6d $\mathcal{N} = (1,0)$ anomalies on S^1 and F-theory implications

Pierre Corvilain

Instituto de Física Teórica
UAM-CSIC

Based on

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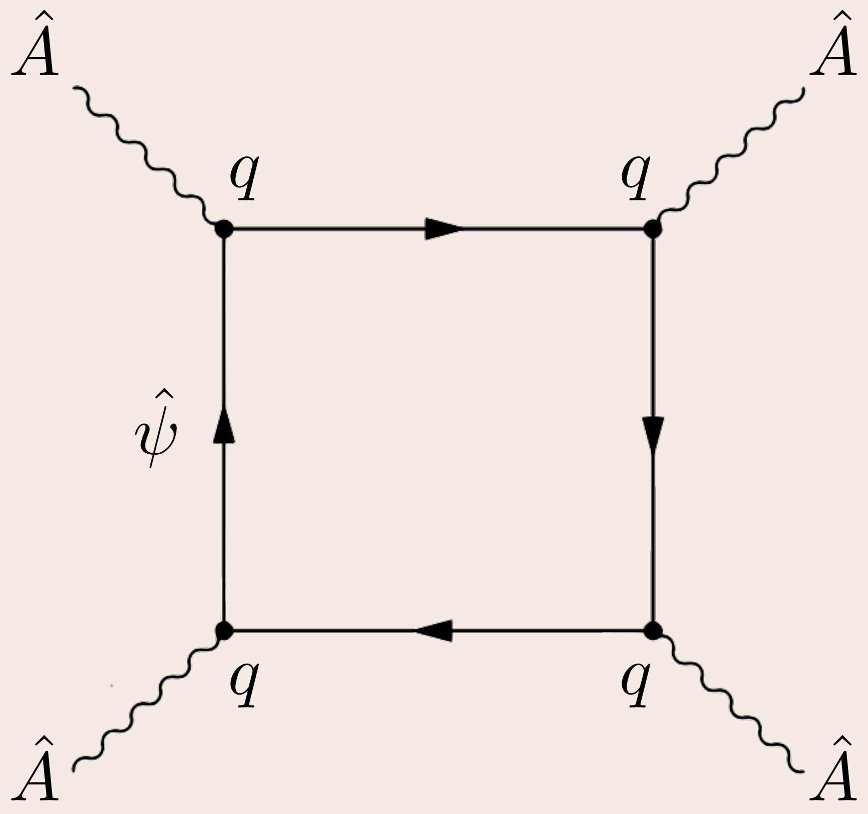
Seminar Series on String Pheno - Zoom

6d Anomalies

6d chiral fermion with **charge** q under **abelian gauge field** $\hat{A}$

Anomaly $$q^4 \int \hat{\lambda}\, \hat{F} \wedge \hat{F} \wedge \hat{F}$$



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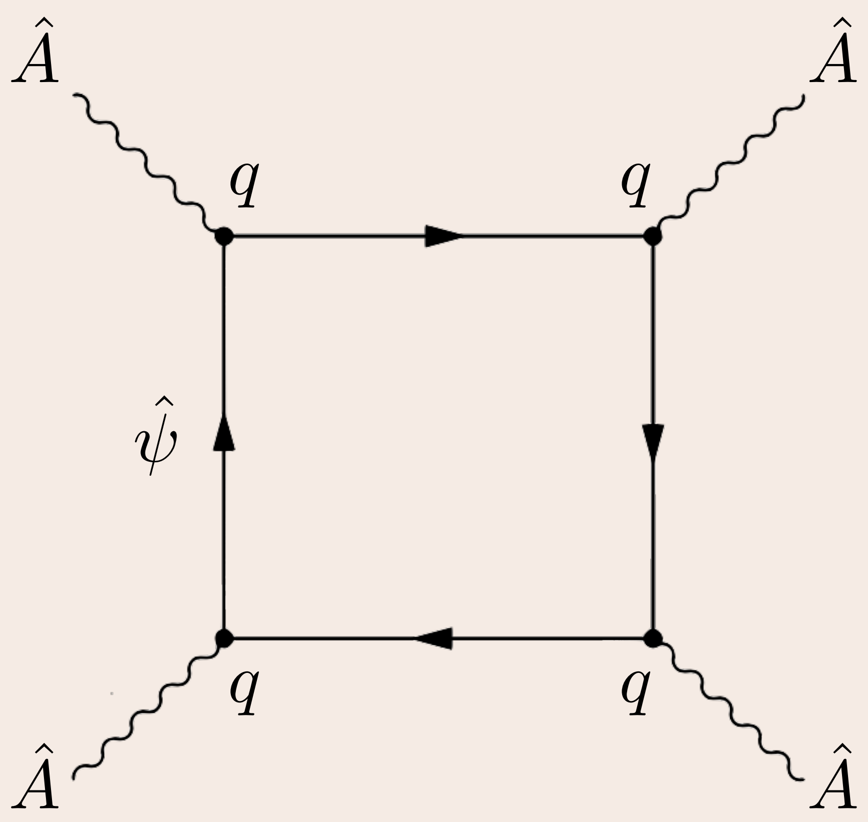


