Duality and Axionic Weak Gravity

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[based on: SA, Huang, Noumi, Ooguri, Shiu ’20 — 2004.13721]

StringPheno Summer series

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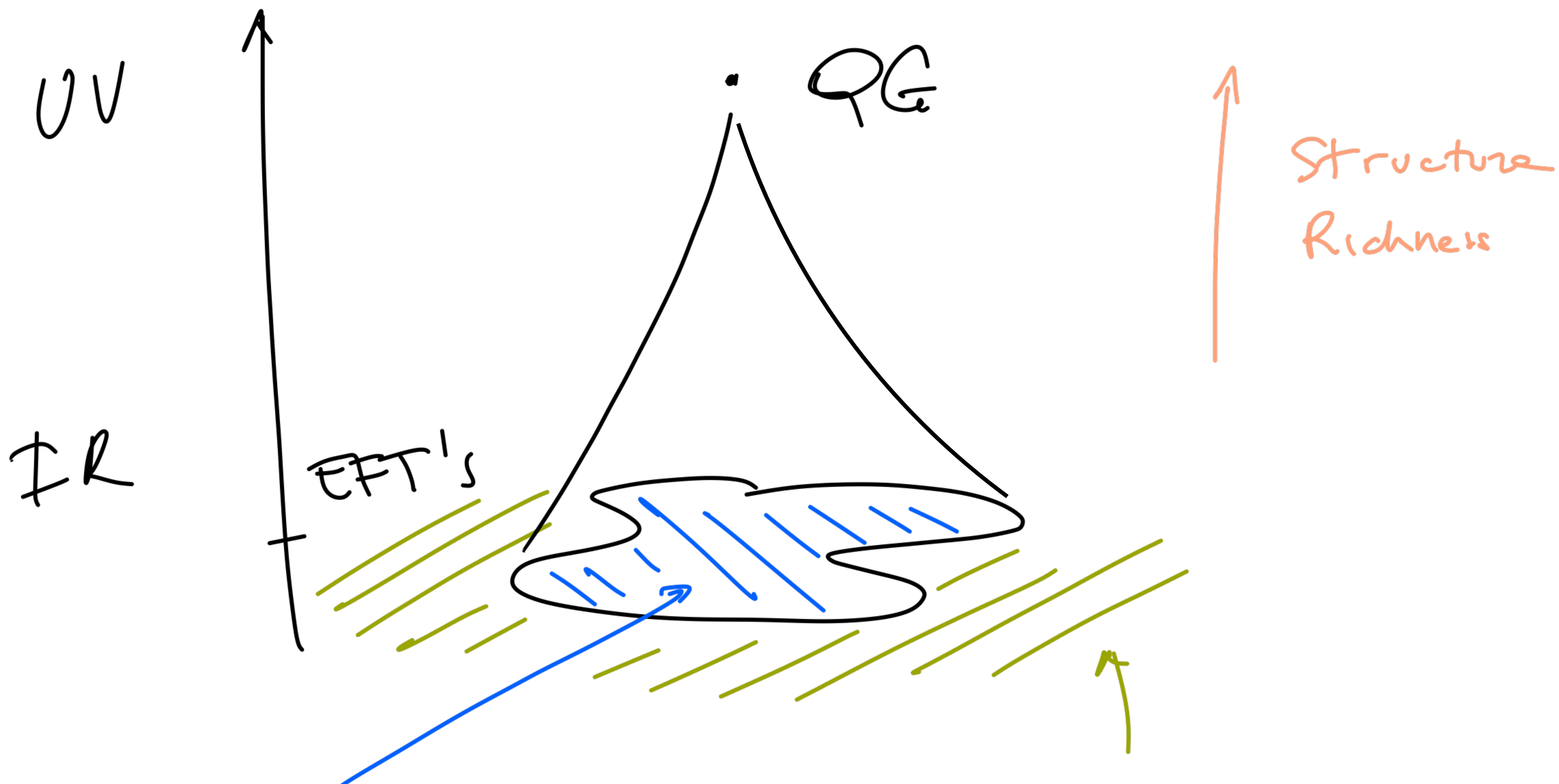
KU LEUVEN

THE SWAMPLAND

[Vafa '05, Ooguri,Vafa '06]



