Towards Realistic Matter Spectra in F-theory

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With M. Bies, M. Cvetič, R. Donagi, M. Ong – ongoing project

Motivation

- Construct string vacua whose effective field theory reproduce (MS)SM.
- In particular: need to fix vector-like pairs to accommodate the Higgs and avoid exotic vector-like matter.
- Vector-like spectra: Characteristic features of 4d vacua.

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  - describing strongly coupled IIB-string theory including non-perturbative effects.
  - translating physics concepts to geometric subjects in elliptic 4-fold $\pi\colon CY_4 \twoheadrightarrow B_3$.
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- Global F-theory compactifications: vector-like spectra hard to get from topological data.

$\Rightarrow$ How can we control the vector-like spectra in F-theory?

Outline

1. Chiral and vector-like spectra in F-theory
2. Vector-like spectra in realistic F-theory geometries:
    - The appearance of root bundles.
    - Limit roots constructions.
3. Apply limit roots constructions to realistic F-theory geometry.
4. Outlook and strategy.

Construct standard model physics

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Find (MS)SM in F-theory – exact chiral spectra

- F-theory compactification is described by singular elliptic Calabi-Yau fibration $\pi\colon Y_4 \twoheadrightarrow \mathcal{B}_3$. In practise, we focus on smooth $\hat{Y}_4$ from the resolution of singular Y_4.
- Chiral matter states are represented by fibrations over localized curves $C_{\mathrm{R}} \subset \mathcal{B}_3$.
- Turn on non-trival field strength $G_4 = dC_3 \in H^{2,2}_{\mathbb{Z}}(\hat{Y}_4)$:
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Past string constructions with exact chiral spectra

- $E_8 \times E_8$ [Candelas Horowitz Strominger Witten '85], [Greene Kirklin Miron Ross '86], [Braun He Ovrut Pantev '05], [Bouchard Donagi '05], [Anderson Gray He Lukas '10], ...
- Type II [Berkooz Douglas Leigh '96], [Aldazabal Franco Ibanez Rabadan Uranga '00], [Ibanez Marchesano Rabadan '00], [Blumenhagen Kors Lust Ott '01], [Cvetič Shiu Uranga '01], ...
- F-theory [Krause Mayrhofer Weigand '12], [Cvetič Klevers Mayorga Oehlmann Reuter '15], [Lin Weigand '16], [Cvetič Lin Liu Oehlmann '18], [Cvetič Halverson Lin Liu Tian '19], [Taylor Turner '19], [Raghuram Taylor Turner '19], ...

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- Massless (anti-)chiral modes depend on the F-theory equivalent M-theory 3-form gauge potential $C_3 \in H^4_D\left(\widehat{Y}_4, \mathbb{Z}(2)\right)$:

$$0 \to J^2(\hat{Y}_4) \to H^4_D\left(\widehat{Y}_4, \mathbb{Z}(2)\right) \to H^{2,2}(\widehat{Y}_4, \mathbb{Z}) \to 0\,.$$

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- On $\hat{Y}_4$, Chow ring $CH^2(\hat{Y}_4)$ provides parameterization of (subset of) $H^4_D(\hat{Y}_4, \mathbb{Z}(2))$ and is computationally more feasible. [M.B. Mayrhofer Pehle Weigand '14], [M.B. Mayrhofer Weigand '17], [M.B. '18]

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- The full gauge information $A \in CH^2(\hat{Y}_4, \mathbb{Z})$ define line bundles $\mathcal{L}_{\mathrm{R}}$ over C_{R}.

$\Rightarrow$ $h^i\left(C_{\mathrm{R}}, \mathcal{L}_{\mathrm{R}}\right)$ count the vector-like spectra:

$$\text{chiral} \leftrightarrow h^0\left(C_{\mathrm{R}}, \mathcal{L}_{\mathrm{R}}\right), \quad \text{anti-chiral} \leftrightarrow h^1\left(C_{\mathrm{R}}, \mathcal{L}_{\mathrm{R}}\right).$$

$\chi = h^0 - h^1$ is topological invariant, and only depends on $G_4 \in H^{2,2}(\hat{Y}_4, \mathbb{Z})$.

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Challenges to determine $h^i(C_{\mathrm{R}}, \mathcal{L}_{\mathrm{R}})$:

- In computationally simple toric surface dP_3, we use topological data to predict jumps of vector-like pairs and reach 95% accuracy. [Bies Cvetič Donagi Lin Liu Ruehle '20]
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- Heterotic [Anderson Gray Lukas Palti '10 & '11 and subsequent works]
- F-theory: Preliminary works [Bies Mayrhofer Pehle Weigand '14], [Bies Mayrhofer Weigand '17], [M.B. '18], [Bies Cvetič Donagi Lin Liu Ruehle '20]. Full construction not (yet) known.

