F-theory vacua at large complex structure

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SEMINAR SERIES ON STRING PHENOMENOLOGY

Contents

F-Theory Potential at large complex structure

Manifold and Moduli

- Our setup: F-theory compactified on a Calabi–Yau four-fold Y_4 + internal background four-form flux G_4 where Y_4 is a smooth elliptic fibration over a three-fold base.
- Compactification gives rise to Kahler and complex structure moduli.

$$K = \underbrace{-2\log\mathcal{V}_3}_{\substack{\text{Kähler moduli}\\ \downarrow \\ \text{D-term potential}}} \underbrace{-\log\int_{Y_4}\Omega\wedge\bar{\Omega}}_{\substack{\text{Cplx. Structure moduli}\\ \downarrow \\ \text{F-term potential}}}$$

- We need a basis for the lattice of flux quanta. This lattice pairs up with the horizontal subspace of middle cohomology of the four fold.[1]

$$\dim H^4_H(Y_4) = 2 + 2h^{(3,1)}(Y_4) + \dim H^{(2,2)}_H(Y_4)$$

[1] A. Strominger, *SPECIAL GEOMETRY*, Commun. Math. Phys. 133 (1990) 163.

Mirror Symmetry and Central Charges

- Use homological mirror symmetry and consider X_4 mirror four fold of Y_4.[2]

$$\text{Periods of } \Omega \text{ in } Y_4 \overset{LCS}{\Longleftrightarrow} \begin{cases} \text{Central charges of D(2p)-branes} \\ \text{wrapping holomorphic 2p-cycles on } X_4 \end{cases}$$

- Tricky part: We build a basis of integral four-form classes $[\sigma_\mu]$ from the intersections of Nef divisors $[D_i \cdot D_j] = \zeta^\mu_{ij}[\sigma_\mu]$.
  Let $\eta_{\mu\nu} \equiv [\sigma_\mu][\sigma_\nu]$, then we have $\mathcal{K}_{ijkl} = \zeta^\mu_{ij}\eta_{\mu\nu}\zeta^\nu_{kl}$.
- The central charges are

$$\Pi_0 = 1\,, \quad \Pi_2^i = -T^i\,, \quad \Pi_{4\,\mu} = \frac{1}{2}\eta_{\mu\nu}\zeta^\nu_{ij}T^iT^j\,, \quad \Pi_{6\,i} = -\frac{1}{6}\mathcal{K}_{ijkl}T^jT^kT^l\,,$$
$$\Pi_8 = \frac{1}{24}\mathcal{K}_{ijkl}T^iT^jT^kT^l\,.$$

[2] C. F. Cota, A. Klemm and T. Schimannek, *Modular Amplitudes and Flux-Superpotentials on elliptic Calabi-Yau fourfolds*, JHEP 01 (2018) 086 .

Flux Quanta and superpotential

- Using mirror symmetry to go back to Y_4, we expand Ω in a basis $\{\alpha, \alpha_i, \sigma_\mu^Y, \beta^i, \beta\}$ of $H_H^4(Y_4)$.

$$\Omega = \alpha\pi_0 + \alpha_i\pi_2^i + \sigma_\mu^Y\pi_4^\mu + \beta^i\pi_{6i} + \beta\pi_8$$

- Expanding the four-form flux G_4 in the same basis we arrive to the flux quanta

$$G_4 = m\alpha - m^i\alpha_i + \hat{m}^\mu\sigma_\mu^Y - e_i\beta^i + e\beta$$

$$W = \int_{Y_4}\Omega\wedge G_4 = e + e_i T^i + \frac{1}{2}\,\hat{m}^\mu\zeta_{\mu,kl}\,T^k T^l + \frac{1}{6}\,\mathcal{K}_{ijkl}\,m^i T^j T^k T^l + \frac{m}{24}\,\mathcal{K}_{ijkl}\,T^i T^j T^k T^l$$

- We can also compute the complex structure sector of the Kähler potential.

$$K_{\mathrm{cs}} = -\log\int_{Y_4}\Omega\wedge\bar{\Omega} = \log(\frac{2}{3}\mathcal{K}_{ijkl}t^i t^j t^k t^l)$$

where $T^i = b^i + it^i$.

