Gravitational waves in a matrix model of dark sector confinement

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Seminar Series on String Pheno

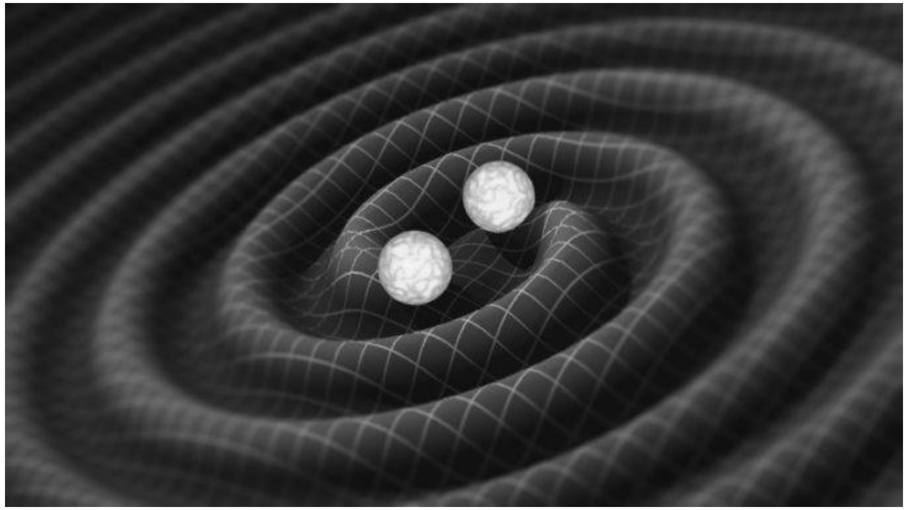
Northeastern University

Outline

- Intro: Gravitational waves from phase transitions
- Modelling confinement in pure Yang-Mills
  - Lattice thermodynamics
  - Symmetries and some group theory
- The GW signal
  - Sources and cosmic evolution
  - Future experiments

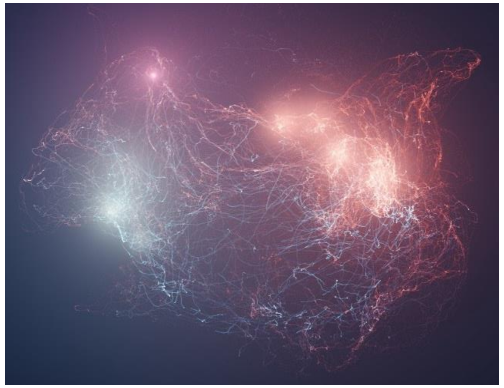
Stochastic background of GWs

- Gravitational waves from astrophysical events, such as black hole/neutron star mergers



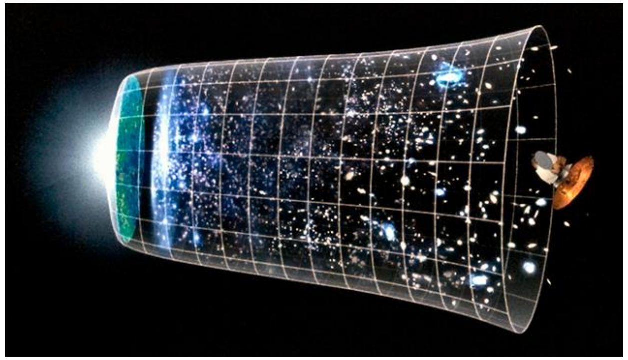
e.g., LIGO

- Stochastic signals



topological defects

inflation



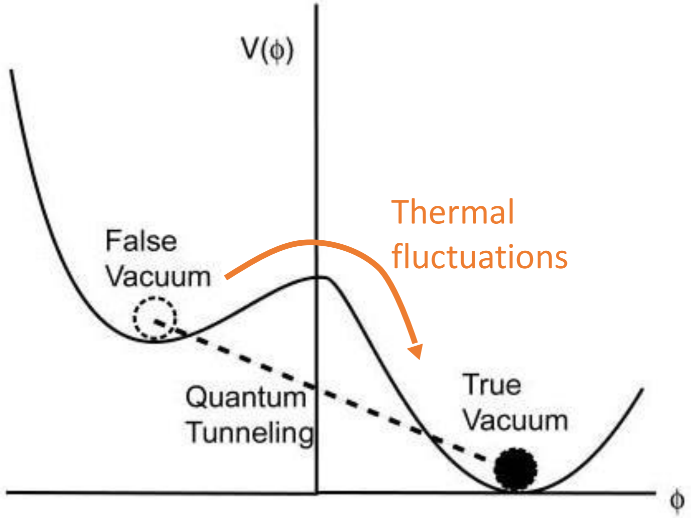
First-order phase transitions
this talk

First-order phase transition

- Symmetric phase (vacuum at high Ts)
Broken phase (vacuum at low Ts)

exist simultaneously for a range of temperatures

- Thermal fluctuations/quantum tunneling
- Latent heat is produced
- Nucleation and subsequent expansion of bubbles
- Bubble collisions and motion of plasma produce GWs



- Electroweak and QCD PTs: first-order only in extensions of the Standard Model

Many dark gauge sectors

- String theory commonly gives many gauge sectors, confinement scale “free” parameter

- F-theory: geometrical gauge sectors have many factors

- Example: Tree ensemble [Halverson, Long, Sung 2017]

$$\mathcal{G} \geq E_8^{10} \times F_4^{18} \times U^9 \times F_4^{H_2} \times G_2^{H_3} \times A_1^{H_4}$$

with U a basis dependent gauge group and H_is integers

- Many of these are pure YM sectors
  - Dark gluons above the critical temperature T_c
  - Glueballs bellow T_c

confining transition is first-order for most gauge groups $\mathcal{G}$

Confinement in pure Yang-Mills

- Order parameter: Polyakov loop in the fundamental representation

$$l = \frac{1}{d_f}\mathrm{Tr}\mathbf{L} \ , \quad \mathbf{L} = \mathcal{P}\exp\left(ig\int_0^\beta A_0^a(\vec{x},\tau)T^a d\tau \right)$$

