# Infinite Distance, Emergent Strings and Quantum Corrections in 4D $\mathcal{N} = 1$

DK, S.J. Lee, T. Weigand, M. Wiesner arXiv:2011.00024


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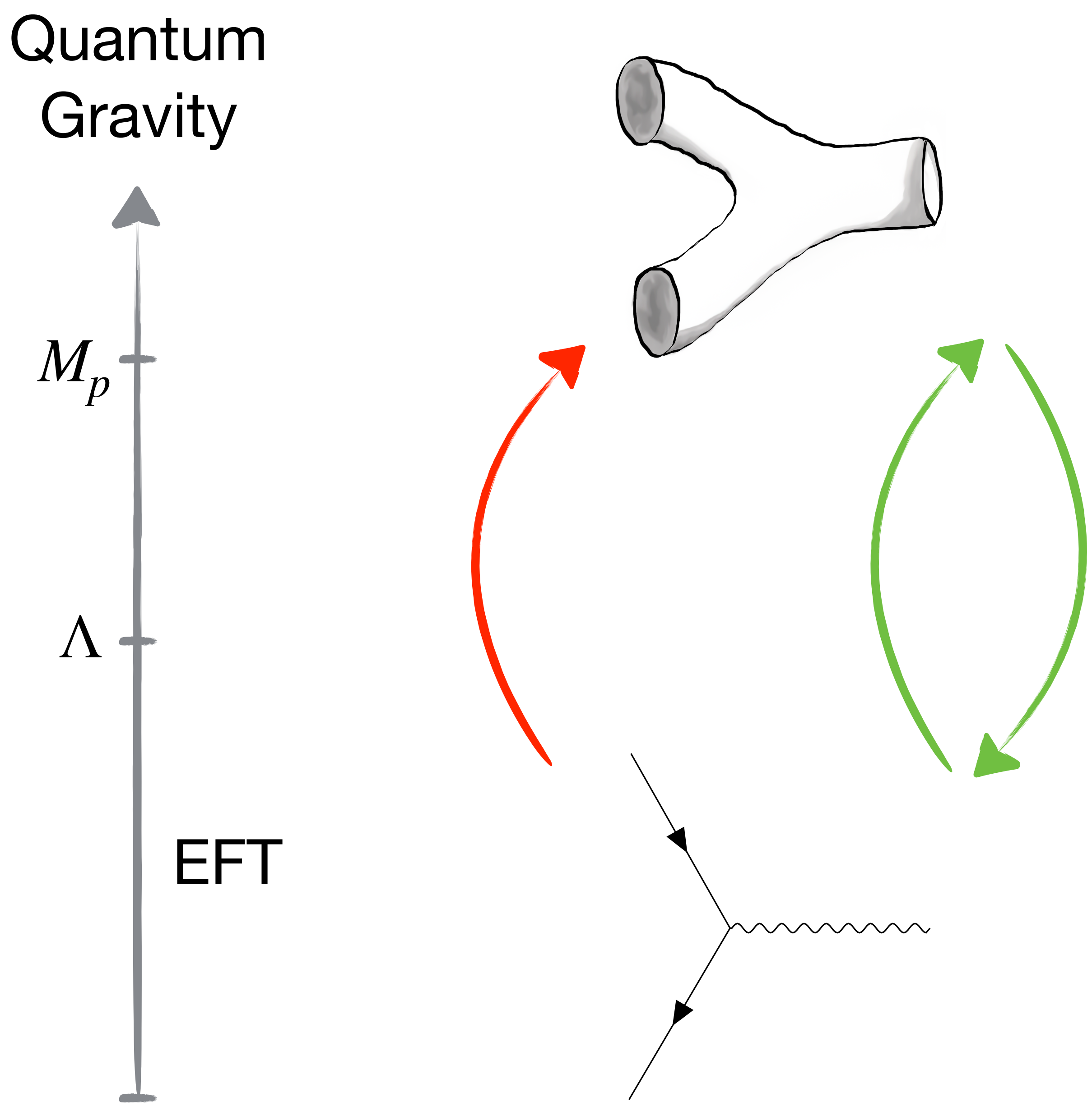
# Outline

- Swampland essentials
- Infinite distance limits in F-theory on $CY_4$
- Uniqueness of emergent string
- Quantum obstructions
- Weak Gravity Conjecture

