Non-Simply-Connected Symmetries in SUGRA & 6D SCFTs

Paul-Konstantin Oehlmann

Uppsala Universitet

Based on
- arXiv:1910.04095 with: Nadir Hajouji
- arXiv:2005.12929 with: Markus Dierigl and Fabian Ruehle

Summer Series on String Phenomenology
June 23rd 2020

What is a field theory with $\pi_1(G) \neq 1$?
Example: The group $SU(2)$ admits a $\mathbb{Z}_2$ center and reps carry charges

Matter

Rep	**1**	**2**	**3**	**4**	...

What is a field theory with $\pi_1(G) \neq 1$?
Example: The group $SU(2)$ admits a $\mathbb{Z}_2$ center and reps carry charges

Matter

Rep	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{4}$	$\ldots$
q_{cent}	0	1	0	1	$\ldots$

What is a field theory with $\pi_1(G) \neq 1$?
Example: The group $SU(2)$ admits a $\mathbb{Z}_2$ center and reps carry charges

Matter

Rep	**1**	**2**	**3**	**4**	...
q_{cent}	0✓	1✗	0✓	1✗	...

- Matter with $q_{cent} = 0$ only: electric 1-form symmetry [Gaiotto, Kapustin, Seiberg, Willet'14]
- **Gauging** this symmetry one obtains $SU(2)/\mathbb{Z}_2 \sim SO(3)$
- Gauging impacts the **spectrum** of Wilson Line and t'Hooft surface operators [Aharony, Seiberg, Tachikawa'13]

What is a field theory with $\pi_1(G) \neq 1$?
Example: The group $SU(2)$ admits a $\mathbb{Z}_2$ center and reps carry charges

Matter

Rep	**1**	**2**	**3**	**4**	...
q_{cent}	0✓	1✗	0✓	1✗	...

- Matter with $q_{cent} = 0$ only: electric 1-form symmetry **[Gaiotto, Kapustin, Seiberg, Willet'14]**
- **Gauging** this symmetry one obtains $SU(2)/\mathbb{Z}_2 \sim SO(3)$
- Gauging impacts the **spectrum** of Wilson Line and t'Hooft surface operators **[Aharony, Seiberg, Tachikawa'13]**

Lie Group	Center
SU(p)	$\mathbb{Z}_p$
SO(4n)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
SO(4n+2)	$\mathbb{Z}_4$
E_6	$\mathbb{Z}_3$
E_7	$\mathbb{Z}_2$
E_8	$\mathbb{Z}_1$

What is a field theory with $\pi_1(G) \neq 1$?
Example: The group $SU(2)$ admits a $\mathbb{Z}_2$ center and reps carry charges

Matter

Rep	**1**	**2**	**3**	**4**	...
q_{cent}	0✓	1✗	0✓	1✗	...

- Matter with $q_{cent} = 0$ only: electric 1-form symmetry [Gaiotto, Kapustin, Seiberg, Willet'14]
- **Gauging** this symmetry one obtains $SU(2)/\mathbb{Z}_2 \sim SO(3)$
- Gauging impacts the **spectrum** of Wilson Line and t'Hooft surface operators [Aharony, Seiberg, Tachikawa'13]

Lie Group	Center
SU(p)	$\mathbb{Z}_p$
SO(4n)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
SO(4n+2)	$\mathbb{Z}_4$
E_6	$\mathbb{Z}_3$
E_7	$\mathbb{Z}_2$
E_8	$\mathbb{Z}_1$

$\longleftarrow$ **Can take arbitrary high quotient SU(p)/$\mathbb{Z}_p$?!**

Landscape: There should be a cutoff!

- We want to **systematically** construct SQFTs and SUGRA theories with Gauge groups $G = G^*/T$
- **Flexible** (all sorts of G^*) and **consistent** using geometric arguments
- **Tool of choice is F-theory**: allows (especially in 6D) to construct a large patch of the string landscape and **relate swampland questions to geometry**

- We want to **systematically** construct SQFTs and SUGRA theories with Gauge groups $G = G^*/T$
- **Flexible** (all sorts of G^*) and **consistent** using geometric arguments
- **Tool of choice is F-theory**: allows (especially in 6D) to construct a large patch of the string landscape and **relate swampland questions to geometry**

- $G^* = ABCDE$ algebras geometrically realized by torus fiber singularities [Kodaira'68] ; $\pi_1(G)$ encoded in finite Mordell-Weil group [Aspinwall,Morrison'98; Mayrhofer,Morrison,Till,Weigand'14]
- **M-theory dual** for higher form symmetries in **lower dimensions** [Morrison,Schafer-Nameki,Willet;Albertini,del Zotto,Etxebarria,Hosseini'20]
- Other **fun geometric games**: (Fiberwise-mirror duality, smooth quotients...) [Klevers, Mayorga, O. Piragua, Reuter'14; O.Reuter,Schimanneck; Cvetic,Klevers,Piragua, Poretschkin'16;Anderson,Gray,O.'19]

- We want to **systematically** construct SQFTs and SUGRA theories with Gauge groups $G = G^*/T$
- **Flexible** (all sorts of G^*) and **consistent** using geometric arguments
- **Tool of choice is F-theory**: allows (especially in 6D) to construct a large patch of the string landscape and **relate swampland questions to geometry**

- $G^* = ABCDE$ algebras geometrically realized by torus fiber singularities [Kodaira'68] ; $\pi_1(G)$ encoded in finite Mordell-Weil group [Aspinwall,Morrison'98; Mayrhofer,Morrison,Till,Weigand'14]
- **M-theory dual** for higher form symmetries in **lower dimensions** [Morrison,Schafer-Nameki,Willet;Albertini,del Zotto,Etxebarria,Hosseini'20]
- Other **fun geometric games**: (Fiberwise-mirror duality, smooth quotients...) [Klevers, Mayorga, O. Piragua, Reuter'14; O.Reuter,Schimanneck; Cvetic,Klevers,Piragua, Poretschkin'16;Anderson,Gray,O.'19]
- **Question 1:** what $T = \pi_1(G)$ in a **SUGRA** are allowed?
- **Question 2:** What structure does an **SCFT** with $T = \pi_1(G_F)$ have?

1 Geometrize via F-theory

$\downarrow$

Finite MW group $MW(YX) = T$

$\downarrow$

Restricted Monodromies: $\Gamma_1(p)$

$\downarrow$

Modular curves $X_1(p)$

*(G always non-Abelian!)

