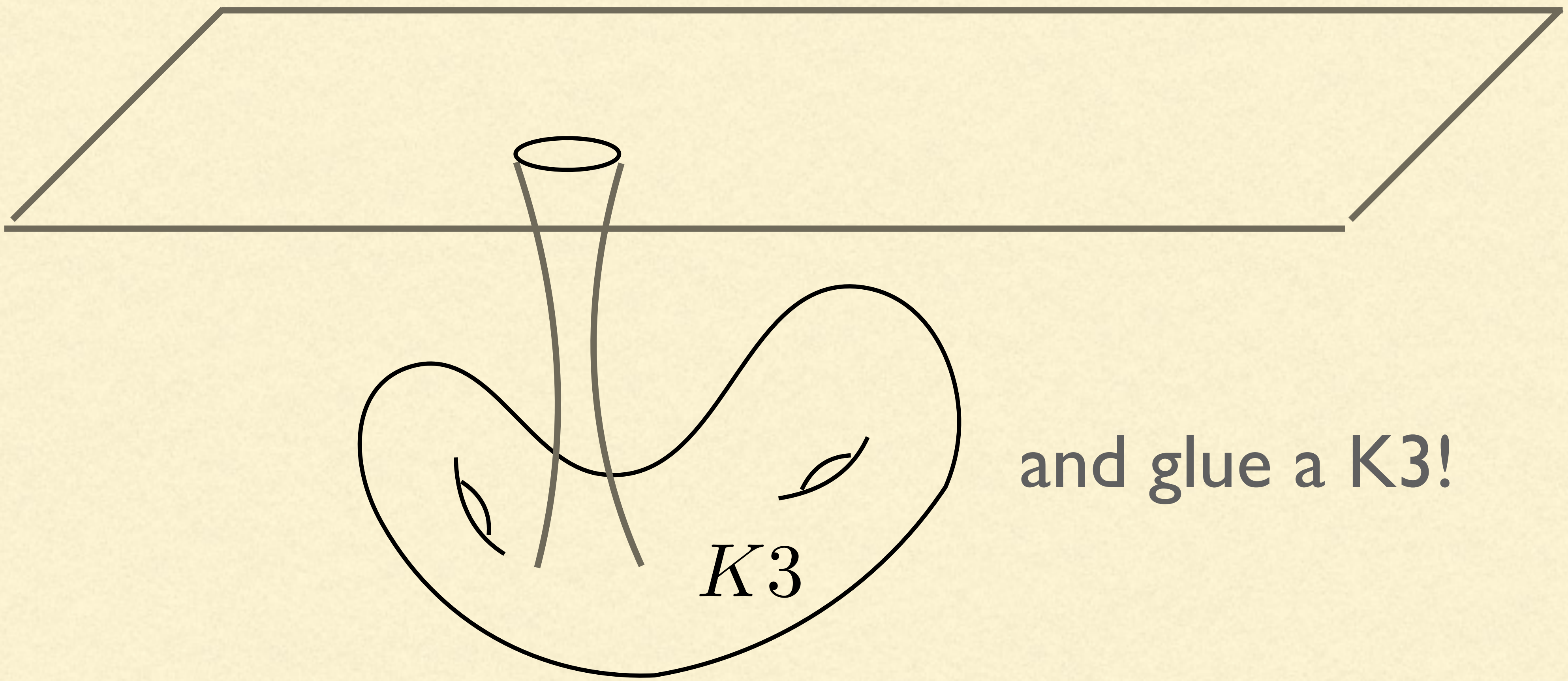

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