Geometric approach for 3d interfaces at strong coupling

Based on: 2005.05983 with J.J. Heckman, T. B. Rochais, E. Torres

String Pheno Seminar - July 7, 2020

Markus Dierigl

DFG Deutsche Forschungsgemeinschaft

Penn Arts & Sciences

Outline

- Non-dynamical example: **Topological insulator**
- The fundamental domain of Maxwell theory and its **T-invariant subspace**
- Generalization to **other duality groups** $\Gamma \subset \mathrm{SL}(2, \mathbb{Z})$
- A **six-dimensional interpretation**
- Other construction of interfaces in four dimensions
- Conclusions and Outlook

Topological insulator

[Kane, Mele ’05,’06], [Bernevig, Hughes, Zhang ’06], [Moore, Balents, ’06], …

System with **unbroken global U(1) and time-reversal symmetry.**

➔ Couple U(1) to **background gauge field** A with local term

$$\frac{\theta}{8\pi^2} F \wedge F$$

Breaks time-reversal invariance unless

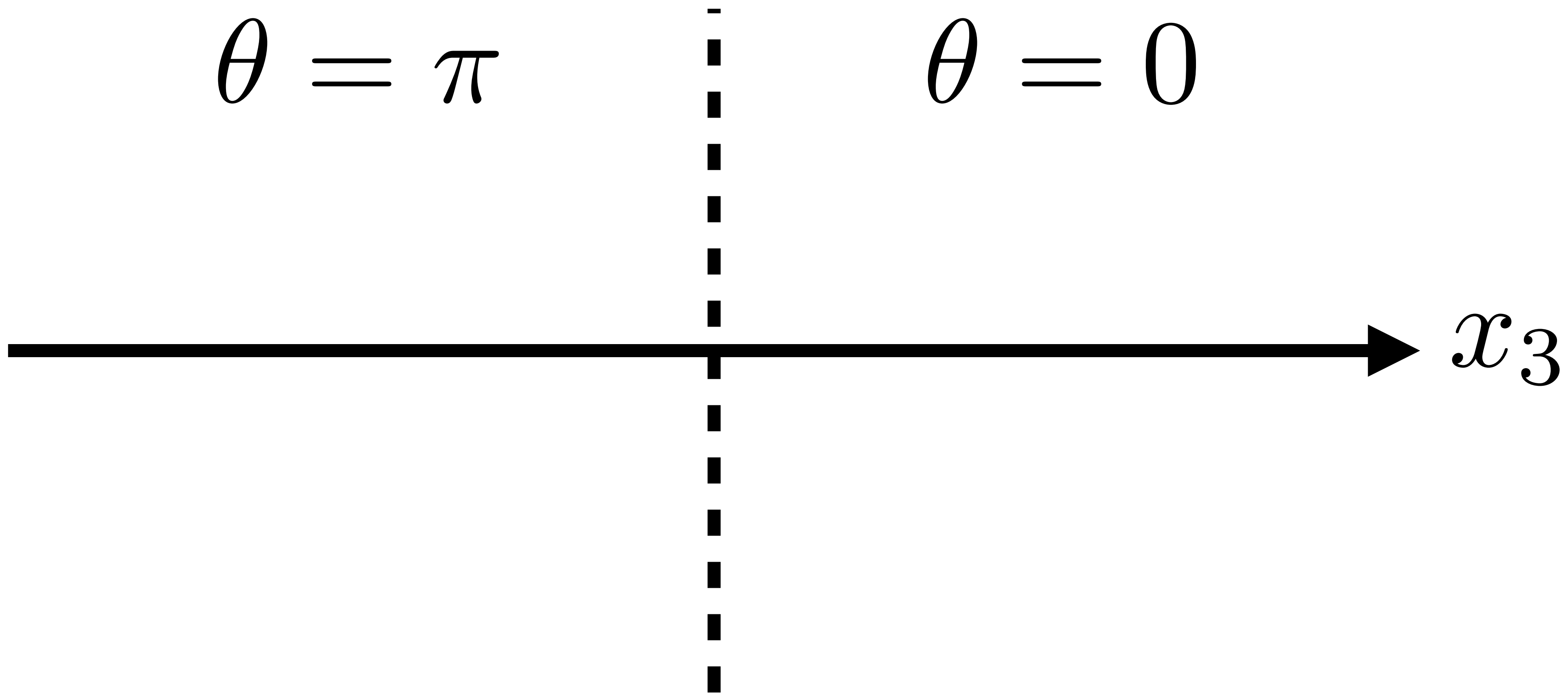
$$\theta \in \{0, \pi\} \mod 2\pi$$

trivial phase **topological insulator** phase

Constructing an interface

