Non-flat elliptic fourfolds, three-form cohomology and 4D strongly coupled theories

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Uppsala University

Based on **arXiv:2102.10722**

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Hodge diamonds of Calabi-Yau Geometries

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 0 & & 0 & & & \\
 & & 0 & & h^{1,1} & & 0 & & \\
 & 0 & & & & & & 0 & \\
1 & & h^{3,1} & & & & h^{3,1} & & 1 \\
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Fourfolds admit **Kahler** and **complex structure moduli** counted by $h^{1,1}$ and $h^{3,1}$

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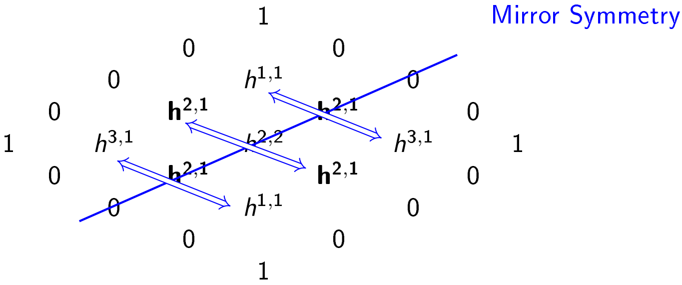

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- $h^{2,1}$ is **self-mirror** $\rightarrow$ **deserves some attention** [Greiner,Grimm'15]

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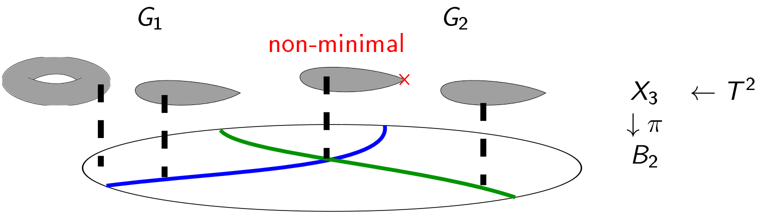
A: Compactifications of certain 6D SCFT sectors

Resolved elliptic threefolds



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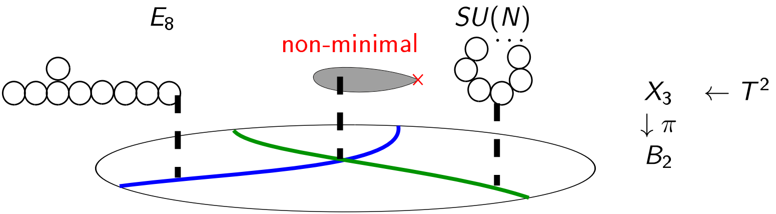

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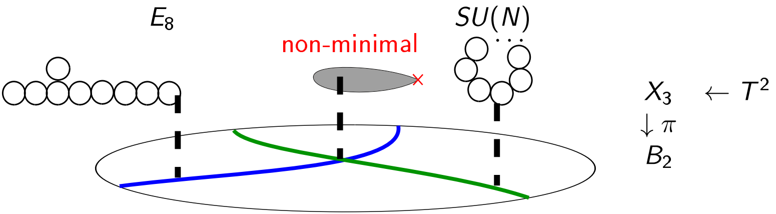

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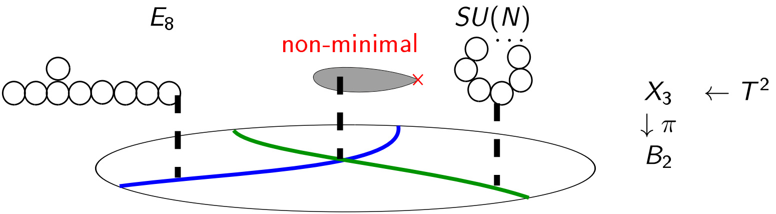