# Swampland Essentials

# Swampland Program

Vafa ‘05
Review: Palti ‘19




**Landscape:**

EFT + quantum gravity =

**Swampland:**

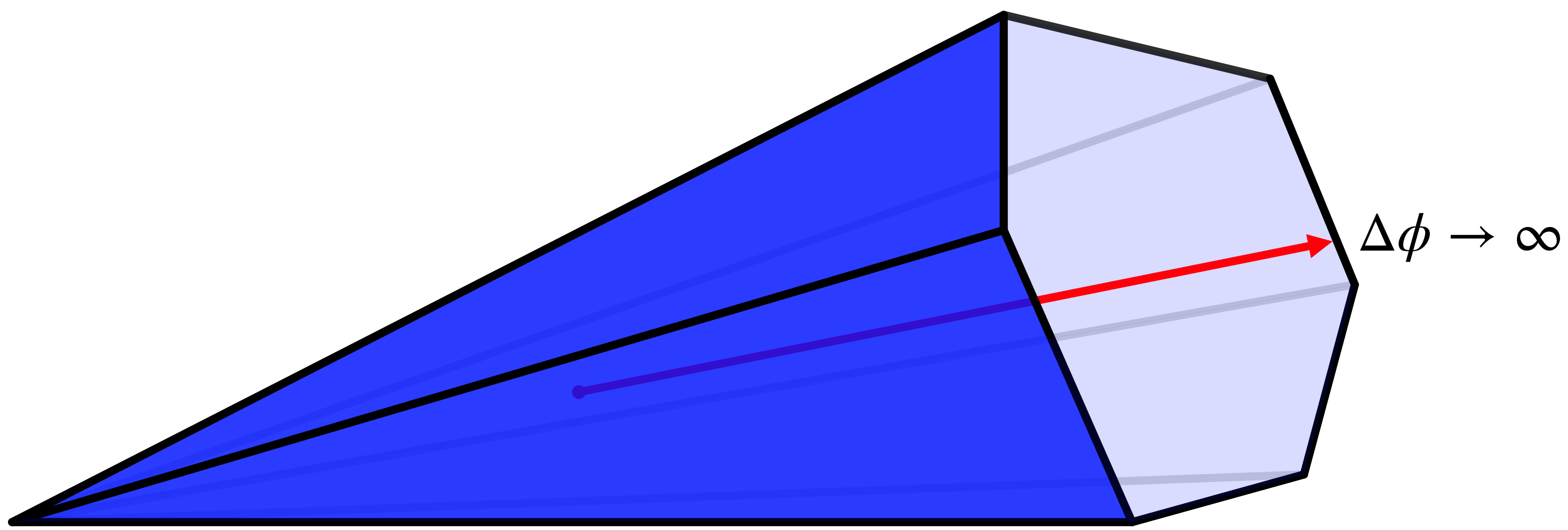
EFT + quantum gravity =

**Swampland conjectures:**

landscape vs. swampland

# Distance Conjecture

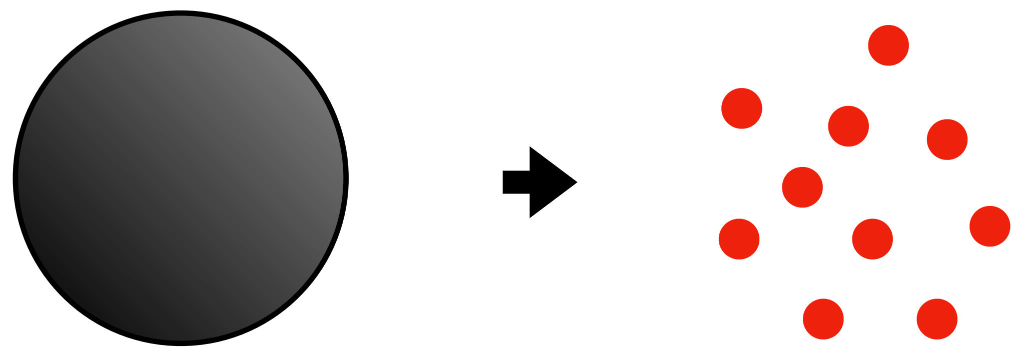
Ooguri, Vafa '06



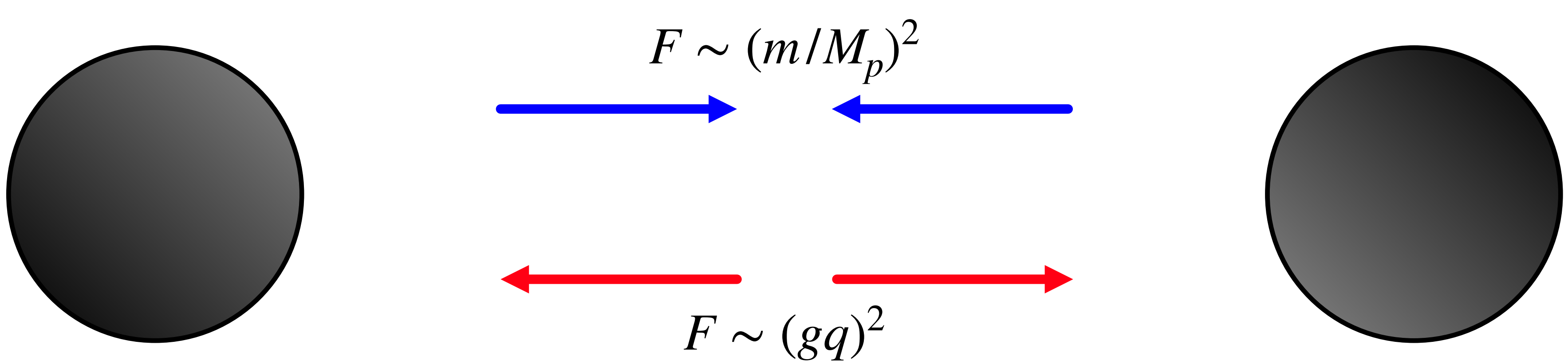

$\exists\,!$ **Infinite tower of states with** $m \sim e^{-\alpha\Delta\phi} M_p$

# Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06



Extremal BH should decay $\Rightarrow$ Need particle with $m \lesssim gqM_p$



# Weak Coupling At Infinite Distance

Heidenreich, Reece, Rudelius ’15, ’16, ’17, ‘18

DK, Palti ’16

Lee, Lerche, Weigand ’18, ’19

- Connection: if $g \sim e^{-\gamma\phi}$, sub-lattice WGC predicts objects with $m_n \lesssim q_n e^{-\alpha\phi} M_p$
- Both consequence of approaching $\infty$ distance loci in field space