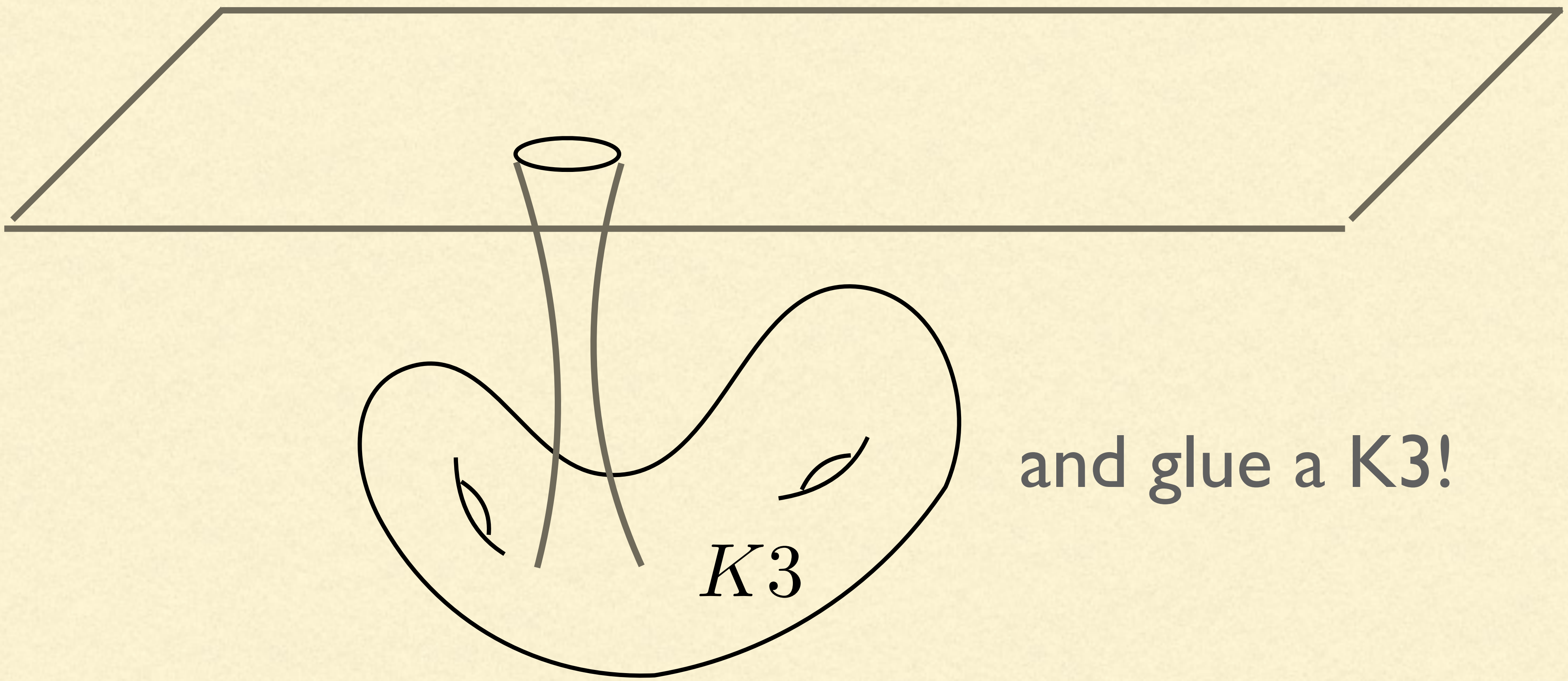
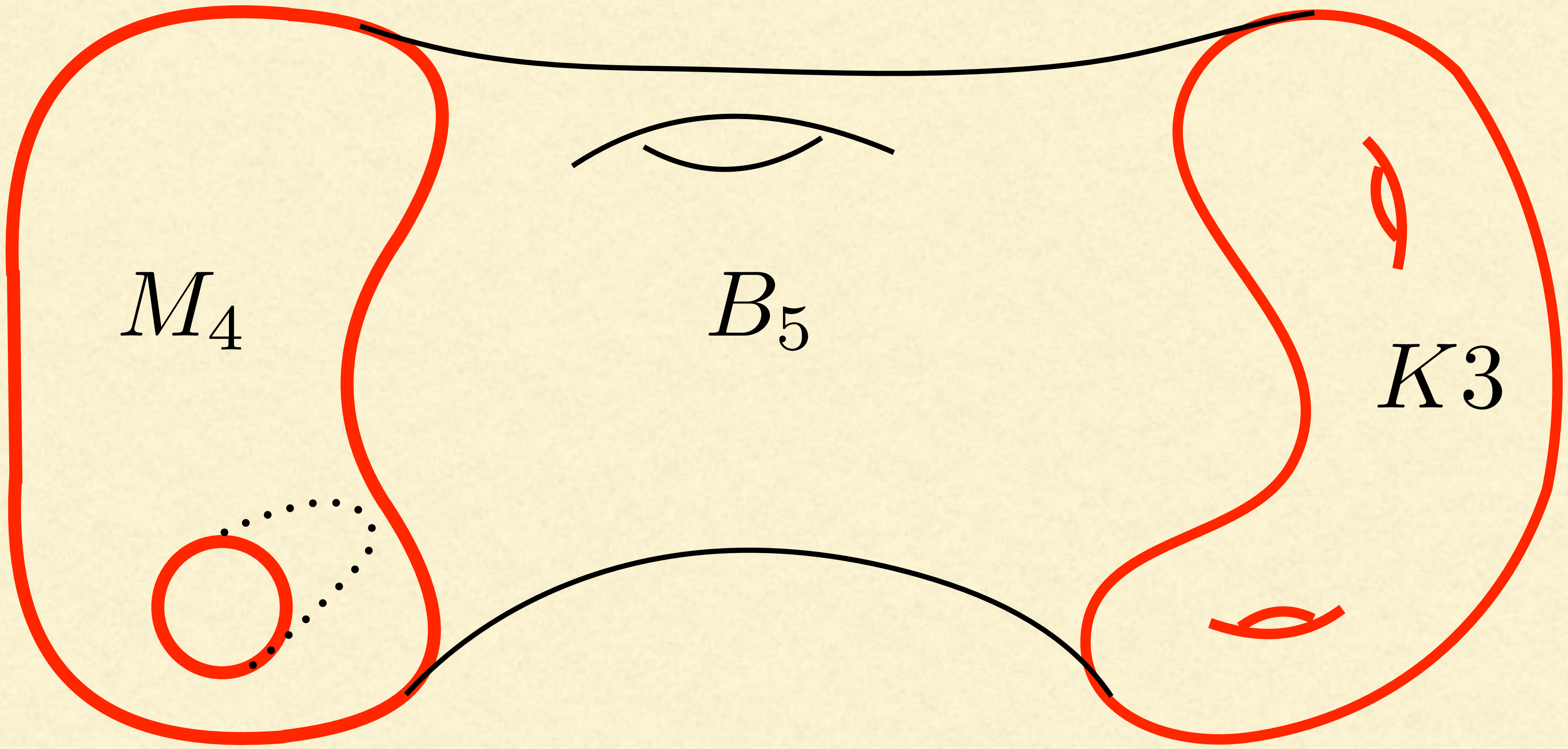