2 SUGRA: $G = G^*/T$

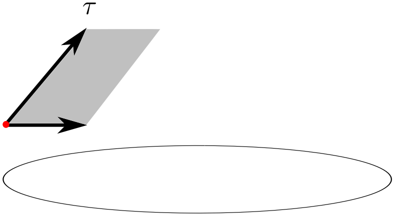
(swampland) Question:

$\boxed{\text{which } T \text{ are possible?}}$

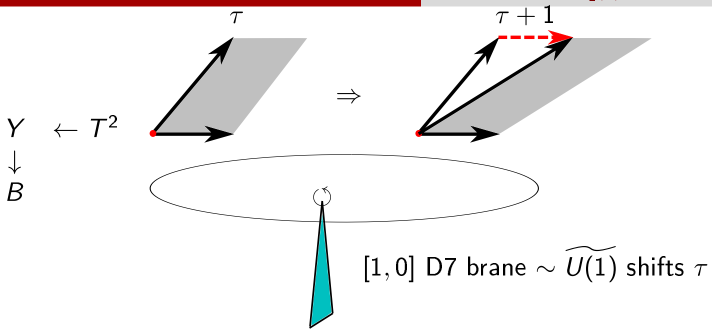
3 6D SCFT: G_F/T

SCFTs with modified global flavor structure

$$\begin{array}{lcl} Y & \leftarrow & T^2 \\ \downarrow & & \\ B & & \end{array}$$



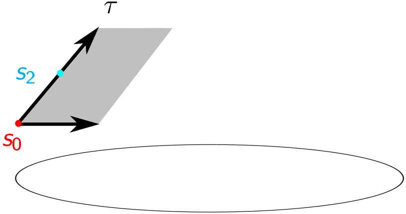
[p,q] 7 Branes act as $SL(2,\mathbb{Z})$ defects on the IIB axio-dilaton τ



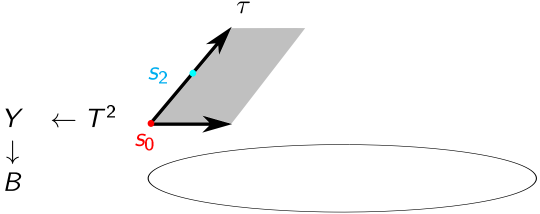
[p,q] 7 Branes act as $SL(2,\mathbb{Z})$ defects on the IIB axio-dilaton τ

- A [1,0] 7-brane acts on τ by a trafo $\mathcal{T} \in SL(2,\mathbb{Z})$
- A [p,q] 7-branes engineered by combinations of $\mathcal{S}, \mathcal{T} \in SL(2,\mathbb{Z})$

$$\begin{array}{ccc} Y & \leftarrow & T^2 \\ \downarrow & & \\ B & & \end{array}$$



Fix an additional point s_p of order p on the elliptic curve $\mathcal{E}$



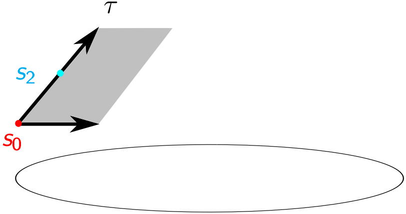
Fix an additional point s_p of order p on the elliptic curve $\mathcal{E}$

- Second point s_p generates an addition law and a group on the elliptic curve
- If $s_p \oplus^{(p)} s_p = s_0$ the group is finite $MW(\mathcal{E}) = T \sim \mathbb{Z}_p$

MW Addition Law

Neutral Element

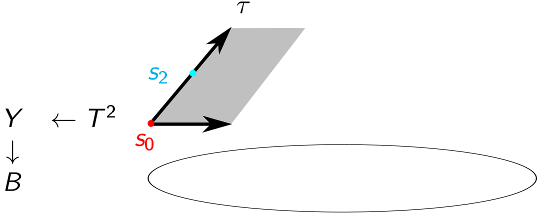
$$\begin{array}{ccc} Y & \leftarrow & T^2 \\ \downarrow & & \\ B & & \end{array}$$



Fix an additional point s_p of order p on the elliptic curve $\mathcal{E}$

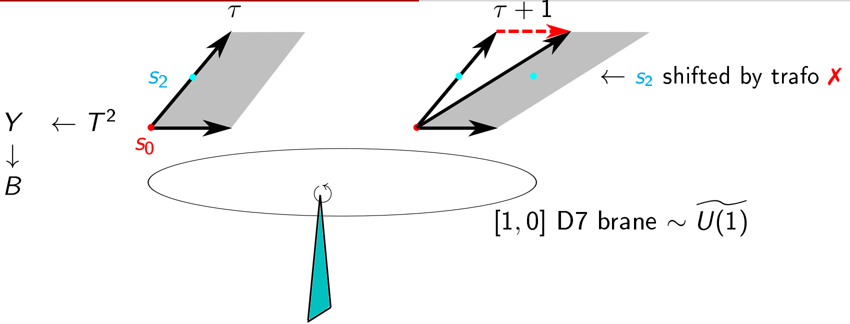
- Second point s_p generates an addition law and a group on the elliptic curve
- If $s_p \oplus^{(p)} s_p = s_0$ the group is finite $MW(\mathcal{E}) = T \sim \mathbb{Z}_p$
- If torsion s_p preserved over all B **construct projector** on states $\sigma(s_p)$ $MW(\mathcal{E}) \hookrightarrow NS(Y_{res})$, [Mayrhofer,Morrison,Till,Weigand'14]

$\boxed{\sigma(s_p) \text{ allows only compatible matter representations } \checkmark}$



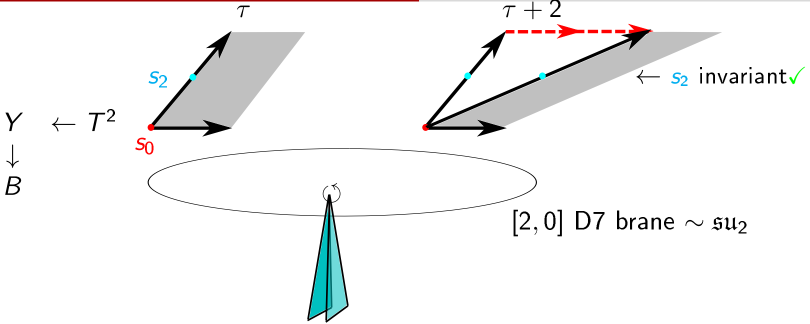
Fix an additional point s_p of order p on the elliptic curve $\mathcal{E}$

- Second point s_p generates an addition law and a group on the elliptic curve
- If $s_p \oplus^{(p)} s_p = s_0$ the group is finite $MW(\mathcal{E}) = T \sim \mathbb{Z}_p$
- **Different perspective:** which $SL(2,\mathbb{Z})$ elements **stabilize** the $\mathbb{Z}_p$ torsion point



