Branes, fermions & superspace

Ander Retolaza

IPhT Saclay

String Pheno zoom seminars, 4^{th} May 2021

2105.XXXXX : AR, J. Rogers, R. Tatar & F. Tonioni

Outline

Motivation

- Gaugino condensates in KKLT from 10D:
  after [Hamada,Hebecker,Shiu,Soler] interest on 4-fermion terms.
  - [Dine,Rohm,Seiberg,Witten] :(roughly) in several SUGRA theories (e.g. Heterotic) **Supersymmetry** implies

$$\mathcal{S} \supset \int (F - \lambda\lambda)^2$$

  - That's λ in codim=0.
  - [Horava,Witten] speculate with codim=1: higher dim picture unclear
  - Speculation about e.g. fermion terms in brane actions

Motivation

- Gaugino condensates in KKLT from 10D:
  after [Hamada,Hebecker,Shiu,Soler] interest on 4-fermion terms.
  - [Dine,Rohm,Seiberg,Witten] :(roughly) in several SUGRA theories (e.g. Heterotic) **Supersymmetry** implies

$$\mathcal{S} \supset \int (F - \lambda\lambda)^2$$

  - That's λ in codim=0.
  - [Horava,Witten] speculate with codim=1: higher dim picture unclear
  - Speculation about e.g. fermion terms in brane actions
- Possible to say something from first principles? Yes!

Today:

- Goal: fermion expansion of brane actions
- This is general physics, this is not a KKLT talk

Branes and superspace

- Branes = solitonic solutions in supergravity theories that spontaneously break 1/2 supersymmetries $\Rightarrow$ **Goldstinos**

Branes and superspace

- Branes $=$ solitonic solutions in supergravity theories that spontaneously break 1/2 supersymmetries $\Rightarrow$ **Goldstinos**
- [Bergshoeff,Sezgin,Townsend] M2-brane action: M2 is 3D hypersurface in $(11|32)$ superspace $\quad (Z^M = (x^m, \theta^\mu) \quad m = 0, 1, ..., 10$ & $\mu = 1, ..., 32)$

$$\mathcal{S}_{M2} = -T_{M2} \int d^3\zeta \sqrt{-\det\left(P[G](Z)\right)} + \mu_{M2} \int P[A_3](Z)$$

Branes and superspace

- Branes $=$ solitonic solutions in supergravity theories that spontaneously break 1/2 supersymmetries $\Rightarrow$ **Goldstinos**
- [Bergshoeff,Sezgin,Townsend] M2-brane action: M2 is 3D hypersurface in (11|32) superspace $\quad (Z^M = (x^m, \theta^\mu) \quad m = 0, 1, ..., 10 \ \& \ \mu = 1, ..., 32)$

$$\mathcal{S}_{M2} = -T_{M2} \int d^3\zeta \sqrt{-\det\left(P[G](Z)\right)} + \mu_{M2} \int P[A_3](Z)$$

$$= -T_{M2} \int d^3\zeta \left(\sqrt{-\det(G_{ij}(Z))} - \frac{1}{6} \epsilon^{ijk} A_{ijk}(Z) \right)$$

where

$$G_{ij}(Z) = E_i^a(Z) E_j^b(Z) \eta_{ab} \quad , \quad A_{ijk}(Z) = E_i^A(Z) E_j^B(Z) E_k^C(Z) A_{ABC}(Z)$$

$$E_i^A(Z) = \frac{\partial Z^M}{\partial \zeta^i} E_M^A(Z)$$

and $\quad A = (a, \alpha) \quad (a = 0, 1, ..., 10 \ \& \ \alpha = 1, ..., 32) \qquad i, j, k = 1, 2, 3$

Branes and superspace

Bulk: $1/2$ supercharges ($Q_\alpha \;\; \alpha = 1, .., 32$) spontaneoulsy broken
Brane worldvolume: $\mathcal{S}(Z)$ built as product of *off-shell superfields*: it has a θ expansion to order 32

Branes and superspace

Bulk: $1/2$ supercharges (Q_α $\alpha = 1, .., 32$) spontaneoulsy broken
Brane worldvolume: $\mathcal{S}(Z)$ built as product of *off-shell superfields*: it has a θ expansion to order 32

- Bulk on-shell: 1/2 of $\theta \Rightarrow$ Goldstinos = fermions on brane
- The other half are redundancies: κ-symmetry [Bergshoeff,Sezgin,Townsend] (for M2-case, but it's general)

$$\delta_\kappa Z^M \text{ (at } \theta = 0 \text{)} = \begin{cases} \delta_\kappa x = 0 \\ \delta_\kappa \theta = \frac{1}{2}(1 + \Gamma_{M2(Dp)})\kappa \end{cases} \quad , \quad \delta_\kappa \mathcal{S} = 0$$