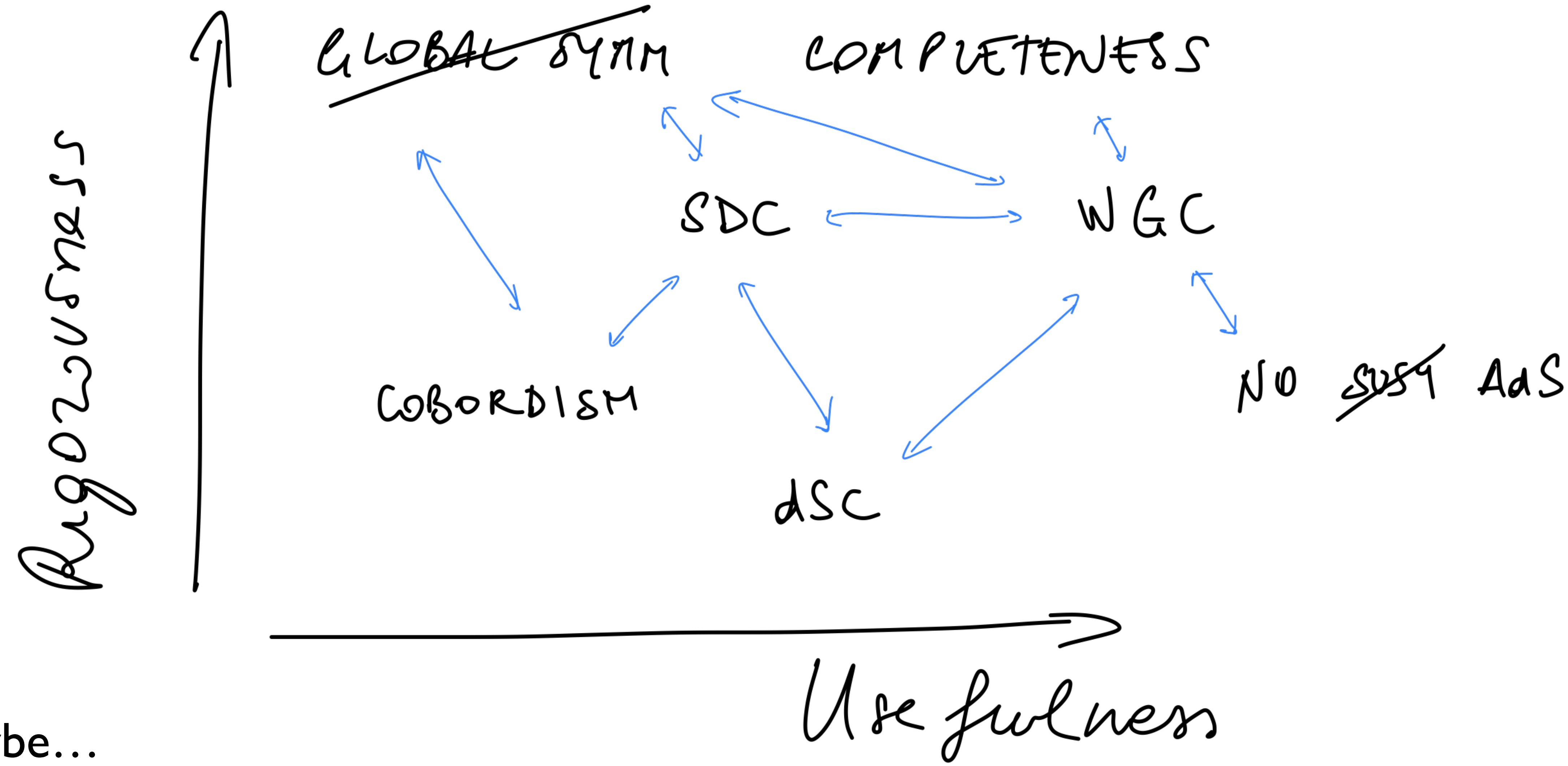
Landscape:
EFT's with QG completion

Swampland:
EFT's that do **not** have QG completion

Boundary defined by ***Swampland criteria***

WEB OF CONJECTURES

[Reviews:
Brennan, Carta, Vafa 1711.00864
Palti 1903.06239]