**Tower**

Kaluza-Klein

$m_n \sim n\, m_0$

String

$m_n \sim \sqrt{n}\, m_0$

# Emergent String Conjecture

Lee, Lerche, Weigand ’19; DK, Lee, Weigand, Wiesner ‘20

Infinite distance limits in the moduli space of a consistent theory of quantum gravity fall into two classes:

A) Decompactification (KK tower dominant)

B) Emergent String (tensionless critical string dominant)

# Emergent String Conjecture

This has been checked in many settings:

F-Theory on $CY_3$ Kähler moduli: D3 on $\mathbb{P}^1$ — Lee, Lerche, Weigand '18

Type IIB on K3: D3 on $E_\tau$ — Lee, Lerche, Weigand '19

Type IIB on $CY_3$ hypermultiplet moduli: F1, D1, D3 on $E_\tau$ — Baume, Marchesano, Wiesner '19

IIA / M-Theory on $CY_3$ Kähler moduli: NS5 / M5 on $K3$ or $T^4$ — Lee, Lerche, Weigand '19

**F-Theory on $CY_4$ Kähler moduli: D3 on curve $C_0$** — Lee, Lerche, Weigand '19; DK, Lee, Weigand, Wiesner '20

# 4D N=1 F-Theory

# Infinite Distance Limits

# F-Theory Realisation

Lee, Lerche, Weigand '18, '19

- F-Theory on elliptic $E_\tau \to CY_4 \to B_3 \;\Rightarrow\;$ 4D $\mathcal{N} = 1$
- Interested in $h^{1,1}(B_3)$ Kähler moduli

$$J' = (v')^\alpha J_\alpha \quad \Rightarrow \quad \mathscr{V}_{B_3} = \frac{1}{3!}\int (J')^3 = \frac{1}{3!} k_{\alpha\beta\gamma}(v')^\alpha (v')^\beta (v')^\gamma$$

- Trivial $\infty$ distance limit: $J' = \mu\, J$ with $\mu \to \infty$

$\longrightarrow$ decompactification with $M_{KK}/M_s \sim \mu^{-1/2} \to 0$

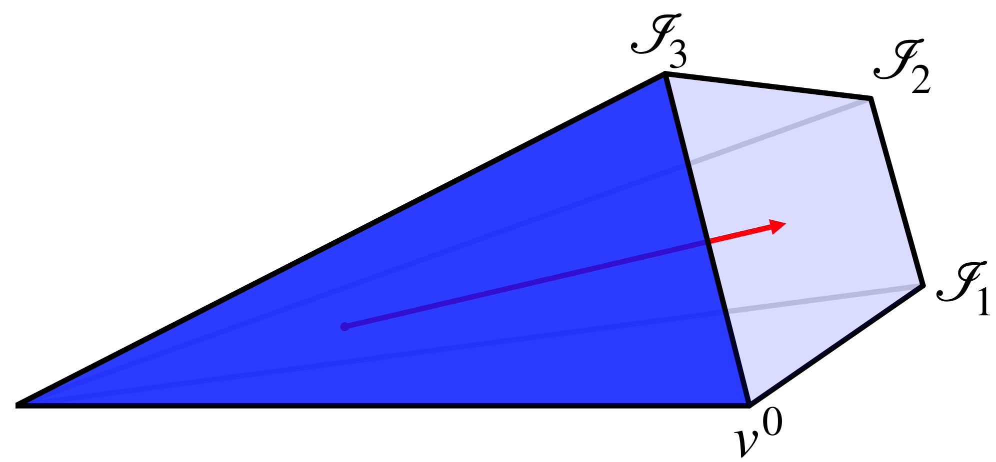
- If $J_0^3 = 0$ for some $J_0$ we may have non-trivial **finite volume limit**
- General limit will be superposition:

$$J' = \mu\left(\lambda J_0 + \sum_i v^i(\lambda)\, J_i\right) = \mu\, J$$

# F-Theory Realisation

Lee, Lerche, Weigand ’18, ‘19

$$J = \lambda J_0 + \sum_{\mu \in \mathcal{I}_1} v^\mu J_\mu + \sum_{\nu \in \mathcal{I}_2} v^\nu J_\nu + \sum_{r \in \mathcal{I}_3} v^r J_r$$