Induces a **Chern-Simons term:**

$$\frac{1}{8\pi} A \wedge F$$



➔ **breaks time-reversal invariance**
(half-integer level)

To restore it, one needs to **add fields on the interface**

- charged 3d Dirac fermion
- topological field theory (gapped phases) **e.g. [Seiberg, Witten '16]**

What if U(1) is dynamical?

$$\mathcal{L} = \frac{1}{2g^2} F \wedge *F + \frac{\theta}{8\pi^2} F \wedge F$$

Has **SL(2,Z) duality** [Montonen, Olive ’77], [Witten ’95]

$$\tau \to \frac{a\tau + b}{c\tau + d}\,, \qquad \begin{pmatrix} q_e \\ q_m \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} q_e \\ q_m \end{pmatrix}$$

with **complexified coupling:**

$$\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$$

Time-reversal acts as:

$$\theta \to -\theta\,, \qquad \tau \to -\overline{\tau}$$

anti-holomorphic involution

Time-reversal revisited

Why is $\theta = \pi$ time-reversal invariant?

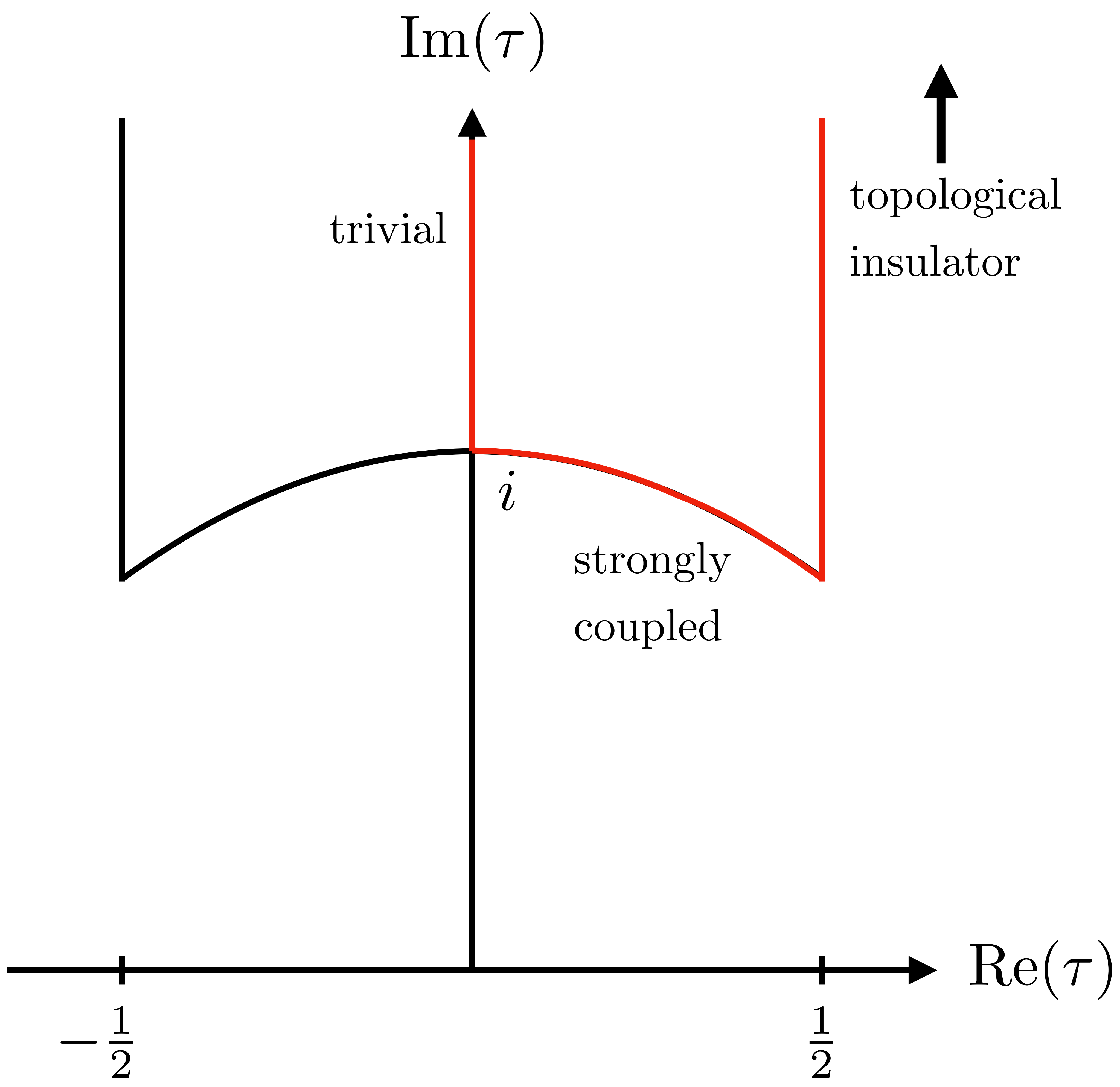
$$\theta = \pi \rightarrow -\pi \sim -\pi + 2\pi = \pi$$ using **T generator** of SL(2,Z)

