Weak Gravity and Dualities

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[1909.01352] and [2006.06696] w/ G. Shiu and T. Noumi

Summer Series on String Phenomenology
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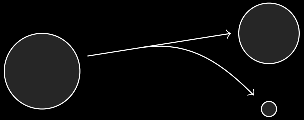
The plan

- Weak gravity conjecture
- IR/UV inputs & Einstein-Maxwell
- With massless scalar(s):
  - No symmetries
  - $\mathrm{SL}(2;\mathbb{R})$ and $\mathrm{O}(d,d;\mathbb{R})$
- Current thoughts

Weak gravity conjecture



[Arkani-Hamed,Motl,Nicolis,Vafa–07]
[Heidenreich,Reece,Rudelius–16]
[Andriolo,Junghans,Noumi,Shiu–18]
⋮

- $\exists$ state with $q \gtrsim m$
- Stronger forms (tower WGC, sublattice WGC, *light* state, …)
- Mild form: $Q \gtrsim M$ for black hole state
  Enough to show:

$$z \leq 1 \quad \xrightarrow[\text{corrections}]{\text{h.d.}} \quad z \leq 1 + \Delta z_{\text{ext}} \quad \text{with} \quad \Delta z_{\text{ext}} > 0$$

Under what assumptions can one show $\Delta z_{\text{ext}} > 0$?

IR/UV inputs

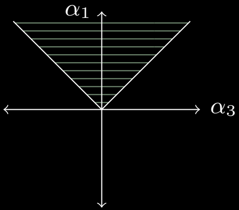
- Unitarity and causality [Hamada,Noumi,Shiu–18]
- RG effects in deep IR [Charles–19]
- Modular invariance [Montero,Shiu,Soler–16], [Aalsma,Cole,Shiu–19]
- Via holography [Montero–19]
- Connection to cosmic censorship [Horowitz,Santos–19]

Einstein-Maxwell

$$\mathcal{L}_{\text{h.d.}} = \frac{\alpha_1}{4}\big(F_{\mu\nu}F^{\mu\nu}\big)^2 + \frac{\alpha_2}{4}\big(F_{\mu\nu}\widetilde{F}^{\mu\nu}\big)^2 + \frac{\alpha_3}{2}F_{\mu\nu}F_{\rho\sigma}W^{\mu\nu\rho\sigma}$$

$$\Delta z_{\text{ext}} = \frac{1}{5(q^2+p^2)^3}\left[2\big(q^2-p^2\big)^2\alpha_1 + 8q^2p^2\alpha_2 - \big(q^4-p^4\big)\alpha_3\right]$$

- Necessary: $\alpha_2 \geq 0$, $2\alpha_1 \geq |\alpha_3|$



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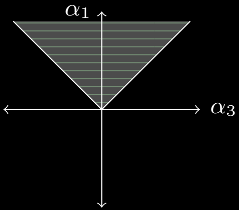
  $$\alpha_2 \geq 0, \quad 2\alpha_1 \pm \alpha_3 \geq 0$$



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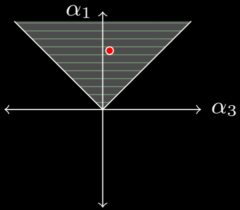

Massless scalars

Q: How does this story change when there are massless scalars?

Massless scalars

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A: The usual assumptions are not sufficient.
What additional ingredients are needed?

Symmetries/dualities are sufficient.

See July 28 talk of Stefano for a similar story
for the axionic weak gravity conjecture.

