On string vacua without supersymmetry

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Lessons from SUSY breaking

naturalness + ~~SUSY~~ $\longrightarrow$ high scales

- **Support from swampland** (Cribiori, Lüst, Scalisi, 2021)

static + ~~SUSY~~ $\longrightarrow$ instabilities

...but our universe expands

- **Dynamics: a way out?**

Game plan

➔ Non-SUSY strings

➔ Flux vacua...

➔ ...and brane dynamics

➔ Holographic interpretation

➔ Phenomenology?

Building a vantage point: string-scale SUSY breaking

Heterotic model (Alvarez–Gaume, Ginsparg, Moore, Vafa, 1986)

$$E_8 \times E_8 \to SO(16) \times SO(16)$$

➔ ~~SUSY~~: $\Lambda_{\text{quantum}} > 0$

Orientifold models: IIB or 0B

O9 + 32 $\overline{\text{D9}}$

➔ $USp(32)$ (Sugimoto, 1999) or $U(32)$ (Sagnotti, 1995)

Brane SUSY breaking

SUSY ✓ in 10d bulk + SUSY ✗ on branes (Antoniadis, Dudas, Sagnotti, 1999)

Low-energy EFT: **goldstino + couplings** (Dudas, Mourad; Pradisi, Riccioni, 2000)

O9 + 32 $\overline{\text{D9}}$: **residual tension** $V(\phi) = T\, e^{\gamma\, \phi}$

No tachyons but back-reaction!

Low-energy description

$$\mathcal{S}_{\text{eff}} = \int d^{10}x \, \sqrt{-g} \left(\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{e^{\alpha\phi}}{12} \, H_3^2 + \ldots \right)$$

Taming back-reaction

IR ingredients: electric and magnetic fluxes ($H_3 = dC_2$ or dB_2)

UV ingredients: p-branes (Dudas, Mourad, Sagnotti, 2001)

Compactifications

Maximally symmetric space-time X

$$ds_{10}^2 = e^{2u(y)}\, ds_X^2 + ds_Y^2$$

No-go theorem (IB, Lanza, 2020)

$$\boxed{\Lambda_X \propto \left(1 - (q-1)\,\frac{\gamma}{\alpha}\right) \int_Y dy\,\sqrt{g_Y}\, e^{2cu(y)}\, V(\phi)}$$

$|\partial \mathcal{V}| \geq \mathcal{O}(1)\, \mathcal{V}$: dS conjecture holds!

...AdS compactifications (Mourad, Sagnotti, 2016)

- Constant dilaton
- $\mathrm{AdS}_3 \times \mathbb{S}^7$ (orientifolds), $\mathrm{AdS}_7 \times \mathbb{S}^3$ (heterotic)
- electric vs magnetic flux N of $H_3 = dC_2$ or dB_2

$$N \gg 1 \qquad \longrightarrow \qquad e^{\phi}\,,\ (\alpha' \mathcal{R}) \ll 1$$

Orientifold models: $\mathrm{AdS}_3 \times \mathbb{S}^7$

Parameters: $V = T\, e^{\frac{3}{2}\phi}$, coupling $\alpha = 1$ to R-R 3-form

$$\text{residual tension: } T = 2k_{10}^2 \times \begin{cases} 64\, T_{\mathsf{D9}} & USp(32) \\ 32\, T_{\mathsf{D9}} & U(32) \end{cases}$$

electric flux

$$N = \int_{\mathbb{S}^7} \star\, e^{\phi}\, H_3$$

(super)gravity regime

$$L_3\,,\, R_7 \propto N^{3/16} \qquad e^{\phi} \propto N^{-1/4}$$

Heterotic model: $\mathrm{AdS}_7 \times \mathbb{S}^3$

Parameters: $V = \Lambda\, e^{\frac{5}{2}\phi}$, coupling $\alpha = -1$ to NS-NS 3-form

1-loop vacuum energy: $\Lambda =$ (modular integral) $= \frac{\mathcal{O}(1)}{\alpha'}$