Branes and superspace

Bulk: $1/2$ supercharges ($Q_\alpha \;\; \alpha = 1, .., 32$) spontaneoulsy broken
Brane worldvolume: $\mathcal{S}(Z)$ built as product of *off-shell superfields*: it has a θ expansion to order 32

- Bulk on-shell: 1/2 of $\theta \Rightarrow$ Goldstinos = fermions on brane
- The other half are redundancies: κ-symmetry [Bergshoeff,Sezgin,Townsend] (for M2-case, but it's general)

$$\delta_\kappa Z^M \text{ (at } \theta = 0 \text{)} = \begin{cases} \delta_\kappa x = 0 \\ \delta_\kappa \theta = \frac{1}{2}(1 + \Gamma_{M2(Dp)})\kappa \end{cases} \quad , \quad \delta_\kappa \mathcal{S} = 0$$

Indeed, in these backgrounds we can decompose

$$\kappa = \frac{1}{2}(1 + \Gamma_{M2(Dp)})\kappa + \frac{1}{2}(1 - \Gamma_{M2(Dp)})\kappa \equiv \epsilon + \theta$$

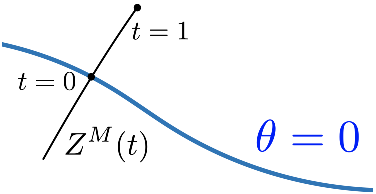
(WV κ-symmetry transformation = surviving bulk SUSY transformation)

A superspace geometric approach to obtain θ expansions

Goal: θ expansion of $\mathcal{S}_{M2}(Z)$
$\Rightarrow$ We obtain the dependence on any point in superspace by perfoming a Taylor expansion about a point where we have information
(Normal coordinate system method / background field method)
[Alvarez-Gaume,Freedmann,Mukhi; McArthur; Atick,Dhar; Grisaru,Knutt; ...]

A superspace geometric approach to obtain θ expansions

Goal: θ expansion of $\mathcal{S}_{M2}(Z)$
$\Rightarrow$ We obtain the dependence on any point in superspace by perfoming a Taylor expansion about a point where we have information
(Normal coordinate system method / background field method)
[Alvarez-Gaume,Freedmann,Mukhi; McArthur; Atick,Dhar; Grisaru,Knutt; ...]



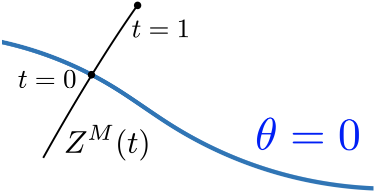
$Z^M(t)$ is a particular geodesic in curved superspace satisfying:

$$v^B \nabla_B v^A = 0 \quad , \; v^A(t=0) = (0, y^\alpha)$$

$$v^A(t) = \frac{dZ^M}{dt}(Z(t)) E_M{}^A(Z(t))$$

A superspace geometric approach to obtain θ expansions

Goal: θ expansion of $\mathcal{S}_{M2}(Z)$
$\Rightarrow$ We obtain the dependence on any point in superspace by perfoming a Taylor expansion about a point where we have information
(Normal coordinate system method / background field method)
[Alvarez-Gaume,Freedmann,Mukhi; McArthur; Atick,Dhar; Grisaru,Knutt; ...]



$Z^M(t)$ is a particular geodesic in curved superspace satisfying:

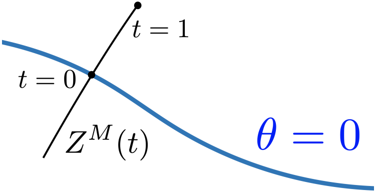
$$v^B \nabla_B v^A = 0 \quad , \; v^A(t=0) = (0, y^\alpha)$$

$$v^A(t) = \frac{dZ^M}{dt}(Z(t)) E_M{}^A(Z(t))$$

$$\mathcal{S}(Z(t)) = \sum_n \frac{t^n}{n!} \left(\frac{\delta^n \mathcal{S}}{\delta t^n} \right)_{t=0} \quad \& \quad \frac{\delta \mathcal{S}}{\delta t} = \frac{\delta Z^M}{\delta t} \frac{\delta \mathcal{S}}{\delta Z^M} = \mathcal{L}_v \mathcal{S}$$

A superspace geometric approach to obtain θ expansions

Goal: θ expansion of $\mathcal{S}_{M2}(Z)$
$\Rightarrow$ We obtain the dependence on any point in superspace by perfoming a Taylor expansion about a point where we have information
(Normal coordinate system method / background field method)
[Alvarez-Gaume,Freedmann,Mukhi; McArthur; Atick,Dhar; Grisaru,Knutt; ...]