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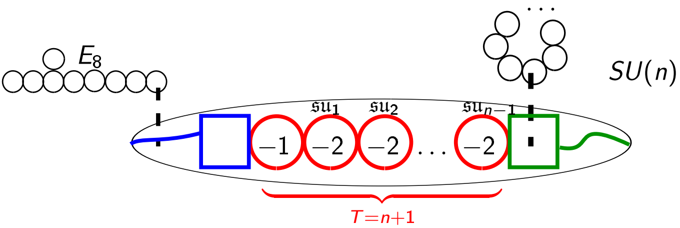
Two (crepant) ways to resolve the threefold

Resolution via base blow-up



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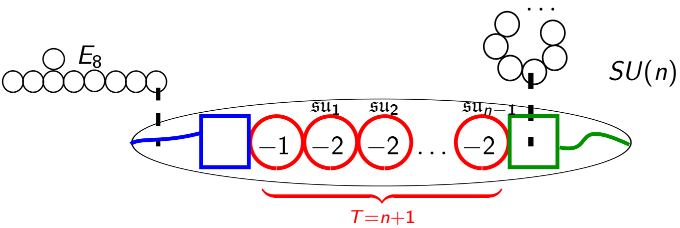

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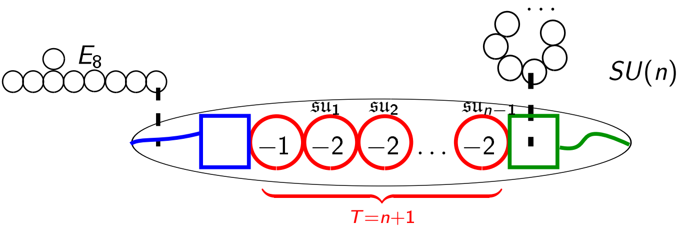

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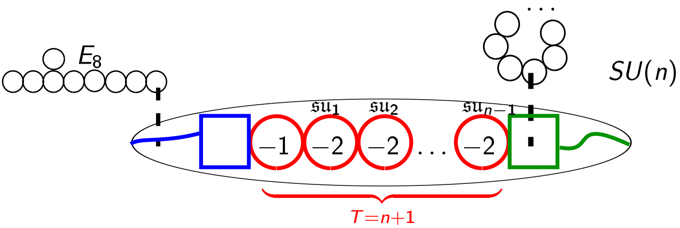

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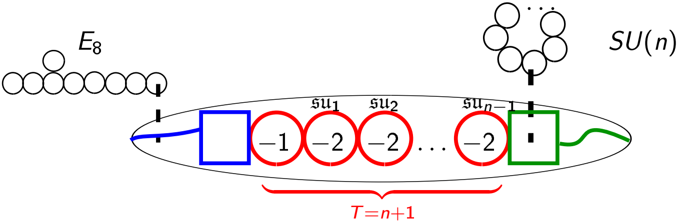
There exists an **alternative resolution**

Non-flat resolutions



Non-minimal singularities admits a **non-flat resolution** [Schafer-Nameki, Lawrie'12; Dierigl, Oehlmann, Ruehle'18]

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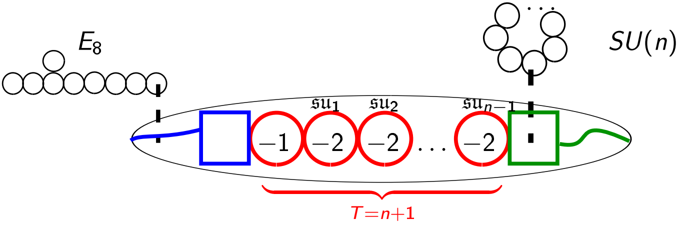

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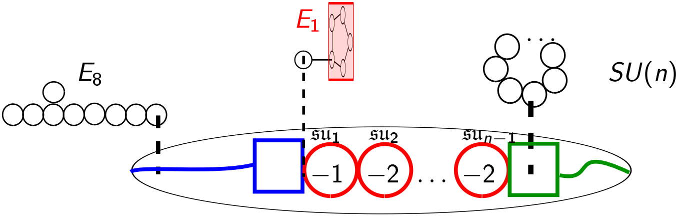

