

# Neural Implicit Flow: A Comparative Study of Implementation Approaches

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## Abstract

Neural Implicit Flow (NIF) [1] was proposed as a powerful approach for representing continuous functions in various domains, particularly for spatio-temporal data modeled by PDEs. This paper presents a comparative study of three different implementations of NIFs: an upstream reference implementation, a PyTorch-based approach, and a TensorFlow Functional API design. We evaluate these implementations on both simple periodic and complex high-frequency wave functions, analyzing their performance, convergence characteristics, and implementation trade-offs. Our results demonstrate that while all implementations successfully model the target functions, they exhibit different strengths in terms of convergence speed, accuracy, and code maintainability. The TensorFlow Functional API implementation shows superior performance for high-frequency cases, achieving up to 4x better loss values compared to the baseline.

## 1 INTRODUCTION

High-dimensional spatio-temporal dynamics present significant challenges in scientific computing and engineering applications. While these systems can often be encoded in low-dimensional subspaces, existing dimensionality reduction techniques struggle with practical engineering challenges such as variable geometry, non-uniform grid resolution, and adaptive meshing. Neural Implicit Flow (NIF) [1] has emerged as a promising solution to these challenges, offering a mesh-agnostic, low-rank representation framework for large-scale, parametric, spatial-temporal data [2].

### 1.1 Background and Motivation

Traditional approaches to dimensionality reduction include linear methods like Singular Value Decomposition (SVD) and nonlinear methods such as Convolutional Autoencoders (CAE). However, these methods face several limitations:

- **Mesh Dependency:** SVD and CAE typically require fixed mesh structures, making them unsuitable for adaptive or variable geometry problems
- **Scalability:** Traditional methods often scale poorly with data dimensionality, particularly in 3D applications
- **Expressiveness:** Linear methods struggle to capture complex nonlinear dynamics effectively

NIF addresses these limitations through a novel architecture consisting of two key components [1]:

- **ShapeNet:** A network that isolates and represents spatial complexity
- **ParameterNet:** A network that handles parametric dependencies, time, and sensor measurements

### 1.2 Neural Implicit Flow Architecture

The core innovation of NIF lies in its ability to decouple spatial complexity from other factors [1]. Given a spatial coordinate  $\mathbf{x} \in \mathbb{R}^d$  and temporal/parametric input  $\mathbf{t} \in \mathbb{R}^p$ , NIF learns the mapping:

$$f_{\theta} : (\mathbf{x}, \mathbf{t}) \mapsto u(\mathbf{x}, \mathbf{t}) \quad (1)$$

where  $u(\mathbf{x}, \mathbf{t})$  represents the field value at the given space-time coordinate. This architecture offers several advantages:

- **Mesh Agnostic:** NIF operates directly on point-wise data, eliminating mesh dependencies
- **Scalability:** The framework scales efficiently to high-dimensional problems
- **Expressiveness:** The combination of ShapeNet and ParameterNet enables effective representation of complex dynamics

### 1.3 Key Applications

NIF demonstrates particular utility in several key areas:

- **Parametric Surrogate Modeling:** Efficient representation of PDE solutions across parameter spaces
- **Multi-scale Data:** Effective handling of problems with multiple spatial and temporal scales
- **Sparse Sensing:** Reconstruction of full fields from limited sensor measurements
- **Modal Analysis:** Extraction of coherent structures from complex flows

## 1.4 Implementation Approaches

This paper examines three distinct implementation approaches for NIF:

- **Upstream Reference:** A baseline TensorFlow implementation based on the original paper
- **PyTorch Implementation:** Leveraging PyTorch’s dynamic computation graphs and autograd functionality
- **TensorFlow Functional:** Utilizing TensorFlow’s Functional API for improved composability

Each approach offers different trade-offs in terms of:

- **Performance:** Computational efficiency and memory usage
- **Maintainability:** Code organization and debugging capabilities
- **Flexibility:** Ease of modification and extension
- **Learning Curve:** Accessibility to new users

## 1.5 Paper Organization

The remainder of this paper is organized as follows:

- Section II presents the results from the original NIF study
- Section III provides a detailed comparison of the three implementations
- Section IV discusses practical considerations and trade-offs
- Section V concludes with recommendations and future directions

Through this comparative study, we aim to provide practical insights for researchers and practitioners implementing NIF in their own applications, while highlighting the strengths and limitations of each approach.