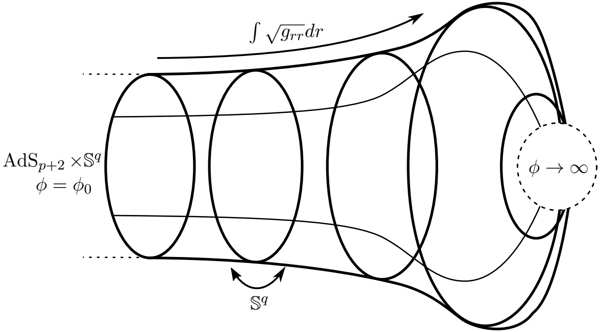
magnetic flux

$$N = \int_{\mathbb{S}^3} H_3$$

(super)gravity regime

$$L_7\,,\, R_3 \propto N^{5/8} \qquad e^{\phi} \propto N^{-1/2}$$

A picture made of branes: near-horizon and pinch-off (Antonelli, IB, 2019)



$SO(1,p) \times SO(q)$ symmetry $\quad \to \quad \phi(r)\,,\, v(r)\,,\, b(r)$

$$ds^2 = e^{\frac{2}{p+1}v - \frac{2q}{p}b}\, dx_{1,p}^2 + e^{2v - \frac{2q}{p}b}\, dr^2 + e^{2b}\, R_0^2\, d\Omega_q^2\,,$$

$$H_{p+2} = \frac{N}{e^{\alpha\phi}(R_0\, e^b)^q}\, e^{2v - \frac{q}{p}(p+2)b}\, d^{p+1}x \wedge dr$$

Perturbative instabilities (IB, Mourad, Sagnotti, 2018)

AdS vacua: unstable scalar KK modes *(BF-bound violations)*

Orientifolds	$\longrightarrow$	$\ell = 2\,,\,3\,,\,4$	$\mathrm{AdS}_3 \times \mathbb{S}^7$
Heterotic	$\longrightarrow$	$\ell = 1$	$\mathrm{AdS}_7 \times \mathbb{S}^3$

Workarounds:

$\mathbb{S}^q \longrightarrow \mathcal{M}_q$ ...need spectrum of $\Delta_{\mathcal{M}_q}$

Orbifold? ...ok for $\mathbb{S}^3$, hard for $\mathbb{S}^7$

Non-perturbative instabilities: brane picture (Antonelli, IB, 2019)

AdS vacua $\rightarrow$ *flux tunneling* (Brown, Teitelboim, 1987-1988), (Blanco-Pillado, Schwartz-Perlov, Vilenkin, 2009)

$$\mathcal{E}_{\text{vac}} \propto -N^{-3} \quad \text{or} \quad -N^{-2} \qquad N \longrightarrow N - \delta N \quad : \quad \text{out of EFT}$$

Instantons $\longleftrightarrow$ branes (D1 or NS5)? right charge & dim.

AdS $\longrightarrow$ **near-horizon** of brane stack...

$\longrightarrow$ brane-antibrane nucleation

$$S^E_{\text{brane}} = \left[\tau_p \, \text{Area} - \frac{N \mu_p}{e^{\alpha\phi} \, R^q} \, \text{Vol}\right]_{\text{extremum}} = B_{\text{CdL}}$$

Consistency: the right branes

$$S^E_{\text{brane}} = \tau_p\, \Omega_{p+1}\, L^{p+1} \left[\frac{1}{(\beta^2-1)^{\frac{p+1}{2}}} - \frac{p+1}{2}\, \beta \int_0^{\frac{1}{\beta^2-1}} \frac{x^{\frac{p}{2}}}{\sqrt{1+x}}\, dx \right]$$

Consistency:

- *existence*: nucl. parameter $\beta \equiv v_0 \left(\frac{\mu_p}{\tau_p}\right) g_s^{-\frac{\alpha}{2}} > 1$

