

# The Holographic Swampland

Seminar Series on String Phenomenology

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Based on: 2006.01021 [Joseph Conlon, FR]

+ other works in progress

(Also in collaboration with Joseph Conlon and Sirui Ning)

# Introduction

Swampland program: criteria to distinguish low energy Lagrangians admitting a UV completion in ST (QG)

Topic of this talk:

AdS/CFT

Swampland constraints  
on 4d EFTS



CFT inconsistencies  
or  
reformulation

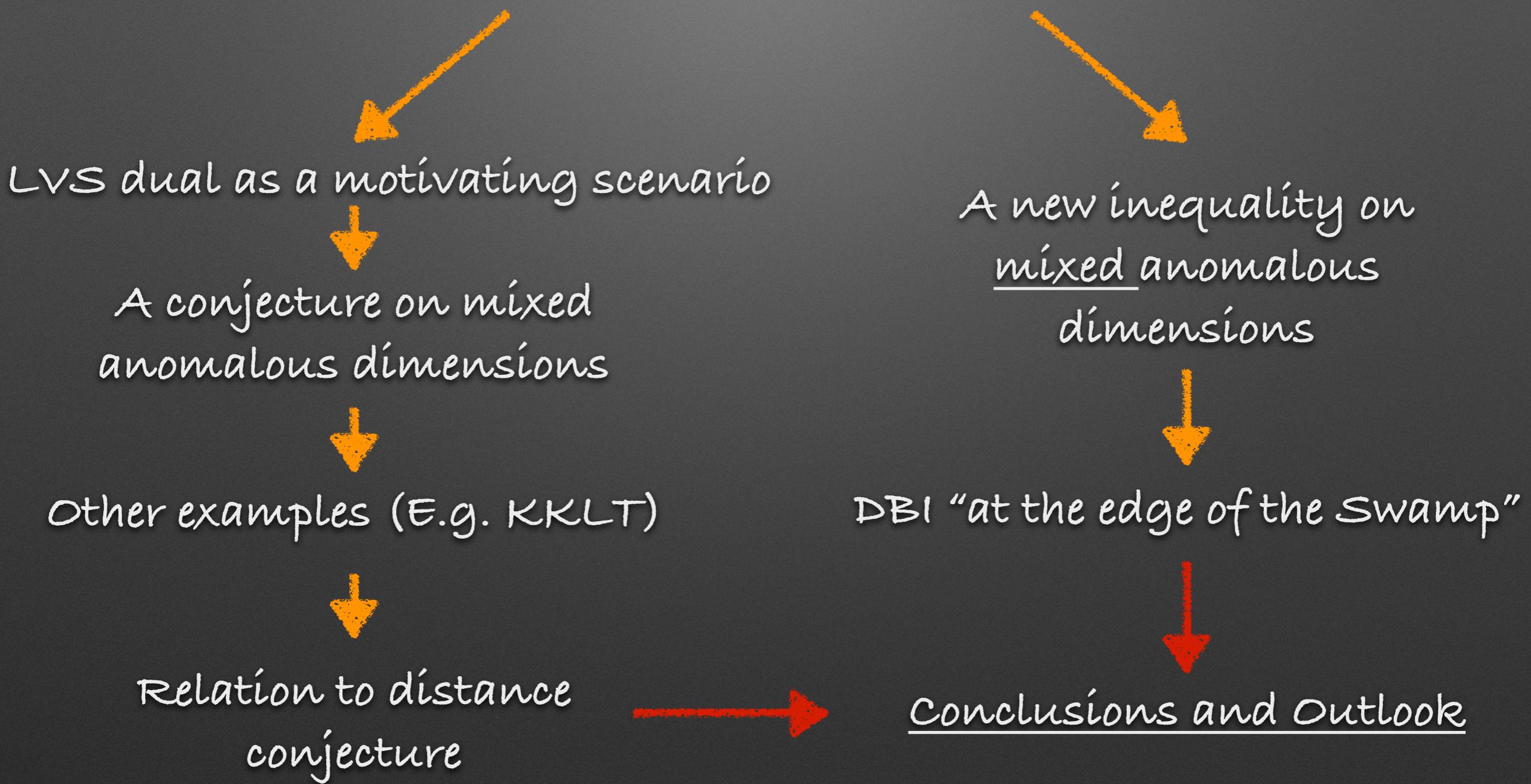
Why?

- Develop independent framework to test **debated constructions**
- AdS/CFT universal tool for QG, not just ST
- Bootstrap very successful in mapping space of allowed theories

# Plan of the talk

Intro: Holographic CFTs

CFT positivity bounds and how to apply them



# Holographic CFTs

Gravity dual is weakly coupled and amenable to perturbative analysis

$$\lambda = g^2 N \rightarrow (R/\ell_S)^d \gg 1$$

Expansion in large  $N$  parametr or equivalent

Large gap in the spectrum  $\Delta_{gap} \gg 1$

Single trace primaries  $\mathcal{O}_1, \mathcal{O}_2$       Dimension  $\Delta_1, \Delta_2$

**OPE:**  $\mathcal{O}_1 \times \mathcal{O}_2 \supset \mathbb{1}, \mathcal{O}_1, \mathcal{O}_2, [\mathcal{O}_1 \mathcal{O}_2]_{n,l}$

Double trace operators:  $[\mathcal{O}_1 \mathcal{O}_2]_{n,l} \sim \mathcal{O}_1 \square^n \partial_{\mu_1} \partial_{\mu_2} \dots \partial_{\mu_l} \mathcal{O}_2$

$$\Delta = \Delta_1 + \Delta_2 + 2n + \ell + \gamma(n, l) \longrightarrow \text{anomalous dimension}$$

# The bootstrap

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_1(x_2) \mathcal{O}_2(x_3) \mathcal{O}_2(x_4) \rangle = \sum_{\mathcal{O}} C_{11\mathcal{O}} C_{22\mathcal{O}} \frac{G_{\Delta,\ell}(u,v)}{|x_{12}|^{2\Delta_1} |x_{34}|^{2\Delta_2}}$$

Bootstrap Equation:

$$u^\Delta \left( 1 + \sum_{\Delta,\ell} c_{\Delta,\ell}^2 G_{\Delta,\ell}(v,u) \right) = v^\Delta \left( 1 + \sum_{\Delta,\ell} c_{\Delta,\ell}^2 G_{\Delta,\ell}(u,v) \right)$$

