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Advice and a better formulation

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Mon, Dec 28, 2015 at 2:15 AM

Dear Christoph,

We hope Christmas was a happy family celebration for you and yours.
We had a nice time here.

We also hope your flight back was not badly delayed by the weather.
There was one more day of rain, but now it's generally clear. These
storms came from the far north. When you come back, we all may
be getting very wet from the effect of El Niño coming from the south!

Sorry, but I had to think more about category theory in free logic! And I just
today thought to look into this book I have known for a long time:

Categories, Allegories (North-Holland Mathematical Library) (Englisch)

von [Peter J. Freyd](#) (Autor), Andre Scedrov (Autor)

Gebundene Ausgabe
EUR 55,59

Gebundene Ausgabe: 293 Seiten

Verlag: Elsevier Ltd; Auflage: New. (8. November 1990)

Sprache: Englisch

ISBN-10: [0444703683](#)

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That was information from [Amazon.de](#). I recommend that you buy it at once!

Freyd has a very algebraic approach which is perfect for free logic -- an idea
he only uses very informally. But, the book contains many, many examples
just perfectly waiting for formal proofs! In other words, this gives a complete
plan for a masters thesis! It is all there in ONE fairly nice book. But of course
it would not be necessary to formalize everything in the book.

Now, referring to the earlier messages I sent about my version of the axioms,
I thought of a more "symmetric" way of getting at domains (codomains).

We can characterize identity maps in a category by this property:

$$\text{Id}(i) \iff i \circ i = i \ \& \ (\text{all } x, y, z) [x \circ i \circ y = z \implies x \circ i = x \ \& \ i \circ y = y]$$

In other words, this is about identity maps in themselves not tied to any specific other maps. But I claim:

$$\text{dom}(x) = i \iff \text{Id}(i) \ \& \ x \circ i = x$$

Because suppose both i and j have that property on the right above. Then $x \circ i \circ j = x$, and so $i \circ j = j$. Then $x \circ j \circ i = x$, and so $j \circ i = i$. But then $j \circ i \circ j = j$, and so $j \circ i = j$. Therefore, $i = j$. QED.

Well, one needs a lemma showing $\text{Id}(\text{dom}(x))$ from my axioms to see that the brief proof I gave tells us all we need to know.

I think this is looking quite neat. Yes?

-- DANA

P.S. Freyd gives no bibliography. What a bad boy he is!

Sent from my iPad