8d Supergravity, Reconstruction of Internal Geometry and the Swampland

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with Cumrun Vafa arXiv:2104.05724

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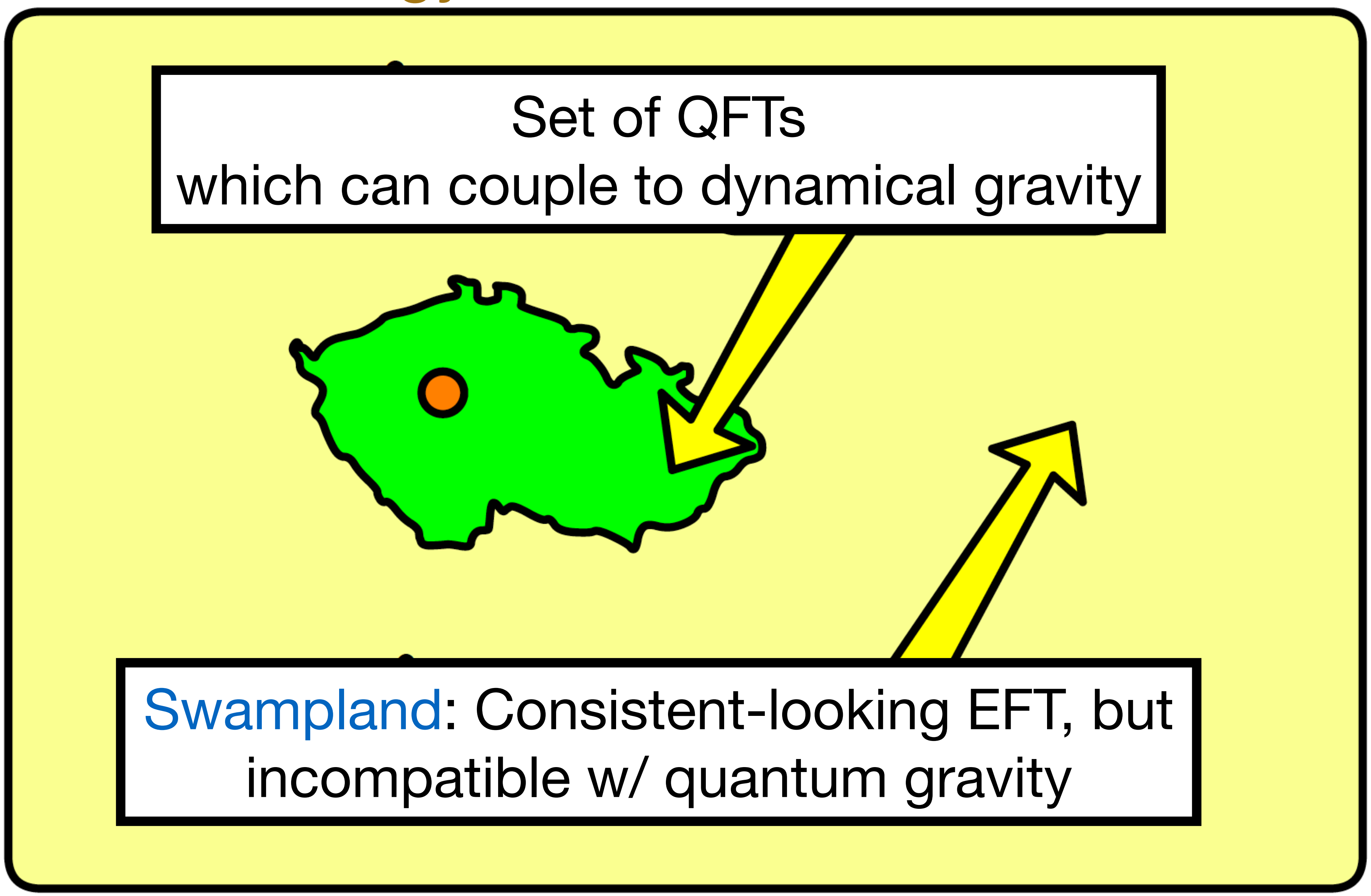
[Vafa ’05, Ooguri-Vafa ’06]