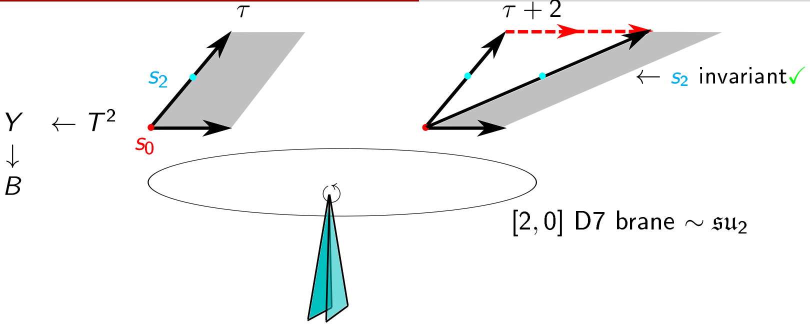
Fix an additional point s_p of order p on the elliptic curve $\mathcal{E}$

- Second point s_p generates an addition law and a group on the elliptic curve
- If $s_p \oplus^{(p)} s_p = s_0$ the group is finite $MW(\mathcal{E}) = T \sim \mathbb{Z}_p$
- **Different perspective:** which $SL(2,\mathbb{Z})$ elements **stabilize** the $\mathbb{Z}_p$ torsion point



Fix an additional point s_p of order p on the elliptic curve $\mathcal{E}$

- Second point s_p generates an addition law and a group on the elliptic curve
- If $s_p \oplus^{(p)} s_p = s_0$ the group is finite $MW(\mathcal{E}) = T \sim \mathbb{Z}_p$
- **Different perspective:** which $SL(2,\mathbb{Z})$ elements **stabilize** the $\mathbb{Z}_p$ torsion point



Fix an additional point s_p of order p on the elliptic curve $\mathcal{E}$

- Second point s_p generates an addition law and a group on the elliptic curve
- If $s_p \oplus^{(p)} s_p = s_0$ the group is finite $MW(\mathcal{E}) = T \sim \mathbb{Z}_p$
- **Different perspective:** which $SL(2,\mathbb{Z})$ elements **stabilize** the $\mathbb{Z}_p$ torsion point
- This is the **congruence subgroup** $\Gamma_1(p) \in SL(2,\mathbb{Z})$

$$\Gamma_1(p) = \{\gamma \in \mathrm{SL}(2,\mathbb{Z}) : \gamma \equiv \begin{pmatrix} 1 & 0 \\ * & 1 \end{pmatrix} \bmod p\}\,. \quad (1)$$

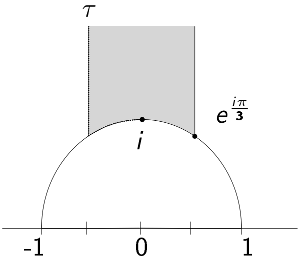
Fiber	(f, g, Δ)	Monodromy	Subgroups	Group	center
II^*	$(\geq 1, 1, 2)$	$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$	-	$-$	-
I_n	$(0, 0, n)$	$\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$	$\Gamma_1(n), \Gamma(n)$	$SU(n)$	$\mathbb{Z}_n$
III	$(\geq 1, \geq 2, 3)$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$\Gamma_1(2)$	$SU(2)$	$\mathbb{Z}_2$
IV	$(\geq 2, 2, 4)$	$\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$	$\Gamma_1(3)$	$SU(3)$	$\mathbb{Z}_3$
I_{2n}^*	$(\geq 2, 3, 6+2n)$	$\begin{pmatrix} -1 & -2n \\ 0 & -1 \end{pmatrix}$	$\Gamma_1(2), \Gamma(2)$	$SO(8+4n)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
I_{2n+1}^*	$(\geq 2, 3, 7+2n)$	$\begin{pmatrix} -1 & -2n-1 \\ 0 & -1 \end{pmatrix}$	$\Gamma_1(2), \Gamma_1(4)$	$SO(10+4n)$	$\mathbb{Z}_4$
IV^*	$(\geq 3, 4, 8)$	$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$	$\Gamma_1(3)$	E_6	$\mathbb{Z}_3$
III^*	$(3, \geq 5, 9)$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$\Gamma_1(2)$	E_7	$\mathbb{Z}_2$
II^*	$(4, \geq 5, 10)$	$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$	-	E_8	-

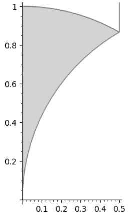
7 Brane monodromies consistent with center of the covering group ($\Gamma(p)$ fixes a $\mathbb{Z}_p \times \mathbb{Z}_p$ group)

Fiber	(f,g,Δ)	Monodromy	Subgroups	Group	center
II^*	$(\geq 1,1,2)$	$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$	-	$-$	-
I_n	$(0,0,n)$	$\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$	$\Gamma_1(n), \Gamma(n)$	$SU(n)$	$\mathbb{Z}_n$
III	$(\geq 1, \geq 2, 3)$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$\Gamma_1(2)$	$SU(2)$	$\mathbb{Z}_2$
IV	$(\geq 2,2,4)$	$\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$	$\Gamma_1(3)$	$SU(3)$	$\mathbb{Z}_3$
I_{2n}^*	$(\geq 2,3,6+2n)$	$\begin{pmatrix} -1 & -2n \\ 0 & -1 \end{pmatrix}$	$\Gamma_1(2), \Gamma(2)$	$SO(8+4n)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
I_{2n+1}^*	$(\geq 2,3,7+2n)$	$\begin{pmatrix} -1 & -2n-1 \\ 0 & -1 \end{pmatrix}$	$\Gamma_1(2), \Gamma_1(4)$	$SO(10+4n)$	$\mathbb{Z}_4$
IV^*	$(\geq 3,4,8)$	$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$	$\Gamma_1(3)$	E_6	$\mathbb{Z}_3$
III^*	$(3, \geq 5, 9)$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$\Gamma_1(2)$	E_7	$\mathbb{Z}_2$
II^*	$(4, \geq 5, 10)$	$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$	-	E_8	-

7 Brane monodromies consistent with center of the covering group ($\Gamma(p)$ fixes a $\mathbb{Z}_p \times \mathbb{Z}_p$ group)

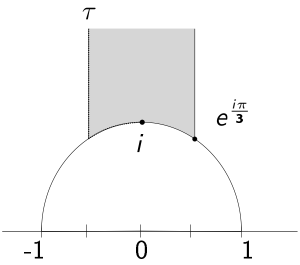
Q: **What is the highest torsion point s_p a compact fibration can preserve?**
A: $T^2 : \mathbb{Z}_{12}$ $\quad$ $K3 : \mathbb{Z}_8$, $\quad$ $\mathrm{CY}_{3/4}$?? [Mazur'78; Oguiso, Shioda'91; Shimada'00]

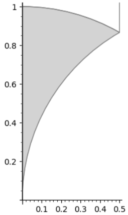




What is the **moduli space** $X(1)$ of an elliptic curve $\mathcal{E}_\tau$?