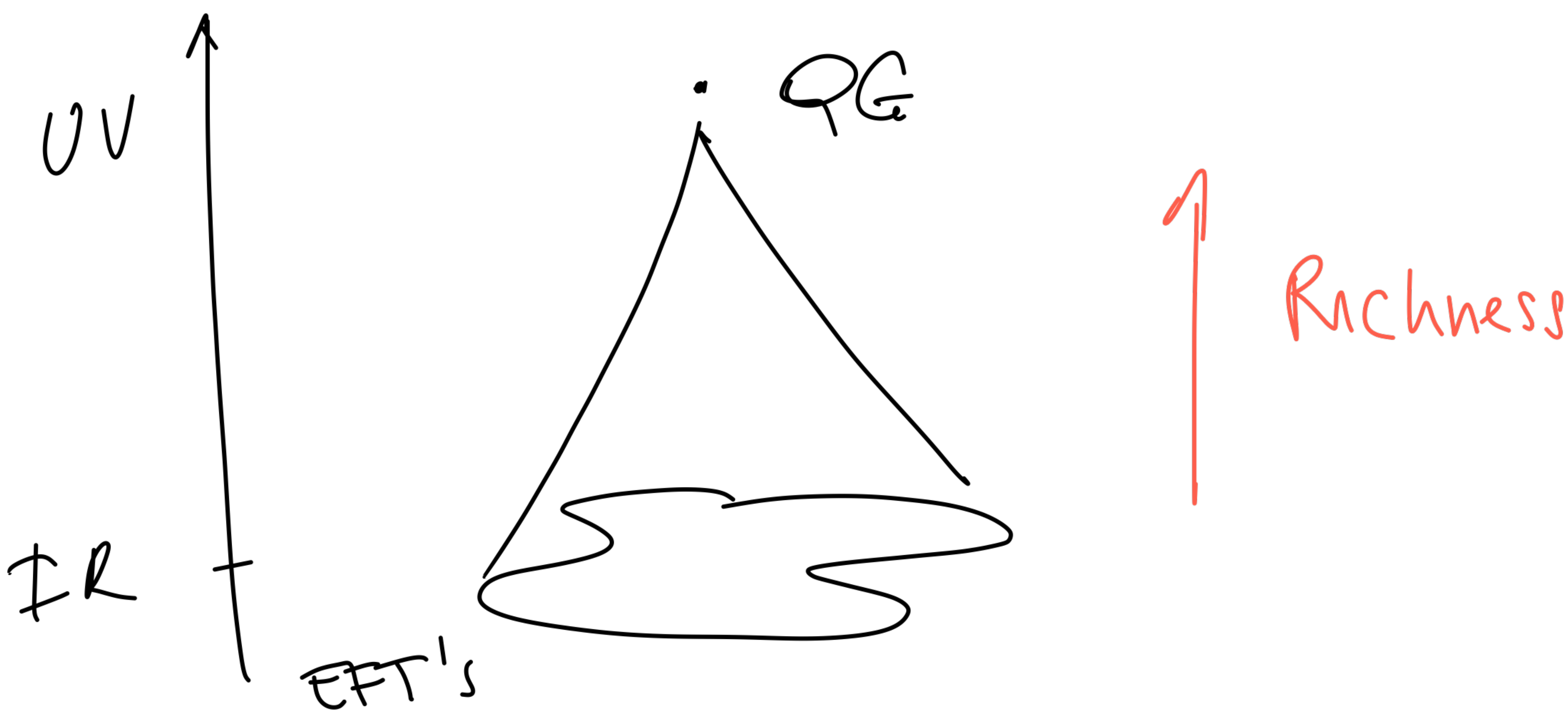
Maybe…

String Lamppost Principle:

all consistent QG theories are part of the string landscape

MOTIVATIONS OF OUR WORK

- **Test** swampland criteria:
  - *self-consistency*: Linking conjectures in the web
  - consistency with *other principles* Unitarity, causality, locality, analyticity, duality, BH physics, SUSY, holography, anomalies,…



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- Highlight the **relevant properties/principles** of QG
- Understand what makes **string theory** so special (QG unique?)

“string theory so complete/rich
=
insurance with full options”

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THE PUNCH-LINE

- Analyse **WGC** (axionic version) **vs positivity** (unitarity, analyticity, locality)
  [Cheung,Remmen '14, Andriolo,Junghans,Noumi,Shiu '18, Hamada,Noumi,Shiu '18,…]
- Result:
  - in simple systems: positivity is sufficient to imply the WGC
  - more often: positivity alone is not enough, but specifying *some* **UV info** is sufficient to satisfy the **WGC** (e.g., SL(2,R) symm)

positivity + UV info	$\Rightarrow$	WGC

[Heidenreich, Reece,Rudelius '16, Montero,Shiu,Soler '16, Aalsma, Cole, Shiu '19]

See also Gregory's talk on September 1st! [Loges,Noumi,Shiu '19, '20]

OUTLINE

- Review of **WGC** and its **axionic** version (**A**WGC)
- **Question** addressed
- Illustration of **setup**
- **Positivity** vs AWGC
- Adding **SL(2,R)** and implications

WGC and AWGC

- Standard formulation of **WGC**: [Arkani-Hamed, Motl, Nicolis, Vafa '06]

  - "An EFT with gauge U(1)+gravity is QG-consistent if it admits at least a **state** with charge-to-mass ratio greater than that of an extremal black hole (EBH)"

    (here D=4) $$\frac{qgM_P}{m} \geq 1$$ since $M_{EBH} = gM_PQ_{EBH}$

  - Motivated by requiring instability and decay of EBH's

  - As swampland criterium, trivial for $M_P \to \infty$

  - Encapsulates "no global symm in QG", since never satisfied for $g \to 0$

  - Generalized to multiple U(1)'s *[Tower/(sub-)lattice WGC]*

    [Heidenreich, Reece, Rudelius '15,'16, Montero,Shiu,Soler '16, SA, Junghans, Noumi, Shiu '18]

  - Generalized to other dimensions and abelian p-forms potentials

- **AWGC** is the generalisation to **p=0** form potential (=**axion**):

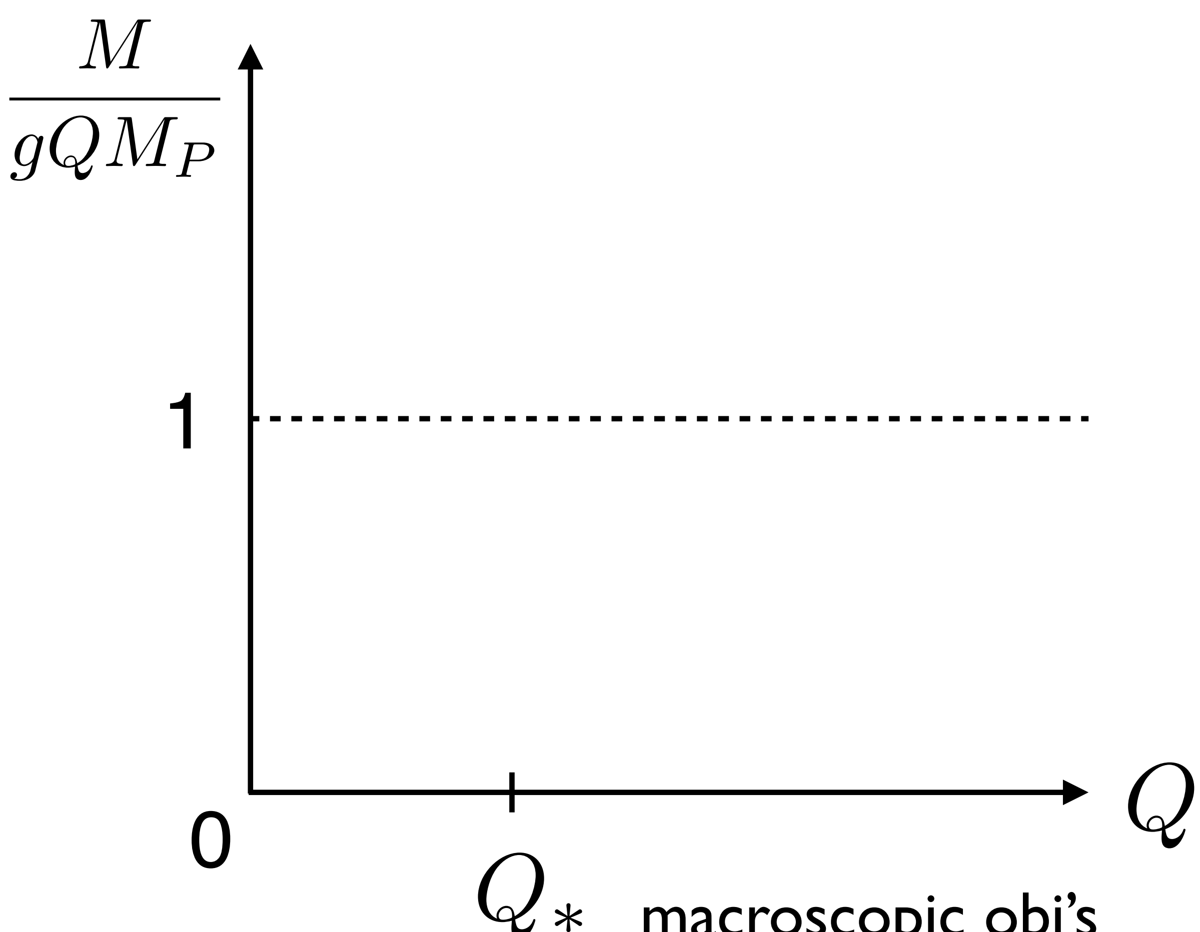
(here D=4)