$J_0^3 = 0$

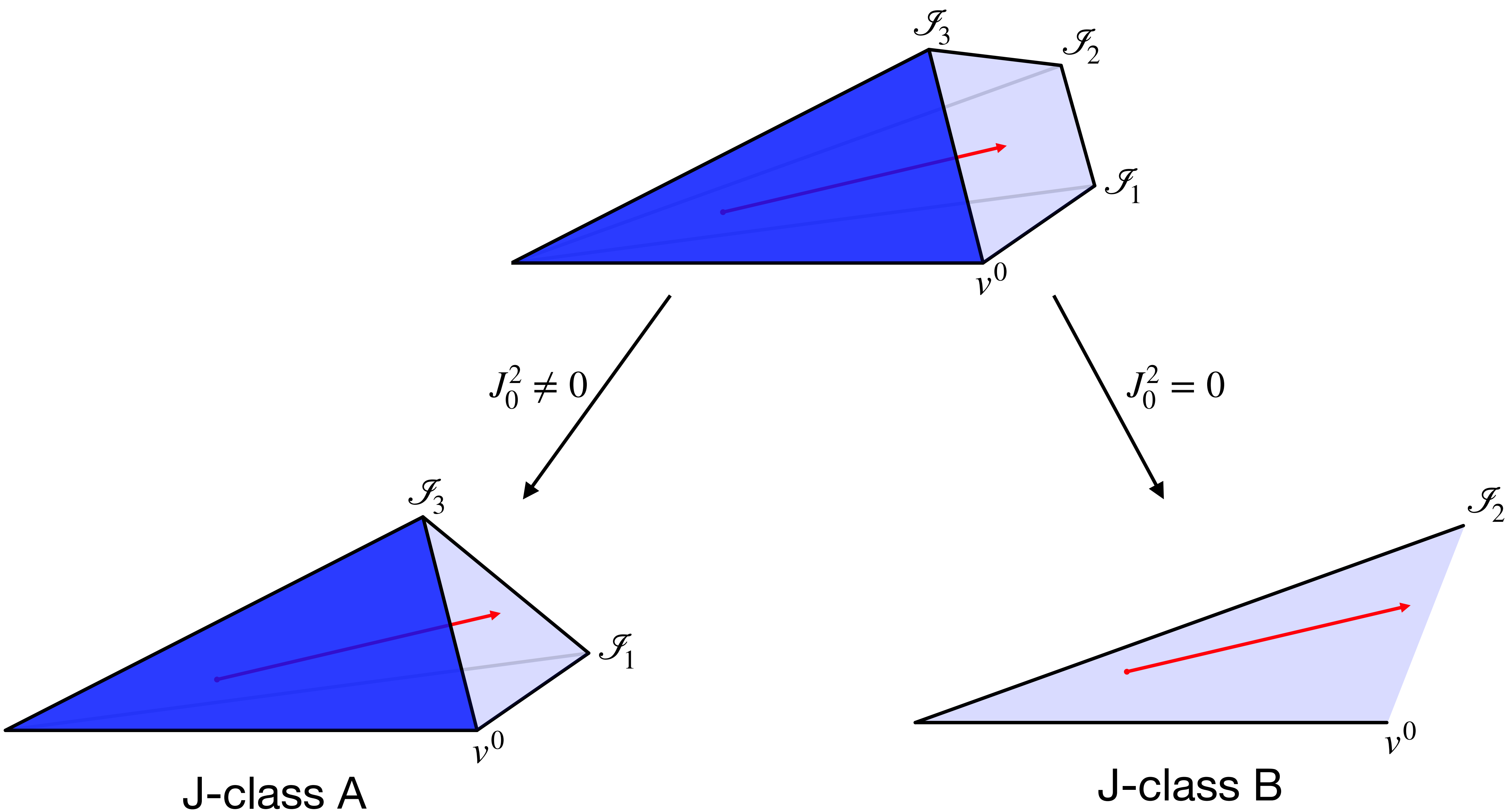
$J_0^2 \cdot J_\mu \neq 0 \qquad \forall \mu \in \mathcal{I}_1$

$J_0^2 \cdot J_\nu = 0 \qquad \forall \nu \in \mathcal{I}_2 \qquad$ and $\exists \nu' \in \mathcal{I}_2 : \; J_0 \cdot J_\nu \cdot J_{\nu'} \neq 0$

$J_0^2 \cdot J_r = 0 \qquad \forall r \in \mathcal{I}_3 \qquad$ and $\forall i \in \mathcal{I}_2 \cup \mathcal{I}_3 : \; J_0 \cdot J_r \cdot J_i = 0$

