Eternal Inflation and (Anti) de Sitter Bounds from Dimensional Reduction

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Based on 1905.05198, 2101.11617

Outline

- (Anti) de Sitter Swampland Bounds
- Dimensional Reduction and Swampland Bounds
- Conditions for Eternal Inflation
- Speculations

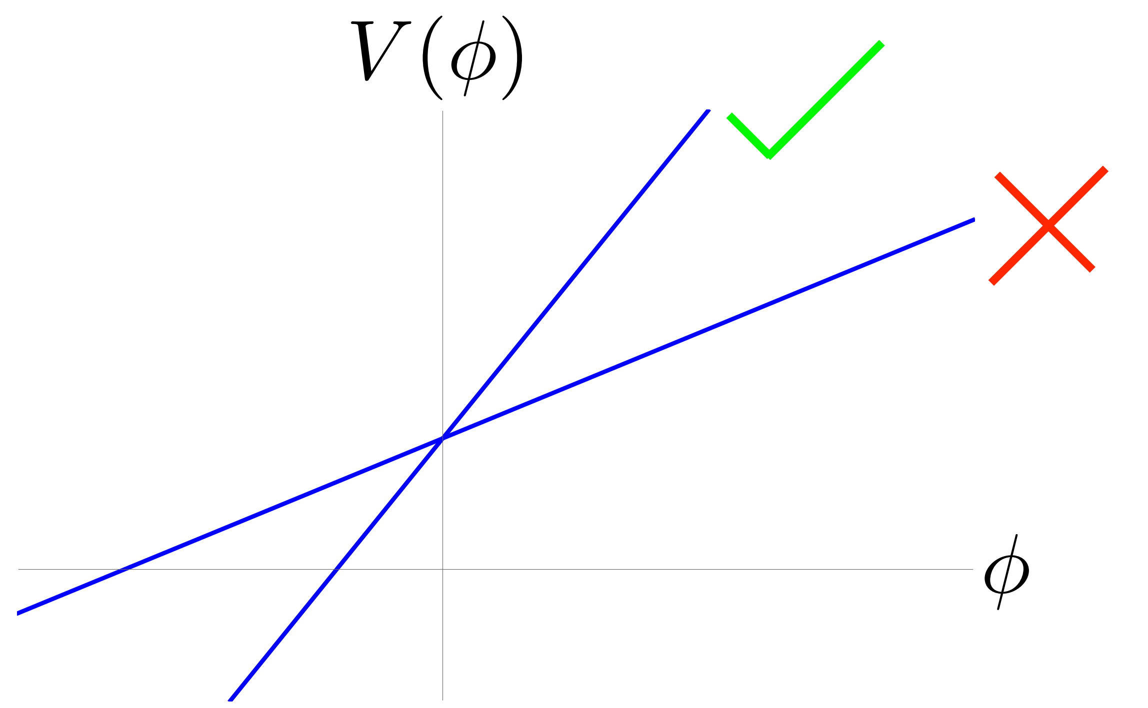
(Anti) de Sitter
Swampland Bounds

de Sitter Conjectures

dSC:

(1) $$|\nabla V| \geq c \cdot V$$

Obied, Ooguri, Spodyneiko, Vafa ’18

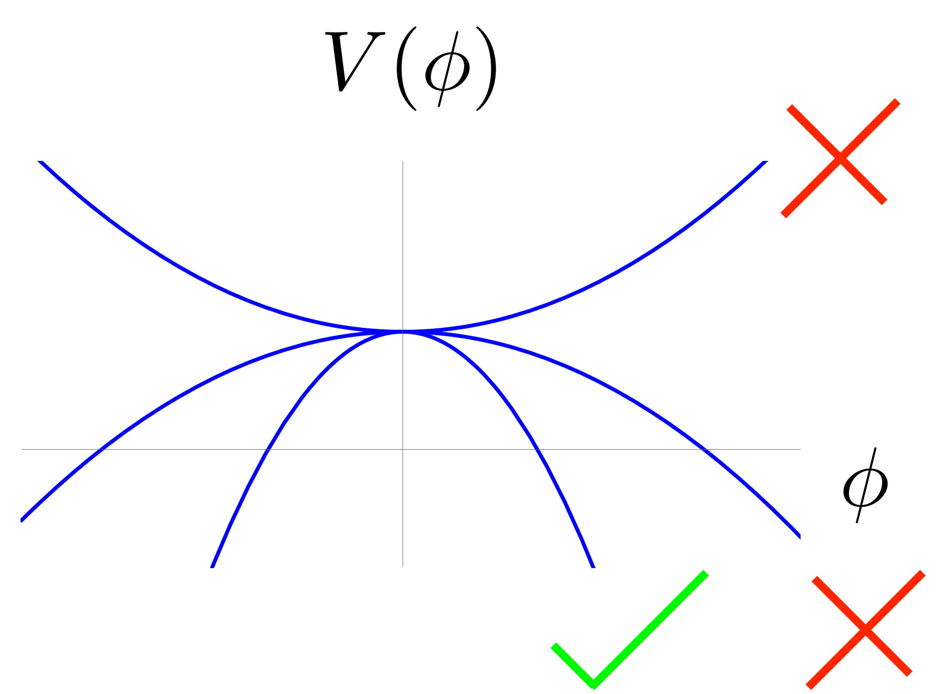


RdSC: (1) or

(2) $$\min(\nabla_i \nabla_j V) \leq -c' \cdot V$$

Garg, Krishnan ’18

Palti, Shiu, Ooguri, Vafa ’18

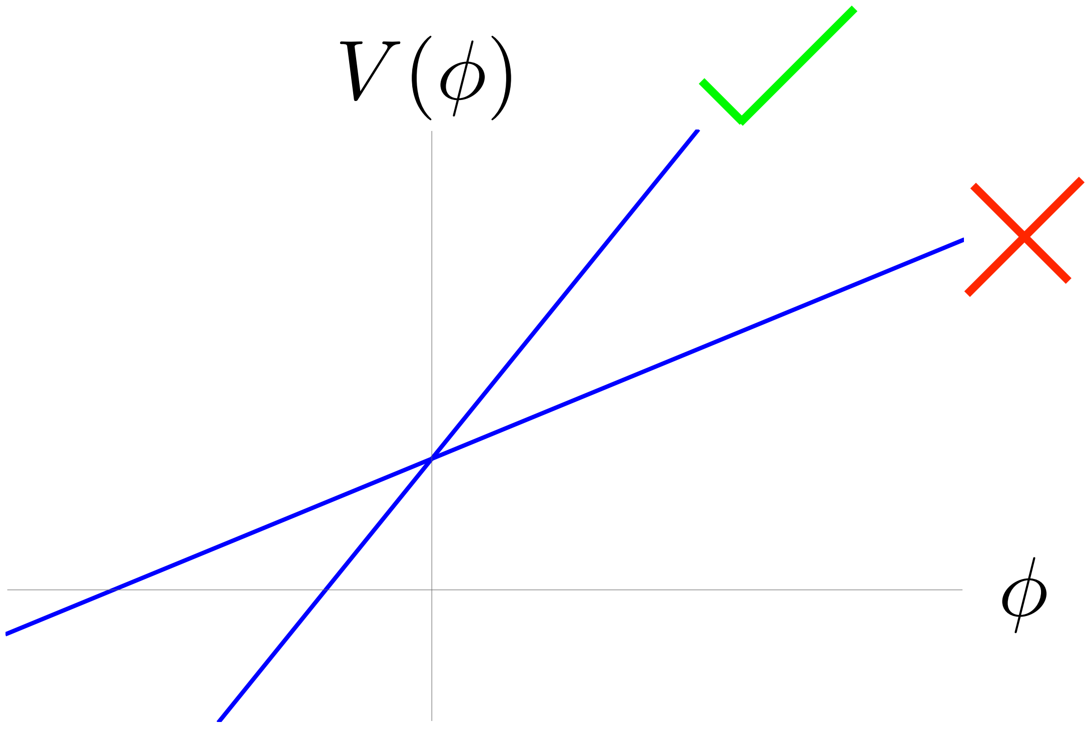


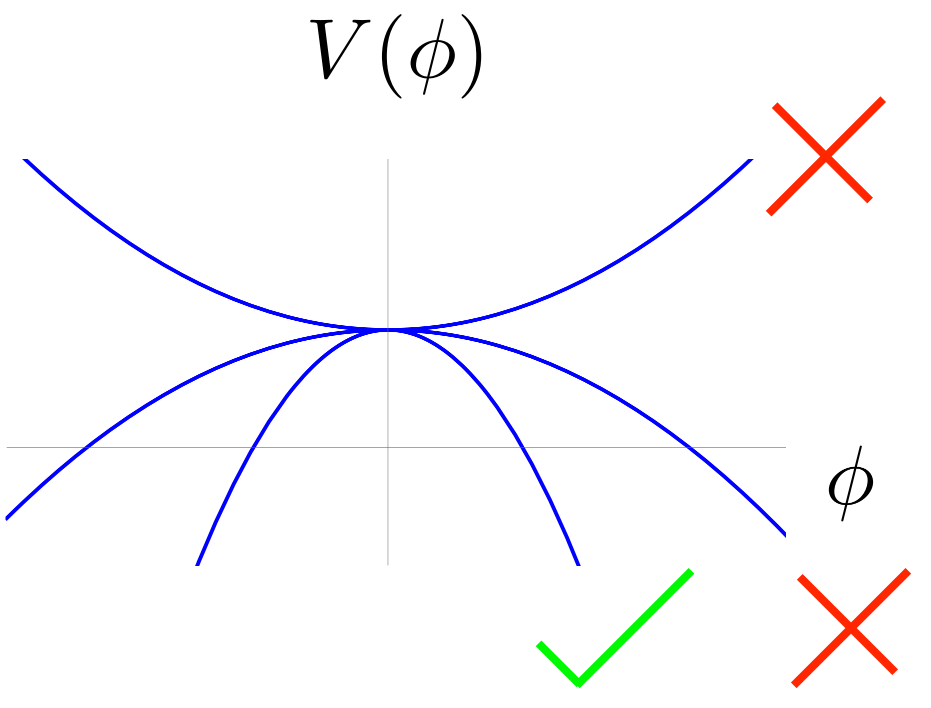
de Sitter Conjectures

(3) RdSC*:

$$\left(\frac{|\nabla V|}{V}\right)^q - a\frac{\min\nabla_i\nabla_j V}{V} \geq b\,, \qquad \text{with } a+b=1,\quad a,b>0,\quad q>2,$$

Andriot '18

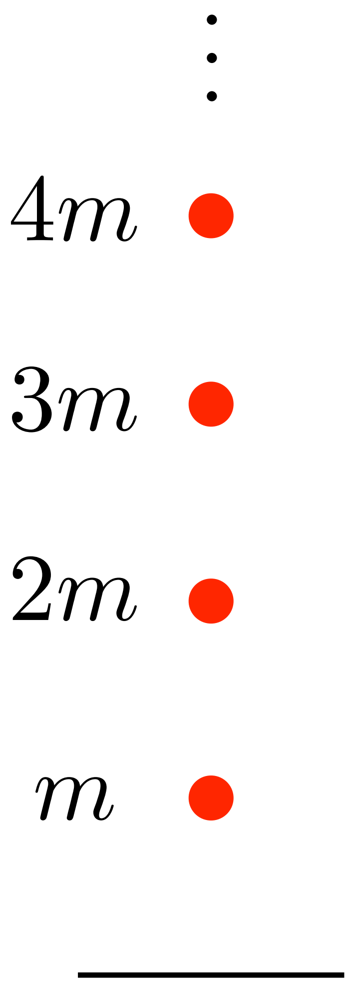




(Anti) de Sitter Distance Conjecture

$$m \lesssim |\Lambda_d|^{\alpha_d}$$

Lüst, Palti, Vafa '19



Remarks

- If true, these conjectures would have *enormous* implications for how we understand our universe
- However, the evidence for them is tenuous—they are true under the lamppost, but there is a high likelihood of a lamppost effect
- In order to understand their regime of validity in string theory, it is crucial to understand their underlying physical motivation

Contrast with WGC

dSC	WGC
unclear physical motivation	clear motivation: black holes must decay
$\lvert\nabla V\rvert \geq c \cdot V \qquad V'' \leq -c' \cdot V$	$\frac{q}{m} \geq \gamma_d M_{\mathrm{Pl};d}^{(2-d)/2}$
unknown O(1) coefficients	sharp number

Dimensional Reduction and Swampland Bounds

Dimensional Reduction and the WGC

$$S = \frac{1}{2\kappa_D^2}\int d^D x\sqrt{-g}\left(R_D - \frac{1}{2}(\nabla\phi)^2\right) - \frac{1}{2e_{P;D}^2}\int d^D x\sqrt{-g}e^{-\alpha_{P;D}\phi}F_{P+1}^2$$

- WGC bound in D dimensions:

$$e_{P;D}^2 q^2 \geq \gamma_{P;D} T_P^2 \qquad \gamma_{P;D} = \frac{\alpha_{P;D}^2}{2} + \frac{P(D-P-2)}{D-2}$$

- After dimensional reduction on S^1:

$$\gamma_{P;D} = \gamma_{p;d} = \gamma_{P,d}\,, \quad p = P-1\,,\ d = D-1$$

- WGC bound exactly preserved!