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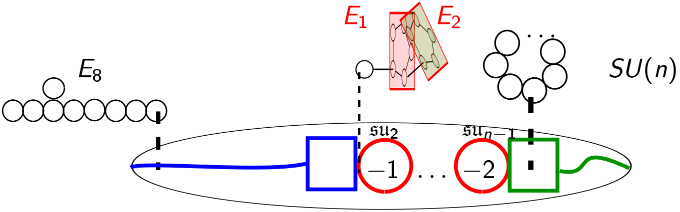

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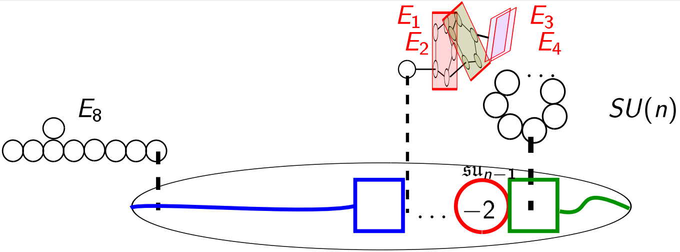

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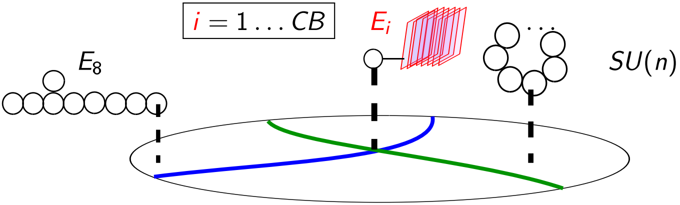

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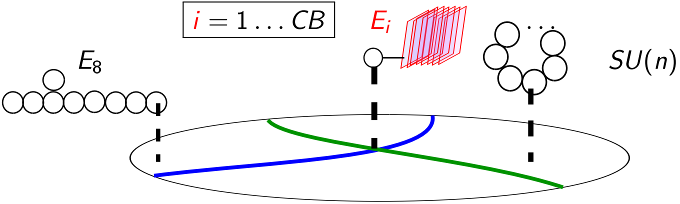

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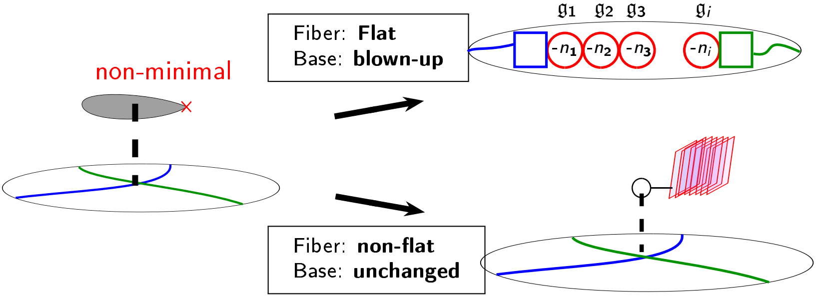
- **Replace** fiber with **several** *surfaces* $\dim_{\mathbb{C}}(E_i) = 2$
- Also needs CB **non-flat** surfaces!
- **Consistent** with 5D **M-theory duality** of **6D SCFT** compactifications [Apruzzi, Lin, Mayrhover; Apruzzi, Lin, Schafer-Nameki, Wang, Lawrie; Hubner'18,19]

Phases of non-minimal singularities



Both resolutions part of **extended Kahler cone** of the threefold

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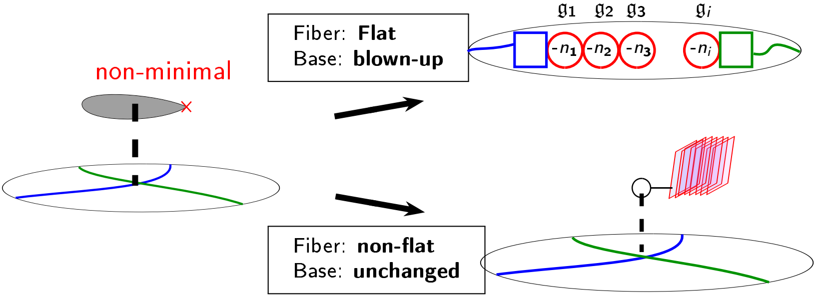

