The Standard Model Quiver from Singular de Sitter String Compactifications

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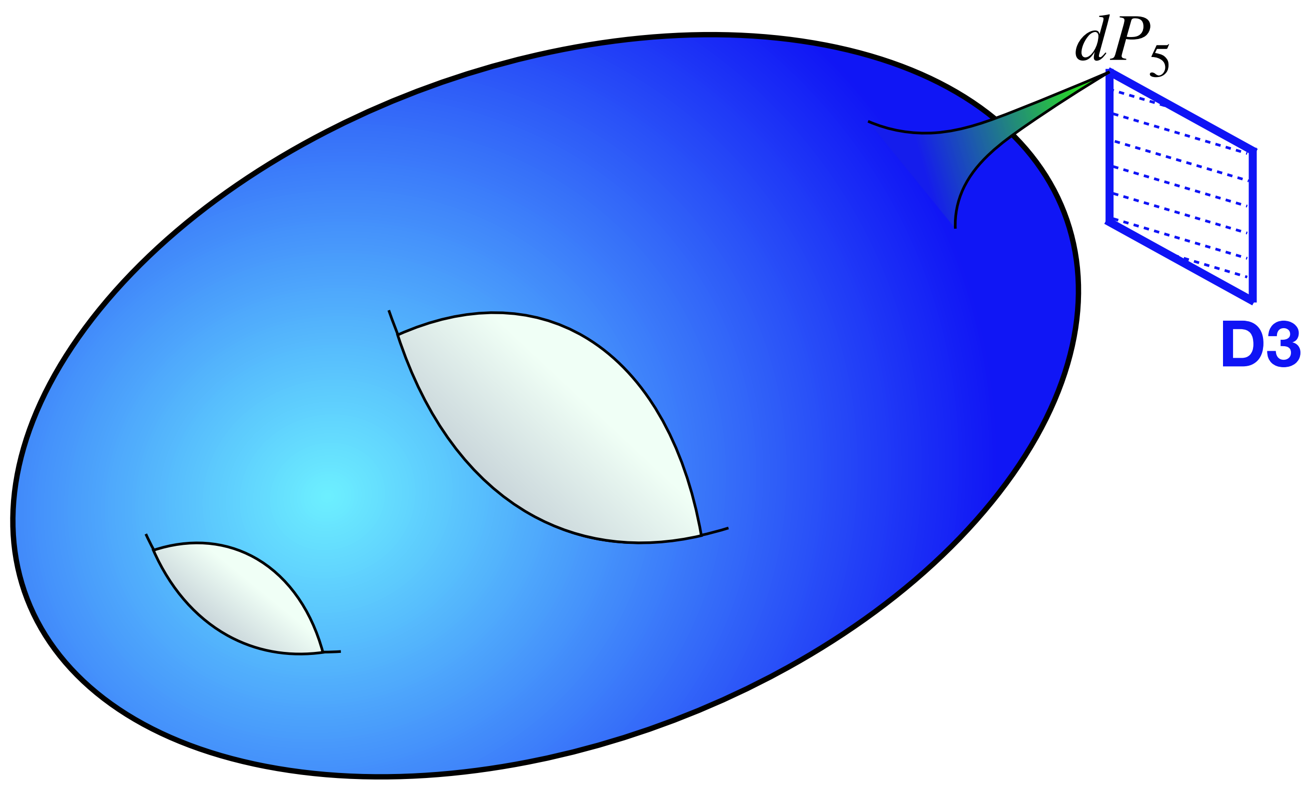
ArXiv: 2105.XXXX

Branes at del Pezzo singularities



Motivation:

- Visible sector localised at a single point in the compactification space
- Global and local issues can be addressed separately
- Largely unexplored class of models from D3-branes at del Pezzo singularities

del Pezzo surfaces:

Complex 2-dimensional Fano surfaces, i.e., projective algebraic surfaces with ample anti-canonical divisor class $-K \cdot C > 0$ for every curve C. We distinguish $\mathbb{P}^1 \times \mathbb{P}^1$ or dP_n corresponding to $\mathbb{P}^2$ blown up at n points.

We define generators of $H_2(dP_n, \mathbb{Z})$

H Hyperplane class

E_i $\quad i = 1,\ldots,n$ Exceptional curves

Intersection numbers

$$H \cdot H = 1\,, \quad E_i \cdot E_j = -\delta_{ij}\,, \quad H \cdot E_i = 0$$

Canonical class

$$K(dP_n) = -3H + \sum_{i=1}^{n} E_i$$

Degree of a dP_n

$$K \cdot K = 9 - n$$

Attractive features of D-branes at dP_n singularities:

A. Huge variety of gauge theories from a single D3 brane

B. Oriented quiver always comes with 2 anomalous $U(1)$'s

C. Chiral spectrum from intersections of 2- and 4-cycles

To a point shrinkable 4-cycle must be of del Pezzo type by **Grauter's criterion**

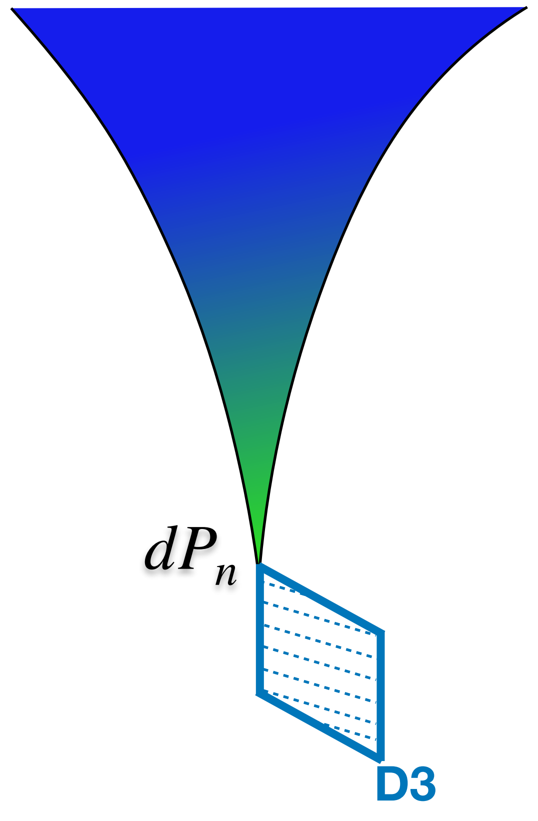
See e.g. Cordova: 0910.2955
Malyshev, Verlinde:0711.2451

Previous global constructions:

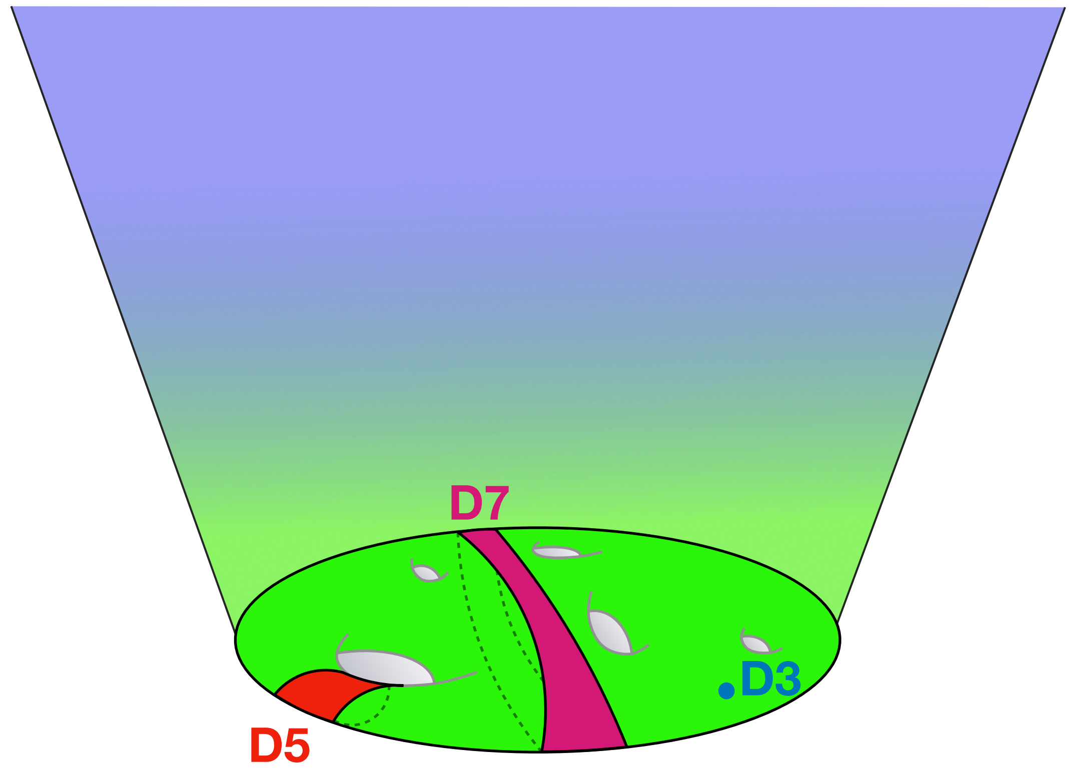
Dolan, Krippendorf, Maharana, Quevedo: 1002.1790
Dolan, Krippendorf, Quevedo: 1106.6039
Cicoli, Krippendorf, Mayrhofer, Quevedo, Valandro: 1206.5237, 1304.0022, 1304.2771
Cicoli, Klevers, Krippendorf, Mayrhofer, Quevedo, Valandro: 1312.0014
Cicoli, García Etxebarria, Mayrhofer, Quevedo, Shukla, Valandro: 1706.06128

D3-branes at singularities — Fractional branes

Geometric description of local model in the large volume limit:



Deform Kähler structure



See for example: Wijnholt: hep-th/0212021, hep-th/0703047
Verlinde, Wijnholt: hep-th/0508089
Malyshev, Verlinde:0711.2451

Fractional branes:
D-branes supported on cycles that can be described as sheaves

For dP_n, there are $n+3$ fractional branes

Fractional branes are characterised by their Chern character or RR-charge vector

$$\mathrm{ch}(F_i) = (\mathrm{rk}(F_i), c_1(F_i), \mathrm{ch}_2(F_i))$$

Wrapping number of D7 around dP_n

D5 component

Instanton number

Massless spectrum of open strings stretched between F_i and F_j encoded by $Ext^k(F_i, F_j)$ **groups** as the generalisations of cohomology groups to sheaves

Number of fields counted by the relative Euler character

$$\chi(F_i, F_j) = \sum_k (-1)^k \dim(\mathrm{Ext}^k(F_i, F_j))$$

See e.g. Verlinde, Wijnholt: hep-th/0508089

For **exceptional collection** $\{F_1, \ldots, F_{n+3}\}$, spectrum encoded in the **anti-symmetrised intersection matrix**

$$\chi_-(F_i, F_j) = \chi(F_i, F_j) - \chi(F_j, F_i) = \mathrm{rk}(F_j)\deg(F_i) - \mathrm{rk}(F_i)\deg(F_j)$$