Heidenreich, Reece, TR ’15

Dimensional Reduction and the dSC

$$S = \int d^D x \sqrt{-g} \left[\frac{M_D^{D-2}}{2} R_D - \frac{1}{2} G_{ij}(\phi) \nabla\phi_i \nabla\phi_j - V_D(\phi) \right]$$

- Assume bound of the form:

$$\frac{|\nabla V_D|}{V_D^{\gamma_D}} \geq c_D M_D^{D(1-\gamma_D)-(D-2)/2}$$

- Compactify on S^1 of radius R, find that R-dependence cancels, bound is exactly preserved for

$$\gamma_D = \gamma_d = 1\,, \quad c_d^2 = \beta + \frac{4}{d-2}$$

Matches dSC value!

$\beta = 0$, forbids accelerated expansion

Dimensional Reduction and the (A)dSDC

$$S = \int d^D x \sqrt{-g} \left[\frac{M_D^{D-2}}{2} R_D - \Lambda_D - \frac{1}{2} \sum_n \left((\nabla \phi_n)^2 + m_n^2 \phi_n^2 \right) \right]$$

- Assume tower of states with mass scale:

$$m \lesssim |\Lambda_D|^{\alpha_D} M_D^{1-D\alpha_D} \, .$$

- Find bound is exactly preserved for

$$\alpha_D = \alpha_d = 1/2 \, .$$

Matches "strong AdSDC" value, true for SUSY AdS vacua with well-understood 10/11d uplifts

Lüst, Palti, Vafa '19

Dimensional Reduction and the (A)dSDC

$$S = \int d^D x \sqrt{-g} \left[\frac{M_D^{D-2}}{2} R_D - \Lambda_D - \frac{1}{2} \sum_n \left((\nabla \phi_n)^2 + m_n^2 \phi_n^2 \right) \right]$$

- To stabilize radion, should include effects of Casimir energy:

$$V_C(\lambda) = \mp \frac{2}{(2\pi R)^d \Omega_d} \zeta(d+1) e^{\frac{d(d-1)}{2(d-2)}\lambda}, \quad \Omega_d = \frac{2\pi^{(d+1)/2}}{\Gamma(\frac{d+1}{2})}$$

- This distinguishes the value

$$\alpha_d = 1/d$$

- Possible connection to neutrinos?

$$m_\nu \sim \Lambda_4^{1/4} \sim 0.01 \text{ eV}$$

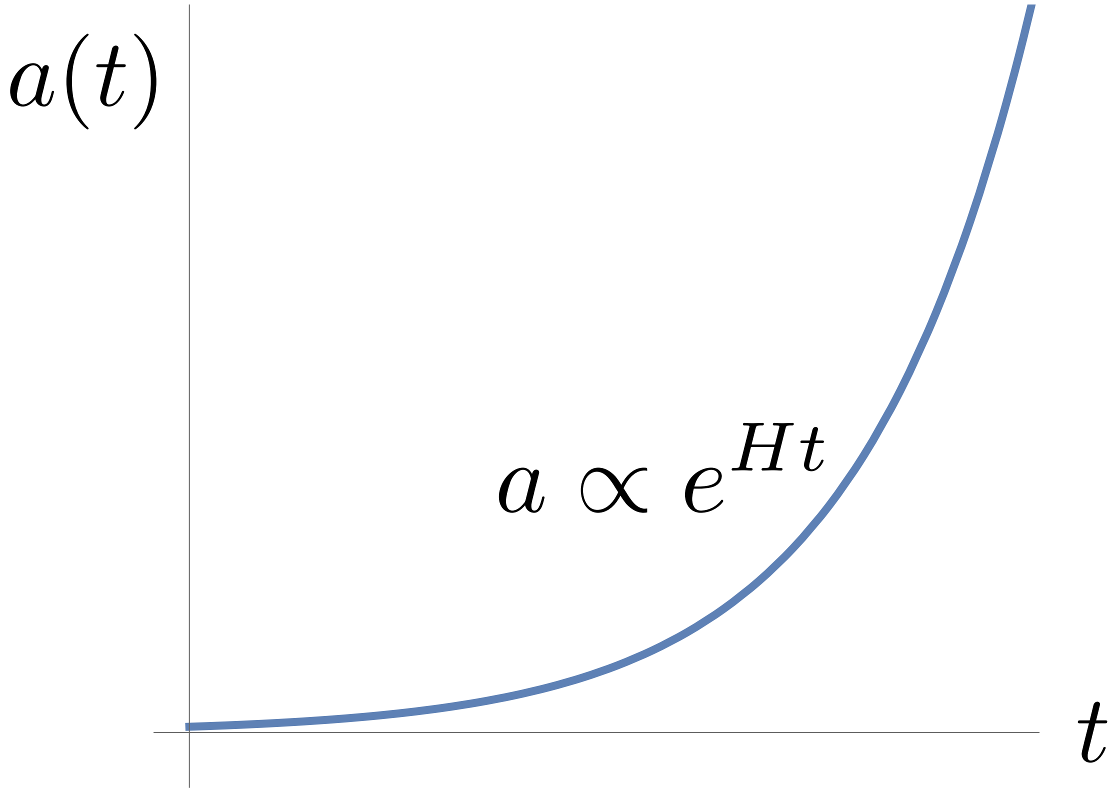
see also Ibañez, Martin-Lozano, Valenzuela ’17, Gonzalo, Herráez, Ibañez ’18