[Andriolo,Huang,Noumi,Ooguri,Shiu–20]

Einstein-Maxwell-dilaton

[GL,Noumi,Shiu–19]

$$\mathcal{L} = \frac{R}{2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}e^{-2\lambda\phi}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\text{h.d.}}$$

$$\begin{aligned}\mathcal{L}_{\text{h.d.}} = &\tfrac{\alpha_1}{4}e^{-6\lambda\phi}(F^2)^2 + \tfrac{\alpha_2}{4}e^{-6\lambda\phi}(F\widetilde{F})^2 + \tfrac{\alpha_3}{2}e^{-4\lambda\phi}(FFW) + \tfrac{\alpha_4}{2}e^{-2\lambda\phi}E^2 \\ &+ \tfrac{\alpha_5}{4}e^{-2\lambda\phi}(\partial\phi)^4 + \tfrac{\alpha_6}{4}e^{-4\lambda\phi}(\partial\phi)^2(F^2) + \tfrac{\alpha_7}{4}e^{-4\lambda\phi}(\partial\phi\partial\phi FF)\end{aligned}$$

Explicit solutions with $\alpha = 0$ only for special cases of λ, q, p.
e.g. $\lambda^2 = \frac{1}{2}$:

$$\mathrm{d}s^2 = -f(r)\,\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + (r+\kappa_1)(r+\kappa_2)\,\mathrm{d}\Omega_2^2$$

$$f(r) = \frac{r(r-r_+)}{(r+\kappa_1)(r+\kappa_2)} \qquad e^{-2\lambda\phi} = \frac{r+\kappa_1}{r+\kappa_2}$$

$$F = \frac{q}{(r+\kappa_1)^2}\,\mathrm{d}t\wedge\mathrm{d}r + p\sin\theta\,\mathrm{d}\theta\wedge\mathrm{d}\varphi$$

Computing corrections

Goal: $z \simeq \frac{Q}{M}$ at (corrected) extremality, i.e. $T = 0$.

[Reall,Santos–19]

$$\big(I_0[X] + I_{\mathrm{h.d}}[X] = \big) \quad I[X] = I[X(\alpha = 0)] + \mathcal{O}(\alpha^2)$$

$$\begin{aligned} G &= TI = M - TS - Q\Phi \\ \mathrm{d}G &= -S\,\mathrm{d}T - Q\,\mathrm{d}\Phi + \Psi\,\mathrm{d}P \\ \mathrm{d}M &= T\,\mathrm{d}S + \Phi\,\mathrm{d}Q + \Psi\,\mathrm{d}P \end{aligned}$$

$$S(T,\Phi,P) = -\left(\frac{\partial G}{\partial T}\right)_{\Phi,P}, \quad Q(T,\Phi,P) = -\left(\frac{\partial G}{\partial \Phi}\right)_{T,P}, \quad \cdots$$

Want: $M(T,Q,P)$ and then extremal limit $M(0,Q,P)$.

Computing corrections ($\lambda^2 = \frac{1}{2}$)

Example: $\alpha_2 (F\widetilde{F})^2$

$$G = \frac{1-\Phi^2}{2T} + \frac{P^2 T}{2(1-\Phi^2)} - \frac{64\pi^2\alpha_2 P^2\Phi^2 T^3\big[(1-\Phi^2)^2 - P^2T^2\big]^2}{5(1-\Phi^2)^9}$$

$$\downarrow$$

$$Q = \Big(\tfrac{\Phi}{T} - \tfrac{P^2\Phi T}{(1-\Phi^2)^2}\Big)\Big[1 + \tfrac{128\pi^2\alpha_2\Phi P^2 T^3}{5(1-\Phi^2)^6}\Big((1+4\Phi^2) - \tfrac{P^2T^2(1+8\Phi^2)}{(1-\Phi^2)^2}\Big)\Big]$$

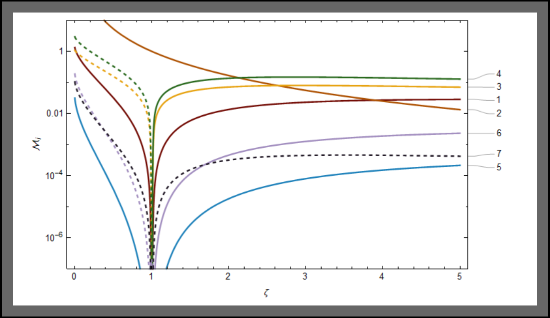
$$M = \tfrac{1}{T} - \tfrac{P^2\Phi^2 T}{(1-\Phi^2)^2} + \tfrac{128\pi^2\alpha_2\Phi^2 P^2 T^3}{5(1-\Phi^2)^6}\Big((2+3\Phi^2) - \tfrac{2P^2T^2(3+4\Phi^2)}{(1-\Phi^2)^2} + \tfrac{P^4T^4(4+5\Phi^2)}{(1-\Phi^2)^4}\Big)$$