Its expectation value gives the energy of a static quark-antiquark pair

$$\langle l \rangle \sim \exp(-\beta F_{q\bar{q}}/2)$$

→

confined state $\langle l \rangle = 0$

deconfined state $\langle l \rangle \neq 0$

- Center symmetry: confined state preserves center, deconfined breaks it

Necessary for confinement? No. G_2 confines on the lattice

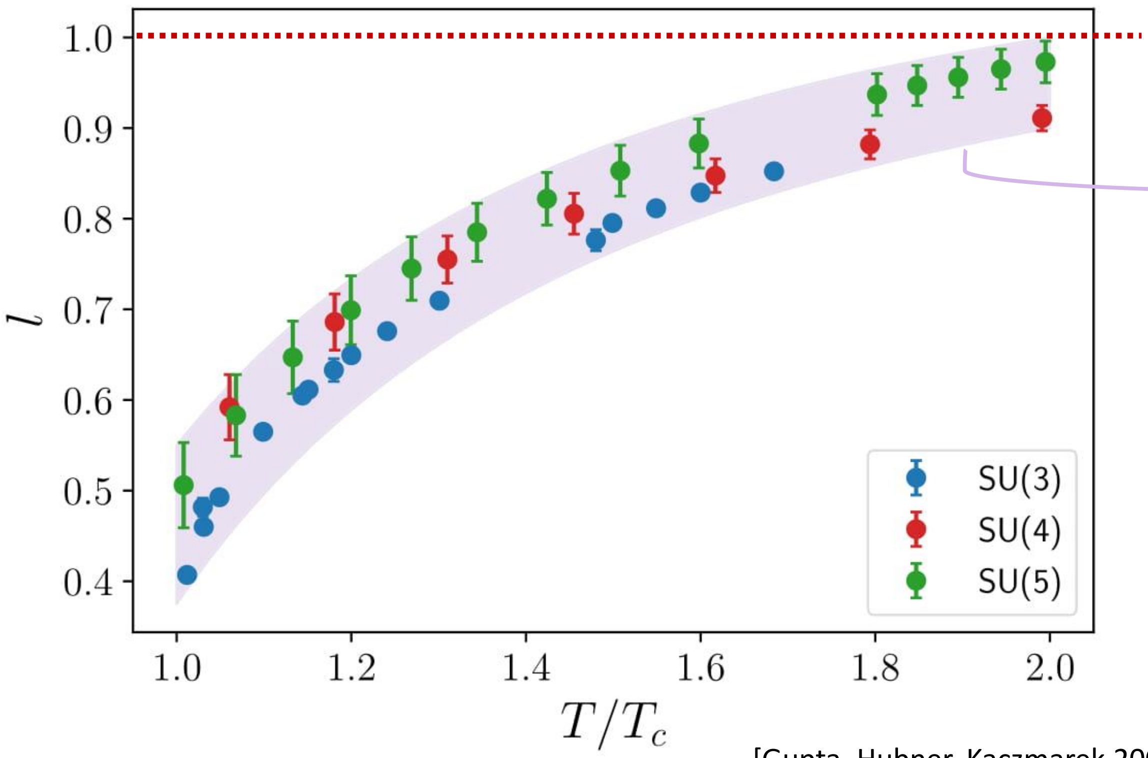
trivial center

Semi-QGP

- Perturbative vacuum: $\langle l(T \to \infty) \rangle = 1$

Lattice gives $\langle l(T_c^+) \rangle < 1$

transition at T_c is between confined state and intermediate state



perturbative vacuum

expected region

semi quark gluon plasma

$$T_c \leq T \lesssim 4T_c$$

theory is partially deconfined

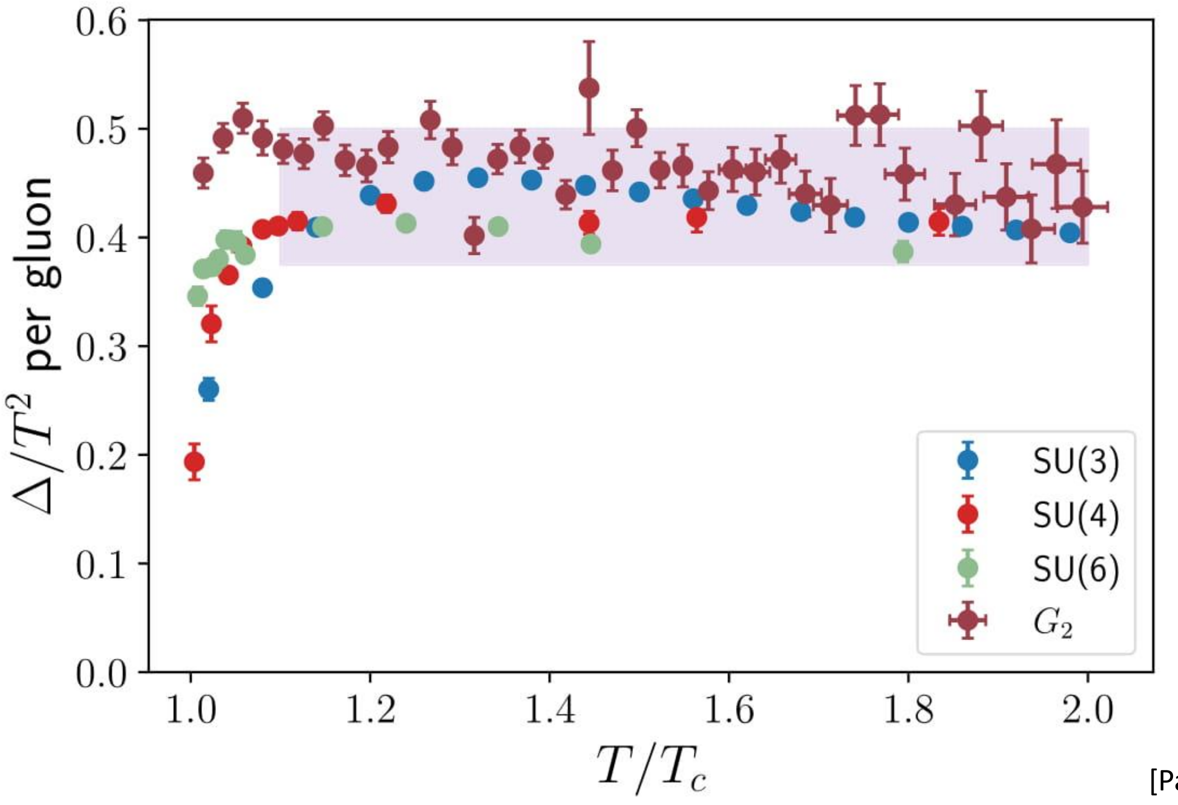
[Gupta, Hubner, Kaczmarek 2008]
[Mykkanen, Panero, Rummukainen 2012]

