On the Finiteness of Supergravity Landscape in d = 6

Based on work with Cumrun Vafa & HCT to appear

and previous work with: Hee-Cheol Kim , HCT, Cumrun Vafa, 1912.06144

Sheldon Katz, Hee-Cheol Kim , HCT, Cumrun Vafa, 2004.14401

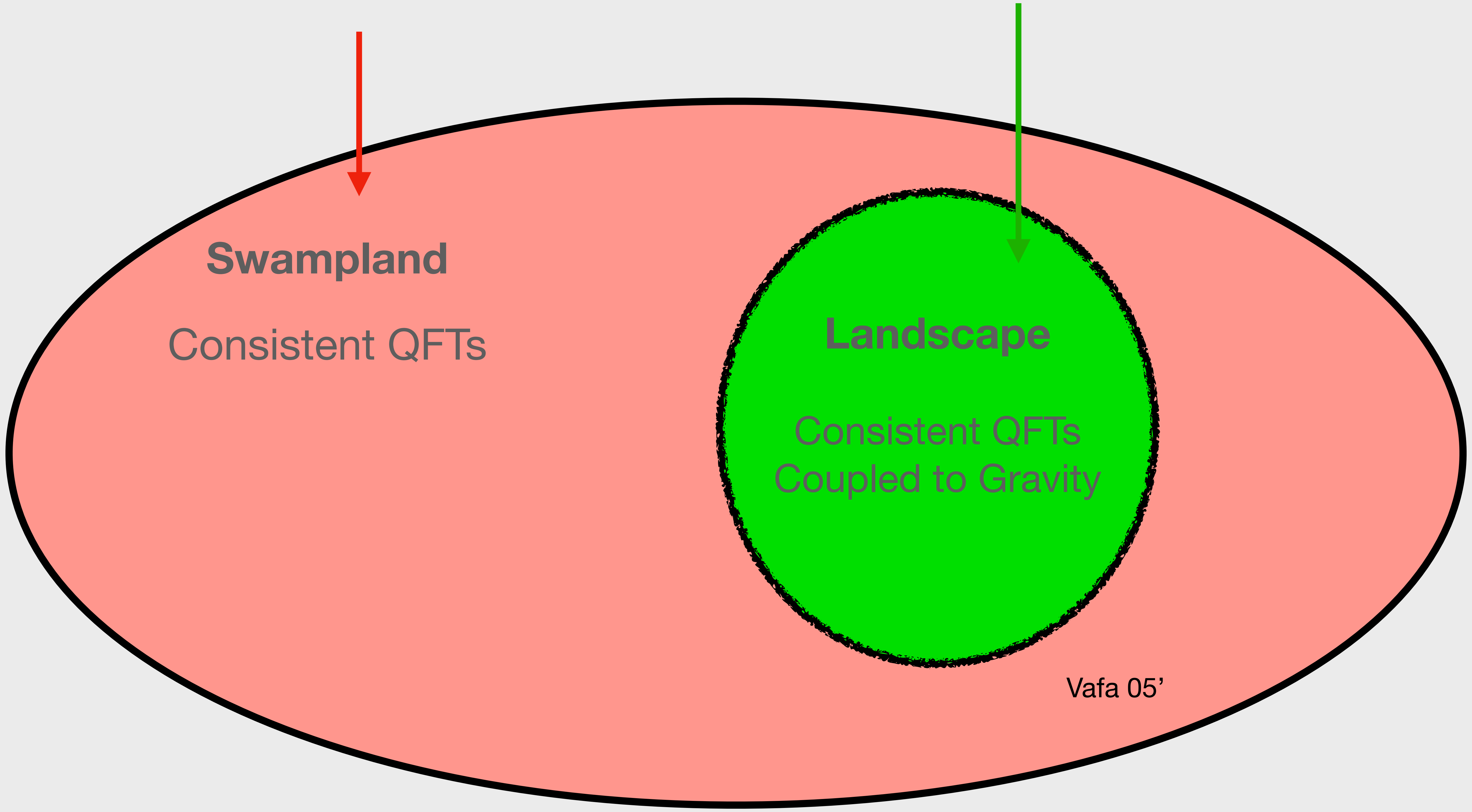
Houri Christina Tarazi

Harvard University

May 11th, 2021

Swampland Program:

What conditions can we use to distinguish between
consistent QFT's which cannot couple to gravity and those that can arise in the low energy limit of a quantum gravity?