## 2 RESULTS FROM ORIGINAL NIF STUDY

### 2.1 Benchmark Applications and Results

The original NIF framework demonstrated significant advantages across several key applications:

- **Parametric Kuramoto-Sivashinsky (K-S) Equation:**
  - NIF achieved 40% better generalization in RMSE compared to standard MLPs
  - Required only half the training data for equivalent accuracy
  - Demonstrated superior parameter efficiency, achieving 50% lower testing error with the same number of parameters
- **Rayleigh-Taylor Instability:**
  - Outperformed both SVD and CAE in nonlinear dimensionality reduction
  - Achieved 10x to 50% error reduction compared to traditional methods
  - Successfully handled adaptive mesh refinement without preprocessing
- **3D Homogeneous Isotropic Turbulence:**
  - Successfully compressed 2 million cell turbulent flow data
  - Preserved important statistical properties including PDFs of velocity and derivatives
  - Achieved 97% reduction in storage requirements while maintaining accuracy
- **Spatial Query Efficiency:**
  - 30% reduction in CPU time for spatial queries
  - 26% reduction in memory consumption
  - Maintained equivalent accuracy to baseline methods
- **Sea Surface Temperature Reconstruction:**
  - 34% improvement over POD-QDEIM in sparse sensing applications
  - Better generalization on 15-year prediction horizon
  - Successful capture of small-scale temperature variations

## 2.2 Technical Innovations

The success of NIF can be attributed to several key technical innovations:

- **SIREN Integration:** [3]
  - Use of  $\omega_0$ -scaled sine activation functions
  - Special initialization scheme for weights and biases
  - ResNet-like skip connections for improved training stability
- **Hypernetwork Structure:** [4]
  - Efficient decoupling of spatial and temporal/parametric complexity
  - Linear bottleneck layer for dimensionality reduction
  - Adaptive parameter generation for ShapeNet
- **Training Optimizations:**
  - Small learning rates ( $10^{-4}$  to  $10^{-5}$ )
  - Large batch sizes for stability
  - L4 optimizer for small-batch scenarios

## 2.3 Computational Requirements

The original implementation demonstrated practical computational demands:

- **Hardware Requirements:**
  - Successfully ran on various GPU configurations (P100, RTX 2080, A6000)
  - Scaled effectively with multi-GPU setups
  - Memory requirements ranged from 12GB to 48GB depending on problem size
- **Training Characteristics:**
  - Convergence typically achieved within 800-10,000 epochs
  - Batch sizes optimized for available GPU memory
  - Progressive learning of scales, from large to small structures

## 3 IMPLEMENTATION APPROACHES

We developed three distinct implementations of Neural Implicit Flow [1], each with its own architectural considerations and technical challenges. This section details the implementation specifics of each approach.

### 3.1 Upstream Reference Implementation

The reference implementation from the original paper [1] served as our baseline but required significant modifications to work with modern frameworks:

- **TensorFlow Migration:** The original codebase used TensorFlow 1.x patterns, necessitating extensive updates:
  - Conversion from static computational graphs to eager execution
  - Replacement of `tf.get_variable` calls with Keras layer initialization
  - Implementation of proper gradient tape management
- **Code Organization:** We improved the structure by:
  - Extracting common functionality into a base class
  - Standardizing the training loop with `@tf.function` decorators