No constraints from crossing symmetry alone

Solution to bootstrap  
equations

[Heemskerk, Penedones,  
Polchinski, Sully '05]

Quartic (derivative)  
vertices

conformal blocks

OPE coefficients

# CFT positivity bounds

Minimal twist operators dominate Lorentzian OPE on the light cone

$$\tau = \Delta - \ell$$

$$\frac{\tau_{\ell_3}^* - \tau_{\ell_1}^*}{\ell_3 - \ell_1} \leq \frac{\tau_{\ell_2}^* - \tau_{\ell_1}^*}{\ell_2 - \ell_1}$$

$\gamma(0, \ell)$  for identical operators  
convex & negative for  $\ell \geq \ell_c$

Analytical bootstrap [Komargodski, Zhiboedov '12]

Inversion formula [Caron-Huot '17] + [Costa, Hansen, Penedones '17]

$$\ell_c \geq 2$$

causality arguments  
in the CFT:

$$\gamma(0, 2) \leq 0$$

[Hartman, Jain, Kundu '16]

$$\mathcal{L} = \frac{g}{\Lambda^4} (\nabla \varphi)^4 \quad g > 0$$

on AdS

Generalization of flat  
space S-matrix bounds

[Adams, Arkani-Hamed, Dubovsky,  
Nicolis, Rattazzi '06]

# Plan of the talk

Intro: Holographic CFTs 

CFT positivity bounds and how to apply them 

LVS dual as a motivating scenario



A conjecture on mixed anomalous dimensions



Other examples (E.g. KKLT)



Relation to distance conjecture



A new inequality on mixed anomalous dimensions



DBI “at the edge of the Swamp”



Conclusions and Outlook

# The Large Volume Scenario

Motivating scenario: Low energy dynamics of moduli in ST

LVS: Type IIB flux compactification, with all moduli stabilised at an **exponentially large volume**

$$V = V_0 e^{-\lambda \Phi / M_P} \left( - \left( \frac{\Phi}{M_P} \right)^{3/2} + A \right)$$

$$\Phi = \sqrt{\frac{2}{3}} \ln \mathcal{V}$$

↑  
canonically normalised  
volume modulus

AdS vacuum:

$$V_{\min} = -3M_P^2 R_{AdS}^{-2}$$

$$m_\Phi = \sqrt{\frac{3}{2}} \frac{\lambda}{R_{AdS}}$$

[Balasubramanian,Berglund,Conlon,Quevedo '05]

[Conlon,Quevedo,Suruliz '05]

# Holographic reformulation of LVS

AdS/CFT

4d moduli EFT in AdS

Crucial property of LVS:

$$m_\phi \ll m_{3/2} \text{ as } \mathcal{V} \rightarrow \infty$$

Putative 3d CFT dual

+

$$m^2 R_{AdS}^2 = \Delta(\Delta - 3)$$

Small number of low lying operators:

Mode	Spin	Parity	Conformal dimension
$T_{\mu\nu}$	2	+	3
$a$	0	-	3
$\Phi$	0	+	$8.038 = \frac{3}{2}(1 + \sqrt{19})$

# LVS Effective Lagrangian

All interactions fixed in term of  $R_{AdS}$  only-  
uniquely determined theory in large  $\mathcal{V}$  limit

$$\mathcal{L}_{(\delta\Phi)^n} = (-\lambda)^n \frac{3M_P^2}{R_{AdS}^2} \frac{n-1}{n!} \left( \frac{\delta\Phi}{M_P} \right)^n \left( 1 + \mathcal{O}\left(\frac{1}{\lambda\langle\Phi\rangle}\right) \right)$$

  $\mathcal{L}_{(\delta\Phi)^{n-2}aa} = \left( + \sqrt{\frac{8}{3}} \right)^{(n-2)} \frac{1}{2(n-2)!} \left( \frac{\delta\Phi}{M_P} \right)^{n-2} \partial_\mu a \partial^\mu a$

Some modifications certainly in the Swampland

Sign flip in axion kinetic term



Divergent  $f_a$  as  $\mathcal{V} \rightarrow \infty$

$$\mathcal{L} \supset \frac{3}{4} e^{+\sqrt{\frac{8}{3}} \frac{\Phi}{M_P}} \partial_\mu a \partial^\mu a$$



What happens to the CFT?

# can we apply the positivity bounds?

Contact diagrams:

Finite support in the spin

$$\ell \leq \frac{\#\partial}{2}$$

[Heemskerk,Penedones,  
Polchinski,Sully '05]

[Fitzpatrick,Katz,Poland,  
Simmons-Duffin '12]

VS

Exchange diagrams:

Arbitrary large spin

BUT

Manifestly negative

$$\gamma(0, \ell) \propto -\lambda^2$$

For identical operators, not quite...

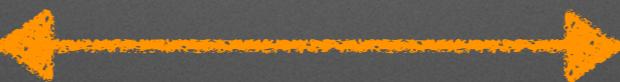
Claim:  $\gamma^{\varphi a}(0, \ell)$  correlates with swampland conditions

# Mixed anomalous dimensions

$$\mathcal{L}_{LVS} = g\varphi^3 - \mu\varphi\partial_\mu a\partial^\mu a$$



$\mathcal{A}(u, v)$



$M(s, t)$

Mandelstam-like variables

$$\gamma(0, \ell) \underset{\ell \gg 1}{\simeq} - \int_{-i\infty}^{+i\infty} \frac{ds}{2\pi i} M(s, 0) \left(\frac{s}{2}\right)^2 \frac{\Gamma(\Delta_1)\Gamma(\Delta_2)}{\ell^s}$$

[Costa, Goncalves, Penedones '12]

Lowest lying pole in **s-channel** dominates the integral

$$M(s, t) \simeq \sum_{m=0}^{\infty} \frac{\mathcal{Q}_{\ell, m}(t)}{s - \Delta + \ell - 2m} + P_{\ell-1}(t) + s \leftrightarrow t + s \leftrightarrow u$$

# Consequences for LVS

$$\gamma^{\varphi a}(0, \ell) \propto -\frac{g\mu(\Delta_\varphi - 6)}{\Gamma(\frac{6-\Delta_\varphi}{2})} \frac{1}{M_P^2 R_{AdS}^2} \frac{1}{\ell^{\Delta_\varphi}}$$