- *semi-classical:* $\beta = \mathcal{O}(N^0) \longrightarrow \tau_p = T_p\, g_s^{-\frac{\alpha}{2}}$

$$\tau_p^{\text{string}} = \frac{T_p}{g_s^{\sigma}} \qquad \boxed{\sigma = 1 + \frac{1}{2}\, \alpha^{\text{string}}_{\text{electric}}}$$

After tunneling: Lorentzian evolution

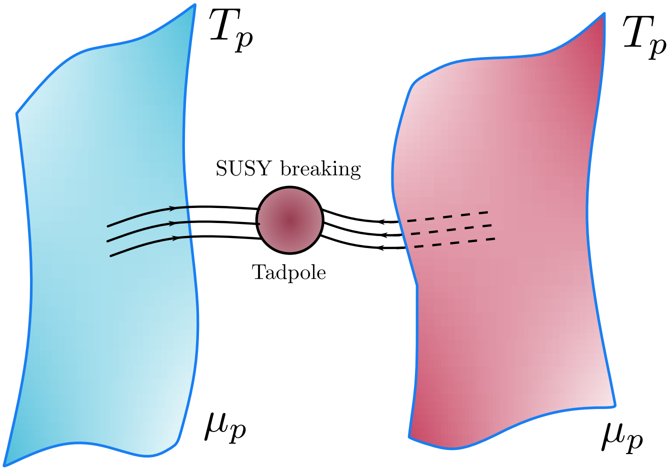
probe $p/\bar{p}$-brane in (Poincaré) AdS throat at pos. Z:

$$V_{\text{probe}} = \tau_p \left(\frac{L}{Z}\right)^{p+1} \left(1 \pm v_0 \, \frac{\mu_p}{T_p}\right)$$

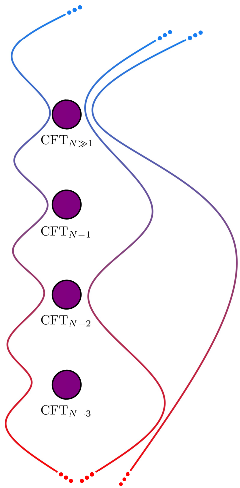
for our string models: $v_0 > 1$!

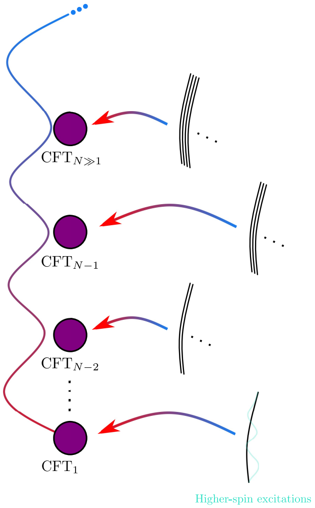
$$\boxed{\textbf{WGC} : \nearrow \left(\frac{\text{charge}}{\text{tension}}\right)_{\text{eff}} \nearrow}$$

$\longrightarrow$ these non-SUSY branes **feel the right forces**

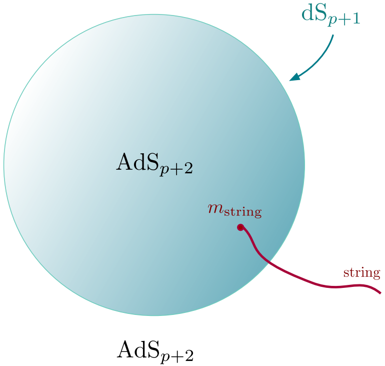

T_p
T_p
SUSY breaking
Tadpole
μ_p
μ_p

Holographic interpretation (Antonelli, Basile, IB, 2018)





Riding the bubble: de Sitter cosmology



World-volume geometry

$$ds^2_{\textsf{bubble}} = -\,dt^2 + a^2(t)\,d\Omega_p^2$$

Junction conditions

Friedmann equations for bubble radius

$$\left(\frac{\dot{a}}{a}\right)^2 = -\,\frac{1}{a^2} + \frac{\beta^2 - 1}{L^2}$$

Hints of EFT (IB, Lanza, 2020)