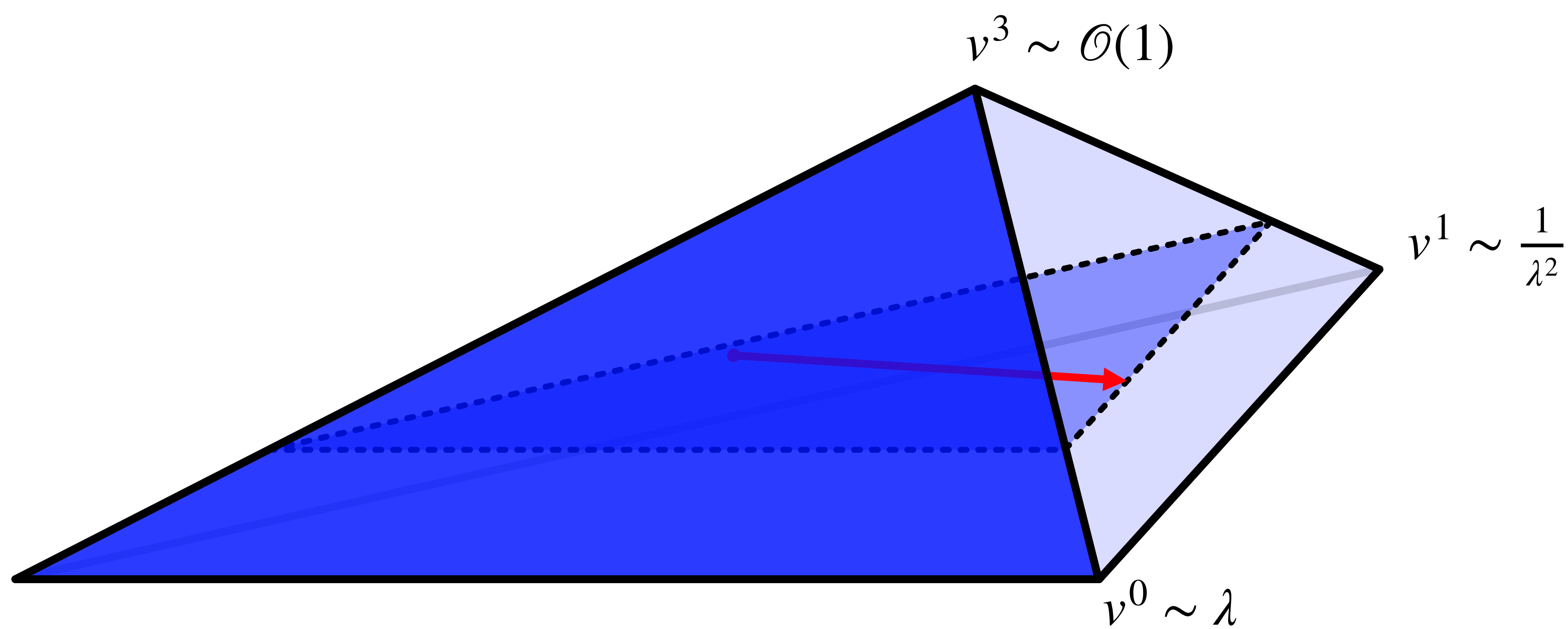
# J-class A and J-class B

Lee, Lerche, Weigand ’18, ‘19



# Focus: J-class A

$J_0^3 = 0$ $\quad J_0^2 \cdot J_1 \neq 0$ $\quad J_0^2 \cdot J_3 = J_0 \cdot J_3^2 = J_3^3 = 0$



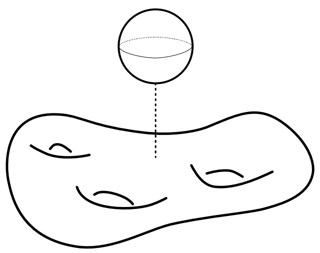

$$\mathscr{V}_{B_3} \sim J^3 \sim \lambda^0 J_0^2 \cdot J_1 + \mathcal{O}\left(\frac{1}{\lambda}\right)$$

# Emergent String

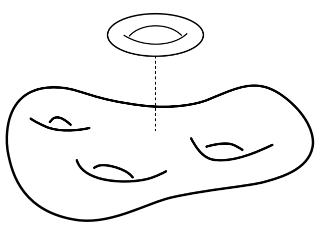
Lee, Lerche, Weigand '18, '19

- We have a distinguished curve class $C_0 = J_0 \cdot J_0$ with $\mathcal{V}_{C_0} \sim \frac{1}{\lambda^2} \to 0$
- Two options: $\bar{K}_{B_3} \cdot C_0 = 2 - 2g \geq 0$

1) $\bar{K}_{B_3} \cdot C_0 = 2 \quad \Rightarrow \quad \mathbb{P}^1 \to B_3 \to B_2$



2) $\bar{K}_{B_3} \cdot C_0 = 0 \quad \Rightarrow \quad T^2 \to B_3 \to B_2$



- Here we **focus on the first case**
- D3-brane wrapped on $C_0$ gives rise to emergent heterotic string

# Some Puzzles…

- Multiple emergent strings becoming light at the same rate?

e.g. $B_3 = \underbrace{\mathbb{P}^1}_{\lambda} \times \underbrace{\mathbb{P}^1}_{\frac{1}{\lambda^{1/2}}} \times \underbrace{\mathbb{P}^1}_{\frac{1}{\lambda^{1/2}}}$ (J-class B)

- Compute: $M^2_{\text{KK}}/M^2_{\text{het}} \sim \lambda \to \infty$

→ Naively, this limit decouples the KK tower and we are left with an intrinsically 4D N=1 heterotic string

# Uniqueness of Emergent String

# Uniqueness of Emergent String

DK, Lee, Weigand, Wiesner ‘20

Back to our example: $$B_3 = \underbrace{\mathbb{P}^1_A}_{\lambda} \times \underbrace{\mathbb{P}^1_B}_{\frac{1}{\lambda^{1/2}}} \times \underbrace{\mathbb{P}^1_C}_{\frac{1}{\lambda^{1/2}}}$$

- We have: $$M_{KK}^2 \sim \frac{M_s^2}{\mathcal{V}_{\mathbb{P}^1_A}} \sim \frac{M_s^2}{\lambda} \qquad T_{\text{str,A}} \sim T_{\text{str,B}} \sim M_s^2 \cdot \mathcal{V}_{\mathbb{P}^1_A} \sim \frac{M_s^2}{\lambda^{1/2}}$$
- Clearly the **KK tower dominates** and we never enter the heterotic duality frame.
- We prove that this is the case in general.

# Uniqueness of Emergent String

DK, Lee, Weigand, Wiesner ‘20

Sketch of Proof (J-class A):

- Suppose $B_3$ admits another rational or genus-one fibration with generic fiber $\tilde{C}_0$ that shrinks in the limit.