Swampland

Space of low energy EFT

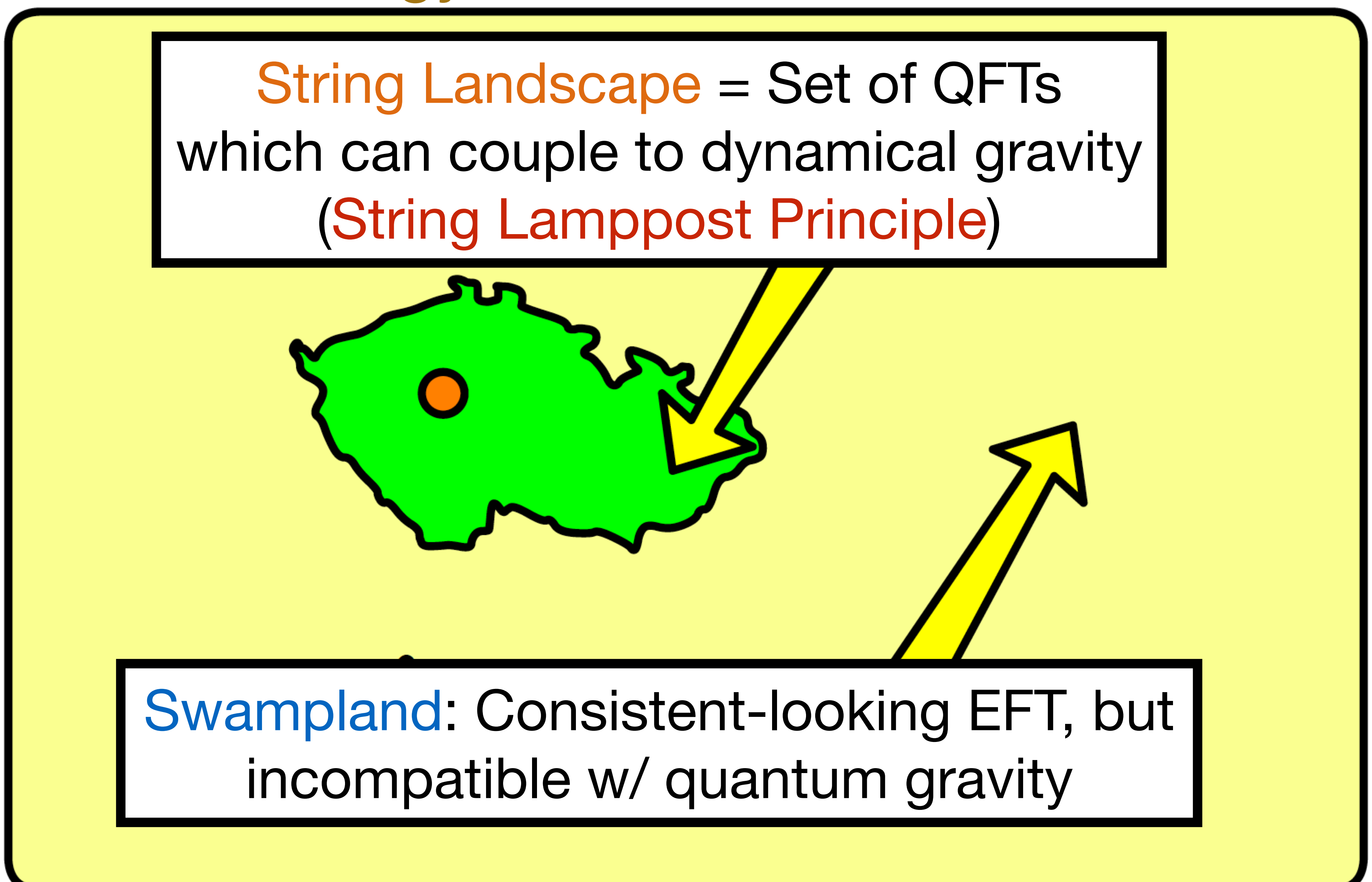


[ArkaniHamed-Motl-Nicolis-Vafa ‘06]

[Vafa ’05, Ooguri-Vafa ’06]

String Lamppost Principle

Space of low energy EFT



[ArkaniHamed-Motl-Nicolis-Vafa ‘06]

String Lamppost Principle (SLP)

Theories with high SUSY is good playground to test the SLP.