Brane bending $\longrightarrow$ **Einstein eqs.** (Banerjee, Danielsson, Dibitetto, Giri, Schillo, 2019)

Brane action vs Einstein-Hilbert: $$M_{\mathrm{dS}}^{1-p} = \beta\,(\beta+1)\,\frac{M_{\mathrm{AdS}}^{-p}}{\delta L_{\mathrm{AdS}}}$$

$$\boxed{\frac{\mathcal{E}_{\mathrm{dS}}}{M_{\mathrm{dS}}^{p+1}} \propto N^{1-\frac{\gamma\left(1+\frac{q}{p}\right)}{(q-1)\gamma-\alpha}} \ll 1}$$

Extra stuff: matter, radiation, curvature corrections...

Outlook

- ~~SUSY~~ brane interactions $\longrightarrow$ matching in **various regimes** ✓

  ➔ *Probe, amplitudes, near-horizon geometry, gauge theory*

- **Swampland implications** ✓

  ➔ *Weak gravity, distance conjecture, emergent strings, dS vacua...*

- **Non-extremal** branes: back-reaction & fine-tuning

- Explore **dS brane-world** EFT *("Swampland on the brane"?)*

Take-home message

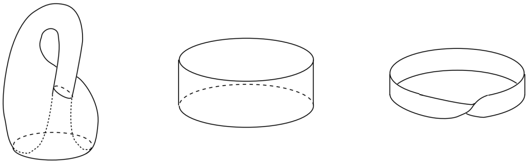
(Natural) SUSY breaking $\longrightarrow$ Dynamics $\longrightarrow$ Phenomenology?

Backup slides

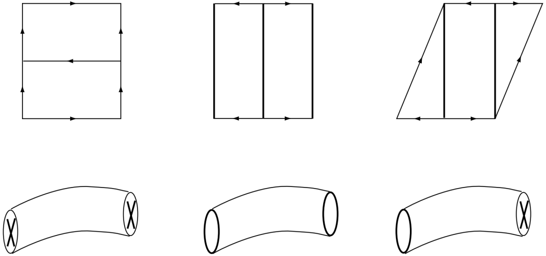
Heterotic torus amplitude

$$\mathcal{T}_{SO(16)\times SO(16)} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^6} \frac{1}{|\eta(\tau)|^{16}} \Big[O_8\, \overline{(V_{16}\, C_{16} + C_{16}\, V_{16})} + V_8\, \overline{(O_{16}\, O_{16} + S_{16}\, S_{16})} \\ - S_8\, \overline{(O_{16}\, S_{16} + S_{16}\, O_{16})} - C_8\, \overline{(V_{16}\, V_{16} + C_{16}\, C_{16})} \Big]$$

- No massless states (**level matching**)
- Gravitational sector + adjoint vector
- $(\mathbf{128}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{128})$ spinor
- $(\mathbf{16}, \mathbf{16})$ spinor



Klein bottle Annulus Mӧbius strip



Orientifold amplitudes

$$\mathcal{K} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^6}\,\frac{(V_8 - S_8)(2i\tau_2)}{\eta^8(2i\tau_2)}\,, \qquad \mathcal{A} = \frac{N^2}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^6}\,\frac{(V_8 - S_8)\left(\frac{i\tau_2}{2}\right)}{\eta^8\left(\frac{i\tau_2}{2}\right)}$$

Orientifold of IIB $\longrightarrow$ Klein bottle, annulus, **Möbius strip** $(N = 32)$

$$\mathcal{M} = \frac{\varepsilon\, N}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^6}\,\frac{\left(\widehat{V}_8 - \widehat{S}_8\right)\left(\frac{i\tau_2}{2} + \frac{1}{2}\right)}{\widehat{\eta}^8\left(\frac{i\tau_2}{2} + \frac{1}{2}\right)}\,, \qquad \mathcal{M}_{\text{BSB}} = \frac{\varepsilon\, N}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^6}\,\frac{\left(\widehat{V}_8 + \widehat{S}_8\right)\left(\frac{i\tau_2}{2} + \frac{1}{2}\right)}{\widehat{\eta}^8\left(\frac{i\tau_2}{2} + \frac{1}{2}\right)}$$

Tadpoles and back-reaction

String-scale back-reaction from tadpoles

Workarounds or alternatives?