	WGC	AWGC
form field potential	photon A_μ	axion/2-form dual $\theta/B_{\mu\nu}$
charged states	particles & black holes	instantons & grav. instantons
coupling	gauge coupling g	$\frac{1}{f}$ f=axion decay constant
relevant quantities	mass, charge (m, q)	action, charge (S, q)
WGC bound Exists a state s.t.	$\frac{m}{qgM_P} < 1$	$\frac{Sf}{nM_P} < \mathcal{O}(1)$
Extremal obj's	EBH's	**regular** solutions **[Eucl. wormholes]**
Interpretation	Instability of EBH's	tunneling process via collection of smaller instantons favoured over single instanton w/ same tot q

OBSERVATION…

Higher order (string) corrections modify the classical BH extremality bound in a way that the same EBH's (Q,M) can be the WGC states

classically $$\frac{M}{gQM_P} = 1$$



macroscopic obj's

$M \gg M_P$

(semi-classical reasonings)

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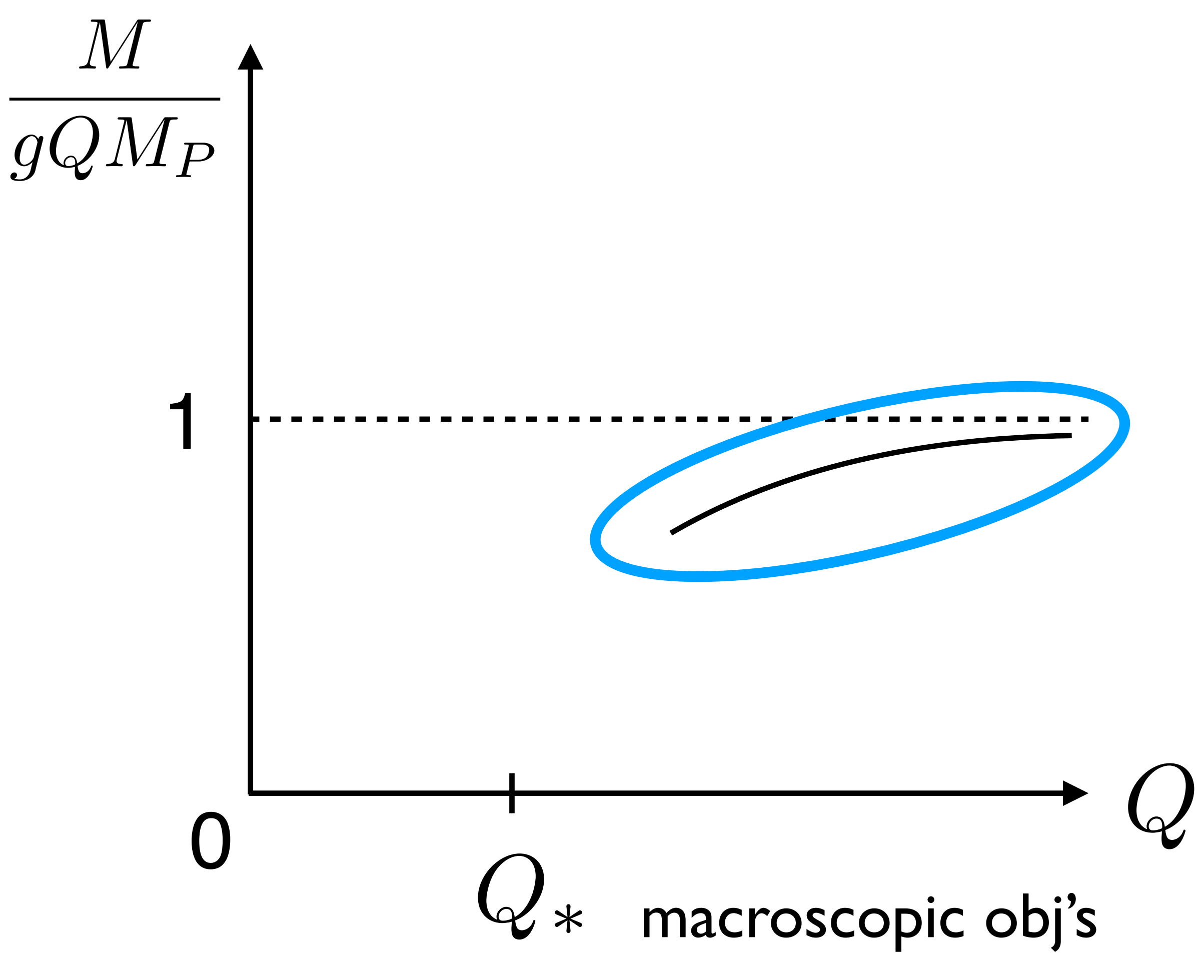