$$\cancel{\sum_{\mathsf{f}} q_{\mathsf{f}}^4 = 0}$$

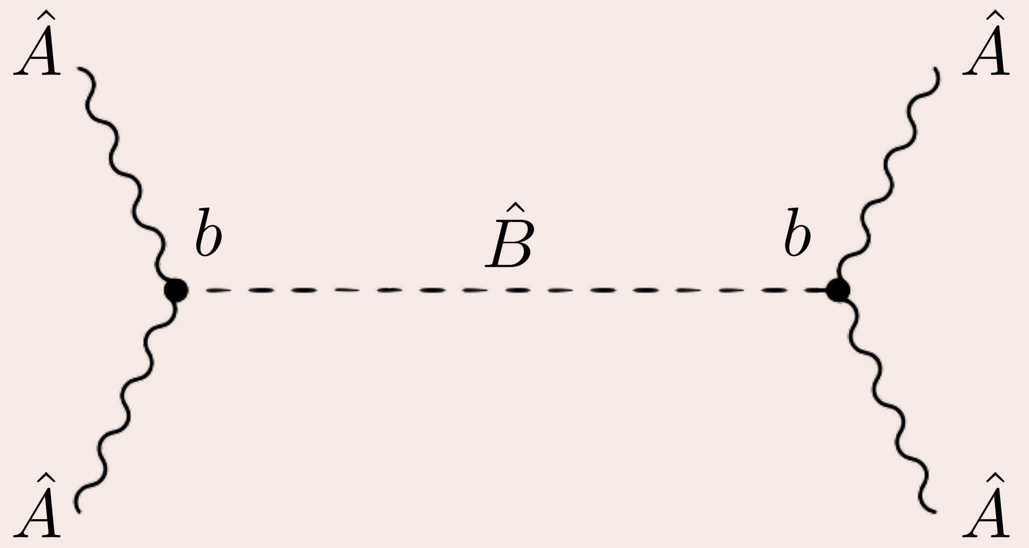
Cannot add fermions with suitable q's to cancel the anomaly, unlike 4d

Green-Schwarz Mechanism

Need to use the Green-Schwarz mechanism to cancel the anomaly:

Add to the action $-b\,\hat{B} \wedge \hat{F} \wedge \hat{F}$

$\hat{B}$ is a **2-form** which varies as $\delta \hat{B} = 2b\hat{\lambda}\hat{F}$



Classically non-invariant + δ(1-loop) = 0

Cancellation condition

$$\boxed{\sum_{\mathrm{f}} q_{\mathrm{f}}^4 = \frac{1}{3} b^2}$$

Circle reduction

What if not M_6 but $M_5 \times S^1$?

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In [PC, Grimm, Regalado '17] we looked at this question in 4d

4d **anomalous**

$$\Big\downarrow S^1$$

3d regularization **matters**

4d **anomaly free** $\sum_{\mathsf{f}} q_{\mathsf{f}}^3 = 0$

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3d regularization does **not** matter

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6d anomaly free: 1PI variation ⟷ GS term classically non-invariant

⟶ Expect 5d regularization to always matter

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Implications for F-theory on Calabi-Yau threefolds. Focus on susy theories

6d $\mathcal{N}$=(1,0) anomalies

Gravity multiplet + n_T tensors + n_V vectors + n_H hypers

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n_H^* charged fermions, with charge q_i^{f}, $\quad \mathsf{f} = 1,\ldots,n_H^*$ under $\hat{A}^i$

Anomaly

$$\propto \sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\, q_l^{\mathsf{f}}$$

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$n_T + 1$ two forms $\hat{B}^\alpha$

Green-Schwarz term $-b_{ij} \cdot \hat{B} \wedge \hat{F}^i \wedge \hat{F}^j$ where $\cdot$ means contraction with $SO(1, n_T)$ metric $\Omega_{\alpha\beta}$

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Anomaly cancellation condition reads

$$\boxed{\sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\, q_l^{\mathsf{f}} = b_{ijkl}}$$

where $b_{ijkl} \equiv b_{ij} \cdot b_{kl} + b_{ik} \cdot b_{jl} + b_{il} \cdot b_{jk}$

6d $\mathscr{N}$=(1,0) anomalies

Pure gauge $$\sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\, q_l^{\mathsf{f}} = b_{ijkl}$$

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Mixed

$$-\frac{1}{6}\sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}} = a \cdot b_{ij}$$

Green-Schwarz coefficient

$$-\tfrac{1}{2}\, a \cdot \hat{B} \wedge \mathrm{tr}\, \hat{R}$$

Pure gravitational

$$9 - n_T = a \cdot a$$

$$n_H - n_V = 273 - 29\, n_T$$

Erler '93

Park, Taylor '11

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$$S_6 = \frac{1}{2} M_{\mathrm{pl},6}^4 \int_{\mathcal{M}_6} R_6 \,\hat{\star}\, 1 - h_{UV}\, \mathrm{d}\hat{q}^U \wedge \hat{\star}\, \mathrm{d}\hat{q}^V - \hat{g}_{\alpha\beta} \left(\mathrm{d}j^\alpha \wedge \hat{\star}\, \mathrm{d}j^\beta + \frac{1}{2} \hat{G}^\alpha \wedge \hat{\star}\, \hat{G}^\beta \right)$$
$$-2\, b_{ij} \bullet j\ \hat{F}^i \wedge \hat{\star}\, \hat{F}^j - \hat{B} \wedge \bullet \left(\frac{1}{2}\, a\ \operatorname{tr} \hat{\mathcal{R}}^2 + 2\, b_{ij}\, \hat{F}^i \wedge \hat{F}^j \right)$$

Reduce it on S^1

Classical S^1 reduction

Reduction Ansätze

$$\hat{A}^i = A^i + \zeta^i \,(\mathrm{d}y - A^0)$$

$$\hat{B}^\alpha = 4\,B^\alpha - \big(4\,A^\alpha - 2\,b^\alpha_{ij}\,\zeta^i A^j\big) \wedge (\mathrm{d}y - A^0)$$

A^0 is the graviphoton

ζ^i are the Wilson lines

5d: dualize B^α into A^α

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Anomalies $\longrightarrow$ focus on the **Chern-Simons terms**

$$\mathcal{L}^{\mathrm{gi}} = -\frac{1}{4}A^0\wedge F\wedge\bullet F + \frac{1}{4}\,b_{ij}\bullet A\wedge F^i\wedge F^j$$

gauge invariant

$$\mathcal{L}^{\mathrm{ngi}} = -\frac{1}{16}\,b_{ijkl}\left(\zeta^i A^j\wedge F^k\wedge F^l - \zeta^i\zeta^j A^k\wedge F^l\wedge F^0 + \frac{1}{3}\,\zeta^i\zeta^j\zeta^k A^l\wedge F^0\wedge F^0\right)$$

Non gauge invariant: come from the GS terms

Canonical form

Expect $\mathcal{N}=1$ (8 supercharges) in 5d. Canonical form (vector part)

$$S_5^{\text{can}} = -G_{IJ}\left(\mathrm{d}\phi^I \wedge \star\mathrm{d}\phi^J + \bar{F}^I \wedge \star\bar{F}^J\right) - \frac{1}{6}\,k_{IJK}\bar{A}^I \wedge \bar{F}^J \wedge \bar{F}^K$$