- Non-tachyonic compactification of **tachyonic** models (Faraggi, Matyas, Percival, 2019-2021)
- **Misaligned SUSY** (Dienes, 1994-2001) (Cribiori, Parameswaran, Tonioni, Wrase, 2020)
- Suppression of vacuum energy (Dienes, 1990) (Kachru, Kumar, Silverstein, 1999) (Angelantonj, Cardella, 2004)

  ➔ *Exponential:* (Abel, Dienes, Mavroudi, Stewart, 2015-2017)

- New expansion parameters (fluxes)

Perturbations and mixings

Linearized analysis: AdS tensors + angular momenta ℓ (IB, Mourad, Sagnotti, 2018)

- **Tensors:** no mixing $\longrightarrow$ stable ✓
- **Vectors:** mixing, still stable ✓ $\qquad \delta g_{\mu i} \qquad \delta B_{\mu i}$
- **Scalars:** Einstein eqs. $\longrightarrow$ 2 constraints!

$$\delta\phi \qquad \delta B_2 = \star_3 \, dB$$

$$\delta g_{\mu\nu} = A \, g^{(0)}_{\mu\nu} \qquad \delta g_{ij} = C \, g^{(0)}_{ij} \qquad \delta g_{\mu i} = \nabla_\mu \nabla_i D$$

Linearized scalar equations: orientifold case

$$L_3^2\,\Box\,A - \left[4 + 3\,\sigma_3 + \frac{\ell_3}{3}\,(\sigma_3 - 1)\right] A + \frac{7}{2}\,\alpha\,\sigma_3\,\delta\phi - \frac{\ell_3}{2}\,(\sigma_3 - 1)\,B = 0$$

$$L_3^2\,\Box\,\delta\phi + 2\,\alpha\,\sigma_3\,A - \left[2\,\alpha^2\,\sigma_3 + \tau_3 + \frac{\ell_3}{3}\,(\sigma_3 - 1)\right] \delta\phi + \alpha\,\frac{\ell_3}{3}\,(\sigma_3 - 1)\,B = 0$$

$$L_3^2\,\Box\,B - 8\,\sigma_3\,A + 4\,\alpha\,\sigma_3\,\delta\phi - \frac{\ell_3}{3}\,(\sigma_3 - 1)\,B = 0$$

where:

$$\ell_3 = \ell\,(\ell + 6) \qquad \sigma_3 = 1 + 3\,\frac{L_3^2}{R_7^2} \qquad \tau_3 = L_3^2\,V_0''$$

Linearized scalar equations: heterotic case

$$L_7^2\,\Box\,A - [\ell_7\,(\sigma_7-3)+5\,\sigma_7+12]\,A + \frac{5}{2}\,\alpha\,\sigma_7\,\delta\phi - \frac{3\,\ell_7}{2}\,(\sigma_7-3)\,B = 0$$

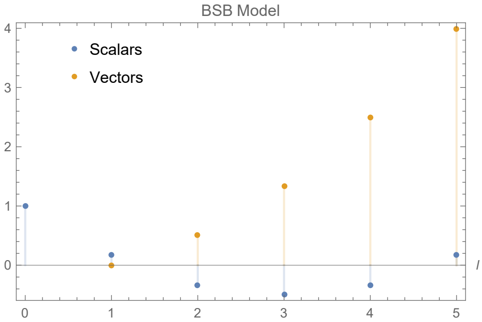
$$L_7^2\,\Box\,\delta\phi + 6\,\alpha\,\sigma_7\,A - \left[2\,\alpha^2\,\sigma_7+\tau_7+\ell_7\,(\sigma_7-3)\right]\delta\phi + \alpha\,\ell_7\,(\sigma_7-3)\,B = 0$$