Effective model of the semi-QGP

- Pressure goes from ≈ 0 at T_c to $\approx 0.7 p_{\mathrm{SB}}$ at $2T_c$

ideal gas

- Interaction measure $\Delta = e - 3p$ is proportional to T^2 right above T_c



[Panero 2009]
[Bruno, Caselle, Panero, Pellegrini 2015]
[Caselle, Nada, Panero 2018]

perturbative contribution

$$\Delta_{\mathrm{HTL}} \ll \Delta_{\mathrm{lattice}}$$

nonperturbative contribution requires leading term

$$V_{\mathrm{npt}} \propto T^2$$

in the effective potential

Matrix model

- Simplest effective model: $V(l)$ (Polyakov loop models)
  - explains first-order PT for SU(3)
  - but it doesn't for, e.g., SU($N > 3$)

- Matrix model: $V(\mathbf{L})$ Domain: gauge group $\mathcal{G}$ $\rightarrow$ Lie algebra $\mathfrak{g}$

[Meisinger, Miller, Ogilvie 2002]
[Dumitru, Guo, Hidaka, Altes, Pisarski 2011 & 2012]

constant background field diagonalized by gauge transformation $\mathbf{A_0} = (2\pi T/g)H \in \mathfrak{h}$

Cartan subalgebra

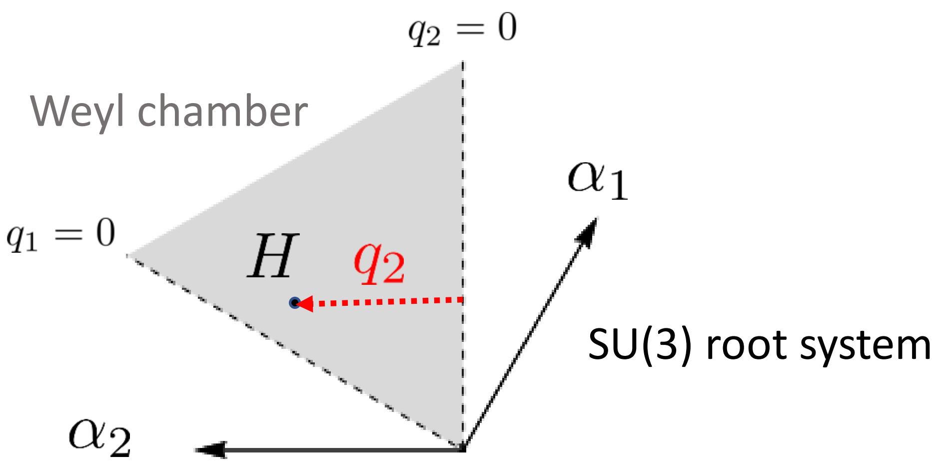
write $H \in \mathfrak{h}$ in terms of a basis $\{H_1, \ldots, H_r\}$ w/ $\exp(2\pi \mathrm{i} H_i) \in \mathcal{C}(\mathcal{G})$

simple roots

coordinates in $\mathfrak{h}$

$$H = \alpha_i(H) H_i \equiv q_i H_i$$

$$\boxed{V(q) \equiv V(q_1, \ldots, q_r)}$$



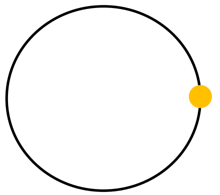
Perturbative and nonperturbative terms

- To one-loop order, the perturbative potential is

Bernoulli polynomial

$$\frac{V_{\mathrm{pt}}}{T^4} = -\frac{p_{\mathrm{SB}}}{T^4} + \frac{2\pi^2}{3}\sum_{\alpha} B_4(q_\alpha)$$

minimum at $H = 0$, no confinement
all eigenvalues of $\mathbf{L}$ are one



- Add nonperturbative term describing transition to confined phase

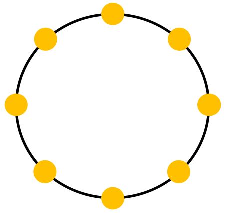
V_{npt} generates eigenvalue repulsion

e.g., SU(N)
eigenvalues of $\mathbf{L}$ uniformly distributed along unit circle

Symmetries
center symmetry + Weyl group invariance

reflections about hyperplanes perpendicular to simple roots

qs are periodic:
$q_i \sim q_i + 1$



Constructing V_{npt}

- SU(N)
  - all simple roots have same length
  - Weyl group transformations generate permutations of all roots