Scalar potential

- Due to the no-scale properties of F-theory compactifications the F-term potential takes the simple form
$$V = e^{K} \sum_{i,j} K^{i\bar{j}} D_i W D_{\bar{j}} \overline{W}\,.$$
- We can express the scalar potential in terms of a set of axion polynomials ρ linear on the flux quanta.
$$V = \frac{1}{2} Z^{AB} \rho_A \rho_B\,,$$
Up to exponentially suppressed terms Z^{AB} only depends on the saxions.

Scalar potential

$$\boxed{V = \frac{1}{2} Z^{AB} \rho_A \rho_B}$$

$$2\mathcal{V}_3^2 Z = \begin{pmatrix} \frac{\mathcal{K}}{24} & 0 & 0 & 0 & 0 \\ 0 & \frac{\mathcal{K}}{6} g_{ij} & 0 & 0 & 0 \\ 0 & 0 & g_{\mu\nu} & 0 & 0 \\ 0 & 0 & 0 & \frac{6}{\mathcal{K}} g^{ij} & 0 \\ 0 & 0 & 0 & 0 & \frac{24}{\mathcal{K}} \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\delta^i_j & 0 \\ 0 & 0 & \eta_{\mu\nu} & 0 & 0 \\ 0 & -\delta^j_i & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{\rho} = m\,,$$

$$\tilde{\rho}^i = m^i + m b^i\,,$$

$$\rho_i = e_i + \hat{m}^\mu \zeta_{\mu,il} b^l + \frac{1}{2}\mathcal{K}_{ijkl} m^j b^k b^l + \frac{1}{6} m \mathcal{K}_{ijkl} b^j b^k b^l\,,$$

$$\hat{\rho}^\mu = \hat{m}^\mu + \zeta^\mu_{ij} b^i m^j + \frac{1}{2}\zeta^\mu_{ij} b^i b^j\,,$$

$$\rho = e + e_i b^i + \frac{1}{2}\hat{m}^\mu \zeta_{\mu,kl} b^k b^l + \frac{1}{6}\mathcal{K}_{ijkl} m^i b^j b^k b^l + \frac{1}{24} m \mathcal{K}_{ijkl} b^i b^j b^k b^l\,.$$

Adding corrections

- We extend the analysis to regions where the complex structure saxions are only moderately large, so that the exponential corrections can still be neglected.
- Go back to type IIA compactified in the mirror four-fold X_4. The Kähler sector gets polynomial curvature corrections, which are encoded in the central charges[3].
- Correction to the intersection numbers can be absorbed by a redefinition of the fluxes.

$$\begin{aligned} \bar{m}^\mu &= \hat{m}^\mu - \frac{1}{2}\zeta^\mu_{ii} m^i + \frac{m}{12} c_2^\mu \,, \\ \bar{e}_j &= e_j + \frac{m^i}{6}\mathcal{K}_{jjji} + m^i K^{(2)}_{ij} - \tfrac{1}{2}\left(\mathcal{K}_{jkkl} + \mathcal{K}_{jkll}\right) m^{kl} \,, \\ \bar{e} &= e + m^{jk}\lambda_{jk} - m^i\left(\frac{1}{24}\mathcal{K}_{iiii} + \frac{1}{2}K^{(2)}_{ii}\right) + mK^{(0)} \,. \end{aligned}$$

[3] A. Gerhardus and H. Jockers, *Quantum periods of Calabi–Yau fourfolds*, Nucl. Phys. B 913 (2016) 425 [1604.05325].

Adding corrections

- In addition there are corrections to the Kähler potential that depend on the third Chern form $K_i^{(3)} \equiv \frac{\zeta(3)}{8\pi^3} \int_{X_4} c_3(X_4) \wedge D_i$.

$$K_{\mathrm{cs}}^{\mathrm{corr}} = -\log\left(\frac{2}{3}\mathcal{K}_{ijkl} t^i t^j t^k t^l + 4K_i^{(3)} t^i\right) ,$$

$$W^{\mathrm{corr}} = \bar{e} + \bar{e}_i T^i + \frac{1}{2}\bar{m}^\mu \zeta_{\mu,kl} T^k T^l + \frac{1}{6}\mathcal{K}_{ijkl}\, m^i T^j T^k T^l + \frac{m}{24}\mathcal{K}_{ijkl}\, T^i T^j T^k T^l - iK_i^{(3)}\left(m^i + mT^i\right) .$$

- These changes respect the factorisation between axions and saxions, and therefore the bilinear structure $V = \frac{1}{2} Z^{AB} \rho_A \rho_B$.
- The diagonal structure of Z is broken.