For details on exceptional collections, see:
Herzog, Walcher: hep-th/0306298
Hanany, Herzog, Vegh: hep-th/0602041

$\deg(F_i) = -c_1(F_i)\cdot K$ is the intersection of the D5 component of F_i with the del Pezzo

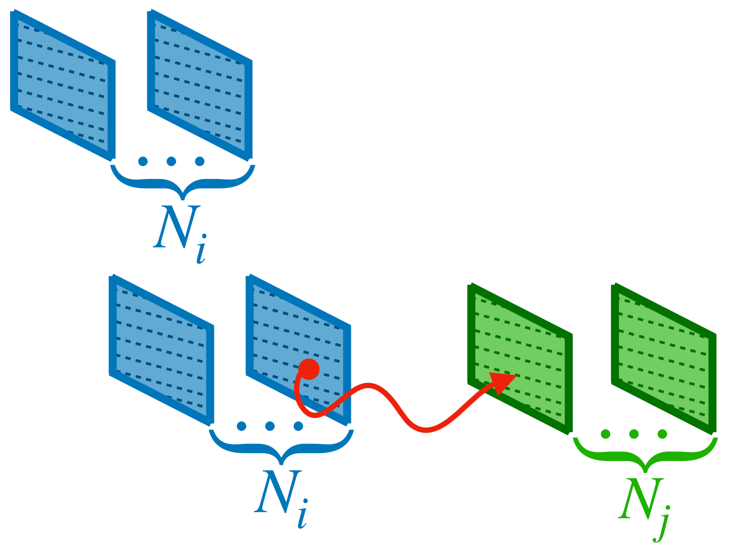
The quiver diagram for D3-branes at dP_5 singularities

We follow the seminal work of M. Wijnholt
"*Geometry of Particle Physics*" hep-th/0703047

Interpret intersection matrix diagrammatically in terms of quiver diagrams:

$\mathrm{U}(N_i)$ (i) N_i copies of a fractional brane F_i

$\mathrm{U}(N_i)$ (i) $\longrightarrow$ (j) $\mathrm{U}(N_j)$ Chiral multiplet in bi-fundamental of $U(N_i)\times U(N_j)$



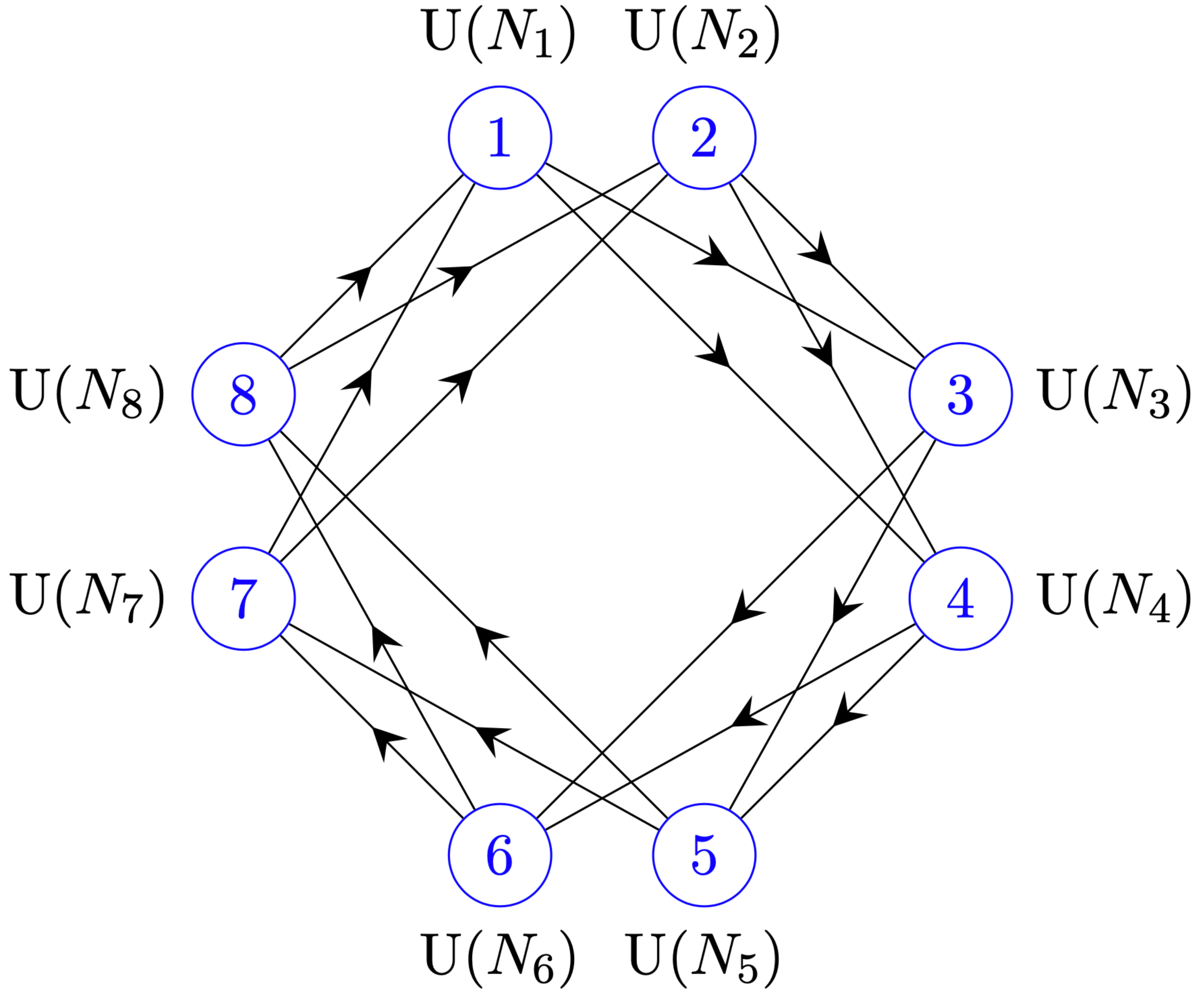
Exceptional collection for dP_5

$$\mathrm{ch}(F_1) = (1, H - E_1, 0) \qquad \deg(F_1) = -2$$
$$\mathrm{ch}(F_2) = (1, H - E_2, 0) \qquad \deg(F_2) = -2$$
$$\mathrm{ch}(F_3) = -\left(1, 2H - E_1 - E_2 - E_4, \frac{1}{2}\right) \qquad \deg(F_3) = 3$$
$$\mathrm{ch}(F_4) = -\left(1, 2H - E_1 - E_2 - E_5, \frac{1}{2}\right) \qquad \deg(F_4) = 3$$
$$\mathrm{ch}(F_5) = -\,(1, H - E_3, 0)\ , \qquad \deg(F_5) = 2$$
$$\mathrm{ch}(F_6) = -\,(1, E_4 + E_5, -1) \qquad \deg(F_6) = 2$$
$$\mathrm{ch}(F_7) = \left(1, H, \frac{1}{2}\right) \qquad \deg(F_7) = -3$$
$$\mathrm{ch}(F_8) = \left(1, 2H - E_1 - E_2 - E_3, \frac{1}{2}\right) \qquad \deg(F_8) = -3$$

The full chiral spectrum is obtained from

$$\chi_-(F_i, F_j) = \mathrm{rk}(F_j)\deg(F_i) - \mathrm{rk}(F_i)\deg(F_j) = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & -1 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \end{pmatrix}$$

dP_5 quiver diagram:



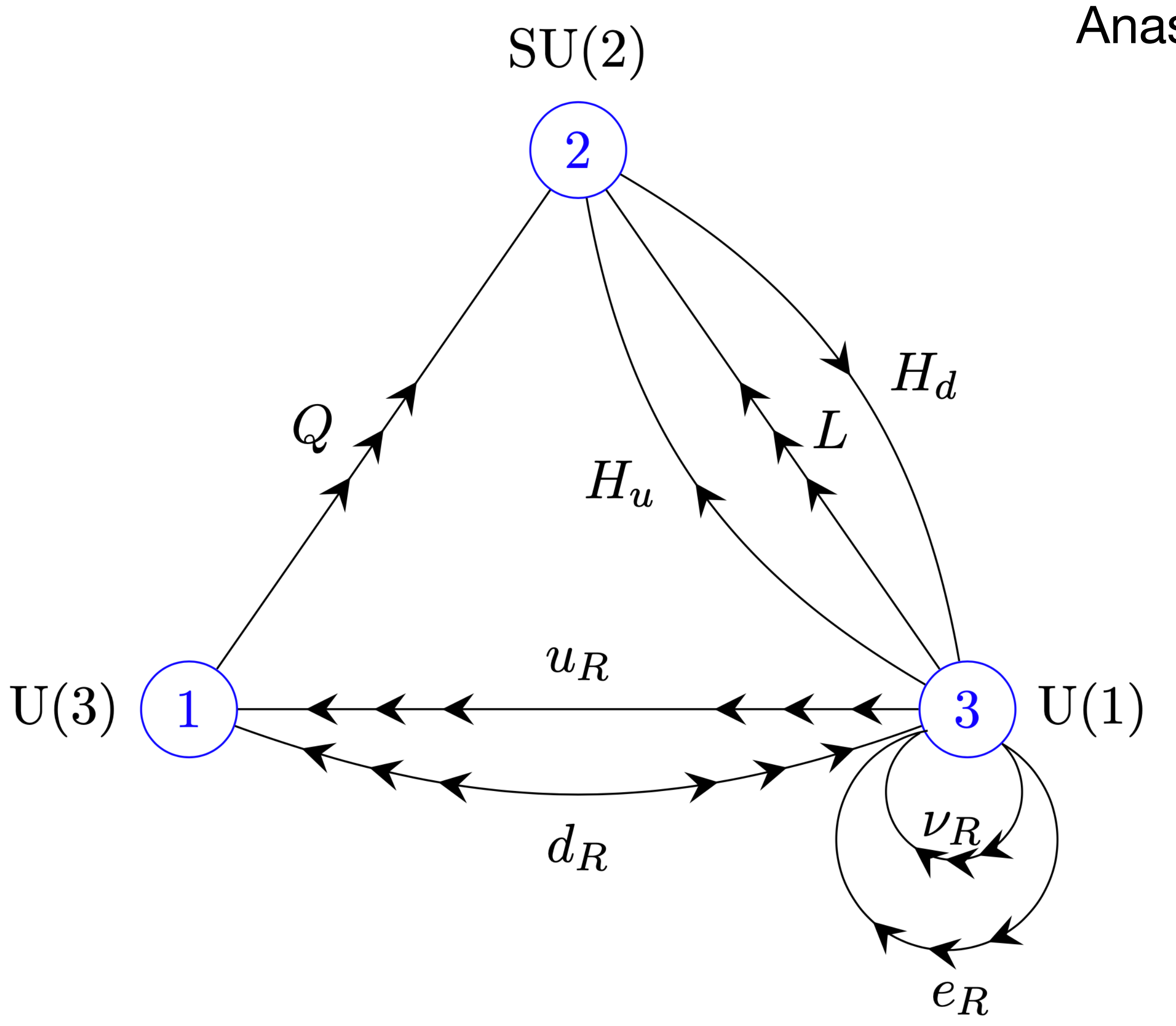
Anomaly cancellation:
$N_1 + N_2 = N_5 + N_6$
$N_3 + N_4 = N_7 + N_8$