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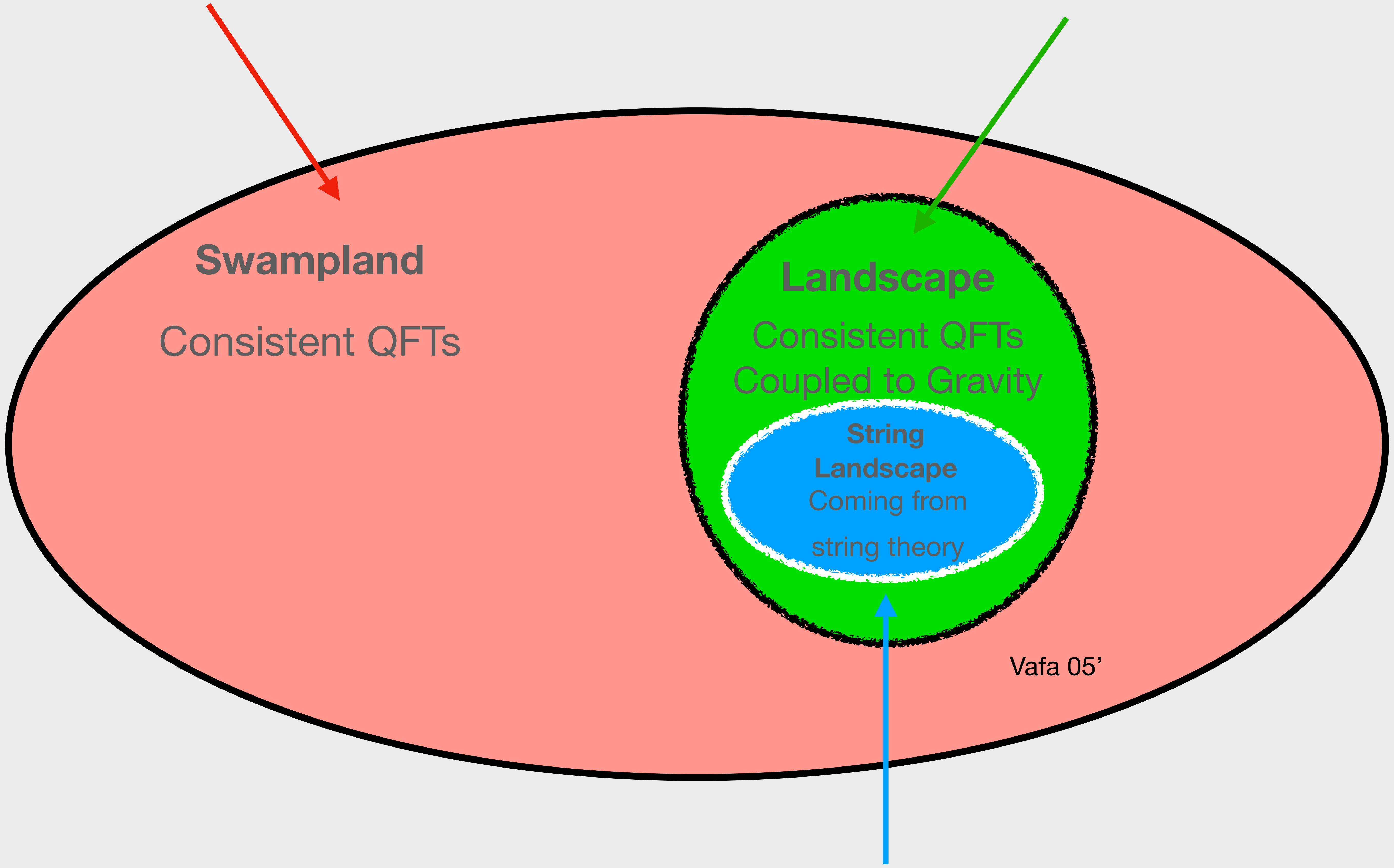

Do all the theories in the Landscape also belong in the String Landscape (SLP: YES)?

Is there a way to use fundamental ideas that are believed to be true for Quantum gravitational theories to construct such condition?