Main results

Using the bilinear formulation of F-theory compactifications we will:

1. Systematically analyse the vacuum conditions for an arbitrary number of moduli.
2. Show that the tadpole condition severely constrains the possible flux choices to find vacua in the LCS regime.
3. Identify two families of vacua with very distinct properties:
   - Moduli stabilisation is achieved including the $K^{(3)}$ corrections and the saxionic vev are bounded by the choice of fluxes.
   - Moduli stabilisation can be achieved through the leading expression, the saxionic vevs are unbounded and there is a single contribution to the tadpole.

Tadpoles and Vacua

Equations of motion

- The scalar potential is the sum of three positive definite quantities.

$$V = e^K \left[4\left(\rho - \frac{\mathcal{K}}{24}\tilde{\rho}\right)^2 + g^{ij}\left(\rho_i + \frac{\mathcal{K}}{6} g_{ik}\tilde{\rho}^k\right)\left(\rho_j + \frac{\mathcal{K}}{6} g_{jl}\tilde{\rho}^l\right) + g_P^{ij}\zeta_{\mu i}\zeta_{\nu j}\hat{\rho}^\mu\hat{\rho}^\nu \right]$$

Therefore its minima correspond to Minkowski vacua where these three terms vanish.

$$\boxed{\begin{aligned} \rho &= \frac{1}{24}\mathcal{K}\tilde{\rho} \\ \rho_i &= -\frac{1}{6}\mathcal{K} g_{ij}\tilde{\rho}^j \\ 0 &= \left(\mathcal{K}\zeta_{\mu i} - \mathcal{K}_i\zeta_\mu\right)\hat{\rho}^\mu \end{aligned}}$$

- Similar procedure can be used to derive the corrected vacuum equations.

Tadpole and flux quanta

- In any consistent F-theory compactification on a four-fold Y_4 one must satisfy the D3-brane tadpole condition

$$N_{\rm flux} = \frac{1}{2}\int_{Y_4} G_4 \wedge G_4 = \frac{\chi(Y_4)}{24} - N_{\rm D3}\,.$$

$$\left.\begin{array}{lr} \text{Stability of Minkowski vacua requires} & N_{D3} > 0 \\ \text{On-shell relation } G_4 = \star G_4 \text{ requires} & N_{\rm Flux} > 0 \end{array}\right\} \quad 0 \leq N_{\rm flux} \leq \chi(Y_4)/24$$

- $N_{\rm flux}$ equals a bilinear of flux-axion polynomials

$$N_{\rm flux} \equiv \bar{e}m - \bar{e}_i m^i + \frac{1}{2}\eta_{\mu\nu}\bar{m}^\mu \bar{m}^\nu \overset{\rho' s\ def}{\Rightarrow} N_{\rm flux} = \bar{\rho}\tilde{\rho} - \bar{\rho}_i\tilde{\rho}^i + \frac{1}{2}\eta_{\mu\nu}\bar{\rho}^\mu\bar{\rho}^\nu$$

$$\overset{eom' s}{\Rightarrow} N_{\rm flux} \overset{\rm vac}{=} \frac{\mathcal{K}}{24}\left(\tilde{\rho}^2 + 4g_{ij}\tilde{\rho}^i\tilde{\rho}^j\right) + \frac{1}{2}g_{\mu\nu}\bar{\rho}^\mu\bar{\rho}^\nu$$

- At large complex structure $\mathcal{K} \to \infty$ but $N_{\rm flux}$ remains finite. This greatly constrains the allowed choice of flux quanta.