Prepotential

$$\boxed{\mathcal{F} = \frac{1}{3!}\,k_{IJK}\,\phi^I\phi^J\phi^K}$$

$$G_{IJ} = -\frac{1}{2}\frac{\partial^2 \log\mathcal{F}}{\partial\phi^I\partial\phi^J}\bigg|_{\mathcal{F}=1} \qquad\qquad k_{IJK} = \partial_I\partial_J\partial_K\mathcal{F}$$

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$$\begin{aligned}\phi^0 &= c\,r^{-4/3}\\ \phi^\alpha &= \frac{c}{4}\,r^{2/3}\left(j^\alpha + 2\,r^{-2}\,b^\alpha_{ij}\,\zeta^i\zeta^j\right)\\ \phi^i &= c\,r^{-4/3}\,\zeta^i\end{aligned} \qquad\qquad \mathcal{F} = \frac{1}{2}\,\phi^0\,\phi\bullet\phi - \frac{1}{2}\,b_{ij}\bullet\phi\,\phi^i\phi^j + \frac{1}{24}\,b_{ijkl}\,\frac{\phi^i\phi^j\phi^k\phi^l}{\phi^0}\,,$$

$$c = 32^{1/3}$$

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$$\mathcal{F} = \underbrace{\frac{1}{2}\,\phi^0\,\phi\cdot\phi - \frac{1}{2}\,b_{ij}\cdot\phi\,\phi^i\phi^j}_{\text{cubic } \checkmark} + \underbrace{\frac{1}{24}\,b_{ijkl}\,\frac{\phi^i\phi^j\phi^k\phi^l}{\phi^0}}_{\text{Not cubic } \times},$$

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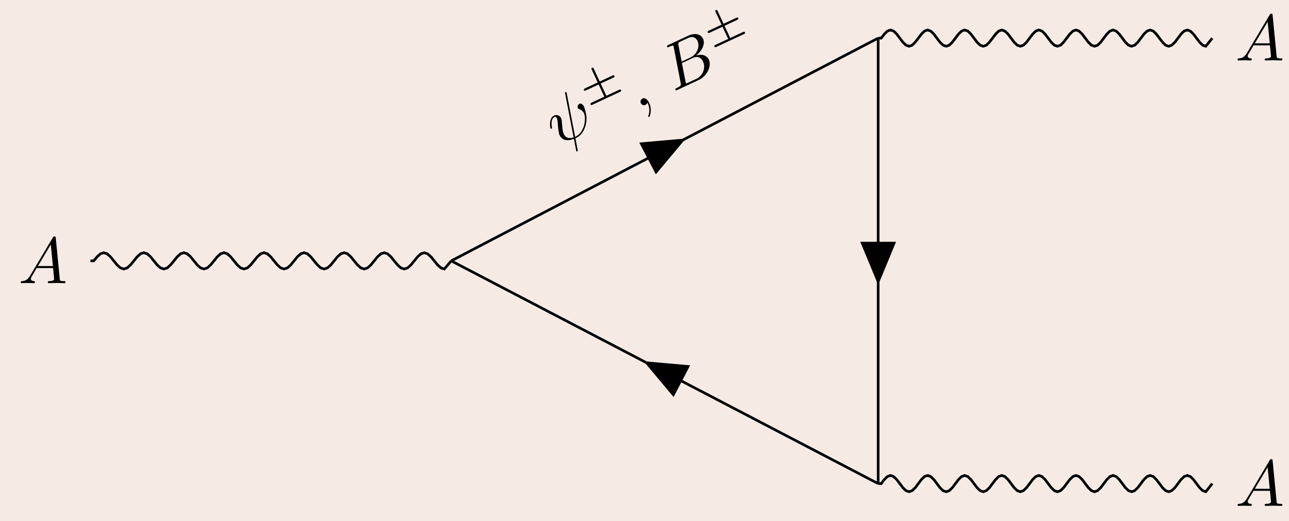
One-loop corrections



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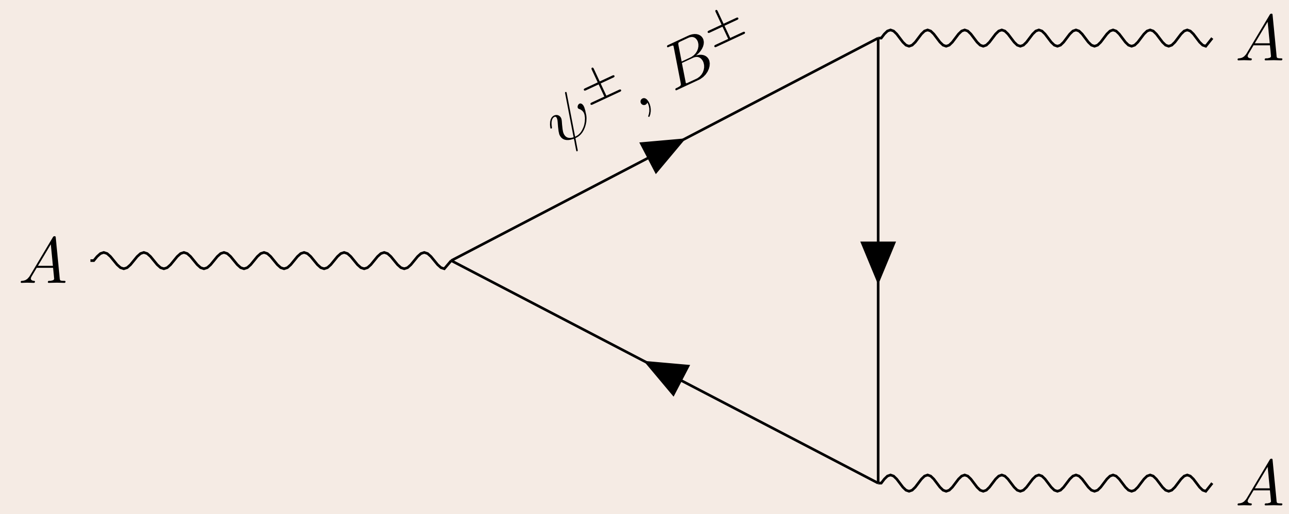