and glue a K3!

Then $$\int p_1 = \int_{K3} \mathrm{tr}(R^2)$$ is a conserved topological **global** charge in supergravity

- p1 is actually a **bordism invariant**



$$p_1(K3) \sim p_1(M_4)$$

- More generally, these new charges are classified by **bordism groups**

e.g. $\Omega_4^{\mathrm{Spin}} = \mathbb{Z}$ or $\Omega_2^{\mathrm{Pin}^-} = \mathbb{Z}_8$

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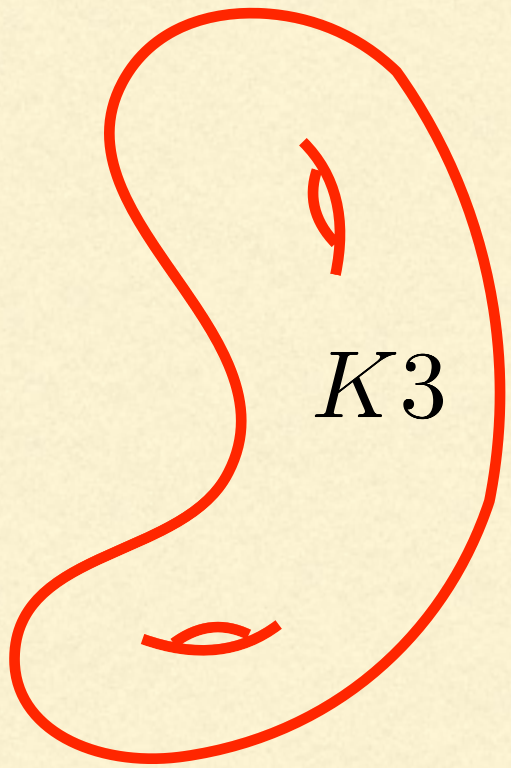