consider polynomial terms P_i:

$$\sum_{\alpha} P_1[\alpha(H)], \qquad \sum_{\alpha\neq\alpha'} P_2[\alpha(H),\alpha'(H)], \qquad \ldots$$

sums over all roots guarantees Weyl invariance

combine Weyl transform $q_i \to -q_i$
w/ periodicity $q_i \to q_i + 1$

potential is invariant under $q_i \to 1 - q_i$

Bernoulli polynomials $B_n(1-q) = (-1)^n B_n(q)$ → n even

Constructing $V_{\rm npt}$

In the semi-QGP, dynamics dominated by term $\propto T^2$

$$V_{\rm npt}(q) = T_c^2 T^2 \underbrace{(c_0 + c_1 V_1 + c_2 V_2 + c_3 V_3)}_{\text{generates eigenvalue repulsion}} + d_1 T_c^4 + d_2 \frac{T_c^6}{T^2}$$

fits latent heat

fits Polyakov loop

with all polynomial terms up to order four in the coordinates q

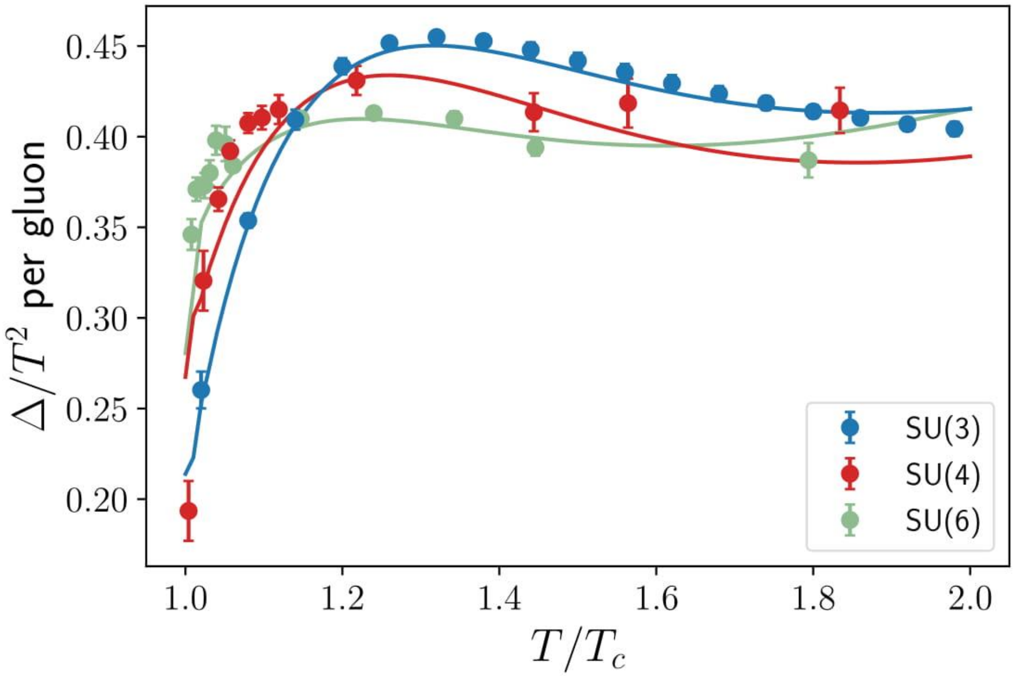
$$V_1 = \frac{1}{2}\sum_{\alpha} B_2(\alpha)\ ,\ V_2 = \frac{1}{8}\sum_{\alpha\neq\alpha'} B_2(\alpha)B_2(\alpha')\ ,\ V_3 = \frac{1}{2}\sum_{\alpha} B_4(\alpha)$$

Constraints: • PT happens at $T = T_c$ with approximately vanishing pressure in both the symmetric and broken phases • Reproduces latent heat from lattice

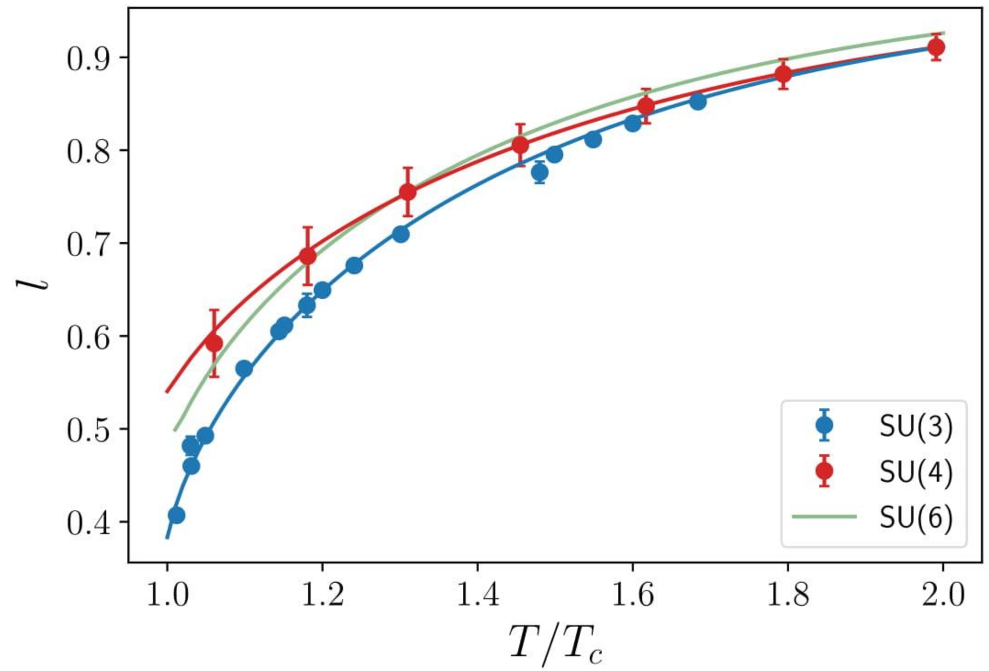
→ left w/ two free coefficients,
fit to p, Δ and l from lattice (if available) or expected (if not)