Conditions for Eternal Inflation

Inflation

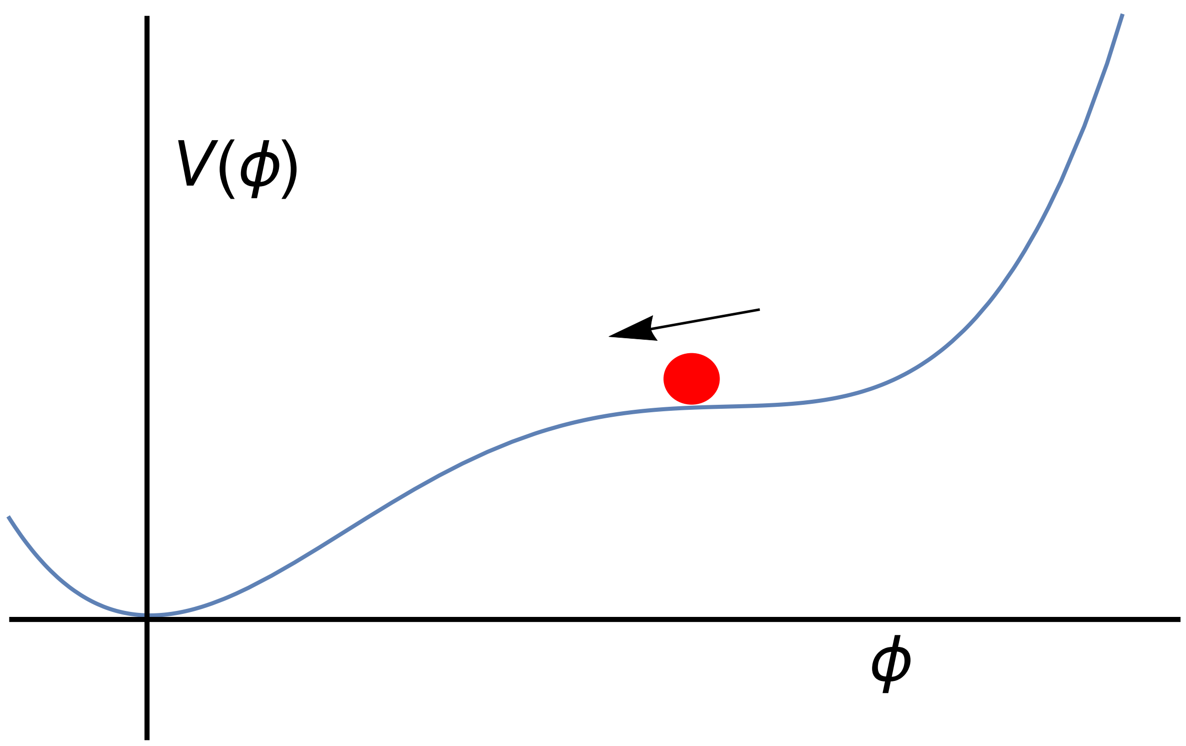
- Inflation describes a period of quasi-exponential growth of the universe:

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2 \qquad\qquad H \approx \text{const.}$$



Inflation in Field Theory

- Inflation can be thought of as the theory of a ball rolling down a hill with friction



$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0 \qquad H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right).$$

Slow-Roll Inflation

- Slow-roll approximation:

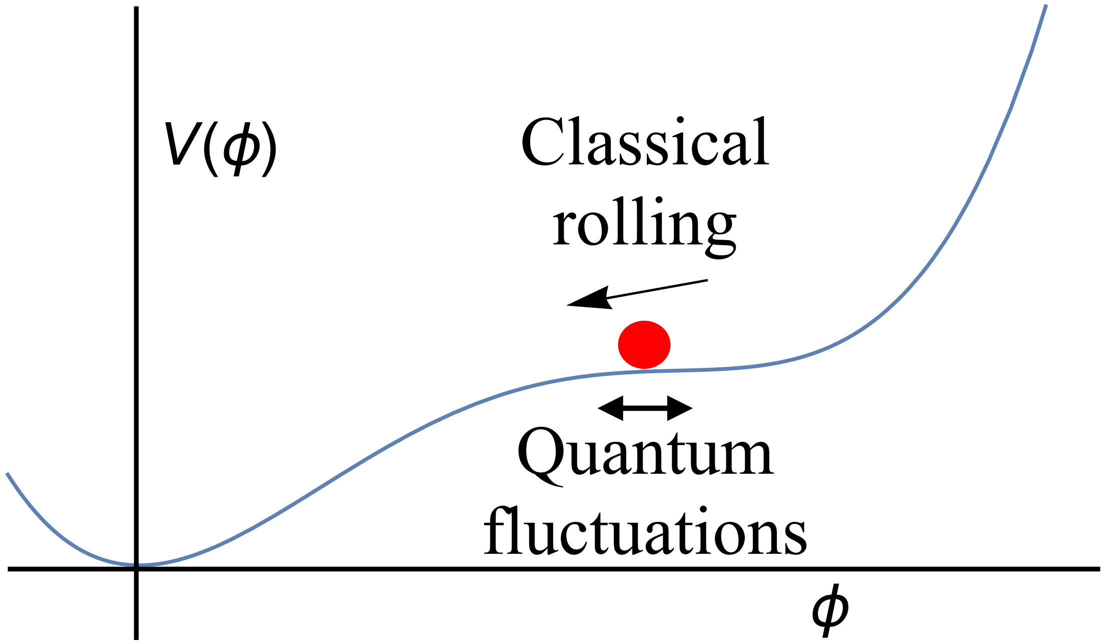
$$\dot{\phi}^2 \ll V(\phi) \qquad\qquad |\ddot{\phi}| \ll |3H\dot{\phi}|,\ |V_{,\phi}|$$

$$\Rightarrow \quad 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0\,, \quad H^2 = \frac{1}{3}V(\phi).$$

Stochastic Inflation

- Dynamics of scalar field governed by combination of classical rolling and quantum fluctuations:

$$\delta\phi = -\frac{1}{3H}V'(\phi)\delta t + \delta\phi_q(\delta t), \quad \delta\phi_q(\delta t) \sim \mathcal{N}(0, H^3\delta t/(2\pi)^2)$$



Fokker-Planck Equation

- Can define p.d.f. describing probability of finding field in $(\phi, \phi + \delta\phi)$ at time t
- Evolution of density described by Fokker-Planck equation:

$$\dot{P}[\phi, t] = \frac{1}{2}\left(\frac{H^3}{4\pi^2}\right)\partial_\phi^2 P[\phi, t] + \frac{1}{3H}\partial_\phi\Big((\partial_\phi V(\phi))P[\phi, t]\Big)$$

Quantum fluctuations

Classical evolution

Solving the FP Equation

- In general, the Fokker-Planck equation must be solved numerically.
- However, under the assumption $H \approx const.$, there are two cases in which the equation can be solved analytically: a linear potential or a quadratic potential.
- Fortunately, these are precisely the cases of most interest to us.

