Control Issues of KKLT

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Outline

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Introduction

dS vacua in string theory?

Long debated question: Is dS possible in string theory?

- Plausible and much studied scenarios such as KKLT, LVS
  But not fully explicit

Kachru, Kallosh, Linde, Trivedi 03
Balasubramanian, Berglund, Conlon, Quevedo 05

- Many no-go theorems in particular corners of string theory

Maldacena, Nuñez 00
Hertzberg, Kachru, Taylor, Tegmark 07
+ many more

No-dS conspiracy?

Brennan, Carta, Vafa 17
Danielsson, Van Riet 18

e.g. (refined) dS conjecture

Obied, Ooguri, Spodyneiko, Vafa 18
Ooguri, Palti, Shiu, Vafa 18
Andriot 18; Garg, Krishnan 18
+ many more

Can this be true?

→ Crucial to construct explicit models realizing the scenarios
or identify potential problems

Today: focus on earliest and most studied proposal, the KKLT scenario

KKLT scenario

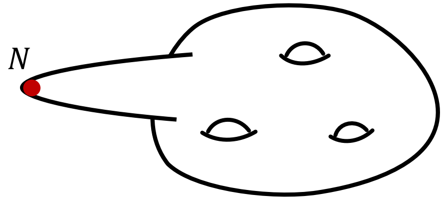
Proposal: meta-stable dS vacua in 3 steps:

Kachru, Kallosh, Linde, Trivedi 03

- IIB flux vacua with strongly warped throat
  modelled locally by Klebanov-Strassler solution

  Klebanov, Strassler 00
  Giddings, Kachru, Polchinski 01

  Fluxes K, M carry D3 charge $N = KM$
  localized at the tip



- Kähler modulus T stabilized by non-perturbative effects
  (E3 instanton or gaugino condensate on N_{c} D7 branes)

  SUSY AdS vacua with vacuum energy density

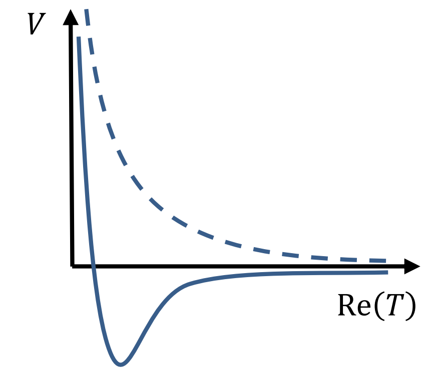
$$V_{\mathrm{AdS}} \sim -e^{-\mathrm{Re}(T)/N_{\mathrm{c}}}$$

  (up to non-exponential effects)

  In the following: set $N_{\mathrm{c}} = 1$ (comments on $N_{\mathrm{c}} \neq 1$ later)

KKLT scenario

- Uplift to dS by placing anti-D3 brane in the throat
  energy density redshifted due to strong warping



$$V_{\mathrm{uplift}} \sim e^{-K/g_s M}$$

Meta-stable if uplift energy is not too large:

$$V_{\mathrm{uplift}} \sim |V_{\mathrm{AdS}}| \qquad \leftrightarrow \qquad e^{-K/g_s M} \sim e^{-\mathrm{Re}(T)}$$

$$\boxed{\mathrm{Re}(T) \sim \frac{N}{g_s M^2}}$$

$g_s M \gtrsim 1$ (small curvature at KS tip), $M > 12$ (meta-stability)

Klebanov, Strassler 00
Kachru, Pearson, Verlinde 01

$g_s M^2 > (6.8)^2$ (conifold)

Bena, Dudas, Graña, Lüst 18
Blumenhagen, Kläwer, Schlechter 19
Bena, Buchel, Lüst 19; Dudas, Lüst 19
Randall 19

$\rightarrow$ Treat $g_s M^2 \gg 1$ as large parameter

Strong warping

Observation: Carta, Moritz, Westphal 19

For $g_s M^2 \gg 1$, strongly warped throat does not “fit” into weakly warped CY bulk

Possible threat of large singularities

But is this really a problem?

A priori, strong warping can be fine with supergravity approximation

→ Need to study warped geometry for $g_s M^2 \gg 1$

The singular-bulk problem

Constraint on the warp factor

IIB on (conformally) CY orientifold X with (string-frame) metric

Giddings, Kachru, Polchinski 01

$$ds_{10}^2 = h(y)^{-1/2}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + h(y)^{1/2}\tilde{g}_{mn}\mathrm{d}y^m\mathrm{d}y^n$$

warp factor

Ricci-flat, $\tilde{V}_X \equiv \int_X \mathrm{d}^6 y\sqrt{\tilde{g}} = 1$

Kähler modulus T is defined in terms of (Einstein-frame) 4-cycle volume wrapped by E3:

$$\mathrm{Re}(T) \sim S_{\mathrm{E3}} \sim \frac{N}{g_s M^2} \quad \leftrightarrow \quad \frac{1}{g_s}\int_\Sigma d^4\xi\sqrt{\tilde{g}}\, h \sim \frac{N}{g_s M^2} \qquad (2\pi\sqrt{\alpha'} = 1)$$

with $\int_\Sigma d^4\xi\sqrt{\tilde{g}} \gtrsim \kappa_{111}{}^{1/3}\tilde{V}_X^{2/3} \sim O(1)$ in our normalization