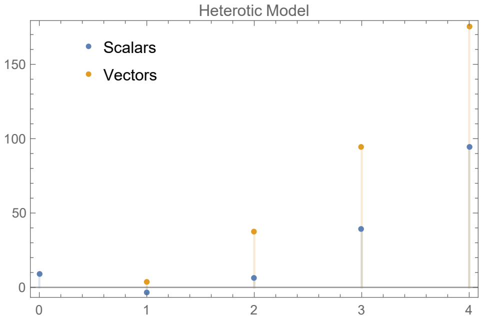
$$L_7^2\,\Box\,B - 8\,\sigma_7\,A + 4\,\alpha\,\sigma_7\,\delta\phi - \ell_7\,(\sigma_7-3)\,B = 0$$

where:

$$\ell_7 = \ell\,(\ell+2) \qquad \sigma_7 = 3 + \frac{L_7^2}{R_3^2} \qquad \tau_7 = L_7^2\,V_0''$$

Results: violations of BF bounds





Orbifolds: *can get rid of unstable modes...* $\longrightarrow$ *vacuum bubbles?* (Horowitz, Orgera, Polchinski, 2008)

Non-perturbative instabilities: flux tunneling

- Gravity in $D = p + 2 + q$ dims + fluxes: $\mathbb{S}^q$ reduction

$$ds^2 = R^{-\frac{2q}{p}}\, ds^2_{p+2} + R^2\, d\Omega_q^2$$

- Reduced action: $(p+2)$-Einstein frame

$$\mathcal{S}_{p+2} = \frac{1}{2\kappa^2_{p+2}} \int d^{p+2}x \sqrt{-g_{p+2}} \left(\mathcal{R}_{p+2} - 2\Lambda\, R^{-\frac{2q}{p}}\right)$$

vacuum energy $\longrightarrow \mathcal{E}_{\text{vac}} \propto -R^{-\frac{2q}{p}-2}$

$\mathcal{E}_{\text{vac}}$ depends on flux...

...higher-dim. instantons, flux transitions (Blanco-Pillado, Schwartz-Perlov, Vilenkin, 2009)

Many branes: background geometry

$SO(1,p) \times SO(q)$ symmetry: $\phi(r)\,,\ v(r)\,,\ b(r)$

$$ds^2 = e^{\frac{2}{p+1}v - \frac{2q}{p}b}\,dx^2_{1,p} + e^{2v - \frac{2q}{p}b}\,dr^2 + e^{2b}\,R_0^2\,d\Omega_q^2$$

$$H_{p+2} = c\,e^{2v - \frac{q}{p}(p+2)b}\,d^{p+1}x \wedge dr\,, \qquad c \equiv \frac{N}{e^{\alpha\phi}(R_0\,e^b)^q}$$

(Constrained) Toda-like system: $(+,-,+)$ kinetic term w/ potential

$$\boxed{U = -\,T\,e^{\gamma\phi + 2v - \frac{2q}{p}b} - \frac{n^2}{2R_0^{2q}}\,e^{-\alpha\phi + 2v - \frac{2q(p+1)}{p}b} + \frac{q(q-1)}{R_0^2}\,e^{2v - \frac{2(D-2)}{p}b}}$$

Geometry: near-horizon

Recover original $\mathrm{AdS}_{p+2} \times \mathbb{S}^q$ with $[r < 0]$

$$\phi = \phi_0 \, , \qquad e^v = \frac{L}{p+1} \left(\frac{R}{R_0} \right)^{-\frac{q}{p}} \frac{1}{-r} \, , \qquad e^b = \frac{R}{R_0}$$

Radial perturbations: $\quad \delta\phi \, , \, \delta v \, , \, \delta b \propto (-r)^\lambda$

$$\{\lambda\}_{\text{orient}} = \left\{ -1 \, , \, \frac{1 \pm \sqrt{13}}{2} \, , \, \frac{1 \pm \sqrt{5}}{2} \right\} \, , \; \{\lambda\}_{\text{het}} = \left\{ -1 \, , \, \pm 2\sqrt{\frac{2}{3}} \, , \, 1 \pm 2\sqrt{\frac{2}{3}} \right\}$$

$\longrightarrow$ 2 *extremality-breaking* deformations

[2 asymptotic fine-tunings?]