Unitarity

Every Quantum Gravity should be Unitary

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Completeness of Spectrum

In a quantum gravity for any p-form gauge field, every charged (p-1)-brane state should appear in the spectrum

Polchincki 03'
Banks, Seiberg 10'
Harlow, Ooguri 18'

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In a quantum gravity at infinite distance in the scalar moduli space an infinite tower of states becomes exponentially light

Ooguri, Vafa 06'

Many more Swampland conditions exist ……..

Is there a way to use fundamental ideas that are believed to be true for Quantum gravitational theories to construct such conditions?

- **Theories with 32 Supercharges:** Low-energy limit completely determined by supersymmetry and unique except d=10

  - d=11: realized through M-theory
  - d=10: Chiral $\mathcal{N} = (2,0)$: realized through IIB string theory

    Non-Chiral $\mathcal{N} = (1,1)$: realized through IIA string theory
  - d<10: circle compactification of d+1

All of them also belong to the String Landscape!
An evidence for SLP

Is there a way to use fundamental ideas that are believed to be true for Quantum gravitational theories to construct such condition?

- **Theories with 16 Supercharges:**

- d=10: Anomaly cancellation gives us gauge groups $G = E_8 \times E_8, SO(32), E_8 \times \underbrace{U(1)^{248}}, U(1)^{496}$

Kim, Shiu, Vafa 19'
Adams, Dewolfe, Taylor 10'

- d<10: All non-chiral except for 6d (2,0), which is unique and realized through IIB on K3

Non-chiral: the rank of the gauge group is bounded by $rank(G) \leq 26 - d$

(For example N=4, d=4 with $rank > 22$ belongs to the Swampland.)

Kim, HCT, Vafa 19'

d=9: $rank(G) = 1,9,17$ d=8: $rank(G) = 2,10,18$ d=7: $rank(G) = 1 \mod 2$

Alvarez-Gaume, Witten 84

Montero, Vafa 20'

- $r = 1$, M-theory on KB or IIB on DP
- $r = 9$: CHL string
- $r = 17$: Heterotic on S^1

On S^1

$r = 3,5,7,11,19$
Can be realized.

Can we also find restrictions on the type of gauge groups that could appear?

String Theory allows only for $8d$: $SU(N), SO(2N), Sp(N), e_6, e_7, e_8$ but not $\underbrace{SO(2N+1)}, \underbrace{f_4, g_2}$

Garc´ıa-Etxebarria, Hayashi ,Ohmori, Tachikawa, Yonekura 17'
Hamada, Vafa 21'

Is there a way to use fundamental ideas that are believed to be true for Quantum gravitational theories to construct such condition?

- **Theories with 8 Supercharges:**

  - d=6: Anomalies are very constraining and many theories can be ruled out. Morrison, ,Kumar, Taylor,…..09'/10'/..

    -Extra conditions using extended objects (which is the approach we will follow here) Kim, Shiu, Vafa 19' Lee,Weigand 19'

    Today's talk will mostly focus on the d=6 and on the finiteness of massless modes for these theories.

  - d=5: Much harder since no anomalies but we can still have some constraints:

    For example for theories of the form $U(1) \times G$ that can Higgs to the quintic we have: $rank(G) \leq 52$

    For the Higgs branch we can put a bound of the form: $\sum_i dimR_i - dimG \leq 36D^3 + 80$

    where D is the divisor class associated with a black string.

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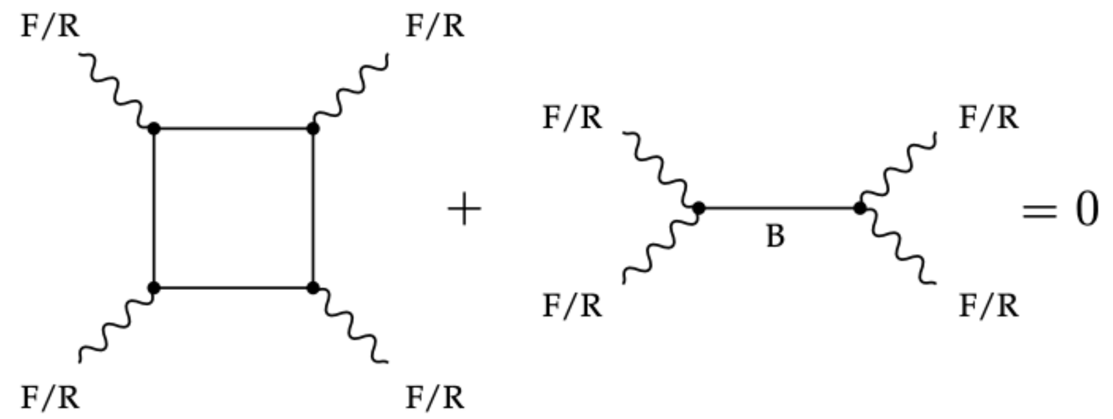
- 6d $\mathcal{N}=1$ Supergravity theories