Fitting lattice

Interaction measure:

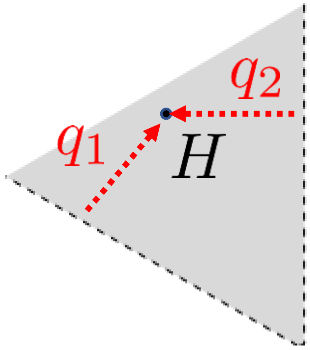


Polyakov loop:



simplifying assumption: *uniform eigenvalue ansatz*

$$q_1 = \ldots = q_{N-1}$$



Constructing $V_{\rm npt}$

- G_2 and F_4
  - simple roots: short roots α_S and long roots α_L
  - Weyl group generates permutations of short roots and of long roots only

Less symmetry, more terms allowed

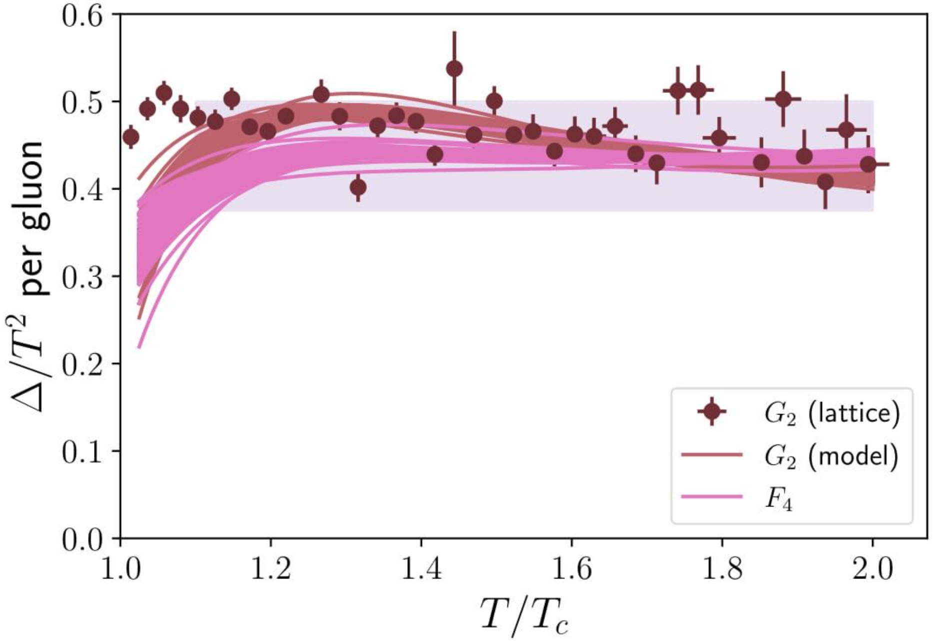
$$\begin{cases} V_1^L = \frac{1}{2}\sum_{\alpha\in\alpha_L} B_2(\alpha)\ ,\ V_2^L = \frac{1}{8}\sum_{\substack{\alpha\neq\alpha' \\ \alpha,\alpha'\in\alpha_L}} B_2(\alpha)B_2(\alpha')\ ,\ V_3^L = \frac{1}{2}\sum_{\alpha\in\alpha_L} B_4(\alpha) \\ V_1^S = \frac{1}{2}\sum_{\alpha\in\alpha_S} B_2(\alpha)\ ,\ V_2^S = \frac{1}{8}\sum_{\substack{\alpha\neq\alpha' \\ \alpha,\alpha'\in\alpha_S}} B_2(\alpha)B_2(\alpha')\ ,\ V_3^S = \frac{1}{2}\sum_{\alpha\in\alpha_S} B_4(\alpha) \end{cases}$$

$$\Downarrow$$

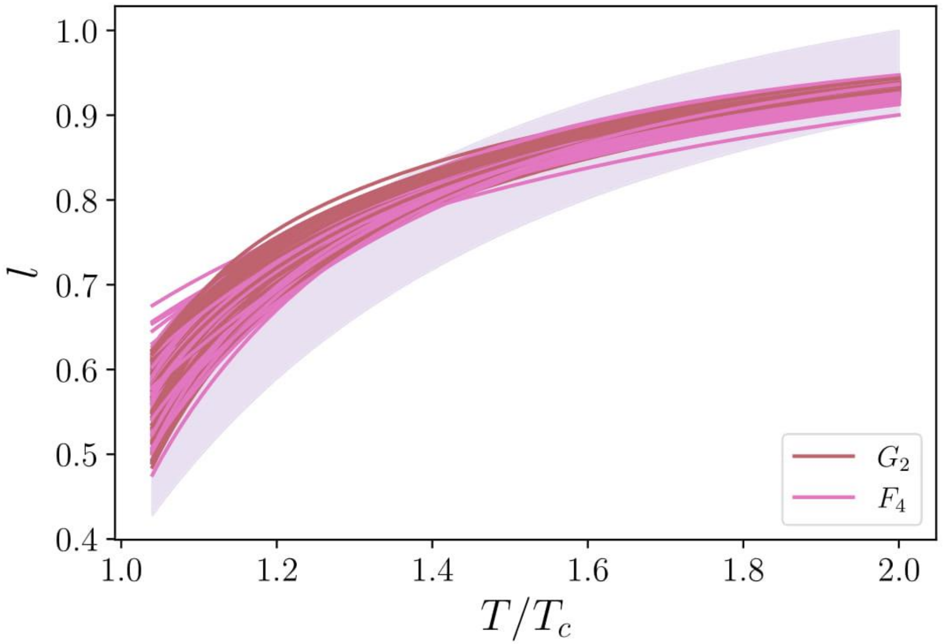
$$\begin{aligned} V_{\rm npt}(q) = T_c^2 T^2 (c_0 + c_1^L V_1^L + c_2^L V_2^L + c_3^L V_3^L + c_1^S V_1^S + c_2^S V_2^S + c_3^S V_3^S + c^{LS} V_1^L V_1^S) \\ + d_1 T_c^4 + d_2 \frac{T_c^6}{T^2} \end{aligned}$$