- "The Key Hole" no structure: $X(1)^* = \tau \in \mathcal{H}/SL(2,\mathbb{Z})$
- There is a **cusp** at infinity where $\Delta = 4f^3 + 27g^2 = 0$
- Adding the point at infinity we obtain the compactified modular curve $X(1)$ topologically a $\mathbb{P}^1$.



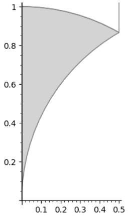


What is the **moduli space** $X(1)$ of an elliptic curve $\mathcal{E}_\tau$?

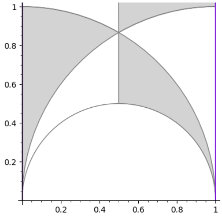
- "The Key Hole" no structure: $X(1)^* = \tau \in \mathcal{H}/SL(2,\mathbb{Z})$
- There is a **cusp** at infinity where $\Delta = 4f^3 + 27g^2 = 0$
- Adding the point at infinity we obtain the compactified modular curve $X(1)$ topologically a $\mathbb{P}^1$.

Over a compact base space you **will always hit** the I_1 **cusp** somewhere

$X(1)$



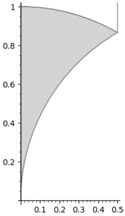
$X_1(2)$



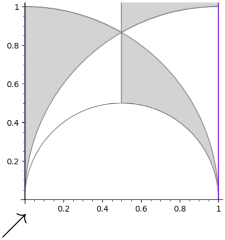
What is the moduli space $X_1(p)$ of an elliptic curve and a torsion point s_p?

- **Tessellation** into triangular regions of modular curve $X_1(p) = \tau \in \mathcal{H}/\Gamma_1(p)$

$X(1)$



$X_1(2)$

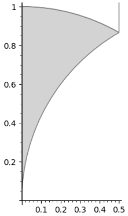


new I_2 cusp from 2 triangles meeting

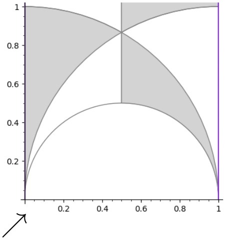
What is the moduli space $X_1(p)$ of an elliptic curve and a torsion point s_p?

- **Tessellation** into triangular regions of modular curve $X_1(p) = \tau \in \mathcal{H}/\Gamma_1(p)$
- **Surprise: additional cusps of higher order** where triangles meet

$X(1)$



$X_1(2)$



new I_2 cusp from 2 triangles meeting

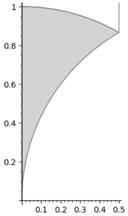
What is the moduli space $X_1(p)$ of an elliptic curve and a torsion point s_p?

- **Tessellation** into triangular regions of modular curve $X_1(p) = \tau \in \mathcal{H}/\Gamma_1(p)$
- **Surprise: additional cusps of higher order** where triangles meet

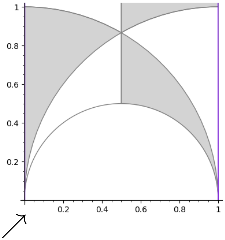
Equivalent construct the Weierstrass model explicitly [Aspinwall, Morrison'98]

s_2@$y = x = 0$: $\quad y^2 = x(x + a_2xz^2 + a_4z^4)\,,\quad \Delta = \underbrace{a_4^2}_{I_2}\underbrace{(a^2 + a_4)}_{I_1} \quad a_i \in c_1(B)^i$

$X(1)$



$X_1(2)$



new I_2 cusp from 2 triangles meeting

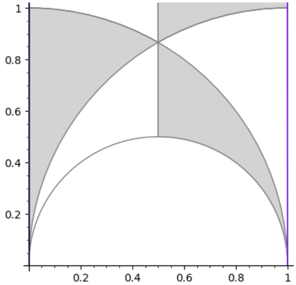
What is the moduli space $X_1(p)$ of an elliptic curve and a torsion point s_p?

- **Tessellation** into triangular regions of modular curve $X_1(p) = \tau \in \mathcal{H}/\Gamma_1(p)$
- **Surprise: additional cusps of higher order** where triangles meet

Equivalent construct the Weierstrass model explicitly [Aspinwall,Morrison'98]

$$s_2 @ y = x = 0 : \quad y^2 = x(x + a_2 x z^2 + a_4 z^4)\,, \quad \Delta = \underbrace{a_4^2}_{I_2}\underbrace{(a^2 + a_4)}_{I_1} \quad a_i \in c_1(B)^i$$

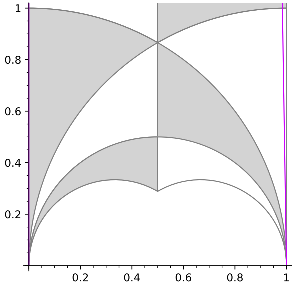
$\boxed{\text{Physics: } \pi_1(G^*/T) = \mathbb{Z}_2 \text{ requires } G^* = SU(2) \text{ at least!}}$



$\Gamma_1(p)$	T	G_{min}
$\Gamma_1(2)$	$\mathbb{Z}_2$	$SU(2)/\mathbb{Z}_2$

Cusps in the modular curve **enforce minimal gauge groups** G with $\pi(G) = T$!

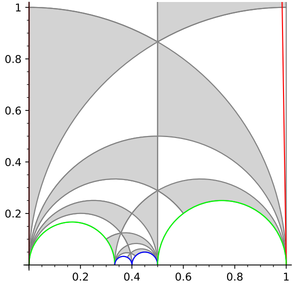
- How does this moduli space look like for large torsion?
- Larger torsion enforces **multiple** gauge algebra factors!



$\Gamma_1(p)$	T	G_{min}
$\Gamma_1(2)$	$\mathbb{Z}_2$	$SU(2)/\mathbb{Z}_2$
$\Gamma_1(3)$	$\mathbb{Z}_3$	$SU(3)/\mathbb{Z}_3$

Cusps in the modular curve **enforce minimal gauge groups** G with $\pi(G) = T$!

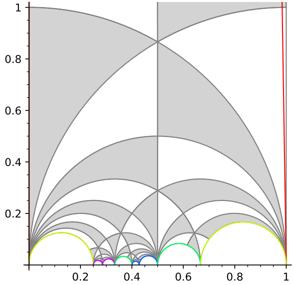
- How does this moduli space look like for large torsion?
- Larger torsion enforces **multiple** gauge algebra factors!



$\Gamma_1(p)$	T	G_{min}
$\Gamma_1(2)$	$\mathbb{Z}_2$	$SU(2)/\mathbb{Z}_2$
$\Gamma_1(3)$	$\mathbb{Z}_3$	$SU(3)/\mathbb{Z}_3$
$\Gamma_1(5)$	$\mathbb{Z}_5$	$SU(5)^2/\mathbb{Z}_5$

Cusps in the modular curve **enforce minimal gauge groups** G with $\pi(G) = T$!