Orientifolded dP_5 quiver

For details on orientifolded quiver, see
Franco, Hanany, Krefl, Park, Uranga, Vegh: 0707.0298
García Etxebarria, Quevedo, Valandro: 1512.06926
Collinucci, García-Etxebarria: 1612.06874
Bianchi, Bufalini, Mancani, Riccioni: 2003.09620

Minimal Quiver Standard Model (MQSM):

Anastasopoulos, Dijkstra, Kiritsis, Schellekens: hep-th/0605226
Berenstein, Pinansky: hep-th/0610104

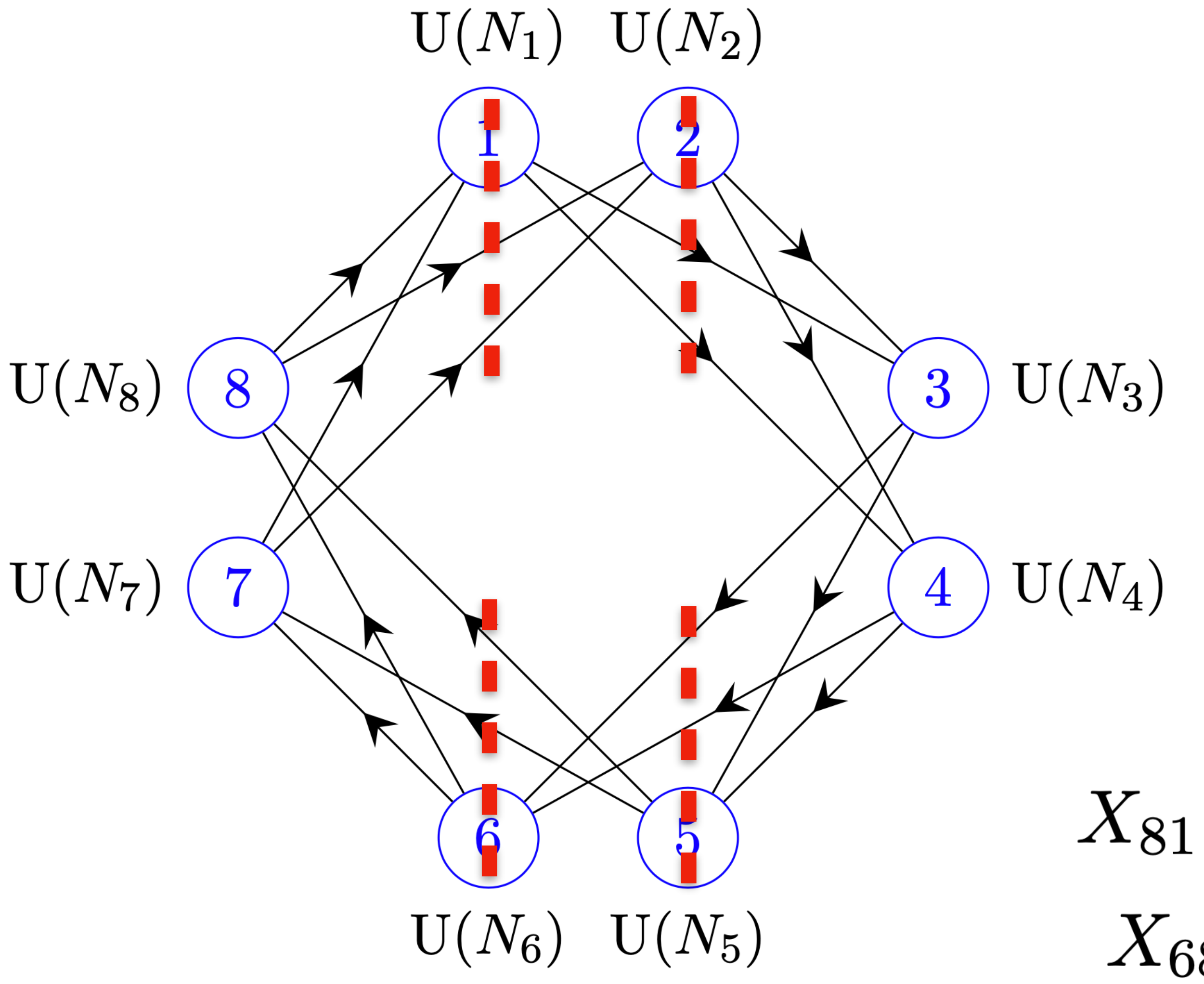


The Standard Model quiver itself is unoriented!

Orientifolding quiver gauge theories: The O-plane involution

1. defines $\mathbb{Z}_2$ automorphism of the quiver which changes arrow direction
2. leaves original superpotential invariant
3. identifies chiral and vector superfields on either side

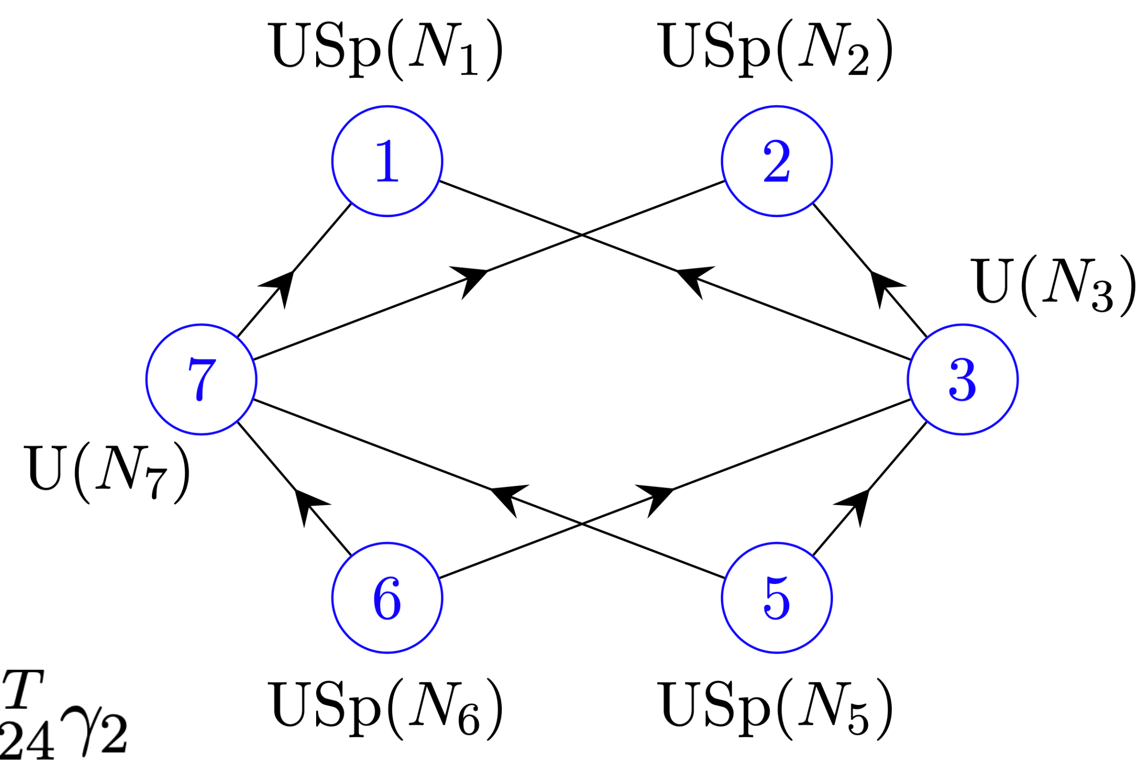
See e.g. García Etxebarria, Heidenreich, Wrase: 1210.7799



We focus on *USp*-**projections!**

$$1 \leftrightarrow 1^*\,, \quad 2 \leftrightarrow 2^*\,, \quad \boxed{3 \leftrightarrow 8^*}$$

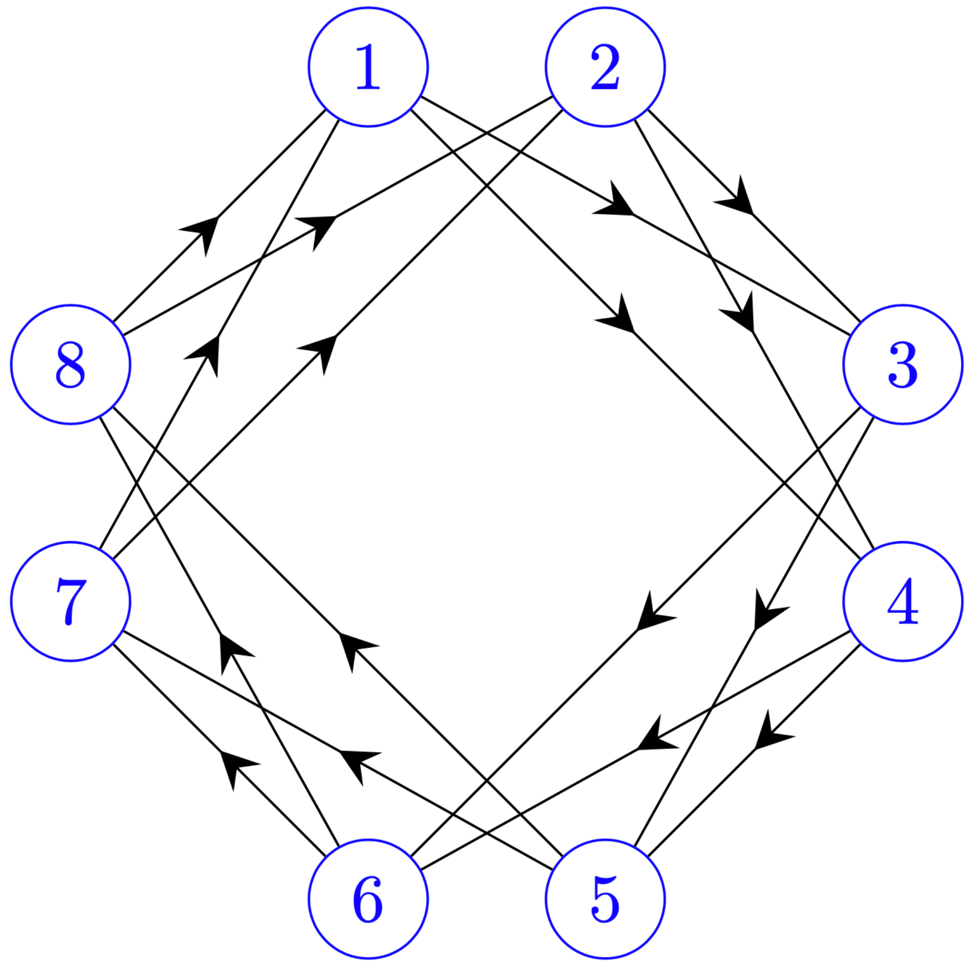
$$\boxed{4 \leftrightarrow 7^*}\,, \quad 5 \leftrightarrow 5^*\,, \quad 6 \leftrightarrow 6^*$$