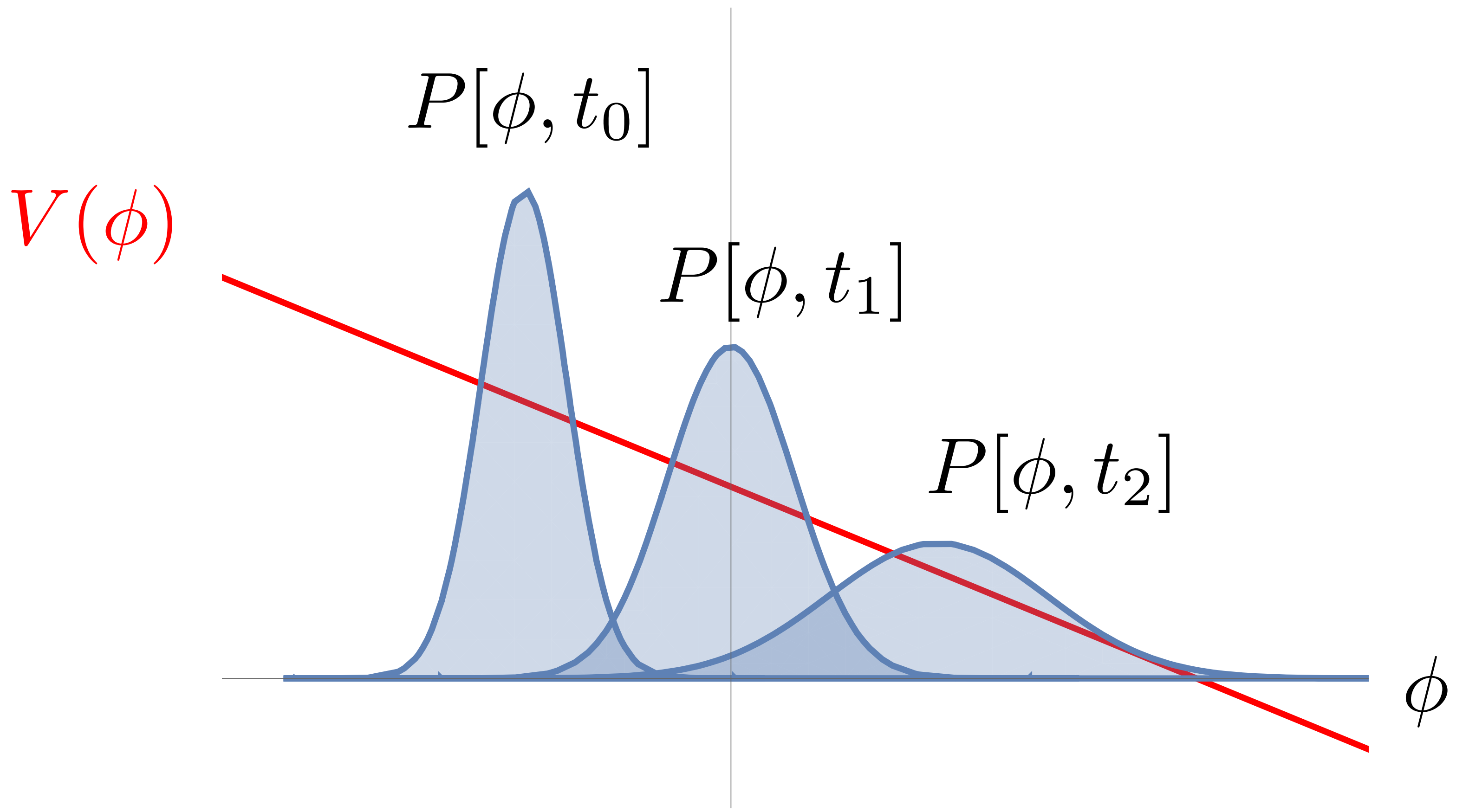
Solutions to the FP Equation

$$P[\phi, t] = \frac{1}{\sigma(t)\sqrt{2\pi}} \exp\left[-\frac{(\phi - \mu(t))^2}{2\sigma(t)^2}\right]$$

Linear Potential	Tachyonic Potential
$V(\phi) = V_0 - \alpha\phi$	$V(\phi) = V_0 - \frac{1}{2}m^2\phi^2$
$\mu(t) = \frac{\alpha}{3H}t$	$\mu(t) = 0$
$\sigma^2(t) = \frac{H^3}{4\pi^2}t$	$\sigma^2(t) = \frac{3H^4}{8\pi^2 m^2}\left(1 - e^{-\frac{2m^2}{3H}t}\right)$