  $\Rightarrow$ $\exists$ nef divisor $\tilde{D} \in \overline{\mathscr{K}(B_3)}$ such that $\tilde{D}^2 = \tilde{n}\, \tilde{C}_0$ and $\tilde{D}^3 = 0$.

- We can expand $\tilde{D} = p^0 J_0 + \sum_{\mu \in \mathscr{I}_1} p^\mu J_\mu + \sum_{\mu \in \mathscr{I}_3} p^r J_r$

- We show that at least one $p^{\mu_0} > 0$ and $p^0 = p^r = 0$

- Now one can show $\mathscr{V}'_{\tilde{D}^2} = J' \cdot \tilde{D}^2 \to 0$ implies $k_{00\mu_0} = 0 \;\; \Rightarrow$ **contradiction**

# Quantum Obstructions

# Quantum Corrections

Shrinking divisor volumes ⇒ Corrections to the effective action, e.g.

1) Perturbative $\alpha'$-corrections to the Kähler potential

2) Non-perturbative (instanton) corrections to the superpotential

The superpotential 2) can receive corrections from D3-branes wrapping vertical divisors of the fibration of $B_3$. Intuitively, at least for the shrinking $\mathbb{P}^1$ limit, we nevertheless expect that the heterotic superpotential vanishes.

$$W_{\text{np}} \sim e^{-S_{\text{het}}} \to 0$$

We argue that the superpotential indeed vanishes in all limits which are not already obstructed by 1).

⇒ study perturbative $\alpha'$-corrections

# The $\alpha'$-Obstruction

Grimm, Keitel, Mayer, Pugh, Savelli, Weissenbacher 2013–2019

- Higher derivative corrections to M-theory lead to corrections to the 4D F-theory Kähler potential and coordinates

$$e^{K/2} = \mathcal{V}_{B_3} = \mathcal{V}^0_{B_3} + \alpha^2 \left( (\tilde{\kappa}_1 + \tilde{\kappa}_2)\mathcal{Z} + \tilde{\kappa}_2 \mathcal{T} \right)$$

$$T_\alpha = \frac{K_\alpha}{2} + \alpha^2 \left( (\kappa_3 + \kappa_5)\frac{K_\alpha \mathcal{Z}}{2\mathcal{V}^0_{B_3}} + \kappa_5 \frac{K_\alpha \mathcal{T}}{2\mathcal{V}^0_{B_3}} + \kappa_4 \mathcal{Z}_\alpha \log(\mathcal{V}_{B_3}) + \kappa_6 \mathcal{T}_\alpha + \kappa_7 \mathcal{Z}_\alpha \right)$$

$$\mathcal{Z}_\alpha = \int_{Y_4} c_3(Y_4) \wedge \pi^*(J_\alpha) \,, \qquad \mathcal{Z} = v^\alpha \mathcal{Z}_\alpha$$

$$\mathcal{T}_\alpha \overset{*}{=} -18(1+\alpha_2)\frac{1}{T^0_\alpha} \int_{D_\alpha} c_1(B_3) \wedge J \int_{D_\alpha} J_\alpha \wedge J \,, \qquad \mathcal{T} = v^\alpha \mathcal{T}_\alpha$$

*smooth Weierstrass model

# The $\alpha'$-Obstruction

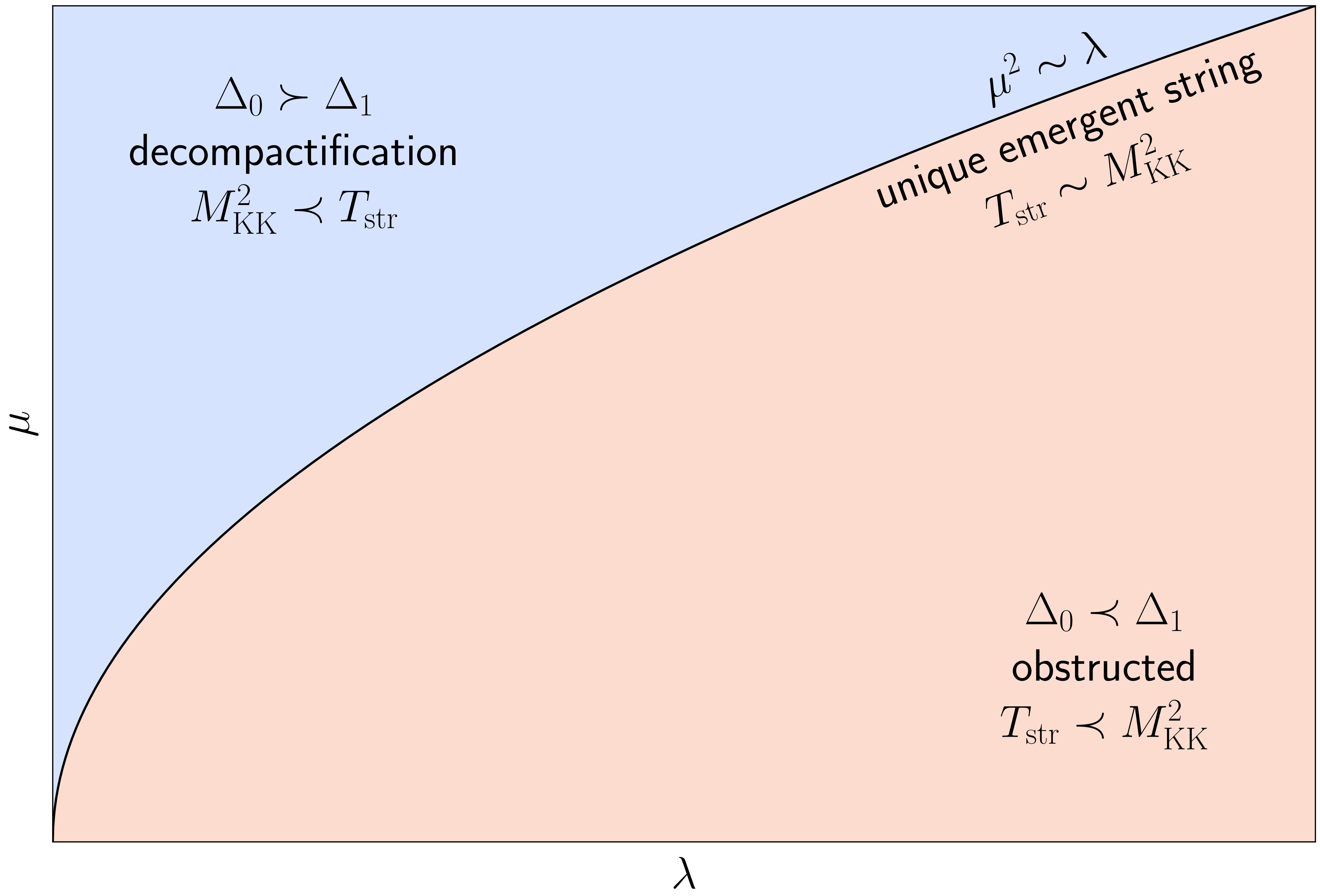
$$e^{K/2} = \mathcal{V}_{B_3} = \mathcal{V}_{B_3}^0 + \alpha^2 \left( (\tilde{\kappa}_1 + \tilde{\kappa}_2)\mathcal{Z} + \tilde{\kappa}_2 \mathcal{T} \right)$$