$Z^M(t)$ is a particular geodesic in curved superspace satisfying:

$$v^B \nabla_B v^A = 0 \quad , \; v^A(t=0) = (0, y^\alpha)$$

$$v^A(t) = \frac{dZ^M}{dt}(Z(t)) E_M^{\ A}(Z(t))$$

$$\mathcal{S}(Z(t)) = \sum_n \frac{t^n}{n!} \left(\frac{\delta^n \mathcal{S}}{\delta t^n} \right)_{t=0} \qquad \& \qquad \frac{\delta \mathcal{S}}{\delta t} = \frac{\delta Z^M}{\delta t} \frac{\delta \mathcal{S}}{\delta Z^M} = \mathcal{L}_v \mathcal{S}$$

So $\quad \mathcal{S}(Z(t=1)) = \sum_n \frac{1}{n!} ((\mathcal{L}_y)^n \mathcal{S})_{t=0} = \left(e^{\mathcal{L}_y} \mathcal{S} \right)_{t=0}$

A superspace geometric approach to obtain θ expansions

$\mathcal{S}(Z(t=1)) = \left(e^{\mathcal{L}_y}\mathcal{S}\right)_{t=0}$: what do we do with it?

1. Apply derivatives: $\mathcal{L}_y E_M{}^A = \nabla_M y^A + y^C E_M{}^B T_{BC}{}^A$

A superspace geometric approach to obtain θ expansions

$\mathcal{S}(Z(t=1)) = \left(e^{\mathcal{L}_y}\mathcal{S}\right)_{t=0}$: what do we do with it?

1. Apply derivatives: $\quad \mathcal{L}_y E_M{}^A = \nabla_M y^A + y^C E_M{}^B T_{BC}{}^A$
2. Evaluate at $\theta = 0$: use tangent space props (e.g. $\omega_{Ma}{}^{\beta} = 0$),

   WZ gauge $\; E_M{}^A(\theta = 0) = \begin{pmatrix} e_m{}^a(x) & \psi_m^\alpha(x) \\ 0 & \delta_\mu^\alpha \end{pmatrix}, \quad y^A = (0, y^\alpha)$

A superspace geometric approach to obtain θ expansions

$\mathcal{S}(Z(t=1)) = \left(e^{\mathcal{L}_y}\mathcal{S}\right)_{t=0}$: what do we do with it?

1. Apply derivatives: $\mathcal{L}_y E_M{}^A = \nabla_M y^A + y^C E_M{}^B T_{BC}{}^A$
2. Evaluate at $\theta = 0$: use tangent space props (e.g. $\omega_{Ma}{}^{\beta} = 0$),

   WZ gauge $E_M^A(\theta=0) = \begin{pmatrix} e_m^a(x) & \psi_m^\alpha(x) \\ 0 & \delta_\mu^\alpha \end{pmatrix}$, $y^A = (0, y^\alpha)$
3. Use superspace Bianchi IDs $dH_4 = 0$, $\nabla T^A = E^B R_B^A$, $\nabla R_B^A = 0$ to write superspace objects into familiar spacetime fields

$$\mathcal{L}_y E_m{}^a = -i y^\alpha (\Gamma^a)_{\alpha\beta}\psi_m^\beta \quad , \quad \mathcal{L}_y E_m{}^\alpha = \nabla_m y^\alpha + e_m{}^b (T_b)^\alpha{}_\beta y^\beta$$

A superspace geometric approach to obtain θ expansions

$\mathcal{S}(Z(t=1)) = \left(e^{\mathcal{L}_y}\mathcal{S}\right)_{t=0}$: what do we do with it?

1. Apply derivatives: $\mathcal{L}_y E_M^{\ A} = \nabla_M y^A + y^C E_M^{\ B} T_{BC}^{\ \ A}$
2. Evaluate at $\theta = 0$: use tangent space props (e.g. $\omega_{Ma}^{\ \ \beta} = 0$),

   WZ gauge $E_M^{\ A}(\theta = 0) = \begin{pmatrix} e_m^{\ a}(x) & \psi_m^\alpha(x) \\ 0 & \delta_\mu^\alpha \end{pmatrix}$, $\quad y^A = (0, y^\alpha)$
3. Use superspace Bianchi IDs $dH_4 = 0$, $\nabla T^A = E^B R_B^{\ A}$, $\nabla R_B^{\ A} = 0$ to write superspace objects into familiar spacetime fields

$$\mathcal{L}_y E_m^{\ a} = -iy^\alpha(\Gamma^a)_{\alpha\beta}\psi_m^\beta \quad , \quad \mathcal{L}_y E_m^{\ \alpha} = \nabla_m y^\alpha + e_m^{\ b}(T_b)^\alpha_{\ \beta} y^\beta$$