macroscopic obj's

$M \gg M_P$

(semi-classical reasonings)

classically $$\frac{M}{gQM_P} = 1$$

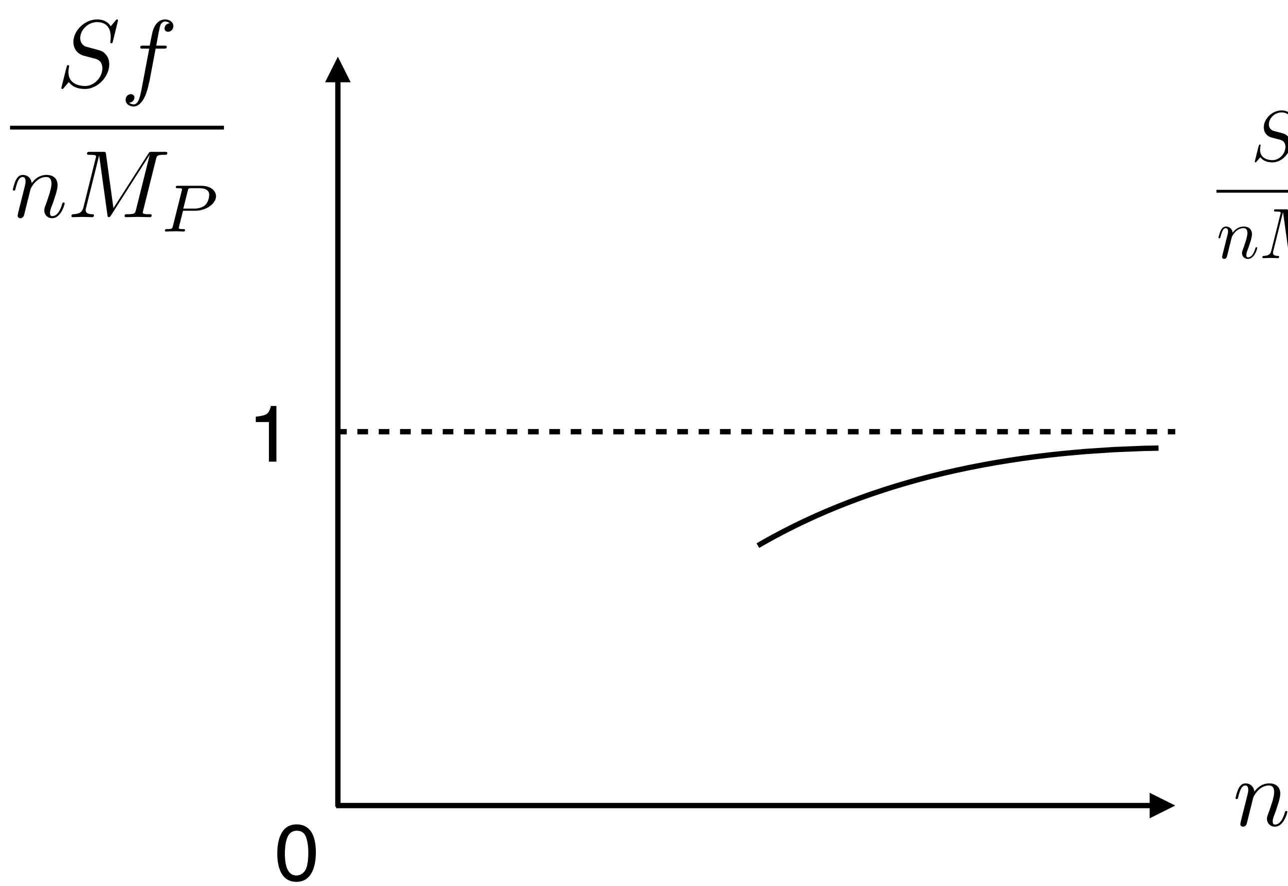
HO corrections $\Delta M < 0$

$$\left.\frac{M}{gQM_P}\right|_{HO} = 1 + \frac{\Delta M}{gQM_P} < 1$$

[Kats, Motl, Padi '06]