- Theories w/ 32 supercharges. SLP is established.
- 10d theories w/ 16 supercharges.
  Among $SO(32)$, $E_8 \times E_8$, $E_8 \times U(1)^{248}$ and $U(1)^{496}$ gauge theories, only first two can couple to gravity consistently.
  SLP is established. [Adams-DeWolf-Taylor ’10, Kim-Shiu-Vafa ‘19]
- Lower dimension w/ 16 supercharges. Various swampland constraints (I will talk later), but SLP is not complete yet.
- This talk focus on 8d supergravity, and gives further evidence of SLP.

Talk Plan

1. 8d supergravity with 16 supercharges

2. Swampland constraints so far

3. Probe 3-brane constraints [YH-Vafa 2104.05724]

8d supergravity

Minimal 8d SUSY theories have 16 supercharges.

- Gravity multiplet $(g_{\mu\nu}, B_{\mu\nu}, 2A_\mu, \sigma\,; \psi_{\mu\alpha}, \chi_\alpha)$
- Vector multiplet $(A_\mu, 2\phi, \lambda_\alpha)$

Perturbative gauge anomaly comes from pentagon diagram.

In supersymmetric theory, no perturbative gauge anomaly.

The gauge algebras $so(2n+1)$, $\mathfrak{f}_4$ suffer from a global gauge anomaly. [Gracia-Etxebarria, Hayashi, Ohmori, Tachikawa, Yonekura '17]

8d String Landscape

All cases are realized by F-theory [Vafa '96] geometry involving an elliptic K3.

- Rank=18

F-theory on elliptic K3. Heterotic/type I on T^2.

Full list of gauge group is known [Shimada '05, Font-Fraiman-Grana-Nunez-De Freitas '20].

- Rank=10

F-theory on elliptic K3 with one $O7_+$ plane (frozen singularity). CHL string.

Full list of gauge group is studied recently [Font-Fraiman-Grana-Nunez-De Freitas '21, YH-Vafa '21].

- Rank=2

F-theory on elliptic K3 with two $O7_+$ planes.

Only a few possibilities of gauge groups, $2\,u(1)$, $u(1) \oplus su(2)$, $2\,su(2)$.

[Kim-Shiu-Vafa ’19, Lee-Weigand ’19, Kim-Tarazi-Vafa ’19, Kaz-Kim-Tarazi-Vafa ’20]

BPS Completeness Hypothesis: 1-brane

Probe branes are useful for getting swampland constraints.

8d supergravity 16 supercharges come along with a 2-form gauge field B_2.

1-brane: electrically charged brane under B_2

BPS completeness hypothesis:

this brane is BPS leading to 8 supercharges on its worldsheet.

[Kim-Shiu-Vafa ’19, Lee-Weigand ’19, Kim-Tarazi-Vafa ’19, Kaz-Kim-Tarazi-Vafa ’20]

Probe 1-brane

Combined with anomaly inflow and unitarity, one gets

$$c_{\mathfrak{g}} = \Sigma_i \frac{l_i d_{\mathfrak{g}_i}}{l_i + h^\vee} \leq \begin{cases} 18 & \text{for} \quad \kappa = 1 \\ 2 & \text{for} \quad \kappa = 0 \end{cases},$$

where κ and l_i are positive integers:

B_2 frame: $dH_3 = l_i \,\mathrm{Tr}F_i^2 - \kappa\, \mathrm{tr}R^2$,

B_4 frame: $\int B_4 \wedge \left(l_i \mathrm{Tr}F_i^2 - \kappa\, \mathrm{tr}R^2\right)$.