4. Replace tangent vector y^A for θ and write superfield expansion:

$$E_m^{\ a}(Z) = e_m^{\ a}(x) - i\bar{\theta}\Gamma^a\psi_m(x) + \ldots \quad , \quad E_m^{\ \alpha}(Z) = \psi_m^{\ \alpha}(x) + (D_m(x)\theta)^\alpha + \ldots$$

5. We'll be interested in bosonic backgrounds, so we can take $\psi_m \to 0$

The M2-brane action (order $(\theta)^2$)

- We can do the same in 11D supergravity, Type IIA & IIB.
- Better: do it only in 11D for M2-brane

$$\mathcal{S}_{M2} = -T_{M2} \int d^3\zeta \left(\sqrt{-\det(G_{ij}(Z))} - \frac{1}{6}\epsilon^{ijk} A_{ijk}(Z) \right)$$

where $G_{mn}(Z) = E_m^a(Z) E_n^b(Z) \eta_{ab}$, $A_{mnp}(Z) = E_{[m}^A(Z) E_n^B(Z) E_{p]}^C(Z) A_{ABC}(Z)$

The M2-brane action (order $(\theta)^2$)

- We can do the same in 11D supergravity, Type IIA & IIB.
- Better: do it only in 11D for M2-brane

$$\mathcal{S}_{M2} = -T_{M2} \int d^3\zeta \Big(\sqrt{-\det(G_{ij}(Z))} - \frac{1}{6} \epsilon^{ijk} A_{ijk}(Z) \Big)$$

where $G_{mn}(Z) = E_m^a(Z) E_n^b(Z) \eta_{ab}$, $A_{mnp}(Z) = E_{[m}^A(Z) E_n^B(Z) E_{p]}^C(Z) A_{ABC}(Z)$

- Less fields in 11D SUGRA (= easier to find structure)
- We only need θ expansion of $E_M^A(Z)$ [Grisaru,Knutt]

The M2-brane action (order $(\theta)^2$)

- We can do the same in 11D supergravity, Type IIA & IIB.
- Better: do it only in 11D for M2-brane

$$\mathcal{S}_{M2} = -T_{M2} \int d^3\zeta \left(\sqrt{-\det(G_{ij}(Z))} - \frac{1}{6}\epsilon^{ijk} A_{ijk}(Z) \right)$$

where $\quad G_{mn}(Z) = E^a_m(Z) E^b_n(Z) \eta_{ab} \quad , \quad A_{mnp}(Z) = E^A_{[m}(Z) E^B_n(Z) E^C_{p]}(Z) A_{ABC}(Z)$

- Less fields in 11D SUGRA (= easier to find structure)
- We only need θ expansion of $E_M^A(Z)$ [Grisaru,Knutt]
- Terms in $\mathcal{S}_{M2}$ up to order $(\theta)^2$ by [(Marolf,)Martucci,Silva]

$$\mathbf{G}_{mn} = g_{mn} - i(\bar{\theta}\Gamma_{(m} D_{n)}\theta) \quad , \quad \mathbf{A}_{mnp} = A_{mnp} - \frac{3i}{2}(\bar{\theta}\Gamma_{[mn} D_{p]}\theta)$$

It involves Γ matrices and (11D) supercovariant derivatives ($\delta\psi_m = D_m\epsilon$)

$$\mathcal{S}^{(2)}_{M2} = -T_{M2} \int d^3\zeta \left(\sqrt{-\det(\mathbf{G}_{ij})} - \frac{1}{6}\epsilon^{ijk} \mathbf{A}_{ijk} \right)$$

The M2-brane action (order $(\theta)^4$)

The M2-brane action is obtained by performing a Taylor expansion of

$$
\begin{aligned}
\mathbf{G}_{mn} &= g_{mn} - i(\bar{\theta}\Gamma_{(m}D_{n)}\theta) - \frac{1}{4}(\bar{\theta}\Gamma_a D_{(m}\theta)(\bar{\theta}\Gamma^a D_{n)}\theta) \\
&\quad + \frac{1}{12}(\bar{\theta}\Gamma_{(m|}\mathcal{T}_b{}^{dfgh}\theta)(\bar{\theta}\mathcal{H}^b{}_{|n)dfgh}\theta) + \frac{1}{12}(\bar{\theta}\Gamma_{(m}\Gamma^{bc}\theta)(\bar{\theta}\mathcal{W}_{|n)bc}\theta) \\
\mathbf{A}_{mnp} &= A_{mnp} - \frac{3i}{2}(\bar{\theta}\Gamma_{[mn}D_{p]}\theta) - \frac{3}{4}(\bar{\theta}\Gamma_{a[m}D_n\theta)(\bar{\theta}\Gamma^a D_{p]}\theta) \\
&\quad + \frac{1}{8}(\bar{\theta}\Gamma_{[mn|}\mathcal{T}_b{}^{dfgh}\theta)(\bar{\theta}\mathcal{H}^b{}_{|p]dfgh}\theta) + \frac{1}{8}(\bar{\theta}\Gamma_{[mn}\Gamma^{bc}\theta)(\bar{\theta}\mathcal{W}_{p]bc}\theta)
\end{aligned}
$$