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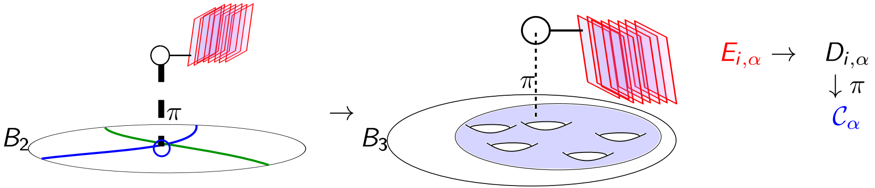
Non-flatness describes **resolved** non-minimal singularities **without changing the base**

From three-to-fourfolds



Extend the base from $B_2 \rightarrow B_3$: threefold $\rightarrow$ fourfold X_4

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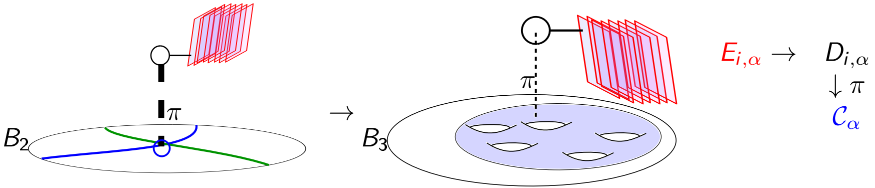

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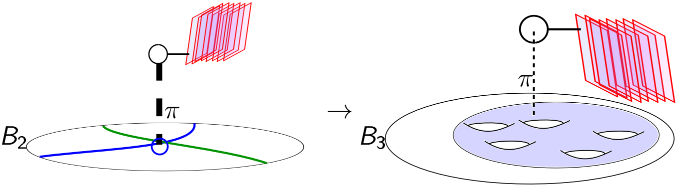

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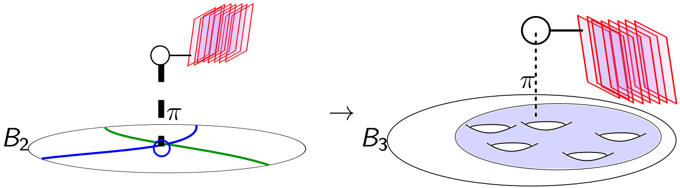

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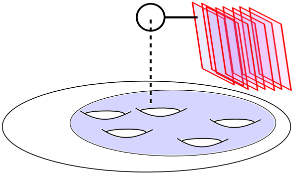
- **Cohomology** of $D_{i,\alpha}$ from **Leray-Hirsch spectral sequence:**

$$\begin{aligned} h^{0,0}(D_{i,\alpha}) =& h^{0,0}(\mathcal{C}_\alpha) \cdot h^{0,0}(E_{i,\alpha}) = 1 \\ h^{1,0}(D_{i,\alpha}) =& h^{1,0}(\mathcal{C}_\alpha) \cdot h^{0,0}(E_{i,\alpha}) = g_\alpha \end{aligned}$$

Non-flat fourfold cohomology



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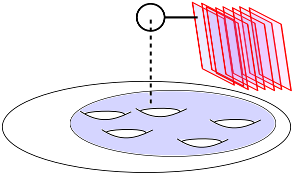

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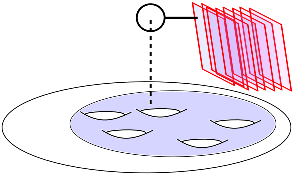