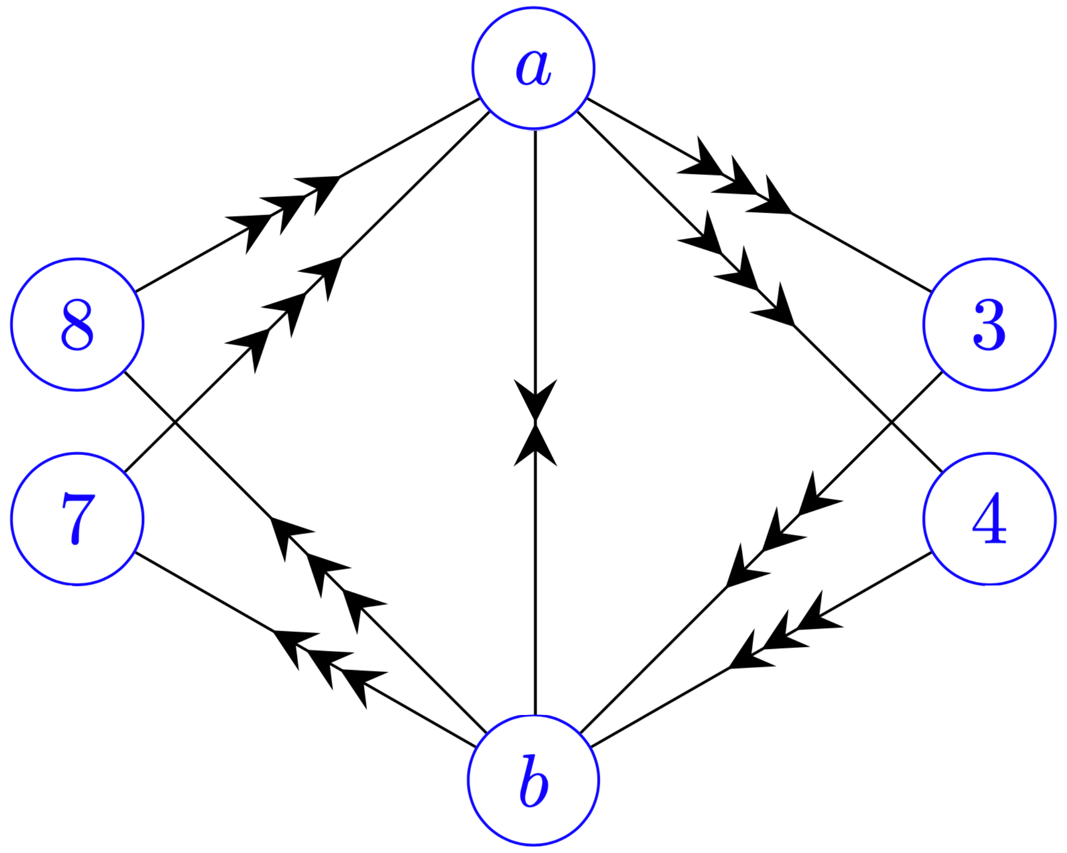
$$X_{81} = X_{13}^T \gamma_1\,, \quad X_{71} = a^{-1} X_{14}^T \gamma_1\,, \quad X_{82} = b^{-1} X_{23}^T \gamma_2\,, \quad X_{72} = X_{24}^T \gamma_2$$

$$X_{68} = \gamma_6^{-1} X_{36}^T\,, \quad X_{58} = \gamma_5^{-1} X_{35}^T\,, \quad X_{67} = \gamma_6^{-1} X_{46}^T\,, \quad X_{57} = \gamma_5^{-1} X_{45}^T$$

Higgsing the dP_5 quiver gauge theory



Higgsing
=
Bound state of fractional branes



Missing features:

A. Chiral spectrum with 3 families

B. Standard Model gauge group

C. Non-chiral matter

All of the above is achieved by **turning on VEVs for bi-fundamental fields!**

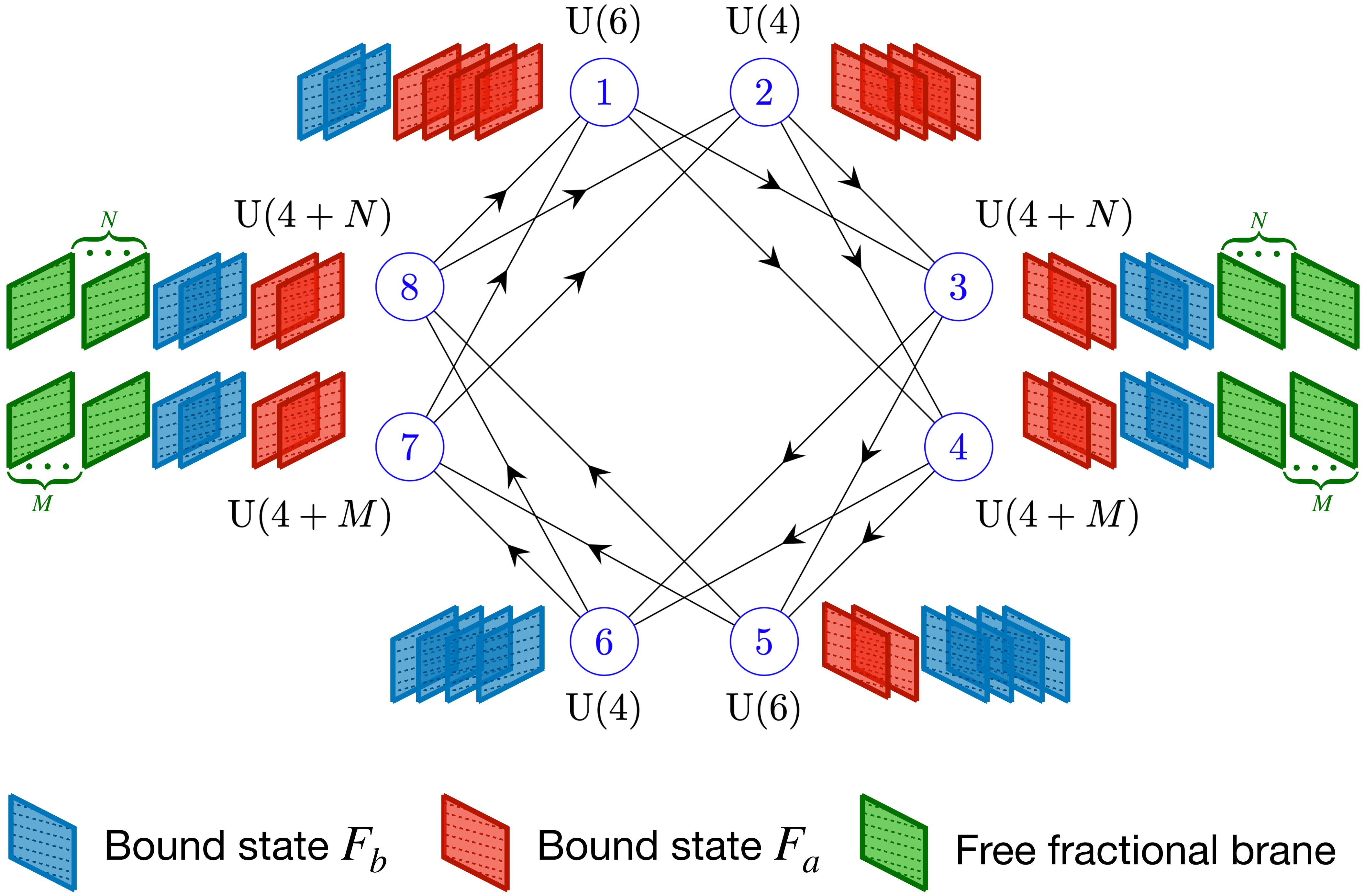
Interpretation:

- **Fractional branes perspective:** bound states of fractional branes

$$\{F_1, \ldots, F_8\} \to \{F_a, F_3, F_4, F_b, F_7, F_8\}$$

- **Geometrically:** partial resolution of singularity

Original quiver



$$\mathrm{ch}(F_b) = \sum_{i=1}^{8} n_i^{(b)} \mathrm{ch}(F_i) = -(3, 2H - E_2 - E_3 + E_4 + E_5, -2) \quad \deg(F_b) = 6$$

$$\mathrm{ch}(F_a) = \sum_{i=1}^{8} n_i^{(a)} \mathrm{ch}(F_i) = (3, 2H - E_1 - E_2 + E_4 + E_5, 0) \quad \deg(F_a) = -6$$

The breaking pattern is given by

$$G = (\mathrm{U}(6) \times \mathrm{U}(4+N) \times \mathrm{U}(4+M) \times \mathrm{U}(4))^2 \to H = (\mathrm{U}(2) \times \mathrm{U}(N) \times \mathrm{U}(M))^2$$

We computed the mass matrix for gauge bosons to confirm this breaking pattern!