- Perturbative control: $\frac{\mathcal{Z}}{\mathcal{V}_{B_3}^0} \ll 1$ and $\frac{\mathcal{T}}{\mathcal{V}_{B_3}^0} \ll 1$
- In our limit $J' = \mu\,(\lambda J_0 + \frac{a}{\lambda^2} J_1 + b J_3)$ ( $\mu = 1$ is the finite volume limit )

$$\Rightarrow \quad \frac{\mathcal{Z}}{\mathcal{V}_{B_3}^0} \sim \frac{\mu\lambda\mathcal{Z}_0 + \dots}{\mu^3} \sim \frac{\lambda}{\mu^2}\mathcal{Z}_0$$

- The finite volume limit is not under control! $\mu \sim \lambda^{1/2}$ is marginally OK
- For this limit, the heterotic string and KK scale coincide parametrically

$$M_{\mathrm{KK}}/M_{\mathrm{het}} \to \mathrm{const}\,.$$



$\Delta_0 \succ \Delta_1$
decompactification
$M_{\rm KK}^2 \prec T_{\rm str}$
$\mu^2 \sim \lambda$
unique emergent string
$T_{\rm str} \sim M_{\rm KK}^2$
$\Delta_0 \prec \Delta_1$
obstructed
$T_{\rm str} \prec M_{\rm KK}^2$
$\mu$
$\lambda$

# Weak Gravity Conjecture

# Emergent String and WGC

Lee, Lerche, Weigand '18, '19

- Any divisor $\mathbf{S}$ with $\mathbf{S} \cdot C_0 \equiv 2m \neq 0$ grows in the limit as $\mathcal{V}_{\mathbf{S}} \sim \mu^2 \lambda^2$
- If $\mathbf{S}$ is wrapped by a 7-brane: weak coupling limit for gauge theory
- In order to prove the WGC we need two ingredients:

$$\underbrace{\mathcal{V}_{C_0}}_{M^2_{\text{het}}} \cdot \underbrace{\mathcal{V}_{\mathbf{S}}}_{\frac{1}{g^2_{\text{YM}}}} = 2m \underbrace{\mathcal{V}_{B_3}}_{M^2_p}$$

classical geometry of the limit

+

$\exists$ tower of states with

$$q_k^2 \geq 4mn_k$$

elliptic genus

↓

$$M_k \geq g_{YM}\, q_k\, M_{pl}$$

# WGC States from the Elliptic Genus

Lee, Lerche, Weigand ’18, ’19; DK, Lee, Weigand, Wiesner ‘20

- A tower of WGC states has already been identified in Lee, Lerche, Weigand ’18, ’19
- The relevant states can be read off from the elliptic genus.
- In 6D the elliptic genus is a meromorphic Jacobi-form. This property was used in Lee, Lerche, Weigand ’18 to prove sub-lattice WGC for F-theory on $CY_3$.
- In 4D the elliptic genus is generally not a Jacobi-form, so the result does not carry over directly. As a result, Lee, Lerche, Weigand ’18 were only able to identify a tower as opposed to a sub-lattice of states generically.
- Building on recent results Lee, Lerche, Weigand ’20 on the decomposition of the 4D elliptic genus in terms of Jacobi-form building blocks we are now able to show that the **sub-lattice WGC holds for generic $U(1)$ fluxes**! In the non-generic case, the sub-lattice may be shifted.