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- **Sum** over all CB_α surfaces E_i over each base curve $\mathcal{C}_\alpha$

$$h^{1,1}(X_4)_{\text{non-flat}} = \sum_\alpha CB_\alpha$$

$$h^{2,1}(X_4)_{\text{non-flat}} = \sum_\alpha g_\alpha \cdot CB_\alpha$$

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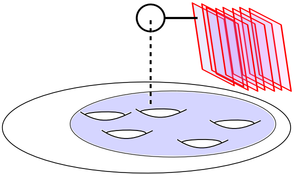

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$$h^{1,1}(X_4)_{\text{non-flat}} = \sum_\alpha CB_\alpha$$

$$h^{2,1}_{\text{fiber}} \ni h^{2,1}(X_4)_{\text{non-flat}} = \sum_\alpha g_\alpha \cdot CB_\alpha$$

Summary

Double check in toric examples via **Batyrev construction** [Klemm, Lian, Roan, Yau'97]

- **E-string theories** on genus $g = 153$ curves ✓
- **Higher rank theories** e.g. $[E_8 x SU(n)]$ superconformal matter over $g >> 1$ curves ✓

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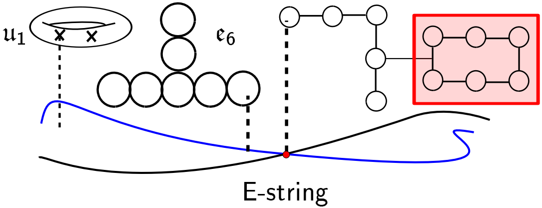
Disclaimer: Fourfold examples X_4 is **compact** and **no $\mathbf{G_4}$ flux!**

Example: $E_6 \times U(1)$ in 6D



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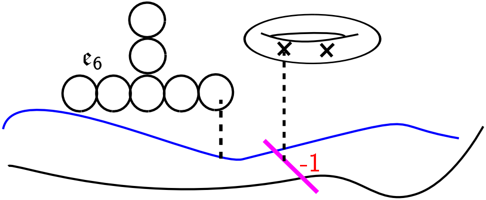

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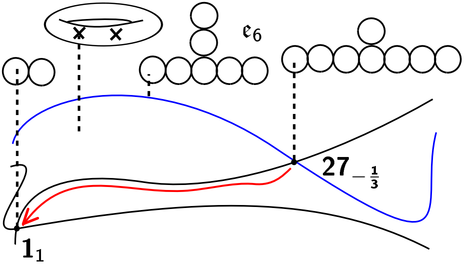

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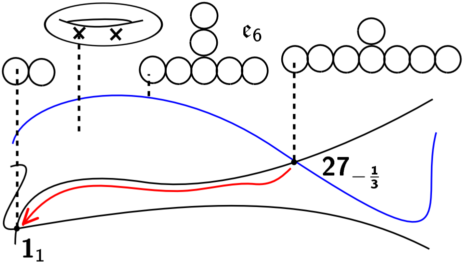

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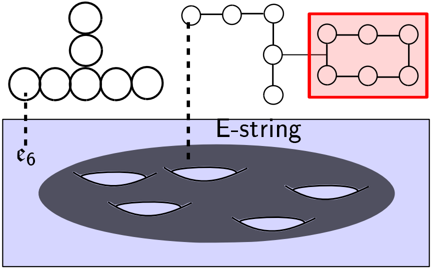
Perform the same transition in a fourfold

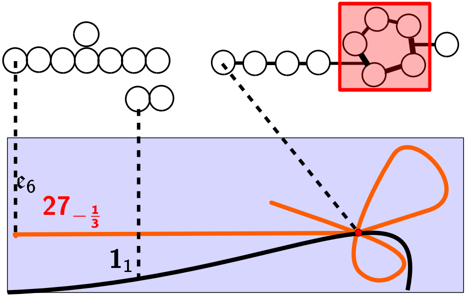
Example: $E_6 \times U(1)$ in 4D



Place E-string type fiber is over Riemann surface of e.g. $g = 5$

- **Phase 1: Deform** the E-string curve into $(\mathbf{27}_{-\frac{1}{3}} + \mathbf{1}_1)$ curves
- Example: **Cohomology change**: $\Delta h^{1,1} = -1, \Delta h^{2,1} = -5$ ✓