What if one uses **S generator** instead?

$$\tau \rightarrow -\frac{1}{\tau} = -\frac{\bar{\tau}}{|\tau|^2}$$ acts as time-reversal for $|\tau| = 1$

Potential for **time-reversal invariant phases at strong coupling**

Time-reversal in pure Maxwell



F-theory toolkit:

Parametrize the physically inequivalent values for τ by the **complex structure** of an **auxiliary torus**

Weierstrass form:

$$y^2 = x^3 + fx + g$$

$$J(\tau) = \frac{j(\tau)}{1728} = \frac{4f^3}{4f^3 + 27g^2} = \frac{4f^3}{\Delta}$$

$$\boxed{\textbf{time-reversal} \longleftrightarrow J \in \mathbb{R}}$$

Building interfaces

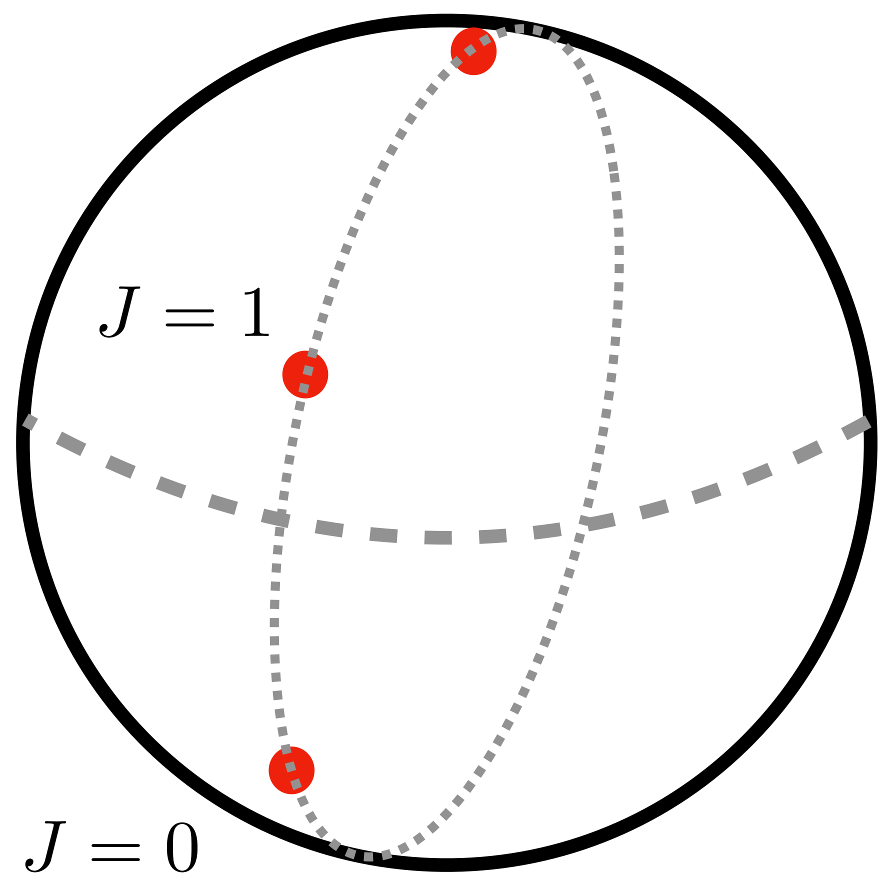
Let τ **vary** with respect to one coordinate in flat space x_3