Turn on VEVs:

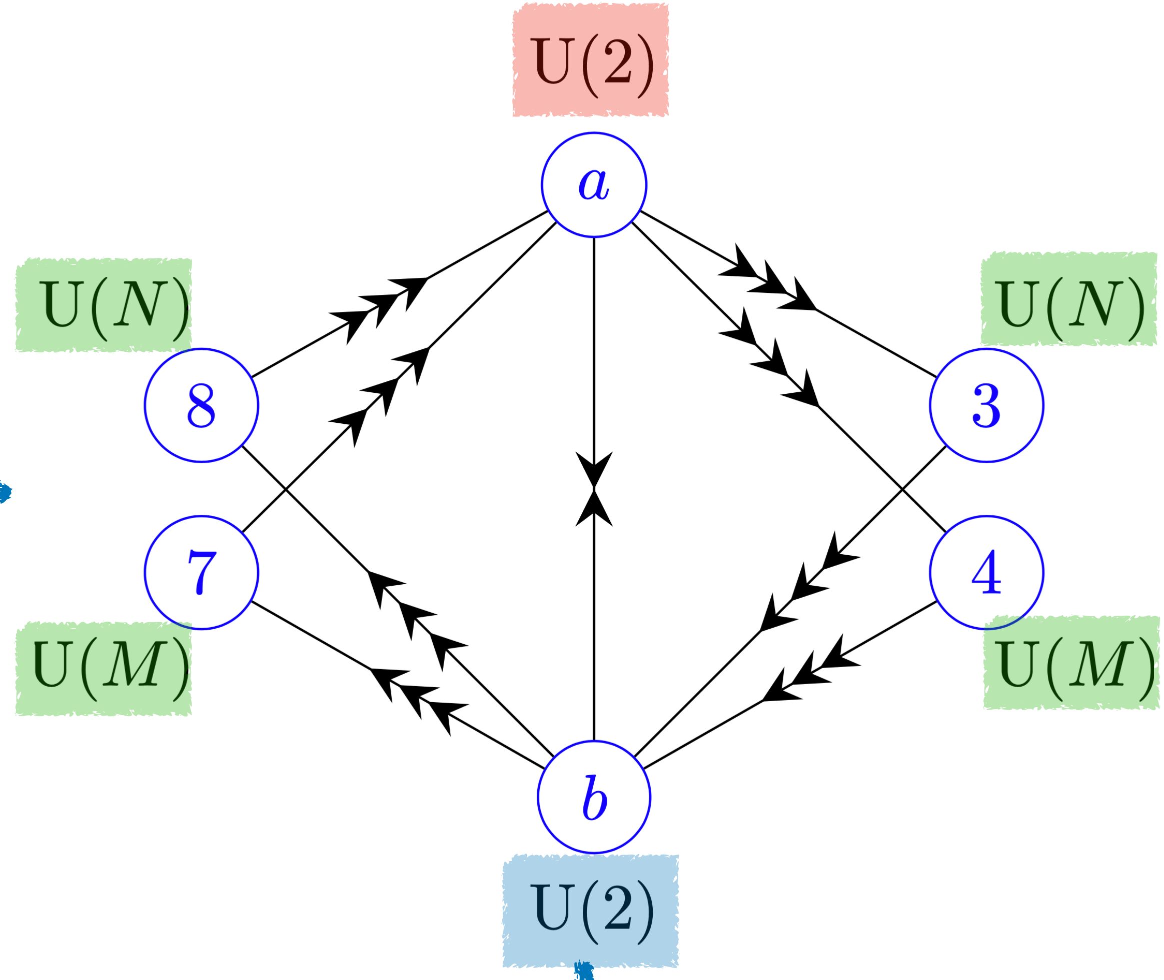
$$\langle X_{13} \rangle = \begin{pmatrix} a_1 & 0 & \cdots \\ 0 & a_2 & \cdots \\ 0 & a_3 & \cdots \end{pmatrix}, \quad \langle X_{14} \rangle = \begin{pmatrix} b_1 & 0 & \cdots \\ 0 & b_2 & \cdots \\ 0 & b_3 & \cdots \end{pmatrix},$$

$$\langle X_{23} \rangle = \begin{pmatrix} 0 & c_1 & \cdots \\ 0 & c_2 & \cdots \end{pmatrix}, \quad \langle X_{24} \rangle = \begin{pmatrix} 0 & d_1 & \cdots \\ 0 & d_2 & \cdots \end{pmatrix},$$

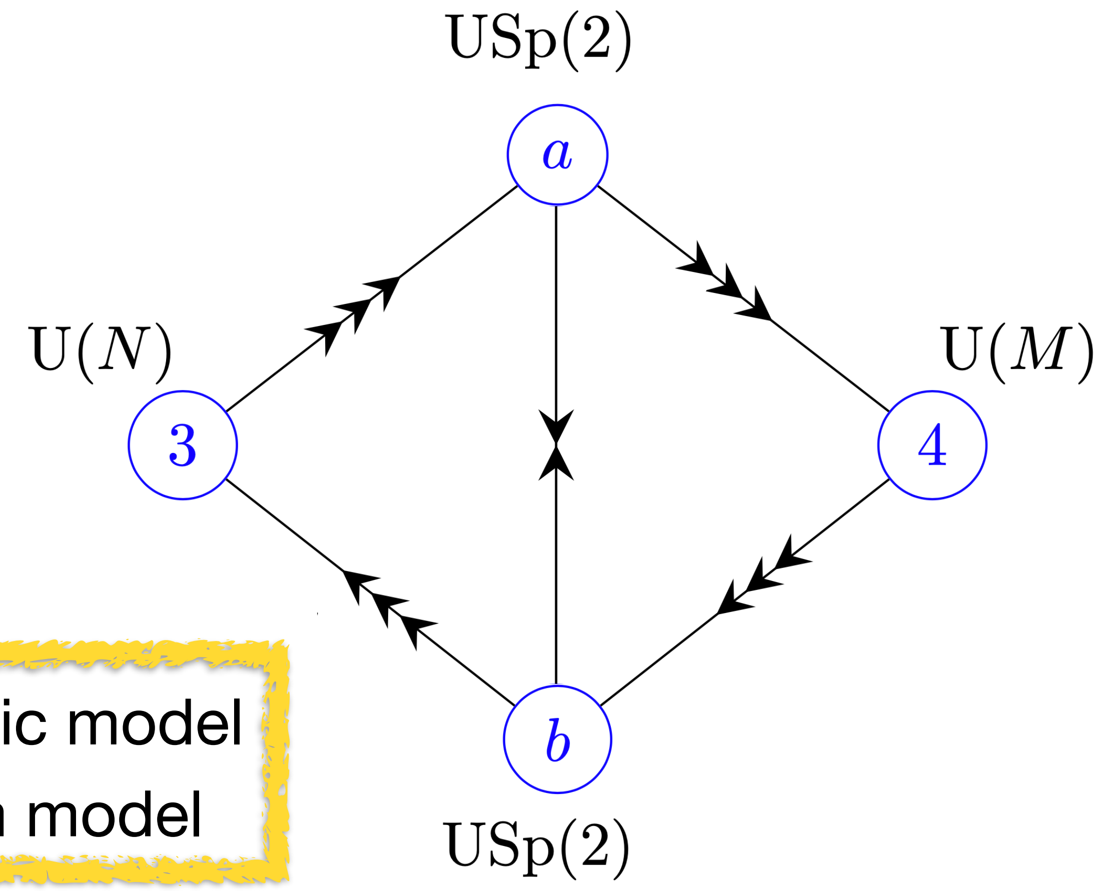
$$\langle X_{35}^T \rangle = \begin{pmatrix} x_1 & 0 & \cdots \\ x_2 & 0 & \cdots \\ 0 & x_3 & \cdots \end{pmatrix}, \quad \langle X_{45}^T \rangle = \begin{pmatrix} z_1 & 0 & \cdots \\ z_2 & 0 & \cdots \\ 0 & z_3 & \cdots \end{pmatrix},$$

$$\langle X_{36}^T \rangle = \begin{pmatrix} y_1 & 0 & \cdots \\ y_2 & 0 & \cdots \end{pmatrix}, \quad \langle X_{46}^T \rangle = \begin{pmatrix} w_1 & 0 & \cdots \\ w_2 & 0 & \cdots \end{pmatrix}$$

Higgsed quiver



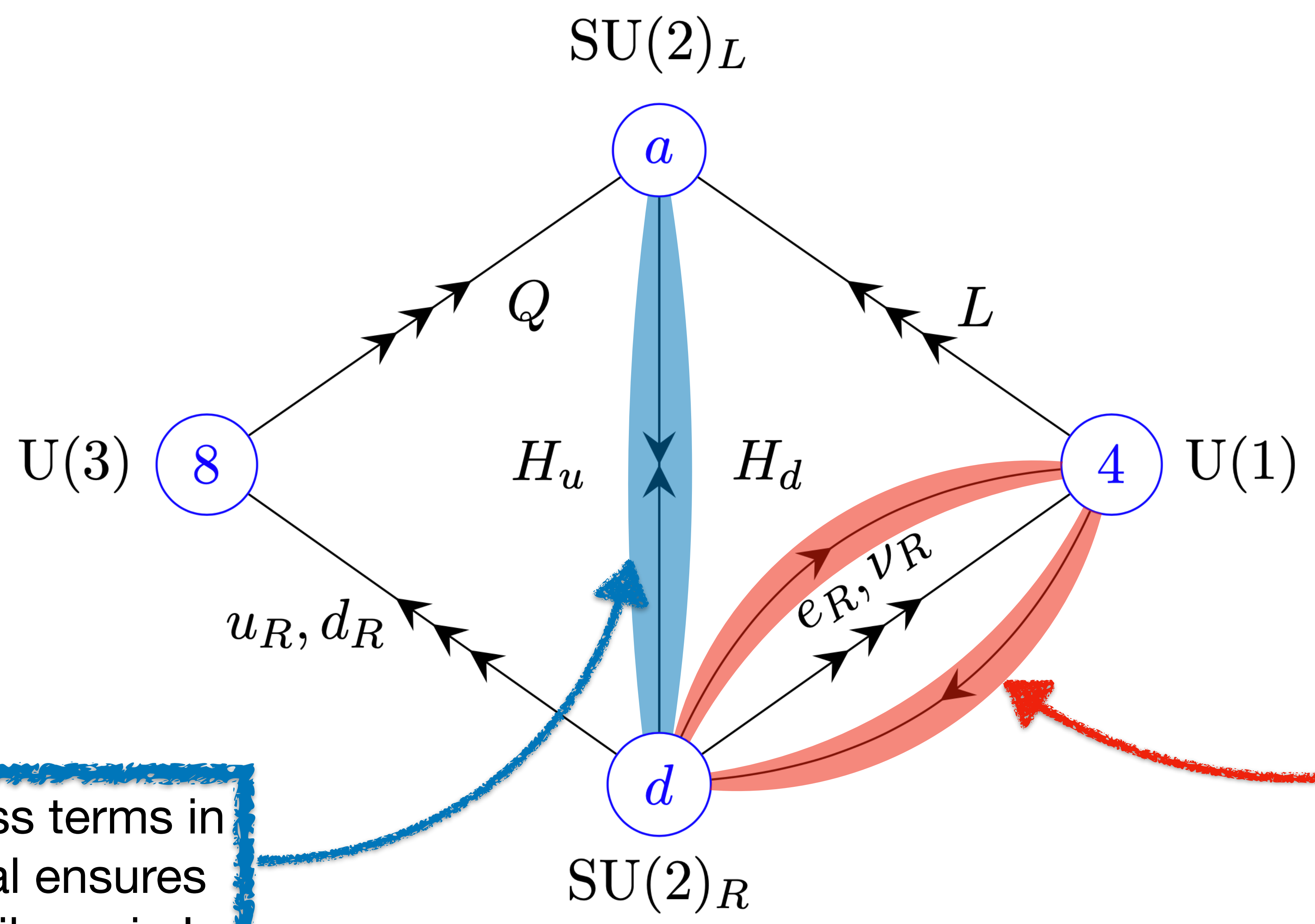
Higgsed, orientifolded quiver



For $N = 3, M = 1$: LR symmetric model
For $N = 0, M = 4$: Pati-Salam model

This is (almost) the spectrum of the **SUSY left-right symmetric model:**

Pati, Salam: Phys. Rev. D10 (1974) 275
Mohapatra, Pati: Phys. Rev. D11 (1974) 566
Senjanovic, Mohapatra: Phys. Rev. D12 (1975) 1502



Generation of mass terms in the superpotential ensures only two vector like pairs!