The $E_6 \times U(1)$ deformed fourfold



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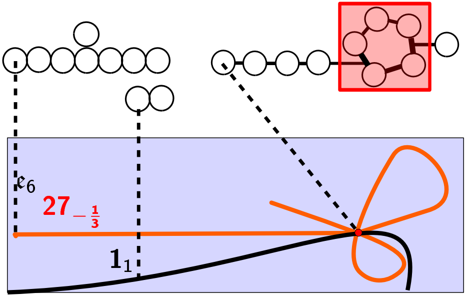
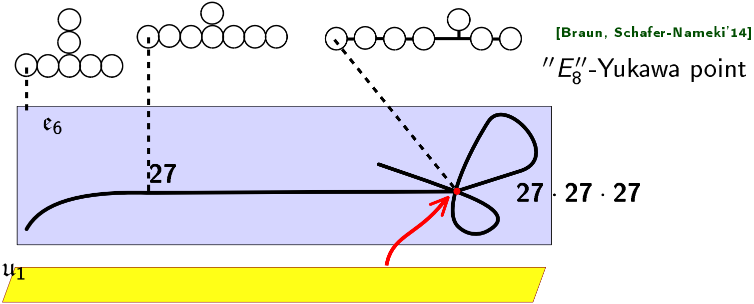


The $\mathbf{27}_{-\frac{1}{3}}$ and $\mathbf{1}_1$ curves **re-intersect** in **codimension 3** non-flat point

- This is a **non-perturbative superpotential** coupling term
- **How to see that?** further **break** the $U(1)$ group

Further break to E_6

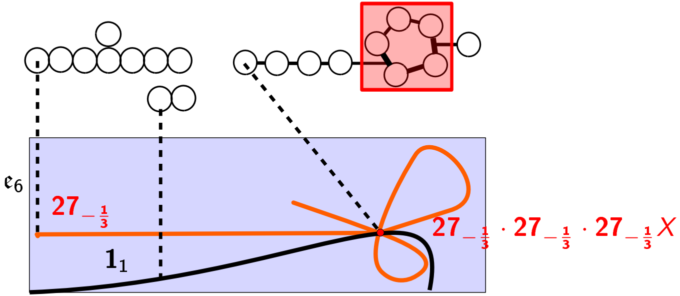
[Esole,Yau; Dolan, Saulina, Schafer-Nameki'11]
[Collinucci, Garcia-Etxebarria'16]



Same locus: **flat fiber** of $\mathbf{E}_8$ bouquet type

- This is a $\mathbf{27}^3$ triple intersection point
- **Adding** U(1) **over enhances** the E_8 point

Adding to $E_6 \times U(1)$ again



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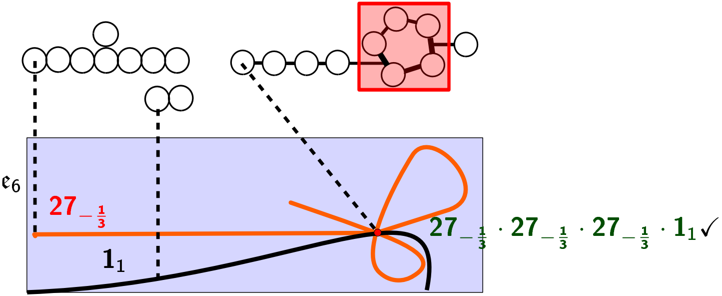