Fitting lattice

Interaction measure:



Polyakov loop:

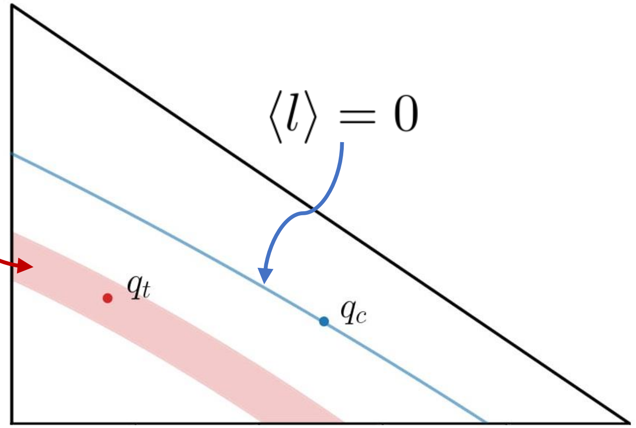


different fits for different choices of confined and broken phases

F_4: *“generalized” uniform eigenvalue ansatz*

$\langle l \rangle$ in the expected region at T_c

e.g., for G_2



Gravitational waves from PTs

- $\Omega_{\rm gw}$ depends on the parameters:

  one bubble per Hubble volume

  - $\alpha = \frac{\Delta(T_n^+) - \Delta(T_n^-)}{3w(T_n^+)}$ with $T_n \lesssim T_c$ $\underbrace{\text{the nucleation temperature}}$

    enthalpy

  - H_* the Hubble parameter at bubble percolation $T_* \approx T_n$

    action of O(3)-symmetric bounce solution

  - $\frac{\beta}{H_*} = T_* \frac{d(S_3/T)}{dT}\Big|_{T=T_*}$ inverse duration of the PT $\beta/H_* \sim \mathcal{O}(10)$ long-lasting
    $\beta/H_* \gg 1$ fast

  - v_w bubble wall velocity, we take ultrarelativistic $v_w \to 1$

Parameters for confining PT

- Confined phase: approx. vanishing pressure and energy density $\begin{array}{l} p(T_c^-) \approx 0 \\ e(T_c^-) \approx 0 \end{array}$

Right above T_c : $p(T_c^+) \approx 0$, energy density changes discontinuously $e(T_c^+) \gg e(T_c^-)$

(i) $\alpha \approx 1/3$, if dark sector dominates at PT
(ii) $\alpha < 1/3$, if many sectors contribute $\longrightarrow$ GW signal is suppressed

to get a maximal signal, assume (i)

- Action for the bounce $S_3 = \int d^3x \left(\frac{1}{2} \mathrm{tr} \mathbf{F}^{\mathbf{2}}_{\mu\nu} + V(q) \right)$

condition for nucleation $\left. \frac{S_3(T)}{T} \right|_{T=T_n} = 2 \log \left(\frac{90 M_{\mathrm{Pl}}^2}{g_* \pi^2 T_n^2} \right) \sim \mathcal{O}(100)$

find $\beta \sim \mathcal{O}(10^4)$

Sources and GW energy density

- Production mechanisms
  - ~~bubble collisions~~ negligible for nonrunaway bubbles
  - sound waves
  - MHD turbulence

- Sound wave contribution (dominant one):

$$h^2\Omega_{\rm sw}(f) = 0.337 F_{\rm gw} K^{3/2} \left(\frac{\overbrace{H_* R_*}^{(8\pi)^{1/3} v_w/\beta}}{\sqrt{c_s}}\right)^2 \overbrace{\tilde{\Omega}_{\rm gw}}^{\sim 10^{-2}} \underbrace{S_{\rm sw}}_{\text{spectral shape}} \left(\frac{f}{\underbrace{f_{\rm sw,0}}_{\text{peak frequency}}}\right)$$

kinetic energy fraction

$$K = \frac{\kappa_v \alpha}{1+\alpha}\,, \quad \kappa_v = \frac{\alpha}{0.73 + 0.083\sqrt{\alpha} + \alpha} \qquad \text{(for } v_w \to 1\text{)}$$

Sources and GW energy density

- Turbulence contribution (subleading):

$$h^2\Omega_{\rm tb}(f) \approx 3.63 F_{\rm gw} K^{3/2}(H_* R_*) S_{\rm tb}\left(\frac{f}{f_{\rm tb,0}}\right)$$