Single spin 1/2 fermion mass m and charge q

$$\mathcal{L}_{\mathrm{CS}}^{\text{1-loop}} = \frac{1}{24}\, q^3 \,\operatorname{sign}(m)\, A \wedge F \wedge F$$

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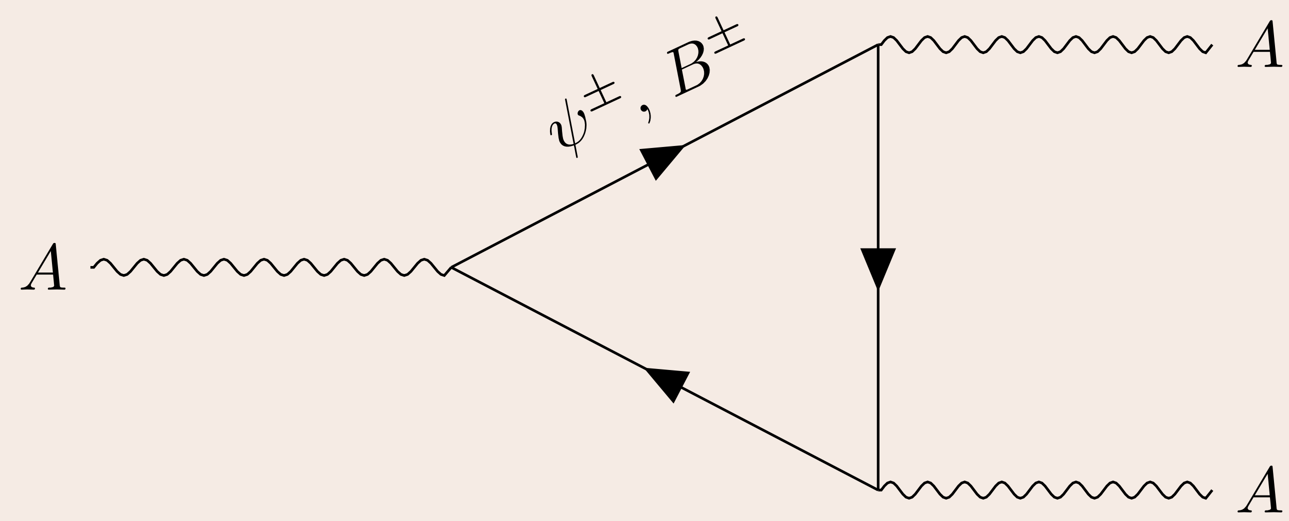

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Contributions of spin 3/2 and tensors computed in Bonetti, Grimm, Hohenegger '13

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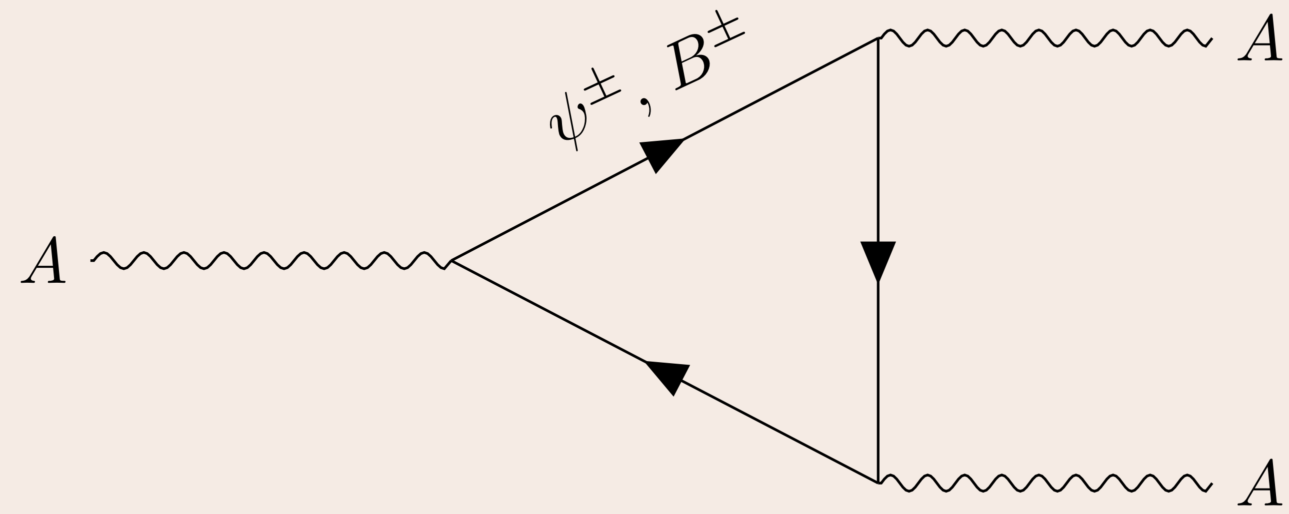

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- If all ext gauge fields are A^0, all KK-modes of spin 1/2, 3/2 and tensors run in the loop
- If at least one ext gauge field is A^i, only charged hyperini run in the loop

One-loop corrections

$$\mathcal{L}_{\text{CS}}^{\text{1-loop}} = -\frac{1}{12}\left(k_{ijk}A^iF^jF^k + 3\,k_{0ij}A^iF^jF^0 + 3\,k_{00i}A^iF^0F^0 + k_{000}A^0F^0F^0\right)$$

$$k_{ijk} = \frac{1}{2}\sum_{n\in\mathbb{Z}}\sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\,\text{sign}(m_n^{\mathsf{f}})$$

$$k_{0ij} = \frac{1}{2}\sum_{n\in\mathbb{Z}} n \sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\,\text{sign}(m_n^{\mathsf{f}})$$