Geometry: "far-horizon"

Away from branes $[r > 0]$: assume $U \sim U_T = -T\, e^{\gamma\phi + 2v - \frac{2q}{p} b}$

Solutions as $r \to \infty$: $\phi\,, v\,, b \propto y(r)$ + subleading

$$y'' \sim \hat{T}\, e^{\Omega\, y + L\, r}$$
$$\frac{1}{2}\,\Omega\, {y'}^2 + L\, y' \sim \hat{T}\, e^{\Omega\, y + L\, r} - M$$

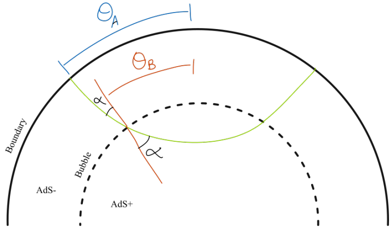
where $\Omega = \frac{D-2}{8}\,\gamma^2 - \frac{2(D-1)}{D-2} = \frac{D-2}{8}\left(\gamma^2 - \gamma^2_{\text{crit}}\right)$ (IB, Mourad, Sagnotti, 2018)

➔ Orientifolds: $\phi\,,\ v\,,\ b \propto r^2$ (due to $\Omega = 0$)

➔ Heterotic: $\phi\,,\ v\,,\ b \propto \log(r_0 - r)$

Our check: bubble entanglement entropy (in 3d)

$$ds_\pm^2 = L_\pm^2 \left(-\cosh^2\rho_\pm\, d\tau_\pm^2 + d\rho_\pm^2 + \sinh^2\rho_\pm\, d\phi_\pm^2\right)$$

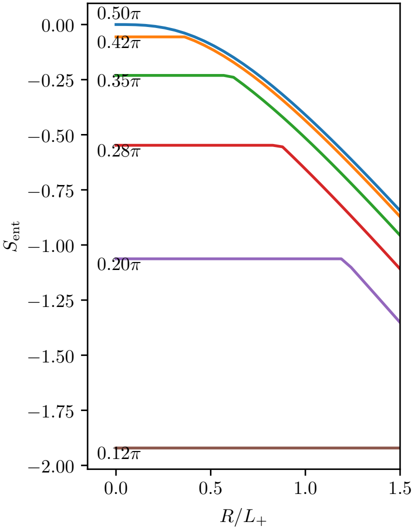


Thin-wall: geodesic is hyperbolic polygonal

$$\text{length} = 2L_+\Lambda + 2L_+\log(\cosh\rho_+ - \sinh\rho_+\cos(\theta_B - \theta_A))$$
$$+ L_-\cosh^{-1}(\cosh^2\rho_- - \sinh^2\rho_-\cos(2\theta_B)) + \mathcal{O}(\Lambda^{-1})$$

➔ Angle at bubble θ_B: **no-kink condition**

c-functions and “bubble RG”



Finite part of $S_{\text{ent}}(\theta_A, R)$ decreases with bubble radius: candidate c-**function**!

Check: trace anomaly

(Henningson, Skenderis, 1998)

$$\langle T^\mu{}_\mu \rangle = \frac{c_{\text{ent}}(R)}{12}\mathcal{R}$$

Moving the bubble:
can use integral geometry

(Antonelli, IB, Bombini, 2018)

Moving the bubble: a primer of integral geometry

Crofton theorem: *space of all lines* $\mathcal{K}$ (Crofton, 1968)

$$\text{length}(\gamma) = \frac{1}{4}\int_{\mathcal{K}} n_{\gamma,\kappa}\,\omega(\kappa) \leftarrow \text{Crofton form}$$

Asymptotically $\mathbb{H}^2$ slices

$$\omega_{\text{bubble}} = \Omega\,\omega_{\mathbb{H}^2}$$

moving bubble via isometry

$\longrightarrow \Omega$ **scalar!**