Tadpole and Large Complex Structure

$$\boxed{N_{\mathrm{flux}} = \frac{\mathcal{K}}{24}\left(\tilde{\rho}^2 + 4 g_{ij}\tilde{\rho}^i\tilde{\rho}^j\right) + \frac{1}{2} g_{\mu\nu}\bar{\rho}^\mu\bar{\rho}^\nu}$$

- We need $\tilde{\rho} = 0$, which means $m = 0$ and $\tilde{\rho}^k = m^k$.

[4] Full classification of all possibilities should follow from the techniques developed in T. W. Grimm, C. Li and I. Valenzuela, *Asymptotic Flux Compactifications and the Swampland*, JHEP 06 (2020) 009 [1910.09549].

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- Does $\mathcal{K}g_{ij}m^im^j = (4\mathcal{K}_i\mathcal{K}_j/\mathcal{K} - 3\mathcal{K}_{ij})m^im^j$ remain bounded? $\rightarrow$ Depends on the topology of Y_4.
- Suppose that we blow up a single moduli[4] $t^i \rightarrow \infty$:

	$\mathcal{K}g_{ii}$	$\mathcal{K}g_{jj}$
$\mathcal{K}_{iiii} \neq 0$	$(t^i)^2$	$(t^i)^2$
$\mathcal{K}_{iiik} \neq 0 \ (k \neq i)$	t^i	t^i
$\mathcal{K}_{iijk} \neq 0 \ (j,k \neq i)$	Constant	$(t^i)^2$
$\mathcal{K}_{ijkl} \neq 0 \ (j,k,l \neq i)$	0	t^i

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$\mathcal{K}_{iijk} \neq 0$ $(j, k \neq i)$	Constant	$(t^i)^2$
$\mathcal{K}_{ijkl} \neq 0$ $(j, k, l \neq i)$	0	t^i

$\rightarrow$We expect to find few vacua with $m^i \neq 0$ when $t^i \gtrsim \frac{1}{2}\sqrt{\chi(Y_4)}$.

Exceptions:

-Linear case when $m^j = 0$ for $j \neq i$

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Solving the vacua equations

- Based on the previous reasoning we take $\vec{q}^{\,t} = (m, m^i, \hat{m}^\mu, \bar{e}_i, \bar{e}) = (0, 0, \hat{m}^\mu, \bar{e}_i, \bar{e})$. Then $N_{\rm flux} = \frac{1}{2}\eta_{\mu\nu}\hat{m}^\mu\hat{m}^\nu$ and the vacua equations can be solved manually.

$$\begin{cases} \bar{\rho} = 0 \\ \bar{\rho}_i = 0 \\ \mathcal{K}\zeta_{\mu i}\hat{m}^\mu = \mathcal{K}_i\zeta_\mu\hat{m}^\mu \end{cases}$$

- At leading order equations for axions and saxions decouple.

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- At leading order equations for axions and saxions decouple.
- We have $2h^{3,1}$ unknowns (b^i, t^i). If $r = \mathrm{rank}(M)$, we can fix r axions.

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$$\begin{cases} \bar{\rho} = 0 & \Longrightarrow \quad \bar{e} + \bar{e}_i b^i + \dfrac{1}{2}\hat{m}^\mu \zeta_{\mu,kl} b^k b^l = 0 \quad \Longrightarrow \quad \bar{e} = -\dfrac{1}{2}\bar{e}_i b^i \\ \bar{\rho}_i = 0 & \Longrightarrow \quad \underbrace{\hat{m}^\mu \zeta_{\mu,ij}}_{M_{ij}} b^j = -\bar{e}_i \\ \mathcal{K}\zeta_{\mu i}\hat{m}^\mu = \mathcal{K}_i \zeta_\mu \hat{m}^\mu & \end{cases}$$

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- At leading order equations for axions and saxions decouple.
- We have $2h^{3,1}$ unknowns (b^i, t^i). If $r = \mathrm{rank}(M)$, we can fix r axions.
- The first equation provides an additional constraint on the choice of fluxes.
- Using the last relation we can fix $r-1$ saxions.

  Even if the $r = h^{3,1}(Y_4)$ there will always be a flat saxionic direction.