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$$k_{000} = \frac{1}{2}\sum_{n\in\mathbb{Z}} n^3 \left[\sum_{\mathsf{f}} \text{sign}(m_n^{\mathsf{f}}) + \left(n_H^0 + n_T - n_V + 2\,(1-n_T) - 5\right)\text{sign}(m_{\text{KK}})\right]$$

$$m_{\text{KK}} = \frac{n}{r}\,\left(\frac{r_0}{r}\right)^{1/3}$$

$$m_n^{\mathsf{f}} = \frac{1}{r}\,\left(\frac{r_0}{r}\right)^{1/3}\left(n + q_i^{\mathsf{f}}\,\langle\zeta^i\rangle\right)$$

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Sums ill-defined $\longrightarrow$ need to regularize

ζ-function regularization

$$k_{ijk}^{\mathrm{reg}} = \sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}} \left(\frac{1}{2} + S_1 - q_l^{\mathsf{f}} \langle \zeta^l \rangle \right)$$

$$k_{0ij}^{\mathrm{reg}} = \sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}} \left(-\frac{1}{12} + S_2 + \frac{1}{2}\, q_k^{\mathsf{f}}\, q_l^{\mathsf{f}} \langle \zeta^k \rangle \langle \zeta^l \rangle \right)$$

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S_i : some integer functions of q_i^{f} and $\langle \zeta^i \rangle$

ζ-function regularization

Not properly quantized $\propto \sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\, q_l^{\mathsf{f}}$

Reminiscent of the anomaly

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Invariant under and $\delta A^i = \mathrm{d}\lambda^i$ and $\delta\zeta^i = n^i$. **Should not** be the case: 1PI should vary.

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$$k^{\text{reg}}_{ijk} = \sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}} \left(\frac{1}{2} + S_1 - q_l^{\mathsf{f}} \langle \zeta^l \rangle\right)$$

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Regularization taking place in the UV $\longrightarrow$ Need to use regularization preserving **6d Lorentz invariance,** e.g. start from 6d Pauli-Villars

6d Lorentz invariant regularization

$$k^{\mathbf{reg}}_{ijk} = \sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}} \left(\frac{1}{2} + S_1 - \frac{3}{4}\, q_l^{\mathsf{f}}\, \zeta^l\right)$$

$$k^{\mathbf{reg}}_{0ij} = \sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}} \left(-\frac{1}{12} + S_2 + \frac{1}{4}\, q_k^{\mathsf{f}}\, q_l^{\mathsf{f}}\, \zeta^k \zeta^l\right)$$

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$$k^{\mathbf{reg}}_{000} = \sum_{\mathsf{f}} S_4 + \frac{1}{120}(n_H - n_V - n_T - 3)$$

S_i : some integer functions of q_i^{f} and $\langle \zeta^i \rangle$

6d Lorentz invariant regularization

$$k^{\mathbf{reg}}_{ijk} = \sum_{\mathsf{f}} q^{\mathsf{f}}_i\, q^{\mathsf{f}}_j\, q^{\mathsf{f}}_k \left(\frac{1}{2} + S_1 - \frac{3}{4}\, q^{\mathsf{f}}_l\, \zeta^l\right)$$

$$k^{\mathbf{reg}}_{0ij} = \sum_{\mathsf{f}} q^{\mathsf{f}}_i\, q^{\mathsf{f}}_j \left(-\frac{1}{12} + S_2 + \frac{1}{4}\, q^{\mathsf{f}}_k\, q^{\mathsf{f}}_l\, \zeta^k \zeta^l\right)$$

$$k^{\mathbf{reg}}_{00i} = \sum_{\mathsf{f}} q^{\mathsf{f}}_i \left(S_3 - \frac{1}{12}\, q^{\mathsf{f}}_j\, q^{\mathsf{f}}_k\, q^{\mathsf{f}}_l\, \zeta^j \zeta^k \zeta^l\right)$$

$$k^{\mathbf{reg}}_{000} = \sum_{\mathsf{f}} S_4 + \frac{1}{120}(n_H - n_V - n_T - 3)$$

Full scalar field, not just vev

Coefficients changed

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Varies as **expected** under $\delta A^i = \mathrm{d}\lambda^i$ and $\delta\zeta^i = n^i$

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$$k^{\mathbf{reg}}_{000} = \sum_{\mathsf{f}} S_4 + \frac{1}{120}(n_H - n_V - n_T - 3)$$

Full scalar field, not just vev

Coefficients changed

S_i : some integer functions of q^{f}_i and $\langle \zeta^i \rangle$

"Anomaly preserved under circle reduction"

Varies as **expected** under $\delta A^i = \mathrm{d}\lambda^i$ and $\delta\zeta^i = n^i$

$$\mathcal{L}^{\text{ngi, quant}} = -\frac{1}{16} \underbrace{\left(b_{ijkl} - \sum_{\mathsf{f}} q^{\mathsf{f}}_i\, q^{\mathsf{f}}_j\, q^{\mathsf{f}}_k\, q^{\mathsf{f}}_l\right)}_{} \left(\zeta^i A^j F^k F^l - \zeta^i \zeta^j A^k F^l F^0 + \frac{1}{3}\, \zeta^i \zeta^j \zeta^k A^l F^0 F^0\right)$$

$= 0$ if anomaly canceled $\longrightarrow$ no field-dep CS terms gauge invariant

CS for an anomaly free theory

Assuming **all** anomalies cancel

$$\mathcal{L}^{\text{CS}} = -\frac{1}{4}A^0 F\bullet F + \frac{1}{4}\, b_{ij}\bullet A F^i F^j - \frac{1}{24}\sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\, A^i F^j F^k - \frac{1}{8}\, a\bullet b_{ij} A^i F^j F^0 - \frac{1}{48}\, a\bullet a\, A^0 F^0 F^0$$

Infer the **fully corrected prepotential**

$$\boxed{\mathcal{F}^{\text{quant}}_{S^1} = \frac{1}{2}\,\phi^0 \phi\bullet\phi - \frac{1}{2}\, b_{ij}\bullet\phi\,\phi^i\phi^j + \frac{1}{12}\sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\,\phi^i\phi^j\phi^k + \frac{1}{4}\, a\bullet b_{ij}\,\phi^0\phi^i\phi^j + \frac{1}{24}\, a\bullet a\,(\phi^0)^3}$$