The M2-brane action (order $(\theta)^4$)

The M2-brane action is obtained by performing a Taylor expansion of

$$\begin{aligned}
\mathbf{G}_{mn} &= g_{mn} - i(\bar\theta\Gamma_{(m}D_{n)}\theta) - \frac{1}{4}(\bar\theta\Gamma_a D_{(m}\theta)(\bar\theta\Gamma^a D_{n)}\theta) \\
&\quad + \frac{1}{12}(\bar\theta\Gamma_{(m|}\mathcal{T}_b{}^{dfgh}\theta)(\bar\theta\mathcal{H}^b{}_{|n)dfgh}\theta) + \frac{1}{12}(\bar\theta\Gamma_{(m}\Gamma^{bc}\theta)(\bar\theta\mathcal{W}_{|n)bc}\theta) \\
\mathbf{A}_{mnp} &= A_{mnp} - \frac{3i}{2}(\bar\theta\Gamma_{[mn}D_{p]}\theta) - \frac{3}{4}(\bar\theta\Gamma_{a[m}D_n\theta)(\bar\theta\Gamma^a D_{p]}\theta) \\
&\quad + \frac{1}{8}(\bar\theta\Gamma_{[mn|}\mathcal{T}_b{}^{dfgh}\theta)(\bar\theta\mathcal{H}^b{}_{|p]dfgh}\theta) + \frac{1}{8}(\bar\theta\Gamma_{[mn}\Gamma^{bc}\theta)(\bar\theta\mathcal{W}_{p]bc}\theta)
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{H}^b{}_{mdfgh} &= \Gamma^b H_{dfgh}D_m - 6e_m^b\Gamma_{df}e_g^p e_h^q[D_p, D_q] \\
\mathcal{W}_{mbc} &= \Sigma_{bc}^{dfgh}H_{dfgh}D_m + \frac{1}{8}\Gamma_f e_m^f e_b^p e_c^q[D_p, D_q] + \frac{1}{4}\Gamma_b e_c^q[D_m, D_q] \\
\Sigma_{bc}^{dfgh} &= \frac{1}{576}\left(\Gamma_{bc}\Gamma^{dfgh} - 8\delta_{[c}^{[d}\Gamma_{b]}\Gamma^{fgh]} - 12\delta_{[c}^{[d}\delta_{b]}^{f}\Gamma^{gh]}\right) \\
\mathcal{T}_c^{dfgh} &= \frac{1}{288}\left(\Gamma_c\Gamma^{dfgh} - 12\delta_c^{[d}\Gamma^{fgh]}\right)
\end{aligned}$$

- (Commutators of) Supercovariant derivatives + Flux

Dimensional reduction and the D2-brane

M2-brane lives in $(11|32)$ superspace, D2 brane in $(10|32)$ superspace

Superspace compactification relates them

Dimensional reduction and the D2-brane

M2-brane lives in (11|32) superspace, D2 brane in (10|32) superspace

Superspace compactification relates them

- Usual S^1 compactification Ansatz in spacetime (not super)

$$\hat{G}_{\hat{m}\hat{n}} = \begin{pmatrix} e^{-2\phi/3}(G_{mn} + e^{2\phi} C_m C_n) & e^{4\phi/3} C_m \\ e^{4\phi/3} C_n & e^{4\phi/3} \end{pmatrix} \quad , \quad \hat{A}_{mnp} = C_{mnp} \quad , \quad \hat{A}_{mn\,10} = B_{mn}$$

Dimensional reduction and the D2-brane

M2-brane lives in $(11|32)$ superspace, D2 brane in $(10|32)$ superspace

Superspace compactification relates them

- Usual S^1 compactification Ansatz in spacetime (not super)

$$\hat{G}_{\hat{m}\hat{n}} = \begin{pmatrix} e^{-2\phi/3}(G_{mn} + e^{2\phi} C_m C_n) & e^{4\phi/3} C_m \\ e^{4\phi/3} C_n & e^{4\phi/3} \end{pmatrix} \quad , \quad \hat{A}_{mnp} = C_{mnp} \quad , \quad \hat{A}_{mn\,10} = B_{mn}$$