Wijnholt: hep-th/0703047

These two vector like pairs are obtained from building more complicated bound states and starting from a larger quiver!

Concerning the two $U(1)$'s:

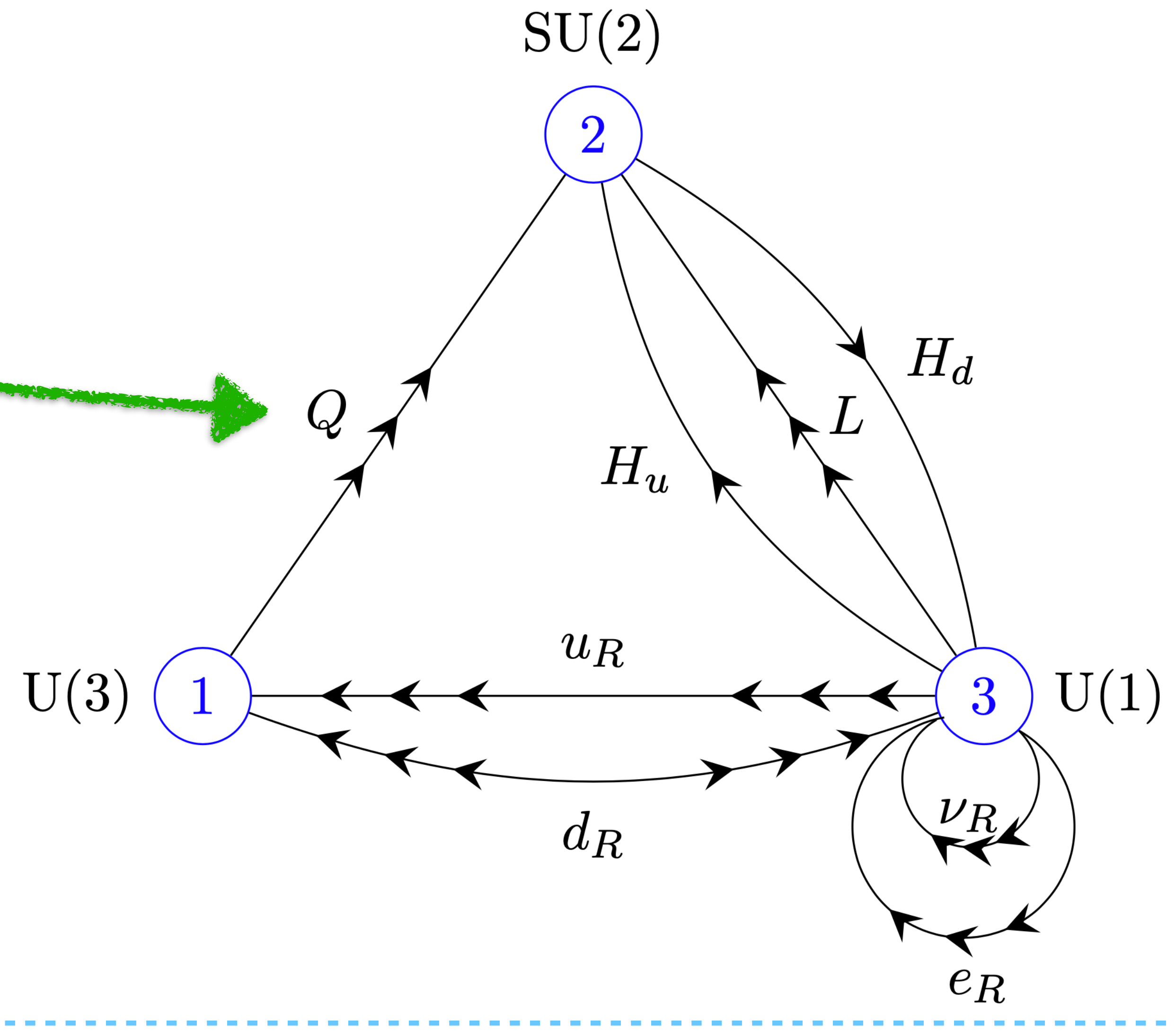
- One combination is non-anomalous corresponding to $U(1)_{B-L}$
- The other anomalous $U(1)$ gains a Stückelberg mass through Green-Schwarz mechanism, but remains as global symmetry

Conventional Higgsing
$$SU(2)_R \times U(1)_{B-L} \to U(1)_Y$$

See e.g. Heckman, Vafa, Verlinde, Wijnholt: 0711.0387

Minimal Quiver Standard Model (MQSM):

Anastasopoulos, Dijkstra, Kiritsis, Schellekens: hep-th/0605226
Berenstein, Pinansky: hep-th/0610104

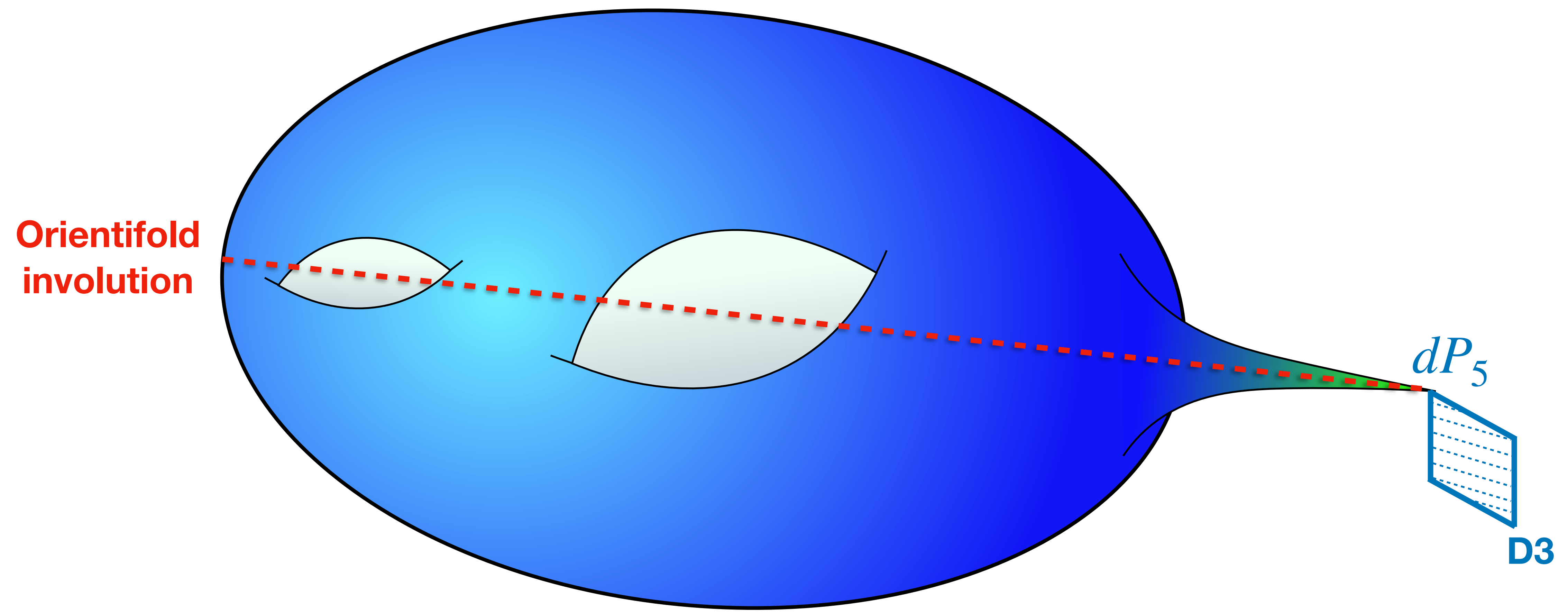


Global Embedding

For similar situation, see e.g.
Cicoli, García-Etxebarria, Mayrhofer,
Quevedo, Shukla, Valandro: 1706.06128

Desirable requirements for our search:

- Diagonal dP_5 divisor ➡ dP_5 singularity
- Additional rigid divisors ➡ support instantons
- Involution leaving dP_5 divisor transversely invariant ➡ realising the local unoriented dP_5 model
- Large negative D3-charge for O7-planes ➡ large tadpole
- T-brane from D7-banes wrapping large cycle ➡ uplift to de Sitter
- Tadpole/anomaly cancellation ➡ consistent compactification
- Non-perturbative superpotential contributions ➡ Kähler moduli stabilisation



The search for a global model in the KS database

For a del Pezzo divisor D_s of type dP_n

$$\int_{X_3} D_s^3 = k_{sss} = 9 - n\,, \qquad \int_{X_3} D_s^2\, D_i \leq 0 \qquad \forall\, i \neq s$$

Diagonality condition for dP_5

$$k_{sss}\; k_{sij} = k_{ssi}\; k_{ssj} \quad \forall\;\, i, j$$

4-cycle volume becomes complete square

$$\tau_s = \frac{1}{2}\, k_{sij} t^i\, t^j = \frac{1}{2\, k_{sss}}\, \left(k_{ssi}\, t^i\right)^2$$

Refer to diagonal dP_n divisor D_s as **ddP_n divisor**

We looked at the divisor structure of CY geometries in the Kreuzer-Skarke (KS) database with $h^{1,1} \leq 5$

Kreuzer, Skarke: hep-th/0002240

$h^{1,1}$	Poly*	Geom*	dP_0	dP_1	dP_2	dP_3	dP_4	dP_5	dP_6	dP_7	dP_8
1	5	5	0	0	0	0	0	0	0	0	0
2	36	39	9	4	0	0	0	0	2	4	5
3	243	305	59	93	4	4	2	9	21	67	71
4	1185	2000	372	880	155	144	55	184	239	689	597
5	4897	13494	2410	8038	2242	2271	947	2190	2459	5462	4692

Distinct favourable polytopes and geometries

$h^{1,1}$	Poly*	Geom* ($n_{\rm CY}$)	ddP_0	ddP_1	ddP_n $2 \leq n \leq 5$	ddP_6	ddP_7	ddP_8	$n_{\rm LVS}$ ($ddP_n \geq 1$)
1	5	5	0	0	0	0	0	0	0
2	36	39	9	2	0	2	4	5	22
3	243	305	59	16	0	17	40	39	171
4	1185	2000	372	144	0	109	277	157	1059
5	4897	13494	2410	944	0	624	827	407	5212

Kreuzer-Skarke ddP_n conjecture:

Calabi-Yau threefolds arising from fine, regular, star triangulations of 4d reflexive polytopes in the Kreuzer-Skarke database do not exhibit diagonal dP_n divisors with $2 \leq n \leq 5$

Further checks of ~200.000 distinct geometries at $6 \leq h^{1,1} \leq 40$ with ***CYTools*** confirm this trend!