Super-multiplets:

Supergravity-multiplet	$(g_{\mu\nu}, B_{\mu\nu}, \psi_\mu^-)$
Tensor-multiplet(T)	$(B_{\mu\nu}, \phi, \chi^+)$
Vector-multiplet(V)	(A_μ, λ^-)
Hyper-multiplet(H)	$(4h, \psi^+)$

Chiral fields contribute to gauge/gravitational anomalies
Cancelled by the **Green-Schwarz-Sagnotti Mechanism**



Anomaly polynomial factorizes as:

$$I_8 = \frac{1}{2}\Omega_{\alpha\beta}X_4^\alpha X_4^\beta$$
$$X^\alpha = \frac{1}{2}a^\alpha trR^2 + \sum_i b_i^\alpha\left(\frac{2}{\lambda_i}trF_i^2\right)$$

$\Omega_{\alpha\beta}$ symmetric of signature (1,T)

$a^\alpha, b_i^\alpha \in \mathbb{R}^{1,T}$

Anomaly Cancellation:

$$R^4: \quad H - V = 273 - 29T$$
$$F^4: \quad 0 = B^i_{Adj} - \sum n^i_R B^i_R$$
$$(R^2)^2: \quad a\cdot a = 9 - T \in \mathbb{Z}$$

$$F^2R^2: \quad a\cdot b_i = \frac{1}{6}\lambda_i\left(A^i_{Adj} - \sum_R n^i_R A^i_R\right) \in \mathbb{Z}$$
$$(F^2)^2: \quad b_i\cdot b_i = \frac{1}{3}\lambda_i^2\left(\sum_R n^i_R C^i_R - C^i_{Adj}\right) \in \mathbb{Z}$$
$$F_i^2F_j^2: \quad b_i\cdot b_j = \sum_{R,S}\lambda_i\lambda_j n^{ij}_{RS}A^i_R A^j_S \in \mathbb{Z}, \quad i\neq j$$

A_R, B_R, C_R group theory coefficients

$$tr_R F^2 = A_R trF^2, \quad tr_R F^4 = B_R trF^4 + C_R (trF^2)^2$$

$n^i_R =$ hypers in number of in R

Taylor, ,Kumar, Morison,….

The number of theories that satisfy these anomaly conditions are infinite.
SLP would though suggest that only a finite number of those can truly be UV-completed in a quantum gravity.

• 6d $\mathcal{N} = 1$ Supergravity theories

Super-multiplets:

Supergravity-multiplet	$(g_{\mu\nu}, B^{+}_{\mu\nu}, \psi^{-}_{\mu})$
Tensor-multiplet(T)	$(B^{-}_{\mu\nu}, \phi, \chi^{+})$
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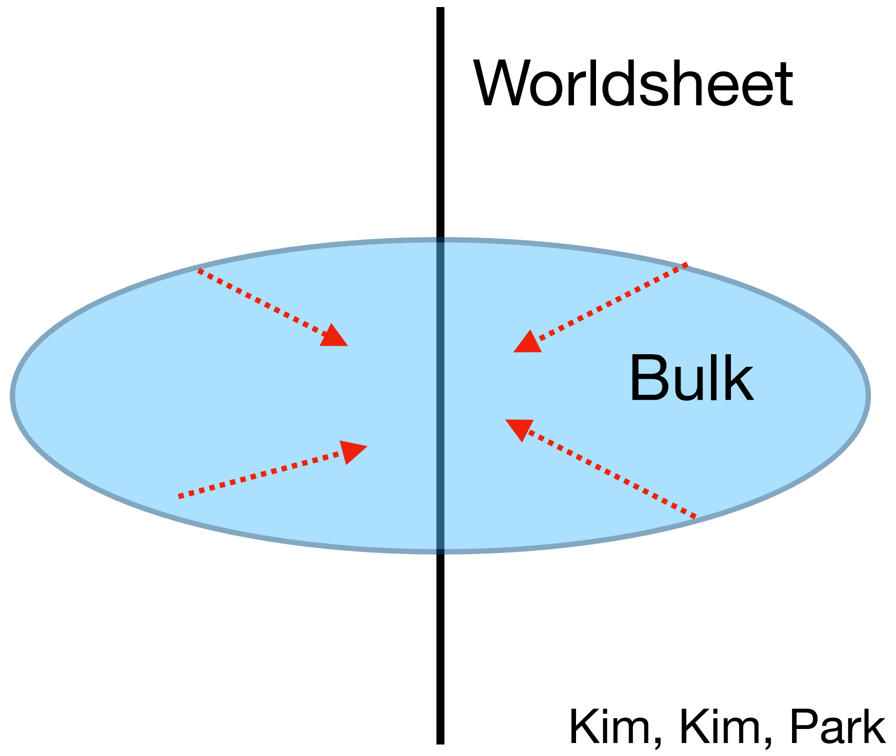

Kim, Kim, Park 16'

Shimizu, Tachikawa 16'

Completeness of Spectrum

- For 6d $\mathcal{N} = 1$ supergravity there exist (anti-)shelf dual BPS strings charged under B_2 → (0,4) SCFT at low energy

Unitarity of the string worldsheet

$$\sum_i c_{G_i} = \frac{k_i \, dimG}{k_i + h_i^{\vee}} \leq c_L$$

Anomaly cancellation for string of charge Q

$$\boxed{-I_4^{inflow} + I_4^{WS} = 0}$$

- Central charges: $c_L = 3Q \cdot Q - 9Q \cdot a + 2,\ c_R = 3Q \cdot Q - 3Q \cdot a \geq 0$
- Levels of G_i, $SU(2)_l$: $k_i = Q \cdot b_i,\ k_l = \frac{1}{2}(Q \cdot Q + Q \cdot a + 2) \geq 0$

Kim, Shiu, Vafa, 19'

• 6d $\mathcal{N}=1$ Supergravity theories

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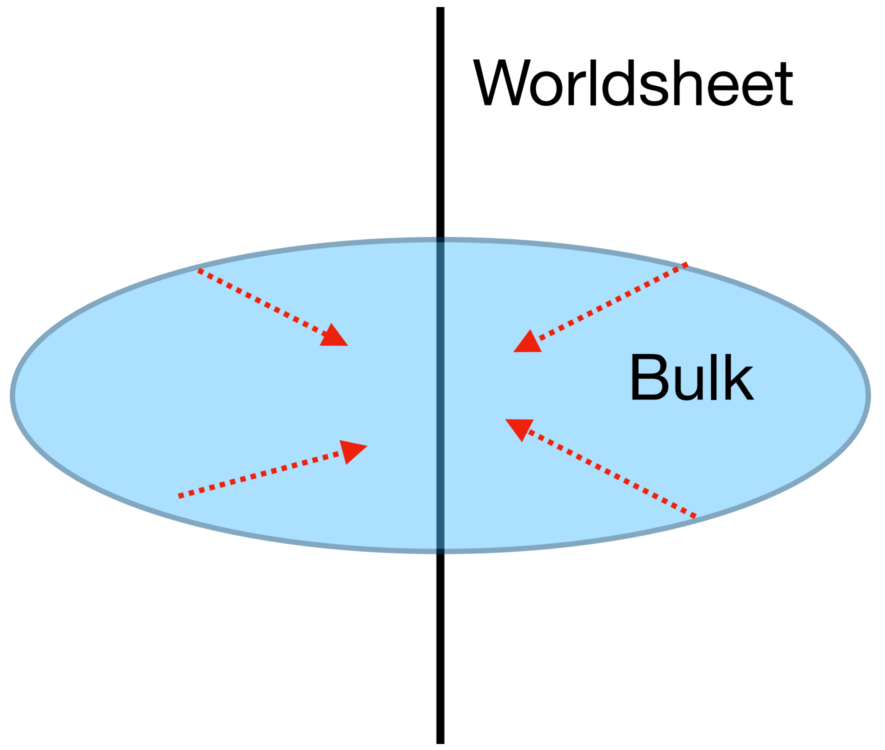