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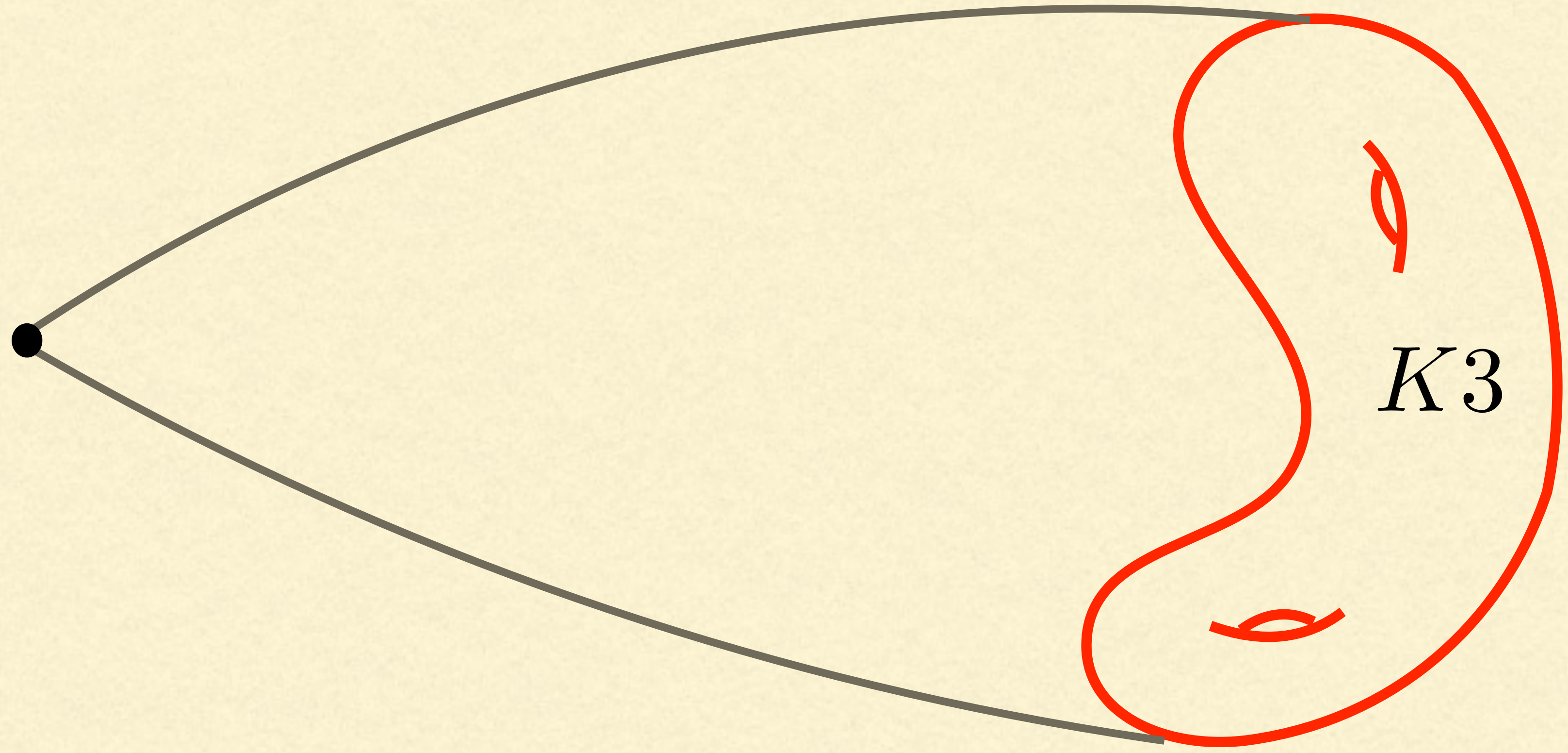