Rank $\leq \begin{cases} 18 & \text{for} \quad \kappa = 1 \\ 2 & \text{for} \quad \kappa = 0 \end{cases}$

κ must be 0 or 1. l_i is level of worldsheet current algebra.

Rank periodicity

9, 8, 7d theories with 16 supercharges.

(Cobordism Conjecture) → Existence of I-fold.

Compactification to lower dimension w/ I-fold.

Anomaly cancellation leads to

$$\mathrm{Rank} = 2, 10, 18 \text{ for 8d case.}$$

All of which are realized in string theory.

Swampland bounds

There are four classes of theory consistent with Swampland constraints.

- Rank=18 with $\kappa = 1$

Only the simply-laced algebras with the level one or $u(1)$'s are allowed.

- Rank=10 with $\kappa = 1$

In addition to the simply-laced algebras and $u(1)$, $sp(n)$ and $\mathfrak{g}_2$ are allowed.

The level is constrained.

- Rank=2 with $\kappa = 0$

Only the simply-laced algebras with the level one or $u(1)$'s are allowed.

- Rank=2 with $\kappa = 1$

No known string constructions, but their existence has not been ruled out.

Open Questions

Swampland constraints so far allow, e.g., $18\,\mathrm{su}(2)$ and $6\,\mathrm{su}(4)$. $18\,\mathrm{su}(2)$ does not show up in the Landscape while $6\,\mathrm{su}(4)$ does appear though these two have the same rank 18. Similarly, there are no known string theory constructions of 8d supergravity theories with $\mathfrak{g}_2$ gauge symmetry although there are no known Swampland arguments to exclude them.

We provide an explanation based on the consistency of probe 3-branes.

BPS Completeness Hypothesis: 3-brane

8d supergravity theories come along with a
2-form gauge field $B_2 \leftrightarrow$ 4-form gauge field B_4.
3-brane: magnetically charged brane under B_2.

By completeness hypothesis such a 3-brane should exist.
BPS completeness hypothesis:
This brane is BPS leading to $\mathcal{N} = 2$ supersymmetry on its worldvolume.
Motivation: gauge instanton 3-brane solutions with $\mathcal{N} = 2$ supersymmetry do exist in supergravity setup.

Zero size instanton

Zero size instantons of a gauge group G inherits G as a global symmetry (e.g. [Callan-Harvey-Strominger '91]).
The finite size instantons break this global symmetry.
Therefore, finite size instanton is described by the Higgs branch of the theory. This is a local fact (with or without dynamical gravity) about gauge instantons and is consistent with what one sees in string theory [Witten '95, Ganor-Hanany '96, Seiberg-Witten '96].

Higgs branch is one-instanton moduli space of G.

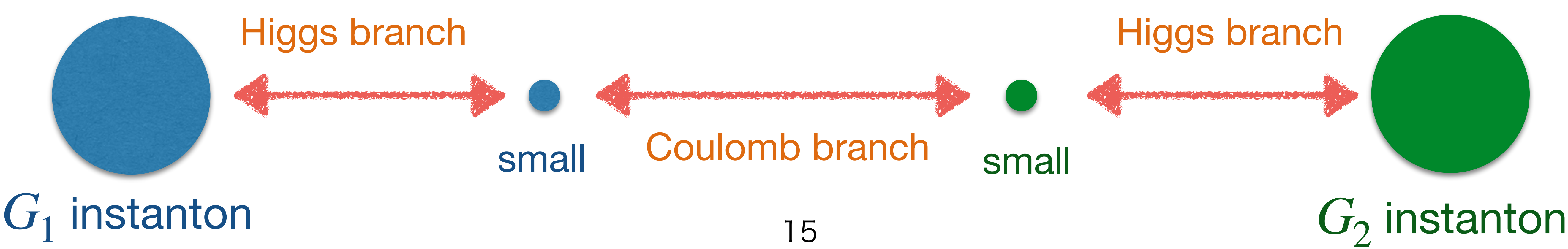
SUSY deformation

B_4 frame: $\int B_4 \wedge \left(l_i \mathrm{Tr} F_i^2 - \kappa \, \mathrm{tr} R^2 \right)$.

