[Cvetic, Heckman, Rochais, Torres, GZ: 2003.13682]

GIANLUCA ZOCCARATO

GEOMETRIC UNIFICATION OF HIGGS BUNDLE VACUA – PART I

July 14th, 2020

Summer Series on String Phenomenology

INTRODUCTION

Rich interplay between string theory and geometry

Central players are special holonomy manifolds

- Calabi-Yau threefolds: type I, heterotic, type II (with branes)
- Calabi-Yau fourfolds: F-theory
- G_2 manifolds: M-theory

One outlier: Spin(7) manifolds

M-theory on Spin(7) manifolds give N=1 susy in 3D

[Bonetti, Grimm, (Palti), Pugh '14]

However it can lead to interesting cosmological solutions

[Heckman, Lawrie, Ling, (Sakstein), GZ '18-'19]

INTRODUCTION

Construction compact of Spin(7) manifolds (and G_2 manifolds) is notoriously difficult

[Kovalev '03]
[Corti, Haskins, Nordström, Pacini '13-'15]
[Braun, Schäfer-Nameki '18]

The general advantage of type II strings (and their strong coupling limits) is that they contain localised gauge sectors

In this talk: study localised gauge sectors for M-theory on Spin(7) manifolds

This system provides a unification of other known local systems

It can be used to build interfaces between 4D vacua

LOCAL SYSTEMS

Give the gauge sector localised on branes

Lagrangian and susy transformations can be obtained by truncation of 10D SYM

$$S = \int d^{10}x \, \mathrm{Tr} \left(\frac{1}{2} F_{MN} F^{MN} - i \overline{\Psi} \Gamma^M D_M \Psi \right)$$

$$\delta A_M = i\overline{\varepsilon}\,\Gamma_M \Psi \qquad \delta\Psi = \frac{1}{2}\Gamma^{MN} F_{MN} \varepsilon$$

After truncation: gauge field and adjoint scalars ("transverse directions") plus fermions

Place systems on curved manifolds and consider susy configurations

7D SYM

Start from 10D SYM and truncate in 3 directions

Bosonic sector has gauge fields and 3 adjoint scalars

Global symmetries of the system is

R-symmetry

$$Spin(1,6) \otimes Spin(3)_R$$

Gauge interactions on a stack of D6-branes

ADE singularity in M-theory

Next: consider theory on curved four manifold

7D SYM ON A FOUR MANIFOLD

[Heckman, Lawrie, Ling, GZ '18]

We put 7D SYM on a generic four manifold M

Global symmetry breaks as $Spin(1,2) \otimes Spin(4) \otimes Spin(3)_R$

$$\downarrow$$

$$SU(2)_l \otimes \boxed{SU(2)_r}$$

Topological twist: embed R-symmetry in $SU(2)_r$

Scalars become adjoint valued self-dual two forms

System preserves 3D N=1 supersymmetry

Internal geometry: M-theory on local Spin(7), IIA on a G_2 manifold with D6-branes

7D SYM ON A FOUR MANIFOLD

[Heckman, Lawrie, Ling, GZ '18]

BPS equations $D_A\phi = 0$ $F_{\mathrm{SD}} + \phi \times \phi = 0$ [Vafa, Witten '94]

Self-dual part

Cross product is inherited from the three form on $\Omega^2_+(M)$

Equivalently it can be defined as

$$(\phi \times \phi)_{ij} = \frac{1}{4}[\phi_{ik}, \phi_{jl}]g^{kl}$$

Supersymmetry conditions can be derived from 3D superpotential

$$W_{\mathrm{Spin}(7)} = \int_M \mathrm{Tr}\left(\phi \wedge \left[F + \frac{1}{3}\phi \times \phi\right]\right)$$

5D N=1 VACUA

Starting point: 7D SYM (M-theory on ADE singularity or D6-branes)

Place system on complex curve

After twisting: complex adjoint valued 1-form

BPS equations are:

$$\overline{\partial}_A \phi = 0$$

$$F = \frac{i}{2}[\phi, \phi^\dagger]$$

This is the famous Hitchin system

[Hitchin '87]

Can also be obtained using 7-branes (gives 6D N=1 vacua)

4D N=1 VACUA - PART I

[Beasley, Heckman, Vafa '08]

Starting point: 8D SYM (F-theory on ADE singularity or D7-branes)