…QUESTION

Can the same happen for **Euclidean wormholes?**



$$\left.\frac{Sf}{nM_P}\right|_{HO} = 1 + \frac{\Delta Sf}{nM_P} < 1 \quad ?$$

Under which **circumstances** $\Delta S < 0$ **?**

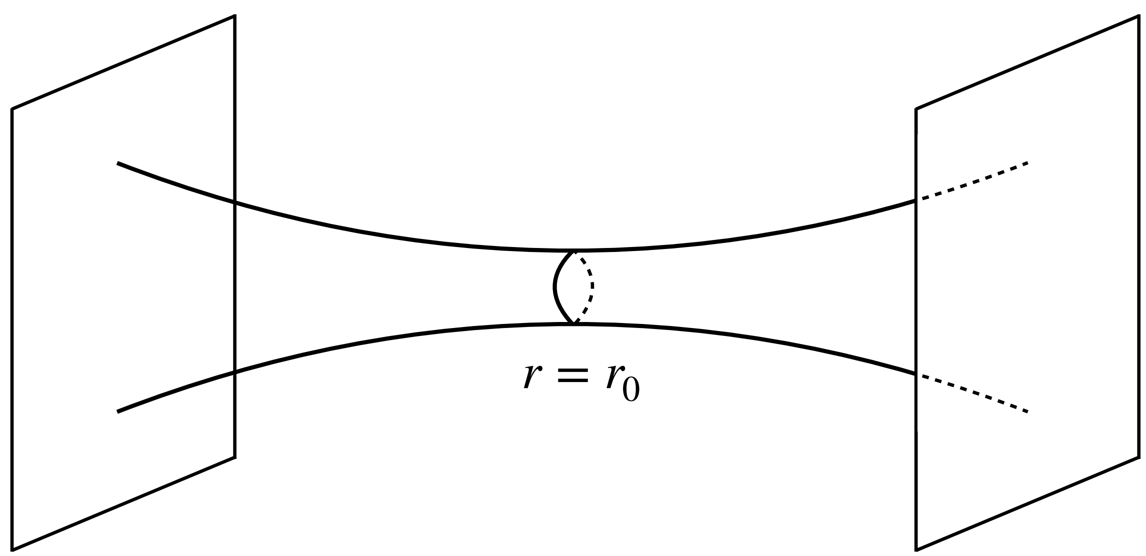
SETUP

- Classical **Axio-dilaton-gravity** (ADG) $\xrightarrow{\beta=0}$ **Axion-gravity** (AG)

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}(\partial_\mu \phi)^2 - \frac{f^2}{2} e^{\beta\phi}(\partial_\mu \theta)^2\right]$$

- **Euclidean wormhole** solutions (<u>non-singular</u> class of solutions for $\beta < \frac{4}{\sqrt{6}}$)

[reviews: Hebecker,Mangat,Theisen,Witkowski '16, Hebecker-Mikhail-Soler '18, Van Riet '20]



can be regarded as instanton—anti-instanton pair

$$ds^2 = \frac{dr^2}{1 - \frac{r_0^4}{r^4}} + r^2 d\Omega_3^2$$

$$r_0^4 = \frac{n^2 f^2}{24\pi^4} \cos^2\left[\frac{\sqrt{6}}{4}\beta \cdot \frac{\pi}{2}\right]$$

semiwormhole (instanton) **action**

$$S = \frac{2|n|M_P}{\beta f} \sin\left[\frac{\sqrt{6}}{4}\beta \cdot \frac{\pi}{2}\right] \xrightarrow{\beta=0} \frac{\sqrt{6}}{4}\pi \cdot \frac{|n|M_P}{f}$$

- Classical **Axio-dilaton-gravity** (ADG) $\xrightarrow{\beta=0}$ **Axion-gravity** (AG)

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- **Euclidean wormhole** solutions (<u>non-singular</u> class of solutions for $\beta < \frac{4}{\sqrt{6}}$)

- \+ **HO** *(4-derivative)* **corrections**, generic

$$\Delta S = \int d^4x\sqrt{-g}\Big[a_1(\phi)(\partial_\mu\phi\partial^\mu\phi)^2 + a_2(\phi)f^4(\partial_\mu\theta\partial^\mu\theta)^2$$
$$+ a_3(\phi)f^2(\partial_\mu\phi\partial^\mu\phi)(\partial_\mu\theta\partial^\mu\theta) + a_4(\phi)f^2(\partial_\mu\phi\partial^\mu\theta)^2 + a_5(\phi)W^2 + a_6\theta W\tilde{W}\Big]$$

Evaluation of ΔS gives…

- **AG** system $\beta = 0$: $\Delta S = -24\pi^2 a_2$

- **ADG** system

$$\Delta S = 36\pi^2 \int_0^{\frac{\pi}{2}} dt \cos^3 t \Bigg[- a_1\big(\phi(t)\big) \tan^4\left[\frac{\sqrt{6}}{4}\beta \cdot t\right] - a_2\big(\phi(t)\big) e^{-2\beta\phi(t)} \sec^4\left[\frac{\sqrt{6}}{4}\beta \cdot t\right]$$

$$+ \Big(a_3\big(\phi(t)\big) + a_4\big(\phi(t)\big)\Big) e^{-\beta\phi(t)} \tan^2\left[\frac{\sqrt{6}}{4}\beta \cdot t\right] \sec^2\left[\frac{\sqrt{6}}{4}\beta \cdot t\right] \Bigg]$$

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$$+ \Big(a_3(\phi(t)) + a_4(\phi(t))\Big) e^{-\beta\phi(t)} \tan^2\left[\tfrac{\sqrt{6}}{4}\beta\cdot t\right] \sec^2\left[\tfrac{\sqrt{6}}{4}\beta\cdot t\right] \Big]$$

- we can use **positivity** conditions to determine (for any bg $\phi = \phi_*$)

$$a_1 \geq 0\,, \quad a_2 \geq 0\,, \quad a_4 \geq 0\,, \quad -a_4 - 2\sqrt{a_1 a_2} \leq a_3 \leq 2\sqrt{a_1 a_2}$$