Vacua equations with corrections

- Corrections change the Kähler potential and superpotential. We compute the eom's expanding this corrections to linear order in $\epsilon_i \equiv 6K_i^{(3)}/\mathcal{K}$.

$$\bar{\rho} = -\frac{3}{8}\epsilon_i t^i \zeta_\mu \hat{m}^\mu \,,$$

$$\bar{\rho}_i = 0 \,,$$

$$\left(\mathcal{K}\zeta_{\mu i} - \mathcal{K}_i \zeta_\mu\right) \hat{m}^\mu = \frac{1}{4}\left(\mathcal{K}\epsilon_i - \epsilon_k t^k \mathcal{K}_i\right) \zeta_\mu \hat{m}^\mu \,.$$

- Saxions and axions no longer decoupled. If $r = h^{3,1}(Y_4)$ we can stabilise all moduli.

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- Estimate there is an integer $p \leq h^{3,1}(X_4)$ satisfying $N_{\rm flux}^p \bar{\rho} \gtrsim d^{2p-1}$, with $d = \gcd(\hat{\rm m}^\mu)$.

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- From the last eom, $\hat{m}^\mu = A\zeta^\mu + C^\mu + \mathcal{O}(\epsilon)$ with $C^\mu \zeta_{\mu i} = 0$ and so $N_{\rm flux} \geq \frac{1}{2}A^2\mathcal{K} + \mathcal{O}(\epsilon_i)\,.$

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$$\left(\mathcal{K}\zeta_{\mu i} - \mathcal{K}_i \zeta_\mu\right) \hat{m}^\mu = \frac{1}{4}\left(\mathcal{K}\epsilon_i - \epsilon_k t^k \mathcal{K}_i\right) \zeta_\mu \hat{m}^\mu \,.$$

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- From the last eom, $\hat{m}^\mu = A\zeta^\mu + C^\mu + \mathcal{O}(\epsilon)$ with $C^\mu \zeta_{\mu i} = 0$ and so $N_{\rm flux} \geq \frac{1}{2}A^2\mathcal{K} + \mathcal{O}(\epsilon_i)\,.$
- We get an upper bound on the possible values of the complex structure saxions.

$$\boxed{\mathcal{K} < (N_{\rm flux})^{2p+1} d^{2-4p} (K_i^{(3)} t^i)^2}$$

Type IIB Limit

Type IIB setup

- Type IIB compactifications with background three-form fluxes can be understood as F-theory on $(C_3 \times \mathbb{T}^2)/\mathbb{Z}_2$, with C_3 a Calabi–Yau three-fold.
- Apply previous results by splitting the complex structure index as $i = \{0, a\}$, where T^0 is the complex structure of $\mathbb{T}^2$ and T^a, $a = 1, \ldots, h^{2,1}(C_3)$. We impose

$$\mathcal{K}_{0abc} = \kappa_{abc} \, .$$

- The basis $\{[\sigma_\mu]\}$ in the mirror four-fold $X_4 = (B_3 \times \mathbb{T}^2)/\mathbb{Z}_2$ can be constructed explicitly. We also get explicit expression for ζ^μ_{ij} and $\eta_{\mu\nu}$.

$$\vec{\rho} = \left(\tilde{\rho}, \tilde{\rho}^0, \tilde{\rho}^a, \hat{\rho}^a, \bar{\rho}'_a, \bar{\rho}_a, \bar{\rho}_0, \bar{\rho}\right)$$

Tadpole condition

- We start with the topological quantity and evaluate it at the vacuum.

$$N_{\text{flux}} = \bar{\rho}\tilde{\rho} - \bar{\rho}_i\tilde{\rho}^i + \bar{\rho}'_a\hat{\rho}^a \Rightarrow \frac{t^0\kappa}{6}\left(\tilde{\rho}^2 + \frac{(\tilde{\rho}^0)^2}{(t^0)^2} + \frac{2}{3}g^{\kappa}_{ab}\tilde{\rho}^a\tilde{\rho}^b + \frac{3}{2\kappa^2}g_{\kappa}^{ab}\rho'_a\rho'_b\right)$$

- In the large complex structure limits $\kappa, t^0 \to \infty$ we must set $\tilde{\rho} = 0$. Then $\tilde{\rho}^0 = m^0$, $\tilde{\rho}^a = m^a$.
- Consider a scaling of the form $t^0 \sim \kappa^r \to \infty$, with $r \in \mathbb{R}$.