CS for an anomaly free theory

Assuming **all** anomalies cancel

$$\mathcal{L}^{\rm CS} = -\frac{1}{4} A^0 F \bullet F + \frac{1}{4}\, b_{ij} \bullet A F^i F^j - \frac{1}{24} \sum_{\sf f} q_i^{\sf f}\, q_j^{\sf f}\, q_k^{\sf f}\, A^i F^j F^k - \frac{1}{8}\, a \bullet b_{ij} A^i F^j F^0 - \frac{1}{48}\, a \bullet a\, A^0 F^0 F^0$$

Infer the **fully corrected prepotential**

$$\mathcal{F}^{\rm quant}_{S^1} = \frac{1}{2}\, \phi^0 \phi \bullet \phi - \frac{1}{2}\, b_{ij} \bullet \phi\, \phi^i \phi^j + \frac{1}{12} \sum_{\sf f} q_i^{\sf f}\, q_j^{\sf f}\, q_k^{\sf f}\, \phi^i \phi^j \phi^k + \frac{1}{4}\, a \bullet b_{ij}\, \phi^0 \phi^i \phi^j + \frac{1}{24}\, a \bullet a\, (\phi^0)^3$$

Corrected coordinates

$$\begin{aligned} \phi^0 &= c\, r^{-4/3} \\ \phi^\alpha &= \frac{c}{4}\, r^{2/3} \left(\sqrt{1-\Delta}\, j^\alpha + 2\, r^{-2}\, b^\alpha_{ij}\, \zeta^i \zeta^j \right) \\ \phi^i &= c\, r^{-4/3}\, \zeta^i \end{aligned}$$

$$\Delta = 32\, r^{-4} \left(\frac{1}{24}\, a \bullet a + \frac{1}{4} a \bullet b_{ij}\, \zeta^i \zeta^j + \frac{1}{12} \sum_{\sf f} q_i^{\sf f}\, q_j^{\sf f}\, q_k^{\sf f}\, \zeta^i \zeta^j \zeta^k - \frac{1}{24}\, b_{ijkl}\, \zeta^i \zeta^j \zeta^k \zeta^l \right)$$

Bonus: Quantum corrections to the kinetic terms

Thanks to susy infer quantum corrections to kinetic terms $\quad G_{IJ} = -\frac{1}{2}\frac{\partial^2 \log \mathcal{F}}{\partial\phi^I \partial\phi^J}\bigg|_{\mathcal{F}=1}$

Before corrections

$$\begin{aligned}\mathcal{L}^{\mathrm{kin,class}} = & -\frac{2}{3}\, r^{-2}\, \mathrm{d}r \wedge \star \mathrm{d}r - \frac{1}{4}\, r^{8/3}\, \widetilde{F}^0 \wedge \star \widetilde{F}^0 - h_{UV}\, \mathrm{d}q^U \wedge \star\, \mathrm{d}q^V \\ & - \frac{1}{2}\, g_{\alpha\beta} \left(\mathrm{d}j^a \wedge \star\, \mathrm{d}j^\beta + r^{-4/3}\, \widetilde{F}^\alpha \wedge \star \widetilde{F}^\beta \right) - 2\, j \bullet b_{ij} \left(r^{-2}\, \mathrm{d}\zeta^i \wedge \star\, \mathrm{d}\zeta^j + r^{2/3}\, \widetilde{F}^i \wedge \star\, \widetilde{F}^j \right),\end{aligned}$$

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After corrections

$$\begin{aligned}
\mathcal{L}^{\text{kin, quant}} = \; & -\frac{2}{3}\, r^{-2}\,\frac{1-4\Delta}{1-\Delta}\;\mathrm{d}r\wedge\star\mathrm{d}r + r^{8/3}\Big(-\frac{1}{4}+\frac{1}{2}\,\Delta-\Delta^2\Big)\,\widetilde{F}^0\wedge\star\widetilde{F}^0 \\
& -\Big(\frac{1}{2}-\frac{3}{2}\,\Delta+\Delta^2\Big)\,g_{\alpha\beta}\;\mathrm{d}j^\alpha\wedge\star\mathrm{d}j^\beta + r^{-4/3}\Big(-\frac{1}{2}+\Delta\Big)\,g_{\alpha\beta}\,\widetilde{F}^\alpha\wedge\star\widetilde{F}^\beta \\
& -\left[2\,r^{-2}\,j\bullet b_{ij}\,\sqrt{1-\Delta}-\frac{1}{16}\,\frac{(1-4\Delta)\,\Delta_{ij}+2\,\Delta_i\Delta_j}{1-\Delta}\right]\mathrm{d}\zeta^i\wedge\star\mathrm{d}\zeta^j \\
& -\left[2\,r^{2/3}\,j\bullet b_{ij}\,\sqrt{1-\Delta}-\frac{1}{4}\,r^{8/3}\,\Big(\Delta_{ij}-\Delta_i\Delta_j\Big)\right]\widetilde{F}^i\wedge\star\widetilde{F}^j \\
& - r^{-1}\,\frac{\Delta_i}{1-\Delta}\,\mathrm{d}r\wedge\star\mathrm{d}\zeta^i + r^{8/3}\,\Big(\frac{1}{2}-\Delta\Big)\,\Delta_i\,\widetilde{F}^0\wedge\star\widetilde{F}^i \\
& + \; r^{2/3}\,j_\alpha\,\sqrt{1-\Delta}\,\Big(\Delta_i\,\widetilde{F}^i\wedge\star\widetilde{F}^\alpha-2\,\Delta\,\widetilde{F}^0\wedge\star\widetilde{F}^\alpha\Big)
\end{aligned}$$

$$\Delta = 32\,r^{-4}\Big(\frac{1}{24}\,a\bullet a+\frac{1}{4}a\bullet b_{ij}\,\zeta^i\zeta^j+\frac{1}{12}\sum_{\mathsf{f}} q_i^{\mathsf{f}}\,q_j^{\mathsf{f}}\,q_k^{\mathsf{f}}\,\zeta^i\zeta^j\zeta^k-\frac{1}{24}\,b_{ijkl}\,\zeta^i\zeta^j\zeta^k\zeta^l\Big)$$