- **AG** system $\beta = 0$: $\boxed{\Delta S = -24\pi^2 a_2 < \mathbf{0}}$ ✔

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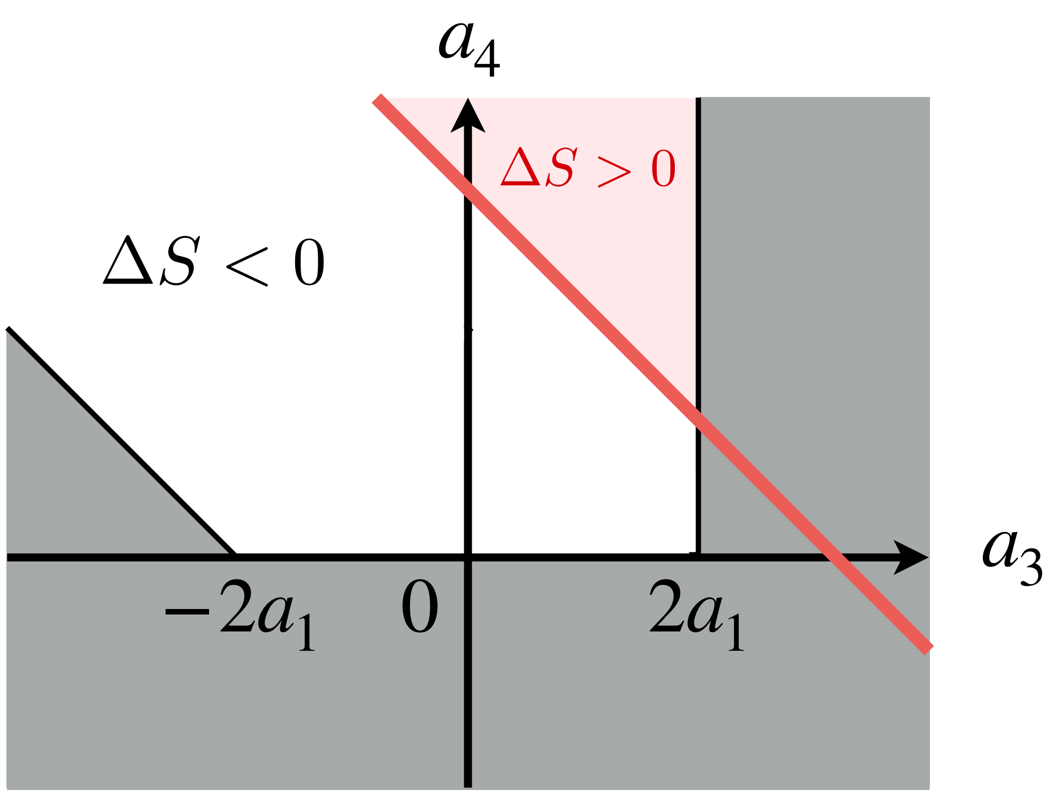


Simplified illustration (in the $a_2 = a_1$ plane)

Prohibited by positivity

Satisfy positivity, but WGC violated

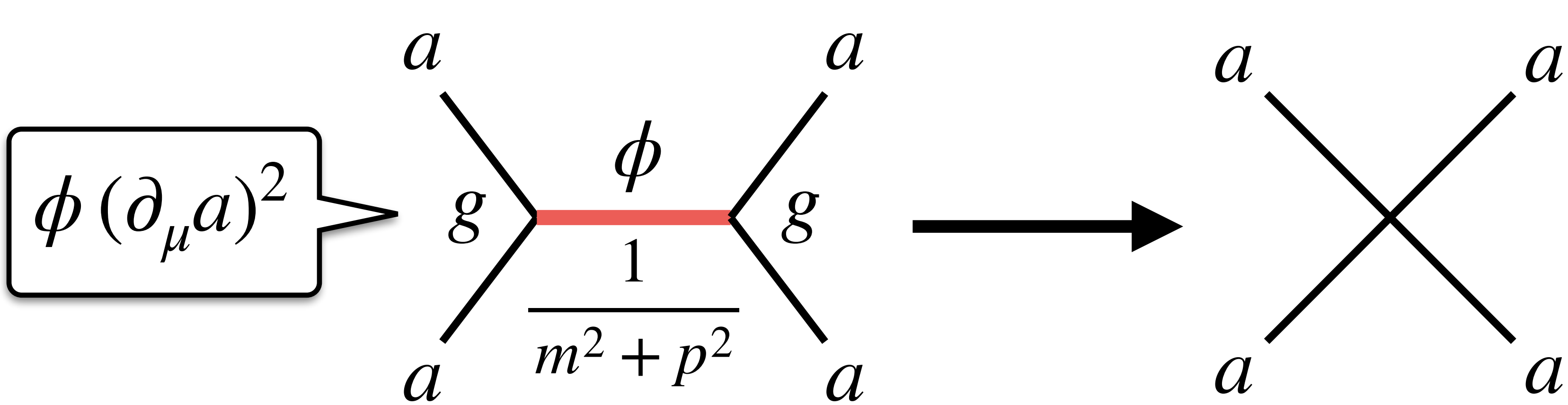
Satisfy positivity and WGC

model-dep

POSITIVITY INTERMEZZO *(presto)*

Axion-gravity EFT $\quad \mathscr{L} = -\frac{1}{2}(\partial_\mu a)^2 + \alpha\,(\partial_\mu a \partial^\mu a)^2 + \cdots$

- where, for instance, α arises after **integrating out** massive scalar ϕ



$\alpha = \frac{g^2}{2m^2} \geq 0$

and the sing of α is related to the sign of propagator (unitarity)

- generically, $\alpha > 0$ follows from unitarity, analyticity, locality of UV scattering amplitudes [Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06]