	Divergence	Flux lattice	N_{flux}	
$r < 1$	$\kappa/t^0,\ t^0\kappa g^{\kappa}_{ab}$	$(0,0,0,0,0,\bar{e}_a,\bar{e}_0,\bar{e})$	0	Generically
		$(0,0,0,\hat{m}^a,m_a,\bar{e}_a,\bar{e}_0,\bar{e})$	$\sum_a \hat{m}^a m_a \neq 0$	With a s.t. $t^0 \sim \kappa g^{\kappa}_{aa}$
$r \geq 1$	$t^0\kappa g^{\kappa}_{ab},\ t^0 g_{\kappa}^{ab}/\kappa$	$(0,m^0,0,\hat{m}^a,0,\bar{e}_a,\bar{e}_0,\bar{e})$	$-m^0\bar{e}_0$	

Type IIB1

- Consider the case $\vec{q}^{\,t} = (0, 0, 0, \hat{m}^a, m_a, \bar{e}_a, \bar{e}_0, \bar{e})$.
- If we include the corrections and demand $S_{ab} \equiv \kappa_{abc}\hat{m}^c$ to be invertible and $S^{ab}m_a m_b \neq 0$, we are able to fix all moduli.
- The total tadpole $N_{\rm flux} = \sum_a m_a \hat{m}^a$ is a sum of positive terms and so it exceeds in value to $h^{2,1}(C_3)$.
- There is tension between tadpole cancellation and full moduli stabilisation for large amount of moduli[5][6].

[5] I. Bena, J. Blabäck, M. Graña and S. Lüst, *The Tadpole Problem*, 2010.10519

[6] P. Betzler and E. Plauschinn, *Type IIB flux vacua and tadpole cancellation*, Fortschritte der Physik 67.11 (2019): 1900065.

Type IIB2

- Consider the case $\vec{q}^{\,t} = (0, m^0, 0, \hat{m}^a, 0, \bar{e}_a, \bar{e}_0, \bar{e})$.
- It is dual, via mirror symmetry, to the type IIA non-supersymmetric Minkowski flux vacua[7].
- Adding the corrections, it can be solved explicitly to fix all moduli.[8]
- This is a counterexample to the Tadpole Conjecture: the flux contribution to the tadpole $N_{\mathrm{flux}} = -m^0 \bar{e}_0$ is independent of the number of complex structure moduli.
  There is no tension between full moduli stabilisation and having an N_{flux} that it is bounded.

[7] E. Palti, G. Tasinato and J. Ward, *WEAKLY-coupled IIA Flux Compactifications*, JHEP 06 (2008) 084 [0804.1248]

[8] D. Escobar, F. Marchesano and W. Staessens, *Type IIA flux vacua and α'-corrections*, JHEP 06 (2019) 129 [1812.08735].

Linear Scenario

Manifold and LCS limit

- Consider a four-fold Y_4 such that at least one complex structure saxion t_L only appears linearly on $\mathcal{K} = \frac{3}{2}e^{-K_{\rm cs}}$ and in the superpotential:

$$\mathcal{K} = 4\mathcal{K}_L t_L + f \,,$$

  with $\mathcal{K}_L \equiv \mathcal{K}_{Labc}t^a t^b t^c$, and $f \equiv f(t^a)$ independent of t_L.

- This kind of Kähler potential is found when the mirror four-fold X_4 is a smooth three-fold fibration over $\mathbb{P}^1$.

- Consider a limit $t_L \sim \mathcal{K}_L \to \infty$ and assume we realise the hierarchy $t_L \gg t^a$. Then $\mathcal{K} \to \infty$ and $\mathcal{K}g_{ab} \to \infty$ and we need $m = m^a = 0$.

- We still can take $m^L \neq 0$ since $\frac{\mathcal{K}}{6}g_{LL} \to \frac{1}{6}\frac{\mathcal{K}_L}{t_L}$. Then $N_{\rm flux} = -m^L \bar{e}_L$.