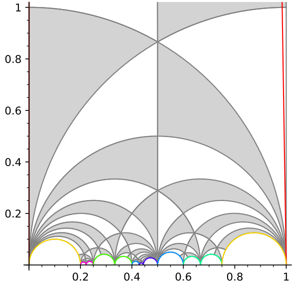
- How does this moduli space look like for large torsion?
- Larger torsion enforces **multiple** gauge algebra factors!



$\Gamma_1(p)$	T	G_{min}
$\Gamma_1(2)$	$\mathbb{Z}_2$	$SU(2)/\mathbb{Z}_2$
$\Gamma_1(3)$	$\mathbb{Z}_3$	$SU(3)/\mathbb{Z}_3$
$\Gamma_1(5)$	$\mathbb{Z}_5$	$SU(5)^2/\mathbb{Z}_5$
$\Gamma_1(7)$	$\mathbb{Z}_7$	$SU(7)^3/\mathbb{Z}_7$

Cusps in the modular curve **enforce minimal gauge groups** G with $\pi(G) = T$!

- How does this moduli space look like for large torsion?
- Larger torsion enforces **multiple** gauge algebra factors!



$\Gamma_1(p)$	T	G_{min}
$\Gamma_1(2)$	$\mathbb{Z}_2$	$SU(2)/\mathbb{Z}_2$
$\Gamma_1(3)$	$\mathbb{Z}_3$	$SU(3)/\mathbb{Z}_3$
$\Gamma_1(5)$	$\mathbb{Z}_5$	$SU(5)^2/\mathbb{Z}_5$
$\Gamma_1(7)$	$\mathbb{Z}_7$	$SU(7)^3/\mathbb{Z}_7$
$\Gamma_1(9)$	$\mathbb{Z}_9$	$SU(9)^4/\mathbb{Z}_9$

Cusps in the modular curve **enforce minimal gauge groups** G with $\pi(G) = T$!

- How does this moduli space look like for large torsion?
- Larger torsion enforces **multiple** gauge algebra factors!

Torsion	$K(\mathbb{P}^1)$	K3	dP_9	$CY_{3/4}$
$\{0\}$	✓	✓	✓	✓
$\{\mathbb{Z}_2\}$	✓	✓	✓	✓
$\{\mathbb{Z}_3\}$	✓	✓	✓	✓
$\{\mathbb{Z}_4\}$	✓	✓	✓	✓
$\{\mathbb{Z}_2 \times \mathbb{Z}_2\}$	✓	✓	✓	✓
$\{\mathbb{Z}_5\}$	✓	✓	✓	✓
$\{\mathbb{Z}_6\}$	✓	✓	✓	✓
$\{\mathbb{Z}_2 \times \mathbb{Z}_4\}$	✓	✓	✓	✓
$\{\mathbb{Z}_3 \times \mathbb{Z}_3\}$	✓	✓	✓	✓
$\{\mathbb{Z}_7\}$	✓	✓	✗	✗
$\{\mathbb{Z}_8\}$	✓	✓	✗	✗
$\{\mathbb{Z}_2 \times \mathbb{Z}_6\}$	✓	✓	✗	✗
$\{\mathbb{Z}_2 \times \mathbb{Z}_8\}$	✓	✓	✗	✗
$\{\mathbb{Z}_4 \times \mathbb{Z}_4\}$	✓	✓	✗	✗
$\{\mathbb{Z}_9\}$	✓	✗	✗	✗
$\{\mathbb{Z}_{10}\}$	✓	✗	✗	✗
$\{\mathbb{Z}_{12}\}$	✓	✗	✗	✗
$\{\mathbb{Z}_3 \times \mathbb{Z}_6\}$	✓	✗	✗	✗
$\{\mathbb{Z}_5 \times \mathbb{Z}_5\}$	✓	✗	✗	✗

Constraints for surfaces:

- rank$(G_{min}) > 8$ no dP$_9$
- rank$(G_{min}) > 16$ no K3

[Miranda,Persson'89] ✓

Torsion	$K(\mathbb{P}^1)$	K3	dP_9	$CY_{3/4}$
$\{0\}$	✓	✓	✓	✓
$\{\mathbb{Z}_2\}$	✓	✓	✓	✓
$\{\mathbb{Z}_3\}$	✓	✓	✓	✓
$\{\mathbb{Z}_4\}$	✓	✓	✓	✓
$\{\mathbb{Z}_2 \times \mathbb{Z}_2\}$	✓	✓	✓	✓
$\{\mathbb{Z}_5\}$	✓	✓	✓	✓
$\{\mathbb{Z}_6\}$	✓	✓	✓	✓
$\{\mathbb{Z}_2 \times \mathbb{Z}_4\}$	✓	✓	✓	✓
$\{\mathbb{Z}_3 \times \mathbb{Z}_3\}$	✓	✓	✓	✓
$\{\mathbb{Z}_7\}$	✓	✓	✗	✗
$\{\mathbb{Z}_8\}$	✓	✓	✗	✗
$\{\mathbb{Z}_2 \times \mathbb{Z}_6\}$	✓	✓	✗	✗
$\{\mathbb{Z}_2 \times \mathbb{Z}_8\}$	✓	✓	✗	✗
$\{\mathbb{Z}_4 \times \mathbb{Z}_4\}$	✓	✓	✗	✗
$\{\mathbb{Z}_9\}$	✓	✗	✗	✗
$\{\mathbb{Z}_{10}\}$	✓	✗	✗	✗
$\{\mathbb{Z}_{12}\}$	✓	✗	✗	✗
$\{\mathbb{Z}_3 \times \mathbb{Z}_6\}$	✓	✗	✗	✗
$\{\mathbb{Z}_5 \times \mathbb{Z}_5\}$	✓	✗	✗	✗

Constraints for surfaces:

- rank(G_{min}) > 8 no dP_9
- rank(G_{min})>16 no K3

[Miranda,Persson'89] ✓

Constraints for $CY_{3/4}$

New non-minimal singularities:

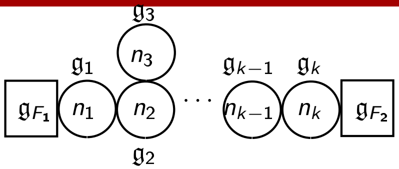
$T = \mathbb{Z}_p$	Singularities
• p$<$ 4	Regular
• 4$<$p$<$ 6	(4,6,12) in codim 2
• 6$<$p $<$ 6	(8,12,24) in codim 2

$\rightarrow$ No crepant resolution ✗

$CY_{3/4}$ folds: $T \in dP_9 \in E_8$

Aspinwall-Morrison'98 is complete!