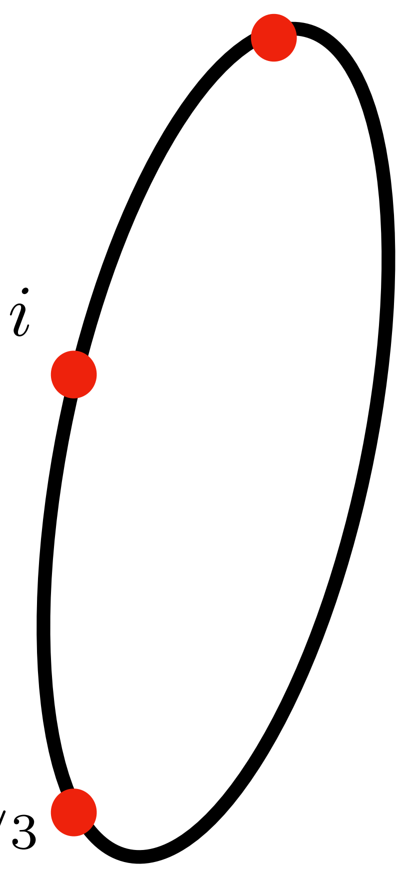
For **time-reversal** invariant system $J(\tau) \in \mathbb{R}$

(can be realized by $f, g \in \mathbb{R}$ $\longrightarrow$ realm of real elliptic curves)

Passing through:

- $J = \infty$ **topological insulator** (one expects localized dynamics and indeed $\Delta = 0$)
- $J = 1$ also leads to $\Delta = 0$ (strongly coupled localized dynamics?)





Other SL(2,Z) subgroups

Assume that the **actual duality group** is a **subgroup of SL(2,Z)**

$$\Gamma_0(N)\,,\Gamma_1(N)\,,\Gamma(N)\subset \mathrm{SL}(2,\mathbb{Z})$$

in the presence of charged fields

e.g. $$\Gamma(N)=\left\{\gamma\in \mathrm{SL}(2,\mathbb{Z}):\gamma=\begin{pmatrix}1&0\\0&1\end{pmatrix}\mod N\right\}$$