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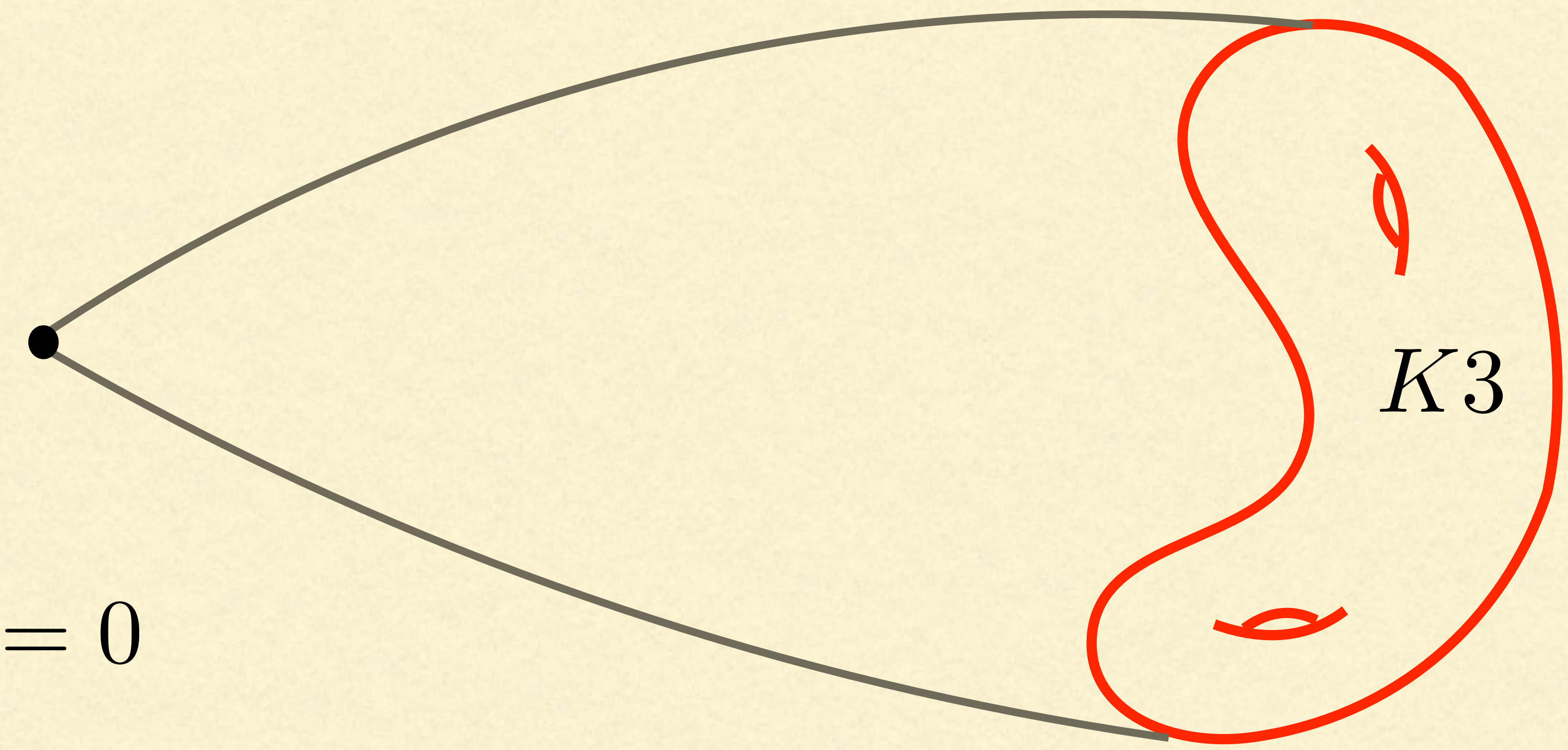

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These defects can be potentially **anomalous**

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We will use this to get **new constraints** on theories with 16 supercharges, in 9,8,7d.