SCFTs with restricted Monodromies

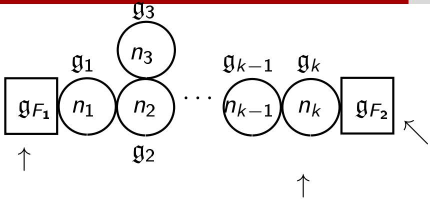


2 dim non-compact Base B_2

6D SCFT construction in IIB: Take B_2 to be **non-compact** and of the form of

- A generalized quiver of k **compact** $\mathbb{P}^1$'s wrapped by 7-branes giving rise to **gauge algebras** $\mathfrak{g}_i$ and bifundamental matter
- **Non-compact Flavor branes** with flavor algebra $\mathfrak{g}_{F_i}$

[Witten'96; Seiberg'96;Harvey, Minasian, Moore,Heckman, Vafa, Hanany, Zaffaroni,del Zotto, Rudelius, Morrison,Mekareeya,Tomasiello,Ohmori, Tachikawa, Zafrir,Park,Bertolini, Merkx, Cordova, Dumitrescu, Intrilligator,Bhardwaj...]



2 dim non-compact Base B_2

Flavor Brane

Flavor Brane

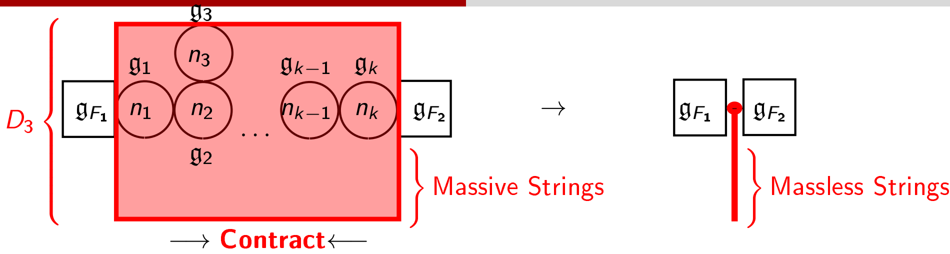
Compact $\mathbb{P}^1$ of selfintersection -n$_k$

and gauge algebra $\mathfrak{g}_k$

6D SCFT construction in IIB: Take B$_2$ to be **non-compact** and of the form of

- A generalized quiver of k **compact** $\mathbb{P}^1$'s wrapped by 7-branes giving rise to **gauge algebras** $\mathfrak{g}_i$ and bifundamental matter
- **Non-compact Flavor branes** with flavor algebra $\mathfrak{g}_{F_i}$

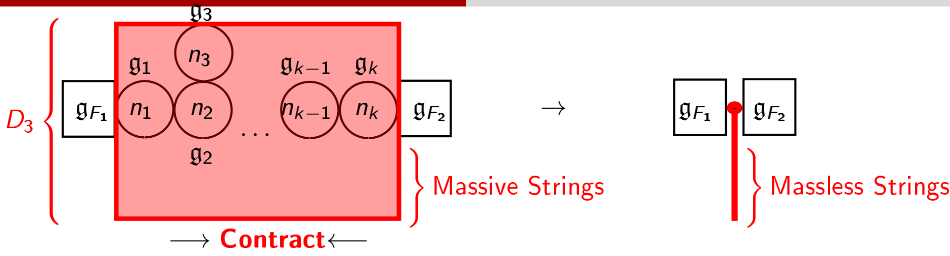
[Witten'96; Seiberg'96;Harvey, Minasian, Moore,Heckman, Vafa, Hanany, Zaffaroni,del Zotto, Rudelius, Morrison,Mekareeya,Tomasiello,Ohmori, Tachikawa, Zafrir,Park,Bertolini, Merkx, Cordova, Dumitrescu, Intrilligator,Bhardwaj...]



$$T_{string} \sim Vol(\mathbb{P}^1_i)$$

6D SCFT construction in IIB: Take B_2 to be **non-compact** and of the form of

- A generalized quiver of k **compact** $\mathbb{P}^1$'s wrapped by 7-branes giving rise to **gauge algebras** $\mathfrak{g}_i$ and bifundamental matter
- **Non-compact Flavor branes** with flavor algebra $\mathfrak{g}_{F_i}$
- **Massive 6D strings** from D3 branes that wrap compact cycles
- **SCFT point** where strings become **tensionless** at the origin of the tensor branch by **contracting all curves** (if possible) [Witten'96; Seiberg'96;Harvey, Minasian, Moore,Heckman, Vafa, Hanany, Zaffaroni,del Zotto, Rudelius, Morrison,Mekareeya,Tomasiello,Ohmori, Tachikawa, Zafrir,Park,Bertolini, Merkx, Cordova, Dumitrescu, Intrilligator,Bhardwaj...]



$$T_{string} \sim Vol(\mathbb{P}^1_i)$$

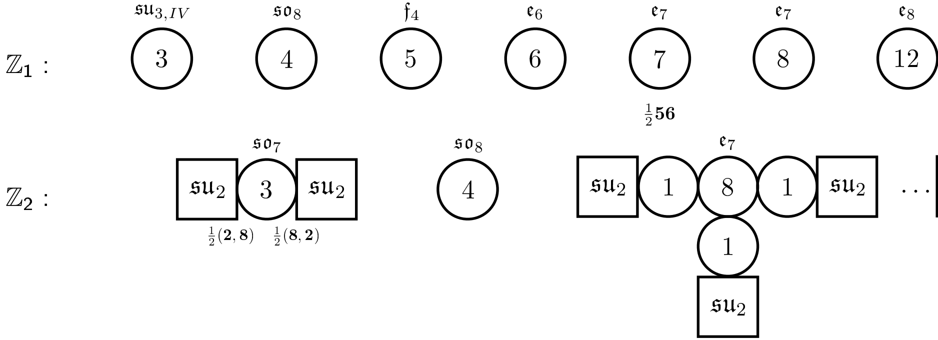
6D SCFT construction in IIB: Take B_2 to be **non-compact** and of the form of

- A generalized quiver of k **compact** $\mathbb{P}^1$'s wrapped by 7-branes giving rise to **gauge algebras** $\mathfrak{g}_i$ and bifundamental matter
- **Non-compact Flavor branes** with flavor algebra $\mathfrak{g}_{F_i}$
- **Massive 6D strings** from D3 branes that wrap compact cycles
- **SCFT point** where strings become **tensionless** at the origin of the tensor branch by **contracting all curves** (if possible) [Witten'96; Seiberg'96;Harvey, Minasian, Moore,Heckman, Vafa, Hanany, Zaffaroni,del Zotto, Rudelius, Morrison,Mekareeya,Tomasiello,Ohmori, Tachikawa, Zafrir,Park,Bertolini, Merkx, Cordova, Dumitrescu, Intrilligator,Bhardwaj...]