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Other consistency conditions:

Moduli space parametrized by $j \in \mathbb{R}^{1,T}$

- $j\cdot b_i\ trF^2 \Longrightarrow j\cdot b_i > 0$
- $j\cdot j > 0$
- Tension: $Q\cdot j \geq 0$
- $-j\cdot a\ trR^2 \Longrightarrow j\cdot a > 0$

Hamada, Noumi, Shiu 18'
Cheung, Remmen 16'

- $Q^2 \geq -1$
- Levels : $k_i = Q \cdot b_i,\ k_l = \frac{1}{2}(Q \cdot Q + Q \cdot a + 2) \geq 0$
- Tension: $Q \cdot j \geq 0$
- $j \cdot b_i > 0$
- $j \cdot j > 0$
- $j \cdot a > 0$

New Ingredient: Bound on the matter

- The vertex operator of the massless modes with representation $\mathbf{R}$ of G with conformal weight $\Delta_{\mathrm{R}} = \frac{C_2(\mathbf{R})}{2(k+h^\vee)}$ where $C_2(\mathbf{R})$ is the second Casimir of the R must obey: $\boxed{\Delta_{\mathbf{R}} \leq 1}$

- The representation R of a primary with highest weight $\mathbf{\Lambda} = (\Lambda_i, \cdots, \Lambda_r)$ where r is the rank of the Lie algebra must satisfy :

$$\boxed{\sum_i^r \Lambda_i \leq k}$$

where k is the level of the current algebra of G on the worldsheet.

Example: $SU(N)$ with $(N-8)$ F $+\,1$ S with $T \leq 10$ and $N \leq 30$ (with $N = 30$ for $T = 1$) $\longleftarrow$ No F-theory realization for any N

This theory has $a \cdot b = 1,\ b \cdot b = -1$

We can choose a basis such that the bilinear form and the vectors a, b are given by: $\Omega = diag(1,(-1)^T),\ a = (-3,1^T),\ b = (0^T, -1)$

One can check that we may choose $Q = (3,0^{T-1},1)$ in which case $k = 1$.

$$\boxed{\sum_i^{N-1} \Lambda_i \leq 1}$$

But symmetric matter has highest weight $(2,0,\cdots,0)$.

•6d $\mathcal{N} = (1,0)$ Supergravity theories Infinite families

- SLP would suggest that there are only finitely many consistent theories when coupled to gravity. It is interesting to investigate whether there is a UV-completion independent reason for the finiteness.

Key Potential Non-abelian Infinite families:

1.Non-abelian theories of fixed T with an infinite number of simple gauge group factors of finite dimension.

A more careful analysis using the gravitational anomaly shows that there can only be a finite number of such theories. Kumar, Morrison, Taylor 10'

2.Non-abelian theories with fixed T and unbounded dimension.

The F^4 anomaly restricts the types of theories that can appear with unbounded dimension: Kumar, Morrison, Taylor 10'

1 Gauge Factor

$SU(N)$	1 Adj	0
	$1\ \square\!\square + 1\ \overset{\square}{\square}$	1
	$2N\ \square$	N^2+1
	$(N+8)\ \square + 1\ \overset{\square}{\square}$	$\frac{1}{2}N^2 + \frac{15}{2}N + 1$
	$(N-8)\ \square + 1\ \square\!\square$	$\frac{1}{2}N^2 - \frac{15}{2}N + 1$
	$16\ \square + 2\ \overset{\square}{\square}$	$15N + 1$
$SO(N)$	$(N-8)\ \square$	$\frac{1}{2}N^2 - \frac{7}{2}N$
$Sp(N)$	$(2N+8)\ \square$	$\frac{1}{2}(2N)^2 + \frac{7}{2}(2N)$
	$16\ \square + 1\ \overset{\square}{\square}$	$15(2N) - 1$

H-V diverges for large N

$H - V \leq 273$

2 Gauge Factors

Gauge Group	Matter content
$SU(N) \times SU(N)$	$2(\square, \bar{\square})$
$SO(2N+8) \times Sp(N)$	$(\square, \square)$
$SU(N) \times SO(N+8)$	$(\square, \square) + (\overset{\square}{\square}, 1)$
$SU(N) \times SU(N+8)$	$(\square, \square) + (\overset{\square}{\square}, 1) + (1, \square\!\square)$
$Sp(N) \times SU(2N+8)$	$(\square, \square) + (1, \square\!\square)$

For fixed $T < 9$ it was shown that the families are finite.