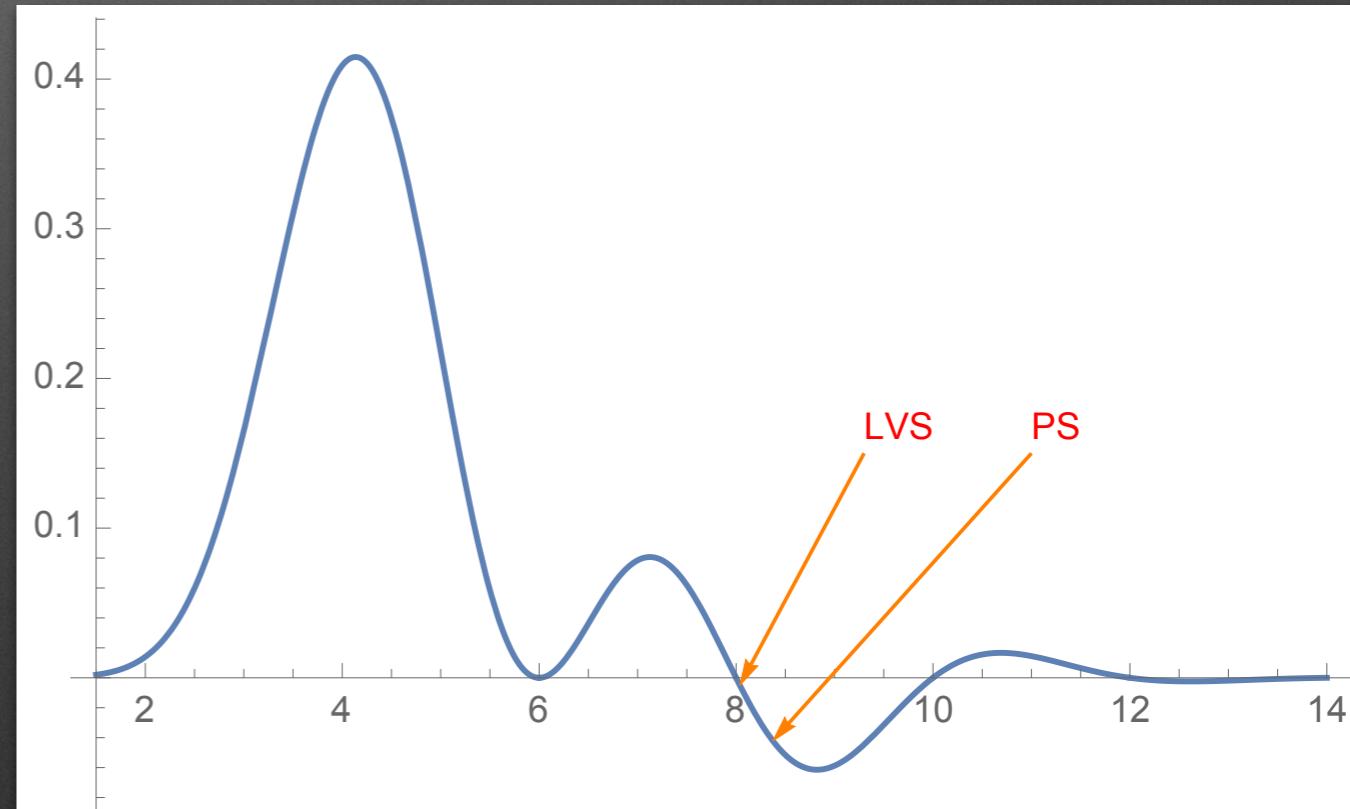
decouples higher order contributions at large volume

With LVS couplings,  $\gamma^{\varphi a}(0, \ell) < 0$

Swampland

sign flip in  $g\mu \longleftrightarrow f_a \sim M_P V^{\frac{2}{3}}$

$\Delta_\varphi \leq 6 \longleftrightarrow V \gtrsim M_S^4$



LVS very close to boundary of the allowed region!

# Other examples

Perturbative Stabilisation: LVS-like: Sign flip has same effect  
 [Berg, Haack, Kors '06,  
 von Gersdorff, Hebecker '05]

$$V_{eff} = Ae^{-\lambda_1 \varphi} - Be^{-\lambda_2 \varphi} \quad \gamma(0, \ell)_{\varphi a} < 0 \text{ for } \lambda_1 = \frac{10}{3}\sqrt{\frac{3}{2}}, \lambda_2 = 3\sqrt{\frac{3}{2}}$$

KKLT (before uplifting): [Kachru, Kallosh, Linde, Trivedi '03]

$$\mathcal{K} = -3 \log [-i(T - \bar{T})] \quad \mathcal{W} = W_0 + Ae^{-\alpha T}$$

Racettrack Stabilisation: [Krasnikov '87, Taylor '90,  
 De Carlos, Casas, Munoz '93]

$$\mathcal{K} = -3 \log[-i(T - \bar{T})] \quad \mathcal{W} = Ae^{-\alpha T} - Be^{-\beta T}$$

For both  $\gamma(0, \ell)_{\varphi a} < 0$  in their regime of validity

$$\sigma_c \gg 1, \sigma_c \alpha \gg 1, (\sigma_c \beta \gg 1)$$

Qualitatively  
 different from LVS

Potential for a   $\Delta_a > 3$   
 $\mathcal{L} \supset \sigma^3, \sigma a^2, \sigma \partial_\mu a \partial^\mu a$

# Connection to the distance conjecture

Interactions with heavy modes:

$$\mathcal{L}_{\psi\bar{\psi}} = K_{\psi\bar{\psi}} \partial_\mu \psi \partial^\mu \bar{\psi} + \frac{1}{V^2} K^{\psi\bar{\psi}} \psi \bar{\psi}$$

Since superpotential  $W$   
does depend on  $T$



E.g. KK mode:  $K_{\psi\bar{\psi}} \sim V^{-1/3}$

$$f_{\varphi\psi\psi} = \frac{\Gamma\left(\frac{2\Delta_\psi + \Delta_\varphi - 3}{2}\right)}{2\Gamma(\Delta_\varphi)\Gamma(\Delta_\psi)^2} \left( \sqrt{\frac{1}{6}} (\Delta_\varphi + 2\Delta_\psi - \Delta_\psi^2 - 3) + \frac{5}{\sqrt{6}} \Delta_\psi (\Delta_\psi - 3) \right)$$

$$\left. \begin{array}{l} f_{\varphi\psi\psi} > 0 \equiv \frac{\partial m^2(\psi)}{\partial \varphi} < 0 \\ f_{\varphi\psi\psi} < 0 \equiv \frac{\partial m^2(\psi)}{\partial \varphi} > 0 \end{array} \right\}$$

