Automatic Enhancement in 6D Supergravity and F-theory Models

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Introduction

- 6D $\mathcal{N} = 1$ supergravity provides a rich arena in which to explore the relationship between possible low-energy theories of quantum gravity and their UV completions in string theory
  - Largest spacetime dimension in which matter representations other than the adjoint appear
  - Still very manageable due to strong constraints from anomaly cancellation
- F-theory is a powerful nonperturbative tool for exploring the global moduli space of 6D $\mathcal{N} = 1$ supergravity
- We are interested in investigating the apparent swampland of low-energy 6D supergravity theories that satisfy all known quantum consistency conditions yet have no known F-theory construction
  - The apparent swampland contains quite a few models that we would like to rule out if possible
- Here, we formulate a conjecture that explains the lack of F-theory constructions for a large class of swampland theories via automatic enhancement

Outline

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Statement

Automatic Enhancement Conjecture

An anomaly-consistent 6D supergravity theory with gauge group G and massless matter content M *cannot* be realized in F-theory if there exists another anomaly-consistent theory with gauge group $G' \supset G$ and matter content $M' \supseteq M$ that *is* realized in F-theory but cannot be broken through a supersymmetry-preserving Higgsing process to the theory with gauge group G.

$$\begin{array}{c} G', M' \in \text{F-theory} \\ \big\downarrow\!\!\!\!\times \\ G, M \quad \notin \text{F-theory} \end{array}$$

Equivalently, trying to tune the gauge group and matter content G, M in an F-theory Weierstrass model leads to an automatic enhancement of the singularity types to a theory with gauge group and matter content G', M'.

Remarks

- The theory G', M' contains G, M in the sense that all other features are the same, i.e., the string charge lattice and positivity cone must be the same, and the anomaly coefficients of both models must match
- In practice, automatic enhancements typically occur when the required choice of divisor classes causes the elliptic fibration to take on some special structure (ineffective or trivial divisor classes, or classes that only admit reducible sections)
- Specific models often admit a stronger formulation of the Automatic Enhancement Conjecture that depends on the gauge algebra, stated in the form of a positivity/effectiveness condition, e.g.,
  - $\mathfrak{su}(2)$ with $-3a - b < 0 \Rightarrow$ at least additional $\mathfrak{su}(2)$ with AC coefficient $b' = -4a - b$

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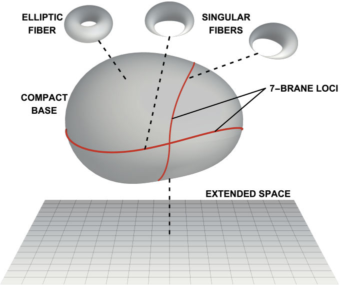
6D $\mathcal{N} = 1$ SUGRA

Data:

- Gauge group G
- Fixed number of tensor multiplets T
- Hypermultiplets M
- Anomaly coefficients $a, b_\kappa, \tilde{b}_{ij} \in \Lambda$ associated with terms $B \wedge R \wedge R$, $B \wedge F_\kappa \wedge F_\kappa$, $B \wedge F_i \wedge F_j$, where F_κ, F_i are nonabelian and abelian field strengths

F-theory overview

- Elliptically fibered Calabi–Yau n-fold X:
  - Torus over each point in base B, $\pi\colon X \to B$
  - Has a section, $\sigma\colon B \to X$ s.t. $\pi\sigma = \mathrm{Id}_B$
  - Complex structure τ encodes Type IIB axiodilaton



[modified from Raghuram]

- Described by Weierstrass model: hypersurface

$$y^2 = x^3 + fxz^4 + gz^6$$

in ambient $\mathbb{P}^{2,3,1}_{[x:y:z]}$, where f, g are sections of $-4K_B, -6K_B$

- Fiber singularities:
  - Codimension one (7-branes) $\longleftrightarrow$ nonabelian gauge algebras
  - Codimension two $\longleftrightarrow$ massless matter

Geometric dictionary

6D SUGRA	F-theory
nonabelian G_κ	codim.-one singularities
$\mathrm{U}(1)_i$	generating sections
M	codim.-two singularities
T	$h^{1,1}(B) - 1$
Λ	$H_{1,1}(B, \mathbb{Z})$
positivity cone	effective cone
$a, b_\kappa, \tilde{b}_{ij}$	$K_B, C_\kappa, \tilde{b}_{ij}$

Consistency conditions

6D SUGRA:

- Local and global anomaly cancellation
- $-a, b_\kappa, \tilde{b}_{ii} \in \Lambda$ lie in positivity cone
- Λ is a unimodular lattice [Seiberg, Taylor '11]
- $a \cdot b_\kappa + b_\kappa \cdot b_\kappa \in 2\mathbb{Z}$, $\tilde{b}_{ii} \in 2\mathbb{Z}$, ... [Monnier, Moore, Park '18]

F-theory:

- $a \cdot x + x \cdot x \in 2\mathbb{Z}$ for all $x \in \Lambda$
- $-12a - \sum_\kappa \nu_\kappa b_\kappa \geq 0$ [Kumar, Morrison, Taylor '10]
- Charge completeness [Morrison, Taylor ('21)] (proven for quantum gravity theories with a holographic dual [Harlow, Ooguri '18])
- Further bounds on spectra, number of $\mathrm{U}(1)$ factors, etc. [Kim, Shiu, Vafa '19; Lee, Weigand '19]

Outline

Non-Higgsable clusters

- When there exists irreducible, effective $C \in H_{1,1}(B, \mathbb{Z})$ with $C \cdot C < -2$ and $g = 0$, any F-theory construction over B will have a gauge algebra of at least $\mathfrak{su}(3)$ supported on C
- In 6D SUGRA, there is no obvious inconsistency to a primitive, positive $b \in \Lambda$ with $b \cdot b < -2$ and $b \cdot a + b \cdot b = -2$ with no associated gauge algebra
- Simple examples of the Automatic Enhancement Conjecture with, e.g., $G = 1$ and $G' = \mathrm{SU}(3)$ in the case $b \cdot b = -3$
    - In this case, the Automatic Enhancement Conjecture is stated purely locally
    - Complete proof of the Automatic Enhancement Conjecture in F-theory for these cases

$\mathfrak{su}(2)$ enhancements

Consider the Weierstrass model [Morrison, Taylor '12] given by

$$f = -\frac{1}{48}\phi^2 + \sigma f_1\,,$$
$$g = \frac{1}{864}\phi^3 - \frac{1}{12}\phi\sigma f_1 + \sigma^2 g_2\,,$$

with discriminant

$$\Delta = \frac{1}{16}\sigma^2\phi^2\big(\phi g_2 - f_1^2\big) + O(\sigma^3)\,.$$

This model has an $\mathfrak{su}(2)$ algebra supported on $\sigma = 0$. When b is sufficiently large, $[g_2] = -6a - 2b$ becomes trivial or ineffective, leading to automatic enhancement.

$\mathfrak{su}(2)$ enhancements

Over $B = \mathbb{P}^2$, $1 \leq b \leq 12$:

- $b = 10, 11$:
  - $[g_2] < 0$ and so we must set $g_2 = 0$, giving $\Delta = \frac{1}{16}\sigma^2 f_1^2 \left(64\sigma f_1 - \phi^2\right)$. Thus, there is a new $\mathfrak{su}(2)$ on $f_1 = 0$
  - New torsional section, $[\hat{x} : \hat{y} : \hat{z}] = \left[\frac{\phi}{12} : 0 : 1\right]$
  - Automatic enhancement to $G' = (\mathrm{SU}(2) \times \mathrm{SU}(2))/\mathbb{Z}_2$
- $b = 9$:
  - $[g_2] = 0$ and so g_2 is a constant and we can write $g_2 = \gamma^2$
  - New generating section, $[\hat{x} : \hat{y} : \hat{z}] = \left[\frac{\phi}{12} : \gamma\sigma : 1\right]$
  - Automatic enhancement to $G' = (\mathrm{SU}(2) \times \mathrm{U}(1))/\mathbb{Z}_2$
- $b = 12$:
  - $[g_2] < 0$ and so $g_2 = 0$, but $[f_1] = 0$ as well, so there is no new $\mathfrak{su}(2)$ factor
  - New gauge group is $G' = \mathrm{SU}(2)/\mathbb{Z}_2 = \mathrm{SO}(3)$, related to Massless Charge Sufficiency Conjecture [Morrison, Taylor ('21)]

$\mathfrak{u}(1)$ infinite families

Consider the infinite family of anomaly-consistent $T = 0$ $\mathfrak{u}(1)$ models [Taylor, APT '18]

$$54 \times (\pm\boldsymbol{q}) + 54 \times (\pm\boldsymbol{r}) + 54 \times (\pm(\boldsymbol{q}+\boldsymbol{r}))\,, \quad q, r \in \mathbb{Z}\,.$$

Cofinitely many of these models are in the apparent swampland.