F-theory implications

F-theory effective actions: use the **F/M-theory duality**:

$$\begin{tikzcd} & \text{F-Theory 12d} \arrow[d, "Y_3"] \\ \text{M-Theory 11d} \arrow[d, "\hat{Y}_3"] & \boxed{\text{6d } \mathcal{N}=(1,0)} \\ \text{5d } \mathcal{N}=1 \arrow[ur, bend left, "\text{lift}"] & \end{tikzcd}$$

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$$
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\text{M-Theory 11d} \arrow[d, "\text{classical}"', "\hat{Y}_3"] & \boxed{\text{6d } \mathcal{N}=(1,0)} \arrow[d, "S^1 \quad \text{quantum}"] \\
\text{5d } \mathcal{N}=1 \arrow[ur, bend left, "\text{lift}"] \arrow[r, leftrightarrow, bend right, "\text{compare}"'] & \text{5d } \mathcal{N}=1
\end{tikzcd}
$$

Morrison, Vafa '96
Bonetti, Grimm '11

F-theory implications

F-theory effective actions: use the **F/M-theory duality**:

Vectors come from expending
$\hat{C}_3$ along harmonic (1,1) forms

$$\hat{C}_3 = A^I \wedge \omega_I + \dots \quad I = 1, \dots, h^{1,1}(\hat{Y}_3)$$

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 & \text{5d } \mathcal{N} = 1 \\
\text{5d } \mathcal{N} = 1 \arrow[uur, bend left, "\text{lift}"] \arrow[ur, leftrightarrow, bend right, "\text{compare}"'] &
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Morrison, Vafa '96
Bonetti, Grimm '11

Chern-Simon terms

$$S^{\mathrm{CS}} = -\frac{1}{12} \int_{\mathcal{M}_5 \times Y_3} C_3 \wedge G_4 \wedge G_4 = -\frac{1}{12} \mathcal{K}_{IJK} \int_{\mathcal{M}_5} A^I \wedge F^J \wedge F^k \,,$$

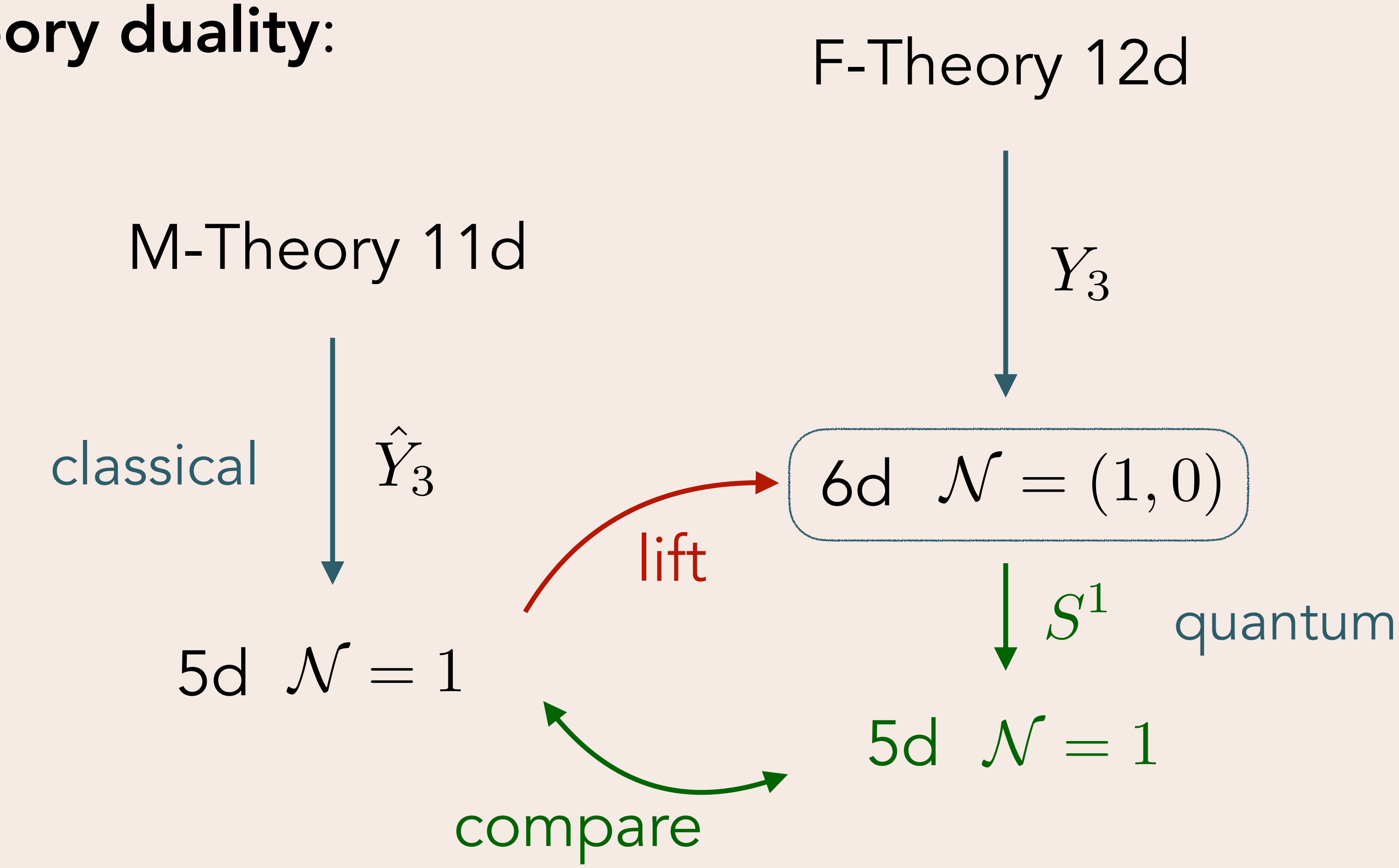
Intersection numbers

$$\boxed{\mathcal{K}_{IJK} = \int_{Y_3} \omega_I \wedge \omega_J \wedge \omega_K = D_I \cap D_J \cap D_K \,.}$$