Negative  $\gamma(0, \ell)$  requires  
heavy states to become light  
in LV limit



Distance Conjecture

# caveats

LVS with a fibred direction

$$\mathcal{V} = \alpha \left( \tau_1 \sqrt{\tau_2} - \tau_3^{3/2} \right)$$

$\downarrow$  Flat direction - lifted by loop

effects but still  $m R_{AdS} \xrightarrow{\mathcal{V} \rightarrow +\infty} 0$

$$\varphi_1 = \sqrt{\frac{2}{3}} \log(\mathcal{V})$$

$$\varphi_2 = \frac{\sqrt{3}}{2} \log(\tau_1 \mathcal{V}^{-2/3})$$

$$\mathcal{L}_{kin} \supset \frac{1}{4} e^{-2\sqrt{\frac{2}{3}}(\varphi_1 + 2\varphi_2)} \partial_u a_1 \partial^u a_1 + \frac{\alpha^2}{2} e^{\left(\sqrt{\frac{2}{3}} - 2\sqrt{\frac{3}{2}}\right)\varphi_1} e^{2\sqrt{\frac{2}{3}}\varphi_2} \partial_u a_2 \partial^u a_2$$

$$\gamma^{a_i \varphi_1}(0, \ell) < 0$$

$$\gamma^{a_1 a_2}(0, \ell) > 0$$

Positivity only for volume modulus?

# Plan of the talk

Intro: Holographic CFTs 

CFT positivity bounds and how to apply them 

LVS dual as a motivating scenario 

↓  
A conjecture on mixed anomalous dimensions 

↓  
Other examples (E.g. KKLT) 

↓  
Relation to distance conjecture 

↓  
A new inequality on mixed anomalous dimensions 

↓  
DBI “at the edge of the Swamp” 

Conclusions and Outlook

# CFT positivity bounds again

For even spins  $\ell \geq 2$

$$\gamma_{12}(0, \ell) \geq \frac{1}{2} [\gamma_{11}(0, \ell) + \gamma_{22}(0, \ell)] \quad [\text{Kundu '20}]$$

Trivially satisfied at large spin, where it becomes equivalent to

$$\sum_{\mathcal{O}} (C_{11\mathcal{O}} - C_{22\mathcal{O}})^2 \geq 0$$

can however be used to constrain multi-derivative quartic vertices

$$\sum_{\ell=0}^{\infty} \gamma_{ij}(0, \ell) \left( f_{[ij]_0, \ell}^{i,j} \right)^2 Q_{\ell,0}(-s + \Delta_1 - \Delta_3) \alpha(\Delta_i, \ell) = -4M_{ii \rightarrow jj}(s, \Delta_1 + \Delta_3)$$

“Forward” Mellin amplitude  $\sim s^k$  contributes up to spin  $k$

# The constraints

$$\begin{aligned}\mathcal{L} \supset & \frac{c_{11}}{\Lambda^4} (\partial\varphi_1)^4 + \frac{c_{12}}{\Lambda^4} (\partial\varphi_1)^2 (\partial\varphi_2)^2 + \frac{c_{22}}{\Lambda^4} (\partial\varphi_2)^4 \\ & + \frac{\tilde{c}_{12}}{\Lambda^4} (\partial\varphi_1 \partial\varphi_2)^2\end{aligned}$$

(Assuming no cubic terms)

$$c_{11} f_d(\Delta_1, \Delta_1) - \left( c_{12} + \frac{\tilde{c}_{12}}{2} \right) f_d(\Delta_1, \Delta_2) + c_{22} f_d(\Delta_2, \Delta_2) \geq 0$$

$$f_d(x, y) = \frac{\Gamma(x+2)\Gamma(y+2)\Gamma\left(x+y+2-\frac{d}{2}\right)}{\Gamma\left(x+\frac{d}{2}-1\right)\Gamma\left(y+\frac{d}{2}-1\right)\Gamma(x+y+4)}$$

Correct flat spacetime limit for  $m(R)R \rightarrow 0$

# Application: multi-field DBI

Probe D3-brane moving along warped throat

$$ds^2 = \textcolor{red}{f}^{-1/2}(y^K) g_{\mu\nu} dx^\mu dx^\nu + \textcolor{red}{f}^{1/2}(y^K) G_{IJ}(y^K) dy^I dy^J$$

Compact coordinates give rise to scalars in 4d -  
inherently stringy and “universal” EFT

Many applications, E.g. DBI inflation  
[Silverstein, Tong '04]  
[Alishahiha, Silverstein, Tong '04]

Full multifield action:

[Langlois, Renaux-Petel, Steer, Tanaka '08]

$$S = - \int d^4x \sqrt{-g} \frac{1}{\textcolor{red}{f}(\varphi^I)} \left( \sqrt{\mathcal{D}_S} - 1 \right)$$

$$\mathcal{D}_S = 1 - 2\textcolor{red}{f} G_{IJ} X^{IJ} + 4\textcolor{red}{f}^2 X_I^{[I} X_J^{J]} + \mathcal{O}(X^3) \quad X^{IJ} = -\frac{1}{2} \partial_\mu \varphi^I \partial^\mu \varphi^J$$

# DBI at the “edge of the Swamp”

$$\mathcal{L} \supset \frac{1}{2f(\varphi^I)} \sum_I (\partial\varphi_I)^4 + \frac{1}{f(\varphi^I)} \sum_{I < J} 4(\partial\varphi_I \partial\varphi_J)^2 - 3(\partial\varphi_I)^2 (\partial\varphi_J)^2$$