⟶ Changes **fundamental domain** or **modular curve**

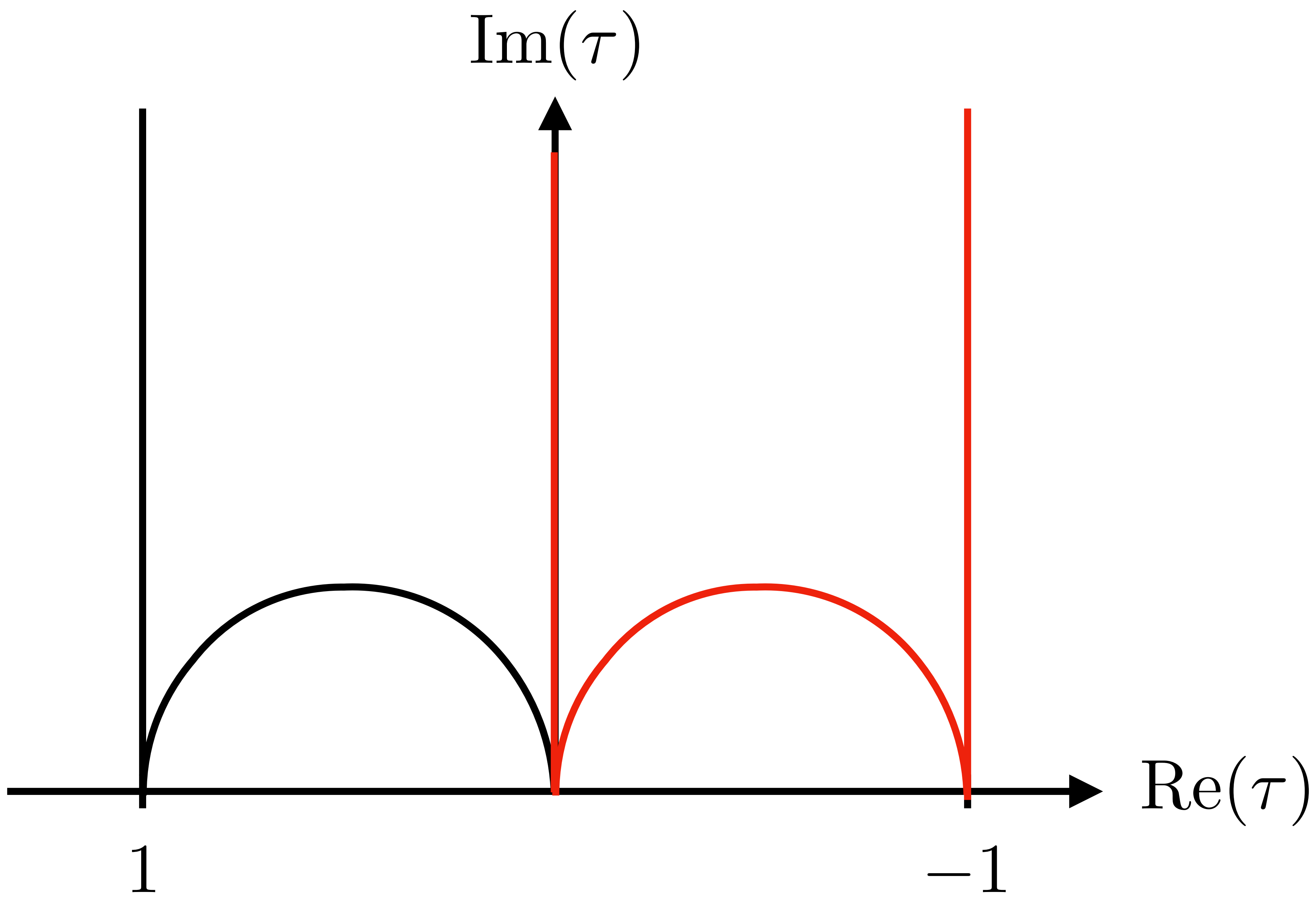
We are interested in the **time-reversal invariant subspace:**

$$X(\Gamma)_{\mathbb{R}}=\{\tau\in X(\Gamma):-\overline{\tau}=\tau\}=\{\tau\in\overline{\mathbb{H}}:-\overline{\tau}=\gamma\tau\ \text{ with }\ \gamma\in\Gamma\}$$

compactification

Example:

$\Gamma = \Gamma(2)$ generated by $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}, \quad \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$



Single closed component with **three cusps:**

- $\tau \to i\infty$ topological insulator interface
  electric states
- $\tau = 0$ related via $S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
  magnetic states
- $\tau = 1$ related via $\gamma = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$
  dyonic states

equivalent of the J-function, the Hauptmodul, given by elliptic λ-function

Seiberg-Witten theory [Seiberg, Witten '94]