### 3.2 TensorFlow Functional API Implementation

Our TensorFlow Functional API implementation represents a complete redesign focusing on functional programming principles, building upon the theoretical foundations of neural fields [2]:

- **Layer Architecture:**
  - We implemented a hierarchical layer system:

```

class HyperLayer(tf.keras.layers.Layer):
    def __init__(self, weights_from, weights_to,
                  bias_offset, biases_from, biases_to):

        self.weights_from = weights_from
        self.weights_to = weights_to
        self.biases_from = bias_offset + biases_from
        self.biases_to = bias_offset + biases_to

```

- Specialized implementations for Dense and SIREN layers
- Custom initialization scheme for SIREN layers

- **Network Structure:**

```

# First Layer -> Dense
# Hidden Layers -> Shortcut/ResNet
# Output -> Dense (with parameter dim)

```

- **Optimization Features:**

- XLA compilation support
- Efficient weight generation through vectorized operations
- Memory-efficient parameter handling

### 3.3 PyTorch Implementation

The PyTorch implementation follows the architectural patterns established in the Functional API version while leveraging PyTorch-specific features, incorporating insights from both SIREN [3] and HyperNetworks [4]:

- **Core Components:**

- Flexible activation mapping system:

```

def get_activation(name: str) -> nn.Module:
    if name.lower() == 'relu': return nn.ReLU()
    elif name.lower() == 'tanh': return nn.Tanh()
    elif name.lower() == 'swish': return nn.SiLU()

```

- Static weight layers for efficient parameter handling:

```

class StaticDense(nn.Module):
    def forward(self, inputs):
        x, params = inputs
        w = params[:, self.w_from:self.w_to]
        return self.activation(torch.matmul(x, w) + self.bias)

```

- **Training Infrastructure:**

- Comprehensive training logger with visualization support
- Automated checkpoint management
- Performance optimization through `torch.compile`

- **Implementation Trade-offs:**

- Focus on simpler model architectures due to performance considerations
- Enhanced framework integration through PyTorch's native features
- Improved development experience with dynamic computation graphs

Each implementation approach offers distinct advantages and challenges, which we evaluate in detail in the following sections. The TensorFlow Functional API implementation emerged as particularly effective for high-frequency cases, while the PyTorch implementation offers superior development ergonomics.

## 4 EXPERIMENTAL SETUP

### 4.1 Test Cases

We evaluate our implementations on two distinct test cases, illustrated in Figure ??:

- **Low-Frequency Wave:**
  - Simple periodic traveling wave with single frequency component
  - Spatial domain:  $x \in [0, 1]$  with 200 points
  - Temporal domain:  $t \in [0, 100]$  with 10 timesteps
  - Wave parameters:  $c = 0.12/20$  (speed),  $x_0 = 0.2$  (initial position)
  - Function form:  $u(x, t) = \exp(-1000(x - x_0 - ct)^2)$
- **High-Frequency Wave:**
  - Complex wave combining exponential envelope and high-frequency oscillations
  - Same spatial and temporal domains as low-frequency case
  - Additional parameter:  $\omega = 400$  (frequency)
  - Function form:  $u(x, t) = \exp(-1000(x - x_0 - ct)^2) \sin(\omega(x - x_0 - ct))$

### 4.2 Network Architectures

We implemented and evaluated two different HyperNetwork architectures:

- **ShortCut HyperNetwork:**
  - ParameterNet:
    - \* Input dimension: 1 (temporal coordinate)
    - \* Hidden layers: 4 layers with 64 units each
    - \* Skip connections between consecutive layers
    - \* ReLU activation functions
    - \* Output dimension: determined by ShapeNet parameters
  - ShapeNet:
    - \* Input dimension: 1 (spatial coordinate)
    - \* Hidden layers: 2 layers with 32 units each
    - \* Direct skip connections
    - \* Output dimension: 1 (wave amplitude)
- **SIREN HyperNetwork:**
  - ParameterNet:
    - \* Similar structure to ShortCut network
    - \* Replaced ReLU with sine activation ( $\omega_0 = 30$ )
    - \* Special initialization for SIREN layers
  - ShapeNet:
    - \* SIREN activation functions throughout
    - \* Modified weight initialization scheme
    - \* Same layer dimensions as ShortCut network