$$\downarrow$$

$$\Phi = \big(1 - \tfrac{1}{2}PT + \cdots\big) - \tfrac{128\pi^2\alpha_2 Q}{5P^3}\big(1 - \tfrac{15}{4}QT + \cdots\big)$$

$$M = (Q+P)\big(1 + \tfrac{1}{8}QPT^2 + \cdots\big) - \tfrac{64\pi^2\alpha_2 Q^2}{P^3}\big(1 - \tfrac{5}{8}Q(3Q-2P)T^2 + \cdots\big)$$

Computing corrections ($\lambda^2 = \frac{1}{2}$)



$$\Delta z_{\text{ext}} = \frac{32\pi^2}{5QP}\alpha_i\mathcal{M}_i$$

e.g. $\mathcal{M}_2 = \frac{2Q^3}{P^2(Q+P)}$

Positivity bounds: $\alpha_1, \alpha_2, \alpha_5, \alpha_7 \geq 0$

$$\mathcal{L}_{\text{h.d.}} = \frac{\alpha_1}{4}e^{-6\lambda\phi}(F^2)^2 + \frac{\alpha_2}{4}e^{-6\lambda\phi}(F\widetilde{F})^2 + \frac{\alpha_3}{2}e^{-4\lambda\phi}(FFW) + \frac{\alpha_4}{2}e^{-2\lambda\phi}E^2$$
$$+ \frac{\alpha_5}{4}e^{-2\lambda\phi}(\partial\phi)^4 + \frac{\alpha_6}{4}e^{-4\lambda\phi}(\partial\phi)^2(F^2) + \frac{\alpha_7}{4}e^{-4\lambda\phi}(\partial\phi\partial\phi FF)$$

Computing corrections ($\lambda^2 = \frac{1}{2}$)

Favorite examples of (partial) UV completion are fine:

- Neutral scalars: $\alpha_{1,2,5,6} \sim \mathcal{O}(\frac{1}{m^2})$ ✓
- Charged: $\alpha_{1,2} \gg$ (others) exactly when we have control over $\alpha_{1,2} > 0$.
- Open-string theory: $\alpha_{1,2} \gg$ (others)

No ingredient so far seems to forbid having α_7 large, spoiling the WGC.

Other IR/UV inputs

Higher-derivative terms can respect symmetries of two-derivative action.

[GL,Noumi,Shiu–20]

- $\mathrm{SL}(2;\mathbb{R})$ w/ axion (S-duality)
- $\mathrm{O}(d,d;\mathbb{R})$ w/ 2-form (T-duality)

If symmetry is only approximate, still kept away from "problematic" region of parameter space.

$$\text{e.g.} \qquad \mathrm{SL}(2;\mathbb{R}) \;\to\; \mathrm{SL}(2;\mathbb{Z})$$

$\mathrm{SL}(2;\mathbb{R})$

$$R - \frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{2(\operatorname{Im}\tau)^2} - \operatorname{Im}\left(\tau F^-_{\mu\nu} F^{-\mu\nu}\right)$$