Place system on a complex surface

After twisting: complex adjoint (2,0)-form

BPS equations are:
$$\overline{\partial}_A \phi = 0 \qquad J \wedge F = \frac{i}{2}[\phi, \phi^\dagger]$$
$$F_{(0,2)} = 0$$

Kähler form

We will call this system BHV system

Extensively used for F-theory model building

Holomorphic structure makes construction of solutions feasible (even non-abelian)

4D N=1 VACUA - PART II

[Pantev, Wijnholt '09]

Starting point: 7D SYM (M-theory on ADE singularity or D6-branes)

Place system on a three-manifold

After twisting: adjoint 1-forms

BPS equations are: $D_A\phi = 0 \qquad D_A * \phi = 0$

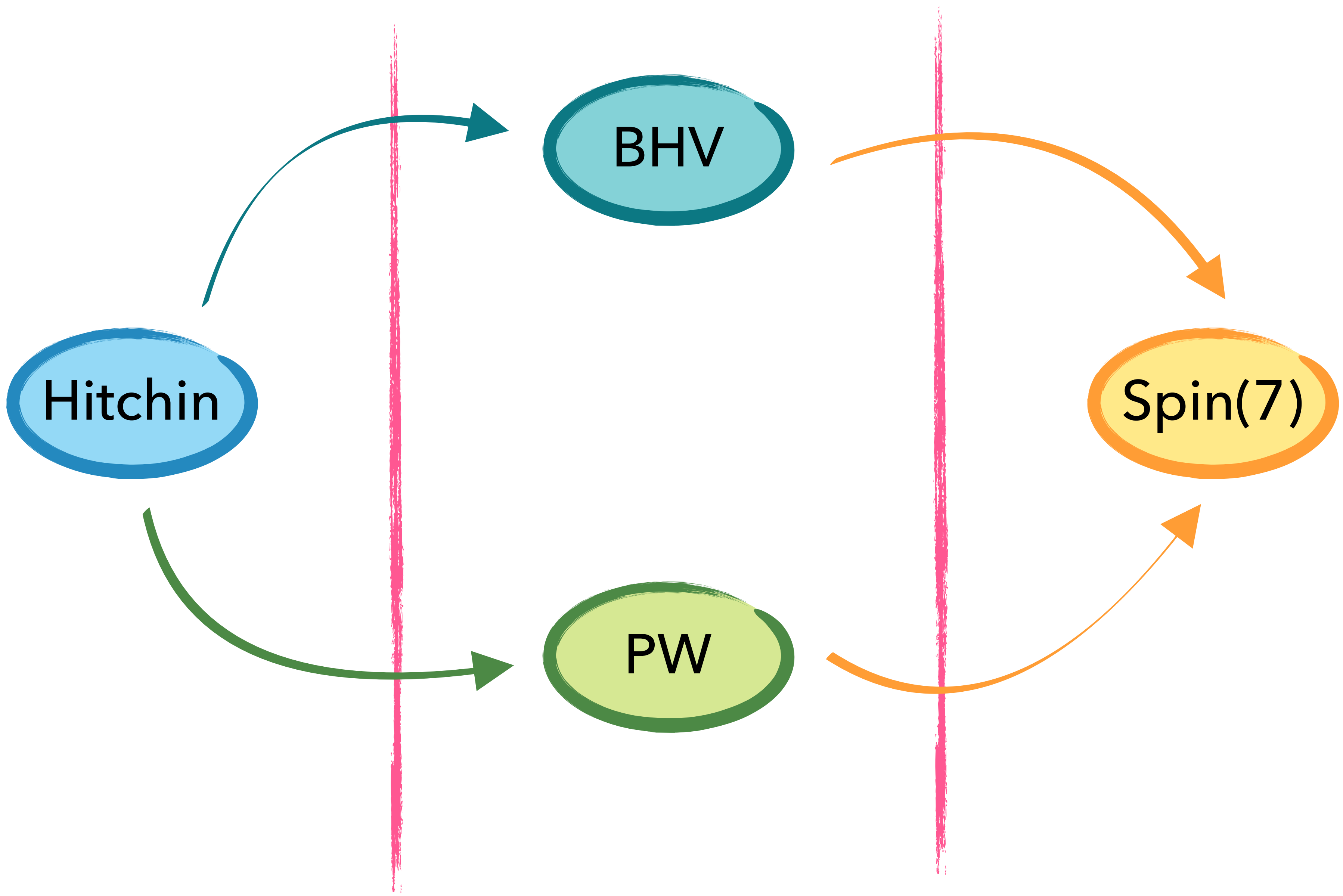
$$F = [\phi, \phi]$$

We will call this system PW system

Difficult to build non-abelian solutions

[Barbosa, Cvetic, Heckman, Lawrie, Torres, GZ '19]