$$c_{11} = c_{22} = 1$$

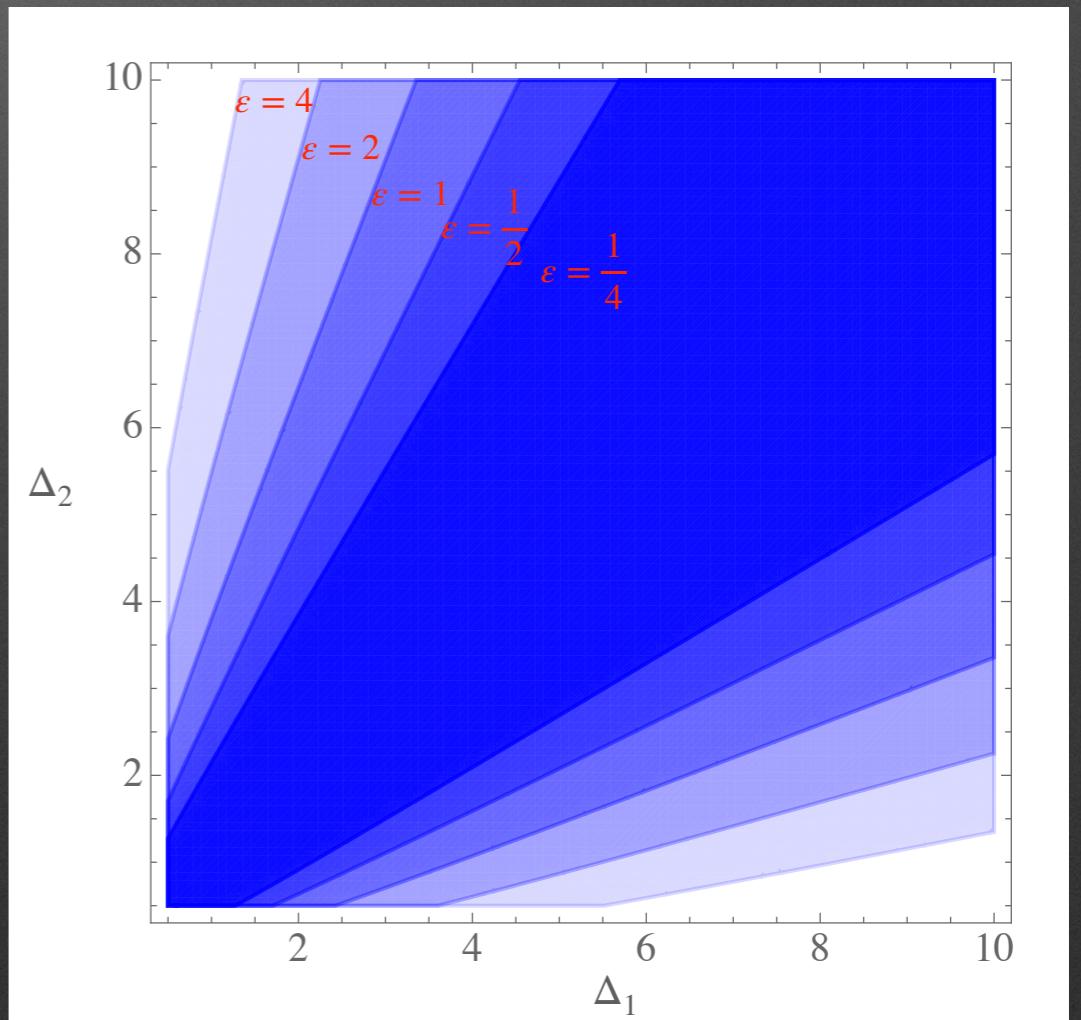
$$c_{\tilde{1}2} = 8$$

$$c_{12} = -6$$

Bounds satisfied for any value of  $\Delta_1, \Delta_2$

$$\left\{ \begin{array}{l} \text{Not true if we add small perturbation} \\ \\ c_{12} + \frac{c_{\tilde{1}2}}{2} = -2 - \varepsilon \end{array} \right.$$

Boundary theories special! (E.g. Ising)



# Conclusions and outlook

CFT techniques promising tool to address AdS stringy EFTs

- Sign of  $\gamma^{\text{mix}}(0, l)$  correlates with Swampland conditions  
Not proven, but very similar to known positivity bounds
- Applications: LVS, KKLT, Racetrack, Perturbative stabilisation
- Connection to the distance conjecture
- Just a feature of volume modulus?  
Work in progress on more examples:  
Fibred models, Type IIA...

- New CFT inequality on mixed & identical anomalous dimensions gives non-trivial constraints on higher derivative theories
- Immediate application: DBI at the edge of the Swamp  
Applications to Cosmology?

Long term goal: invert the relationship to “navigate” the Swampland

Thank you for your attention!

# Back-up slides

# The Large volume Scenario

“Big” and “small” Kähler moduli  $T_b, T_s$

$$\mathcal{V} = \frac{1}{\kappa} \left( \tau_b^{3/2} - \tau_s^{3/2} \right)$$

$$K = -2 \ln \left( \frac{1}{\kappa} \left( \left( \frac{T_b + \bar{T}_b}{2} \right)^{3/2} - \left( \frac{T_s + \bar{T}_s}{2} \right)^{3/2} \right) + \frac{\xi}{g_s^{3/2}} \right)$$

$$W = W_0 + A_s e^{-a_s T_s}$$

$\alpha'^3$  correction

Minimum for an exponentially large volume  $\langle \mathcal{V} \rangle \sim e^{a_s \langle \tau_s \rangle}$