$\mathcal{N}=2$ SYM in 4d with gauge algebra $\mathfrak{su}_2 \longrightarrow \mathrm{U}(1)$ gauge theory in IR

$$y^2 = (x-u)(x-\Lambda^2)(x+\Lambda^2)\,, \qquad u = \tfrac{1}{2}\mathrm{tr}\phi^2$$

duality group given by: $\Gamma = \Gamma(2)$

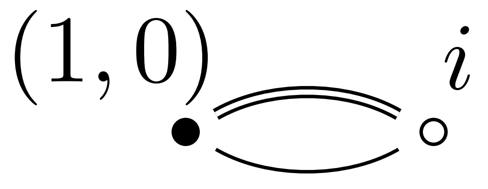
- interface crossing $\tau = i\infty$ **localized electric fields** (W-bosons)
- interface crossing $\tau = 0$ **localized magnetic fields** (monopole point)
- interface crossing $\tau = 1$ **localized dyonic fields** (dyon point)

Similar analysis for other $\mathcal{N}=2$ theories (more exotic AD theories)

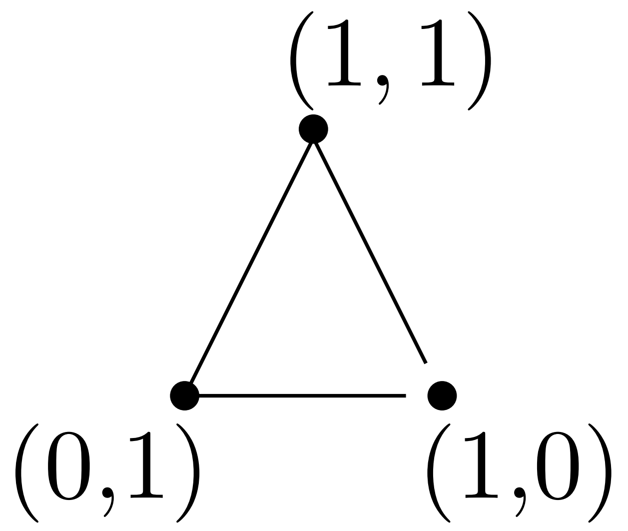
Mathematical classification [Snowden ’11]

Real components of modular curves classified:

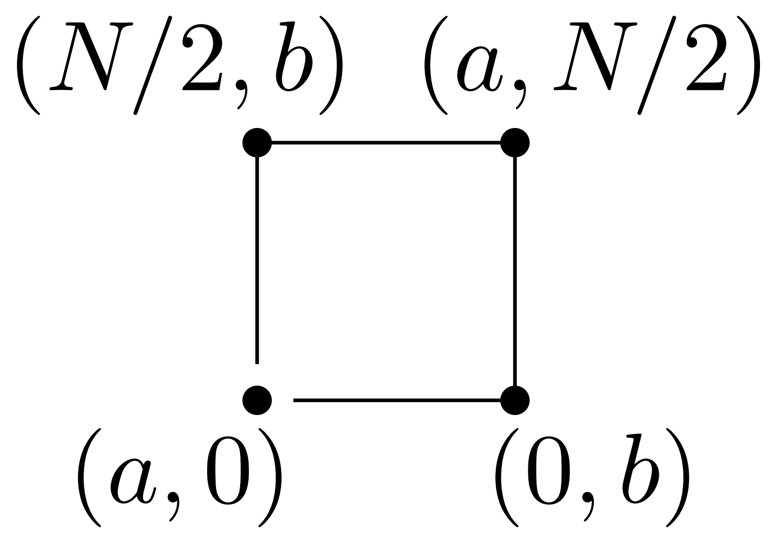
Splits into **disconnected sets of** topological **circles** with “special points”