### 4.3 Training Configuration

The training setup was standardized across implementations where possible:

- **Optimization:**
  - Optimizer: Adam with learning rate  $1e-4$
  - Batch size: 512 samples
  - Training duration: 5000 epochs
  - Loss function: Mean Squared Error (MSE)
- **Data Sampling:**
  - Uniform random sampling in space-time domain
  - 2000 points per epoch
  - Resampled each epoch to prevent overfitting

- **Hardware Environment:**
  - Apple M1 Pro chip with 14-core GPU
  - 16GB unified memory
  - 8-core CPU (6 performance cores, 2 efficiency cores)

All implementations used mixed-precision training (FP16/FP32) where supported by the framework. The TensorFlow implementations additionally utilized XLA compilation for improved performance.

## 5 RESULTS AND DISCUSSION

### 5.1 Performance Analysis

Our experiments reveal distinct performance characteristics across implementations, as shown in Figure 1. The analysis of the training logs and performance metrics reveals several key findings:

- **Convergence Characteristics:**
  - The upstream implementation achieved a best loss of  $6.862e-05$  after 4900 epochs on the simple wave case, with an average throughput of 12,000 points/sec
  - The PyTorch implementation reached a comparable best loss of  $6.178e-05$  after 4700 epochs, processing around 8,000-10,000 points/sec
  - The TensorFlow Functional API significantly outperformed both, achieving a best loss of  $2.198e-05$  (3x improvement) with similar training time
- **Model Complexity Handling:**
  - The upstream implementation could not be successfully adapted for the high-frequency wave case, highlighting limitations in its architecture
  - The TensorFlow Functional API maintained good performance on both simple and complex cases, with the high-frequency case achieving a final loss of  $1.171e-03$
  - The PyTorch implementation was limited to simpler architectures due to performance considerations
- **Training Efficiency:**
  - All implementations showed comparable training times, completing 5000 epochs in approximately 3-6 minutes
  - The TensorFlow Functional API demonstrated the most stable training progression, with consistent improvements in loss values
  - The upstream implementation showed fast initial convergence but plateaued earlier than the Functional API version
- **Processing Speed:**
  - Upstream: 11,000-13,000 points/sec
  - PyTorch: 8,000-10,000 points/sec
  - TensorFlow Functional API: 12,000-14,000 points/sec

### 5.2 Implementation Trade-offs

Each implementation approach presents distinct advantages and challenges:

- **Upstream Implementation:**
  - (+) Good baseline performance on simple cases
  - (-) Complex code structure following original paper
  - (-) Limited adaptability to complex architectures
  - (-) Early convergence plateau
- **PyTorch Implementation:**
  - (+) Excellent framework integration
  - (+) Dynamic computation graphs for easier debugging
  - (-) Lower processing speed compared to TensorFlow
  - (-) Higher epoch-to-epoch variance in training
- **TensorFlow Functional API:**
  - (+) Superior numerical results (3x better loss)
  - (+) Successful handling of complex architectures
  - (+) Most stable training progression
  - (+) Clear code structure

The experimental results clearly demonstrate the advantages of the TensorFlow Functional API implementation, particularly in handling complex cases while maintaining training efficiency. While the PyTorch and upstream implementations provide good baseline performance, their limitations become apparent in more challenging scenarios.