However, there is also a $T = 0$ $\mathfrak{u}(1) \oplus \mathfrak{u}(1)$ model [Cvetič, Klevers, Piragua '13; Cvetič, Klevers, Piragua, Taylor '15], constructible in F-theory, with spectrum

$$54 \times (\mathbf{1}, \mathbf{0}) + 54 \times (\mathbf{0}, \mathbf{1}) + 54 \times (\mathbf{1}, -\mathbf{1})\,.$$

Under a change of basis $\left(\begin{smallmatrix} q & -r \\ 0 & 1 \end{smallmatrix}\right)$ for the two $\mathfrak{u}(1)$ factors, this becomes

$$54 \times (\boldsymbol{q}, \mathbf{0}) + 54 \times (\boldsymbol{r}, -\mathbf{1}) + 54 \times (\boldsymbol{q}+\boldsymbol{r}, -\mathbf{1})\,.$$

This model cannot be Higgsed down to only the first $\mathrm{U}(1)$ factor for nontrivial q, r, and so the Automatic Enhancement Conjecture applies.

We can explicitly verify the enhancement for $(r, q) = (1, 1), (2, 1), (3, 1)$.
[Morrison, Park '12; Klevers, Mayorga Peña, Oehlmann, Piragua, Reuter '15; Raghuram '18]

Exotic matter

F-theory $\mathfrak{su}(2)$ models support $\frac{1}{2}\mathbf{4}+\mathbf{2}$ at triple point singularities of the gauge divisor [Klevers, Taylor '16; Klevers, Morrison, Raghuram, Taylor '17]. However, although the $T=0$ spectrum

$$2 \times \frac{1}{2}\mathbf{4} + 84 \times \mathbf{3} + 104 \times \mathbf{1}$$

satisfies anomaly cancellation with $b=5$, it cannot be realized in F-theory on $B=\mathbb{P}^2$ because an irreducible quintic curve cannot have two triple points. The only way to achieve two triple points is for the quintic to be reducible, in which case the model has gauge algebra $\mathfrak{su}(2)\oplus\mathfrak{su}(2)$ and spectrum

$$2 \times \frac{1}{2}(\mathbf{2},\mathbf{3}) + 1 \times (\mathbf{1},\mathbf{3}) + 19 \times (\mathbf{2},\mathbf{1}) + 64 \times (\mathbf{1},\mathbf{2}) + 104 \times (\mathbf{1},\mathbf{1})\,.$$

This model would branch to the desired model after giving a VEV to bifundamental matter, but it contains no bifundamentals, consistent with the Automatic Enhancement Conjecture.

Other examples

- Various $\mathfrak{su}(N)$ models for large b/large N [Katz, Morrison, Schafer-Nameki, Sully '11; Morrison, Taylor '12; Johnson, Taylor '16]
- $\mathfrak{sp}(N)$ and $\mathfrak{so}(N)$ examples
- Other infinite $\mathfrak{u}(1)$ families
- Universal $(\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1))/\mathbb{Z}_6$ model [Raghuram, Taylor, APT '20]
- Other exotic matter examples [Seiberg '97; Heckman, Morrison, Vafa '14; Del Zotto, Heckman, Tomasiello, Vafa '15; Anderson, Gray, Raghuram, Taylor '16; Klevers, Morrison, Raghuram, Taylor '17; Cvetič, Heckman, Lin '18; Tian, Wang '18]
- Discrete gauge groups [Braun, Morrison '14; Morrison, Taylor '14; Klevers, Mayorga Peña, Oehlmann, Piragua, Reuter '15; Monnier, Moore '18]

Conclusions and further directions

- Identified a range of situations in which tuning a certain gauge group and matter content in F-theory results in an automatic enhancement to a larger gauge group and matter content (or inconsistency)
- Applies to many cases that lie in the apparent swampland
- Can we explicitly see automatic enhancement for additional infinite $\mathfrak{u}(1)$ and $\mathfrak{su}(13) \oplus \mathfrak{u}(1)$ families that have quadratic charge structure?
- Can we prove the Automatic Enhancement Conjecture fully in F-theory?
- Does the Automatic Enhancement Conjecture hold more generally for 6D supergravity?
- Can we unify the Automatic Enhancement Conjecture and the Massless Charge Sufficiency Conjecture in 6D supergravity?