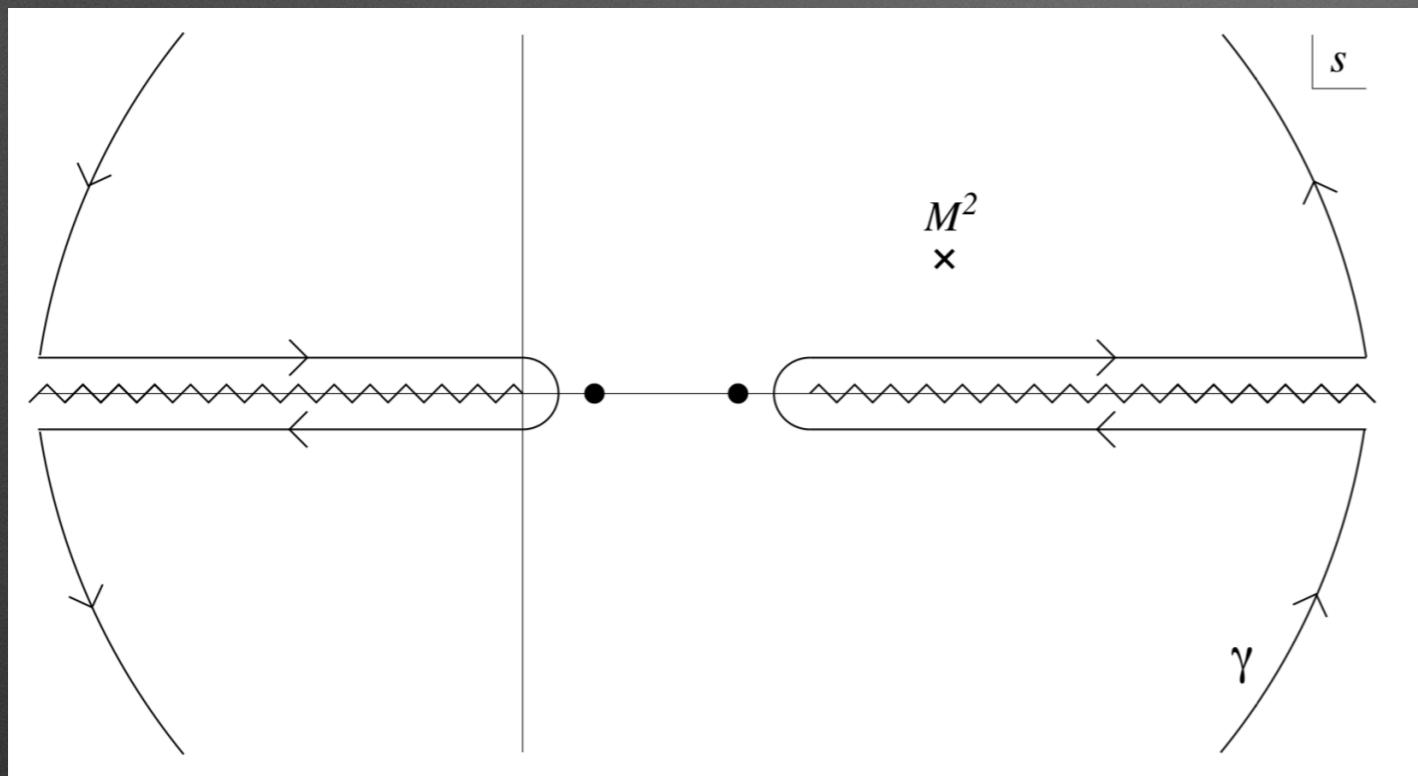
$$V_{eff} = \frac{1}{\mathcal{V}^3} \left( -A (\ln \mathcal{V})^{3/2} + \frac{B}{g_s^{3/2}} \right) \quad \langle \tau_s \rangle \sim \frac{\zeta^{2/3}}{g_s}$$

# Intermezzo: S-Matrix positivity

unitarity + analyticity + weak coupling expansion

$$\mathcal{A}(s)_{tree} = g(c_2 s^2 + c_4 s^4 \dots)$$

$$g \, c_i > 0$$



E.g.

$$\mathcal{L} = \frac{g}{\Lambda^4} (\partial\varphi)^4 \quad g > 0$$

"QFT Swampland"

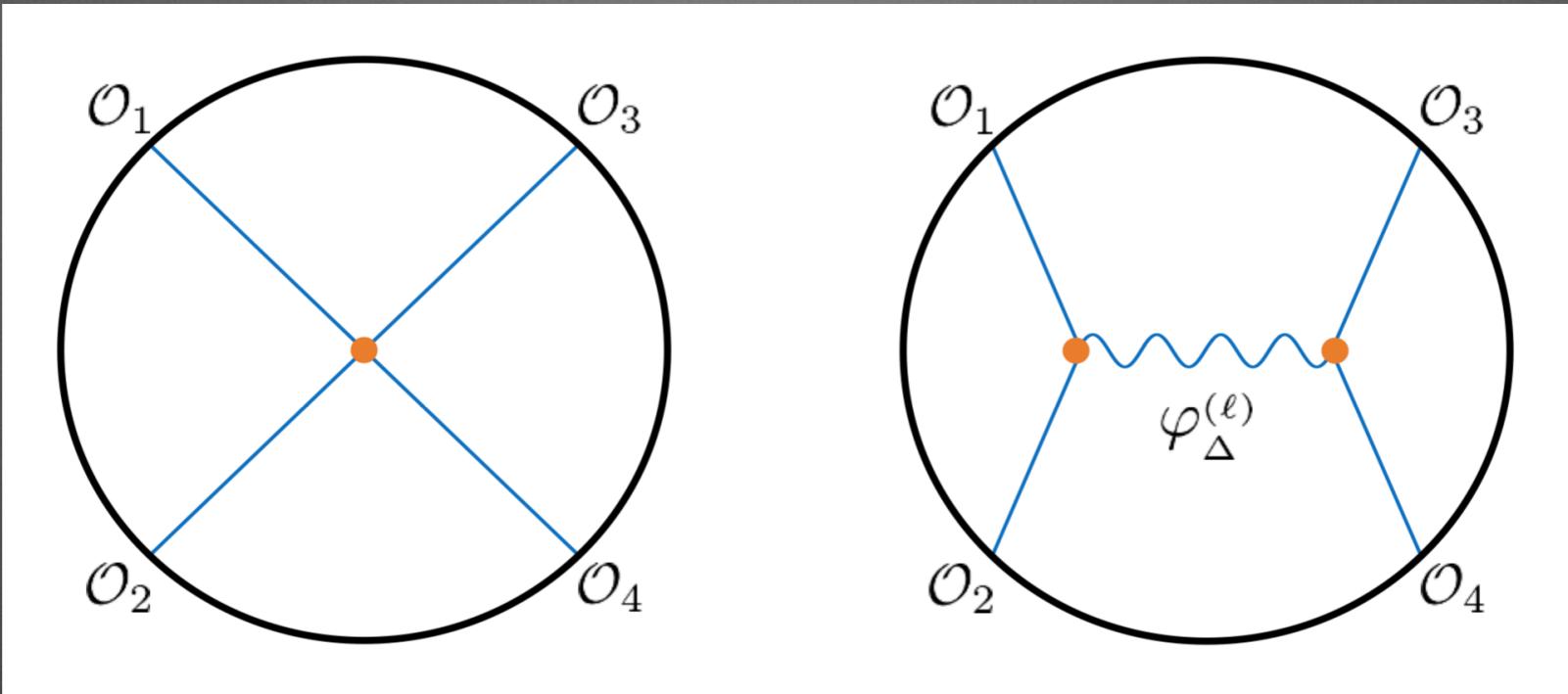
CAVEAT:  $\mathcal{A}_{grav}(s, t) \supset \frac{1}{M_{PL}^2} \frac{s^2}{t}$   $\longrightarrow \Lambda \ll M_{PL}$