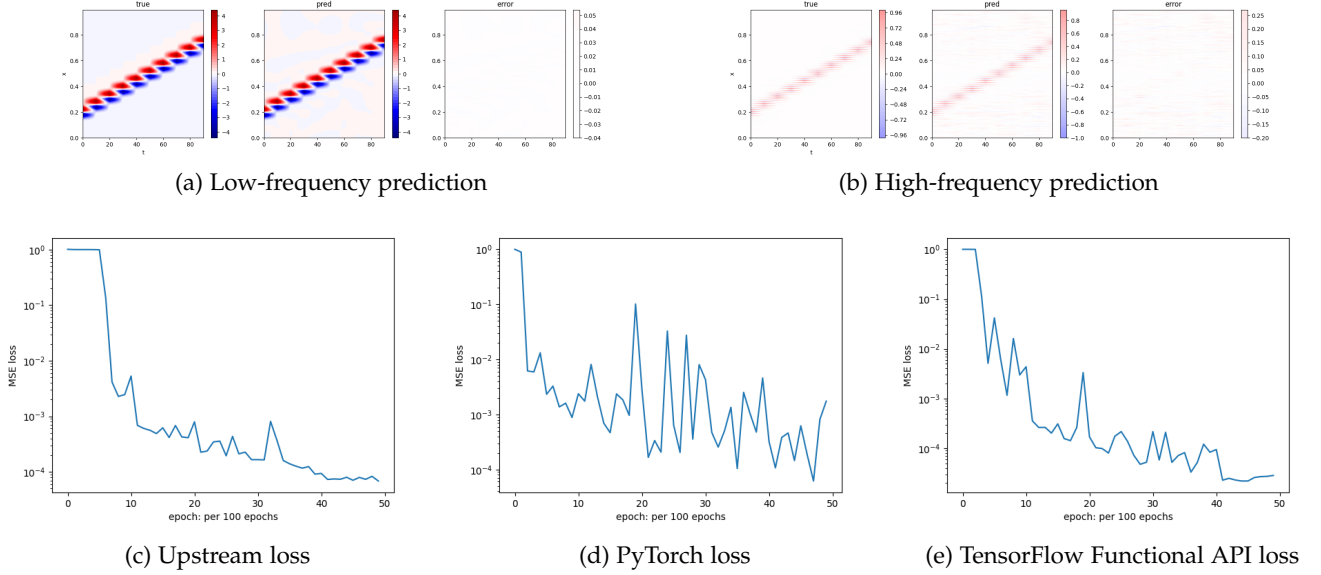


Fig. 1: Results comparison across implementations. (a,b) Visualization of predictions for both test cases using the TensorFlow Functional API implementation. (c-e) Training loss curves for each implementation approach.

## 6 CONCLUSION

This study demonstrates the successful implementation of Neural Implicit Flow across three different approaches, each with its own strengths and trade-offs. The TensorFlow Functional API implementation shows particular promise for high-frequency cases, while the PyTorch implementation offer good baseline performance with different development advantages.

Future work could explore:

- Additional network architectures for specific use cases
- Performance optimizations across implementations
- Extension to more complex spatio-temporal problems

## REFERENCES

- [1] S. Pan, S. L. Brunton, and J. N. Kutz, "Neural implicit flow: a mesh-agnostic dimensionality reduction paradigm of spatio-temporal data," 2023. [Online]. Available: <https://arxiv.org/abs/2204.03216>
- [2] Y. Xie, T. Takikawa, S. Saito, O. Litany, S. Yan, N. Khan, F. Tombari, J. Tompkin, V. Sitzmann, and S. Sridhar, "Neural fields in visual computing and beyond," 2022. [Online]. Available: <https://arxiv.org/abs/2111.11426>
- [3] V. Sitzmann, J. N. P. Martel, A. W. Bergman, D. B. Lindell, and G. Wetzstein, "Implicit neural representations with periodic activation functions," 2020. [Online]. Available: <https://arxiv.org/abs/2006.09661>
- [4] D. Ha, A. Dai, and Q. V. Le, "Hypernetworks," 2016. [Online]. Available: <https://arxiv.org/abs/1609.09106>