CICY orientifolds and GV invariants

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Based on...

1. A landscape of orientifold vacua
    - F.C, J. Moritz, A. Westphal. 2020
2. Gopakumar-Vafa hierarchies in winding inflation and uplifts
    - F.C, A. Mininno, N. Righi, A. Westphal. 2021

Motivation

- One of the main goals of String Phenomenology is to make concrete string models which:
  1. Reproduce the esablished low-energy physics (SM, ΛCDM...)
  2. Extend the esablished low-energy physics (DM, SUSY, Inflation...)
- Type II string theory on $\mathbb{R}^{1,3} \times X_6$, with X_6 compact CY 3-fold.
- 4d EFT is $\mathcal{N} = 2$ SUGRA. More realistic $\mathcal{N} = 1$ EFTs arise from modding out by an orientifold action $\Omega\mathcal{R}(-1)^{F_L}$
- Focus on IIB, with orientifold action allowing for $O3/O7$ planes. Typical setup for the flux landscape (GKP '01. KKLT '03)
- Properties of the CY orientifold fix properties of the low energy 4d EFT.

Even and odd cohomology

The orientifold action induces a splitting in cohomology

$$H^{p,q}(X) = H^{p,q}_{+}(X) \oplus H^{p,q}_{-}(X)$$

$h^{p,q}_{\pm}$ counts the number of fields in the 4d EFT (Grimm, Louis '04).

- $h^{2,1}_{-}$ complex structure moduli. Stabilized by fluxes.
- $h^{1,1}_{-}$ C_2 and B_2 axions. Usually light (DM? Inflation?)
- $h^{2,1}_{+}$ U(1) vector multiplets. (Dark photons?)
- $h^{1,1}_{+}$ kahler moduli. Stabilized by non-perturbative effects.

Few orientifolds with $h^{1,1}_{-} \neq 0$ appear in the literature (Gao, Shukla '13). This motivated us to systematically look for such cases, on a large scale search.

Complete intersection CYs

- Defined as the zero locus of a set of homogeneous polynomials $p_i[z], (i = 1, \dots K)$ in an ambient space $\mathcal{A} = \mathbb{P}^{n_1} \times \dots \mathbb{P}^{n_s}$
- Multidegrees of $p_i[z]$ encoded in the configuration matrix.

x^i	$\mathbb{P}^2$	0	2	0	1
y^i	$\mathbb{P}^2$	2	1	0	0
w^i	$\mathbb{P}^3$	1	1	1	1

$$\sum_{i=1}^{s} n_i - K = 3$$

$$\begin{aligned} p_1[z] &= a_{ijk} y^i y^j w^k, \\ p_2[z] &= b_{ijkl} x^i x^j y^k w^l, \\ p_3[z] &= c_i w^i, \\ p_4[z] &= d_{ij} x^i w^j. \end{aligned} \tag{1}$$

- List of at most 7890 (possibly) distinct CY constructed in this way. (Candelas, Dale, Lutken, Schimmrigk '87) (Green, Hubsch, Lutken '89) (Anderson, Gao, Gray, Lee '17)

Looking for geometric involutions $\mathcal{R}$

- Consider all ambient space involutions
  1. Swap of identical $\mathbb{P}^n$ factors
  2. Invert some coordinates ($z \to -z$) of non-swapped $\mathbb{P}^n$ factors.
- Consider all the ways in which the $p_i[z]$ can map into each other.
  1. $p_i[z] \to \pm p_i[z]$
  2. $p_i[z] \to p_j[z]$

Most choices give a singular manifold. Many others are equivalent to each other. Ex. Take $\mathbb{P}^2$ and consider the two $\mathbb{Z}_2$ actions

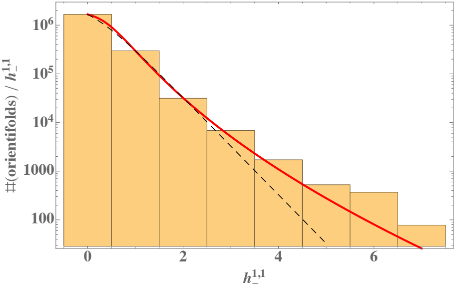
$$[z_0, z_1, z_2] \to [-z_0, z_1, z_2] \tag{2}$$