$\mathrm{SL}(2,\mathbb{Z})$



$\Gamma(2)$



several components

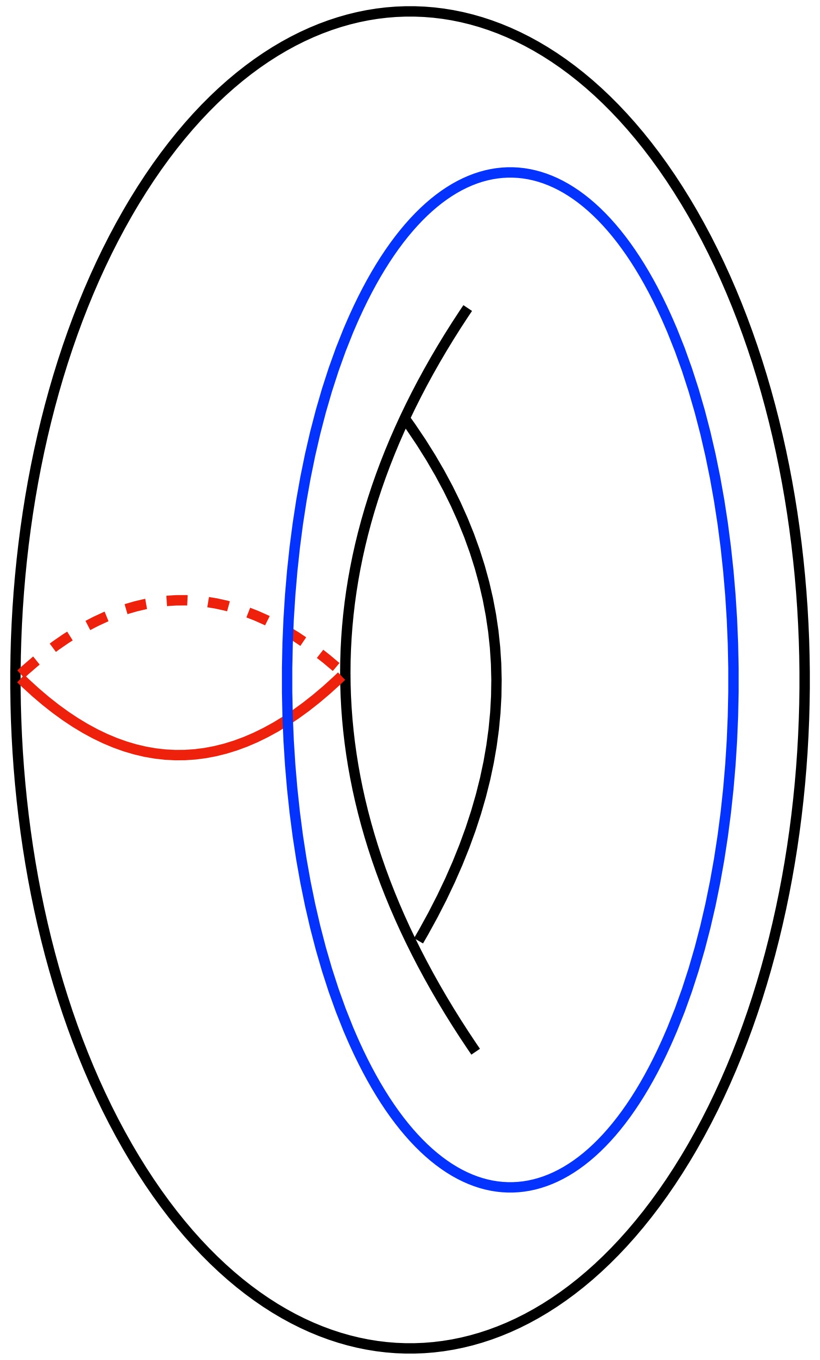
$\Gamma(N) = \Gamma(4n)$

- **time-reversal invariant** configuration sticks to **one component**
- only **certain combinations of interfaces** realizable
- **localized states from coset representatives** (interesting statistics between surfaces)