- Bosonic brane action ($p_i = \partial_i x^{10} + \partial_i x^m C_m$, $f_2 = dA_1$)

$$\begin{aligned} S_{M2} + LM &= -\int d^3\zeta \left(\sqrt{-\det(\hat{G}_{ij})} - \frac{1}{6}\epsilon^{ijk}\hat{A}_{ijk} \right) + \int d^3\zeta \, \frac{\epsilon^{ijk}}{6} \left[3(p_i - C_i) f_{jk} \right] \\ &= -\int d^3\zeta \, e^{-\phi} \sqrt{-g} \sqrt{1 + e^{2\phi} p^2} + \int d^3\zeta \, \frac{\epsilon^{ijk}}{6} \left[C_{ijk} + 3(p_i - C_i)(B_{jk} + f_{jk}) \right] \\ &= -\int d^3\zeta \, e^{-\phi} \sqrt{-\det(g_{ij} + B_{ij} + f_{ij})} + \int (C_3 - C_1 \wedge (B_2 + f_2)) \\ &= S_{D2} \end{aligned}$$

Dimensional reduction and the D2-brane

M2-brane lives in (11|32) superspace, D2 brane in (10|32) superspace

Superspace compactification relates them

- Usual S^1 compactification Ansatz in spacetime (not super)

$$\hat{G}_{\hat{m}\hat{n}} = \begin{pmatrix} e^{-2\phi/3}(G_{mn} + e^{2\phi} C_m C_n) & e^{4\phi/3} C_m \\ e^{4\phi/3} C_n & e^{4\phi/3} \end{pmatrix} \quad , \quad \hat{A}_{mnp} = C_{mnp} \quad , \quad \hat{A}_{mn\,10} = B_{mn}$$

- Bosonic brane action ($p_i = \partial_i x^{10} + \partial_i x^m C_m$, $f_2 = dA_1$)

$$\begin{aligned} S_{M2} + LM &= -\int d^3\zeta \left(\sqrt{-\det(\hat{G}_{ij})} - \frac{1}{6}\epsilon^{ijk}\hat{A}_{ijk} \right) + \int d^3\zeta \, \frac{\epsilon^{ijk}}{6} \left[3(p_i - C_i) f_{jk} \right] \\ &= -\int d^3\zeta \, e^{-\phi} \sqrt{-g} \sqrt{1 + e^{2\phi} p^2} + \int d^3\zeta \, \frac{\epsilon^{ijk}}{6} \left[C_{ijk} + 3(p_i - C_i)(B_{jk} + f_{jk}) \right] \\ &= -\int d^3\zeta \, e^{-\phi} \sqrt{-\det(g_{ij} + B_{ij} + f_{ij})} + \int (C_3 - C_1 \wedge (B_2 + f_2)) \\ &= S_{D2} \end{aligned}$$

- For superspace: fields $\Rightarrow$ superfields

Dimensional reduction and the D2-brane

M2-brane lives in $(11|32)$ superspace, D2 brane in $(10|32)$ superspace

Superspace compactification relates them

- S^1 compactification Ansatz in superspace

$$\hat{\mathbf{G}}_{\hat{m}\hat{n}} = \begin{pmatrix} e^{-2\Phi/3}(\mathbf{G}_{mn} + e^{2\Phi}\mathbf{C}_m\mathbf{C}_n) & e^{4\Phi/3}\mathbf{C}_m \\ e^{4\Phi/3}\mathbf{C}_n & e^{4\Phi/3} \end{pmatrix} \quad , \quad \hat{\mathbf{A}}_{mnp} = \mathbf{C}_{mnp} \quad , \quad \hat{\mathbf{A}}_{mn\,10} = \mathbf{B}_{mn}$$

- Superspace brane action ($\mathbf{p}_i = \partial_i x^{10} + \partial_i x^m \mathbf{C}_m$, $f_2 = dA_1$)

$$\begin{aligned} S_{M2} + LM &= -\int d^3\zeta \left(\sqrt{-\det(\hat{\mathbf{G}}_{ij})} - \frac{1}{6}\epsilon^{ijk}\hat{\mathbf{A}}_{ijk} \right) + \int d^3\zeta\, \frac{\epsilon^{ijk}}{6}\left[3(\mathbf{p}_i - \mathbf{C}_i)f_{jk}\right] \\ &= -\int d^3\zeta\, e^{-\Phi}\sqrt{-\mathbf{g}}\sqrt{1 + e^{2\Phi}\mathbf{p}^2} + \int d^3\zeta\, \frac{\epsilon^{ijk}}{6}\left[\mathbf{C}_{ijk} + 3(\mathbf{p}_i - \mathbf{C}_i)(\mathbf{B}_{jk} + f_{jk})\right] \\ &= -\int d^3\zeta\, e^{-\Phi}\sqrt{-\det(\mathbf{g}_{ij} + \mathbf{B}_{ij} + f_{ij})} + \int (\mathbf{C}_3 - \mathbf{C}_1 \wedge (\mathbf{B}_2 + f_2)) \\ &= S_{D2} \end{aligned}$$