Construct SCFTs with global quotient structure in the Flavor group, by restricting to a congruence subgroup!

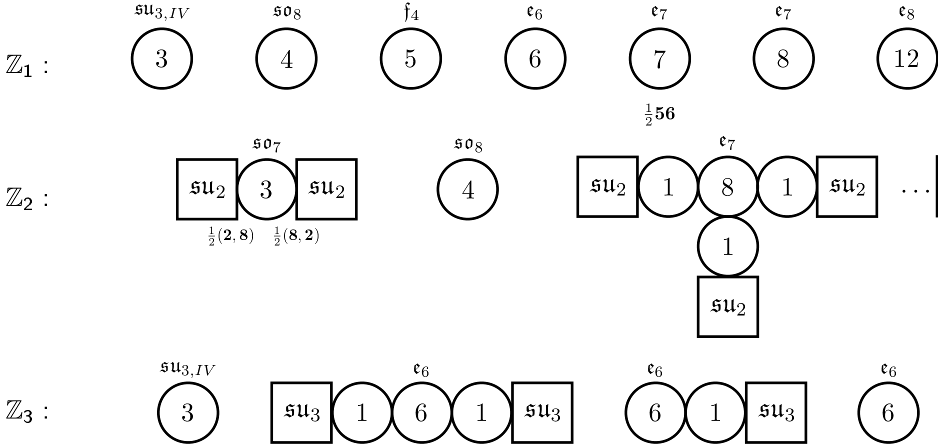
	$\mathfrak{su}_{3,IV}$	$\mathfrak{so}_8$	$\mathfrak{f}_4$	$\mathfrak{e}_6$	$\mathfrak{e}_7$	$\mathfrak{e}_7$	$\mathfrak{e}_8$
$\mathbb{Z}_1$:	3	4	5	6	7	8	12
					$\frac{1}{2}\mathbf{56}$		

1. Basic building block: Non-Higgsable Clusters [Taylor Morrison'12]
(chains) of $\mathbb{P}^1$'s with high negative self intersection $n > 2$



1. Basic building block: Non-Higgsable Clusters [Taylor Morrison'12]
(chains) of $\mathbb{P}^1$'s with high negative self intersection $n > 2$

- Combine $n > 2$ curves with torsion models!



1. Basic building block: Non-Higgsable Clusters [Taylor Morrison'12]
(chains) of $\mathbb{P}^1$'s with high negative self intersection $n > 2$

- Combine $n > 2$ curves with torsion models!
- Torsion model **enhances** the NHC to a compatible gauge group

$$\mathbb{Z}_1 : \quad \boxed{\mathfrak{su}_1} - \boxed{\mathfrak{e}_8}$$

2. Buidling Block: When bifundamental matter is not perturbatively allowed, it becomes superconformal matter, a 6D SCFT by itself

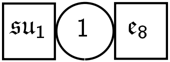
- **Simplest case**, E-string theory $\mathfrak{su}_1 \times \mathfrak{e}_8$ collision
- Changed to **discrete holonomy instanton** theories [Aspinwall,Morrison'98]

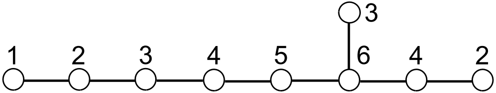
$$\mathbb{Z}_1 : \quad \boxed{\mathfrak{su}_1} \, \textcircled{1} \, \boxed{\mathfrak{e}_8}$$

2. Buidling Block: When bifundamental matter is not perturbatively allowed, it becomes superconformal matter, a 6D SCFT by itself

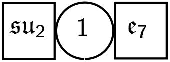
- **Simplest case**, E-string theory $\mathfrak{su}_1 \times \mathfrak{e}_8$ collision
- Changed to **discrete holonomy instanton** theories [Aspinwall,Morrison'98]

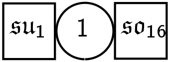
$\mathbb{Z}_1$:





$\mathbb{Z}_2$:

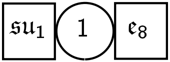


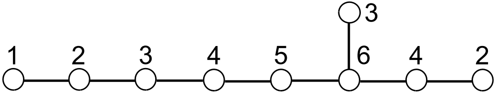


2. Buidling Block: When bifundamental matter is not perturbatively allowed, it becomes superconformal matter, a 6D SCFT by itself

- **Simplest case**, E-string theory $\mathfrak{su}_1 \times \mathfrak{e}_8$ collision
- Changed to **discrete holonomy instanton** theories [Aspinwall,Morrison'98]

$\mathbb{Z}_1$:





$\mathbb{Z}_2$:

$\mathbb{Z}_3$:

2. Buidling Block: When bifundamental matter is not perturbatively allowed, it becomes superconformal matter, a 6D SCFT by itself

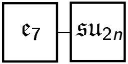
- **Simplest case**, E-string theory $\mathfrak{su}_1 \times \mathfrak{e}_8$ collision
- Changed to **discrete holonomy instanton** theories [Aspinwall,Morrison'98]

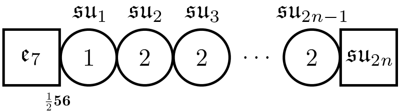
$\mathbb{Z}_1 : \quad \boxed{\mathfrak{e}_7} - \boxed{\mathfrak{su}_{2n}}$

Higher rank collisions can be systematically constructed and their tensor branch can be analyzed

- Example:$[\mathfrak{e}_7] \times [\mathfrak{su}_{2n}]$ admits an diagonal $\mathbb{Z}_2$ center
  Inconsistent with modding: $\mathfrak{su}_{2k+1}$ factors, $\mathbf{56}$ matter, $(\mathbf{2k}, \overline{\mathbf{2k-1}})$ matter
- $([E_7] \times [SU(2n)])/\mathbb{Z}_2$ admits a **different tensor branch** gauge group and matter compatible with global modding [Ohmori, Tachikawa, Zafir'18]

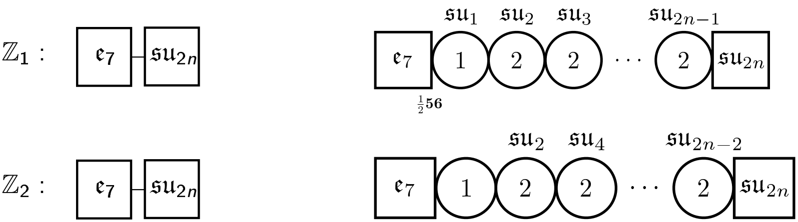
$\mathbb{Z}_1$:





Higher rank collisions can be systematically constructed and their tensor branch can be analyzed

