MPAS-CICE Workflow #1: 3D Tensor operations, momentum on vertices

1. Solve momentum equation in 2D at vertex (MPAS-CICE subroutine)

$$\mathbf{u}_{v} = \begin{bmatrix} u_{v} \\ v_{v} \end{bmatrix}$$

basis: $(\mathbf{e}_1, \mathbf{e}_2)$ $\mathbf{u}_v = 0$ at boundary

2. Interpolate to edge, rotate

$$\mathbf{u}_e = u_e \mathbf{n}_e + v_e \tilde{\mathbf{n}}_e \qquad \qquad \text{basis: edge normal} \\ & \text{\& tangent in } \mathbf{R}^3$$

$$\varepsilon_i = \left[\nabla_s u \right]_i =$$

basis: **R**³

4. Rotate to 2D

$$\varepsilon_i = \left[\nabla_s u \right]_i = \left[\begin{array}{cc} \bullet & \bullet \\ & \bullet \end{array} \right]$$

basis: $(\mathbf{e}_1, \mathbf{e}_2)$

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(MPAS-CICE subroutine) $\sigma_i = \begin{bmatrix} \bullet & \bullet \\ \bullet & \end{bmatrix}$

5. Stress Tensor, 2D at edge

$$\sigma_e = \begin{bmatrix} \bullet & \bullet & \bullet \\ & \bullet & \bullet \\ & & \bullet \end{bmatrix}$$
 basis: \mathbf{R}^3

7. Divergence of Stress Tensor in 3D from edge to vertex (Tensor operation subroutine)

edge to vertex (Tensor operation subroutine)
$$\left[\nabla \cdot \sigma \right]_{v} = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \end{bmatrix}$$
 basis: \mathbf{R}^{3}

8. Rotate to 2D

$$\left[\nabla \bullet \sigma\right]_{v} = \left[\begin{array}{c} \bullet \\ \bullet \end{array}\right]$$

basis: $(\mathbf{e}_1, \mathbf{e}_2)$