- For superspace: fields $\Rightarrow$ superfields

Dimensional reduction and the D2-brane

But how do those 10D superfields look like? We can use Normal coordinate methods in Type IIA.
Better: read them off from dimensional reduction

$$
\begin{aligned}
\hat{\mathbf{G}}_{10\,10} &= e^{4\Phi/3} = e^{4(\phi+\rho^{(2)}+\dots)/3} \\
\hat{G}_{10\,10} - i\hat{\bar{\theta}}\hat{\Gamma}_{10}\hat{D}_{10}\hat{\theta} + \dots &= e^{4\phi/3}\left(1 + \frac{4\rho^{(2)}}{3} + \dots\right)
\end{aligned}
$$

Dimensional reduction of 11D supercovariant derivative requires dim reduction of 11D gravitino ($\delta_\epsilon \lambda = \Delta\epsilon$)

$$
\hat{D}_{10} = \frac{e^{\phi}}{3}\Gamma_*\Delta \quad , \quad \hat{\Gamma}_{10} = e^{2\phi/3}\Gamma_* \quad , \quad \hat{\theta} = e^{-\phi/6}\theta = e^{-\phi/6}(\theta_+ + \theta_-)
$$

So

$$
\rho^{(2)} = \frac{-i}{4}\bar{\theta}\Delta\theta \quad \Rightarrow \quad \Phi = \phi - \frac{i}{4}\bar{\theta}\Delta\theta + \dots
$$

Dimensional reduction and the D2-brane

So the D2-brane action is

$$S_{D2} = -\int d^3\zeta\, e^{-\Phi}\sqrt{-\det(\mathbf{g}_{ij} + \mathbf{B}_{ij} + f_{ij})} + \int (\mathbf{C}_3 - \mathbf{C}_1 \wedge (\mathbf{B}_2 + f_2))$$

where

$$\begin{aligned}
\mathbf{g}_{ij} &= g_{ij} - i\,\bar{\theta}\Gamma_{(i}D_{j)}\theta + \dots \\
\Phi &= \phi - \frac{i}{4}\,\bar{\theta}\Delta\theta + \dots \\
\mathbf{C}_i &= C_i - \frac{i}{2}e^{-\phi}\,\bar{\theta}\Gamma_*\left(D_i - \frac{1}{2}\Gamma_i\Delta\right)\theta + \dots \\
\mathbf{C}_{ijk} &= C_{ijk} - \frac{3i}{2}e^{-\phi}\,\bar{\theta}\left(\Gamma_{[ij}D_{k]} - \frac{1}{6}\Gamma_{ijk}\Delta\right)\theta - 3iC_{[i}\,\bar{\theta}\Gamma_*\Gamma_j D_{k]}\theta + \dots \\
\mathbf{B}_{ij} &= B_{ij} - i\,\bar{\theta}\Gamma_*\Gamma_{[i}D_{j]}\theta + \dots
\end{aligned}$$

- Full agreement with [Marolf,Martucci,Silva] at order $(\theta)^2$

Dimensional reduction and the D2-brane

So the D2-brane action is

$$S_{D2} = -\int d^3\zeta\, e^{-\Phi}\sqrt{-\det(\mathbf{g}_{ij} + \mathbf{B}_{ij} + f_{ij})} + \int (\mathbf{C}_3 - \mathbf{C}_1 \wedge (\mathbf{B}_2 + f_2))$$

where

$$\begin{aligned}
\mathbf{g}_{ij} &= g_{ij} - i\,\bar{\theta}\Gamma_{(i}D_{j)}\theta + \ldots \\
\mathbf{\Phi} &= \phi - \frac{i}{4}\,\bar{\theta}\Delta\theta + \ldots \\
\mathbf{C}_i &= C_i - \frac{i}{2}e^{-\phi}\,\bar{\theta}\Gamma_*\Big(D_i - \frac{1}{2}\Gamma_i\Delta\Big)\theta + \ldots \\
\mathbf{C}_{ijk} &= C_{ijk} - \frac{3i}{2}e^{-\phi}\,\bar{\theta}\Big(\Gamma_{[ij}D_{k]} - \frac{1}{6}\Gamma_{ijk}\Delta\Big)\theta - 3iC_{[i}\,\bar{\theta}\Gamma_*\Gamma_j D_{k]}\theta + \ldots \\
\mathbf{B}_{ij} &= B_{ij} - i\,\bar{\theta}\Gamma_*\Gamma_{[i}D_{j]}\theta + \ldots
\end{aligned}$$