6d interpretation

Maxwell theory as anti-symmetric 6d tensor B **on a torus**

$$A = \int_{C} B \,, \quad A_D = -\int_{C} B$$



charged fields form **strings** coupling to B

τ literally the **complex structure** of **torus**

$\Delta = 0$ ⟶ 1-cycle pinches ⟶ massless states

charges by type of 1-cycle $C = q_e C - q_m C$

Congruence subgroups

Demand **invariance of** certain **line operators** in 4d

$$\exp\left(i\int_{L\times(r\textcolor{blue}{C}+s\textcolor{red}{C})} B\right) = \exp\left(ir\int_L A - is\int_L A_D\right)$$

restricts the identifications: $\mathrm{SL}(2,\mathbb{Z}) \to \Gamma$

(example $\Gamma(2)$ if r and s are measured mod 2 and the line operators are invariant)

Alternatively: as **torsion points** on Jacobian

$$\mathcal{J}(E) = H^1(E,\mathbb{R})/H^1(E,\mathbb{Z}) \simeq \widetilde{E}$$

natural generalization to higher-genus

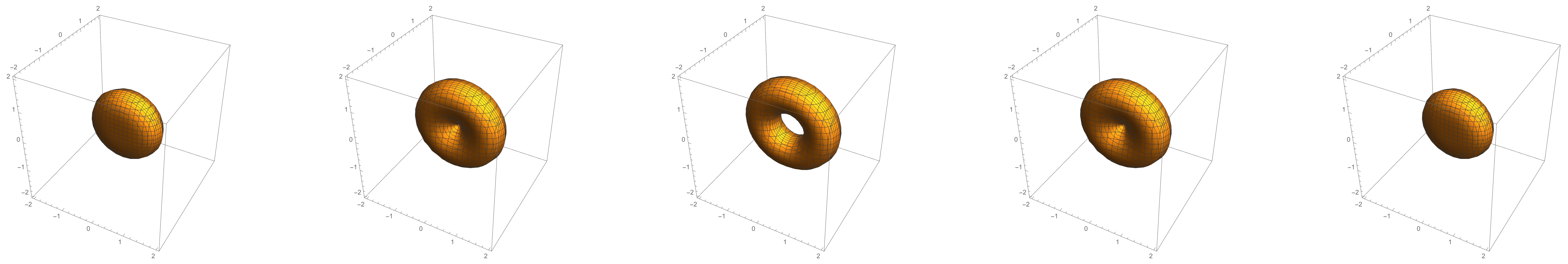
with decomposition: $N\sigma \in H^1(E,\mathbb{Z})$: $B \sim \tilde{A} \wedge \sigma$

→ direct **relation** to **congruence subgroups** (example: $\Gamma(2)$ preserves $\mathbb{Z}_2 \times \mathbb{Z}_2$ torsion)

Other geometrical interfaces

So far: varied the **shape of a torus** along one direction (+ subtleties involving time-reversal)

Why stop there?



Vary instead: **genus, fluxes, …**

Some **control over chiral degrees of freedom** via **anomaly**

Again use **interplay between real geometry** and **time-reversal** to secure **protection of localized states**

Conclusions

- Use **duality** to explore **time-reversal invariant regions** in moduli space
- Uncovers connection to **real elliptic curves** for **Maxwell theory**
- Deduce **localized degrees of freedom** from **time-reversal invariance** of **interfaces** (associated to special points in moduli space, e.g. cusps)
- Passes tests where we have control ($\mathcal{N} = 2$ supersymmetric theories)
- Embedding into **higher-dimensional theories** makes the **geometric perspective** more apparent
- Explore further (more exotic) possibilities for geometrically engineered interfaces

Outlook

- **Anomalies in duality groups** [Tachikawa, Yonekura '17], [Cordova, Freed, Lam, Seiberg '19], [Hsieh, Tachikawa, Yonekura '19]
- **Non-Abelian groups with 1-form center symmetries** [Aharony, Seiberg, Tachikawa '13],… (relation to MW-torsion, e.g. [Hajouji, Oehlmann '19])
- **Realization within higher-dimensional internal space (Spin-7 manifolds)** e.g. recent progress in [Cvetic, Heckman, Rochais, Torres, Zoccarato '20]
- **Tests beyond SUSY**