# How do we compute ?

Correlation functions



Witten Diagrams in AdS



Other techniques:

- Bootstrapping correlators (good for exchanges)  
[Alday,Bissi,Perlmutter '17 + others]
- Eigenvalues of dilatation operator:  $\gamma(n, \ell) = \langle n, \ell | \Delta H | n, \ell \rangle$   
[Fitzpatrick,Katz,Poland,Simmons-Duffin '12]
- Mellin amplitude formula for  $\gamma(0, \ell)$   
[Costa,Goncalves,Penedones '12]

# Perturbative stabilisation

$$V_{eff} = Ae^{-\lambda_1 \varphi} - Be^{-\lambda_2 \varphi} \quad \varphi_{min} = \frac{1}{\lambda_1 - \lambda_2} \log \left( \frac{A\lambda_1}{B\lambda_2} \right)$$

*string loops*       $\alpha'^3$  correction      typical example

$\sim \mathcal{V}^{-\frac{10}{3}}$        $\sim \mathcal{V}^{-3}$

$$\Delta_\varphi = \frac{3}{2} \left( 1 \pm \sqrt{1 + \frac{4}{3}\lambda_1\lambda_2} \right) \quad \Delta_a = 3$$

$$\gamma^{\varphi a}(0, \ell) \propto -\lambda_1\lambda_2(\lambda_1 + \lambda_2) \frac{g\mu(\Delta_\varphi - 6)}{\Gamma\left(\frac{6-\Delta_\varphi}{2}\right)} \frac{1}{M_P^2 R_{AdS}^2}$$

$$\lambda_1\lambda_2 \geq \frac{40}{3} ? \quad \text{De Sitter conjecture} \longrightarrow \lambda_i > C$$

[Obied, Ooguri, Spodyneiko, Vafa '18]

# KKLT

Well known example of **ds vacuum** - we consider it before uplifting

$$\mathcal{K} = -3 \log(-i(T - \bar{T}))$$

$$\mathcal{W} = W_0 + A e^{-\alpha T}$$

[Kachru,Kallosh,  
Linde,Trivedi '03]

**SUSY vacuum**

$$\frac{W_0}{A} = -e^{-a\sigma_c} \left(1 + \frac{2}{3}a\sigma_c\right)$$

$$\begin{aligned}\sigma_c a &>> 1 \\ \sigma &>> 1\end{aligned}$$

Potential for  $a$   $\longrightarrow$   $\Delta_a > 3$

$$\mathcal{L} \supset \sigma^3, \sigma a^2, \sigma \partial_\mu a \partial^\mu a$$

} qualitatively  
different

Negative anomalous dimensions in its regime of validity

$$\begin{aligned}\gamma^{\sigma a}(0, \ell) \propto & - \left[ a\sigma_c(a\sigma_c + 2)(2a\sigma_c + 3) \right. \\ & \left. + 2\Delta_\varphi(a\sigma_c) - 2\Delta_a^2(a\sigma_c) + 4\Delta_a(a\sigma_c) - 6 \right]\end{aligned}$$

# Racetrack

$$\mathcal{K} = -3 \log -i(T - \bar{T})$$

$$\mathcal{W} = Ae^{-aT} - Be^{-bT}$$

[Krasnikov '87, Taylor '90,  
De Carlos, Casas, Munoz '93]

Potential for  $a$   $\longrightarrow \Delta_a > 3$

$\mathcal{L} \supset \sigma^3, \sigma a^2, \sigma \partial_\mu a \partial^\mu a$

$\left. \right\}$  similar  
to KKLT

Negative anomalous dimensions for  $\sigma_c \gtrsim \frac{1}{a}, \frac{1}{b}$

$\Delta_a < \Delta_\varphi < 2\Delta_a$   $\longrightarrow$  no need to decouple  
higher order contributions

# Mellin Amplitudes (1)

Convenient representation of correlators

$$A(x_i) \supset \langle \mathcal{O}_1(x_1) \dots \dots \mathcal{O}_n(x_n) \rangle_c = \prod_{1 \leq i < j \leq n} \int_{-i\infty}^{+i\infty} \frac{d\delta_{ij}}{2\pi i} M(\delta_{ij}) \Gamma(\delta_{ij}) (x_{ij}^2)^{-\delta_{ij}}$$

$$e^{-s_i s_j P_{ij}} = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma_{ij}}{2\pi i} \Gamma(\gamma_{ij}) (s_i s_j P_{ij})^{-\gamma_{ij}}$$

arises naturally due to  
Euler star formula

with  $\delta_{ii} = -\Delta_i$   $\sum_i \delta_{ij} = 0$   $\longrightarrow$   $n(n-3)/2$  independent variables - analogue to  $n$ -particle scattering

can introduce fictitious moments s.t.

$$p_i \cdot p_j = \delta_{ij} \quad \sum_{i=1}^n p_i = 0 \quad s_{ij} = -(p_i + p_j)^2 = \Delta_i + \Delta_j - 2\delta_{ij}$$

# Mellin Amplitudes (2)

$$\mathcal{A}(u, v) = \int_{-i\infty}^{+i\infty} \frac{dt ds}{(4\pi i)^2} M(s, t) u^{s/2} v^{-(s+t)/2} \Gamma\left(\frac{\Delta_1 + \Delta_2 - s}{2}\right) \Gamma\left(\frac{\Delta_3 + \Delta_4 - s}{2}\right) \\ \Gamma\left(\frac{\Delta_{34} - t}{2}\right) \Gamma\left(\frac{-\Delta_{12} - t}{2}\right) \Gamma\left(\frac{t+s}{2}\right) \Gamma\left(\frac{t+s+\Delta_{12}-\Delta_{34}}{2}\right)$$

Contact interactions:

$$M(s, t) = \frac{g\pi^{\frac{d}{2}}}{2} \Gamma\left(\frac{\sum \Delta_i - d}{2}\right) \prod_{i=1}^n \frac{1}{\Gamma(\Delta_i)} \quad \text{scalar vertex}$$

$$M(s, t) = \frac{g\pi^{\frac{d}{2}}}{2} \Gamma\left(\frac{\sum \Delta_i - d}{2}\right) \prod_{i=1}^n \frac{1}{\Gamma(\Delta_i + \beta_i)} \prod_{i < j}^n (-2\delta_{ij})^{\alpha_{ij}} \quad \text{derivative vertex}$$

Closest analogue to scattering amplitudes in AdS!