- Full agreement with [Marolf,Martucci,Silva] at order $(\theta)^2$
- Approach works at all orders and we computed $(\theta)^4$ terms

T-duality and Dp-branes

Dp-brane lives in certain $(10|32)$ superspace, D$(p \pm 1)$ brane in a related $(10|32)$ superspace

T-duality relates these superspaces

Approach:

- Promote (bosonic) T-duality relations to superfield level
- Taylor expand and identify terms order by order
- Use T-duality rules for fermions [Hassan]

T-duality and Dp-branes

Dp-brane lives in certain (10|32) superspace, D($p \pm 1$) brane in a related (10|32) superspace

T-duality relates these superspaces

Approach:

- Promote (bosonic) T-duality relations to superfield level
- Taylor expand and identify terms order by order
- Use T-duality rules for fermions [Hassan]

$$\tilde{\mathbf{G}}_{99} = \frac{1}{\mathbf{G}_{99}} \quad , \quad \tilde{\Phi} = \Phi - \frac{1}{2}\log(\mathbf{G}_{99}) \quad , \quad \tilde{\mathbf{G}}_{9m} = -\frac{\mathbf{B}_{9m}}{\mathbf{G}_{99}} \quad , \quad \tilde{\mathbf{B}}_{9m} = -\frac{\mathbf{G}_{9m}}{\mathbf{G}_{99}}$$

$$\tilde{\mathbf{G}}_{mn} = \mathbf{G}_{mn} - \frac{\mathbf{G}_{9m}\mathbf{G}_{9n} - \mathbf{B}_{9m}\mathbf{B}_{9n}}{\mathbf{G}_{99}} \quad , \quad \tilde{\mathbf{B}}_{mn} = \mathbf{B}_{mn} - \frac{\mathbf{G}_{9m}\mathbf{B}_{9n} - \mathbf{B}_{Bm}\mathbf{G}_{9n}}{\mathbf{G}_{99}}$$

$$\left(\left(\sum_q \tilde{\mathbf{C}}_q\right) e^{-\tilde{\mathbf{B}}}\right)_{9m_1\ldots m_p} = \left(\left(\sum_q \mathbf{C}_q\right) e^{-\mathbf{B}}\right)_{m_1\ldots m_p}$$

T-duality and Dp-branes

An example in some detail (order $(\theta)^2$)

$$(\mathbf{G}_{99})^A = \left(\frac{1}{\mathbf{G}_{99}}\right)^B$$

$$\left(G_{99} - i\bar{\theta}\Gamma_9 D_9\theta + ...\right)^A = \left(\frac{1}{G_{99} + \gamma^{(2)}_{mn} + ...}\right)^B = \left(\frac{1}{G_{99}} - \frac{\gamma^{(2)}_{mn}}{(G_{99})^2} + ...\right)^B$$

At order $(\theta)^2$:

$$\left(-i\bar{\theta}\Gamma_9 D_9\theta\right)^A = \{\text{[Hassan] rules}\} = -\left(\frac{-i\bar{\theta}\Gamma_9 D_9\theta}{(G_{99})^2}\right)^B = -\left(\frac{\gamma^{(2)}_{mn}}{(G_{99})^2}\right)^B$$

Outcome:

- NSNS superfield expansions look exactly the same at all orders
- RR superfield expansions need to be computed

T-duality and Dp-branes

With this we can compute the θ expansion for any Dp-brane

$$S_{Dp} = -\int d^{p+1}\zeta\, e^{-\Phi}\sqrt{-\det(\mathbf{g}_{ij}+\mathbf{B}_{ij}+f_{ij})} + \int \left(\sum_q \mathbf{C}_q e^{-(\mathbf{B}_2+f_2)}\right)$$

Summary

- Fermions in branes are goldstinos of on-shell bulk configuration (asymptotically flat solutions)
- Branes are hypersurfaces in superspace
- θ expansion of brane action from expansion of superfields involved
- For M2-brane: $E_M^A(Z)$ is enough
- Normal coordinate method to obtain $E_M^A(Z)$ to order $(\theta)^4$
- D2-brane action from superspace dimensional reduction
- Dp-brane actions from superspace T-duality relations

Extra: KKLT and perfect squares

Approach:

1. Take D9-brane action and fix gauge for κ-symm:
    - [Grana,Kovensky,AR] : D9-brane action knows that it lives in Type I and not Type IIB
    - Identify $(\theta)^4$ terms leading to perfect square structure. What happens with flux?
2. Check those terms in the D7-brane (after gauge gixing). Perfect square?
3. Previous points were background independent for a 10D/8D gaugino condensate in Type I/KKLT. Next: compactify and consider 4D gaugino condensates