# $\alpha'$-Corrections and the WGC

- Another subtlety in 4D comes from $\alpha'$-corrections to the relation $\underbrace{\mathscr{V}_{C_0}}_{M^2_{\text{het}}} \cdot \underbrace{\mathscr{V}_{\mathbf{S}}}_{\frac{1}{g^2_{\text{YM}}}} = 2m \underbrace{\mathscr{V}_{B_3}}_{M^2_p}$
- We find $\dfrac{\mathscr{V}_{\mathbf{S}} \cdot \mathscr{V}_{C_0}}{2m\mathscr{V}_{B_3}} = 1 + \Delta_0 + \Delta_1 + \ldots,$

  where $\Delta_0$ represents corrections from the classical geometry away from the limit and $\Delta_1$ are the leading $\alpha'$-corrections. They can be mapped to heterotic threshold corrections.
- Before including $\alpha'$-corrections the WGC tower satisfies the repulsive force condition

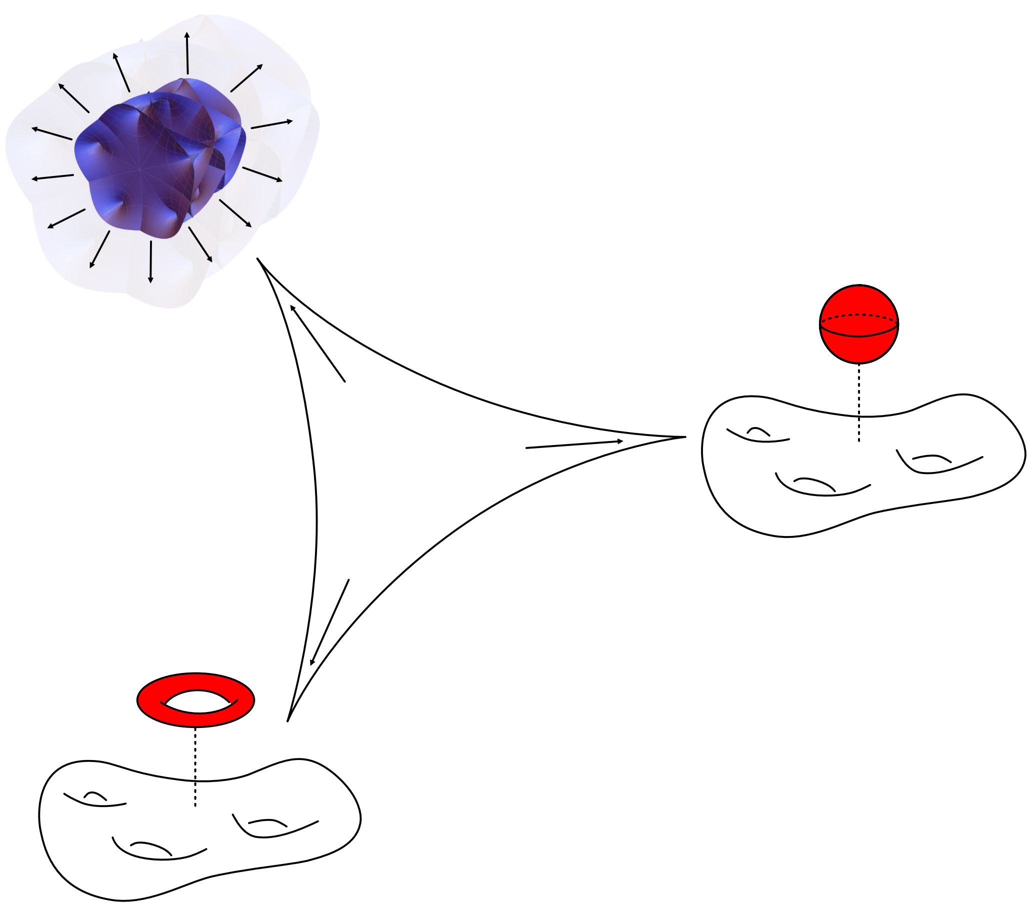
$$|F_{\text{Coulomb}}| \geq |F_{\text{Grav}}| + |F_{\text{Yuk}}| \qquad \Rightarrow \qquad g^2_{YM}\, q_k^2 \geq \frac{M_k^2}{M_{pl}^2}\left(\frac{d-3}{d-2} + \frac{1}{4}\frac{M_{pl}^4}{M_k^4} g^{rs}\partial_r\left(\frac{M_k^2}{M_{pl}^2}\right)\partial_s\left(\frac{M_k^2}{M_{pl}^2}\right)\right)$$

- If we demand that this still holds after including the $\alpha'$-corrections, this implies that the masses $M_k$ are also corrected.

As a result, we conclude that the WGC relation is modified:

$$\frac{g^2_{YM} q_k^2}{M_k^2} \geq \frac{1}{M_{pl}^2}\left(1 - \tfrac{1}{2}(\Delta_0 + \Delta_1)\right)$$

# Summary



- Infinite distance limits in the Kähler moduli space of F-Theory on elliptic $CY_4$ can be classified by the dominant tower of states that becomes light:

1. Excitations of unique heterotic string
2. Excitations of unique type II string
3. Kaluza-Klein tower

- In a regime of controlled $\alpha'$-corrections, the KK tower can never be decoupled.
- Generically, we can show that the excitations of the heterotic string furnish a sub-lattice WGC tower. We analyse $\alpha'$-corrections to the WGC relation.

# Thank You!