CONNECTIONS BETWEEN SYSTEMS



PW TO SPIN(7)

To relate the two consider the case of Spin(7) on a four-manifold

$$M = Q \times S^1$$

Then write the SD forms as $\phi_{\rm SD} = \phi_{\rm PW} \wedge dt + *_3\, \phi_{\rm PW}$

The Spin(7) BPS equations become

$$F - [\phi_{\rm PW}, \phi_{\rm PW}] + *_3(D_t A - d_3 A_t) = 0$$

$$D_A \phi_{\rm PW} + *_3 D_t \phi_{\rm PW} = 0 \qquad D_A *_3 \phi_{\rm PW} = 0$$

One recovers PW if $A_t = \partial_t A = \partial_t \phi = 0$

PW is the dimensional reduction of Spin(7) along the additional direction

BHV TO SPIN(7)

We take the four manifold to be a Kähler manifold

In a Kähler manifold SD two forms can be decomposed as

$$\phi_{\mathrm{SD}} = \phi_{(2,0)} \oplus \phi_{(1,1)} \oplus \phi^{\dagger}_{(0,2)}$$

Non-primitive

The BPS equations become $F_{(0,2)} - \frac{i}{2}\phi_{(1,1)} \times \phi^{\dagger}_{(0,2)} = 0$

$$\bar{\partial}_A \phi_{(2,0)} - \frac{i}{2}\partial_A \phi_{(1,1)} = 0 \qquad J \wedge F = \frac{i}{2}[\phi_{(2,0)}, \phi^{\dagger}_{(0,2)}]$$

BHV is recovered for configurations with the (1,1)-component set to zero

DOMAIN WALLS IN 4D N=1 THEORIES

The 1/2-BPS equations for a domain wall are

Phase

$$D_t\phi^i = e^{i\eta} G^{i\bar{j}} \partial_{\bar{j}} \overline{W}$$

Chiral multiplets

Superpotential

Kähler metric

Domain walls interpolate between different vacua

Exact moduli (no superpotential term) remain constant

Phase is fixed by values of superpotential in the two vacua

$$\Delta\ \mathrm{Im}\left(e^{-i\eta} W\right) = 0$$

SPIN(7) AS PW DOMAIN WALL

PW superpotential:

$$W = \int_Q \mathrm{Tr}\left(\mathcal{A}\wedge d\mathcal{A} + \frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right) \qquad \mathcal{A} = A + i\phi_{PW}$$

[Pantev, Wijnholt '09]

Domain wall equations are

$$D_t\mathcal{A} = e^{i\eta} *_3 \overline{\mathcal{F}}$$

Expanding in real and imaginary parts one gets the Spin(7) equations choosing a "temporal gauge" and with zero phase

SPIN(7) AS BHV DOMAIN WALL

BHV superpotential:

$$W_{\text{BHV}} = \int \text{Tr}\left(\phi_{(2,0)} \wedge F\right)$$

[Beasley, Heckman, Vafa '08]

One puzzle: both BHV and Spin(7) are realised on a four-manifold

Take all 4D fields constant along the domain wall direction

Breaking the 4D Lorentz group is due to a component of the gauge field along the domain wall direction

Going from BHV to Spin(7) involves one T-duality (usual M-theory to type IIB duality)

$$A_t \sim \phi_{(1,1)}$$

DOMAIN WALLS VS INTERFACES

Spin(7) equations are PW/BHV domain wall equations

But modes that flow are actually massive (presumably at KK scale)

In EFT they are not present (need full 7D/8D theory)

These configurations are better interpreted as interfaces

One implication for EFT is position dependent couplings

$$\mathcal{L}_{\text{eff}} \supset \sum_i c_i(t) \frac{\mathcal{O}_i(x,t)}{\Lambda^{\Delta_i - D}}$$

CONCLUSIONS

Higgs bundles appear in different string vacua

Spin(7) Higgs bundle provides a unification of other systems

1/2-BPS domain walls in PW/BHV obey the Spin(7) system of equations

A better interpretation of these configurations is in terms of interfaces

Up next: analysis/construction of solutions, explicit interpolations,…

THANK YOU!