Demirtas, McAllister, Rios-Tascon: 2008.01730

Global models with diagonal dP_5 divisor

Crucial inside: The dP_5 surfaces can be constructed as the bi-quadric in $\mathbb{P}^4$

HY_1	HY_2	x_1	x_2	x_3	x_4	x_5
2	2	1	1	1	1	1

Stanley-Reisner ideal

$$SR = \{x_1 x_2 x_3 x_4 x_5\}$$

Plan for constructing CY threefolds:
CY threefolds as complete intersections of 2 equations in a 5-dimensional toric space

Let us focus on specific model with $(h^{2,1}, h^{1,1}) = (59,3)$

Eq_1	Eq_2	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
4	4	1	1	0	0	2	2	1	1
2	2	0	0	1	0	1	1	0	1
2	2	0	0	0	1	1	1	1	0
		SD1	SD1	dP_5	dP_5	SD2	SD2	SD3	SD3

Stanley-Reisner ideal

$$SR = \{x_3 x_4,\ x_3 x_8,\ x_4 x_7,\ x_1 x_2 x_5 x_6 x_7,\ x_1 x_2 x_5 x_6 x_8\}$$

Euler characteristic

$$\chi = -112$$

We define coordinate divisors as

$$D_i = \{x_i = 0\}$$

Basis of smooth divisors

$$\{D_b, D_3, D_4\} \quad D_b \equiv D_1 + D_3 + D_4$$

Kähler form

$$J = t_b D_b + t_3 D_3 + t_4 D_4$$

4-cycle volumes

$$\tau_b \equiv \mathrm{vol}(D_b) = 2t_b^2\,, \quad \tau_3 \equiv \mathrm{vol}(D_3) = 2t_3^2\,, \quad \tau_4 \equiv \mathrm{vol}(D_4) = 2t_4^2$$

Volume form

$$\mathcal{V} = \frac{1}{3\sqrt{2}}\left(\tau_b^{3/2} - \tau_3^{3/2} - \tau_4^{3/2}\right)$$

Orientifold involution

$$\sigma\ :\ x_5 \mapsto -x_5$$

Single O7-plane on

$$D_5 = 2D_b - D_3 - D_4$$

We checked that the local model can indeed be embedded as well as that the **global and local involutions are consistent!**

We take the singular limit $t_3 \to 0$ which gives rise to a dP_5 singularity!

In our paper, we present **6 additional CY threefolds** with $h^{1,1} = 2,3,4$

D-brane setup

Essentially two ingredients:

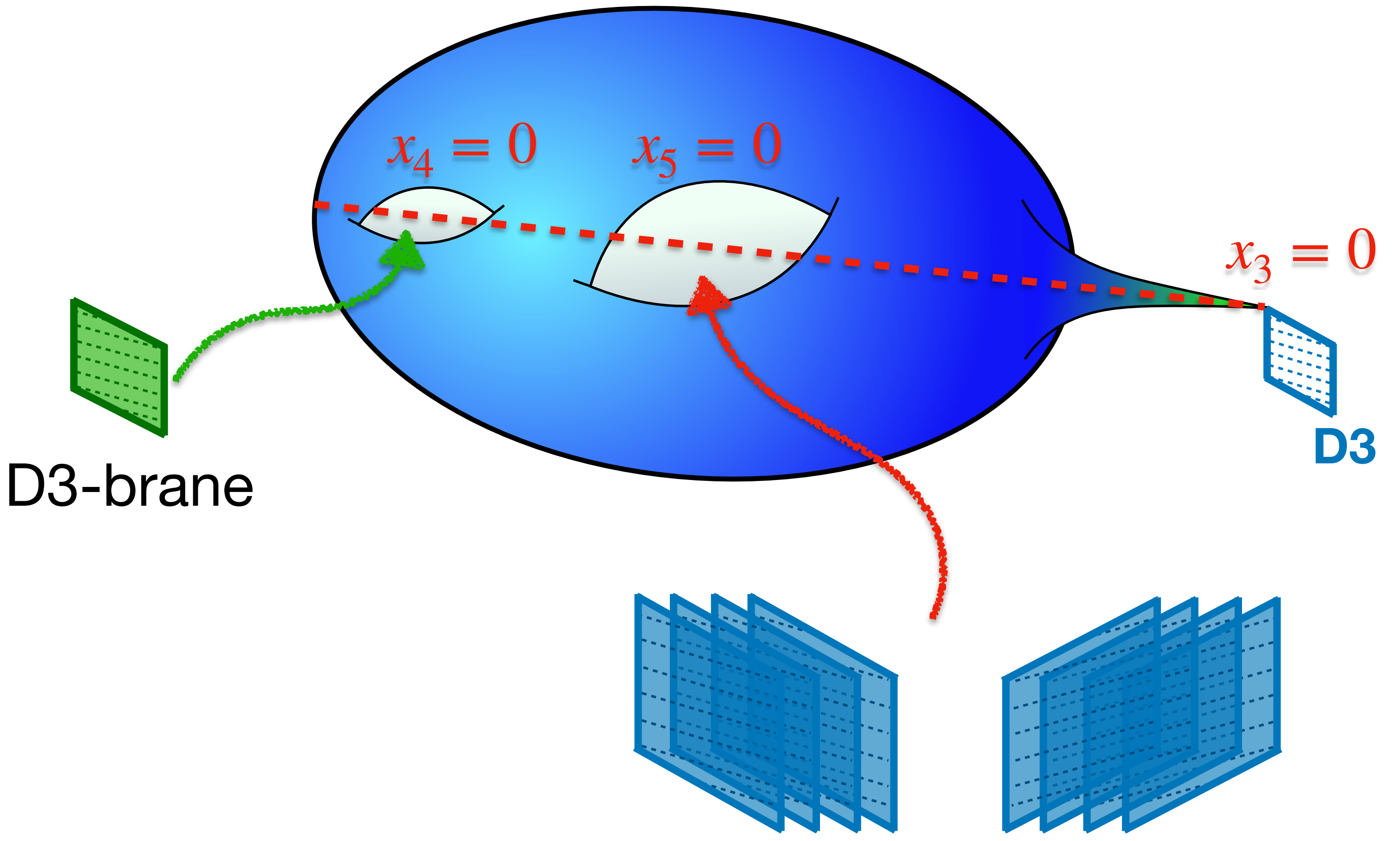
A. D7-branes to cancel D7-tadpole induced by O7-plane
B. D3-brane on additional dP_5 divisor for non-perturbative effects

Stack of 4 D7-branes (plus their 4 images) wrapping $x_5 = 0$ giving rise to SO(8) gauge group

We turn on flux on D7-branes (pullback implicit):

$$\mathcal{F} = F - B = \left(n_b - \frac{1}{2}\right) D_b + \left(n_3 + \frac{1}{2}\right) D_3 + n_4 D_4 \qquad n_b, n_3, n_4 \in \mathbb{Z}$$

We put D3-brane on dP_5 divisor D_4 at $x_4 = 0$



$O(1)$ **instanton** is generated by wrapping D3-brane on invariant divisor D with zero flux

$$\mathcal{F}_{E3} \equiv F_{E3} - \iota_D^* B = 0$$

Freed-Witten quantisation condition

$$F + \frac{c_1(D)}{2} \in H^2(D, \mathbb{Z})$$

Freed, Witten: hep-th/9907189

Since dP_5 is non-spin, i.e., $c_1(D_4)$ is odd, the **flux must be half-integral**:

$$B = -\frac{D_4}{2} + \frac{D_b}{2}$$

Superpotential contribution necessitates **absence of chiral zero modes at $D7 \cap E3$**

$$0 = \int_{D7 \cap E3} \mathcal{F} - \mathcal{F}_{E3} \longrightarrow n_4 = 0$$

Summary

SO(8) D7-brane configuration wrapping $x_5 = 0$ with **O(1) E3-instanton** on dP_5 at $x_4 = 0$

T-brane background

Flux breaks SO(8) to U(4) with Fayet-Iliopoulos (FI) terms

$$\xi_{D7} = \frac{1}{4\pi\mathcal{V}}\int_{D7}\mathcal{F}\wedge J \xrightarrow{t_3\to 0} \frac{1}{\pi\mathcal{V}}(2n_b-1)t_b$$

SUSY equation of motion in 8 dimensions with flux

$$J\wedge\mathcal{F}_{D7} + \left[\Phi,\Phi^\dagger\right] dvol_4 = 0 \qquad \Rightarrow \qquad \langle\Phi\rangle \neq 0$$

In 4d, this implies a vanishing D-term!

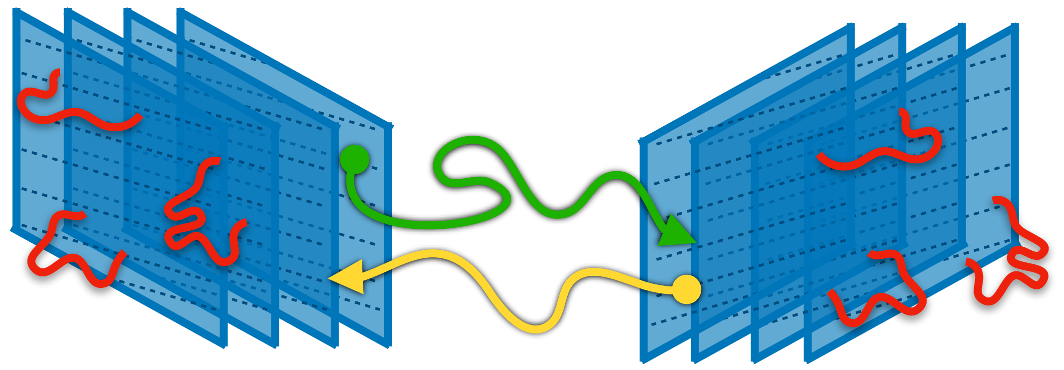
Non-vanishing FI-term implies we have to switch on VEV for adjoint complex scalar Φ living on D7 stack
$\rightarrow$ **T-brane background**

Stability of and further details on T-branes:
Collinucci, Savelli: 1410.4178
Marchesano, Savelli, Schwieger: 1707.03797

The adjoint Φ is broken under $SO(8)\to U(4)$ as

$$\mathbf{28}\to\mathbf{16}_0\oplus\mathbf{6}_{+2}\oplus\mathbf{6}_{-2}$$

$$\Phi = \begin{pmatrix}\phi_{\mathbf{16}_0} & \phi_{\mathbf{6}_{+2}}\\ \phi_{\mathbf{6}_{-2}} & -\phi^T_{\mathbf{16}_0}\end{pmatrix}$$