→ warp-factor average over Σ:

$$\boxed{\langle h\rangle_\Sigma \sim \frac{N}{M^2}} \qquad \langle h\rangle_\Sigma \equiv \frac{\int_\Sigma d^4\xi\sqrt{\tilde{g}}\, h}{\int_\Sigma d^4\xi\sqrt{\tilde{g}}}$$

Warp-factor variation

$\langle h \rangle_\Sigma \sim \frac{N}{M^2}$ implies a neighborhood on Σ where

$$h \lesssim \frac{N}{M^2}$$

Warp factor satisfies Poisson equation:

Giddings, Kachru, Polchinski 01
Giddings, Maharana 05

$$\tilde{\nabla}^2 h = -g_s \tilde{\rho}_{\mathrm{D3}}$$

D3-charge density

Variation of the warp factor due to D3 charge N at the KS tip:

$$\left|\widetilde{\partial h}\right| \sim g_s N \qquad \text{(at } O(1) \text{ distance in } \tilde{g}\text{)}$$

with $\left|\widetilde{\partial h}\right| \equiv \sqrt{(\partial_m h)(\partial_n h)\tilde{g}^{mn}}$

→ neighborhood on Σ with $\boxed{\frac{\left|\widetilde{\partial h}\right|}{h} \gtrsim g_s M^2 \gg 1}$

Singularity

Use $\frac{|\widetilde{\partial h}|}{h} \gtrsim g_s M^2$ with $h(y_0 + \delta y) \approx h(y_0) + \partial_m h(y_0)\, \delta y^m$

Recall $h = g_s N \times \text{"}O(1) \text{ function"}$

→ singularity $\boxed{h \leq 0}$ at $\left|\widetilde{\delta y}\right| \lesssim 1/g_s M^2 \ll 1$

Recap:

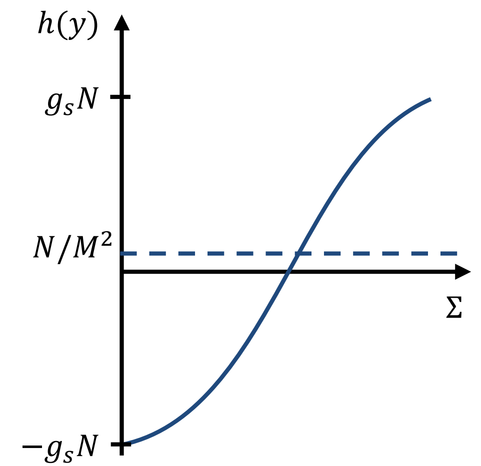
- Step 1 of the KKLT proposal requires a flux compactification with a volume modulus $\mathrm{Re}(T) \sim \frac{N}{g_s M^2}$ and a conifold region hosting a D3 charge N
- $\mathrm{Re}(T)$ is too small (relative to N) to ensure small curvature; instead, the D3 charge creates singularities in the bulk

Size of the singularity

How large is the singular region?

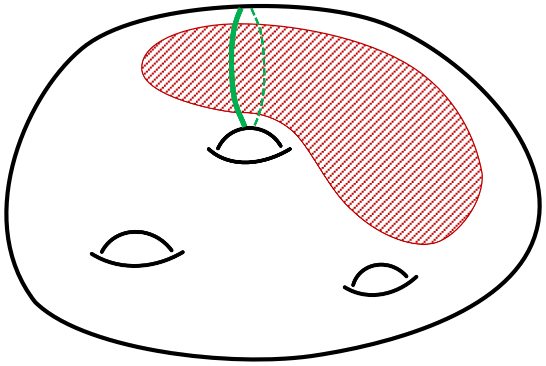
Variation of h much larger than its average $\langle h \rangle_\Sigma \sim \frac{N}{M^2}$

$\rightarrow h < 0$ on $O(1)$ fraction of E3 volume (in $\tilde{g}$)



Generically, it will then also spread over

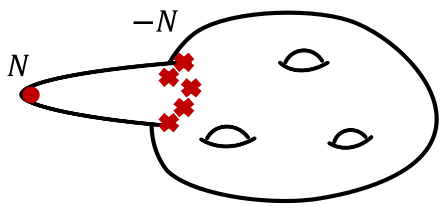
an $O(1)$ distance into the transverse space



Complementary argument: coarse-grained warp factor (see paper)

Escape routes

Escape routes?