Tadpoles and vacua

- The leading-order vacua equations read

$$\left.\begin{aligned} \bar{\rho} &= 0 \\ \bar{e}_L + \frac{\mathcal{K}}{6} g_{LL} m^L &= 0 \\ \bar{\rho}_a - \varepsilon_a \bar{e}_L &= 0 \\ \hat{\rho}^a &= 0 \end{aligned}\right\} \qquad \begin{aligned} b^0 &= -\frac{1}{3e_L (m^L)^2}\left(\mathcal{K}_{Labc}\hat{m}^a\hat{m}^b\hat{m}^c - 3e_a\hat{m}^a m^L\right) - \bar{e} \\ b^a &= -\frac{\hat{m}^a}{m^L} \\ \varepsilon_a &= \frac{m^L \bar{e}_a - \frac{1}{2}\mathcal{K}_{Labc}\hat{m}^b\hat{m}^c}{m^L \bar{e}_L} \end{aligned}$$

where $\epsilon_a \equiv \partial_a \left(\frac{f}{4\mathcal{K}_L}\right)$.

- They suffice to find a set of vacua with full moduli fixing. In addition we get the following inequalities

$$N_{\rm flux}|\varepsilon_a| \gtrsim 1 \qquad \frac{\mathcal{K}}{6} g_{LL}|\varepsilon_a| \gtrsim N_{\rm flux}^{-2}\,.$$

- Corrections will also contribute but do not deform significantly the set of vacua equations. In some cases they are needed to understand the implications of the inequalities.

Realisation of the linear scenario

- We take the mirror manifold X_4 to be a triple fibration $\mathbb{T}^2 \to \mathbb{P}^1 \to \mathbb{P}^1 \to \mathbb{P}^1$.
- The intersection polynomial is given by

$$I(Y_4) = \left(8D_0^3 + D_0 D_1 D_2 + D_0 D_2^2 + 2D_0^2 D_1 + 3D_0^2 D_2\right) D_L + 6D_0^2 D_2 D_1 + 2D_0 D_2 D_1^2 + 2D_0 D_2^2 D_1 + 16D_0^3 D_1 + 2D_0 D_2^3 + 4D_0^2 D_1^2 + 6D_0^2 D_2^2 + 18D_0^3 D_2 + 52D_0^4 .$$

- To find vacua in the limit $t_L \sim \mathcal{K}_L \to \infty$ we set $m = m^a = 0$ in order not to violate the tadpole constraint.
- We focus on the overall rescaling

$$t^a = v^a \lambda , \qquad v^a \sim \mathcal{O}(1) , \qquad \lambda \to \infty ,$$

together with $t_L \sim \lambda^3 \to \infty$.

- We manage to fix the saxions and find that $N_{\text{flux}} |\varepsilon_a| \geq 1$ is trivially satisfied and $\frac{t_L}{\lambda^3} \lesssim N_{\text{flux}}$.

Conclusions

- We analysed flux potentials and their vacua for F-theory compactifications on smooth elliptically fibered Calabi–Yau four-folds.
- Using mirror symmetry, we provided an explicit bilinear expression for the scalar potential that allows for a systematic study of its vacua.
- We need to restrict the choice of fluxes in order not to violate tadpole cancellation parametrically in the LCS regime.
- The generic choice of fluxes compatible with the tadpole cancellation is too constrained and at least one saxionic direction necessarily remains flat.
- The correction $K_i^{(3)}$ generically stabilises all the complex structure fields.

Conclusions

- In the generic flux scenario saxion vevs are bounded from above by $|K^{(3)}|N_{\text{flux}}^{p+\frac{1}{2}}$.
- Reducing our general F-theory setup to type IIB, we connected with several existing results in the literature. [9]
- We found a second class of vacua arising for a different pattern of flux quanta when at least one of the complex structure fields only enters linearly in e^{-K} and the superpotential.
- For this flux choice only a pair of flux quanta contribute to the tadpole.
- In the linear scenario the full moduli stabilisation can be achieved provided the matrix Z^{AB} entering the scalar potential has enough off-diagonal components.

[9] J. J. Blanco-Pillado, K. Sousa, M. A. Urkiola and J. M. Wachter, *Towards a complete mass spectrum of type-IIB flux vacua at large complex structure*, JHEP 04 (2021) 149 [2007.10381].