Appearance of root bundles

- Explicit construction of quadrillion MSSM three family F-theory models presented in [Cvetič Halverson Lin Liu Tian '19] induces:

$$G_4 = -\frac{3}{\overline{K}_{B_3}^3}\left(5[e_1] \wedge [e_4] - 3[e_1] \wedge [\overline{K}_{B_3}] - 2[e_2] \wedge [\overline{K}_{B_3}] - 6[e_4] \wedge [\overline{K}_{B_3}]s \cdots\right) .$$

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$\Rightarrow$ Actual G_4 induces L_{R} subject to

$$L_{\mathrm{R}}^{\otimes \overline{K}^3_{\mathcal{B}_3}} = \mathcal{L}_{\mathrm{R}} .$$

i.e., L_{R} is a (special) $\overline{K}^3_{\mathcal{B}_3}$-root of $\mathcal{L}_{\mathrm{R}}$.

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- Mathematicans realized that on nodal curves, such n-th root bundles can explicitly be constructed by the so-called limit roots.
  [Caporaso Casagrande Cornalba '04] Thanks Marielle for pointing out this paper!
- Nodal curve $\leftrightarrow$ locally looks like $x \cdot y = 0$.

Root bundles on a realistic F-theory geometry $(Y_1 = V(s_3, s_5, s_9))$

curve	g	$\mathcal{L}$	d	BN-theory		
				h^0	h^1	ρ
$C_{(3,2)_{1/6}} = V(s_3, s_9)$	10	$\mathcal{L}^{\otimes 36}_{(3,2)_{1/6}} = K^{\otimes 24}_{C_{(3,2)_{1/6}}}$	12	3	0	10
				4	1	6
				5	2	0
				h^0	h^1	ρ
$C_{(1,2)_{-1/2}} = V\left(s_3, s_2 s_5^2 + s_1(s_1 s_9 - s_5 s_6)\right)$	82	$\mathcal{L}^{\otimes 36}_{(1,2)_{-1/2}} = K^{\otimes 22}_{C_{(1,2)_{-1/2}}} \otimes \mathcal{O}_{C_{(1,2)_{-1/2}}}(-30 \cdot Y_1)$	84	3	0	82
				4	1	78
				$\vdots$	$\vdots$	$\vdots$
				10	7	12
$C_{(\overline{3},1)_{-2/3}} = V(s_5, s_9)$	$\ldots$					
$\vdots$	$\ddots$					

Example: exactly 3 quarks on the quark-doublet curve

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curve	equation	genus	$\deg\left(2K_C\|_{C_i}\right)$
C_1	$V(x_1, s_9)$	1	6
C_3	$V(x_3, s_9)$	1	6
C_6	$V(x_6, s_9)$	0	12
C_{11}	$V(x_{11}, s_9)$	0	12
C_2	$V(x_2, s_9)$	0	0
$\left\{C_8^{(i)}\right\}_{1\leq i\leq 6}$	$V(x_8, x_1 - \alpha_i x_3)$	0	0
$\left\{C_{10}^{(i)}\right\}_{1\leq i\leq 6}$	$V(x_{10}, x_1 - \alpha_i x_3)$	0	0

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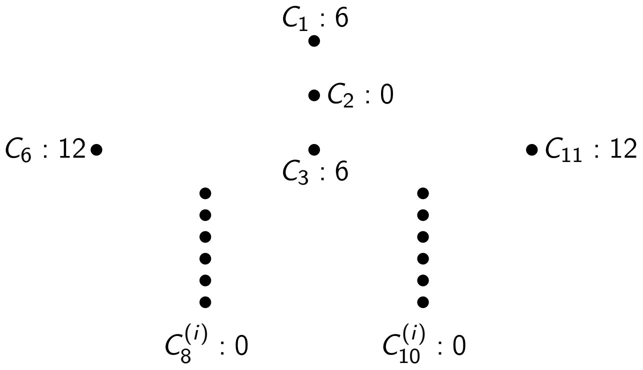

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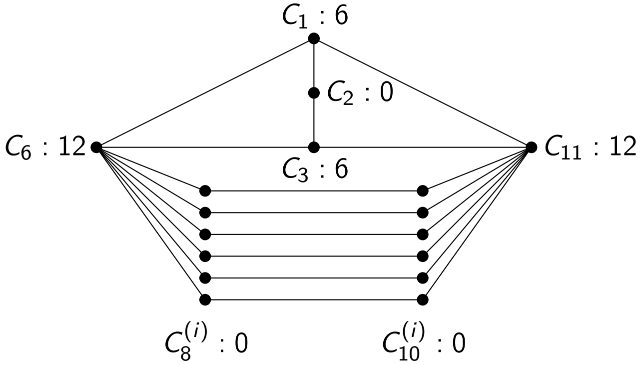

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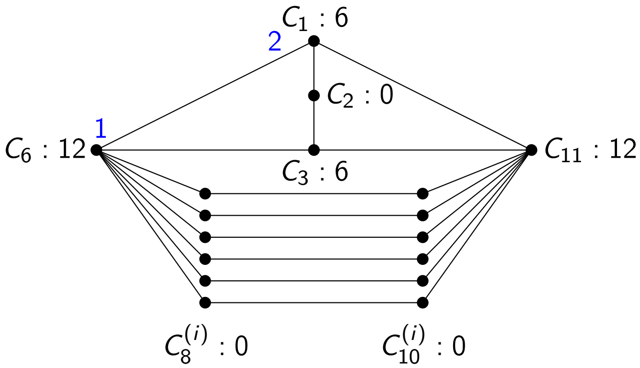