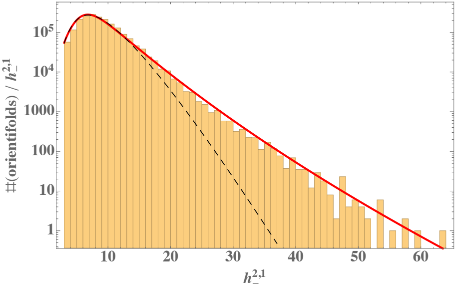
$$[z_0, z_1, z_2] \to [z_0, -z_1, -z_2] \tag{3}$$

They are equivalent because of the usual $\mathbb{P}^2$ scaling.
Write a code to list all inequivalent involutions.

Results

Explicit database of **2.004.513 orientifolds**. List available at www.desy.de/∼westphal/orientifold webpage/cicy_orientifolds.html





Histograms for the number of orientifolds with non-zero $h^{1,1}_-$ and $h^{2,1}_-$. $h^{1,1}_-$ appears to be quite generic! Good for model building with B_2 and C_2 axions!

Singularities at codimension 3: frozen conifolds

- Singularities at codimension 1, and codimension 3.
- We identify and drop from the database the cases of singularities of codimension 1. Non-trivial implementation for this.
- Codimension 3 singularities are interesting: conifold points that lie on the fixed surfaces, i.e. conifold points on top of the O-planes.
- Let f^i_- be a set of antisymmetric polynomials across the fixed locus, whenever $df^1_-[z] \wedge df^2_-[z] \wedge ... = 0$ we have such singularities.
- The orientifold projects out the deformation branch.

$$\det \begin{pmatrix} x & v \\ u & y \end{pmatrix} = \epsilon \tag{4}$$

$(v, y) \to (-v, -y)$ is a symmetry only for $\epsilon = 0$.

Resolving frozen conifolds

- Let $[\alpha, \beta]$ be the homogeneous coordinates of the resolution $\mathbb{P}^1$.
- **A-type resolution** (locally)

$$\begin{pmatrix} x & v \\ u & y \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0 \tag{5}$$

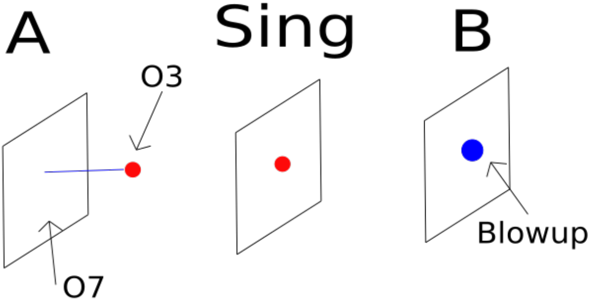
- **B-type resolution** (locally)

$$\begin{pmatrix} x & u \\ v & y \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0 \tag{6}$$

- A-type resolution. Action of the orientifold $[\alpha, \beta] \to [-\alpha, \beta]$. Two fixed points: $[1, 0], [0, 1]$. Now, $[1, 0]$ lies on the fixed divisor, while $[0, 1]$ is isolated fixed point. This is an $O7$-plane on $\mathbb{C}^2$, plus a distant $O3$.
- B-type resolution. Action of the orientifold $[\alpha, \beta] \to [\alpha, \beta]$. Whole resolution $\mathbb{P}^1$ is fixed. This is an $O7$-plane on $\mathbb{C}^2$ blown up at a point.

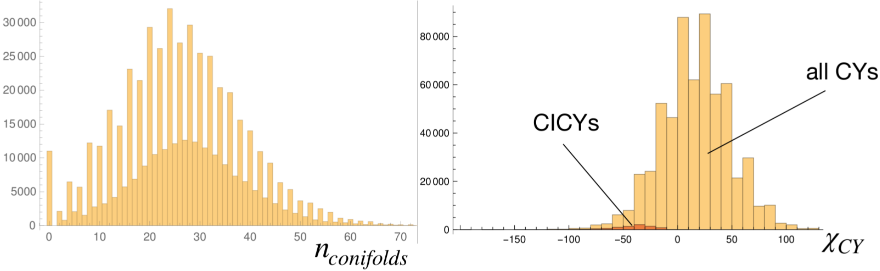
Frozen conifold transitions

- A distant $O3$ approaches the $O7$. We are in the resolved A-side.
- The $O3$ on top of the $O7$. Singular frozen conifold point on the $O7$.
- The $O7$ eats the $O3$, converting it into geometry. Now the topology of the fixed divisor wrapped by the $O7$s, there is one blowup. We are in the resolved B-side.
- Notice it is consistent with modification of $D3$ tadpole.