One-instanton 3-brane has

- B_4 charge l_i.
- Preserves $\mathcal{N} = 2$ supersymmetry.

If there are two gauge algebras (G_1 and G_2) and having same level,
they are connected by a SUSY deformation
(stronger cobordism conjecture [McNamara-Vafa '19 + unpublished]).



1d Coulomb Branch (CB)

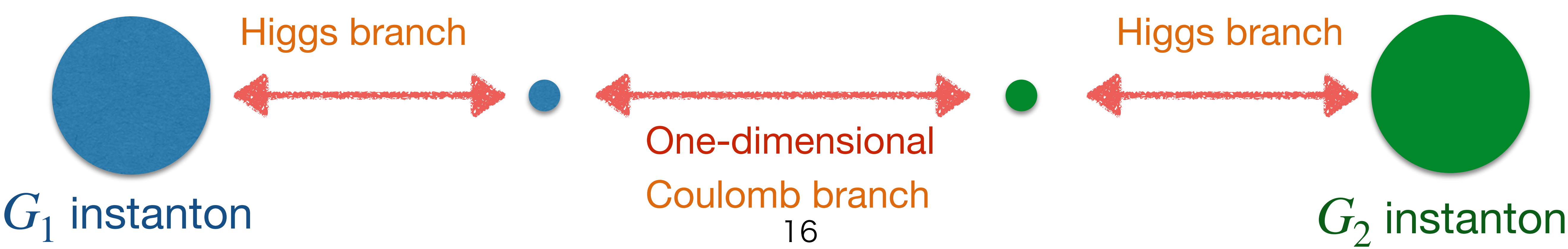
Expectation: Given Higgs branch and global symmetry, the CB structure is highly constrained, and the dimension of CB is fixed.

(Note: This is purely field theory question without dynamical gravity.)

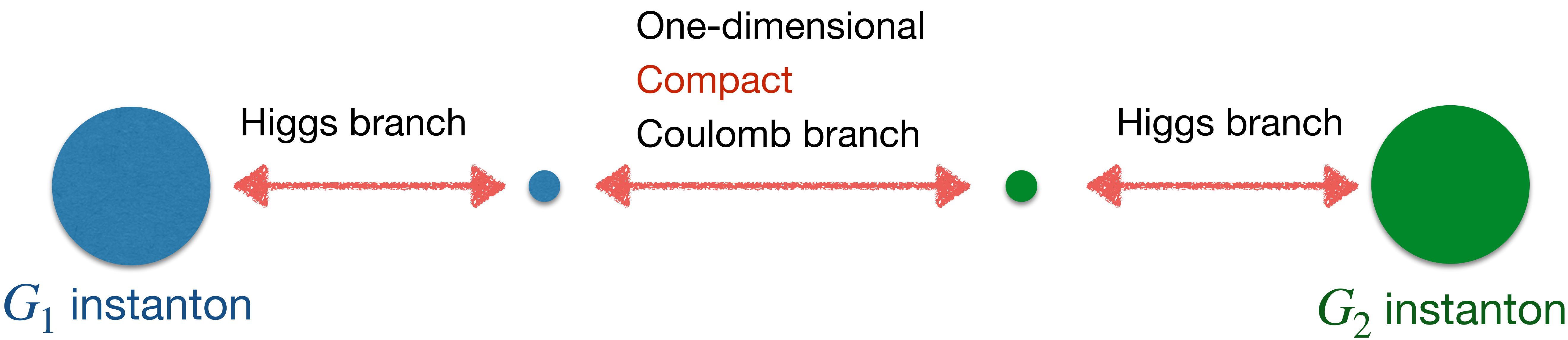
Supports:

- Central charges of SCFT are determined.[Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees ’13, Shimizu, Tachikawa, Zafrir ’17]
- ADHM provides an unique constructions of instanton configuration including zero size limit.