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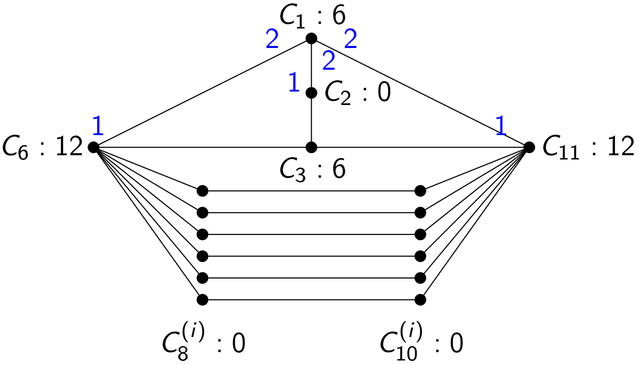
- Place all weights on node $C_{\mathbf{1}}$.
- $deg(C_{\mathbf{1}})^w = deg(2K_C|_{C_{\mathbf{1}}}) - \sum_j w_{\mathbf{1}}^j$
  $= 6 - 2 - 2 - 2 = 0$.



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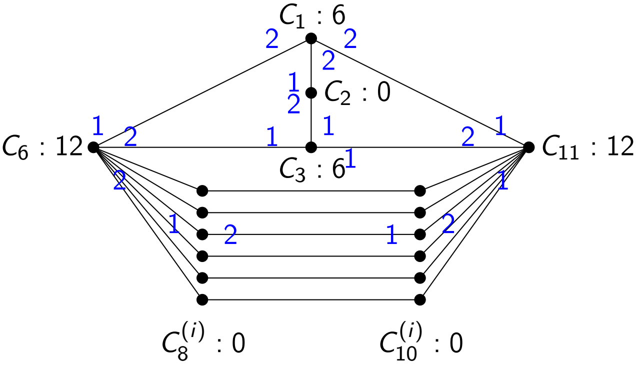
- Extend this to the rest of the diagram.



Global section counting

- Suppose there are n_i weights attached to curve component C_i, the degree of line bundle on C_i turns into $deg(C_i)^w = deg(2K_C|_{C_i}) - \sum_j w_i^j$.

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- The associated line bundle has multidegree $\deg(l) = (0, -3, 3, -3, -18, -18, 3)$. Consequently, the corresponding 3rd limit root has degree

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- We conclude that the number of global sections is given by the sum of local sections as followed:

$$h^0(C_1) = \begin{cases} 1 & \text{if } \mathcal{L}_{C_1} \equiv 0 \\ 0 & \text{otherwise} \end{cases}\,, \qquad h^0(C_3) = 1\,, \qquad h^0(C_{11}) = 2\,.$$

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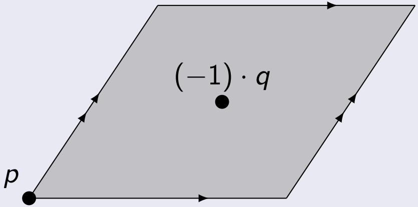
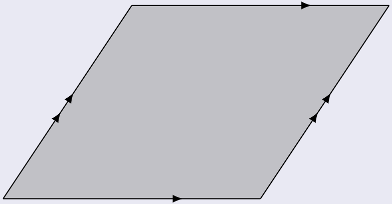
h^0 counting on torus $C_1 \cong \mathbb{C}/\Lambda$





$$h^0\left(\mathcal{O}_{C_1}(p-q)\right) = 0 \quad \rightarrow \quad h^0\left(\mathcal{O}_{C_1}(0)\right) = 1$$

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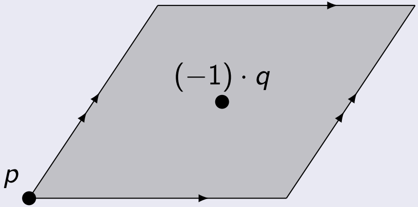

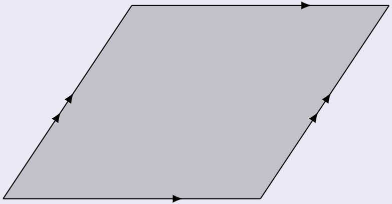

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There exsits non-trivial line bundle on C_1 such that $h^0(C_1) = 0$.
$\Rightarrow$ Existence of 3rd limit roots of $2K_C$ with $h^0 = 3$.

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$\Rightarrow$ **If such** $\widetilde{Y}_4$ **exsit** $\rightarrow$ Find MSSM vector-like spectra.

Outlook and strategy – II

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3. Extend to construct a database of $\widetilde{Y}_4$ admiting MSSM vector-like spectra including full gauge data. `https://github.com/homalg-project/ToricVarieties_project`

Thanks for your attention!