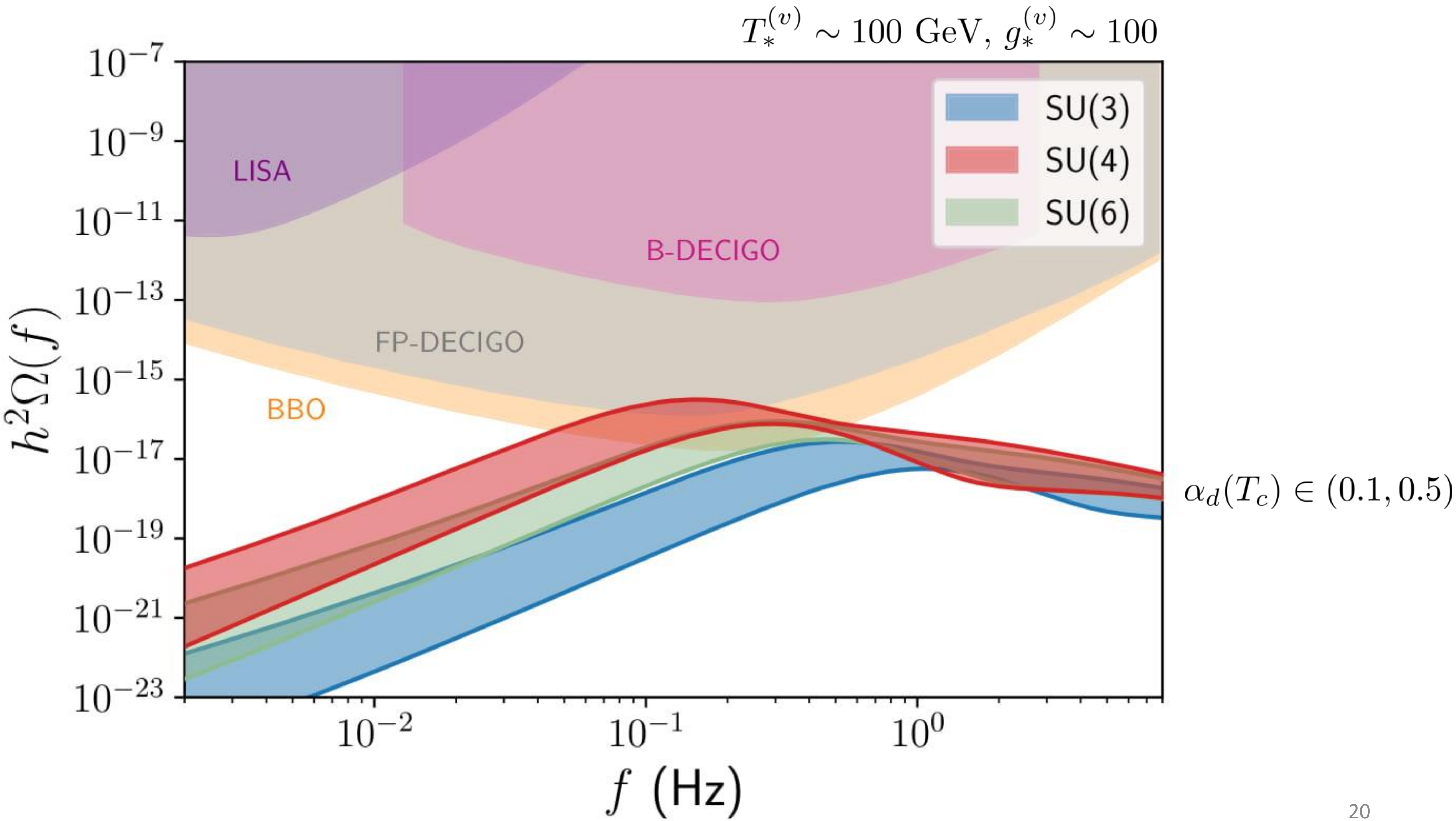
$> f_{\rm sw,0}$

- Factor $F_{\rm gw}$ accounts for redshift from production to today
  - bellow T_c, glueballs dominate the energy density of the universe $\longrightarrow$ early matter domination
  - these must decay before the onset of BBN
  - matter-dominated phase suppresses the GW spectrum

assume glueballs decay to visible sector immediately after PT

after decay, photon temperature is $T_\gamma = T_*^{(v)}$ with $g_*^{(v)}$ relativistic degrees of freedom