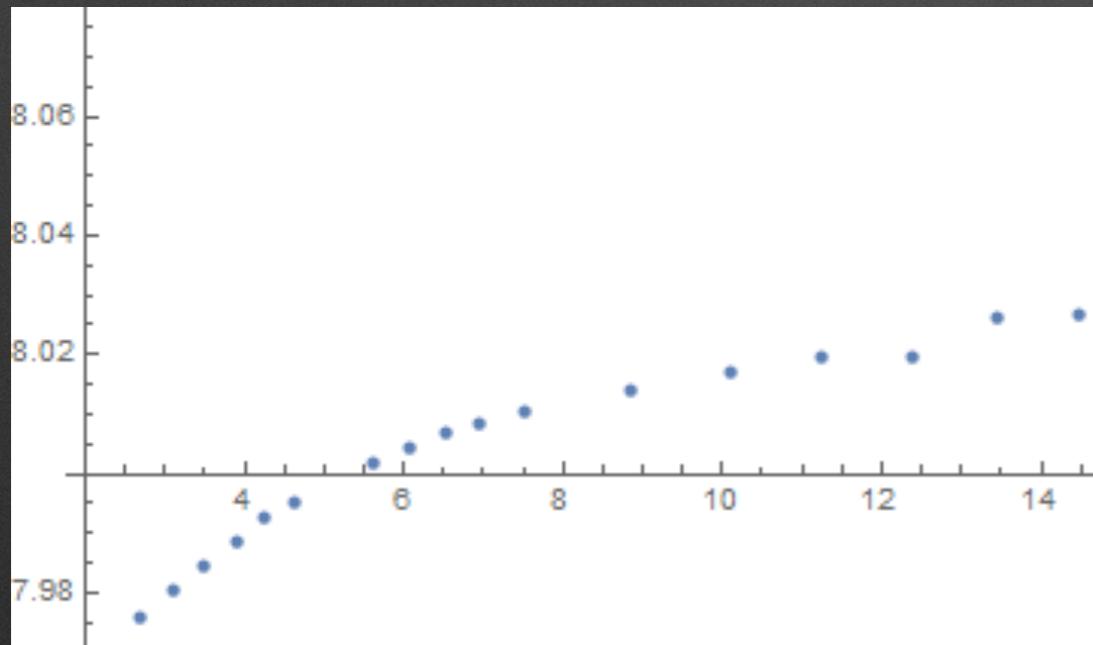
# Finite volume

At small volumes,  $\Delta_\varphi$  can in principle drop below 8

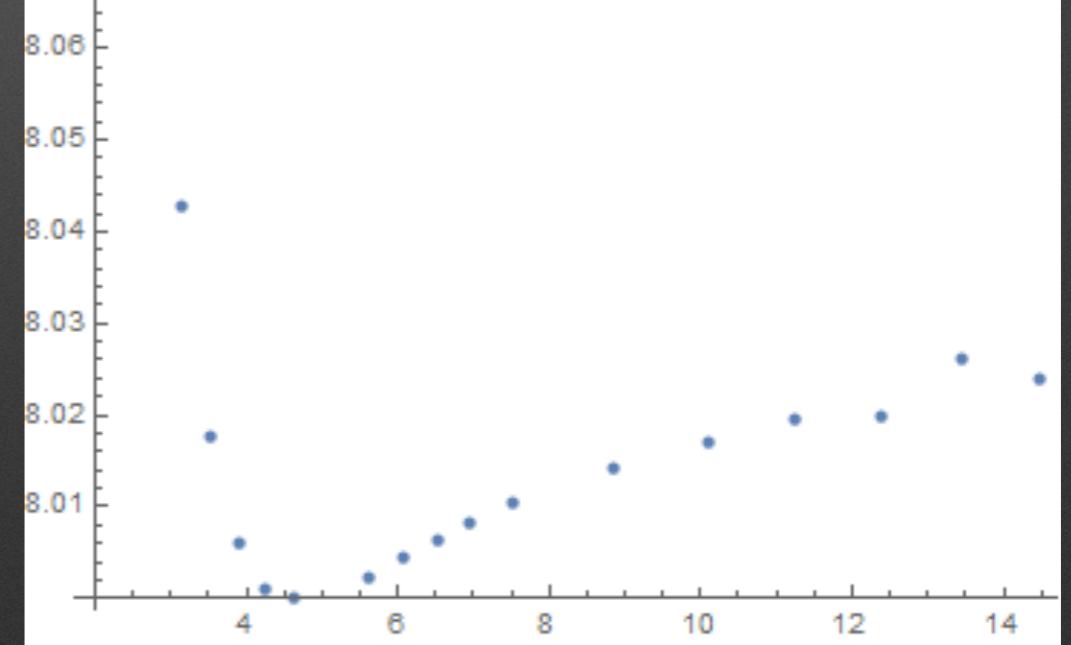
$$\Delta = \frac{3(1 + \sqrt{19})}{2} \left( 1 - \sqrt{\frac{2}{27}} \frac{1}{\langle \Phi \rangle} + \mathcal{O}\left(\frac{1}{\langle \Phi \rangle}\right)^2 \right)$$

However, subleading volume corrections become important

$$K = -2 \ln(\mathcal{V} + \xi) \rightarrow K = -2 \ln \left( \mathcal{V} + \xi + \frac{\xi^2}{\mathcal{V}} \right)$$



Pure LVS



With corrections

# Flat spacetime límit

Flat space bounds

$$\left\{ \begin{array}{l} \alpha^2 c_{11} + \alpha \beta \left( c_{12} + \frac{\tilde{c}_{12}}{2} \right) + \beta^2 c_{22} \geq 0 \quad \forall \alpha, \beta \\ \tilde{c}_{12} \geq 0 \\ -2\sqrt{c_{11}c_{22}} - \tilde{c}_{12} \leq c_{12} \leq 2\sqrt{c_{11}c_{22}} \end{array} \right.$$

$$\gamma(0, 2)_{R \rightarrow +\infty} \sim \frac{1}{R^{3d/2}} \left[ m^{4-3d/2} + \frac{m^{3-3d/2}}{R} + \frac{m^{2-3d/2}}{R^2} + \dots \right]$$

only for  $m(R)R \rightarrow 0$  as  $R \rightarrow +\infty$  the AdS bounds reduce to flat space

AdS Distance Conjecture? [Lust,Palti,Vafa '19]