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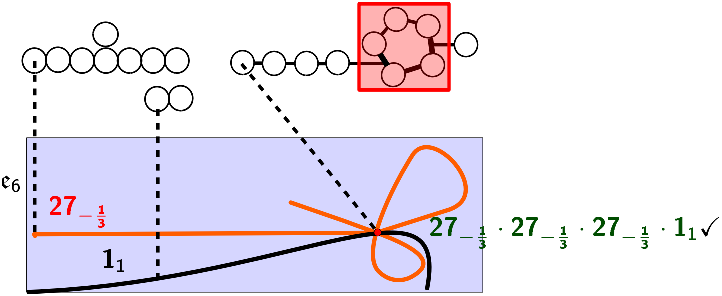

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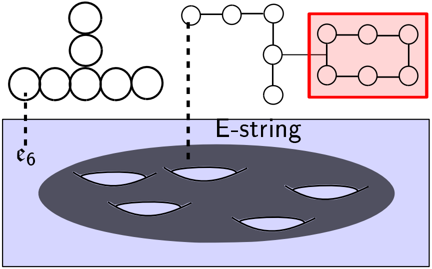
A remnant of the deformed E-string curve

Back to E-string curve



Phase 2: Blow-ups of B_3 to **remove E-string curve**

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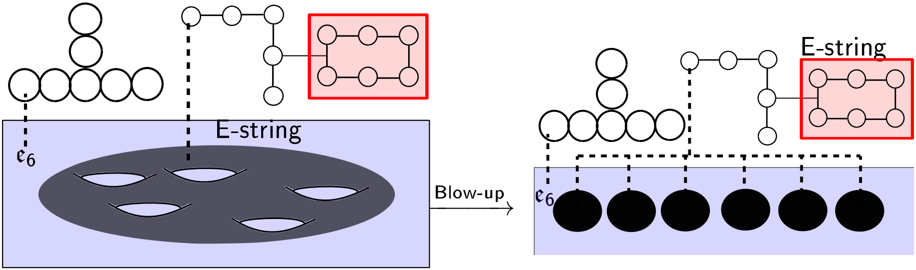

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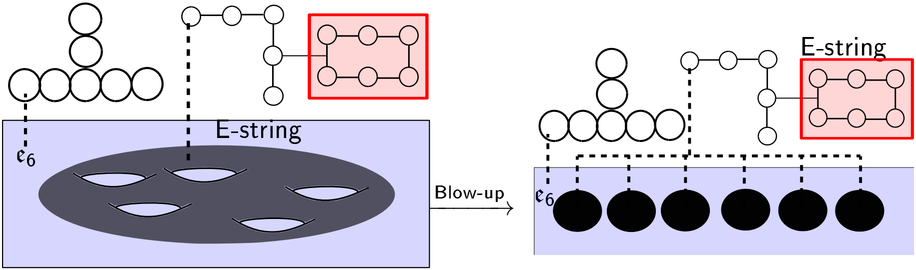

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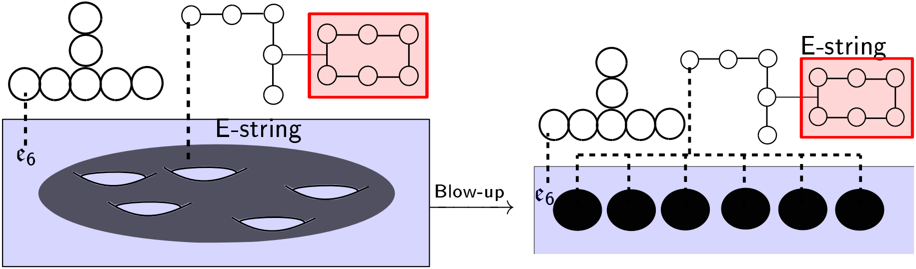

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- **Euler number invariant**: $\Delta\chi = 6(8 + \Delta h^{1,1} + \Delta h^{3,1} - \Delta h^{2,1}) = 0$
- **D3 tadpole** unchanged: $\Delta \int G_4^2 = \Delta n_{d3}/12$

Non-flat fibers in codim 2

- **Resolve** non-minimal **fiber singularities, sacrificing equi-dimensionality**
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Thank You Very Much

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