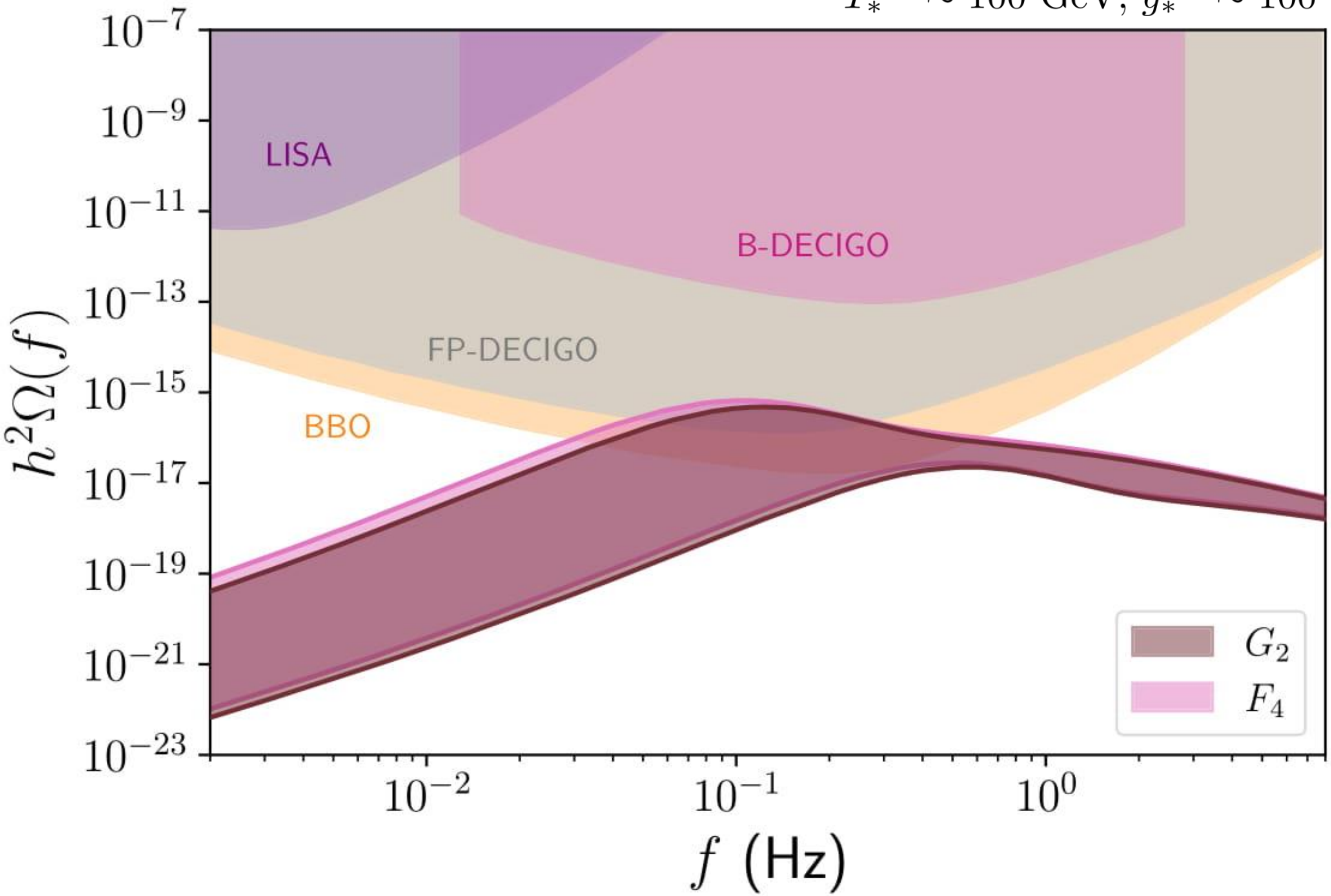
GW spectra



$T_*^{(v)} \sim 100$ GeV, $g_*^{(v)} \sim 100$
SU(3)
SU(4)
SU(6)
LISA
B-DECIGO
FP-DECIGO
BBO
$h^2\Omega(f)$
f (Hz)
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-19}
10^{-21}
10^{-23}
10^{-2}
10^{-1}
10^{0}
$\alpha_d(T_c) \in (0.1, 0.5)$

GW spectra

$T_*^{(v)} \sim 100$ GeV, $g_*^{(v)} \sim 100$



$\alpha_d(T_c) \in (0.1, 0.5)$

+

diff. choices
for q_c and q_t

Next-to-next generation searches

- Signal is many orders of magnitude bellow next generation of experiments, e.g., LISA
- Suppression is commonly observed in effective models of PTs, also finding fast transitions with $\beta \sim \mathcal{O}(10^4)$ or higher
- However, future searches such as the Big Bang Observer (BBO) and Deci-Hertz Interferometer Gravitational Wave Observatory (DECIGO) are sensitive to them
- Lesson: many dark sectors with confining PTs are not super noisy (gravitationally)
- Going to even larger gauge groups does not seem to change this

E_8 transition seems to be even faster (preliminary)

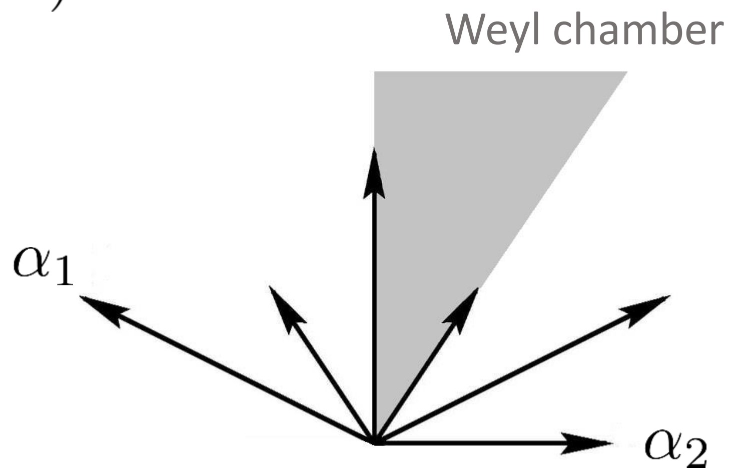
Summary

- Many pure YM dark sectors in string theory
- Thermodynamics of confining transition can be described by an effective model
- We assumed similar thermodynamical behavior for all gauge groups, motivated by universal behavior on the lattice
- Symmetries (center + Weyl) determine behavior particular to each group
- GW signal is produced, but not strong enough for next-generation searches (LISA)
- Reheating of many sectors suppresses signal even more
- These are, however, accessible to futuristic experiments (BBO, DECIGO)


Thank you for attention

Generalizing the uniform eigenvalue ansatz

- SU(N): $q_1 = \dots = q_{N-1}$ → global minimum of V is equidistant from the hyperplanes at boundary of Weyl chamber, defined by $\alpha_i(H) = 0$



G_2 root system

- G_2: boundary planes are at $\alpha_1(H) = 0$
  $\alpha_2(H) = 0$

no symmetry under interchange $\alpha_1 \leftrightarrow \alpha_2$
$q_1 = q_2$ not a valid ansatz

$V(q_1, q_2)$ intrinsically 2D

- F_4: if $\alpha_1, \alpha_2 \in \alpha_L$, $\alpha_3, \alpha_4 \in \alpha_S$ it is possible to take $q_1 = q_2$ and $q_3 = q_4$

"generalized" uniform eigenvalue ansatz