Stack of 4 D7s

4 image D7s

We give a **VEV to $\phi_{\mathbf{6}_{+2}}$** which recombines some D7s with image D7s

$$\left.\begin{array}{r}\text{Positive FI-term } \xi_{D7}>0\\ \text{Effective line bundle } 0\leq\int_{D_5}J\wedge(D_5-2\mathcal{F})\end{array}\right\} n_b = 1$$

Details on the application of T-branes:
Cicoli, Quevedo, Valandro: 1512.04558
Cicoli, García-Etxebarria, Mayrhofer, Quevedo, Shukla, Valandro: 1706.06128

Total D3-charge coming from D-branes and O-planes is

$$\Gamma = \Gamma_{D7}+\Gamma_{O7} = 8D_b^3(2-n_b+n_b^2)-2D_3^3(1+2n_3+2n_3^2)-D_4^3 \qquad Q_{D3} = -\int_X \Gamma|_{6-\text{form}} = -(52-16n_3-16n_3^2)$$

Moduli stabilisation — Fluxes + D-terms

$(\alpha')^3$-corrected Kähler potential

$$K = -\ln\left(S+\bar{S}\right) - \ln\left(-i\int_X \Omega\wedge\bar{\Omega}\right) - 2\ln\left(\mathcal{V}+\frac{\zeta}{2}\right) + K_Q \qquad \zeta = -\frac{\chi(X)\,\zeta(3)}{2(2\pi)^3\,g_s^{3/2}}$$

Superpotential with $a_3 = 2\pi$

$$W = \int_X G_3\wedge\Omega + A_3\,e^{-a_3 T_3} + W_Q(X,U)$$

Full D/F-term scalar potential for bulk and quiver:

$$V = V_F^{\text{Flux}} + V_F^{\text{quiver}} + V_D^{\text{quiver}} + V_D^{\text{bulk}} + V_F^{\text{LVS}} + V_{\text{soft}}$$

Supersymmetric minima at leading order in the volume $\mathcal{O}(\mathcal{V}^{-2})$

$$V_F^{\text{Flux}} = V_F^{\text{quiver}} = V_D^{\text{quiver}} = V_D^{\text{bulk}} = 0$$

Quiver D/F-term potentials

Non-abelian quiver D-term conditions

$$D_i^a = X_{ij}T^a_{jk}X^\dagger_{ki} - Y_{ij}T^a_{jk}Y^\dagger_{ki} = 0$$

Ingoing arrows   Outgoing arrows

4 constraints on 40 real parameters in VEVs!

Abelian quiver D-term conditions

$$D_i = Q_i^{(ab)}\,\mathrm{Tr}(X^\dagger_{ab}X_{ab}) = 0$$

Vanish automatically! No FI-terms for anomalous $U(1)$!

Quiver F-term conditions

$$D_{X_{ab}}W = \frac{\partial W}{\partial X_{ab}} + \frac{\partial K}{\partial X_{ab}}W = 0$$

36 unfixed real parameters from VEVs plus fluxes to satisfy these conditions!

Bulk D/F-term potentials

Flux scalar potential

$$V_F^{\text{Flux}} = \mathrm{e}^{K_{\text{tree}}}\left(|D_S W|^2 + \sum_{\alpha=1}^{h_-^{1,2}} |D_{U_\alpha}W|^2\right)$$

D-term for D7-brane for a single open string modulus

$$V_D^{\text{bulk}} = \frac{c_1}{\mathcal{V}^{2/3}}\left(q_\varphi|\varphi|^2 - \frac{c_2}{\mathcal{V}^{2/3}}\right)^2$$

Vanishing D-term implies

$$|\varphi|^2 = \frac{c_2}{q_\varphi\mathcal{V}^{2/3}}$$

$c_1 = 0.7552$

$c_2 = 0.2295$

$q_\varphi = 2$

Moduli stabilisation - Kähler moduli

LVS scalar potential

$$V_F^{\rm LVS} = \left(\frac{4\sqrt{2}\,a_3^2\sqrt{\tau_3}\,|A_3|^2\,e^{-2a_3\tau_3}}{\mathcal{V}} + \frac{4a_3|W_0|\,|A_3|\,\tau_3\,e^{-a_3\tau_3}\,\cos\left(a_3\rho_3+\theta_0-\theta_3\right)}{\mathcal{V}^2}\right)+\frac{3\,\zeta\,|W_0|^2}{4\mathcal{V}^3}$$

Contributions from soft scalar masses:

$$V_{\rm soft} = m_\varphi^2|\varphi|^2 \;=\frac{c_2\,m_{3/2}^2}{q_\varphi\mathcal{V}^{2/3}}$$

Total scalar potential for Kähler moduli

$$V_{\rm tot} = \frac{e^{K_{\rm cs}}}{2\,{\rm Re}(S)}\left(V_F^{\rm LVS}+\boxed{\frac{\mathcal{F}_{\rm up}\,|W_0|^2}{\mathcal{V}^{8/3}}}\right)\,,\qquad \mathcal{F}_{\rm up}=\frac{c_2}{q_\varphi}>0$$

Contribution from soft scalar masses

Vacuum energy at non-SUSY LVS minima

$$\langle V_{\rm tot}\rangle \simeq \frac{e^{K_{\rm cs}}|W_0|^2}{18\,{\rm Re}(S)\,\mathcal{V}^3}\left[\mathcal{F}_{\rm up}\mathcal{V}^{1/3}-\frac{9\sqrt{2}}{4a_3}\sqrt{\tau_3}\right]$$

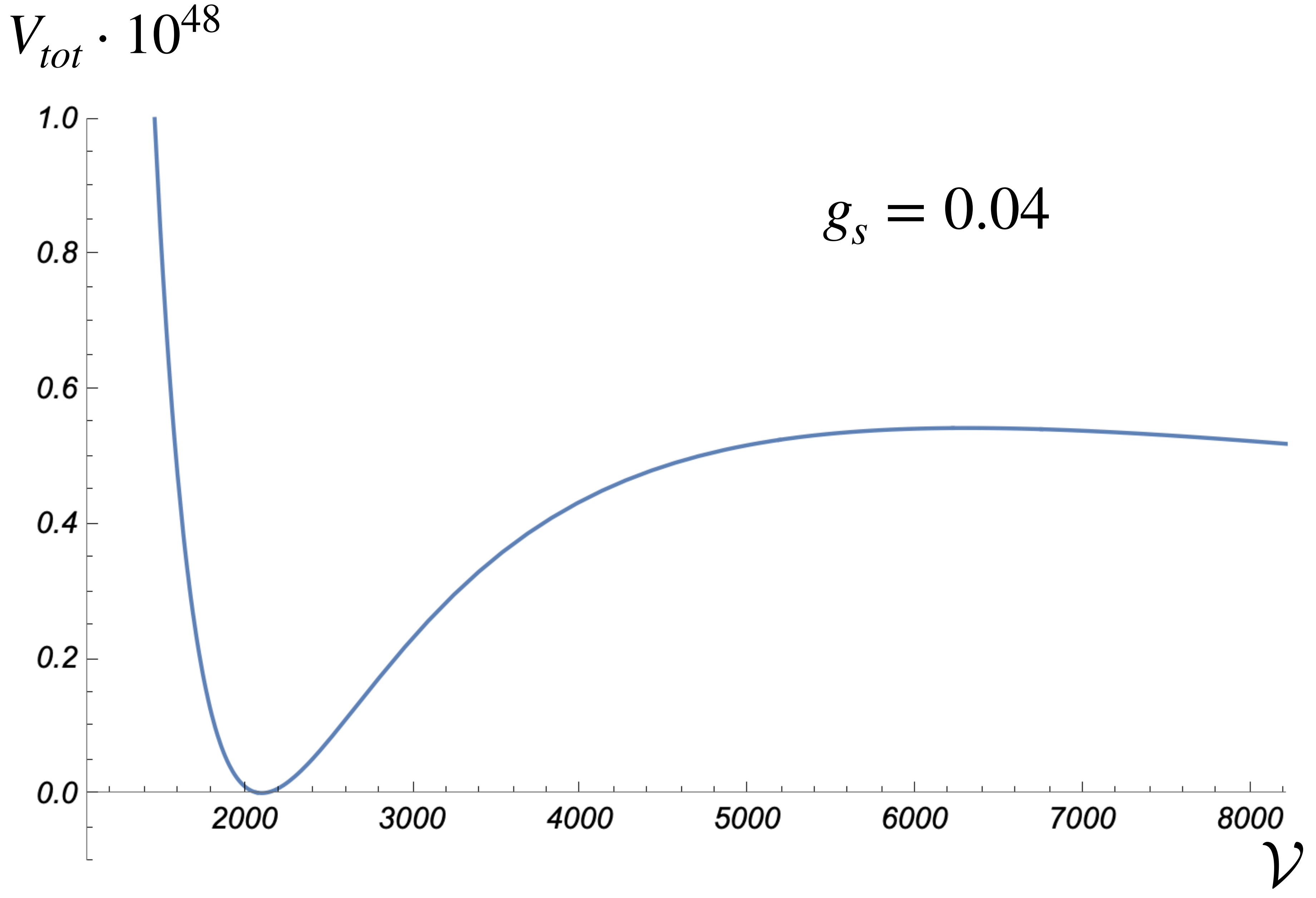
Balasubramanian, Berglund, Conlon, Quevedo: hep-th/0502058

Values for Minkowski minima

g_s	$\lvert W_0\rvert/\lvert A_3\rvert$	$\langle\tau_3\rangle$	$\langle\mathcal{V}\rangle$
0.10	$3.59\cdot10^{-7}$	3.73	579.83
0.08	$2.70\cdot10^{-9}$	4.54	788.07
0.06	$7.25\cdot10^{-13}$	5.89	1179.58
0.04	$4.44\cdot10^{-20}$	8.60	2106.06
0.02	$6.20\cdot10^{-42}$	16.72	5787.28

Comments on these findings:

- **Small W_0** could be achieved with
  A. analytic solutions — Demirtas, Kim, McAllister, Moritz: 1912.10047
  B. stochastic search optimisation — Cole, AS, Shiu: 1907.10072
- Situation can be improved
  1. in models with **more moduli $h^{1,1}>3$**
  2. by using **gaugino condensation**
  3. with other **uplifting source**



Outlook and Conclusions

Summary:

- Standard Model quiver from a single D3-brane at dP_5 singularity
- Explicit construction of compact CY threefolds together with consistent global embedding
- Moduli stabilisation in de Sitter minimum from T-brane uplift

Investigations of the orientifold action on exceptional collections

Combination with inflationary model building (with potentially higher $h^{1,1} > 3$)

To my knowledge, no understanding of these models in F-theory with singular base

Future directions

Systematic approach towards model building with branes at dP_n singularities

Explore other dP_n models with $n \geq 6$

$h^{1,1}$	Poly*	Geom* (n_{CY})	ddP_6	ddP_7	ddP_8
1	5	5	0	0	0
2	36	39	2	4	5
3	243	305	17	40	39
4	1185	2000	109	277	157
5	4897	13494	624	827	407

Thank you!