Linear Potential

$$V(\phi) = V_0 - \alpha\phi$$

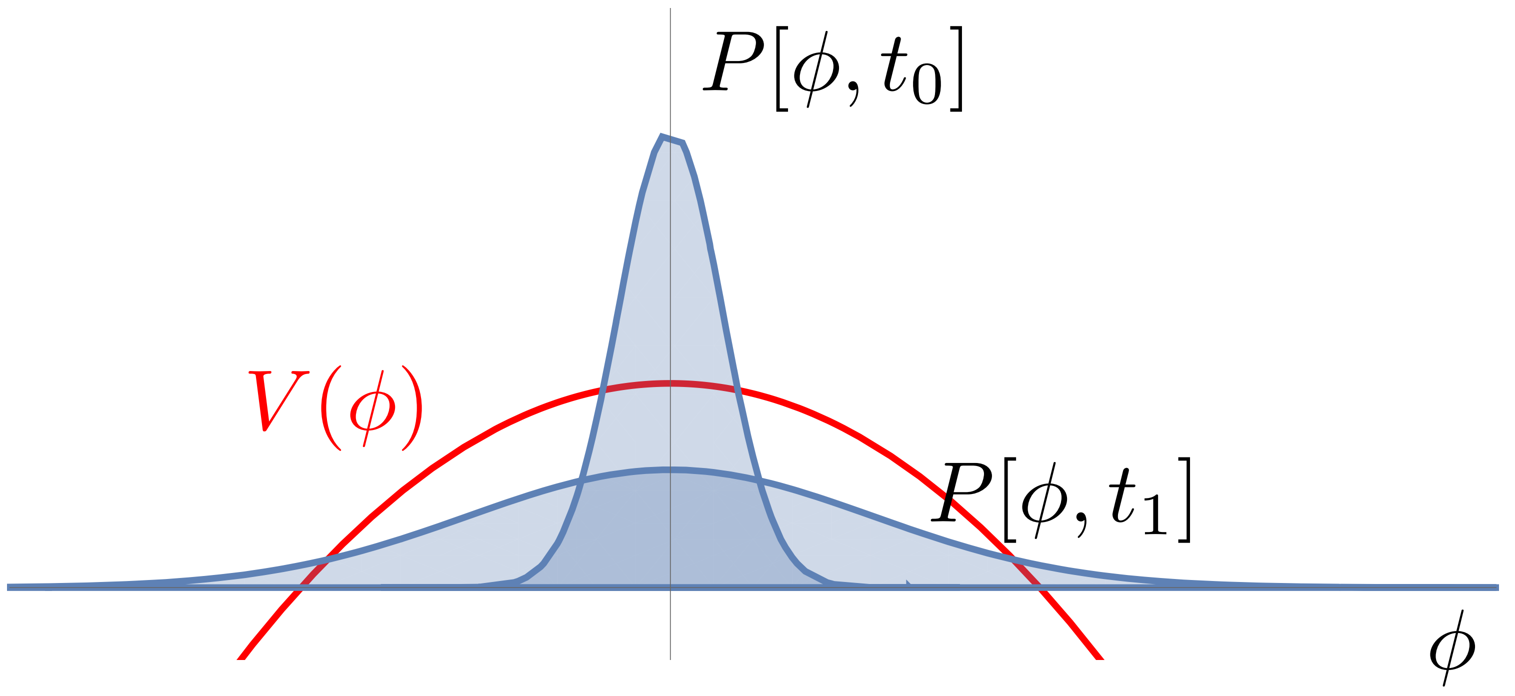


$$\mu(t) = \frac{\alpha}{3H}t \qquad \sigma^2(t) = \frac{H^3}{4\pi^2}t$$

(see also Rey '87)

Tachyonic Potential

$$V(\phi) = V_0 - \frac{1}{2}m^2\phi^2$$



$$\mu(t) = 0 \qquad \sigma^2(t) = \frac{3H^4}{8\pi^2 m^2}\left(-1 + e^{\frac{2m^2}{3H}t}\right)$$

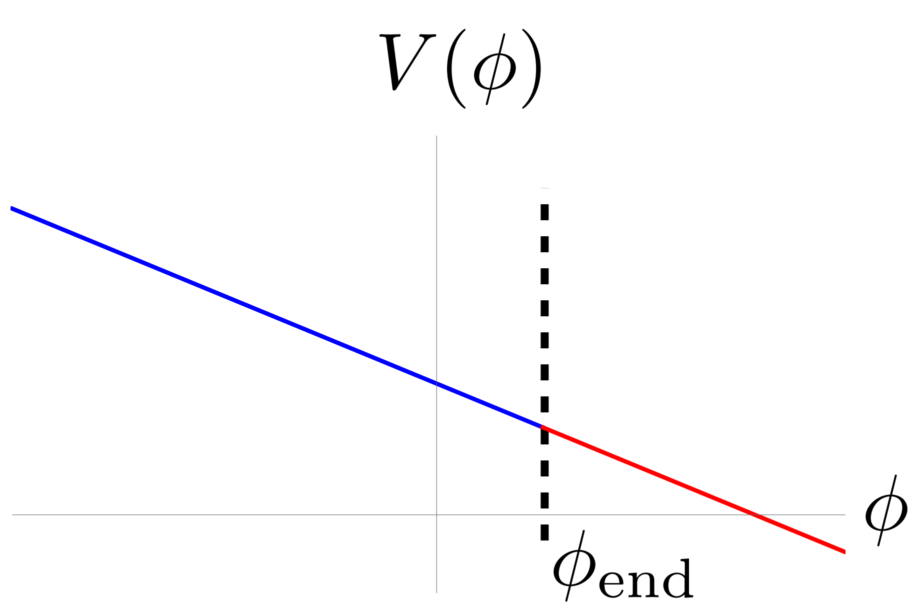
(see also Rey '87)

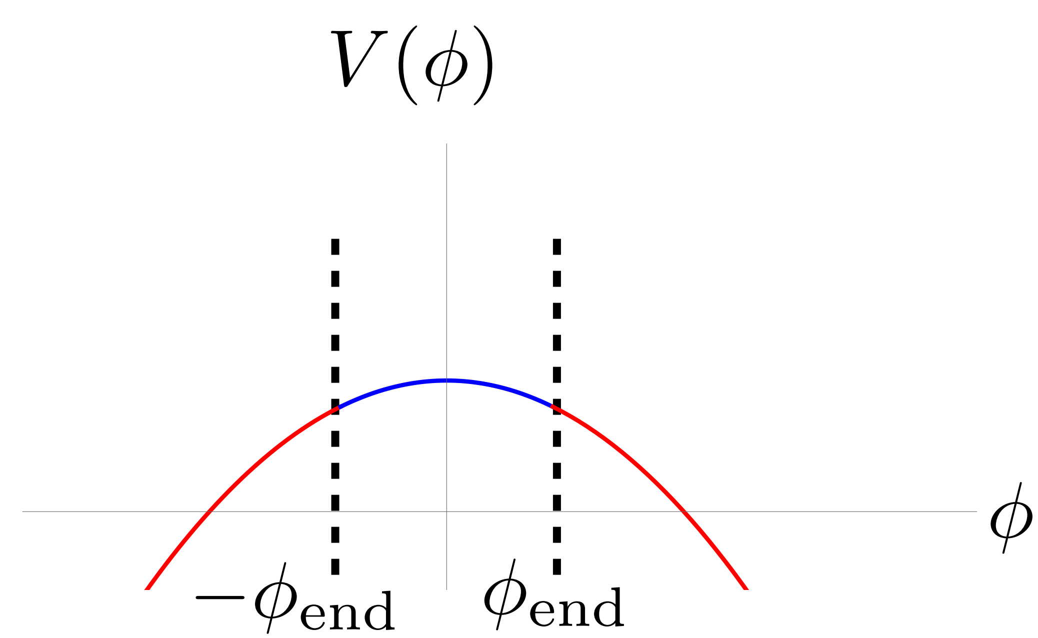
Conditions for Inflation

- First slow-roll parameter ϵ computed from derivative of the potential:

$$\epsilon = \frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2$$

- Inflation requires $\epsilon \ll 1$, ends when $\epsilon \gtrsim 1$:

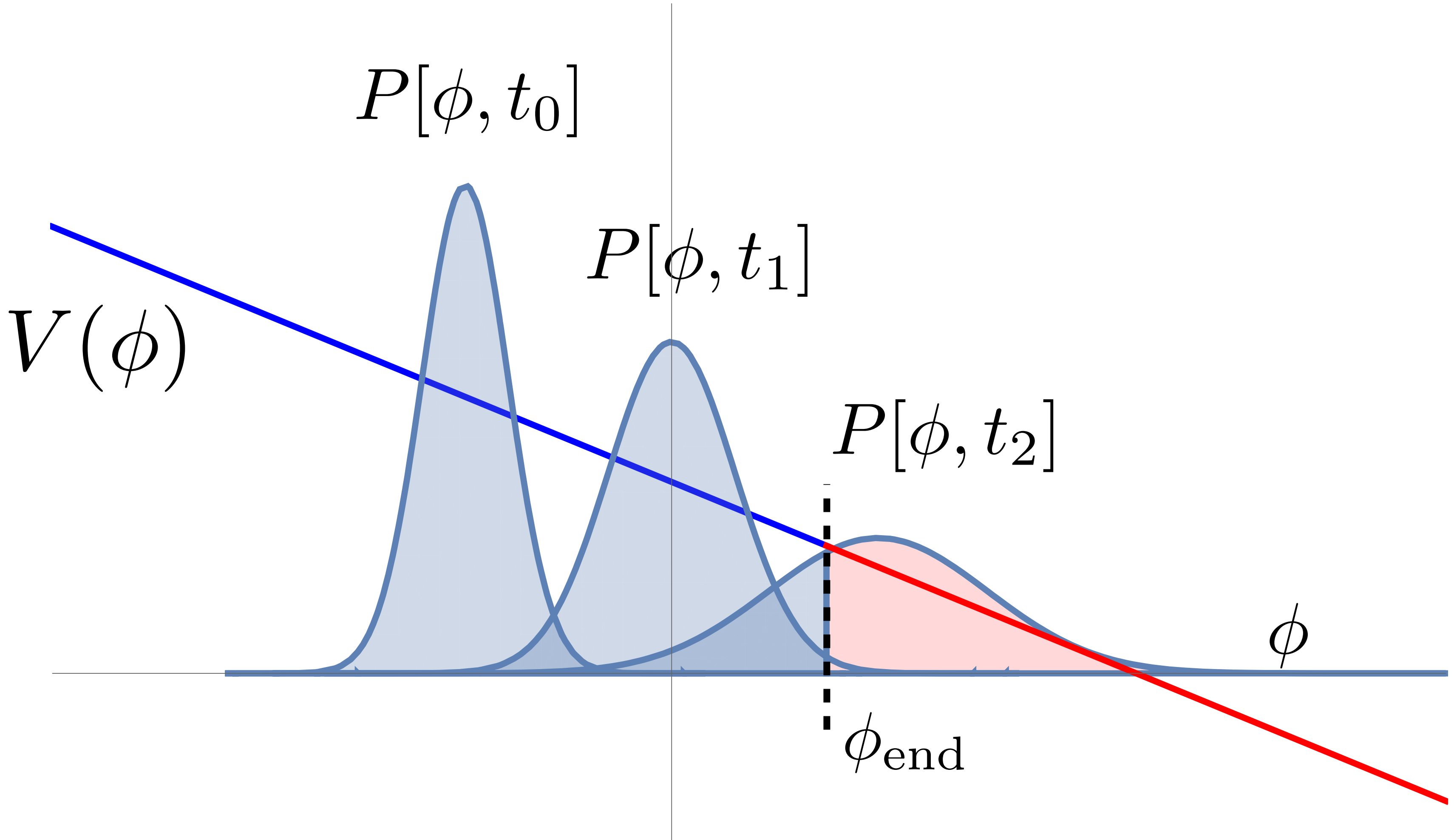




Probability of Inflation

- Linear model probability of inflation at time t:

$$\Pr[\phi < \phi_{\rm end}, t] = \int_{-\infty}^{\phi_{\rm end}} d\phi\, P[\phi, t]$$



Eternal Inflation?

- Probability of inflation at time t:

$$
\begin{aligned}
\Pr[\phi < \phi_{\rm end}, t] &= \frac{1}{2}\mathrm{erfc}\left[\frac{\mu(t) - \phi_{\rm end}}{\sigma(t)\sqrt{2}}\right] \\
&= \frac{1}{2}\mathrm{erfc}\left[\frac{\frac{\alpha}{3H}t - \phi_{\rm end}}{\frac{H}{2\pi}\sqrt{2Ht}}\right] \\
&= C(t)\exp\left[-\frac{4\pi^2\alpha^2}{18H^5}t\right] \\
&\to 0 \text{ as } t \to \infty
\end{aligned}
$$

Eternal Inflation!

- Probability of inflation at time t:

$$\Pr[\phi < \phi_{\rm end}, t] \;= C(t)\exp\left[-\frac{4\pi^2\alpha^2}{18H^5}t\right] \;\rightarrow 0$$

- But, also have exponential growth of inflating volume:

$$U(t) \sim e^{3Ht}$$

- So, total inflating volume:

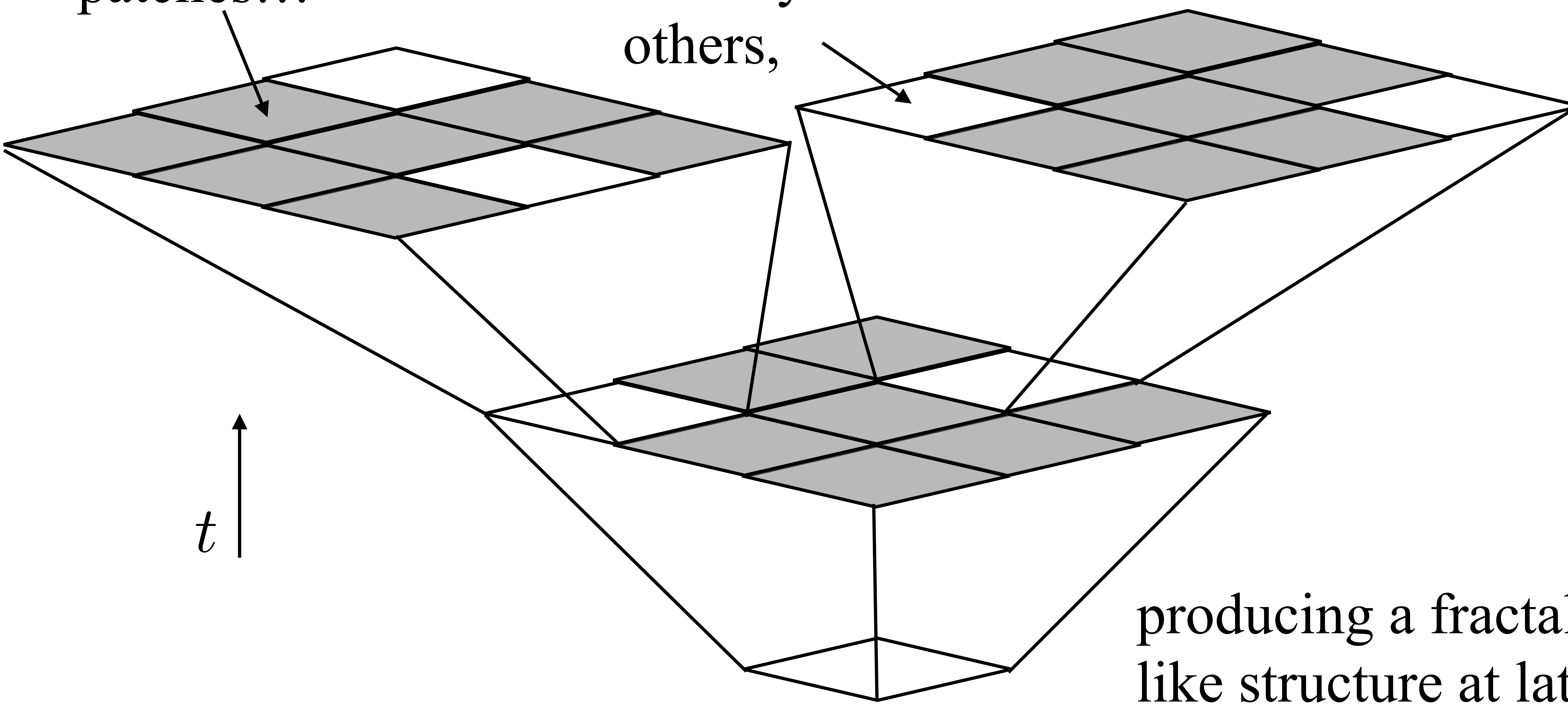
$$U_{\rm inf}(t) \sim \;\Pr[\phi < \phi_{\rm end}, t]\times U(t) \sim \exp\left[3Ht - \frac{4\pi^2\alpha^2}{18H^5}t\right]$$

$$\rightarrow \infty, \text{ if } 3H > \tfrac{4\pi^2\alpha^2}{18H^5}$$

EI and the Multiverse

Inflation ends in most Hubble patches…

…but, continues indefinitely in others,



producing a fractal-like structure at late times

First Condition for EI

- Condition for eternal inflation:

$$3H > \frac{4\pi^2\alpha^2}{18H^5}$$

- Using $H^2 M_{\rm Pl}^2 = V/3$, $\alpha = V'(\phi)$, this becomes

$$\boxed{\frac{|V'|}{V^{3/2}} < \frac{\sqrt{2}}{2\pi}\frac{1}{M_{\rm Pl}^3}}$$

cf. Creminelli, Dubovsky, Nicolis, Senatore, Zaldarriaga ‘08

Second Condition for EI

- Similar analysis for tachyonic potential gives:

$$U_{\rm inf}(t) = \Pr[\phi > \phi_{\rm end}, t] \times U(t) \sim \exp\left[3Ht - \frac{m^2}{3H}t\right]$$

- Condition for eternal inflation:

$$3H > \frac{m^2}{3H} \quad \overset{V'' = -m^2}{\underset{H^2 M_{\rm Pl}^2 = V/3}{\Longrightarrow}} \quad \boxed{\frac{V''}{V} > -\frac{3}{M_{\rm Pl}^2}}$$

T.R. '19

Generalization to Higher d

- Conditions for EI in four dimensions:

$$\frac{|V'|}{V^{3/2}} < \frac{\sqrt{2}}{2\pi}\frac{1}{M_{\rm Pl}^3} \qquad\qquad \frac{V''}{V} > -\frac{3}{M_{\rm Pl}^2}$$

- Can be generalized to d spacetime dimensions:

$$\frac{|V'|^2}{V^{(d+2)/2}} < \frac{2(d-1)^3}{\pi\Omega_{d-2}}\left(\frac{2}{(d-1)(d-2)M_d^{d-2}}\right)^{(d+2)/2}$$

$$\frac{V''}{V} > -\frac{2(d-1)}{d-2}\frac{1}{M_d^{d-2}}$$

T.R. '21

Speculations

Possible Motivations for the (R)dSC

(1) No Eternal Inflation TR '19

$$\frac{|\nabla V|}{V^{3/2}} \geq \frac{\sqrt{2}}{2\pi} \frac{1}{M_{\mathrm{Pl}}^3} \qquad \frac{V''}{V} < -\frac{3}{M_{\mathrm{Pl}}^2}$$

(2) Transplanckian Cosmic Censorship Bedroya, Vafa '19

$$\frac{|V'|}{V} \geq \frac{2}{\sqrt{(d-1)(d-2)}} \frac{1}{M_d^{(d-2)/2}}$$

(3) No Accelerated Expansion TR '21

$$\frac{|\nabla V|}{V} \geq \sqrt{\frac{4}{d-2}} \frac{1}{M_d^{(d-2)/2}}$$

Possible Versions of the (A)dSDC

(1) $m \lesssim |\Lambda|^{1/2}$: possibly related to SUSY, but violated in our universe

(2) $m \lesssim |\Lambda|^{1/d}$: motivation unclear, but possibly related to neutrino masses, and (via the see-saw mechanism) correlates the hierarchy problem and the cosmological constant problem

Summary

- Eternal inflation imposes constraints on first+second derivatives of the scalar field potential, reminiscent of the (refined) de Sitter Conjectures
- Dimensional reduction distinguishes the correct O(1) coefficients in the WGC bound, and it distinguishes certain physically-relevant values of the coefficients in the RdSC and the (A)dSDC
- Whether or not these values are “correct” remains to be seen

Thank You