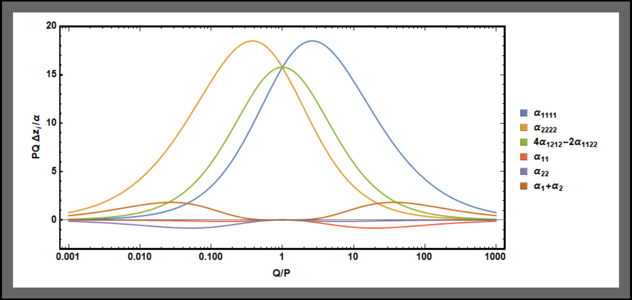
$$\tau = \theta + i e^{-2\phi}$$
$$F^\pm = \tfrac{1}{2}\big(F \pm i\widetilde{F}\big)$$

$$g_{\mu\nu} \to g_{\mu\nu} \qquad \tau \to \frac{a\tau + b}{c\tau + d} \qquad F^-_{\mu\nu} \to (c\tau + d) F^-_{\mu\nu}$$

Symmetry constrains h.d. terms:

$$\mathcal{L}_{\text{h.d.}} = (\operatorname{Im}\tau)^2 \alpha_{abcd} (F^-_{a\,\mu\nu} F_b^{-\mu\nu})(F^+_{c\,\rho\sigma} F_d^{+\rho\sigma}) + (\operatorname{Im}\tau)^{-1} \alpha_{ab} (\partial_\mu \tau \partial_\nu \bar{\tau}\, F_a^{-\mu\rho} F_b^{+\nu}{}_\rho)$$
$$+ (\operatorname{Im}\tau)^{-4} \Big[\alpha_1 (\partial_\mu \tau \partial^\mu \bar{\tau})^2 + \alpha_2 |\partial_\mu \tau \partial^\mu \tau|^2\Big] + \alpha_3 E^2$$

$SL(2;\mathbb{R})$ corrections



Positivity bounds:

$$\alpha_1 + \alpha_2 \geq 0 \qquad \alpha_{11}, \alpha_{22} \leq 0 \qquad \alpha_{1212} \geq 0$$

$$\alpha_{1111} - 2x^2\alpha_{1122} + x^4\alpha_{2222} \geq 0 \quad \forall x \in \mathbb{R}$$

$$\partial_\mu \tau \partial_\nu \bar{\tau} F^{-\mu\rho} F^{+\nu}{}_{\rho} \sim (\partial\phi)^2 F^2 - \partial\phi\partial\phi F F + \cdots$$

$\mathrm{O}(d, d; \mathbb{R})$

$$D: \qquad e^{-2\Phi}\Big(R+4\partial_{\hat\mu}\Phi\partial^{\hat\mu}\Phi-\tfrac{1}{12}H_{\hat\mu\hat\nu\hat\rho}H^{\hat\mu\hat\nu\hat\rho}\Big)$$

$$\Big\downarrow \text{torus}$$

$$d: \qquad e^{-2\phi}\Big(R+4\partial_\mu\phi\partial^\mu\phi-\tfrac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}+\tfrac{1}{8}\,\mathrm{Tr}(\eta\,\partial_\mu\mathcal{H}\,\eta\,\partial^\mu\mathcal{H})-\tfrac{1}{4}\mathcal{F}^M_{\mu\nu}\mathcal{H}_{MN}\mathcal{F}^{\mu\nu\,N}\Big)$$

$$\Big\downarrow \text{torus}$$

$$1: \qquad e^{-2\phi}\Big(-4\dot\phi^2-\tfrac{1}{8}\,\mathrm{Tr}(\eta\dot{\mathcal{H}}\eta\dot{\mathcal{H}})\Big)$$

$$\mathcal{H} = \begin{pmatrix} g_{ij} - B_{ik}g^{kl}B_{lj} & B_{ik}g^{kj} \\ -g^{ik}B_{kj} & g^{ij} \end{pmatrix} \in \mathrm{O}(d', d'; \mathbb{R})$$

$\mathcal{H}\eta\mathcal{H} = \eta$ with quadratic form $\eta = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}$

$$\mathcal{H} \to \Omega\mathcal{H}\Omega^{\mathrm{T}} \qquad\qquad \mathcal{F} \to \Omega\mathcal{F}$$