- Special geometries avoiding our parametric estimates
  e.g. screen KS charge by special O-plane arrangement

- Large N_{c} helps:

  Carta, Moritz, Westphal 19

$$V_{\mathrm{AdS}} \sim -e^{-\mathrm{Re}(T)/N_{\mathrm{c}}} \qquad \leftrightarrow \qquad \frac{|\widetilde{\partial h}|}{h} \gtrsim \frac{g_s M^2}{N_{\mathrm{c}}}$$

  But: D7 tadpole constraints bound $N_{\mathrm{c}} < O(10)\, h^{1,1}$

  Louis, Rummel, Valandro, Westphal 12

- Variants of KKLT with $h^{1,1} \neq 1$

  All models suffer from a singular-bulk problem

  Possible exception:
  Parametrically large $h^{1,1} \gtrsim (g_s M^2)^{3/5} \gg 1$
  and $N_{\mathrm{c}} \sim O(h^{1,1})$ D7 stack on most 4-cycles (so $O((h^{1,1})^2)$ D7 branes in total)

  D7 tadpole?

Conclusions

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- Flux compactifications admitting a KKLT-like dS uplift generically have large singularities that extend over an $O(1)$ fraction of the Calabi-Yau
- The singularities arise because the charge N in the KS throat leads to a too large variation of the warp factor in the bulk
- Difficult to escape the conclusion
- LVS appears to avoid the problem. Could there be other hidden problems preventing explicit models?

Conclusions

- Flux compactifications admitting a KKLT-like dS uplift generically have large singularities that extend over an $O(1)$ fraction of the Calabi-Yau

- The singularities arise because the charge N in the KS throat leads to a too large variation of the warp factor in the bulk

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Thank you!

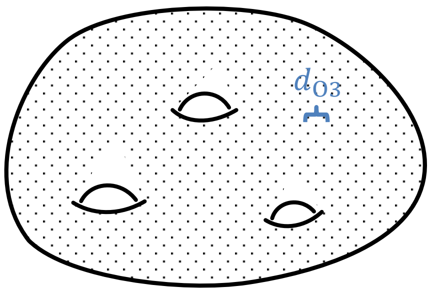
Size of the singularity

At large N, we can consider a coarse-grained warp factor $h_c(y)$

with $\quad d_{\text{O3}} \ll d \ll 1 \qquad$ (in $\tilde{g}$)

coarse-graining scale

avg. O3 distance

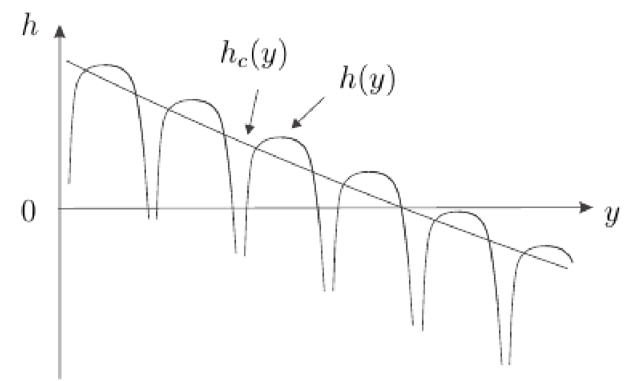


Coarse-grained D3-charge distribution:

Positively-charged lump of diameter d at the tip of the conifold,

uniform negative charge density

→ Negative "spikes" due to O-planes averaged away in h_c

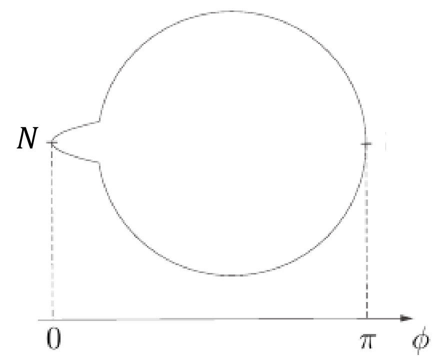


h_c singular by the same arguments as above

$$\frac{|\widetilde{\partial h_c}|}{h_c} \gg 1$$

Toy model

- Compact space: S^6 with polar angle $\phi \in (0, \pi)$
- Point-like source with charge N at $\phi = 0$ ("KS throat")
- Add uniform negative charge density to satisfy Gauss law ("O-planes")



Poisson equation $\tilde{\nabla}^2 h = -g_s \tilde{\rho}_{\mathrm{D3}}$ becomes:

$$\pi^3 [\sin^5 \phi \;\; h(\phi)']' = -g_s N \left(\delta(\phi) - \frac{15}{16} \sin^5 \phi \right)$$

Solution: $\quad h(\phi) = g_s N h_0(\phi) + \text{const.} \qquad h_0(\phi) \sim O(1)$

Fix constant by condition on h at "instanton" position $\phi = \phi_{\mathrm{E3}}$:

$$h(\phi_{\mathrm{E3}}) \sim \frac{N}{M^2}$$