Frozen conifold transitions

- One can access the resolution (Higgs) branch, in two ways.
- How generic? Almost all orientifolds have non-deformable conifolds, whose resolution branches are often outside the CICY set.



- Notice the χ plot is now symmetric. Are we connecting CICYs to mirror CICYs in such a way? Or maybe a different and new class of CYs?

CICY redundancies

- Some of the configuration matrices in the original CICY list define the same CY
- **Wall's Theorem.** If two CY X and Y are such that
  1. $h^{1,1}(X) = h^{1,1}(Y)$
  2. $h^{2,1}(X) = h^{2,1}(Y)$
  3. There exist a choice of bases for $H^2(X,\mathbb{Z})$ and $H^2(Y,\mathbb{Z})$ such that $\int_{D_i} c_2(X) = \int_{D_i} c_2(Y), \ \forall i = 1, ..., h^{1,1}$
  4. In the same base as above, $K_{ijk}(X) = K_{ijk}(Y)$

  then the two CY are diffeomorphic as real manifolds.
- Idea: pick two CICYs with same $h^{1,1}$ and $h^{2,1}$ and naively different $\int_{D_i} c_2$ and K_{ijk}. Find a change of basis to make these quantities equal, or show it can't exist.
- Write a code for this. Find and list 1169 redundancies. Extending the results of (Candelas, He, '90) and (Anderson, He, Lukas, '08)

Genus zero GV invariants

- Topological invariants of the CY 3-fold X. (Gopakumar, Vafa '98)
- Count the number of holomorphic maps from a Riemann surface Σ_g to a 2-cycle class $\beta \in H_2(X)$
- Good to know the value, for pheno model building. I.e. Kahler potential for cs moduli z^i at LCS is

$$
\begin{aligned}
K = & -\ln\Bigg(-\frac{4}{3}k_{ijk}\mathrm{Im}(z^i)\mathrm{Im}(z^j)\mathrm{Im}(z^k) + ic \\
& -2\sum_{\beta}^{\infty} n^0_\beta \left(\mathrm{Li}_3(e^{i\beta_i z^i}) + \mathrm{Li}_3(e^{-i\beta_i \bar{z}^i})\right) + \\
& -2\sum_{\beta}^{\infty} n^0_\beta \beta_i \mathrm{Im}(z^i)\left(\mathrm{Li}_2(e^{i\beta_i z^i}) + \mathrm{Li}_2(e^{-i\beta_i \bar{z}^i})\right)
\end{aligned}
\tag{7}
$$

Computing the genus 0 GV by Mirror symmetry

(Hosono, Klemm, Theisen, Yau, '94)

- $w_0(z) = \sum_{n_1 \geq 0} \cdots \sum_{n_{h^{2,1}} \geq 0} c(n) \prod_{i=1}^{h^{2,1}} z_i^{n_i}$.
  Generic solution of the PF equation for the first entry of the period vector, in terms of data in the configuration matrix.
- $\Pi(z) = \left(w_0(z), \frac{\partial}{\partial \rho_i} w_0(z, \rho) \mid_{\rho=0}, \ldots \right)^t$
- $w_i(z) =$
  $\sum_{n_1 \geq 1} \cdots \sum_{n_{h^{2,1}} \geq 0} \frac{1}{2\pi i} \frac{\partial}{\partial \rho^i} c(n+\rho) \mid_{\rho=0} \prod_{i=1}^{h^{2,1}} z_i^{n_i} + w_0(z) \frac{\ln z_i}{2\pi i}$
- $t^i = \frac{w_i(z)}{w_0(z)}$ mirror map. Relation between Kahler moduli at large radius in the A-model side, with complex structure moduli at LCS in the B-model side.
- Now, invert the mirror map in order to get $z(t)$. Hardest step.