- Example:$[\mathfrak{e}_7] \times [\mathfrak{su}_{2n}]$ admits an diagonal $\mathbb{Z}_2$ center
  Inconsistent with modding: $\mathfrak{su}_{2k+1}$ factors, $\mathbf{56}$ matter, $(\mathbf{2k}, \overline{\mathbf{2k-1}})$ matter
- $([E_7] \times [SU(2n)])/\mathbb{Z}_2$ admits a **different tensor branch** gauge group and matter compatible with global modding [Ohmori, Tachikawa, Zafir'18]



Higher rank collisions can be systematically constructed and their tensor branch can be analyzed

- Example:$[\mathfrak{e}_7] \times [\mathfrak{su}_{2n}]$ admits an diagonal $\mathbb{Z}_2$ center
  Inconsistent with modding: $\mathfrak{su}_{2k+1}$ factors, $\mathbf{56}$ matter, $(\mathbf{2k}, \overline{\mathbf{2k-1}})$ matter
- $([E_7] \times [SU(2n)])/\mathbb{Z}_2$ admits a **different tensor branch** gauge group and matter compatible with global modding [Ohmori, Tachikawa, Zafir'18]

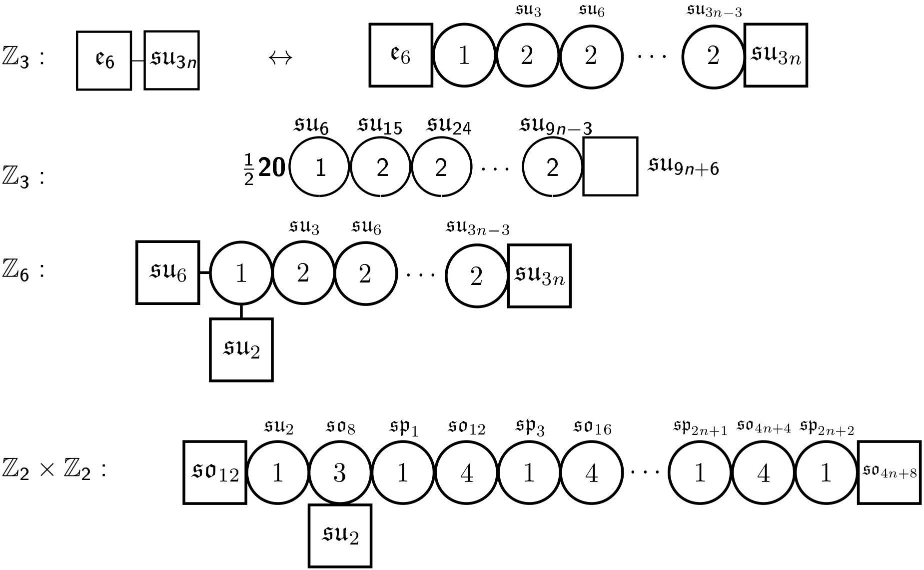
$$\mathbb{Z}_3 : \quad \boxed{\mathfrak{e}_6} - \boxed{\mathfrak{su}_{3n}} \qquad \leftrightarrow \qquad \boxed{\mathfrak{e}_6}\; 1 \; \overset{\mathfrak{su}_3}{2} \; \overset{\mathfrak{su}_6}{2} \; \cdots \; \overset{\mathfrak{su}_{3n-3}}{2} \; \boxed{\mathfrak{su}_{3n}}$$

A wide range of possibilities: exotic Matter, non-prime quotients, multiple generators.....

$$\mathbb{Z}_3: \quad \boxed{\mathfrak{e}_6} - \boxed{\mathfrak{su}_{3n}} \quad \leftrightarrow \quad \boxed{\mathfrak{e}_6}\; 1 \; \overset{\mathfrak{su}_3}{2} \; \overset{\mathfrak{su}_6}{2} \cdots \overset{\mathfrak{su}_{3n-3}}{2} \; \boxed{\mathfrak{su}_{3n}}$$

$$\mathbb{Z}_3: \quad \tfrac{1}{2}\mathbf{20}\; \overset{\mathfrak{su}_6}{1} \; \overset{\mathfrak{su}_{15}}{2} \; \overset{\mathfrak{su}_{24}}{2} \cdots \overset{\mathfrak{su}_{9n-3}}{2} \; \boxed{} \; \mathfrak{su}_{9n+6}$$

A wide range of possibilities: exotic Matter, non-prime quotients, multiple generators.....



A wide range of possibilities: exotic Matter, non-prime quotients, multiple generators.....

Elliptic fibrations with additional points s_p of order p $\pi_1(G) = T = \mathbb{Z}_p$.

Use the **same geometric approach** to construct **SCFTs** with non-simply connected flavor groups $\pi_1(G_F) = T$.

Elliptic fibrations with additional points s_p of order p $\pi_1(G) = T = \mathbb{Z}_p$.

- Fibrations have **restricted monodromies** that lie in a congruence subgroup $\Gamma_1(p)$ (or generalizations)
- **Minimal gauge groups** in compact geometries can be deduced from the modular curve $X_1(n) = \mathcal{H}/\Gamma_1(n)$
- **Swampland criterion** in compact geometries $\pi_1(G)$ must be finite Abelian subgroup of E_8, otherwise non-crepant singularities (modulo 8D)

Use the **same geometric approach** to construct **SCFTs** with non-simply connected flavor groups $\pi_1(G_F) = T$.

Elliptic fibrations with additional points s_p of order p $\pi_1(G) = T = \mathbb{Z}_p$.

- Fibrations have **restricted monodromies** that lie in a congruence subgroup $\Gamma_1(p)$ (or generalizations)
- **Minimal gauge groups** in compact geometries can be deduced from the modular curve $X_1(n) = \mathcal{H}/\Gamma_1(n)$
- **Swampland criterion** in compact geometries $\pi_1(G)$ must be finite Abelian subgroup of E_8, otherwise non-crepant singularities (modulo 8D)

Use the **same geometric approach** to construct **SCFTs** with non-simply connected flavor groups $\pi_1(G_F) = T$.

- **Enhanced** flavor and gauge algebras over **non-Higgsable clusters**
- **Quotient extends** to gauge group and matter at the tensor branch

Elliptic fibrations with additional points s_p of order p $\pi_1(G) = T = \mathbb{Z}_p$.

- Fibrations have **restricted monodromies** that lie in a congruence subgroup $\Gamma_1(p)$ (or generalizations)
- **Minimal gauge groups** in compact geometries can be deduced from the modular curve $X_1(n) = \mathcal{H}/\Gamma_1(n)$
- **Swampland criterion** in compact geometries $\pi_1(G)$ must be finite Abelian subgroup of E_8, otherwise non-crepant singularities (modulo 8D)

Use the **same geometric approach** to construct **SCFTs** with non-simply connected flavor groups $\pi_1(G_F) = T$.

- **Enhanced** flavor and gauge algebras over **non-Higgsable clusters**
- **Quotient extends** to gauge group and matter at the tensor branch

Thank you very much!