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- There are just **two multiplets**
    - Gravity: $g, A_i, B \;\; \phi$
    - Gauge: A, ϕ_i
- Low-energy interactions are **completely fixed** by susy.
- There is a **Narain** moduli space parametrized by the scalars

$$\frac{SO(10-d,r)}{SO(10-d)\times SO(r)}$$

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$$E_8 \times E_8,\ \mathrm{Spin}(32)/\mathbb{Z}_2,\ E_8 \times U(1)^{248},\ U(1)^{496}$$

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We get **string universality**

[Adams-De Wolfe- Taylor '10, Kim-Shiu-Vafa '19]

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We will explain the **modulo 8** periodicity in 8d and 9d

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Assumption: Every N=1 theory in 8d, 9d makes sense on non-orientable manifolds

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These are **orbifolds** or **orientifolds** in examples

We call them **I-folds**

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- This is a 6d (1,0) model. Gravitational anomaly cancellation demands

$$273 = n_T + n_H - n_V$$

$$n_T = 8n_I^T + 1, \quad n_H = 1 + r + 8n_V^I, \quad n_V = 8n_V^I$$

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- Possible subtlety involving a **topological Green-Schwartz mechanism** **[Garcia-Etxebarria-Hayashi-Ohmori-Tachikawa-Yonekura '17]**

- In 7d, compactification on S1/Z2 yields

$$r \equiv 1 \bmod 2$$

so we don't explain the pattern in this case.

- What about the gauge algebra/group?

$$
\begin{array}{c|ccccccccc}
9d & \mathfrak{a}_k & \mathfrak{b}_k & \mathfrak{c}_k & \mathfrak{d}_k & \mathfrak{e}_6 & \mathfrak{e}_7 & \mathfrak{e}_8 & \mathfrak{f}_4 & \mathfrak{g}_2 \\
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- Still, there are constraints on the global form of the gauge group.

- These come from applying cobordism conjecture to (gauge transformation+parity)

Algebra	$\dim(\mathfrak{g}) + \mathrm{rank}(\mathfrak{g})$	$\neq 0$ mod 8?	Group	Real reps?
A_k	k^2+3k	$k \neq 0, 5 \bmod 8$	$SU(k+1)$	✗
			$PSU(k+1)$	✓
B_k	$2k(2k+1)$	$k \equiv 1, 2 \bmod 4$	$Spin(2k+1)$	✗
			$SO(2k+1)$	✓
C_k	$2k(2k+1)$	$k \equiv 1, 2 \bmod 4$	$Sp(k)$	✗
			$Sp(k)/\mathbb{Z}_2$	✓
D_k	$2k^2$	k odd	$Spin(2k)$	✗
			$SO(2k)$	✓
			$Spin(2k)/\mathbb{Z}_4$	✓
E_6	84	✓	E_6	✗
			$E_6/\mathbb{Z}_3$	✓
E_7	140	✓	E_7	✗
			$E_7/\mathbb{Z}_2$	✓
E_8	256	✗	E_8	✓
F_4	56	✗	F_4	✓
G_2	16	✗	G_2	✓

TO WRAP UP

- **Cobordism + anomalies = Swampland**
- Established **Coulomb branch string universality** in 8d and 9d.
- Apply these ides to **models with less susy**.
    - 6d (1,0)
    - SM?

Thank you!

- Bonus track: Tadpoles in T3/Z2

- The orbifold T3/Z2 can carry monopole charge under 9d gauge fields. **If non-integer, it will spoil the argument**

- Integrality can be determined from Dirac quantization and anomaly of dual particle (BPS with 8d supercharges).

$$\eta(\mathbb{RP}^2) = \frac{1}{8} \cdot 8$$

- Argument fails for higher codimension I-folds