  - **Caveat, assumption:** gravitational Regge states are sub-dominant

$$|\alpha| > 1/\left(M_s^2 M_{\rm Pl}^2\right)$$ [Hamada-Noumi-Shiu '18]

Back to the **ADG** system:

Can we assume some **additional property** and show that

$$\Delta S < 0 \qquad ?$$

SL(2,R) SYMMETRY

"duality"
[=SL(2,Z)+axion shift symm]

- Symmetry of the 2-derivative action $\tau = \frac{\beta}{2} f\theta + ie^{-\frac{\beta}{2}\phi}$

$$\tau \to \frac{a\tau + b}{c\tau + d} \quad (a, b, c, d \in \mathbb{R},\ ad - bc = 1)$$

- Extended to the **HO** 4-derivative action **terms**:
only two SL(2,R) invariant operators

$$\lambda_1 \frac{(\partial_\mu \tau \partial^\mu \bar{\tau})^2}{\left(\frac{\beta}{2}\right)^4 (\mathrm{Im}\tau)^4} + \lambda_2 \frac{(\partial_\mu \tau \partial^\mu \tau)(\partial_\mu \bar{\tau} \partial^\mu \bar{\tau})}{\left(\frac{\beta}{2}\right)^4 (\mathrm{Im}\tau)^4} \qquad \lambda_{1,2} = const$$

4d parameter space $\longrightarrow$ **2d** parameter space

(a_1, a_2, a_3, a_4) $\qquad$ (λ_1, λ_2)

We are adding structure to EFT

Evaluation of ΔS gives…

$$\Delta S = -24\pi^2(\lambda_1 + \lambda_2)$$

- Positivity means $\lambda_1 + \lambda_2 \geq 0$ $\lambda_2 \geq 0$

Evaluation of ΔS gives…

$$\boxed{\Delta S = -24\pi^2(\lambda_1 + \lambda_2) < \mathbf{0}}$$

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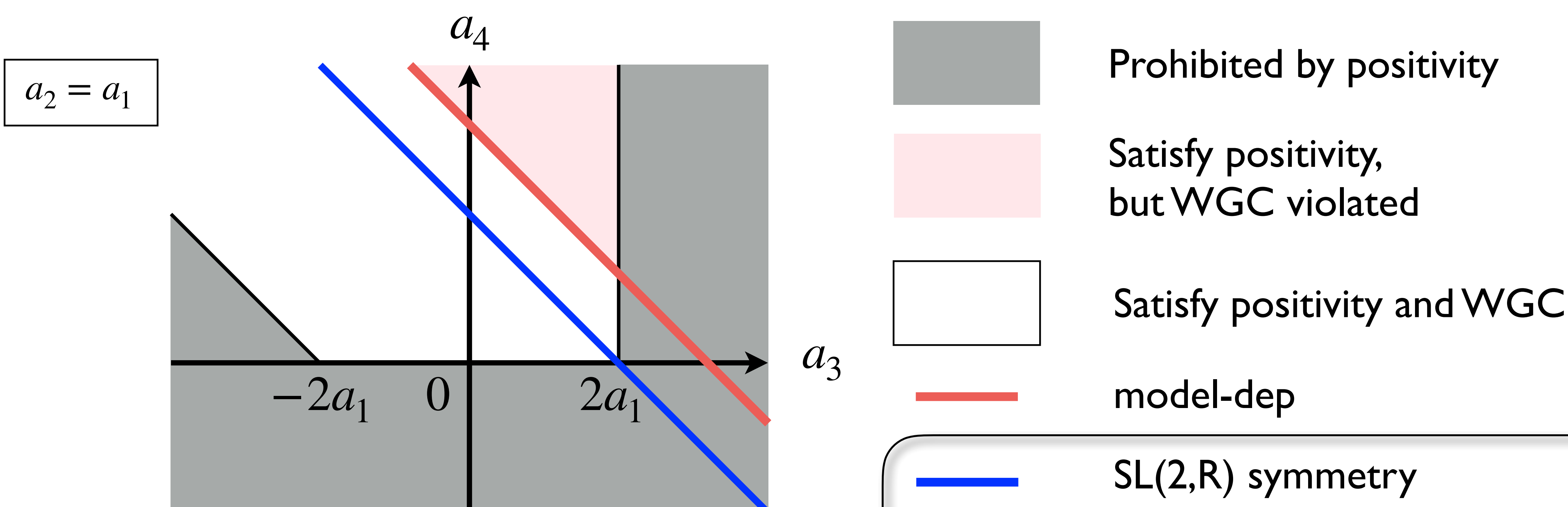

$$a_1 = a_2 = \lambda_1 + \lambda_2$$

$$a_3 + a_4 = 2a_1$$

CONCLUSION

SUMMARY AND OUTLOOK

- We provided some evidence for AWGC (relevant for pheno: inflation,…)
- We did it by studying relationship AWGC vs positivity* in A(D)G:
  - In absence of dilaton: positivity implies AWGC

    *caveat on Regge states
  - With a dilaton: positivity is not enough. In particular, there is a region in the EFT parameter space where WGC is *violated* even if positivity is satisfied!
  - Enriching the EFT structure with SL(2,R) is sufficient for AWGC (i.e., the EFT lies in the region satisfying the AWGC)
- Are there other UV inputs useful to demonstrate WGC? [see Gregory's talk]
- Work on this direction to find which UV properties are necessary/sufficient for WGC or other swampland conjectures: Can we find "what makes string theory tick"? (relevant features)

Thank you