Computing the genus 0 GV by Mirror symmetry

(Hosono, Klemm, Theisen, Yau, 94)

- Having $z(t)$, compute the quantum corrected triple intersection number

$$k_{ijk} = \frac{\partial}{\partial t_i}\frac{\partial}{\partial t_j}\frac{\frac{1}{2}k^0_{kab}\frac{\partial}{\partial \rho_a}\frac{\partial}{\partial \rho_a}w_0(z,\rho)\mid_{\rho=0}}{w_0(z)}(t)$$

- Introduce $q_i = \exp(2\pi i t_i)$, and the general expression for the quantum corrected triple intersection number

$$k_{ijk} = k^0_{ijk} + \sum_{n_1\geq 1}\dots\sum_{n_{h^{2,1}}\geq 0} n_{d_1,\dots,d_{\bar{h}^{1,1}}} d_i d_j d_k \frac{\prod_{l=1}^{\bar{h}^{1,1}} q_l^{d_l}}{1-\prod_{l=1}^{\bar{h}^{1,1}} q_l^{d_l}}$$

- Match and extract the GV.

Instanton program, and the scan

- The above algorithm has been coded in a program called Instanton. (Klemm, Kreuzer)
- Made modification of their code, parallelized it, and let it run.
- Computing time $\approx$ 6 months, on two different clusters (DESY and Madrid IFT)
- List all the genus 0 GV, up to total degree 10, for all favourable CICY up to $h^{1,1} = 9$.
- We find directions in the Picard lattice in which the GV invariants grow at hierarchical rates, as well as "vanishing directions" (Demirtas, Kim, McAllister, Moritz '20) and "periodic directions".
- List at www.desy.de/∼westphal/GV_CICY_webpage/GVInvariants.html

Occupation sites for CICY 7858 ($h^{1,1} = 2$)

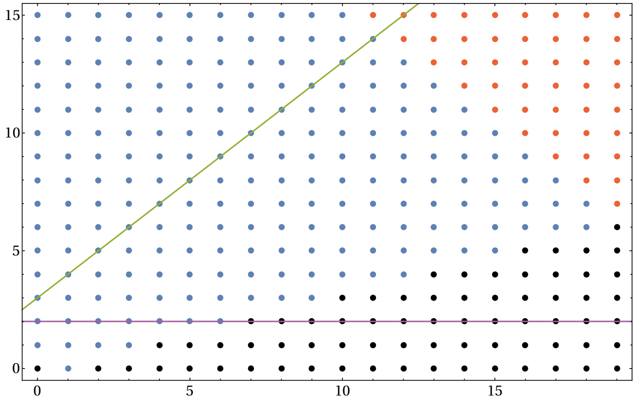


Figure: Blue=non-zero GV. Black = zero GV. Orange = not computed, believed to be non-zero GV. Green = non-vanishing direction. Purple = vanishing direction.

Conclusions

1. We compute a database of all the $O3/O7$ orientifolds of the CICYs that descend from the ambient space involutions. We compute quantities as the $D3$ tadpole, the number of frozen conifolds, etc.
2. We observe the phenomenom of frozen conifold transitions, connectiong the CICYs to a larger set of (potentially new) CYs.
3. We compute a database of all the genus 0 GV invariants for CICYs up to $h^{1,1} = 9$ and total degree 10.
4. We observe the presence of directions in the Picard lattice where the GV growth is hierarchically dominant/suppressed with respect to other directions, as well as periodic directions.
5. We identifiy 1169 redundancies in the CICY list.

Future directions

- Would be nice to do similar scans for Kreuzer-Skarke.
- Are frozen conifold transitions the generic $\mathcal{N} = 1$ geometric transitions? Or are they special to CICYs? Can we see them in Kreuzer-Skarke?
- Compute the topology of the divisors for the CICY orientifolds (important to know which 4-cycles will support D7s for gaugino condensation or ED3)
- Suppose you realize a given CICY by a different, much larger configuration matrix. Do you find more orientifolds?
- Would be nice to do the orientifold scan for the CICYs, for IIA orientifolds. Much harder as the orientifold action is not now-holomorphic. i.e. fixed loci at real codimension 3.