3 Gauge Factors

Gauge group
$SU(N-8) \times SU(N) \times SU(N+8)$
$Sp((N-8)/2) \times SU(N) \times SO(N+8)$

What about $T \geq 9$?

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- $SU(N) + 1Adj$
- $SU(N) + 1S + 1A$

$T = 9$

$a^2 = 0, b^2 = 0, a \cdot b = 0 \implies b = -\lambda a$ with $\lambda > 0$

We can find solutions: $\Omega = diag(+1,(-1)^{10}),\, a = (-3,1^{10})$

Unitarity Condition: $\dfrac{k(N^2-1)}{k+N} \leq c_L = 3Q \cdot Q - 9Q \cdot a + 2$

A string of charge Q needs to satisfy:

- $Q^2 \geq -1$
- Levels : $k_i = Q \cdot b_i,\ k_l = \dfrac{1}{2}(Q \cdot Q + Q \cdot a + 2) \geq 0$
- Tension: $Q \cdot j \geq 0$

A minimal choice is: $Q = (1, -1, -1, 0, \cdots, 0)$

Unitarity Condition: $\dfrac{\lambda(N^2-1)}{\lambda+N} \leq c_L = 8$

Solutions: $(k \geq 1, N = 0,1,2,3), (4 \geq k \geq 1, N = 4), (2 \geq k \geq 1, N = 5), (k = 1, N = 6,7,8,9)$

Note that for $k = 1$ the matter bound becomes: $\sum_i^{N-1} \Lambda_i \leq 1$ but the $S,\ Adj$ do not satisfy this.

Solutions: $(k \geq 2, N = 0,1,2,3), (4 \geq k \geq 2, N = 4), (k = 2 \geq 1, N = 5)$

From the F-theory perspective we have both $a,\ b$ primitive and hence $\lambda = 1$ but as we saw that is not allowed by the matter bound and hence we do not expect any F-theory realizations of those remaining theories.

- What about $T \geq 9$?

2 Gauge Factors

Gauge Group	Matter content
$SU(N) \times SU(N)$	$2(\square, \bar{\square})$
$SO(2N+8) \times Sp(N)$	$(\square, \square)$
$SU(N) \times SO(N+8)$	$(\square, \square) + (\begin{smallmatrix}\square\\\square\end{smallmatrix}, 1)$
$SU(N) \times SU(N+8)$	$(\square, \square) + (\begin{smallmatrix}\square\\\square\end{smallmatrix}, 1) + (1, \square\square)$
$Sp(N) \times SU(2N+8)$	$(\square, \square) + (1, \square\square)$

For $T = 9$ the dimension is bounded by $N \leq 9$ using completeness of spectrum and unitarity

Kim, Shiu, Vafa 19'

- Consider $SO(2N+8) \times Sp(N)$ with $1(F \otimes F)$ valid for $T \leq 10$:

The charge lattice is $\Lambda = \begin{pmatrix} a^2 & -a \cdot b_1 & -a \cdot b_2 \\ -a \cdot b_1 & b_1^2 & b_1 \cdot b_2 \\ -a \cdot b_2 & b_1 \cdot b_2 & b_2^2 \end{pmatrix} = \begin{pmatrix} 9-T & -2 & 1 \\ -2 & -4 & 2 \\ 1 & 2 & -1 \end{pmatrix}$

-For $T = 9$ we have:

$$\left.\begin{aligned} &a \cdot (b_1 + 2b_2) = 0,\ (b_1 + 2b_2)^2 = 0 \Longrightarrow \quad b_1 + 2b_2 = \lambda a \Longrightarrow b_1 = \lambda a - 2b_2 \\ &b_1 \cdot b_2 = 2 \Longrightarrow (\lambda a - 2b_2) \cdot b_2 = -\lambda + 2 \Longrightarrow \lambda = 0 \Longrightarrow j \cdot b_1 = -j \cdot 2b_2 \end{aligned}\right\}$$