Poincaré dual divisors

F-theory implications

F-theory effective actions: use the **F/M-theory duality**:



$$\mathcal{L}^{\rm CS} = -\frac{1}{12}\,\mathcal{K}_{IJK}\, A^I \wedge F^J \wedge F^k$$

$$\boxed{\mathcal{K}_{IJK} = \int_{Y_3} \omega_I \wedge \omega_J \wedge \omega_K = D_I \cap D_J \cap D_K\,.}$$

Intersection numbers are **field-independent**

→ Can only be matched with circle reduction of an anomaly free theory

→ **EFTs from F-theory** obtained via M-theory are automatically **anomaly free**

F-theory basis

We take Y_3 **elliptically fibered**, smooth, with **extra sections** σ_i

The correct basis to lift to F-theory $\qquad D_I = (D_0, D_\alpha, D_i)$

- The **vectors** A^α lifting to two-forms (tensors multiplets) come from **vertical divisors**

$$D_\alpha = \pi^*(D_\alpha^{\mathrm{b}}), \qquad \alpha = 1, ..., h^{1,1}(B_2)$$

- **KK-photon** A^0 comes from the **zero section** σ_0 **shifted as**

$$D_0 = \sigma_0 - \frac{1}{2}\left(\sigma_0 \cap \sigma_0 \cap D^\alpha\right) D_\alpha \ ,$$

Park '11
Grimm, Savelli '11
Bonetti, Grimm '11

- $U(1)$ **vectors** come from the **extra sections** σ_i**'s shifted as**

$$D_i = \sigma_i - \sigma_0 - \Big[(\sigma_i - \sigma_0) \cap \sigma_i \cap D^\alpha\Big] D_\alpha$$

Park '11
Morisson Park '11

Prepotential

Scalars $\varphi^I = \frac{v^I}{\mathcal{V}^{1/3}}$ come from the Kähler form $J = v^I \omega_I$

Prepotential $$\mathcal{F}_M = \frac{1}{3!}\mathcal{K}_{IJK}\varphi^I\varphi^J\varphi^K$$

For the basis $D_I = (D_0, D_\alpha, D_i)$, compute the intersection numbers, find

$$\begin{aligned}\mathcal{F}_M = \ & \frac{1}{2}\varphi^0\,\varphi\circ\varphi + \frac{1}{2}\pi(D_i\cap D_j)\circ\varphi\,\varphi^i\varphi^j \\ & + \frac{1}{6}\mathcal{K}_{ijk}\,\varphi^i\varphi^j\varphi^k - \frac{1}{4}\pi(D_i\cap D_i)\circ K\,\varphi^0\,\varphi^i\varphi^j + \frac{1}{24}K\circ K\,\left(\varphi^0\right)^3\end{aligned}$$

Grimm, Kapfer, Keitel '11

where

- $\circ$ product with $\eta_{\alpha\beta} = D^{\mathrm{b}}_\alpha \cap D^{\mathrm{b}}_\beta$ intersection matrix on the base
- π projection onto the base
- K coefficients of $K^{-1}_{B_2} = K^\alpha D^{\mathrm{b}}_\alpha$

Match

Matches with the prepotential from the circle reduction

$$\mathcal{F}^{\text{quant}}_{S^1} = \frac{1}{2}\,\phi^0\phi\bullet\phi - \frac{1}{2}\,b_{ij}\bullet\phi\,\phi^i\phi^j + \frac{1}{12}\sum_{\mathsf{f}} q_i^{\mathsf{f}}\,q_j^{\mathsf{f}}\,q_k^{\mathsf{f}}\,\phi^i\phi^j\phi^k + \frac{1}{4}\,a\bullet b_{ij}\,\phi^0\phi^i\phi^j + \frac{1}{24}\,a\bullet a\,(\phi^0)^3$$

if one identifies

$$\phi^I = \varphi^I$$
$$\eta_{\alpha\beta} = \Omega_{\alpha\beta}$$

$$a^\alpha = K^\alpha$$
$$b_{ij}^\alpha = -\pi(D_i\cap D_j)^\alpha$$

$$\mathcal{K}_{ijk} = \frac{1}{2}\sum_{\mathsf{f}} q_i^{\mathsf{f}}q_j^{\mathsf{f}}q_k^{\mathsf{f}}\,,$$

Match

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Match with literature

Sadov '96
Park '11
Grimm, Kapfer, Keitel '13

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Matches with the prepotential from the circle reduction

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Match with literature

Sadov '96
Park '11
Grimm, Kapfer, Keitel '13

$$c = 32^{1/3}$$

Justifies

$$\phi^0 = c\,r^{-4/3}$$
$$\phi^\alpha = \frac{c}{4}\,r^{2/3}\left(j^\alpha + 2\,r^{-2}\,b_{ij}^\alpha\,\zeta^i\zeta^j\right)$$
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Match

Matches with the prepotential from the circle reduction

$$\mathcal{F}^{\text{quant}}_{S^1} = \frac{1}{2}\,\phi^0\phi\bullet\phi - \frac{1}{2}\,b_{ij}\bullet\phi\,\phi^i\phi^j + \frac{1}{12}\sum_{\mathsf{f}} q_i^{\mathsf{f}}\,q_j^{\mathsf{f}}\,q_k^{\mathsf{f}}\,\phi^i\phi^j\phi^k + \frac{1}{4}\,a\bullet b_{ij}\,\phi^0\phi^i\phi^j + \frac{1}{24}\,a\bullet a\,(\phi^0)^3$$

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Match only possible if

- All anomalies cancel
- Adequate regularization

Summary

- **6d $\mathcal{N}$=(1,0) anomalies on a circle**
  - Regularization preserving 6d Lorentz invariance
  - Field-dependent Chern-Simons terms
  - Vanish when anomaly cancel via Green-Schwarz mechanism
  - 5d $\mathcal{N}$=1 cubic prepotential for anomaly free theories
  - Read off quantum corrections to the kinetic terms

- **F-theory implications**
  - CS coeff are intersection numbers $\longrightarrow$ EFTs from F-theory are **anomaly free**
  - Perfect match of circle and M-theory prepotentials
  - Correct identification of the coordinates

  - All anomalies cancel
  - Adequate regularization

Thank you!