Therefore, we conclude that the dimension of CB for the probe theory for all the simply-laced groups is one.



Compact moduli space



Compactify 3-branes on T^3, which becomes 5d 0-brane.
This 0-brane can be viewed as black holes (BH).
The spectrum of the Laplacian on brane moduli space corresponds to a state of a black hole.

(the number of BH states in given mass range)= $\int dM\ e^{(\mathrm{BH\,entropy})}$ = finite,

The spectrum of the Laplacian must to be discrete. Compact moduli space.

Coulomb Branch Geometry

We use consistency of Coulomb branch geometry to put bounds.

The metric of the Coulomb branch is

$ds^2 = \tau_2\, du d\bar{u}$.

The curvature $\mathscr{R} = \partial\bar{\partial} \log \tau_2$ satisfies

$$\int \mathscr{R} = \frac{1}{2}\int \frac{d^2\tau}{\tau_2^2} = 2\pi(2 - 2\mathrm{g}),$$

where g is genus. Since $\int \mathscr{R}$ is positive, $\mathrm{g} = 0, 1$.

$\mathrm{g} = 1$ → No non-Abelian symmetries.

$\mathrm{g} = 0$ → Non-Abelian symmetries at singular points.

Bound on number of singular points

From the relation $\int \mathscr{R} = 4\pi$, the number of singular points are bounded.

$$
\begin{aligned}
&10\,\#(II^*) + 9\,\#(III^*) + 8\,\#(IV^*) + 4\,\#(IV) \\
&+3\,\#(III) + 2\,\#(II) + \sum_{n=0} \left(n\#(I_n) + (n+6)\#(I_n^*) \right) = 24.
\end{aligned}
$$

Correspondingly, the brane global symmetry (bulk gauge symmetry) is bounded.

Bounds on gauge algebra

Consistency of 3-brane gives the bound

$$\sum_i \left(a_i\,\mathrm{su}(n_i) \oplus d_i\,\mathrm{so}(2m_i) \oplus f_i\,\mathrm{sp}(k_i) \right) \oplus l_1\mathbf{e}_6 \oplus l_2\mathbf{e}_7 \oplus l_3\mathbf{e}_8$$

with $\sum_i \left(n_i a_i + (m_i+2)d_i + (10+k_i)f_i \right) + 8l_1 + 9l_2 + 10l_3 = 24.$

For example,

$6\,\mathrm{su}(4)$ is allowed, $6\times 4 = 24.$
$18\,\mathrm{su}(2)$ is excluded, $18 \times 2 = 36.$

Theories with $\mathfrak{g}_2$ symmetry

A known relation is [Shapere-Tachikawa '08]

$$\sum_{i=1}^{r} 2D(u_i) - r = 4(2a - c).$$

The central charges of 4d $\mathcal{N} = 2$ SCFT with one-instanton Higgs moduli and $\mathfrak{g}_2$ flavor symmetry are [Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees '13, Shimizu, Tachikawa, Zafrir '17]

$$a = 17/24, \qquad c = 5/6.$$

With unitarity bound $D(u_i) \geq 1$ (Assumption: no complex singularity [Argyres-Lu-Martone '17]),

$$r \leq \sum_{i=1}^{r} 2D(u_i) - r = 4(2a - c) = \frac{7}{3}$$ → rank is less than 3.

For rank-1 case, we obtain $D(u) = 5/3$, but this value does not appear in Kodaira Table. For rank-2, we obtain $D(u_1) + D(u_2) = 13/6$. The allowed scaling dimensions for scale invariant geometries of rank-2 are known [Caorsi-Cecotti '18, Argyres-Martone '18]. It turns out that there are no pairs which add up to $13/6$.

Summary

We sharpen Swampland constraints on 8d supergravity theories by studying consistency conditions on worldvolume theory of 3-brane probes. Combined with a stronger form of the cobordism conjecture, this implies strong restrictions on the gauge algebra.

We also argue that 8d supergravity theories with $\mathfrak{g}_2$ gauge symmetry are in the Swampland.