There is no b_1, b_2 that satisfy positivity conditions

-For $T = 10$ we have: We can find solutions: $a = (-3, 1^{10}),\ b_1 = -2a,\ b_2 = (1, -1, -1, 0^8)$ and $J = (1, 0^{10})$ and $\Omega = diag(+1, (-1)^{10})$

Unitarity Condition: $\dfrac{k_1((2N+8)(2N+7)/2)}{k_1 + (2N+6)} + \dfrac{k_2(2N(2N+1)/2)}{k_2 + (N+1)} \leq c_L = 3Q \cdot Q - 9Q \cdot a + 2$

Consider a string of charge $Q = (q_1, \cdots, q_{11})$, the minimal choice $Q = (1, -1, 0^8, -1)$ with $Q^2 = -1,\ k_2 = 0,\ k_1 = 2$

$$\boxed{\frac{k_1((2N+8)(2N+7)/2)}{k_1 + (2N+6)} \leq 8 \Longrightarrow N <= 1/2}$$

- $N = 0$ we have $SO(8)$
- $N = 1/2$ we have $SO(9) + 1F$

Realized in F-theory

Morrison, Taylor 12'
Taylor 12'

•6d $\mathcal{N} = (1,0)$ Supergravity theories Infinite families

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Similar analysis shows that they are all finite for a given choice of basis!

- There are in fact more theories allowed by anomalies with k number of simple factors and excluded with similar tools as above:

$$Sp((N-8)/2) \underbrace{\times}_{F\otimes F} SU(N) \underbrace{\times}_{F\otimes F} SU(N+8) \times \cdots \underbrace{\times}_{F\otimes F} SO(N+8(k-2))$$

$$\underset{A}{SU(N-8)} \underbrace{\times}_{F\otimes F} SU(N) \underbrace{\times}_{F\otimes F} SU(N+8) \times \cdots \underbrace{\times}_{F\otimes F} \underset{S}{SU(N+8(k-2))}$$

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$$SU(N) \underbrace{\times}_{F\otimes F} SU(N) \times \cdots \underbrace{\times}_{F\otimes F} SU(N)$$

$$F\otimes F$$

Are finite!

More generally consider the unitarity condition: $\sum_i c_{G_i} = \frac{k_i \, dimG}{k_i + h_i^\vee} \leq c_L = 3Q \cdot Q - 9Q \cdot a + 2$

The condition that $k_l = \frac{1}{2}(Q \cdot Q + Q \cdot a + 2) \geq 0$ implies that $-Q \cdot a \leq Q \cdot Q + 2$ and so:

$$\sum_i c_{G_i} = \frac{k_i \, dimG}{k_i + h_i^\vee} \leq c_L \leq 12Q^2 + 20$$

Assumptions:

- There are finitely many inequivalent families of type $F(N) = G_1(N) \times G_2(N) \times \cdots \times G_k(N)$ with the same type of matter at each N.
- The string charge lattice is generated by BPS strings.

Then the vectors a, b_i are independent of N and the charge lattice spanned by BPS states is also independent of N because giving a Vev to a hypermultiplet does not affect the inner product of the charge lattice.

In addition, since we can find generators given by BPS strings Q_i then they can not all have $k_i = 0$ because then they would not generate the whole lattice.

In our examples one can show that $Q = kJ - a$ where k is the number of simple gauge factors G_i is a consistent charge of a BPS string with $Q^2 > 0$ and $k_i > 0$.

Therefore, for each family of type $F(N)$ one can find a minimal choice of Q^2 which is independent of N.

•6d $\mathcal{N} = (1,0)$ Supergravity theories Infinite families

3. Abelian Theories $U(1)^N$

The number of abelian factors is bounded by $N \leq 32$ for $T = 0$, $N \leq 20$ for $T \geq 1$, unless $T > 1$ and $b_i \cdot b_i = 0$ then $N \leq 22$

Park, Taylor 11'

Lee, Weigand 19'

4. Infinite abelian theories with unbounded charges

Taylor, Turner 19'

For example, $U(1)$ with $54 \times (\pm q) + 54(\pm r) + 54 \times (\pm (q + r))$ with $q, r \in \mathbb{Z}$

More work is needed to exclude all infinite families but the number of massless modes is finite!

Thank you very much!