$\mathrm{O}(d,d;\mathbb{R})$

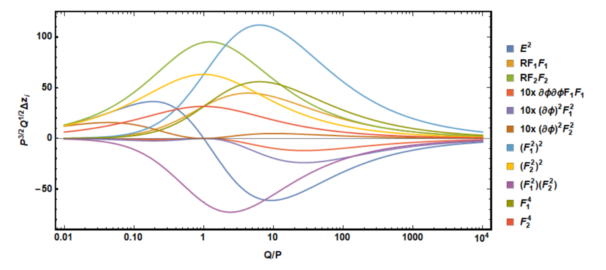
Two independent four-derivative terms respect this symmetry for all $D \to d$. [Eloy,Hohm,Samtleben–20]

$$\mathcal{L}_{\text{h.d.}} = \alpha e^{-2\phi}\Big[R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \tfrac{1}{2}\delta_{MN}(\mathcal{F}^M\mathcal{F}^N R) + \big(\tfrac{1}{8}\delta_{MP}\delta_{NQ} - \tfrac{1}{2}\delta_{MQ}\delta_{NP} + \tfrac{1}{8}\eta_{MP}\eta_{NQ}\big)(\mathcal{F}^M\mathcal{F}^N\mathcal{F}^P\mathcal{F}^Q) + \mathcal{O}(H^2)\Big]$$

$$+\beta e^{-2\phi}\Big[\tfrac{1}{4}\eta_{MN}(\mathcal{F}^M\mathcal{F}^N R) - \tfrac{1}{16}\eta_{MP}\delta_{NQ}(\mathcal{F}^M\cdot\mathcal{F}^N)(\mathcal{F}^P\cdot\mathcal{F}^Q) + \mathcal{O}(H)\Big]$$

Heterotic string has $(\alpha,\beta) = (\frac{\alpha'}{16}, -\frac{\alpha'}{8})$ (i.e. $2\alpha+\beta=0$)

$\mathrm{O}(d, d; \mathbb{R})$ corrections



$$\Delta z_{\text{ext}} = \frac{32\pi^2(2\alpha \pm \beta)}{5P(Q+P)}$$

Check: vanishing corrections for Heterotic $(2\alpha + \beta = 0)$

In fact, $\Delta z_{\text{ext}} \geq 0$ follows from the null energy condition alone:

$$T_{\mu\nu}k^\mu k^\nu = (2\alpha \pm \beta)\,\frac{8q^2 r(r - r_+)}{(r+\kappa_1)^8} \geq 0$$

Ongoing

Inspired by relationship between NEC and WGC, can we reframe the statement of weak gravity in terms of reasonable properties of the effective stress tensor?

For example, Einstein-Maxwell ($\mathrm{d}{\star}F = \mathrm{d}{\star}S$):

$$\binom{\text{EM}}{\text{WGC}} \quad \Longleftrightarrow \quad \int_{r>r_+} \mathrm{d}^{d-1}x\,\sqrt{\gamma}\,\big(\underbrace{F_{t\rho}S^{t\rho} + T^{(\text{h.d.})}{}_t{}^t}_{\delta T_t{}^t}\big)\big|_{z=1} \geq 0$$

Ongoing

More generally,

$$\Delta m \sim \int_\Sigma \mathrm{d}^{d-1}x \sqrt{\gamma}\, n^\mu \delta T_{\mu\nu}\, \xi^\nu \quad \overset{?}{\leq} 0$$

(Quantum) Penrose inequality:

[Bousso,Shahbazi-Moghaddam,Tomašević–19]

$$M \geq \frac{1}{2}\sqrt{\frac{A}{16\pi G_\mathrm{N}^2}} \qquad \longrightarrow \qquad M \overset{?}{\geq} \frac{1}{2}\sqrt{\frac{S_\mathrm{gen}}{4\pi G_\mathrm{N}}}$$

Recap

- Mild form of WGC: $\Delta z_{\text{ext}} \geq 0$
- Common IR/UV assumptions are sufficient to demonstrate for Einstein-Maxwell
- In principle not enough when additional massless scalars
- Adding assumption of (approximate) symmetry *is* enough
- Hints at connection with energy conditions

Thanks!